Research Article

Adaptive Output Feedback Stabilization of Nonholonomic Systems with Nonlinear Parameterization

Yanling Shang,[1] Ye Yuan,[2,3] and Fushun Yuan[2]

[1] *School of Software, Anyang Normal University, Anyang 455000, China*
[2] *School of Mathematics and Statistics, Anyang Normal University, Anyang 455000, China*
[3] *School of Mathematics and Physics, Suzhou University of Science and Technology, Suzhou 215009, China*

Correspondence should be addressed to Yanling Shang; hnnhsyl@126.com

This paper investigates the problem of adaptive output feedback stabilization for a class of nonholonomic systems with nonlinear parameterization and strong nonlinear drifts. A parameter separation technique is introduced to transform nonlinearly parameterized system into a linear-like parameterized system. Then, by using the integrator backstepping approach based on observer and parameter estimator, a constructive design procedure for output feedback adaptive control is given. And a switching strategy is developed to eliminate the phenomenon of uncontrollability. It is shown that, under some conditions, the proposed controller can guarantee that all the system states globally converge to the origin, while other signals remain bounded. An illustrative example is also provided to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Control of nonholonomic systems has received a great deal of attention over the last few years. It has been shown in [1] that mechanical systems with nonholonomic constraints such as mobile robots and wheeled vehicle can be either locally or globally converted to the so-called chained form under a coordinate transformation and a control mapping. As a result, the chained form has been used as a canonical form in analysis and control design for nonholonomic systems. Due to the nonsatisfaction of Brockett's necessary condition [2], nonholonomic systems cannot be asymptotically stabilized by stationary continuous state-feedback, although it is controllable. To overcome this difficulty, a number of approaches have been proposed; see the review paper [3] and references therein. The proposed solutions for stabilization nonholonomic systems include discontinuous feedback, time-varying feedback, and hybrid control laws. Using there valid approaches, the robustness issue for the asymptotic and exponential stability properties has been extensively studied [4–20].

However, it should be noticed that most of these papers were concerned with the systems with linear parameterization. Since nonlinear parameterization is exceptionally difficult to estimate, there are very few reports in literature for adaptive control of nonlinearly parameterized nonholonomic systems. Exceptions include [21–23] (and some references therein), where the authors consider that the whole state vector is measurable and adaptive control for chained form systems with nonlinear parameterization using state feedback.

In this paper, the case of only partial state vector being measurable and adaptive control for chained form systems with nonlinear parameterization by output feedback is considered. The contributions of this paper are listed as follows: (i) a new observer design method is proposed, and, based on the observer, the unmeasurable states of the system involved are reconstructed; (ii) using parameter separation technique [24], the parameters nonlinearities are solved and the resulting adaptive regulator is of minimum dimension (1D) independent of the parameter dimension; (iii) output feedback adaptive control based switching strategy is adopted

to handle the technical problem of uncontrollability at $x_0(t_0) = 0$, which prevents the finite escape of systems and guarantees that all the states converge to the origin and other signals are bounded.

The rest of this paper is organized as follows. In Section 2 preliminary knowledge and the problem formulation are given. Section 3 presents the state-scaling technique and the backstepping design procedure, while Section 4 provides the switching control strategy and the main result. Section 5 gives a simulation example to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section 6.

2. Problem Formulation

In this paper, we present an output feedback adaptive control design procedure for a class of uncertain chained form systems with nonlinear parameterization:

$$
\begin{aligned}
\dot{x}_0 &= u_0 + f_1(x_0, \theta) \\
\dot{x}_i &= u_0 x_{i+1} + f_i(x_0, x, \theta) \quad 1 \le i \le n-1 \\
\dot{x}_n &= u_1 + f_n(x_0, x, \theta) \\
y &= (x_0, x_1)^T,
\end{aligned}
\tag{1}
$$

where $(x_0, x)^T = (x_0, x_1, \ldots, x_n)^T \in R^{n+1}$, $u = (u_0, u_1)^T \in R^2$, and $y \in R^2$ are the system state, control input, and system measurable output, $f_i, i = 0, 1, \ldots, n$ are continuous functions of their arguments, and $\theta \in R^p$ is an unknown constant vector. The function f_i is called the nonlinear drifts of the system (1).

The objective of this paper is to design an output feedback adaptive stabilization control in the form

$$
\begin{aligned}
\dot{\hat{x}} &= \vartheta(\hat{x}, y), & \dot{\mu} &= \nu(x_0, x, \mu) \\
u_0 &= u_0(y, \mu), & u_1 &= u_1(y, \mu)
\end{aligned}
\tag{2}
$$

such that all signals of the closed loop system are bounded. Furthermore, the asymptotic regulation of the states is achieved; that is, $\lim_{t \to \infty}(|x_0(t)| + |x(t)|) = 0$.

A full characterization of the class of system (1) is given by the following assumption, which will be the base of the coming control design and performance analysis.

Assumption 1. For f_0, there exists a smooth nonnegative function φ_0 such that

$$
|f_0(x_0, \theta)| \le |x_0| \varphi_0(x_0, \theta).
\tag{3}
$$

For $1 \le i \le n$, there are smooth nonnegative function φ_i such that

$$
|f_i(x_0, x, \theta)| \le |x_1| \varphi_i(y, \theta).
\tag{4}
$$

Remark 2. Assumptions 1 implies that $f_0(0, \theta) \equiv 0$ and $f_i(0, 0, \theta) \equiv 0$, that is, the origin is the equilibrium point of system (1).

Lemma 3 (see [24]). *For any real-valued continuous function $f(x, y)$, where $x \in R^m, y \in R^n$, there are smooth scalar functions $a(x) \ge 0$, $b(y) \ge 0$, $c(x) \ge 1$ and $d(y) \ge 1$, such that*

$$
\begin{aligned}
|f(x, y)| &\le a(x) + b(y) \\
|f(x, y)| &\le c(x) d(y).
\end{aligned}
\tag{5}
$$

By Assumption 1 and Lemma 3, we easily obtain the following lemma.

Lemma 4. *For $1 \le i \le n$, there are smooth nonnegative function $\overline{\varphi}_i$ and unknown constant $\Theta_i \ge 1$ such that*

$$
\begin{aligned}
|f_0(x_0, \theta)| &\le |x_0| \overline{\varphi}_0(x_0) \Theta_0 \\
|f_i(x_0, x, \theta)| &\le |x_1| \overline{\varphi}_i(y) \Theta_i.
\end{aligned}
\tag{6}
$$

3. Adaptive Output Feedback Control Design

In this section, we focus on designing the control input u_1 via output feedback provided that $x_0(t_0) \ne 0$. The case where the initial $x_0(t_0) = 0$ will be treated in Section 4. The special structure of the system (1) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

Let $\widetilde{\Theta}_0 = \Theta_0 - \widehat{\Theta}_0$, where $\widehat{\Theta}_0$ is an estimate of Θ_0. Assumption 1 leads us to choose the control law u_0 as

$$
u_0(x_0, \widehat{\Theta}_0) = x_0 g_0(x_0, \widehat{\Theta}_0)
\tag{7}
$$

$$
g_0(x_0, \widehat{\Theta}_0) = -\left[\overline{\varphi}_0 \widehat{\Theta}_0 + \sqrt{2k_0^2 + 2(m_0 \overline{\varphi}_0 + \overline{\varphi}_0 \widehat{\Theta}_0)^2} \right],
\tag{8}
$$

where k_0 and m_0 are positive design parameters to be specified later.

Remark 5. Note the control u_0 is an uncertain version of Sontag formula [25]. It is used to stabilize x_0 with uncertainties. Because the particular choice of (8), $g_0(x_0, \widehat{\Theta}_0) \ne 0$ is guaranteed irregardless of the values of x_0. Hence, $1/g_0$ is well defined.

Consider the Lyapunov function candidate

$$
V_0 = \frac{1}{2} x_0^2 + \frac{1}{2} \widetilde{\Theta}_0^2.
\tag{9}
$$

From (7) and (8) and Assumption 1, we have

$$
\begin{aligned}
\dot{V}_0 &= x_0 [u_0 + f_0(x_0, \theta)] - \widetilde{\Theta}_0 \dot{\widehat{\Theta}}_0 \\
&\le g_0 x_0^2 + x_0^2 \overline{\varphi}_0 \Theta - \widetilde{\Theta}_0 \dot{\widehat{\Theta}}_0.
\end{aligned}
\tag{10}
$$

Choose the adaptation law $\widehat{\Theta}_0$ as

$$
\dot{\widehat{\Theta}}_0 = x_0^2 \overline{\varphi}_0(x_0)
\tag{11}
$$

renders

$$
\dot{V}_0 \le -k_0 x_0^2.
\tag{12}
$$

Therefore, x_0 and $\widetilde{\Theta}_0$ (or, equivalently, $\widehat{\Theta}_0$) are bounded; without loss of generality, we assume $|\widetilde{\Theta}_0| \leq M$, so we can choose design parameter m_0 in (8) as $m_0 > M$. Furthermore, we can conclude that $x_0 \to 0$ as $t \to \infty$ by using LaSalle's Invariant Theorem [26].

Under the control law (7), the solution of x_0-subsystem can be expressed as

$$x_0(t) = x_0(t_0) \exp\left(\int_{t_0}^t \frac{u_0 + f_0}{x_0} ds\right) \tag{13}$$

which implies

$$|x_0(t)| \geq |x_0(t_0)| \exp\left(-\int_{t_0}^t \left|\frac{u_0 + f_0}{x_0}\right| ds\right)$$
$$\geq |x_0(t_0)| \exp\left(-\int_{t_0}^t \lambda(s) ds\right), \tag{14}$$

where $\lambda(s) = 2\overline{\varphi}_0 \widehat{\Theta}_0 + \sqrt{2k_0 + 2(m_0\overline{\varphi}_0 + \overline{\varphi}_0\widehat{\Theta}_0)^2} + k_0\overline{\varphi}_0$.

Consequently, x_0 does not cross zero for all $t \in (t_0, \infty)$ provided that $x_0(t_0) \neq 0$.

From the above analysis, we can see the x_0-state in (1) can be globally regulated to zero via u_0 in (7) as $t \to \infty$. It is troublesome in controlling the x-subsystem via the control input u_1 in the limit (i.e., $u_0 = 0$), the x-subsystem is uncontrollable. This problem can be avoided by utilizing the following discontinuous state scaling transformation [4]:

$$z_i = \frac{x_i}{x_0^{n-i}} \quad 1 \leq i \leq n. \tag{15}$$

Under the new z-coordinates, with choice of u_0 as in (7), the x-system is transformed into

$$\dot{z}_1 = g_0 z_2 - (n-1) g_0 z_1 + \frac{f_1}{x_0^{n-1}}$$
$$\dot{z}_i = g_0 z_{i+1} - (n-i) g_0 z_1 + \frac{f_i}{x_0^{n-2}} \tag{16}$$
$$\dot{z}_n = u_1 + f_n.$$

It should be noted that the measurement of state z_1 can be obtained if the to-be-designed control u_0 is only dependent on output y. If $x_0(t) \neq 0$ for every $t \geq 0$, the discontinuous state transformation (15) is applicable.

We design the following observer for the system (16):

$$\dot{\widehat{z}}_1 = g_0 \widehat{z}_2 - (n-1) g_0 \widehat{z}_1 + \frac{k_1 u_0}{x_0} (z_1 - \widehat{z}_1)$$
$$\dot{\widehat{z}}_i = g_0 \widehat{z}_{i+1} - g_0 \widehat{z}_i + \frac{k_i u_0}{x_0} (z_1 - \widehat{z}_1) \tag{17}$$
$$\dot{\widehat{z}}_n = u_1 + \frac{k_n u_0}{x_0} (z_1 - \widehat{z}_1),$$

where $k_1, \ldots, k_n$ are design parameters to be determined later.

The estimation error $\widetilde{z} = z - \widehat{z}$ satisfies the dynamical equations:

$$\dot{\widetilde{z}}_1 = g_0\widetilde{z}_2 - (n-1) g_0\widetilde{z}_1 - k_1 g_0\widetilde{z}_1 + \frac{f_1}{x_0^{n-1}}$$
$$\dot{\widetilde{z}}_i = g_0\widetilde{z}_{i+1} - (n-i) g_0\widetilde{z}_i - k_i g_0\widetilde{z}_1 + \frac{f_i}{x_0^{n-i}} \tag{18}$$
$$\dot{\widetilde{z}}_n = -k_n g_0\widetilde{z}_1 + f_n + \phi_n^d.$$

The differential equations (18) can be rewritten into the compact form

$$\dot{\widetilde{z}} = g_0 A\widetilde{z} + \Phi, \tag{19}$$

where

$$A = \begin{pmatrix} -k_1 - n + 1 & 1 & 0 & 0 & \cdots & 0 \\ -k_2 & -n + 2 & 1 & 0 & \cdots & 0 \\ \vdots & & 0 & -n + 3 & 1 & \ddots & 0 \\ \vdots & & \vdots & & \ddots & \ddots & \ddots & 0 \\ -k_{n-1} & \vdots & & \vdots & & \ddots & -1 & 1 \\ -k_n & 0 & \cdots & & \cdots & 0 & 0 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \dfrac{f_1}{x_0^{n-1}} \\ \dfrac{f_2}{x_0^{n-2}} \\ \vdots \\ \dfrac{f_{n-1}}{x_0} \\ f_n \end{pmatrix}. \tag{20}$$

About matrix A defined by (20), there exists the following lemma.

Lemma 6. *The eigenvalues of the matrix A defined by (20) can be arbitrarily assigned by a proper selection of the design parameters $k_1, \ldots, k_n$.*

Proof. The proof can be found in [16] and thus omitted here. $\square$

Lemma 7. *By Assumption 1 and Lemma 4, we easily obtain that, for every $1 \leq i \leq n$, there exists smooth nonnegative function γ_i such that*

$$\left|\frac{f_i(x_0, x, \theta)}{x_0^{n-i}}\right| \leq |z_1| \gamma_i(x_0, z_1, \widehat{\Theta}_0) \Theta_i. \tag{21}$$

In view of (18), (19), *and* (16), *the overall system to be controlled can be expressed as*

$$\dot{\tilde{z}} = g_0 A \tilde{z} + \Phi$$

$$\dot{z}_1 = g_0 \hat{z}_2 + g_0 \tilde{z}_2 - (n-1) g_0 z_1 + \frac{f_1}{x_0^{n-1}}$$

$$\dot{\hat{z}}_i = g_0 \hat{z}_{i+1} - (n-i) g_0 \hat{z}_i + k_i g_0 \tilde{z}_1 \qquad (22)$$

$$\dot{\hat{z}}_n = u_1 + k_n g_0 \tilde{z}_1.$$

We now turn to the constructive design procedure of the control.

Step 1. This step can be regarded as the initial assignation of the entire design procedure. At this step, we introduce a Lyapunov function for the estimation error $\tilde{z}$. Define $\Theta = \max\{\Theta_i, \Theta_i^2\}$ and $\tilde{\Theta} = \Theta - \hat{\Theta}$, where $\hat{\Theta}$ is the estimate of Θ. Consider the Lyapunov function

$$U_0 = \tilde{z}^T P \tilde{z} + \frac{\tilde{\Theta}^2}{2}, \qquad (23)$$

where P is the positive definite solution of the Riccati equation

$$PA + A^T P = I_n. \qquad (24)$$

Then, taking time-derivation of U_0 along the solution of (19), we have

$$\dot{U}_0 = g_0 \|\tilde{z}\|^2 + 2\tilde{z}^T P \Phi - \tilde{\Theta}\dot{\hat{\Theta}}. \qquad (25)$$

By Lemma 7 and the expression of Φ in (20), we have

$$\|\Phi\|^2 \leq z_1^2 \overline{\Phi}\left(x_0, z_1, \hat{\Theta}_0\right) \Theta, \qquad (26)$$

where $\overline{\Phi} = \sum_{i=1}^{n} \gamma_i^2(x_0, z_1, \hat{\Theta}_0)$ is a nonnegative smooth function. Then, for the terms on the right-hand side of (25), by completing the square, we have

$$2\tilde{z}^T P \Phi = \varepsilon_0^2 \|\tilde{z}\|^2 + \varepsilon_0^{-2} \Phi^T P P \Phi - \varepsilon_0^2 \left\| \tilde{z} - \varepsilon_0^{-2} P \Phi \right\|^2$$

$$\leq \varepsilon_0^2 \|\tilde{z}\|^2 + \varepsilon_0^{-2} \|P\|^2 z_1^2 \overline{\Phi}^2 \Theta, \qquad (27)$$

where ε_0 is a positive design parameter to be specified later.
Substituting (27) into (25) leads to

$$\dot{U}_0 = \left(g_0 + \varepsilon_0^2\right) \|\tilde{z}\|^2 + \Psi_0 \left(\hat{\Theta} + \tilde{\Theta}\right) - \dot{\hat{\Theta}}\tilde{\Theta}, \qquad (28)$$

where $\Psi_0 = \varepsilon_0^{-2} \|P\|^2 z_1^2 \overline{\Phi}^2$.

Step 2. Take $V_1 = U_0 + z_1^2/2$ as the Lyapunov function of this step. Then, by (28) we have

$$\dot{V}_1 \leq \left(g_0 + \varepsilon_0^2\right) \|\tilde{z}\|^2 + \Psi_0 \left(\hat{\Theta} + \tilde{\Theta}\right) - \dot{\hat{\Theta}}\tilde{\Theta}$$

$$+ g_0 z_1 \hat{z}_2 + g_0 z_1 \tilde{z}_2 + z_1^2 \gamma_1 \left(\tilde{\Theta} + \hat{\Theta}\right); \qquad (29)$$

with the choice of the virtual controller

$$\hat{z}_2^* = -\frac{1}{g_0} z_1 \left[n + g_0^2 \varepsilon_1^{-2} + \left(\varepsilon_0^{-2} \|P\|^2 \overline{\Phi}^2 + \gamma_1\right) \sqrt{\hat{\Theta}^2 + 1} \right]$$

$$= z_1 \beta_1 \left(x_0, z_1, \hat{\Theta}\right) \qquad (30)$$

we have

$$\dot{V}_1 \leq \left(g_0 + \varepsilon_0^2 + \varepsilon_1^2\right) \|\tilde{z}\|^2 - n z_1^2 + g_0 z_1 \left(\hat{z}_2 - \hat{z}_2^*\right)$$

$$+ \left(\Psi_1 - \dot{\hat{\Theta}}\right) \left(\tilde{\Theta} + \eta_1\right), \qquad (31)$$

where $\Psi_1 = z_1^2 (\varepsilon_0^{-2} \|P\|^2 \overline{\Phi}^2 + \gamma_1) \geq 0$ and $\eta_1 = 0$.

Step i $(2 \leq I \leq n)$. Suppose at step $i-1$ we have designed a set of smooth virtual controllers $\hat{z}_1^*, \ldots, \hat{z}_i^*$ defined by

$$z_1^* = 0,$$

$$\hat{z}_2^* = \xi_1 \beta_1 \left(x_0, \xi_1, \hat{\Theta}_0, \hat{\Theta}\right),$$

$$\vdots$$

$$\hat{z}_i^* = \xi_{i-1} \beta_{i-1} \left(x_0, \xi_1, \ldots, \xi_{i-1}, \hat{\Theta}_0, \hat{\Theta}\right);$$

$$\xi_1 = z_1 - z_1^*, \qquad (32)$$

$$\xi_2 = \hat{z}_2 - \hat{z}_2^*,$$

$$\vdots$$

$$\xi_i = \hat{z}_i - \hat{z}_i^*$$

with $\beta_1(x_0, \xi_1, \hat{\Theta}) > 0, \ldots, \beta_{i-1}(x_0, \xi_1, \ldots, \xi_{i-1}, \hat{\Theta}) > 0$ being smooth, such that

$$\dot{V}_{i-1} = \left(g_0 + \sum_{k=0}^{i-1} \varepsilon_k^2\right) \|\tilde{z}\|^2 - (n-i+2) \left(\xi_1^2 + \cdots + \xi_{i-1}^2\right)$$

$$+ g_0 \xi_{i-1} \left(\hat{z}_i - \hat{z}_i^*\right) + \left(\Psi_{i-1} - \dot{\hat{\Theta}}\right) \left(\tilde{\Theta} + \eta_{i-1}\right), \qquad (33)$$

where

$$0 \leq \Psi_{i-1} \left(x_0, \xi_1, \ldots, \xi_{i-1}, \hat{\Theta}_0, \hat{\Theta}\right)$$

$$\leq \left(\xi_1^2 + \cdots + \xi_{i-1}^2\right) \alpha_{i-1} \left(x_0, \xi_1, \ldots, \xi_{i-1}, \hat{\Theta}_0, \hat{\Theta}\right) \qquad (34)$$

for $C^\infty \alpha_{i-1} \geq 0$.

We intend to establish a similar property for $(\tilde{z}, z_1, \ldots, \hat{z}_i)$-subsystem. Consider the Lyapunov function

$$V_i \left(x_0, \xi_1, \ldots, \xi_i, \hat{\Theta}_0, \hat{\Theta}\right) = V_{i-1} \left(x_0, \xi_1, \ldots, \xi_{i-1}, \hat{\Theta}_0, \hat{\Theta}\right) + \frac{1}{2}\xi_i^2. \qquad (35)$$

Clearly

$$
\dot{V}_i = \left(g_0 + \sum_{k=0}^{i-1} \varepsilon_k^2 \right) \| \widetilde{z} \|^2 - (n - i + 2) \left(\xi_1^2 + \cdots + \xi_{i-1}^2 \right)
$$

$$
+ g_0 \xi_{i-1} \left(\widehat{z}_i - \widehat{z}_i^* \right) + \left(\Psi_i - \dot{\widehat{\Theta}} \right) \left(\widetilde{\Theta} + \eta_i \right)
$$

$$
+ \xi_i \left\{ g_0 \widehat{z}_{i+1} - (n - i) g_0 \widehat{z}_i + k_i g_0 \widetilde{z}_1 - \frac{\partial z_i^*}{\partial \widehat{\Theta}_0} x_0^2 \overline{\varphi}_0 \right.
$$

$$
- \frac{\partial \widehat{z}_i^*}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}} - \frac{\partial z_i^*}{\partial x_0} (u_0 + f_0)
$$

$$
- \frac{\partial \widehat{z}_i^*}{\partial z_1} \left[g_0 z_2 - (n - 1) g_0 z_1 + \frac{f_1}{x_0^{n-1}} \right]
$$

$$
\left. - \sum_{j=2}^{i-1} \frac{\partial \widehat{z}_i^*}{\partial \widehat{z}_j} \left[g_0 \widehat{z}_{j+1} - (n - j) g_0 \widehat{z}_j + k_j g_0 \widetilde{z}_1 \right] \right\}.
$$

$$(36)$$

Since $\widehat{z}_i^*$ is a smooth function and satisfies

$$
\widehat{z}_i^* \left(x_0, 0, \ldots, 0, \widehat{\Theta}_0, \widehat{\Theta} \right) = 0 \tag{37}
$$

therefore there exist some continuous functions ψ_i, $1 \le i \le n - 1$ such that

$$
\widehat{z}_i^* \left(x_0, \xi_1, \ldots, \xi_{i-1}, \widehat{\Theta}_0, \widehat{\Theta} \right) = \sum_{j=1}^{i-1} \xi_k \psi_j \left(x_0, \xi_1, \ldots, \xi_{i-1}, \widehat{\Theta}_0, \widehat{\Theta} \right). \tag{38}
$$

After lengthy but simple calculations based on the completion of squares, there is a smooth nonnegative function σ_i such that

$$
|\xi_i| \left| (n - i) g_0 \widehat{z}_i - k_i g_0 \widetilde{z}_1 + \frac{\partial z_i^*}{\partial \widehat{\Theta}_0} x_0^2 \overline{\varphi}_0 \right.
$$

$$
+ \frac{\partial \widehat{z}_i^*}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}} + \frac{\partial z_i^*}{\partial x_0} u_0 + \frac{\partial \widehat{z}_i^*}{\partial z_1} \left[g_0 z_2 - (n - 1) g_0 z_1 \right]
$$

$$
\left. + \sum_{j=2}^{i-1} \frac{\partial \widehat{z}_i^*}{\partial \widehat{z}_j} \left[g_0 \widehat{z}_{j+1} - (n - j) g_0 \widehat{z}_j + k_j g_0 \widetilde{z}_1 \right] \right|
$$

$$(39)$$

$$
\le \varepsilon_i^2 \| \widetilde{z} \|^2 + \frac{1}{4} \sum_{j=1}^{i-1} \xi_j^2 + \xi_i^2 \sigma_i \left(x_0, \xi_1, \ldots, \xi_i, \widehat{\Theta}_0, \widehat{\Theta} \right),
$$

where ε_i is a positive design parameter to be specified later.
Using (38) and Lemma 4, we have

$$
\left| \frac{\partial z_i^*}{\partial x_0} f_0 \right| \le \sum_{j=1}^{i} |\xi_j| \rho_i \left(x_0, \xi_1, \ldots, \xi_i, \widehat{\Theta}_0, \widehat{\Theta} \right) \Theta_0, \tag{40}
$$

where $\rho_i \ge 0$ is a smooth function.

By Lemma 7, there is a smooth function $\omega_i \ge 0$ such that

$$
\left| \frac{\partial \widehat{z}_i^*}{\partial z_1} \right| \left| \frac{f_1}{x_0^{n-1}} \right| \le |z_1| \omega_i \left(x_0, z_1, \ldots, \widehat{z}_i, \widehat{\Theta}_0, \widehat{\Theta} \right) \Theta_1. \tag{41}
$$

Based on the completion of squares, it is deduced that there is a smooth function $\varpi_i \ge 0$ satisfying

$$
|\xi_i| \left| \frac{\partial z_i^*}{\partial x_0} f_0 + \frac{\partial \widehat{z}_i^*}{\partial z_1} \frac{f_1}{x_0^{n-1}} \right| \le \left[\frac{\sum_{j=1}^{i-1} \xi_j^2}{2 \left(1 + \widehat{\Theta}^2 \right) \left(1 + \eta_{i-1}^2 \right)} + \xi_i^2 \varpi \right] \Theta
$$

$$
\le \left[\frac{\sum_{j=1}^{i-1} \xi_j^2}{2 \left(1 + \widehat{\Theta}^2 \right) \left(1 + \eta_{i-1}^2 \right)} + \xi_i^2 \varpi_i \right] \widetilde{\Theta}
$$

$$
+ \frac{1}{4} \sum_{j=1}^{i-1} \xi_j^2 + \xi_i^2 \varpi_i \sqrt{\widehat{\Theta}^2 + 1}
$$

$$(42)$$

since

$$
\left| g_0 \xi_{i-1} \left(\widehat{z}_i - \widehat{z}_i^* \right) \right| \le \frac{1}{4} \xi_{i-1}^2 + \xi_i^2 \widetilde{\gamma}_i \left(x_0, \xi_1, \ldots, \xi_i, \widehat{\Theta}_0, \widehat{\Theta} \right) \tag{43}
$$

for a smooth function $\widetilde{\gamma}_i \ge 0$.

Substituting (39), (42), and (43) into (36) gives

$$
\dot{V}_i \le \left(g_0 + \sum_{k=0}^{i} \varepsilon_k^2 \right) \| \widetilde{z} \|^2 - \left(n - i + \frac{5}{4} \right) \left(\xi_1^2 + \cdots + \xi_{i-1}^2 \right)
$$

$$
+ \left(\Psi_{i-1} - \dot{\widehat{\Theta}} \right) \left(\widetilde{\Theta} + \eta_{i-1} \right) - g_0 \xi_i \widehat{z}_{i+1}
$$

$$
+ \xi_i^2 \left[\varpi_i \sqrt{\widehat{\Theta}^2 + 1} + \sigma_i + \widetilde{\gamma}_i \right]
$$

$$
+ \left[\frac{\sum_{j=1}^{i-1} \xi_j^2}{2 \left(1 + \widehat{\Theta}^2 \right) \left(1 + \eta_{i-1}^2 \right)} + \xi_i^2 \varpi_i \right] \widetilde{\Theta} + \xi_i \frac{\partial \widehat{z}_i^*}{\partial \widehat{\Theta}} \dot{\widehat{\Theta}}.
$$

$$(44)$$

Define

$$
\Psi_i = \Psi_{i-1} + \frac{\sum_{j=1}^{i-1} \xi_j^2}{2 \left(1 + \widehat{\Theta}^2 \right) \left(1 + \eta_{i-1}^2 \right)} + \xi_i^2 \varpi_i
$$

$$
\eta_i = \eta_{i-1} + \xi_i \frac{\partial \widehat{z}_i^*}{\partial \widehat{\Theta}}.
$$

$$(45)$$

Using (34), it is not difficult to verify that

$$
0 \le \Psi_i \left(x_0, \xi_1, \ldots, \xi_i, \widehat{\Theta}_0, \widehat{\Theta} \right)
$$

$$
\le \left(\xi_1^2 + \cdots + \xi_i^2 \right) \alpha_i \left(x_0, \xi_1, \ldots, \xi_i, \widehat{\Theta}_0, \widehat{\Theta} \right)
$$

$$(46)$$

for a $C^\infty \alpha_i \ge 0$.

Moreover, (44) can be rewritten as follows:

$$\dot{V}_i \leq \left(g_0 + \sum_{k=0}^{i} \varepsilon_k^2\right)\|\tilde{z}\|^2 - \left(n-i+\frac{5}{4}\right)\left(\xi_1^2 + \cdots + \xi_{i-1}^2\right)$$
$$+ \left(\Psi_i - \dot{\widehat{\Theta}}\right)\left(\widetilde{\Theta} + \eta_i\right) - g_0\xi_i\widehat{z}_{i+1}$$
$$+ \xi_i^2\left[\varpi_i\sqrt{\widehat{\Theta}^2+1} + \sigma_i + \tilde{\gamma}_i\right]$$
$$- \left[\frac{\sum_{j=1}^{i-1}\xi_j^2}{2\left(1+\widehat{\Theta}^2\right)\left(1+\eta_{i-1}^2\right)} + \xi_i^2\varpi_i\right]\eta_{i-1} - \xi_i\frac{\partial\widehat{z}_i^*}{\partial\widehat{\Theta}}\Psi_i. \tag{47}$$

By (46), we have

$$\left|\left[\frac{\sum_{j=1}^{i-1}\xi_j^2}{2\left(1+\widehat{\Theta}^2\right)\left(1+\eta_{i-1}^2\right)} + \xi_i^2\varpi_i\right]\eta_{i-1} + \xi_i\frac{\partial\widehat{z}_i^*}{\partial\widehat{\Theta}}\Psi_i\right|$$
$$\leq \frac{1}{8}\sum_{j=1}^{i-1}\xi_j^2 + \xi_i^2\varpi_i\sqrt{\eta_{i-1}^2+1} + \left|\xi_i\frac{\partial\widehat{z}_i^*}{\partial\widehat{\Theta}}\right|\left(\xi_1^2 + \cdots + \xi_i^2\right)\alpha_i$$
$$\leq \frac{1}{4}\sum_{j=1}^{i-1}\xi_j^2 + \xi_i^2\varpi_i\sqrt{\eta_{i-1}^2+1} + \xi_i^2\sum_{j=1}^{i}\tilde{\beta}_{i,j}. \tag{48}$$

The last inequality from the following relation

$$\left|\xi_i\right|\left|\xi_j^2\frac{\partial\widehat{z}_i^*}{\partial\widehat{\Theta}}\alpha_i\right| \leq \frac{1}{8}\xi_j^2 + \xi_i^2\tilde{\beta}_{i,j}\left(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta}\right), \tag{49}$$

where $\tilde{\beta}_{i,j} \geq 0$, $1 \leq j \leq i-1$ are smooth functions and $\tilde{\beta}_{i,i} \geq |\xi_i(\partial z_i^*/\partial\widehat{\Theta})\alpha_i|$, is a smooth function.

Putting (47) and (48) together, we arrive at

$$\dot{V}_i \leq \left(g_0 + \sum_{k=0}^{i}\varepsilon_k^2\right)\|\tilde{z}\|^2 - (n-i+1)\left(\xi_1^2 + \cdots + \xi_{i-1}^2\right)$$
$$+ \left(\Psi_i - \dot{\widehat{\Theta}}\right)\left(\widetilde{\Theta}+\eta_i\right) + g_0\xi_i\widehat{z}_{i+1}$$
$$+ \xi_i^2\left[\varpi_i\sqrt{\widehat{\Theta}^2+1} + \sigma_i + \tilde{\gamma}_i + \widehat{\gamma}_i\right], \tag{50}$$

where $\widehat{\gamma}_i = \varpi_i\sqrt{\eta_{i-1}^2+1} + \sum_{j=1}^{i}\tilde{\beta}_{i,j}(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta})$.

Now, it easy to see that the smooth virtual controller

$$\widehat{z}_{i+1}^* = -\frac{1}{g_0}\xi_i\left[n-i+1+\kappa_i\left(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta}\right)\right]$$
$$:= \xi_i\beta_i\left(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta}\right) \tag{51}$$
$$\kappa_i = \varpi_i\sqrt{\widehat{\Theta}^2+1} + \sigma_i + \tilde{\gamma}_i + \widehat{\gamma}_i$$

renders

$$\dot{V}_i \leq \left(g_0 + \sum_{k=0}^{i}\varepsilon_k^2\right)\|\tilde{z}\|^2 - (n-i+1)\left(\xi_1^2 + \cdots + \xi_i^2\right)$$
$$+ g_0\xi_i\left(\widehat{z}_{i+1} - \widehat{z}_{i+1}^*\right) + \left(\Psi_i - \dot{\widehat{\Theta}}\right)\left(\widetilde{\Theta}+\eta_i\right). \tag{52}$$

As $i = n$, the last step, we can construct explicitly a change of coordinates $(\xi_1,\ldots,\xi_n)$, a positive-definite and proper Lyapunov function $V_n = V_{n-1} + \xi_n^2/2$, and a smooth controller $u_1^* = -\xi_n\beta_n$ such that

$$\dot{V}_n \leq \left(g_0 + \sum_{k=0}^{n}\varepsilon_k^2\right)\|\tilde{z}\|^2 - \left(\xi_1^2 + \cdots + \xi_n^2\right)$$
$$+ \xi_n\left(u_1 - u_1^*\right) + \left(\Psi_n - \dot{\widehat{\Theta}}\right)\left(\widetilde{\Theta}+\eta_n\right). \tag{53}$$

Therefore, by choosing the smooth actual control u_1 and update law for $\widehat{\Theta}$ as

$$u_1 = -\xi_n\beta_n\left(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta}\right) \tag{54}$$
$$\dot{\widehat{\Theta}} = \Psi_n\left(x_0,\xi_1,\ldots,\xi_i,\widehat{\Theta}_0,\widehat{\Theta}\right) \tag{55}$$

we have

$$\dot{V}_n \leq \left(g_0 + \sum_{k=0}^{n}\varepsilon_k^2\right)\|\tilde{z}\|^2 - \left(\xi_1^2 + \cdots + \xi_n^2\right). \tag{56}$$

In Section 4, we will show that an appropriate choice of the design parameters k_0 and ε_i guarantees the asymptotic convergence of the state (x_0, x) of the original system (1) in closed loop with our control laws (7), (8), (54), and (55) as long as the initial condition $x_0(t_0)$ is nonzero. It will also be shown in the next section how to handle the singular case where $x_0(t_0)$ is zero.

4. Switching Controller and Main Result

In the preceding section, we have given controller design for $x_0(t_0) \neq 0$. Without loss of generality, we can assume that $t_0 = 0$. Now, we discuss how to select the control laws u_0 and u_1 when $x_0(0) = 0$. Choose u_0 as

$$u_0 = x_0g_0 + u_0^*, \qquad 2u_0^* > 0, \tag{57}$$

where g_0 is given by (7).

Choosing the same Lyapunov function (9), its time derivative is given by

$$\dot{V}_0 \leq -k_0x_0^2 + u_0^*x_0 \tag{58}$$

which leads to the boundedness of x_0 and consequently the bound of $\widehat{\Theta}_0$ as well. Consequently, x_0 does not escape and $x_0(t_s) \neq 0$ for any given finite $t_s > 0$. Thus state-scaling for the control design can be carried out.

During the time period $[0, t_s)$, using u_0 defined in (57), new control law $u_1 = u_1^*(x_0, x, \mu)$ and new update law

$\dot{\mu} = \nu^*(x_0, x, \mu)$ can be obtained by following the control procedure described above to the original x-system in (1). Then we can conclude that the x-state of (1) cannot blow up during the time period $[0, t_s)$. Since $x(t_s) \neq 0$ at t_s, we can switch the control input u_0 and u_1 to (7) and (54), respectively.

We are now ready to state the main theorem of our paper.

Theorem 8. *Under Assumption 1, if the above switching control strategy is applied to* (1) *with an appropriate choice of the design parameters k_i and ε_i's, uncertain system* (1) *is globally regulated at the origin. Furthermore, the estimated parameters are bounded.*

Proof. According to the above analysis, it suffices to prove the statement in the case where $x_0(0) \neq 0$.

Since we have already proven that $\lim_{t \to \infty} x_0(t) = 0$ and $\widehat{\Theta}_0$ is bounded in Section 3, we just need to show that $\lim_{t \to \infty} x(t) = 0$ and $\widehat{\Theta}$ is bounded. In this case, as a result of (7), (11), (54), and (55), from (56), it holds that

$$\dot{V}_n \leq \left(g_0 + \sum_{k=0}^{n} \varepsilon_k^2 \right) \|\widetilde{z}\|^2 - \left(\xi_1^2 + \cdots + \xi_n^2 \right). \tag{59}$$

Choose the design parameters k_0 and ε_i's so that

$$k_0 \geq \sum_{k=0}^{n} \varepsilon_k^2. \tag{60}$$

Therefore, we have

$$\dot{V}_n \leq - \left(\xi_1^2 + \cdots + \xi_n^2 \right) \tag{61}$$

which means ξ and $\widehat{\Theta}$ are bounded. Furthermore, we can conclude that $\xi \to 0$ as $t \to \infty$ by using LaSalle's Invariant Theorem [26]. This implies that $z \to 0$ as $t \to \infty$. From the state scaling transformation (15), we conclude that $x \to 0$ as $t \to \infty$. This completes the proof of Theorem 8. □

Remark 9. As seen from (54) and (7), the control law u_1 may exhibit extremely large value when $x_0(t_0) \neq 0$ is sufficiently small. This is unacceptable from a practical point of view. It is therefore recommended to apply (57) in order to enlarge the initial value of x_0 before we appeal to the asymptotically converging controllers (7) and (54).

5. Example

To verify our proposed controller, we consider the following low-dimensional system with parametric uncertainty:

$$\dot{x}_0 = u_0$$

$$\dot{x}_1 = u_0 x_2 + \ln \left(1 + (\theta x_1)^2 \right) \tag{62}$$

$$\dot{x}_2 = u_1,$$

where θ is unknown constant parameter. Define $\Theta = 1 + \theta^2$; obviously, this system satisfies Lemma 4; that is, $|\ln(1 + (\theta x_1)^2)| \leq |x_1|\Theta$. The purpose is to design u_0 and u_1 based on only $y = (x_0, x_1)^T$ such that $(x_0, x_1, x_2)^T \to 0$ as $t \to \infty$.

If $x(0) = 0$, controls u_0 and u_1 are set as in Section 4 in interval $[0, t_s)$, such that $x(t_s) \neq 0$; then we can adopt the controls developed below. Therefore, without loss of generality, we assume that $x(0) \neq 0$.

For simplicity, in the first subsystem, we can choose the control law $u_0 = g_0 x_0$ and introduce the introduce the state scaling transformation:

$$z_1 = \frac{x_1}{x_0}, \qquad z_2 = x_2. \tag{63}$$

In new z-coordinates, the (x_1, x_2)-subsystem of (62) is rewritten as

$$\dot{z}_1 = g_0 (z_2 - z_1) + \frac{1}{x_0} \ln \left(1 + (\theta x_1)^2 \right) \tag{64}$$

$$\dot{z}_2 = u_1.$$

Design observer to reconstruct z_1 and z_2 of system (62) as follows:

$$\dot{\widehat{z}}_1 = g_0 (\widehat{z}_2 - \widehat{z}_1) + g_0 k_1 (z_1 - \widehat{z}_1), \quad k_1 = -4$$

$$\dot{\widehat{z}}_2 = u_1 + k_2 (z_1 - \widehat{z}_1), \quad k_2 = -2. \tag{65}$$

Then, the estimation error $\widetilde{z} = (z_1 - \widehat{z}_1, z_2 - \widehat{z}_2)^T$ satisfies the equation

$$\dot{\widetilde{z}} = g_0 \begin{pmatrix} -k_1 - 1 & 1 \\ k_2 & 0 \end{pmatrix} \widetilde{z} + \frac{1}{x_0} \begin{pmatrix} \ln \left(1 + (\theta x_1)^2 \right) \\ 0 \end{pmatrix}. \tag{66}$$

In this case, we have

$$A = \begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}. \tag{67}$$

Solving the matrix equation $A^T P + PA = I_2$, we obtain

$$P = \begin{pmatrix} 0.500 & 0.500 \\ 0.500 & 0.100 \end{pmatrix}. \tag{68}$$

The eigenvalues of P are $0.1910, 1.3090$, and thus $\|P\| = 1.3090$.

Using the method given in Section 3, we can design the adaptive controller as follows:

$$\widehat{z}_2^* = -\frac{1}{g_0} z_1 \left[3 + g_0^2 \varepsilon_1^{-2} + 1.604 \varepsilon_0^{-2} + \sqrt{\widehat{\Theta}^2 + 1} \right] = z_1 \beta_1$$

$$u_1 = -\xi_2 \left[9 + \beta_1 + \beta_1^4 + \beta_1^2 \sqrt{\widehat{\Theta}^2 + 1} \right]$$

$$\dot{\widehat{\Theta}} = z_1^2 \left(1.604 \varepsilon_0^{-2} + 1 \right)^2 + \xi_2^2 \beta_1^2. \tag{69}$$

The unknown parameter θ is assumed to be 1. In simulation, the design parameters are chosen as $g_0 = -1$ and $\varepsilon_0 = \varepsilon_1 = \varepsilon_2 = 0.25$. The simulation results for initial conditions $(x_0(0), x_1(0), x_2(0)) = (2, 1, -1)$ and $(\widehat{z}_1(0), \widehat{z}_2(0), \widehat{\Theta}(0)) = (1, -1, 1)$ are shown in Figures 1–4. From the figures, it is clear to see all the closed-loop system states are regulated to zero and the parameter estimates are bounded as proven in Theorem 8.
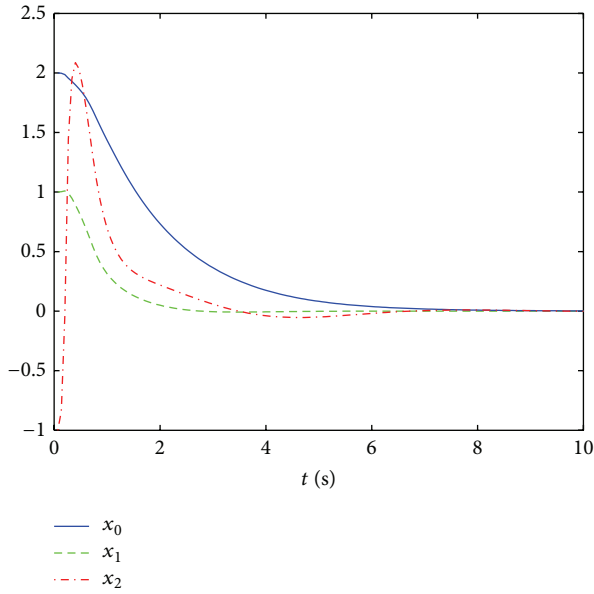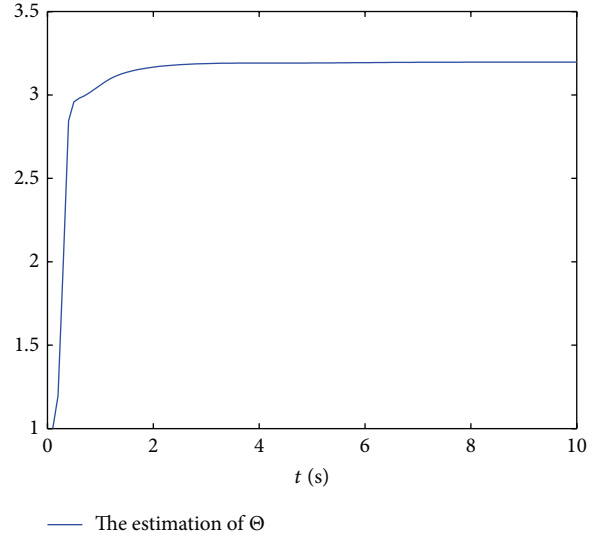
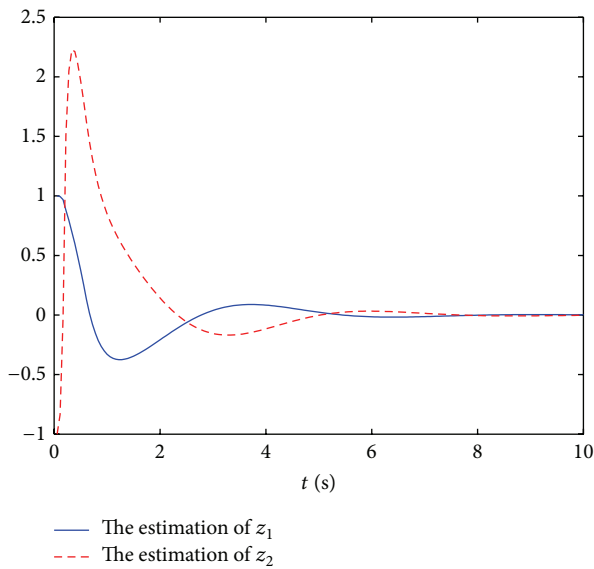FIGURE 1: System states.

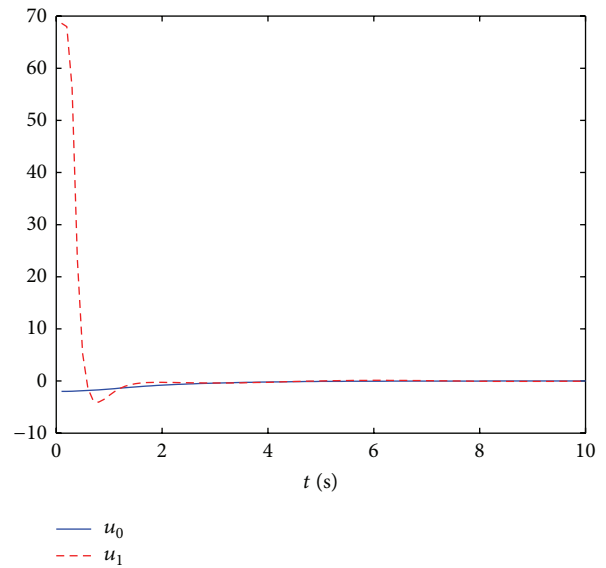FIGURE 3: Parameter estimate.

FIGURE 2: Observer states.

FIGURE 4: Control inputs.

6. Conclusions

In this paper, a constructive adaptive output feedback control strategy is presented for a class of nonlinearly parameterized nonholonomic systems with strong nonlinear drifts. To deal with the nonlinear parameterization problem, a parameter separation technique is introduced to transform the non-linear parameterized nonholonomic system into a linear-like parameterized nonholonomic system. We estimate only Θ, the bound of the unknown parameter rather than the parameter vector θ. This, in turn, results in a minimum dimension (1D) parameter update law. The controller design is developed by using the integrator backstepping approach based on observer and parameter estimator. With a switching strategy eliminating the phenomenon of uncontrollability, the system states asymptotically converge to the origin, while other signals remaining bounded is guaranteed. Simulation results have shown the effectiveness and feasibility of the proposed control strategy.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

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