

# Adaptive Parameter Estimation of Power System Dynamic Model Using Modal Information

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**Abstract**—A novel method for estimating parameters of a dynamic system model is presented using estimates of dynamic system modes (frequency and damping) obtained from wide area measurement systems (WAMS). The parameter estimation scheme is based on weighted least squares (WLS) method that utilizes sensitivities of the measured modal frequencies and damping to the parameters. The paper concentrates on estimating the values of generator inertias but the proposed methodology is general and can be used to identify other generator parameters such as damping coefficients. The methodology has been tested using a wide range of accuracy in the measured modes of oscillations. The results suggest that the methodology is capable of estimating accurately inertias and replicating the dynamic behavior of the power system. It has been shown that the damping measurements do not influence estimation of generator inertia. The method has overcome the problem of observability, when there were fewer measurements than the parameters to be estimated, by including the assumed values of parameters as pseudo-measurements.

**Index Terms**—Dynamic power system modeling, parameter estimation, small signal analysis, synchronous generators, wide area measurements.

## NOMENCLATURE

$\delta$	Rotor angle.
$\omega$	Rotor angular velocity.
$M = 2H$	Rotor inertia constant.
$D$	Damping coefficient.
$K_s$	Synchronizing torque coefficient.
$E'_q$	q-axis voltage behind transient reactance.
$E'_d$	d-axis voltage behind transient reactance.
$E''_q$	q-axis voltage behind subtransient reactance.
$E''_d$	d-axis voltage behind subtransient reactance.
$T'_{d0}$	d-axis open circuit transient time constant.
$T'_{q0}$	q-axis open circuit transient time constant.

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$T''_{d0}$	d-axis open circuit subtransient time constant.
$T''_{q0}$	q-axis open circuit subtransient time constant.
$X'_d$	d-axis transient reactance.
$X'_q$	q-axis transient reactance.
$X''_d$	d-axis subtransient reactance.
$X''_q$	q-axis subtransient reactance.
$I_q$	q-axis armature current.
$I_d$	d-axis armature current.

## I. INTRODUCTION

**K**NOWLEDGE of parameter values for dynamic generator models is of paramount importance for creating accurate models for power system dynamics studies. Traditionally, power systems consisted of a relatively limited number of large power stations and the values of generator parameters were provided by manufacturers and validated by utilities. Recently however, with the increasing penetration of distributed generation, the accuracy of the models and parameters of many small generators connected to the system cannot be guaranteed. This has motivated the effort reported in this paper to develop a methodology to estimate the parameter values from online measurements. One application of such a methodology could be estimation of parameters of dynamic equivalents.

Traditionally dynamic equivalents have been used to represent an external power system [1]. Recently, dynamic equivalents are also used to represent the combined effect of a large number of small (usually renewable) power stations embedded in the distribution network [2]. [3] proposed an artificial neural network (ANN)-based boundary matching technique to derive dynamic equivalents. [4] developed a grey-box approach for validating dynamic equivalents of active distribution network cell. Quite often however, the actual observed oscillations may not match the model that combines a detailed internal network model and an external network equivalent. There could be two possible reasons for the discrepancy. Firstly the external equivalent model could have been derived using inaccurate information. Secondly the external equivalent could have been derived for certain operating conditions that were different from the actual conditions studied or, in other words, the model was not adaptive and therefore could not reflect the actual dynamic system conditions at hand. For example, if some generators in the external system are disconnected, the values of parameters of the equivalent would be changed. An adaptive model is therefore desirable for online system studies, whereby the parameters of the equivalent would change with operating conditions and reveal the physical system characteristics. This paper addresses

the above concerns by developing a methodology for online estimation of the parameter values of a power system dynamic model by employing the values of dynamic system modes, i.e., modal frequencies and damping, calculated from online measurements using wide area measurement systems (WAMS). The aim is to minimize the differences between the observed and modeled modes of oscillation. It should be emphasized that the proposed methodology does not aim at developing the dynamic model itself but rather modifying its parameter values using WAMS measurements. The developed methodology is general and can be used to identify any generator parameters but, as the first step, this paper concentrates on estimating the values of generator inertias.

As the input, the proposed methodology requires the values of modal frequencies and damping calculated from online WAMS measurements. We do not deal in this paper with the problem of how to calculate the modes of oscillation as there are many different approaches proposed in the literature. They usually rely on signal processing methods for extracting the information of power system oscillations from measurements of power flow, frequency or voltage angle difference. Measurement data used can be usually divided into three types [5]: ringdown signal, ambient signal and probing signal. Ringdown signal normally occurs after large system disturbances, while ambient signal is obtained when the system is subject to continuous small system disturbances such as load variation. Probing signal is procured when a system is injected with testing pseudo-random noise intentionally.

Approaches such as Prony analysis utilize ringdown signals following a large disturbance and are well established and widely employed [6]–[8]. In contrast, ambient data (small-signal) based approaches estimate system dynamic modes continuously on a near real time basis using approaches such as Wiener-Hopf equations [9], recursive methods [5], [10]–[12], or empirical mode decomposition [13]. Continuous modal measurements contribute significantly to power system management [14].

We assume in this paper that the dynamic modes (frequency and damping) have been estimated with certain accuracy by one of the established techniques and the focus is on the estimation of the values of generator parameters without considering in detail a particular mode estimation technique.

The problem is stated as follows. It is assumed that the system model, i.e., the models of generators and the transmission network, is known but some of the generator parameters may not be known accurately. To update the values of parameters we use measurements of oscillatory system modes, i.e., the frequency and damping, obtained from WAMS. We compare the observed modes with the calculated ones, obtained using the assumed system model with the initial guesses of parameter values. The objective is to correct the values of parameters such that the observed and model-derived modes are as close as possible. One of the main advantages of the methodology is that it can be used for a continuous updating of the dynamic equivalent model if continuous estimation of frequency and damping is employed from ambient data. The transmission network parameters are assumed to be known.

The stated problem is solved by proposing an effective online updating scheme for dynamic model parameters. Using online estimates of system modes, the modal sensitivity analysis (MSA) can locate the generators in which the parameters varia-

tion causes the change of oscillatory characteristics. The modal assurance criterion (MAC) pairs the observed oscillatory modes with those in the original model. Then a weighted-least-squares (WLS) scheme is implemented for calculating parameters updates based on the iterative Newton-Raphson algorithm. The proposed iterative estimation method is shown to be effective even when the measurements are severely degraded by noise. When the number of measurements is less than the number of parameter values to be estimated, initial guesses of the parameter values are added as pseudo-measurements. This is shown to work well although the estimation accuracy obviously depends on the accuracy of initial values of parameters. Throughout the paper, we assume that no bad measurement data are present.

The rest of the paper is organized as follows. An introduction to power system dynamics analysis is presented in Section II. A novel adaptive parameter estimation method is proposed in Section III. In Section IV, detailed instructions on the simulation setup are given, while simulation results are comprehensively discussed in Section V.

## II. POWER SYSTEM DYNAMICS ANALYSIS

### A. Power System Dynamic Model Description

Under small system disturbances, the nonlinear power system model can be linearized at an equilibrium point and represented as

$$\dot{x} = Ax \quad (1)$$

where  $x$  is the vector of the state variables and  $A$  is the state matrix.

Different synchronous generator models that are fundamental for creating (1) are comprehensively discussed in [1]. In this paper, the sixth-order synchronous generator model is used due to its closeness to reality. The equations for this model are given in the Appendices.

### B. Modal Sensitivity Analysis

In the use of modal information for parameter estimation, the principle is that modal measurements should be sensitive to the parameters. In other words, changes in the parameters should adequately cause changes in modal frequency or damping. It can be interpreted physically that parameters of a generator in a coherent group do not normally affect local modes in another coherent group, but they may affect considerably local modes in their own group and also inter-area modes. Hence, if less sensitive modes are selected for estimating a parameter value, the results of estimation may be inaccurate. Thus, sensitivity analysis is an important step in estimation. For a multi-machine system represented as in (1), the standard eigenvalue analysis results in

$$A\phi_l = \phi_l\lambda_l \quad (2)$$

where

$$\begin{aligned} \phi_l & \quad l\text{th right eigenvector (mode shape);} \\ \lambda_l = \sigma_l \pm j\omega_l & \quad l\text{th eigenvalue;} \end{aligned}$$

$\sigma_l$  modal damping of the  $l$ th mode;  
 $\omega_l$  modal frequency of the  $l$ th mode.

Left-multiplying by left eigenvector  $\psi_l$  and substituting  $\psi_l \phi_l = 1$  gives

$$\lambda_l = \psi_l A \phi_l. \quad (3)$$

The sensitivity of a particular mode to a parameter  $\theta_i$  can then be represented as

$$\frac{\partial \lambda_l}{\partial \theta_i} = \psi_l \frac{\partial A}{\partial \theta_i} \phi_l \quad (4)$$

with the  $l$ th right eigenvector is given as

$$\phi_l = [u_1, \dots, u_l, \dots, u_n]^T_l \quad (5)$$

and the corresponding  $l$ th left eigenvector

$$\psi_l = [v_1, \dots, v_l, \dots, v_n]_l. \quad (6)$$

The full modal sensitivity matrix is then expressed as

$$S = \begin{bmatrix} \frac{\partial \lambda_1}{\partial \theta_1} & \dots & \frac{\partial \lambda_1}{\partial \theta_i} & \dots & \frac{\partial \lambda_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial \lambda_l}{\partial \theta_1} & \dots & \frac{\partial \lambda_l}{\partial \theta_i} & \dots & \frac{\partial \lambda_l}{\partial \theta_n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \frac{\partial \lambda_m}{\partial \theta_1} & \dots & \frac{\partial \lambda_m}{\partial \theta_i} & \dots & \frac{\partial \lambda_m}{\partial \theta_n} \end{bmatrix}. \quad (7)$$

The eigenvalues corresponding to the oscillatory modes are normally complex, with the real part equal to the reciprocal of the damping time constant and the imaginary part equal to the frequency of oscillation. Thus the sensitivity matrix can be further separated into frequency sensitivity  $S_f$  and damping sensitivity  $S_d$ , and the resultant parts are written as

$$S = S_d + jS_f. \quad (8)$$

Practically, it is not necessary, or possible, to obtain the sensitivity for all the modes in the above sensitivity matrices due to incomplete measurements or prior selection of reliable measurements.

The methodology presented in this paper is general and can be used to estimate the values of any generator parameters. The focus of this paper is on the demonstration of the methodology to estimate the values of generator inertias due to their strong influence on the frequency of oscillation.

For the single sixth-order generator described in the Appendices, the sensitivity of the state matrix with respect to inertia constant  $M_i$  is derived in (9) which is used in (4) for determining the modal sensitivity matrix. Index “ $i$ ” has been omitted for all the terms to simplify notation:

$$\frac{\partial A}{\partial M} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_s}{M^2} & \frac{D}{M^2} & 0 & 0 & \frac{Y_q}{M^2} & \frac{Y_d}{M^2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

where  $Y_q = \partial P / \partial E_q''$  and  $Y_d = \partial P / \partial E_d''$ .

### C. Modal Assurance Criterion

As suggested, selected sensitive modes should be used in the estimation methodology. For a continuous estimation process of certain parameters under a particular operating condition, the selected modes should be the same at different time points. Moreover, the iterative algorithm proposed in this paper requires that the selected modes used in each iteration are the same modes. However, in practice it is quite difficult to differentiate one selected mode from unselected ones when damping and frequency values are close. As a consequence of mis-pairing, estimated parameters may be wrong or the algorithm may diverge. To overcome this, MAC is introduced for mode pairing [15]. The MAC between two modes is defined as

$$MAC_{au} = \frac{|\phi_a^T \phi_u|^2}{(\phi_u^T \phi_u)(\phi_a^T \phi_a)} \quad (10)$$

where  $\phi_a$  is a right eigenvector (mode shape vector) calculated from the analytical model with an initial guess and  $\phi_u$  is a measured right eigenvector.

The MAC in (10) takes on values from 0, representing no consistent correspondence, to 1, meaning a consistent correspondence. If the value of MAC is close to 1, the two modes are considered to be highly correlated and therefore may be assumed to be the same mode. If it is close to 0, then they are less correlated. The full description of MAC is outside the scope of this paper but can be found in [16].

## III. PARAMETER ESTIMATION

### A. Weighted Least Squares Estimation

To derive an optimal parameter update scheme, we follow the classical WLS estimation [17]. For a mode  $\lambda(\theta)$ , by neglecting the higher order terms in its full Taylor expansion at a given value  $\theta_k$ , we can derive a Newton-Raphson iterative scheme to calculate the values of parameters for  $k$  iterations:

$$\Delta \lambda_k = S_k \Delta \theta_k, \quad k = 0, 1, 2, \dots \quad (11)$$

$k = 0$  means the initial values of the subscripted variable;  $\Delta \theta_k = \theta_{k+1} - \theta_k$  is the correction to the value of the parameter  $\theta$  calculated in iteration  $k$ ;  $\Delta \lambda_k = \lambda_m - \lambda_k$  is the difference between the measured eigenvalue  $\lambda_m$  and its estimate in  $k$ th iteration,  $\lambda_k$ , calculated using  $\theta_k$ ;  $S_k$  is the value of the modal sensitivity matrix calculated using  $\theta_k$ .

By separating the sensitivity matrix into real and imaginary parts, (11) can be rewritten as

$$\Delta \lambda_k = \Delta \sigma_k + j \Delta \omega_k = S_{dk} \Delta \theta_k + j S_{fk} \Delta \theta_k \quad (12)$$

where  $S_{fk}$  is the frequency sensitivity matrix calculated based on  $\theta_k$  and  $S_{dk}$  is the damping sensitivity matrix—see (8).

If the number of measurements is higher than the number of parameters to be estimated, the optimal solution can be obtained by applying the WLS method, i.e., by minimizing the objective function  $J$  equal to the sum of weighted squares of errors between the measured and estimated value of frequency and damping:

$$J(\Delta \theta) = \varepsilon_f^T W_f \varepsilon_f + \varepsilon_d^T W_d \varepsilon_d \quad (13)$$

where  $\varepsilon_f = \Delta\omega - S_f\Delta\theta$  is the frequency error calculated using the updated parameters;  $\varepsilon_d = \Delta\sigma - S_d\Delta\theta$  is similarly the damping error;  $W_f$  and  $W_d$  are diagonal weighting matrices. The weights are equal to the reciprocals of the variance of the corresponding measurements which depend on a particular methodology used to calculate the values of modal frequency and damping. The two terms in (13) represent different physical quantities (frequency and damping estimation errors) but their influence on the estimation of the parameters can be taken into account together by weighting the squared estimation errors by the measurement variances.

The optimal solution is obtained when  $\partial J(\delta\theta)/\partial\delta\theta = 0$  [17] and is expressed as

$$\Delta\theta = (S_f^T W_f S_f + S_d^T W_d S_d)^{-1} (S_d^T W_d \Delta\sigma + S_f^T W_f \Delta\omega). \quad (14)$$

It should be noted that (14) will be underdetermined when the number of unknown parameters exceeds the number of measured modes. Thus, there would be an infinite number of sets of parameter differences that satisfy (14). To overcome this, initial guesses of the parameter values may be treated as pseudo-measurements so that the objective function (13) is further expanded by adding a term corresponding to the pseudo-measurements

$$J(\Delta\theta) = \varepsilon_f^T W_f \varepsilon_f + \varepsilon_d^T W_d \varepsilon_d + (\theta - \theta_0)^T W_p (\theta - \theta_0) \quad (15)$$

where  $\theta_0$  is the initial guesses for the unknown parameters and  $W_p$  is the parameter weighting matrix which is a diagonal matrix and positive definite.

Adding the pseudo-measurements has an additional advantage that it restricts the parameter changes to values close to the initial guesses. In effect, the assumed values of parameters may not be entirely accurate but they are close to the true values. It will be shown that treating the initial guesses of parameters values as pseudo-measurements significantly improves the accuracy of parameter estimation, even when the set of mode measurements was incomplete. Obviously adding the last term in (15) makes sense only if the initial guesses are accurate. Hence the last term in (15) should only be added for those parameters that are known with relatively good confidence.

The diagonal elements of  $W_p$  are the reciprocals of the estimated variance of the corresponding parameters. To determine the variance, it is useful to have some knowledge of the range of variation of inertia constants from their initial guesses in order to set the levels of uncertainty in the parameters, so that parameters with smaller deviations from their initial guesses are weighted by larger weights.

Finally, it should be added that the approach proposed in this paper is based on standard WLS estimation where no bad data are assumed to be present. Obviously for the methodology to be of practical use the problem of robustness in the presence of bad data must also be addressed. This is a subject of further research.

As the power system model is nonlinear, the optimal solution of (15) can be re-written as an iterative equation

$$\theta_{k+1} = \theta_k + (S_{dk}^T W_d S_{dk} + S_{fk}^T W_f S_{fk} + W_p)^{-1} \times [S_{dk}^T W_d (\sigma_m - \sigma_k) + S_{fk}^T W_f (\omega_m - \omega_k) + (\theta_0 - \theta_k)^T W_p] \quad (16)$$

where  $\theta_{k+1}$  and  $\theta_k$  are the estimated parameters at  $k+1$ th iteration and  $k$ th iteration respectively.  $\omega_k$  and  $\sigma_k$  are the estimated

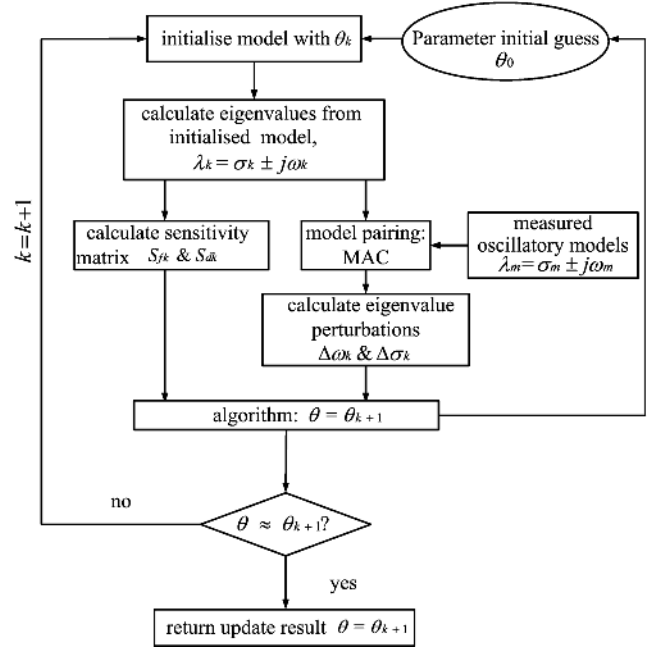


Fig. 1. Iterative parameter estimation.

modal frequency and damping, respectively.  $\omega_m$  and  $\sigma_m$  are the measured modal frequency and damping.

### B. Iterative Parameter Estimation

The iterative parameter estimation with pseudo-measurements is illustrated in Fig. 1. For the variant of the methodology shown by (16) that includes pseudo-measurements, the parameter initial guesses are involved in the “algorithm” block in the figure. When the updated parameters are all close to those from the previous estimate, convergence is reached and the algorithm returns the results of the last estimates.

## IV. SIMULATION SETUP

### A. Problem Description

Before any new methodology can be implemented in practice, it should be tested in a controlled environment to determine its accuracy and efficiency. Hence this paper describes the results of tests in which modal frequencies were calculated not from actual WAMS measurements but rather derived from a power system model. This has allowed us to assess the accuracy of the parameter estimation.

The simulation set up was as follows. Using the system model the corresponding eigenvalues were calculated for the electro-mechanical modes of oscillation. Then real-time modal measurements were simulated by adding random noise to the eigenvalues and were used to estimate the values of model parameters using (16). The initial guesses of parameter values were assumed to be different than the actual ones (which in practice are not known). This setup has made it possible to evaluate the accuracy of estimation by comparing the estimated and the actual values of the parameters.

Generally both frequency and damping could be used for parameter estimation however it is well-known that generator inertias affect the oscillation frequency much more than they do the corresponding damping. This can be easily confirmed by a

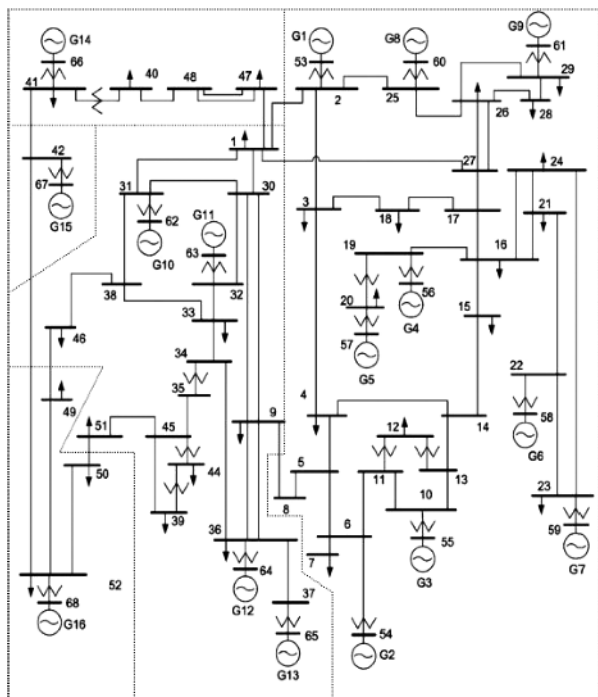


Fig. 2. New York-New England system [18].

TABLE I  
ASSUMED AND ACTUAL INERTIA CONSTANTS IN NY-NE SYSTEM

Generator Inertia	Assumed	Actual	Error (%)
M1	6.8000	7.8000	12.8
M10	5.8210	4.8210	20.7
M14	6.0000	6.8000	11.8
M15	6.0000	6.8000	11.8
M16	8.9000	9.9000	10.1

sensitivity analysis using (4). Consequently, we have used frequency measurements only in the estimation of generator inertias, hence neglecting the damping term in (16). Experiments have confirmed that adding the damping term did not improve significantly the inertia estimation.

In the simulations we have tested two versions of the proposed methodology. The first used frequency measurements only—see (13), while the second utilized both frequency measurements and pseudo-measurements (i.e., assumed values of the parameters)—see (15).

The 16-generator New York-New England (NY-NE) system from [18], shown in Fig. 2, has been used in the simulations. The system contains five coherent groups, shown in Fig. 2, separated by dashed lines. One generator from each coherent group is selected where the inertia constants of G1, G10, G14, G15, and G16 will be the parameters to be estimated. The inertia constants of these generators are indicated as M1, M10, M14, M15, and M16. The assumed and actual values of these parameters (which would normally be unknown) are given in Table I. The percentage differences between them are also presented in the table and are in the range 10%–20%. The remaining generator parameters and the transmission network model are assumed to be perfectly known.

TABLE II  
OSCILLATORY MODES OF NY-NE SYSTEM

mode	assumed	actual
1	$-0.7780 \pm 10.2655j$	$-0.7771 \pm 10.2992j$
2	$-0.6941 \pm 9.8776j$	$-0.6284 \pm 9.2617j$
3	$-0.9151 \pm 8.4890j$	$-0.9148 \pm 8.4888j$
4	$-0.9294 \pm 8.1682j$	$-0.9294 \pm 8.1682j$
5	$-0.7226 \pm 7.3825j$	$-0.7281 \pm 7.3021j$
6	$-0.4201 \pm 7.2777j$	$-0.4343 \pm 8.0015j$
7	$-0.5497 \pm 7.0772j$	$-0.5494 \pm 7.0774j$
8	$-0.4579 \pm 6.4859j$	$-0.4601 \pm 6.4926j$
9	$-0.6317 \pm 6.4001j$	$-0.6317 \pm 6.4001j$
10	$-0.4150 \pm 5.9480j$	$-0.4155 \pm 5.9499j$
11	$-0.3619 \pm 5.6092j$	$-0.3617 \pm 5.6090j$
12	$-0.2694 \pm 4.5177j$	$-0.2622 \pm 4.3268j$
13	$-0.1993 \pm 3.7389j$	$-0.1990 \pm 3.7356j$
14	$-0.1438 \pm 2.9858j$	$-0.1402 \pm 2.8921j$
15	$-0.1305 \pm 2.0801j$	$-0.1271 \pm 2.0401j$

### B. Eigenvalue and Modal Sensitivity Analysis

With the initial guesses for M1, M10, M14, M15, and M16, the oscillatory modes (eigenvalues) have been calculated and are shown in the second column of Table II. As the system contains 16 generators, there are 15 independent modes of electromechanical oscillations. The real and imaginary parts of the eigenvalues are denoted as  $\sigma$  and  $\omega$ , respectively, in (16). The actual oscillatory modes (which would normally be unknown) are given in the third column of the table. The mode measurements  $\sigma_m$  and  $\omega_m$  are simulated by perturbing the actual oscillatory modes shown in the last column of Table II by adding random errors as shown later in the paper. Table II shows that modes 2 and 6 are most affected by the parameter changes while modes 12 and 14 are less affected. The other modes are not significantly affected.

The direct analytical representation of calculating sensitivity of modal frequency to generator inertias can be found in [19]. The sensitivities are presented in Fig. 3 with the vertical axis indicating frequency sensitivity and the horizontal axis indicating total 15 modes. For example the first diagram labelled M1 shows the sensitivities of all the modes to inertia M1. Fig. 3 shows that the modal sensitivity to a particular parameter can vary significantly and confirms the results shown in Table II. As shown in the first two diagrams in Fig. 3, the sensitivities of modes 2 and 6 are the highest and they are influenced by the disturbed inertias M1 and M10. The changes of M14, M15, and M16 can be mainly reflected in the changes of the frequencies of mode 12, 14, and 15. Especially, mode 12 can reveal the changes in M14, M15, and M16 at the same time.

### C. Measurement Errors and Weighting Matrices

Error level of estimation of modal frequency and damping varies and depends on a particular estimation method used, measuring devices and locations. For investigating the robustness and feasibility of the proposed methodology, a range of error levels were studied. In this paper representative results are shown for the frequency errors equal to 1.0%, 5.0%, and 10.0%. The weighting matrix for parameters,  $W_p$ , was calculated based on estimated standard deviations of 10% for M1, M10 and M15, 15% for M14, and 25% for M16 from their initial values.

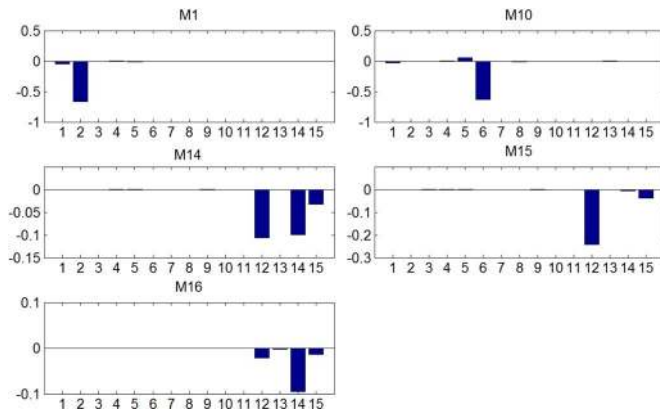


Fig. 3. Frequency sensitivity (vertical axis) of all the modes (shown on the horizontal axis) to inertia constants M1, M10, M14, M15, and M16.

#### D. Observability

For a power system with  $n$  machines there are  $(n - 1)$  independent modes of electromechanical oscillations which could possibly be measured. Hence it is only possible to estimate at the maximum  $(n - 1)$  inertias and we have to assume that at least one inertia value is known with a high accuracy. In practice, the number of measurable modes of oscillation is significantly less as some modes can be weak (i.e., have a small amplitude) or so well-damped that they are difficult to observe.

To test the influence of measurement redundancy, three characteristic sets of measurements have been selected:

$$\begin{aligned} s_1 &= \{\text{full observability}\} \\ s_2 &= \{2, 6, 11, 12, 14, 15\} \\ s_3 &= \{2, 6, 12\}. \end{aligned}$$

The full observability set  $s_1$  contains fifteen dynamic modes shown in Table II which has high redundancy as there are only five inertias to be estimated. The set  $s_2$  contains six measured modes to estimate five inertias. Referring to the modal sensitivities shown in Fig. 3, all the measured modes in  $s_2$ , apart from mode 11, are sensitive to the changes in the selected parameters. As mode 11 gives no information about the parameters, case ii) constitutes marginal observability.

Set  $s_3$  contains only three measured modes so it corresponds to an unobservable case. The three modes in  $s_3$  are highly associated with all parameters to be identified. The observability problem of case  $s_3$  is overcome by adding the pseudo-measurements ( $\theta_0$  in (16)).

### V. SIMULATION RESULTS

#### A. Estimation Using Frequency and Pseudo-Measurements

The inertia estimation errors and standard deviations (SD) (both in %) based on full observability ( $s_1$ ) for different levels of error of the frequency measurements are presented in Table III. The errors of M1, M10, M14, M15 and M16 are indicated as  $e_1$ ,  $e_{10}$ ,  $e_{14}$ ,  $e_{15}$ , and  $e_{16}$ , respectively. All the results in this section are averages obtained from 100 random Monte Carlo simulation runs.

The results show that the proposed methodology works well as the inertia estimation errors have been significantly reduced

TABLE III  
ESTIMATION BASED ON  $s_1$

Frequency Error (%)	1.0		5.0		10.0	
	mean	SD	mean	SD	mean	SD
$e_1$	0.78	1.19	6.90	2.61	10.77	2.07
$e_{10}$	0.57	1.33	9.74	3.01	15.86	2.63
$e_{14}$	0.09	1.63	5.49	2.04	8.06	2.12
$e_{15}$	1.51	1.34	7.65	2.05	9.68	1.67
$e_{16}$	0.15	2.03	0.27	5.10	3.11	5.57

TABLE IV  
ESTIMATION BASED ON  $s_2$

Frequency Error (%)	1.0		5.0		10.0	
	mean	SD	mean	SD	mean	SD
$e_1$	0.59	1.16	7.91	2.35	10.56	2.20
$e_{10}$	0.73	1.21	10.17	3.39	16.46	2.72
$e_{14}$	0.11	1.55	5.22	1.96	8.42	2.05
$e_{15}$	0.86	1.24	7.63	2.08	10.18	1.45
$e_{16}$	0.20	2.03	1.01	5.22	2.91	5.65

TABLE V  
ESTIMATION BASED ON  $s_3$

Frequency Error (%)	1.0		5.0		10.0	
	mean	SD	mean	SD	mean	SD
$e_1$	0.78	1.11	6.90	2.67	10.42	2.13
$e_{10}$	0.68	1.18	10.21	3.34	16.31	2.53
$e_{14}$	0.51	1.11	7.42	2.04	10.02	1.58
$e_{15}$	0.89	1.06	7.58	1.97	10.09	1.51
$e_{16}$	0.16	1.00	6.37	1.80	8.62	1.35

compared with the errors shown in Table I. When the error of the frequency was increased, the estimation accuracy dropped.

The results of the estimation based on  $s_2$  (i.e., marginal observability) are shown in Table IV. The results show a similar pattern to those when estimation was based on  $s_1$ . With the highly associated modes included, the group of measurements which included more redundancy did not contribute much to the accuracy of the estimates nor the estimating speed, since the redundant measurements were not involved in improving the estimation process due to their low sensitivity.

The set  $s_3$  contained fewer measurements than inertia constants to be estimated. The results in Table V showed that the parameters could be still estimated with high accuracy due to inclusion of pseudo-measurements. The accuracy of parameter estimation was similar to that using sets  $s_1$  or  $s_2$ .

Recall that the aim of estimation is minimization of the frequency estimation errors. Hence Tables VI–VIII show the mean errors (in %) between the estimated modal frequencies, calculated using the modified parameter values, and the actual frequencies for  $s_1$ ,  $s_2$ , and  $s_3$ . For a small frequency measurement error (1%) in set  $s_1$ , the large frequency estimation errors are observed for the modes associated with the estimated inertias: 2, 6, 12, 14, 15 (see Fig. 3). The reason is that the pseudo-measurements are forcing the objective function towards the initial values of inertias at the cost of increased frequency errors. When the frequency measurement error was increased to 10%, that effect was reduced as the frequency measurement errors were in the same range as the inertia estimation errors.

TABLE VI  
MEAN ERRORS OF  $s_1$

Measured freq error	1%	5%	10%
mode 1	0.51	2.40	5.43
mode 2	5.38	5.11	5.50
mode 3	0.42	2.64	5.04
mode 4	0.50	2.61	5.26
mode 5	0.80	2.52	4.99
mode 6	6.94	7.09	6.09
mode 7	0.50	2.44	5.05
mode 8	0.54	2.67	5.17
mode 9	0.46	2.56	4.95
mode 10	0.50	2.41	4.64
mode 11	0.51	2.46	5.27
mode 12	5.44	5.73	6.26
mode 13	0.49	2.68	5.55
mode 14	4.84	4.94	6.20
mode 15	2.54	3.20	5.69

TABLE VII  
MEAN ERRORS OF  $s_2$

Measured freq error	1%	5%	10%
mode 2	5.36	5.49	6.81
mode 6	7.03	6.51	6.90
mode 11	0.48	2.62	5.53
mode 12	5.44	5.09	5.89
mode 14	4.90	5.05	5.91
mode 15	2.51	2.77	5.77

TABLE VIII  
MEAN ERRORS OF  $s_3$

Measured freq error	1%	5%	10%
mode 2	5.36	5.74	5.72
mode 6	7.02	7.33	7.00
mode 12	5.75	5.81	6.49

### B. Estimation When Not Using Pseudo-Measurements

The methodology was also tested for using frequency measurements only, i.e., without using the assumed values of inertias as pseudo-measurements. This approach could not be applied to the inadequate measurement case ( $s_3$ ), since the measurement set was underdetermined and therefore an infinite set of solutions could fit the objective function. The estimation using this methodology in terms of  $s_1$  and  $s_2$  showed that it lacked robustness and did not always converge. The reason for non-convergence was the high nonlinearity of the power system dynamic model. It was found that the convergence of the algorithm was highly dependent on initial inertia guesses and the values of frequency measurement errors. Table IX shows the percentage of divergence cases in 100 random Monte Carlo simulation runs. Generally, the number of divergence cases increased with the growing frequency measurement errors. The overall conclusion was therefore that pseudomeasurements have to be added to the measurement set to ensure convergence.

TABLE IX  
PERCENTAGE OF DIVERGENCE

Frequency error	1.0%	5.0%	10.0%
$s_1$	40%	77%	92%
$s_2$	37%	86%	91%

## VI. CONCLUSION

A novel algorithm was developed for estimating the values of dynamic model parameters based on dynamic modal information. The aim is to minimize the differences between the observed and modeled modal frequency and damping. One possible application of the methodology is adapting the parameters of a dynamic equivalent to changing operating conditions which are different to the ones used when developing the equivalent.

The focus of this paper was on the estimation of generator inertia values, however the proposed methodology is general and not confined to inertia estimation. The methodology can therefore be applied to estimation of other parameters such as e.g. the damping coefficients. The methodology is iterative and utilizes the framework of WLS estimation. The methodology effectively circumvented the technical barrier for pairing modal measurements when they were close in frequency or damping by using MAC.

The methodology has been tested using a wide range of accuracies in the measured frequency of oscillations. The results suggest that the methodology is capable of estimating accurately inertias and replicating the dynamic behavior of the power system. Simulations also have shown that it was necessary to use pseudomeasurements of the assumed values of parameters to ensure convergence of the algorithm. The pseudomeasurements also helped to overcome the problem of observability when there were fewer measurements than the parameters to be estimated.

## APPENDIX

The six-order synchronous generator model is given as

$$\begin{bmatrix} \dot{\Delta\delta} \\ \dot{\Delta\omega} \\ \dot{\Delta E'_q} \\ \dot{\Delta E'_d} \\ \dot{\Delta E''_q} \\ \dot{\Delta E''_d} \end{bmatrix} = \begin{bmatrix} 0 & \omega_0 & 0 & 0 & 0 & 0 \\ -\frac{K_s}{M} & -\frac{D}{M} & 0 & 0 & K_{1sq} & K_{1sd} \\ K_{1\delta} & 0 & a_{1q} & 0 & K_{2sq} & K_{2sd} \\ K_{2\delta} & 0 & 0 & a_{1d} & K_{3sq} & K_{3sd} \\ K_{3\delta} & 0 & a_{2q} & 0 & K_{4sq} & K_{4sd} \\ K_{4\delta} & 0 & 0 & a_{2d} & K_{5sq} & K_{5sd} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \Delta E'_q \\ \Delta E'_d \\ \Delta E''_q \\ \Delta E''_d \end{bmatrix}. \quad (17)$$

The elements in the state matrix of (17) are calculated as follows:

$$\begin{aligned} K_{1\delta} &= (T'_{d0})^{-1} \Delta X'_d \frac{\partial I_d}{\partial \delta} \\ K_{2\delta} &= (T'_{q0})^{-1} \Delta X'_q \frac{\partial I_q}{\partial \delta} \\ K_{3\delta} &= (T''_{d0})^{-1} \Delta X''_d \frac{\partial I_d}{\partial \delta} \\ K_{4\delta} &= (T''_{q0})^{-1} \Delta X''_q \frac{\partial I_q}{\partial \delta} \end{aligned} \quad (18)$$

$$\begin{aligned}
 K_{1sq} &= -M^{-1} \frac{\partial P}{\partial E''_q} \\
 K_{2sq} &= (T'_{d0})^{-1} \Delta X'_d \frac{\partial I_d}{\partial E''_q} \\
 K_{3sq} &= (T'_{q0})^{-1} \Delta X'_q \frac{\partial I_q}{\partial E''_q} \\
 K_{4sq} &= (T''_{d0})^{-1} \left( \Delta X''_d \frac{\partial I_d}{\partial E''_q} - 1 \right) \\
 K_{5sq} &= - (T''_{q0})^{-1} \Delta X''_q \frac{\partial I_q}{\partial E''_q}
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 K_{1sd} &= -M^{-1} \frac{\partial P}{\partial E''_d} \\
 K_{2sd} &= (T'_{d0})^{-1} \Delta X'_d \frac{\partial I_d}{\partial E''_d} \\
 K_{3sd} &= (T'_{q0})^{-1} \Delta X'_q \frac{\partial I_q}{\partial E''_d} \\
 K_{4sd} &= (T''_{d0})^{-1} \Delta X''_d \frac{\partial I_d}{\partial E''_d} \\
 K_{5sd} &= - (T''_{q0})^{-1} \left( \Delta X''_q \frac{\partial I_q}{\partial E''_d} + 1 \right)
 \end{aligned} \tag{20}$$

$$\begin{aligned}
 a_{1q} &= (T'_{d0})^{-1} \\
 a_{1d} &= (T'_{q0})^{-1} \\
 a_{2q} &= (T''_{d0})^{-1} \\
 a_{2d} &= (T''_{q0})^{-1} .
 \end{aligned} \tag{21}$$

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