ADAPTIVE SOFT-SWITCHING FILTER FOR IMPULSIVE NOISE SUPPRESSION IN COLOR IMAGES

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ABSTRACT

In this paper a new class of filters designed for the removal of impulsive noise in color images is presented. The proposed filter class is based on the nonparametric estimation of the density probability function of pixels in a sliding filtering window. The comparison of the new filtering method with the standard techniques used for impulsive noise removal, indicates good noise removal capabilities and excellent structure preserving properties.

1. INTRODUCTION

During image *formation*, *acquisition*, *storage* and *transmission* many types of distorsions limit the quality of digital images. Transmission errors, periodic or random motion of the camera system during exposure, electronic instability of the image signal, electromagnetic interferences from natural or man-made sources, sensor malfunctions, optic imperfections, electronics interference or aging of the storage material all disturb the image quality, [8].

Table 1: Kernel functions used for the soft-switching scheme, $x = \langle -1, 1 \rangle$, $h = \langle 0, \infty)$.

$$[f(x)]^+ = \begin{cases} f(x), & \text{if} \left|\frac{x}{h}\right| < 1 \\ 0, & \text{otherwise,} \end{cases} , \mathscr{K}(0) = 1 , \int_{-\infty}^{+\infty} \mathscr{K}(x) dx = 1$$

Kernel	$K(x)$	$\mathscr{K}(x) = \gamma_h K(x)$	γ_1
(L)	$e^{-\left|\frac{x}{h}\right|}$	$\frac{1}{2h} e^{-\left|\frac{x}{h}\right|}$	$\frac{1}{2}$
(G)	$e^{-\frac{x^2}{2h^2}}$	$\frac{1}{\sqrt{2\pi}h} e^{-\frac{x^2}{2h^2}}$	$\frac{1}{\sqrt{2\pi}}$
(C)	$\frac{1}{1+\frac{x^2}{h^2}}$	$\frac{1}{\pi h} \frac{1}{1+\frac{x^2}{h^2}}$	$\frac{1}{\pi}$
(T)	$\left[1 - \left|\frac{x}{h}\right|\right]^+$	$\left[\frac{h(1-\left|\frac{x}{h}\right|)}{2h-1}\right]^+$	1
(E)	$\left[1 - \frac{x^2}{h^2}\right]^+$	$\left[\frac{3h^2\left(1-\frac{x^2}{h^2}\right)}{6h^2-2}\right]^+$	$\frac{3}{4}$
(B)	$\left[\left(1 - \frac{x^2}{h^2}\right)^2\right]^+$	$\left[\frac{15h^4\left(1-\frac{x^2}{h^2}\right)^2}{30h^4-20h^2+6}\right]^+$	$\frac{15}{16}$
(W)	$\left[\left(1 - \frac{x^2}{h^2}\right)^3\right]^+$	$\left[\frac{35h^6\left(1-\frac{x^2}{h^2}\right)^3}{70h^6-70h^4+42h-10}\right]^+$	$\frac{35}{32}$

In many practical situations, images are corrupted by the so called *impulsive noise* caused mainly either by faulty image sensors or due to transmission errors resulting from man-made phenomena such as ignition transients in the vicinity of the receivers or even natural phenomena such as lightning in the atmosphere.

In this paper we address the problem of impulsive noise removal in color images and propose an efficient technique capable of removing the impulsive noise noise and preserving important image features.

2. VECTOR MEDIAN BASED FILTERS

Mathematically, a $N_1 \times N_2$ multichannel image is a mapping $\mathbb{Z}^l \rightarrow \mathbb{Z}^m$ representing a two-dimensional matrix of three-component samples (pixels), $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{im}) \in \mathbb{Z}^l$, where l is the image domain dimension and m denotes the number of channels, (in the case of standard color images, parameters l and m are equal to 2 and 3, respectively). Components x_{ik}, for $k = 1, 2, \ldots, m$ and $i = 1, 2, \ldots, Q$, $Q = N_1 \cdot N_2$, represent the color channel values quantified into the integer domain.

The majority of the nonlinear, multichannel filters are based on the ordering of vectors in a sliding filter window. The output of these filters is defined as the lowest ranked vector according to a specific vector ordering technique, [1, 9].

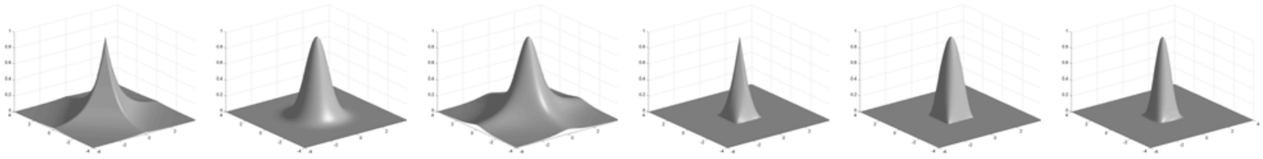
Let the color images be represented in the commonly used RGB color space and let $\mathbf{x}_1$, $\mathbf{x}_2$, …, $\mathbf{x}_N$ be N samples from the sliding filter window W, with $\mathbf{x}_1$ being the central pixel in W. Each of the $\mathbf{x}_i$ is an m-dimensional multichannel vector, (in our case $m = 3$). The goal of the vector ordering is to arrange the set of N vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N\}$ belonging to W using some sorting criterion.

In [9, 13] the ordering based on the cumulative distance function $R(\mathbf{x}_i)$ has been proposed: $R(\mathbf{x}_i) = \sum_{j=1}^{N} \rho(\mathbf{x}_i, \mathbf{x}_j)$, where $\rho(\mathbf{x}_i, \mathbf{x}_j)$ is a function of the distance among $\mathbf{x}_i$ and $\mathbf{x}_j$. The increasing ordering of the scalar quantities $\{R_1, R_2, \ldots, R_N\}$ generates the ordered set of vectors $\{\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \ldots, \mathbf{x}_{(N)}\}$.

One of the most important noise reduction filter is the vector median, [1]. Given a set W of N vectors, the vector median of the set is defined as $\mathbf{x}_{(1)} \in W$ satisfying

$$\sum_j \left\|\mathbf{x}_{(1)} - x_j\right\| \leq \sum_j \left\|\mathbf{x}_i - \mathbf{x}_j\right\|, \forall\ \mathbf{x}_i, \mathbf{x}_j \in W. \quad (1)$$

The orientation difference between two vectors can also be used as their dissimilarity measure. This so-called vector angle criterion is used by the *Basic Directional Filter* (BDF), to remove vectors with atypical directions, [14]. Other techniques combine the distance and angular criteria to achieve better noise suppression results, *Directional Distance Filter* DDF, [10, 14].



(L) (G) (C) (T) (E) (B)

Figure 1: 2D plots of the kernel functions: L-laplacian, G-Gaussian, C-Cauchy, T-Triangle, E-Epanechnikov, B-Biweight function, see also Tab. 1.

3. PROPOSED FILTERING DESIGN

The well known local statistic filters constitute a class of linear minimum mean squared error estimators, based on the non-stationarity of the signal and the noise model, [3, 4, 6]. These filters make use of the local mean and the variance of the input set W and define the filter output for the gray-scale images as

$$y = \hat{x} + \alpha\,(x_1 - \hat{x}) = \alpha x_1 + (1-\alpha)\hat{x}, \qquad (2)$$

where $\hat{x}$ is the arithmetic mean of the image pixels belonging to the filter window W cantered at i and α is a filter parameter usually estimated through, [5, 8, 12]

$$\alpha = \frac{\sigma_x^2}{\sigma_n^2 + \sigma_x^2}\,,\ \hat{x} = \frac{1}{N}\sum_{k=1}^{N} x_k\,,\ \nu^2 = \frac{1}{N}\sum_{k=1}^{N}(x_k - \hat{x})^2\,,$$

$$\sigma_x^2 = \max\left\{0, \nu^2 - \sigma_n^2\right\},\ \alpha = \max\left\{0, 1 - \sigma_n^2/\nu^2\right\}, \qquad (3)$$

where ν^2 is the local variance calculated from the samples in the filter window and σ_n^2 is the estimate of the variance of the noise process. If $\nu \gg \sigma_n$, then $\alpha \approx 1$ and practically no changes are introduced. When $\nu < \sigma_n$, then $\alpha = 0$ and the central pixel is replaced with the local mean. In this way, the filter smooths with the local mean, when the noise is not very intensive and leaves the pixel value unchanged when a strong signal activity is detected. The major drawback of this filter is that it **fails to remove impulses** and leaves noise in the vicinity of high gradient image features. Equation (2) can be rewritten as, [12]

$$y = \alpha x_1 + (1-\alpha)\hat{x} = (1-\alpha)\,(\psi_1 x_1 + x_2 + \ldots + x_N)/N, \qquad (4)$$

with $\psi_1 = (1-\alpha+N\alpha)/(1-\alpha)$, and in this way the local statistic filter (2) is reduced to the *central weighted average*, with an adaptive weighting coefficient ψ_1.

The structure of the **proposed filter** called *Kernel based VMF* (KVMF) is similar to the presented above approach. However, as our aim is to construct a filter capable of removing impulsive noise, instead of the mean value, the VMF output is utilized and the noise intensity estimation mechanism is accomplished through the similarity function, which can be viewed as kernel function, known from the nonparametric probability density estimation, (Tab. 1, Fig. 1).

In this way the proposed technique is a compromise between the VMF and the identity operation. When an impulse is present, then it is detected by the kernel $\mathscr{K} = f(\|\mathbf{x}_1 - \mathbf{x}_{(1)}\|)$, which is a function f of the distance between the central pixel $\mathbf{x}_1$ and the vector median $\mathbf{x}_{(1)}$, and the output $\mathbf{y}_i$ is close to the VMF. If the central pixel is not disturbed by the noise process then the kernel function is close to 1 and the output is near to the original value $\mathbf{x}_1$ of the central pixel.

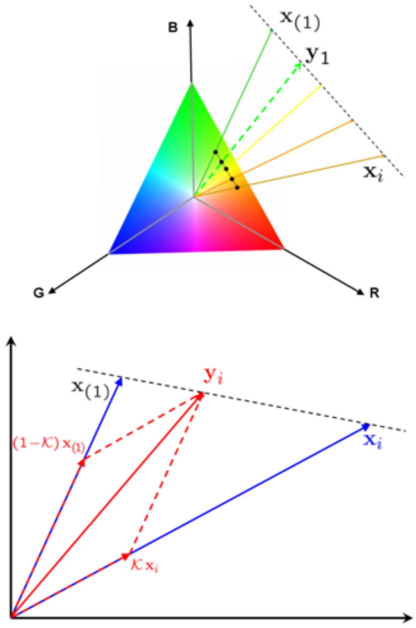


Figure 2: The output vector $\mathbf{y}_i$ lies on the line connecting the vector $\mathbf{x}_i$ and $\mathbf{x}_{(1)}$ in the RGB space.

If the central pixel in W $\mathbf{x}_i$ is denoted as $\mathbf{x}_1$ then:

$$\boxed{\mathbf{y}_i = \mathbf{x}_{(1)} + \mathscr{K}\left(\mathbf{x}_1, \mathbf{x}_{(1)}\right)\cdot\left(\mathbf{x}_1 - \mathbf{x}_{(1)}\right),} \qquad (5)$$

$$\mathbf{y}_i = \mathscr{K}\mathbf{x}_1 + (1-\mathscr{K})\mathbf{x}_{(1)}\,,\ \mathscr{K} = f\left(\|\mathbf{x}_1 - \mathbf{x}_{(1)}\|\right), \qquad (6)$$

which is similar to (2) and the filtering operation can be summarized as

$$\begin{cases} \mathscr{K} = 1 \ \Rightarrow \quad \mathbf{y}_i = \mathbf{x}_1\,, \\ \mathscr{K} = 0 \ \Rightarrow \quad \mathbf{y}_i = \mathbf{x}_{(1)}\,. \end{cases} \qquad (7)$$

If $\left\{\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \ldots, \mathbf{x}_i, \ldots, \mathbf{x}_{(n)}\right\}$ denotes the ordered set of pixels in W, then the weighted structure corresponding to (4) is: $\left\{(1-\mathscr{K})\mathbf{x}_{(1)}, \mathbf{x}_{(2)}, \ldots, \mathscr{K}\mathbf{x}_1, \ldots, \mathbf{x}_{(N)}\right\}$.

In this way the proposed structure can be seen as a modification of the known techniques applied for the suppression of Gaussian noise. In the proposed technique we replace the mean of the pixels in W with the vector median and such an approach proves to be capable of removing strong impulsive noise while preserving important image features like edges, corners and texture.

It is interesting to observe that the filter output $\mathbf{y}_i$ lies on the line joining the vectors $\mathbf{x}_i$ $(\mathbf{x}_1)$ and $\mathbf{x}_{(1)}$ and depending on the value of the kernel $\mathscr{K}$ it slides from the identity operation and the vector median, (Fig. 2).

4. EXPERIMENTAL RESULTS

The noise modelling and evaluation of the efficiency of noise attenuation methods using the widely used test images allows the objective comparison of the noisy, restored and original images.

In this paper we assume a noise model, [9–11] which reflects well the signal corruption and allows to simulate the correlation among noisy image channels. The sample distortion is given by

$$\mathbf{x}_i = \begin{cases} \mathbf{o}_i, & \text{with probability } 1-p, \\ \{v_i, o_{i_2}, o_{i_3}\}, & \text{with probability } p_1 p, \\ \{o_{i_1}, v_i, o_{i_3}\}, & \text{with probability } p_2 p, \\ \{o_{i_1}, o_{i_2}, v_i\}, & \text{with probability } p_3 p, \\ \{v_i, v_i, v_i\}, & \text{with probability } p_4 p, \end{cases} \quad (8)$$

where o is the original signal, p is the sample corruption probability and p_1, p_2, p_3 are corruption probabilities of each color channel, so that $\sum_1^4 p_k = 1$. The impulses v_i are random-valued variables in the range $[0, 255]$ and $p_k = 0.25$.

The efficiency of the proposed filtering approach is summarized in Tab. 2 and also presented graphically in Fig. 3. As can be seen the dependence on the kind of the kernel function is not, as expected, very strong. However, the main problem is to find an optimal bandwidth parameter h, as the proper setting of the bandwidth guarantees good performance of the proposed filtering design.

The experimentally found *rule of thumb* for the value of h called h_{est} is: $h_{est} = \gamma_1/\sqrt{\hat{\sigma}}$, where $\hat{\sigma}$ is the mean value of the approximation of variance, [2, 7] calculated using the whole image or some randomly selected parts, and γ_1 is the coefficient taken from Tab. 1. The comparison of the efficiency of the proposed scheme in terms of PSNR for the optimal values of h and estimated by the *rule of thumb* is shown in Tab. 2 and in Fig. 4. Practically the h_{est} yields the best possible impulsive noise attenuation.

The illustrative examples depicted in Fig. 5 show that the proposed filter efficiently removes the impulses and preserves edges and small image details. Additionally, due to its smoothing nature it is also able to suppress slightly the Gaussian noise present in natural images.

Filtering efficiency in terms of PSNR, [dB] (Lena)						
Noise	$p = 1\%$		$p = 3\%$		$p = 5\%$	
Kernel	h_{opt}	h_{est}	h_{opt}	h_{est}	h_{opt}	h_{est}
L	40.75	40.70	37.92	37.90	36.38	36.35
G	39.22	39.22	36.96	36.95	35.68	35.67
C	39.65	39.39	37.11	37.03	35.72	35.67
T	40.46	40.45	37.76	37.76	36.27	36.27
E	40.87	40.81	37.96	37.94	36.39	36.34
B	40.87	40.82	37.98	37.96	36.41	36.37
W	40.85	40.80	37.98	37.96	36.41	36.38
VMF	33.33		32.94		32.58	
DDF	32.90		32.72		32.25	
BDF	32.04		31.81		31.14	

Table 2: Efficiency of the proposed filtering design in comparison with VMF, DDF and BDF.

5. CONCLUSION

In the paper an adaptive soft-switching scheme based on the vector median and similarity function has been presented. The proposed filtering structure is superior to the standard filtering schemes and can be applied for the removal of impulsive noise in natural images. It is relatively fast and the proposed bandwidth estimator enables automatic filtering independent of noise intensity.

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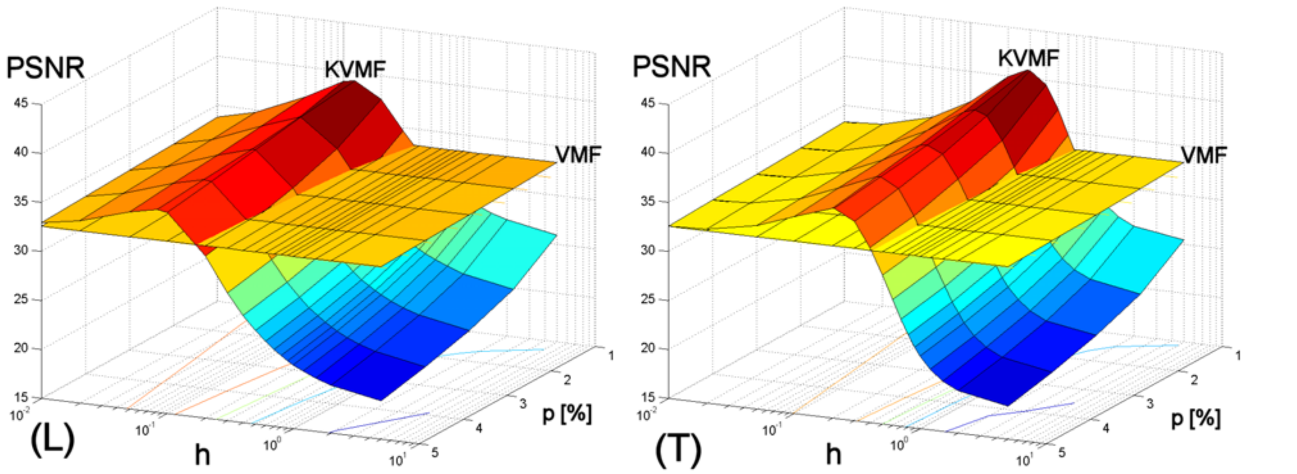

Figure 3: Dependence of the PSNR on the h parameter for the KVMF with the L and T kernels in comparison with the VMV for p ranging from 1% to 5%.

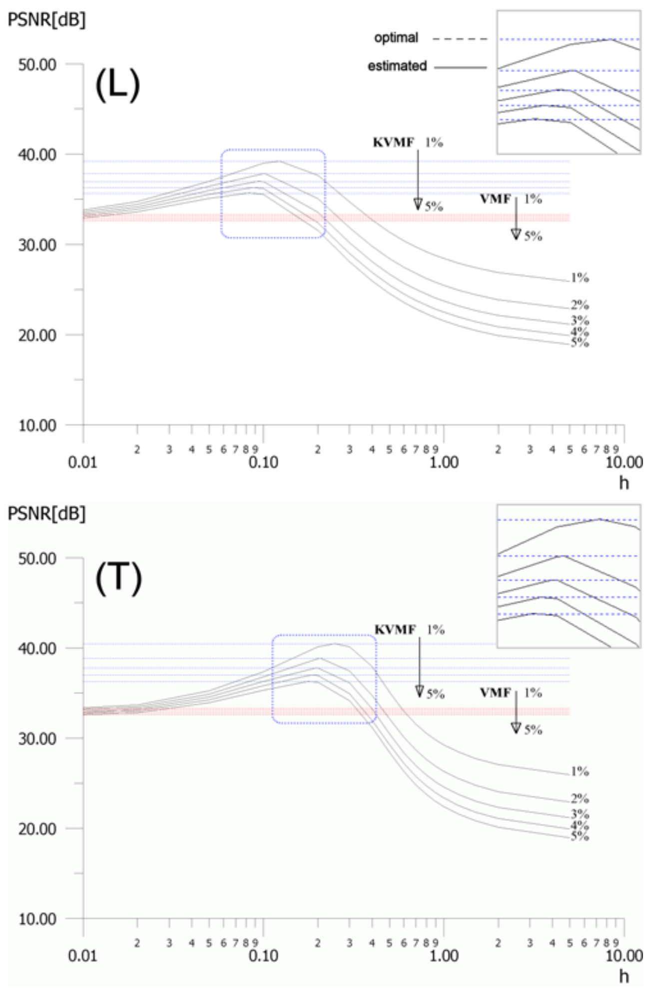

Figure 4: Dependence of the PSNR on the h parameter of the L and T kernel, for $p = 1 - 5\%$ in comparison with the standard VMF. The dashed lines indicate the optimal value of the KVMF filter (blue) and VMF (red). In the corner the magnified parts of the plots, which show the excellent performance of the proposed bandwidth estimator are presented.

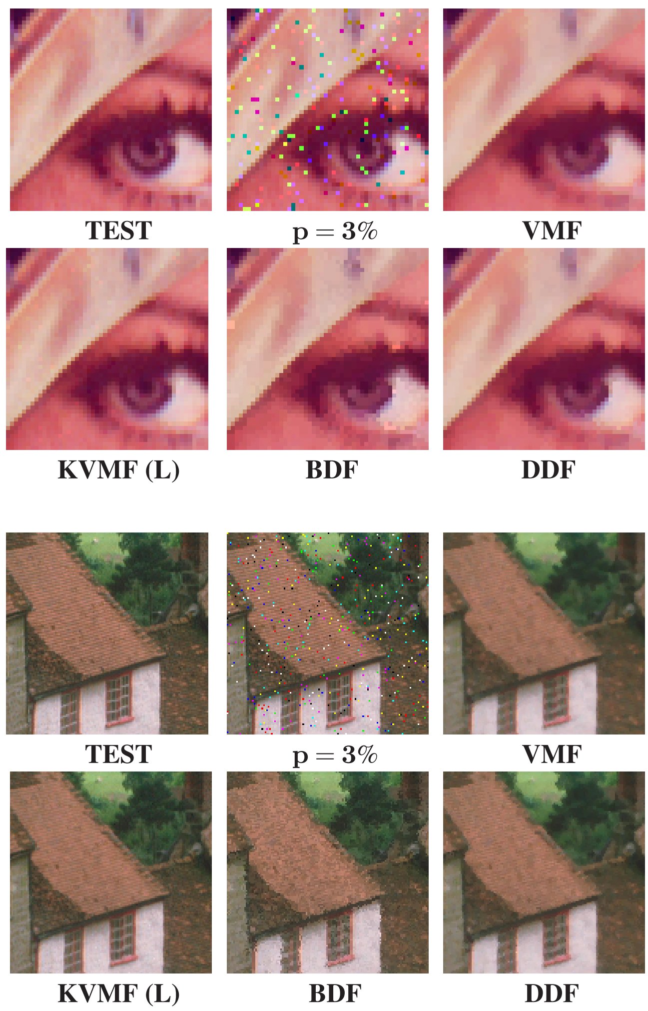

Figure 5: Comparison of the filtering efficiency of the proposed filter with the Laplace kernel with the VMF, BDF and DDF.