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***Abstract*— When a scalar field, such as temperature, is to be estimated from sensor readings corrupted by noise, the estimation accuracy can be improved by judiciously controlling the locations where the sensor readings (samples) are taken. In this paper, we solve the following problem: given a set of static sensors and a group of mobile robots equipped with the same sensors, how to determine the data collecting paths for the mobile robots so that the reconstruction error of the scalar field is minimized. In our scheme, the static sensors are used to provide an initial estimate and the mobile robots refine the estimate by taking additional samples at critical locations. Unfortunately, it is computationally expensive to search for the best set of paths that minimizes the field estimation errors and hence the field reconstruction errors as well). We propose a heuristic to find 'good' paths for the robots. Our approach first partitions the sensing field into equal gain subareas and then we use a single robot planning algorithm to generate a path for each robot separately. The properties of this approach are studied in simulation. Our approach also implicitly solves a multi-robot coordination/task allocation problem, where the robots are homogeneous and the size of task set might be large.**

I. INTRODUCTION

A sensor actuator network (also a robotic sensor network), which consists of both static and mobile nodes, provides a new tool for measuring and monitoring the environment. On the one hand, with less energy consumed, the static sensor nodes are able to provide high resolution temporal sampling. On the other hand, with the ability to move, the mobile nodes (henceforth) are able to change the spatial distribution of the sensor readings leading (if running the appropriate algorithm) to a high density of readings in important areas. The key challenge is an adaptive sampling problem - come up with trajectories that the robots can follow, sampling alone which will improve the field reconstruction. In [1], we proposed an adaptive sampling algorithm for a system consisting of a set of static sensor nodes and one mobile robot, a robotic boat. The system, part of the NAMOS project at USC (http://robotics.usc.edu/ namos), is used for measuring scalar fields, such as temperature, salinity and chlorophyll concentration. We have shown [1] that by combining optimal experimental design and path planning, we are able to achieve an improved estimation performance, i.e., a lower Integrated Mean Square Error (IMSE) with the same (finite) initial energy available to the mobile robot.

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In [1], we assume that the scalar field to be reconstructed changes slowly. That is, during the time the mobile robot is sent out for a data collecting tour, the readings from the static sensors are still valid. However, this might not be true in practice since it takes a while for the mobile robot to finish a tour. One way to overcome this drawback is to use multiple mobile robots in parallel to accomplish the task. If we can generate 'good' paths for all the mobile robots and let them carry out the sampling task simultaneously, the speedup could be significant. Another advantage of a system with multiple mobile robots is energy efficiency. In many cases, the ideal distribution (leading to the best reconstruction of the field) of the sensor readings contains several clusters. Since normally, we already have static sensors covering the whole sensing field, the mobile robots may just need to take readings within each cluster. If only one mobile robot is deployed, it has to move between the clusters. If multiple mobile robots are used and the number of robots is more than the number of clusters, each robot only needs to stay within a cluster and the energy to move from one cluster to another could be saved.

Fig. 1. One of the robotic boats used in the NAMOS project at USC.

In this paper, we investigate the problem of adaptive sampling using multiple mobile robots. Specifically, given a set of static sensor nodes deployed uniformly across the sensing field, and a team of mobile robots each with the same energy, how to exploit the information collected by the static sensors and coordinate the motion of the mobile robots so that error associated with the reconstruction of the underlying scalar field is minimized. Here we assume that all the mobile robots have the same energy consumption profile and the underlying scalar field is continuous and has finite second order derivative at any point.

As in the case of single robot problem [1], we use techniques from optimal experimental design to define a gain associated with each location and then apply a search algorithm to find 'good' paths for the mobile robots. The challenge in the multiple mobile robot case comes from the path planning. Even in the case of a single robot, to find the path with maximum sum of gains is NP-complete. This is called the Orienteering problem, which has been studied in the theoretical computer science and operating research communities. In the case of multiple mobile robots, the situation is even worse since the search space grows exponentially with the number of the mobile robots. Therefore, we will have to compromise performance by approximation so that the path planning can be done quickly enough to be realistically feasible for a real-world situation. In this paper, we propose a divide and conquer approach to solve the problem. Once the gain of each location in the sensing field is computed, we partition the sensing field into subareas with equal gain. Now the path planning problem reduces to a set of problems. Each robot is assigned a subarea and a path planning algorithm for single robot within each region is applied. An advantage of this approach is that the path planning for a single mobile robots can be done in parallel since the subareas do not overlap.

This paper is organized as follows. We first discuss related work in the next section. Then, the optimal experimental design is explained briefly in section III. We discuss the partition strategy in section IV and the simulations results are presented in section V. We discuss future work and conclude in section VI.

II. RELATED WORK

Adaptive sampling and actuated sensing have been studied in the sensor network community. Based on Wavelet theory, Willett [2] proposed an algorithm to extend the life time of a static sensor network by putting some sensors to sleep without significantly increasing the estimation errors. Rahimi [3] proposed a sequential algorithm for a single mobile node to minimize the estimation error with the limitation on the time or distance the mobile node travels. In his approach, more readings are taken in the place where there is higher residual. Krause [4] proposed a near-optimal algorithm to solve the problem of static sensor node deployment. However, it is assumed that the underlying phenomenon is a Gaussian Process and a dense deployment of sensors is needed to find the correlation between readings at any two locations. Based on the same assumption, Singh [5] introduced an approximation algorithm to solve a problem that is similar to ours and also extended to the case with multiple mobile robots by using sequential allocation. Our approach does not assume previous dense deployment of the sensors.

Another problem related to adaptive sampling with multiple mobile robots is the multi-robot exploration and mapping. It is normally assumed that the robots are deployed in an unknown environment and no global information is available. In this case, many techniques of coordinated exploration and mapping are based on the idea of the frontier points or cells [6], [7], [8], [9]. To assign frontier points to individual robots, a market-based approach is used and the target locations are assigned through auctions [7], [8], [9]. There are different ways to evaluate the target locations for each robot. In the case where the energy consumption is important, the assignment with best tradoff between energy and utility is chosen [9]; Fox took the uncertainty of the localization into consideration and proposed a strategy that trades off between the frontier and the hypothesis. Normally, the coordination strategies assign each robot one target location to visit. In the case multiple target locations are to be assigned, a sequential allocation is used [7], [8]. Stroupe [10] proposed a value-based coordination algorithm, which trades off between dynamic target observing, exploration, sampling and communications. However, the energy consumption is not considered. In this paper, we are looking at a slightly different problem. First, we are going to use the static sensors to provide a coarse estimation of the scalar field and hence rough global information is available. Second, multiple target locations are to be assigned to each robot and each robot then visits these locations sequentially. Finally, because of the existence of the static sensor nodes, we assume that communication can be achieved by using multi-hop protocol and hence we do not consider communication as a constraint.

III. ADAPTIVE SAMPLING

In this paper, we assume that no parametric model is available for the scalar field and non-parametric regression is appropriate. A Kernel estimator is one of the most popular non-parametric regression techniques since it is easy to understand, analyze and implement. Local Linear Regression is a kernel estimator where the value of the scalar field at any location $\mathbf{x}$ is estimated by using weighted linear regression. It is assumed that the closer two locations are, the higher the correlation between the values. As a result, when the linear regression is applied to location $\mathbf{x}$, more weight is given to the data points closer to $\mathbf{x}$ while less weight is given to the data points far away. We assume a non-parametric model shown as equation 1.

$$y = m(\mathbf{x}) + \sigma(\mathbf{x})\epsilon, \tag{1}$$

where $\mathbf{x}$ is sensor location, y is the corresponding sensor reading, $\sigma^2(\mathbf{x})$ is the variance of the noise and ϵ is a random variable with zero mean and unit variance and are independent of $\mathbf{x}$. The local linear regression can be represented by equation 2.

$$\hat{m}(\mathbf{x}, H) = e_1 \cdot (X_x^T W_x X_x)^{-1} X_x^T W_x Y, \tag{2}$$

where $\mathbf{X_x} = \begin{bmatrix} 1 & (\mathbf{x}_1 - \mathbf{x})^T \\ \cdots & \cdots \\ 1 & (\mathbf{x}_n - \mathbf{x})^T \end{bmatrix}$, $Y = [y_1, \cdots, y_n]^T$, $W_x = diag\{K_H(\mathbf{x}_1 - \mathbf{x}), \cdots, K_H(\mathbf{x}_n - \mathbf{x})\}$ and $e_1 = [1, 0, \cdots, 0]$. The kernel $K_H(\mathbf{u})$ determines how much weight would be assigned to each data point and is normally defined by a base $K(\mathbf{u})$ and a matrix H,

$$K_H(\mathbf{u}) = |H|^{-1/2} K(H^{-1/2}(\mathbf{u})).$$

H determines the size and shape of the neighborhood and $H^{-1/2}$ is called the bandwidth matrix.

If $K(\mathbf{u})$ satisfies $\int \mathbf{u}\mathbf{u}^T K(\mathbf{u})d\mathbf{u}$ is finite and $\int u_1^{l_1} \cdots u_d^{l_d} K(\mathbf{u})d\mathbf{u} = 0$ for all non-negative integers $l_1, \cdots, l_d$ such that their sum is odd, it has been proved that the estimation error associated with the local linear regression is given by the following equation [11]:

$$\begin{aligned} MSE\{\hat{m}(\mathbf{x}; H)\} &= \frac{C_1\sigma^2(\mathbf{x})}{n|H|^{1/2}f(\mathbf{x})} \\ &\quad + \frac{1}{4}C_2 tr^2\{H\mathrm{H}_m(\mathbf{x})\} \\ &\quad + \mathbf{o}_p\{n^{-1}|H|^{-1/2} + tr^2(H)\}. \end{aligned} \tag{3}$$

where n is the number of samples, $f(\mathbf{x})$ is the density function with $\int f(\mathbf{x})d\mathbf{x} = 1$, and $\mathrm{H}_m(\mathbf{x})$ is the Hessian matrix of $m(\mathbf{x})$ and C_1 and C_2 are the constant depending on kernel $K(\mathbf{u})$.

If $nH^{-1/2}$ is big enough and H is small enough, the infinitesimal can is negligible. By applying Lagrange-Euler differential equation, we can find the optimal bandwidth and corresponding IMSE as follows.

$$h^* = \left(\frac{dR(K)v(\mathbf{x})}{nf(\mathbf{x})\mu_2(K)^2 tr^2\{\mathrm{H}_m(\mathbf{x})\}}\right)^{\frac{1}{d+4}}, \tag{4}$$

$$IMSE(\mathbf{X}_1, \ldots, \mathbf{X}_{n0+n}) \propto \int \left(\frac{tr^d\{\mathrm{H}_m(\mathbf{x})\}v^2(\mathbf{x})}{n^2\hat{f}^2(\mathbf{x})}\right)^{\frac{2}{d+4}} d\mathbf{x}, \tag{5}$$

where $\hat{f}(\mathbf{x}) = n^{-1}\sum_{i=1}^{n} K_H(\mathbf{X_i} - \mathbf{x})$ is the density function.

Assume that initially there are n_0 readings from static sensor nodes $(\mathbf{x}_1, y_1), ..., (\mathbf{x}_{n_0}, y_{n_0})$, the path of the mobile robot passes through the points $\mathbf{x}_{n_0+1}$, ..., $\mathbf{x}_{n_0+n}$, then the optimal path should minimize IMSE. The IMSE can be estimated with the equation 5 and the corresponding density can be estimated by using $\hat{f}(\mathbf{x}) = n^{-1}\sum_{i=1}^{n} K_H(\mathbf{X_i} - \mathbf{x})$. Similar to the information gain defined in robot exploration literature, we define the gain for each point as follows.

$$G(\mathbf{x}) = IMSE(\mathbf{X}_1, \ldots, \mathbf{X}_{n_0}) - IMSE(\mathbf{X}_1, \ldots, \mathbf{X}_{n_0}, \mathbf{x}), \tag{6}$$

The gain associated with the path is defined as

$$\begin{aligned} G(p) &= IMSE(\mathbf{X}_1, \ldots, \mathbf{X}_{n_0}) - \\ &\quad IMSE(\mathbf{X}_1, \ldots, \mathbf{X}_{n_0}, \mathbf{X}_{n_0+1}, \ldots, \mathbf{X}_{n_0+n}). \end{aligned} \tag{7}$$

Now, the problem is to find the path or paths collecting the most gains and hence a path planning problem needs to be solved.

IV. DIVIDE AND CONQUER

A. Discretization

In the problem of adaptive sampling with static sensor nodes and multiple mobile robots, the main constraint is that the energy available to each mobile robot is limited. Therefore, path planning for the mobile robots needs to take into consideration the energy consumption model. The simplest model is to assume that the energy consumed by a mobile robot to move from location A to B is proportional to the length of the line segment connecting the A and B. The energy consumption model used in this paper is the one for the robotic boat of the NAMOS project [12], as shown in Figure 1. In this model, the state is the location and the orientation of the boat. The energy consumed for state transition not only depends on the distance between two states but also depends on the orientations of the two states. The details of the energy consumption model are described in [1] and will not be repeated here.

Algorithm 1: Adaptive Sampling With Multiple Mobile Robots

Construct the state graph G=(V, E);
Collect readings from static sensors $r0$;
for *each vertex* $v \in V$ **do**
    Compute gain $g(v)$;
end
Partition G into subgraphs $G_1, ..., G_m$;
for i = 1 *to* m **do**
    π_i = ABFS(G_i, E);
    Collect new reading r_i;
end
Reconstruct the scalar field from $r_0 \cup r_1 \cup ... \cup r_m$;

In our approach, we first construct a graph from the sensing field and the energy consumption model. Each vertex of the graph represents one state of the mobile robot and the coordinates of any vertex are within the sensing field. The edges between vertices represent a state transition. The length of an edge represents the energy consumed for state transition. An important assumption is that all the mobile robots share the same energy consumption model and are equipped with the same sensors, i.e., the team of the robots is homogeneous. Otherwise, each mobile robot would need its own state transition graph and the graph partition method below can not be used. For each vertex of the graph, the Hessian matrix is estimated by using Local Polynomial Regression using the readings from the static sensors and the gain is computed by using equation 6. Note that the vertices with the same coordinates share the same gain. If the robot visits one location twice but in different orientation, it will only collect the gain once.

B. Graph Partition

Once the graph is constructed and the gain associated with each state is computed, we divide the graph into subgraphs to simplify the problem. The graph is partitioned in such a way that the sum of the gains of the vertices in each sub-graph are the same. The basic idea here is that with the same amount of energy consumed, the same amount of gain is achieved. Another constraint on the partition is that the boundary between two sub-graphs should be as straight as possible. Generally speaking, a complex boundary would result in more energy consumption. The complexity of the boundary can be measured by using the length of the boundary and the length of the boundary in turn can be measured by using the number of cuts of between sub-graphs. Therefore, the partition problem is to find a partition of m sub-graphs so that the total of gain of all the nodes in each sub-graphs is the same, where m is the number of mobile robots.

Graph partition is a well known NP-complete problem and there is no polynomial time algorithm to find the optimal partition. However, many approximation algorithms have been proposed. The approach used in this paper exploits a multilevel paradigm [13]. This approach consists of two stages. In the first stage, the graph is contracted until the size is less than a given threshold. Then, an interactive process of expansion and refinement is performed in such a way that the weight is balanced. This algorithm runs quickly with reasonable graph sizes.

C. Path Planning for a Single Robot

Once the graph is partitioned, the problem is reduced to a set of smaller problems each with a single mobile robot and many search algorithms can be used. The problem of finding the best path such that the gain collected along the path is maximized is called the Orienteering problem. This problem has been well-studied and many approximation algorithms are proposed. Most of the algorithms employ a prime and dual scheme and the typical one is proposed in [14], [15]. In [1], A Bread First Search algorithm was proposed to find a approximate solution. Although the approximation factor is not good, the algorithm runs quickly and the performance is good. The approximate BFS is used here as the single mobile path planning algorithm. However, any other path planning algorithm can well be used here.

In summary, the algorithm is described in Algorithm 1, where m is the number of mobile robots, E is the initial energy for the mobile robots, G is the state graph, G_i is a subgraph and π_i is the path generated in subgraph G_i.

V. SIMULATIONS

We carried out a series of simulations in a unit square. Three scalar fields are used in the simulations. They are shown in Figure 2 and their equation are below.

$$r = \frac{1}{1 + exp(\frac{1}{2}x^2 - y + \frac{1}{5})} \tag{8}$$

$$r = exp(-\frac{1}{2}(4x^2 - 10y + 3)) \tag{9}$$

$$r = exp(-\frac{(5x-1)^2 + (5y-1)^2}{2}) \tag{10}$$

In the first field, there is a boundary across the sensing field and it separates the high and low values. The second scalar field has a ridge across the sensing field. In our simulation, we smooth the readings by applying a local linear regression on the data to reduce the noise associated with it. As a result, in the place where the scalar field is not symmetric, the place with the maximum trace of the Hessian matrix would be moved. The biggest difference between field 1 and the other two is that in field 1, the estimation of the Hessian matrix would not only have errors in magnitude but also in location. Therefore, we expect higher estimation errors in reconstructing field 1.

When sensor readings are taken, either from the static sensor nodes or the mobile robots, we assume a Gaussian noise associated with the readings. Note that all the scalar fields vary from 0 to 1 and we use the ratio of the noise to the variation of the scalar field to describe the noise level. In the simulations, we use a coffeehouse design [16] to determine the locations of the static sensor nodes. The simulations are performed in groups. The sensing field is discretized into a graph consisting of 100 vertices uniformly distributed in the unit square. For each set of initial sensor readings, we estimate the Hessian matrix for each vertex in the graph and then compute the corresponding information gain. Then the graph is partitioned into subgraphs with equal gain. After that, the path planning procedure is called 25 times for each initial energy level and the new readings are collected. The scalar field is estimated and the IMSE is computed for 25 times. The whole process is in turn carried out for 10 times.

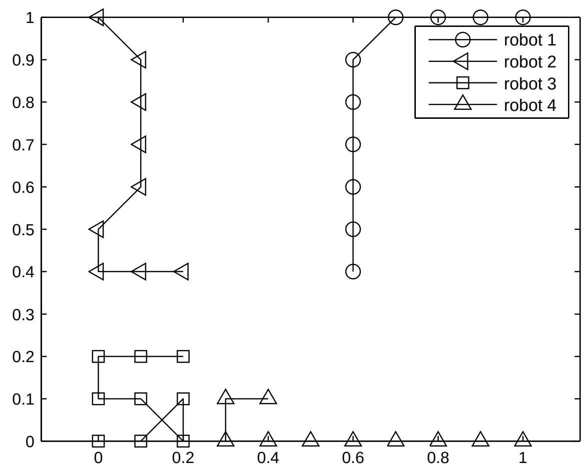
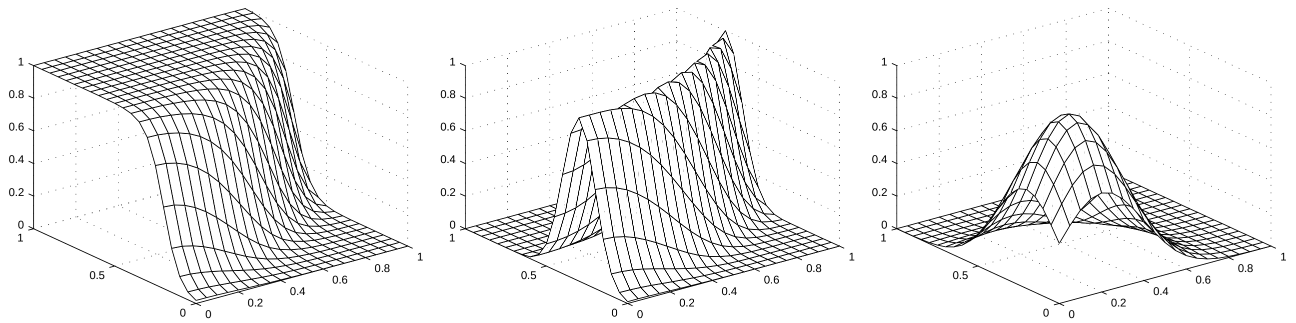


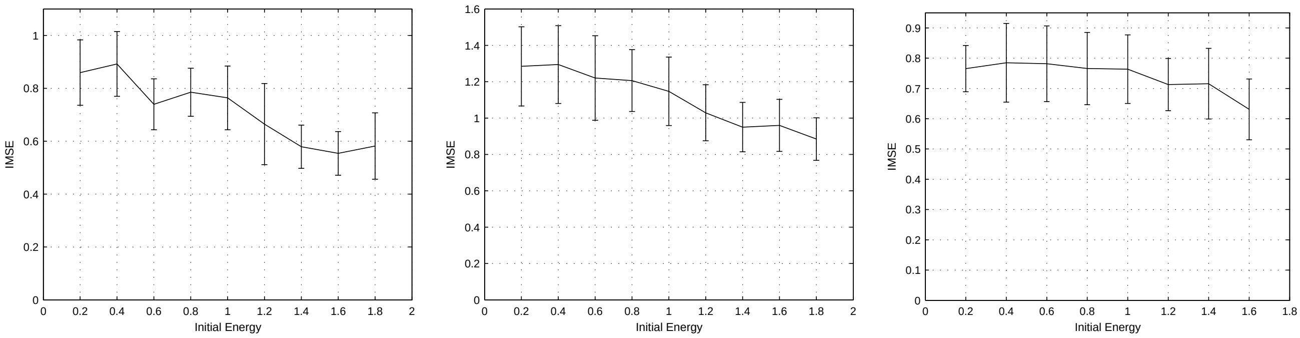
Fig. 3. The paths planned for all 4 robots. The underlying phenomenon is scalar field 3. Note the cluster of samples in lower left where the most variation in the field happens.

One example of the paths planned for all the robots is shown in Figure 3. The underlying phenomenon is defined by equation 10. The paths are generated in such a way that more readings are taken in the lower left part of the sensing field, where the trace of the Hessian matrix is much higher than other places. Figure 4 shows how the IMSE changes with increased initial energy available to the mobile robots in three different settings. In all simulations, the IMSE decreases when the initial energy available to the mobile robots increases. We study the effect of two parameters, the number of static sensors and the noise level. The simulation results show that both of them have an effect on the performances. The simulation with the best performance is shown in Figure 4(a), where 50 static sensors are used and the noise level, 5%, is low. When the initial energy is 1.6 units, which means approximately 36 new readings are taken, the IMSE



(a) (b) (c)

Fig. 2. Scalar fields used in the simulation.



(a) $\sigma = 0.05$, $m = 50$; (b) $\sigma = 0.1$, $m = 50$; (c) $\sigma = 0.05$, $m = 25$;

Fig. 4. The IMSE of the reconstruction of scalar field 1 with different noise levels (σ) and different number of static sensor nodes (m).

decreases to 64% of the initial IMSE. When the noise level of the readings increases, not only does the absolute value of the IMSE increases, the rate at which the IMSE decreases also decreased. As shown in Figure 4(b), with the same initial energy of 1.6, the IMSE is still approximately 75% of the initial IMSE. Fewer initial static sensors also reduce the rate of IMSE decrease, as shown in Figure 4(c), where there only 25 static sensors to provide initial readings. The reason for this is that both parameters affect the accuracy of the estimation of the Hessian matrix. When the number of initial readings is small or the sensor noise is high, the error in the estimation of the Hessian matrix is high, which in turn causes more readings to be taken at the non-critical places.

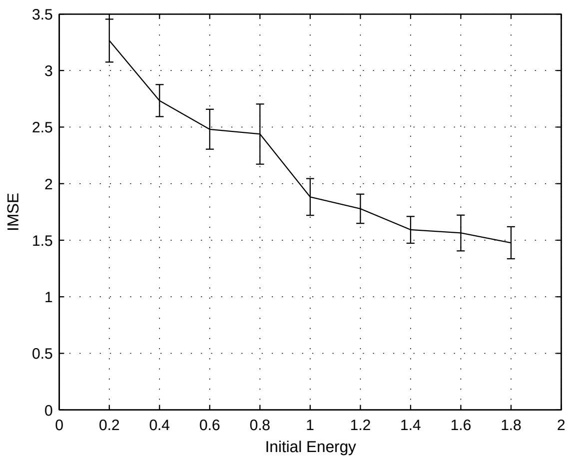
Figure 5 shows the results of simulations on the other two scalar fields. Both sets of simulations are performed with the same noise level and the same number of static sensors as in Figure 4(a). As we discussed above, both of the scalar fields are symmetric and hence estimation of the Hessian matrix only has error in magnitude with much less error on the location. Therefore, IMSE in both situations has a high decrease rate, which is obvious in Figure 5.

VI. CONCLUSION AND FUTURE WORK

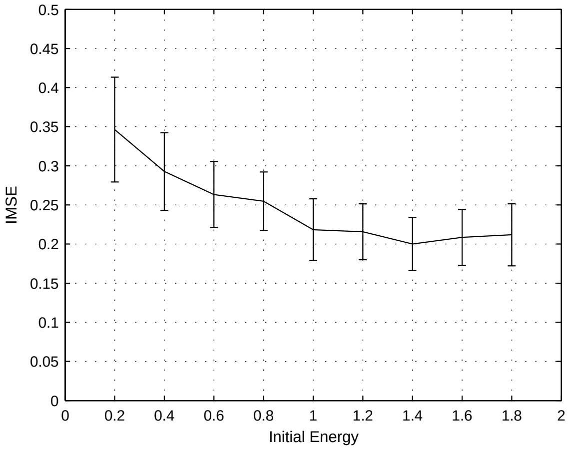
In this paper we presents a simple strategy to coordinate multiple mobile robots to take sensor readings so that errors associated with the reconstruction of a scalar field, would be reduced. Linear local regression is used to estimate the scalar field and an optimal experimental design is used to define the information gain of each location. The sensing field is partitioned into subareas with equal gain.Within each subarea path planning for a single mobile robot is used to generate the path for each individual robot. This properties of the strategy are studied using simulation.

However, equal gain is not the only strategy to partition the sensing field. For example, another even simpler strategy, partitioning the sensing field into subareas with equal area might be a good option. We performed preliminary simulations on the latter strategy and the results show that it might be competitive with the equal gain strategy. We are currently working on the detailed analysis on the second strategy. Figure 6 shows the preliminary simulation results from the equal area strategy. Figure 6(a) shows the paths planned for the four robots in one simulation. Compared with Figure 3, fewer readings are to be taken in the lower left part of the sensing field. However, equal area strategy is still able to achieve a estimation error that is very close to the equal gain strategy, as shown in Figure 6(b).

We plan to apply this approach to the robotic boats in the NAMOS project, as shown in Figure 1. Currently, there are two robotic boats with the same configuration and they are used for measuring physical, chemical and biological parameters on the water surface as well as in depth. Our plan is to test our strategy on the robotic boats in a lake or harbor where there are reasonable variations in the scalar field, such as temperature, on the surface.

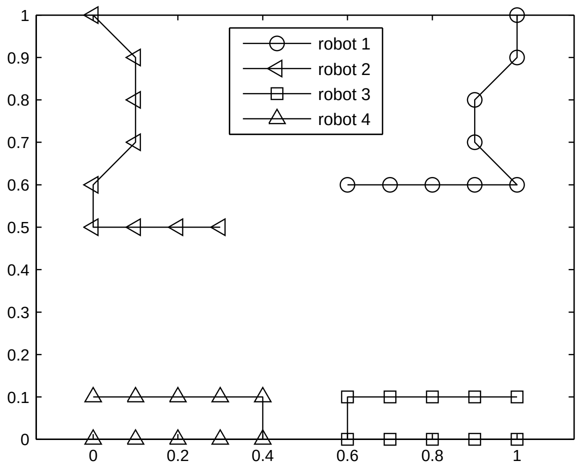


(a)

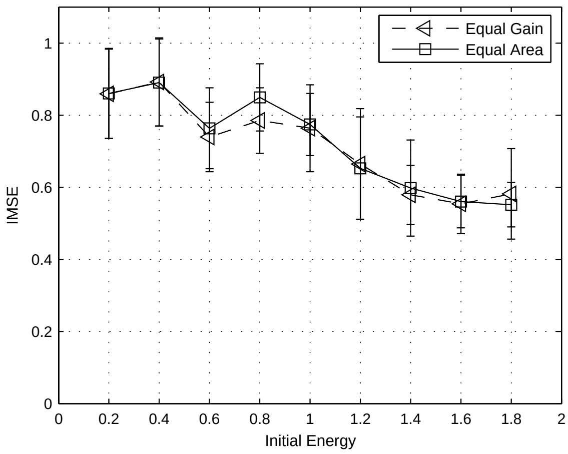


(b)

Fig. 5. The change in IMSE of the reconstruction of scalar field 2(a) and 3(b) with the availability of increased initial energy to the robots.



(a)



(b)

Fig. 6. The preliminary simulation results of the equal area strategy.

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