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Research Article

Adaptive State-Feedback Stabilization for Stochastic Nonholonomic Mobile Robots with Unknown Parameters

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The stabilizing problem of stochastic nonholonomic mobile robots with uncertain parameters is addressed in this paper. The nonholonomic mobile robots with kinematic unknown parameters are extended to the stochastic case. Based on backstepping technique, adaptive state-feedback stabilizing controllers are designed for nonholonomic mobile robots with kinematic unknown parameters whose linear velocity and angular velocity are subject to some stochastic disturbances simultaneously. A switching control strategy for the original system is presented. The proposed controllers that guarantee the states of closed-loop system are asymptotically stabilized at the zero equilibrium point in probability.

1. Introduction

In the past decades, the control of nonholonomic systems has been widely pursued. By the results of Brockett [1], the nonholonomic system cannot be stabilized at a single equilibrium point by any static smooth pure state-feedback controller. To solve this problem, lots of novel approaches have been considered: discontinuous feedback control [2–4], smooth time-varying feedback controller [5], and the method of LMI [6]. The control of nonholonomic mobile robots plays an important role in that of nonholonomic systems because they are a benchmark for these systems. There is much attention devoted to the control of nonholonomic mobile robots. The nonholonomic mobile robots were classified into four types, which were characterized by generic structures of the model equations [7]. Based on the backstepping technique, the control for nonholonomic mobile robots was discussed: tracking problems [8] and stabilizing problems [9, 10]. Hespanha et al. introduced the mobile robot with parametric uncertainties [11], which were further discussed [12, 13]. But all the above articles discussed the nonholonomic systems in the deterministic case, which was not considered a stochastic disturbance.

In recent years, stochastic nonlinear systems have received much attention [14, 15], especially for stochastic control when backstepping designs were firstly introduced [16, 17]. For stochastic nonholonomic systems, there were a few papers. The almost global adaptive asymptotical controllers of stochastic nonholonomic chained form systems were discussed by using discontinuous control [18]. The adaptive stabilization problem of stochastic nonholonomic systems with nonlinear drifts was considered [19–21]. By using state-scaling method, backstepping controllers were proposed to deal with exponential stabilization for nonholonomic mobile robots with stochastic disturbance [22, 23]. But the above two papers did not consider unknown parameters. To our knowledge, the problem of adaptive state-feedback stabilization for nonholonomic mobile robots with kinematic unknown parameters, whose linear velocity and angular velocity are subject to some stochastic disturbances simultaneously, has not been reported. So, there exists a natural problem which is how to extend the models in [11–13] to the stochastic case and design an adaptive state-feedback stabilizing controller for stochastic nonholonomic mobile robots with uncertain parameters.

The purpose of this paper is to design adaptive state-feedback stabilizing controllers for stochastic nonholonomic mobile robots with unknown parameters. The main idea of this paper is highlighted as follows.

(i) We extend the models of nonholonomic mobile robots with unknown parameters in [11–13] to the stochastic case. The stabilizing controllers are designed for stochastic nonholonomic mobile robots with unknown parameters by adaptive state-feedback backstepping technique.

(ii) A switching control strategy for the original system is presented. It guarantees that the states of closed-loop system are asymptotically stabilized at the zero equilibrium point in probability.

The paper is organized as follows. Section 1 begins with the mathematical preliminaries. In Section 2, the adaptive state-feedback backstepping controller is designed. In Section 3, a switching control strategy for the original system is discussed. Finally, a simulation example is given to show the effectiveness of the controller in Section 4.

2. Preliminaries and Problem Formulation

2.1. Preliminaries. Consider the following stochastic nonlinear system:

$$\mathrm{d}x = f(x)\,\mathrm{d}t + g(x)\,\mathrm{d}B, \quad x(t_0) \in \mathbb{R}^n, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, the Borel measurable functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x, and $B \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, P)$.

The following definitions and lemmas will be used in the paper.

Definition 1 (see [16]). For any given $V(x) \in \mathscr{C}^2$, associated with stochastic system (1), the differential operator $\mathscr{L}$ is defined as follows:

$$\mathscr{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2}\operatorname{Tr}\left\{g^T(x)\frac{\partial^2 V}{\partial x^2} g(x)\right\}. \tag{2}$$

Definition 2 (see [24]). The equilibrium $x = 0$ of system (1) is

(i) globally stable in probability if for $\forall \varepsilon > 0$, there exists a class $\mathscr{K}$ function $\gamma(\cdot)$ such that

$$P\{|x(t)| < \gamma(|x(t_0)|)\} \ge 1 - \varepsilon, \quad \forall t \ge 0, x(t_0) \in \mathbb{R}^n \setminus \{0\}; \tag{3}$$

(ii) globally asymptotically stable in probability if it is globally stable in probability and

$$P\left\{\lim_{t\to\infty} |x(t)| = 0\right\} = 1, \quad \forall x(t_0) \in \mathbb{R}^n. \tag{4}$$

Definition 3 (see [25]). A stochastic process $x(t)$ is said to be bounded in probability if the random variable $|x(t)|$ is bounded in probability uniformly in t; that is,

$$\lim_{R\to\infty} \sup_{t>t_0} P\{|x(t)| > R\} = 0. \tag{5}$$

Lemma 4 (see [24]). *Considering the stochastic system (1), if there exist a $\mathscr{C}^2$ function $V(x)$, class $\mathscr{K}_\infty$ functions $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$, constants $c_1 > 0$, $c_2 \ge 0$, and a nonnegative function $W(x)$ such that*

$$\begin{aligned} \alpha_1(|x|) \le V(x) &\le \alpha_2(|x|), \\ \mathscr{L}V(x) = \frac{\partial V}{\partial x} f &+ \frac{1}{2}\operatorname{Tr}\left\{g^T \frac{\partial^2 V}{\partial x^2} g\right\} \\ &\le -c_1 W(x) + c_2, \end{aligned} \tag{6}$$

then

(i) *for (1), there exists an almost surely unique solution on $[t_0, \infty)$ for each $x(t_0) \in \mathbb{R}^n$;*

(ii) *when $c_2 = 0$, $f(0) = 0$, $g(0) = 0$, and $W(x)$ is continuous, then the equilibrium $x = 0$ is globally stable in probability and*

$$P\left\{\lim_{t\to\infty} W(x(t)) = 0\right\} = 1, \quad \text{for } \forall x(t_0) \in \mathbb{R}^n. \tag{7}$$

Lemma 5 (see [26]). *Let x and y be real variables. Then, for any positive integers m, n and any real number $\varepsilon > 0$, the following inequality holds:*

$$|x|^m |y|^n \le \frac{m}{m+n}\varepsilon |x|^{m+n} + \frac{n}{m+n}\varepsilon^{-m/n}|y|^{m+n}. \tag{8}$$

2.2. Problem Formulation. Hespanha et al. introduced the mobile robot with parametric uncertainties [11], which were further discussed in [12, 13] as follows:

$$\begin{aligned} \dot{\theta} = p_1^* \omega, \qquad \dot{x}_c &= p_2^* v \cos\theta, \\ \dot{y}_c &= p_2^* v \sin\theta, \end{aligned} \tag{9}$$

where v and ω are two control inputs to denote the forward velocity and angular velocity, respectively.

Here we assume that the forward velocity v and the angular velocity ω are subject to some stochastic disturbances. Based on the similar methods in [27, Page 1-2], velocity v and the angular velocity ω with stochastic disturbances can be expressed as follows:

$$\begin{aligned} \omega(\theta) &= \omega_1(\theta) + \omega_2(\theta)\dot{B}(t), \\ v(x_c, y_c, \theta) &= v_1(x_c, y_c, \theta) + v_2(x_c, y_c, \theta)\dot{B}(t), \end{aligned} \tag{10}$$

where $\dot{B}(t)$ is the derivative of a Brownian motion $B(t)$.

Remark 6. The second equality of (10) is the same as that of Remark 2 in [19]. Moreover, (10) means that $\omega(\theta)$ can be divided into two parts, with the second parts being stochastic disturbances and the same for $v(x_c, y_c, \theta)$.

Substituting (10) into (9), the system (9) can be transformed into

$$\begin{aligned} \mathrm{d}\theta &= p_1^* \omega_1 \mathrm{d}t + p_1^* \omega_2 \mathrm{d}B, \\ \mathrm{d}x_c &= p_2^* v_1 \cos\theta \mathrm{d}t + p_2^* v_2 \cos\theta \mathrm{d}B, \\ \mathrm{d}y_c &= p_2^* v_1 \sin\theta \mathrm{d}t + p_2^* v_2 \sin\theta \mathrm{d}B, \end{aligned} \tag{11}$$

where p_1^* is unknown parameter taking values in a known interval $[p_{\min}, p_{\max}]$ with $0 < p_{\min} < p_{\max} < \infty$; p_2^* is unknown positive parameter.

For system (11), we introduce the following state and input transformation:

$$x_0 = \theta, \qquad u_0 = \omega_1, \qquad u = v_1,$$
$$x_1 = x_c \sin\theta - y_c \cos\theta, \tag{12}$$
$$x_2 = x_c \cos\theta + y_c \sin\theta,$$

and it is easy to see that

$$\mathrm{d}x_0 = p_1^* u_0 \mathrm{d}t + p_1^* \omega_2(x_0)\,\mathrm{d}B, \tag{13a}$$

$$\begin{aligned}\mathrm{d}x_1 &= p_2^* x_2 u_0 \mathrm{d}t - \frac{1}{2}(p_1^*)^2 x_1 \omega_2^2 \mathrm{d}t \\ &\quad + p_1^* p_2^* v_2 \omega_2 \mathrm{d}t + p_1^* x_2 \omega_2 \mathrm{d}B, \\ \mathrm{d}x_2 &= p_2^* u \mathrm{d}t - \left(p_1^* x_1 u_0 + \frac{1}{2}(p_1^*)^2 x_2 \omega_2^2\right)\mathrm{d}t \\ &\quad + \left(p_2^* v_2 - p_1^* x_1 \omega_2\right)\mathrm{d}B.\end{aligned} \tag{13b}$$

Remark 7. The main difference between this paper and [22] is that the unknown parameter exists in this paper. The controller design of systems (13a) and (13b) will be more difficult.

Remark 8. For system (13a) and (13b), the variable x_2 appears in the term $p_1^* x_2 \omega_2 \mathrm{d}B$ in the first equation of (13b); this is different from the traditional stochastic backstepping technique in [16, 17, 24].

3. Adaptive Controller Design

In this section, we will design state-feedback controllers such that all the signals in closed-loop system are regulated to the origin in probability. The following assumptions are needed.

Assumption 9. For the smooth function $\omega_2(\theta)$, there exists a known positive constant m_1, such that

$$\omega_2(\theta) = m_1 \theta. \tag{14}$$

Assumption 10. For smooth function $v_2(x_c, y_c, \theta)$ and any positive constant C, there exists a known nonnegative constant m_2, such that

$$\left|v_2(x_c, y_c, \theta)\right| \le m_2 \left|x_2 - C\frac{x_1}{u_0}\right|. \tag{15}$$

Remark 11. For the adaptive controllers' design in the following, if we let $C = c_1$, this assumption will change to $|v_2(x_c, y_c, \theta)| \le m_2|\varepsilon_2|$, where ε_2 is defined in (25) and c_1 is the same as that in (28) in the following Section 3.2.

Firstly, we will consider the problem of stabilization for systems (13a) and (13b) under the condition of $x_0(t_0) \ne 0$. The case of $x_0(t_0) = 0$ will be discussed in Section 3.

3.1. The First State Stabilization. Let us consider the subsystem (13a) of stochastic nonholonomic nonlinear systems (13a) and (13b):

$$\mathrm{d}x_0 = p_1^* u_0 \mathrm{d}t + p_1^* \omega_2(x_0)\,\mathrm{d}B. \tag{16}$$

In order to guarantee that x_0 converges to zero, one can take u_0 as follows:

$$u_0 = -\eta_0 x_0, \qquad \eta_0 = \lambda + \frac{3}{2} m_1^2 p_{\max}, \tag{17}$$

where $\lambda > 0$ is a design parameter.

If we employ a Lyapunov function of the form:

$$V_0(x_0) = \frac{1}{4p_1^*} x_0^4. \tag{18}$$

From (13a), (17), (18), and Assumption 9, one can obtain

$$\mathscr{L}V_0 \le x_0^3 u_0 + \frac{3}{2p_1^*} x_0^2 (p_1^*)^2 \omega_2^2 \le -\lambda x_0^4. \tag{19}$$

Theorem 12. *If Assumption 9 holds, one can choose positive constants m_1, λ, and $p_{\max}$ and the controller u_0 as (17), respectively, then*

(i) *the closed-loop subsystem composed by* (13a) *and* (17) *has an almost surely unique solution on $[t_0, \infty)$ for $\forall x_0(t_0)$;*

(ii) *the equilibrium $x_0 = 0$ of the closed-loop subsystem composed by* (13a) *and* (17) *is globally asymptotically stable in probability.*

Proof. Choosing Lyapunov function as (18), by (19), $\lambda > 0$, and Lemma 4, (i) holds and the equilibrium $x_0 = 0$ of the closed-loop subsystem which contained (13a) and (17) is globally stable in probability and for $\forall x_0(t_0) \ne 0$, $P\{\lim_{t\to\infty}(|x_0(t)| = 0)\} = 1$. From Definition 2, (ii) holds. □

Remark 13. From Theorem 12, one has the state x_0 bounded in probability; that is, there exists a positive constant m_3, such that

$$\lim_{m_3 \to \infty} \sup_{t > t_0} P\left\{\left|x_0(t)\right| > m_3\right\} = 0. \tag{20}$$

Substituting (17) into the subsystem (13a), one gets

$$\mathrm{d}x_0 = -p_1^* \eta_0 x_0 \mathrm{d}t + p_1^* m_1 x_0 \mathrm{d}B. \tag{21}$$

Proposition 14. *For initial state $x_0(t_0) \ne 0$, the solution of* (21) *will never reach the zero, which avoids the uncontrollability of the subsystem* (13b).

Proof. From Lemma 2.3 ([27, Page 93]), the following equality will be the solution of (21):

$$x_0(t) = x_0(t_0) \exp\left\{\int_{t_0}^{t} \left(-p_1^* \eta_0 - \frac{1}{2}(p_1^* m_1)^2\right)\mathrm{d}s + \int_{t_0}^{t} p_1^* m_1 \mathrm{d}\omega\right\}. \tag{22}$$

From the above expression of $x_0(t)$ and $x_0(t_0) \neq 0$, it is easy to see that $x_0(t)$ will never cross the origin at the time interval $t \in (t_0, +\infty)$. □

In the following Section 3.2, the other states will be regulated to the origin in probability by the design of the control input u.

3.2. Other States Stabilization. In order to design a smooth adaptive state-feedback controller, the following state-input scaling discontinuous transformation is needed:

$$z_1 = \frac{x_1}{u_0}, \qquad z_2 = x_2. \tag{23}$$

Remark 15. For the initial state $x_0(t_0) \neq 0$, from Proposition 14, one can obtain that transformation (23) is meaningful.

Under the new z-coordinate, the subsystem (13b) is transformed into

$$\begin{aligned} \mathrm{d}z_1 &= p_1^* \left(-\eta_0 z_2 - 0.5 p_1^* z_1 \omega_2^2 - p_1^* z_2 \frac{\omega_2^2}{x_0^2} + \eta_0 z_1 \right. \\ &\qquad \left. + p_2^* v_2 \frac{\omega_2}{x_0} + p_1^* z_1 \frac{\omega_2^2}{x_0^2} \right) \mathrm{d}t \\ &\quad + p_1^* \left(z_2 \frac{\omega_2}{x_0} - z_1 \frac{\omega_2}{x_0} \right) \mathrm{d}B, \\ \mathrm{d}z_2 &= p_2^* u \mathrm{d}t + \left(p_1^* \eta_0 z_1 x_0^2 - 0.5 (p_1^*)^2 z_2 \omega_2^2 \right) \mathrm{d}t \\ &\quad + \left(p_2^* v_2 - p_1^* z_1 x_0 \omega_2 \right) \mathrm{d}B. \end{aligned} \tag{24}$$

To invoke the backstepping method, the error variables ε_1 and ε_2 are given by

$$\varepsilon_1 = z_1, \qquad \varepsilon_2 = z_2 - \alpha_1(\varepsilon_1). \tag{25}$$

Step 1. Define the first Lyapunov candidate function:

$$V_1 = \frac{1}{4 p_1^*} \varepsilon_1^4. \tag{26}$$

By (24)–(26) and Definition 1, one has

$$\begin{aligned} \mathscr{L}V_1 \leq \varepsilon_1^3 & \left\{ -\eta_0 z_2 - \frac{1}{2} p_1^* z_1 \omega_2^2 - p_1^* z_2 \frac{\omega_2^2}{x_0^2} + \eta_0 z_1 \right. \\ & \left. + p_2^* v_2 \frac{\omega_2}{x_0} + p_1^* z_1 \frac{\omega_2^2}{x_0^2} \right\} \\ & + 3\varepsilon_1^2 \left\{ (p_1^*)^2 z_2^2 \frac{\omega_2^2}{x_0^2} + (p_1^*)^2 z_1^2 \frac{\omega_2^2}{x_0^2} \right\}. \end{aligned} \tag{27}$$

The virtual control can be chosen as

$$\alpha_1(\varepsilon_1) = c_1 \varepsilon_1, \tag{28}$$

where c_1 is a positive constant, which will be chosen later. From (27), Lemma 5, and simple operation, we have the following inequalities:

$$\begin{aligned} -\eta_0 z_2 \varepsilon_1^3 &\leq \eta_0 \left\{ \frac{3d}{4} \varepsilon_1^4 + \frac{3}{4d^3} \varepsilon_2^4 \right\} - \eta_0 c_1 \varepsilon_1^4, \\ -\frac{1}{2} \varepsilon_1^3 p_1^* z_1 \omega_2^2 &\leq \frac{1}{2} p_{\max} m_1^2 m_3^2 \varepsilon_1^4, \\ -p_1^* \varepsilon_1^3 z_2 \frac{\omega_2^2}{x_0^2} &\leq p_{\max} m_1^2 \left\{ \frac{3}{4} + c_1 \right\} \varepsilon_1^4 + \frac{1}{4} p_{\max} m_1^2 \varepsilon_2^4, \\ p_2^* v_2 \frac{\omega_2}{x_0} &\leq \frac{1}{4} \varepsilon_1^4 + \frac{3}{4} (m_1 m_2)^{4/3} (p_2^*)^{4/3} \varepsilon_2^4, \\ p_1^* z_1 \frac{\omega_2^2}{x_0^2} &\leq p_{\max} m_1^2 \varepsilon_1^4, \\ 3 (p_1^*)^2 \varepsilon_1^3 z_2^2 \frac{\omega_2^2}{x_0^2} &\leq 3 p_{\max}^2 m_1^2 \left\{ 2c_1^2 + 1 \right\} \varepsilon_1^4 + 3 p_{\max}^2 m_1^2 \varepsilon_2^4, \\ 3 (p_1^*)^2 \varepsilon_1^3 z_1^2 \frac{\omega_2^2}{x_0^2} &\leq 3 p_{\max}^2 m_1^2 \varepsilon_1^4, \end{aligned} \tag{29}$$

where $d > 0$ is a design parameter. Substituting these above inequalities into (27), it is easy to see that

$$\begin{aligned} \mathscr{L}V_1 \leq & \left\{ -\eta_0 c_1 e + \left(1 + \frac{3d}{4} \right) \eta_0 - c_1 \eta_0 (1 - e) \right. \\ & + \frac{p_{\max} m_1^2 \left(2m_3^2 + 4c_1 + 24 p_{\max} c_1^2 \right)}{4} \\ & \left. + \frac{p_{\max} m_1^2 (36 p_{\max} + 7) + 1}{4} \right\} \varepsilon_1^4 \\ & + \left\{ \frac{\eta_0}{4d^3} + \frac{p_{\max} m_1^2}{4} + 3 p_{\max}^2 m_1^2 \right\} \varepsilon_2^4 \\ & + \frac{3}{4} (m_1 m_2)^{4/3} (p_2^*)^{4/3} \varepsilon_2^4, \end{aligned} \tag{30}$$

where e is a design parameter and $0 < e < 1$. If we select parameters η_0 and c_1 to satisfy

$$\begin{aligned} c_1 &\geq \frac{4 + 3d}{4e}, \\ \eta_0 &\geq \frac{p_{\max} m_1^2 \left(2m_3^2 + 4c_1 + 24 p_{\max} c_1^2 + 36 p_{\max} + 7 \right) + 1}{2c_1 (1 - e)}, \end{aligned} \tag{31}$$

one has

$$\begin{aligned} \mathscr{L}V_1 \leq & -\frac{c_1 \eta_0 (1 - e)}{2} \varepsilon_1^4 + \frac{3}{4} (m_1 m_2)^{4/3} (p_2^*)^{4/3} \varepsilon_2^4 \\ & + \left\{ \frac{\eta_0}{4d^3} + \frac{p_{\max} m_1^2}{4} + 3 p_{\max}^2 m_1^2 \right\} \varepsilon_2^4. \end{aligned} \tag{32}$$

Step 2. By (24), (25), (32), and Itô formula (Theorem 6.2, [27, Page 32]), one gets

$$d\varepsilon_2 = p_2^* u\,dt + \Big\{ p_1^* \eta_0 z_1 x_0^2 - \frac{1}{2}(p_1^*)^2 z_2 \omega_2^2 + p_1^* c_1 \eta_0 z_2 + \frac{1}{2}(p_1^*)^2 c_1 z_1 \omega_2^2 - p_1^* c_1 \eta_0 z_1 + (p_1^*)^2 c_1 z_2 \frac{\omega_2^2}{x_0^2} - p_1^* p_2^* c_1 v_2 \frac{\omega_2}{x_0} - (p_1^*)^2 c_1 z_1 \frac{\omega_2^2}{x_0^2} \Big\} dt + \left(p_2^* v_2 - p_1^* z_1 x_0 \omega_2 - p_1^* c_1 z_2 \frac{\omega_2}{x_0} + p_1^* c_1 z_1 \frac{\omega_2}{x_0} \right) dB. \tag{33}$$

To deal with the uncertain parameter p_2^*, define parameter

$$\Theta = \max\left\{ p_2^*, (p_2^*)^{4/3}, \frac{1}{p_2^*}, \left(\frac{1}{p_2^*}\right)^{4/3}, \left(\frac{1}{p_2^*}\right)^{2} \right\}, \tag{34}$$

and $\widetilde{\Theta} = \Theta - \widehat{\Theta}$ being the parameter estimation error, $\widehat{\Theta}$ being the estimate of Θ. Define the second Lyapunov candidate function:

$$V_2 = V_1 + \frac{1}{4p_2^*}\varepsilon_2^4 + \frac{1}{2}\widetilde{\Theta}^2. \tag{35}$$

From (33), (35), and Definition 1, one can obtain

$$\begin{aligned}
\mathscr{L}V_2 \le & -\frac{c_1\eta_0(1-e)}{2}\varepsilon_1^4 + \frac{3}{4}(m_1 m_2)^{4/3}(p_2^*)^{4/3}\varepsilon_2^4 \\
& + \left\{ \frac{\eta_0}{4d^3} + \frac{p_{\max} m_1^2}{4} + 3p_{\max}^2 m_1^2 \right\}\varepsilon_2^4 \\
& + \varepsilon_2^3 \Big\{ u + \frac{p_1^*}{p_2^*}\eta_0 z_1 x_0^2 - \frac{1}{2}\frac{(p_1^*)^2}{p_2^*} z_2 \omega_2^2 \\
& + \frac{p_1^*}{p_2^*} c_1 \eta_0 z_2 + \frac{1}{2}\frac{(p_1^*)^2}{p_2^*} c_1 z_1 \omega_2^2 \\
& - \frac{p_1^*}{p_2^*} c_1 \eta_0 z_1 + \frac{(p_1^*)^2}{p_2^*} c_1 z_2 \frac{\omega_2^2}{x_0^2} \\
& - p_1^* c_1 v_2 \frac{\omega_2}{x_0} - \frac{(p_1^*)^2}{p_2^*} c_1 z_1 \frac{\omega_2^2}{x_0^2} \Big\} dt \\
& + 6\varepsilon_2^2 \Big\{ p_2^* v_2^2 + \frac{(p_1^*)^2}{p_2^*} z_1^2 x_0^2 \omega_2^2 \\
& + \frac{(p_1^*)^2}{p_2^*} c_1^2 z_2^2 \frac{\omega_2^2}{x_0^2} + \frac{(p_1^*)^2}{p_2^*} c_1^2 z_1^2 \frac{\omega_2^2}{x_0^2} \Big\} \\
& - \widetilde{\Theta}\dot{\widehat{\Theta}}.
\end{aligned} \tag{36}$$

By (34), (36), and Lemma 5, we have the following inequalities:

$$\begin{aligned}
& \frac{p_1^*}{p_2^*}\eta_0 z_1 x_0^2 \varepsilon_2^3 \le \Theta \frac{3}{4}\left(p_{\max}\eta_0 m_3^2\right)^{4/3}\varepsilon_2^4 + \frac{1}{4}\varepsilon_1^4, \\
& -\frac{1}{2}\frac{(p_1^*)^2}{p_2^*} z_2 \omega_2^2 \varepsilon_2^4 \\
& \le \Theta\left\{ \frac{1}{2} p_{\max}^2 m_3^2 m_1^2 + \frac{3}{8}\left(c_1 p_{\max}^2 m_3^2 m_1^2\right)^{4/3} \right\}\varepsilon_2^4 + \frac{1}{8}\varepsilon_1^4, \\
& \frac{p_1^*}{p_2^*} c_1 \eta_0 z_2 \varepsilon_2^3 \le \Theta\left\{ c_1 p_{\max}\eta_0 + \frac{3}{4} c_1^{8/3} p_{\max}^{4/3}\eta_0^{4/3} \right\}\varepsilon_2^4 + \frac{1}{4}\varepsilon_1^4, \\
& \frac{1}{2}\frac{(p_1^*)^2}{p_2^*} c_1 z_1 \omega_2^2 \varepsilon_2^3 \le \Theta \frac{3}{8}\left(c_1 p_{\max}^2 m_3^2 m_1^2\right)^{4/3}\varepsilon_2^4 + \frac{1}{8}\varepsilon_1^4, \\
& -\frac{p_1^*}{p_2^*} c_1 \eta_0 z_1 \varepsilon_2^3 \le \Theta \frac{3}{4}\left(c_1 \eta_0 p_{\max}\right)^{4/3}\varepsilon_2^4 + \frac{1}{4}\varepsilon_1^4, \\
& \frac{(p_1^*)^2}{p_2^*} c_1 z_2 \frac{\omega_2^2}{x_0^2}\varepsilon_2^3 \le \Theta c_1 p_{\max}^2 m_1^2 \varepsilon_2^4 + \Theta \frac{3}{4}\left(c_1^2 m_1^2 p_{\max}^2\right)^{4/3}\varepsilon_2^4 \\
& \quad + \frac{1}{4}\varepsilon_1^4, \\
& -p_1^* c_1 v_2 \frac{\omega_2}{x_0}\varepsilon_2^3 \le p_{\max} m_1 m_2 c_1 \varepsilon_2^4, \\
& -\frac{(p_1^*)^2}{p_2^*} c_1 z_1 \frac{\omega_2^2}{x_0^2}\varepsilon_2^3 \le \frac{1}{4}\varepsilon_1^4 + \Theta \frac{3}{4}\left(p_{\max}^2 c_1 m_1^2\right)^{4/3}\varepsilon_2^4, \\
& 6\varepsilon_2^2 \Big\{ p_2^* v_2^2 z_1^2 x_0^2 + \frac{(p_1^*)^2}{p_2^*}\omega_2^2 + \frac{(p_1^*)^2}{p_2^*} c_1^2 z_2^2 \frac{\omega_2^2}{x_0^2} + \frac{(p_1^*)^2}{p_2^*} c_1^2 z_1^2 \frac{\omega_2^2}{x_0^2} \Big\} \\
& \le \Theta\left\{ 6m_2^2 + 3p_{\max}^4 m_3^8 m_1^4 + 12 p_{\max}^2 m_1^2 + 6p_{\max}^4 m_1^4 c_1^4 + 3p_{\max}^4 m_1^4 c_1^4 \right\}\varepsilon_2^4 + 12\varepsilon_1^4.
\end{aligned} \tag{37}$$

Substituting the above inequalities into (36) and adding and subtracting the term $c_2\varepsilon_2^4$ on the right-hand side of (36), we have

$$\begin{aligned}
\mathscr{L}V_2 \le & -\left\{ \frac{c_1\eta_0(1-e)}{2} - 13.5 \right\}\varepsilon_1^4 - c_2\varepsilon_2^4 \\
& + H_{21}\varepsilon_2^4 + \varepsilon_2^3 u + \Theta H_{22}\varepsilon_2^4 - \widetilde{\Theta}\dot{\widehat{\Theta}} \\
\le & -\bar{c}_1\varepsilon_1^4 - c_2\varepsilon_2^4 + \varepsilon_2^3 u + \widetilde{\Theta}\left\{ H_{22}\varepsilon_2^4 - \dot{\widehat{\Theta}} \right\} \\
& + \left\{ H_{21} + \sqrt{1+\widehat{\Theta}^2} H_{22} \right\}\varepsilon_2^4,
\end{aligned} \tag{38}$$

where

$$
\begin{aligned}
H_{21} &= \frac{\eta_0}{4d^3} + \frac{p_{\max} m_1^2}{4} + 3p_{\max}^2 m_1^2 + p_{\max} m_1 m_2 c_1, \\
\bar{c}_1 &= \frac{c_1 \eta_0 (1-e)}{2} - 13.5, \\
H_{22} &= c_2 + \frac{3}{4}\left(p_{\max}\eta_0 m_3^2\right)^{4/3} + \frac{1}{2} p_{\max}^2 m_3^2 m_1^2 \\
&\quad + c_1 p_{\max}\eta_0 + \frac{3}{8}\left(c_1 p_{\max}^2 m_3^2 m_1^2\right)^{4/3} \\
&\quad + \frac{3}{4} c_1^{8/3} p_{\max}^{4/3} \eta_0^{4/3} + \frac{3}{8}\left(c_1 p_{\max}^2 m_3^2 m_1^2\right)^{4/3} \\
&\quad + \frac{3}{4}\left(c_1 \eta_0 p_{\max}\right)^{4/3} + c_1 p_{\max}^2 m_1^2 \\
&\quad + \frac{3}{4}\left(c_1^2 m_1^2 p_{\max}^2\right)^{4/3} + 6m_2^2 + 12 p_{\max}^2 m_1^2 \\
&\quad + 3p_{\max}^4 m_3^8 m_1^4 + 6p_{\max}^4 m_1^4 c_1^4 + 3p_{\max}^4 m_1^4 c_1^4 \\
&\quad + \frac{3}{4}\left(p_{\max}^3 c_1 m_1^2\right)^{4/3} + \frac{3}{4}\left(m_1 m_2\right)^{4/3}.
\end{aligned} \tag{39}
$$

One can choose the actual control law u and the adaptive laws $\widehat{\Theta}$ as follows:

$$
\begin{aligned}
u &= -\left\{ H_{21} + \sqrt{1+\widehat{\Theta}^2} H_{22} \right\} \varepsilon_2, \\
\dot{\widehat{\Theta}} &= H_{22}\varepsilon_2^4.
\end{aligned} \tag{40}
$$

Substituting (40) into (38), one gets

$$
\mathscr{L}V_2 \le -\bar{c}_1 \varepsilon_1^4 - c_2 \varepsilon_2^4. \tag{41}
$$

Choosing the Lyapunov function as

$$
V = V_0 + V_2, \tag{42}
$$

together with (19) and (41), we have

$$
\mathscr{L}V \le -\lambda x_0^2 - \bar{c}_1 \varepsilon_1^4 - c_2 \varepsilon_2^4. \tag{43}
$$

Theorem 16. *If Assumptions 9 and 10 hold, one can choose positive constants λ, m_1, m_2, m_3, and $p_{\max}$, with $d > 0$ and $0 < e < 1$ satisfying $\bar{c}_1 > 0$ and* (31)*; for positive constant c_2, one has the following.*

(i) *The closed-loop system composed by* (13a), (17), (24), *and* (40) *has an almost surely unique solution on $[t_0, \infty)$ for $\forall x_0(t_0), z(t_0)$ and $\widehat{\Theta}(t_0)$.*

(ii) *The equilibrium $(x_0, z, \widetilde{\Theta}) = (0, 0, 0)$ of the closed-loop system is globally stable in probability.*

(iii) *For initial condition $\forall x_0(t_0)$, $z(t_0)$, and $\widehat{\Theta}(t_0)$, $P\{\lim_{t\to\infty}(|x_0(t)| + |z(t)|) = 0\} = 1$, $P\{\lim_{t\to\infty}\widehat{\Theta}(t)$ exists and is finite$\} = 1$, where $\widetilde{\Theta} = \Theta - \widehat{\Theta}$ and $z(t) = (z_1(t), z_2(t))$.*

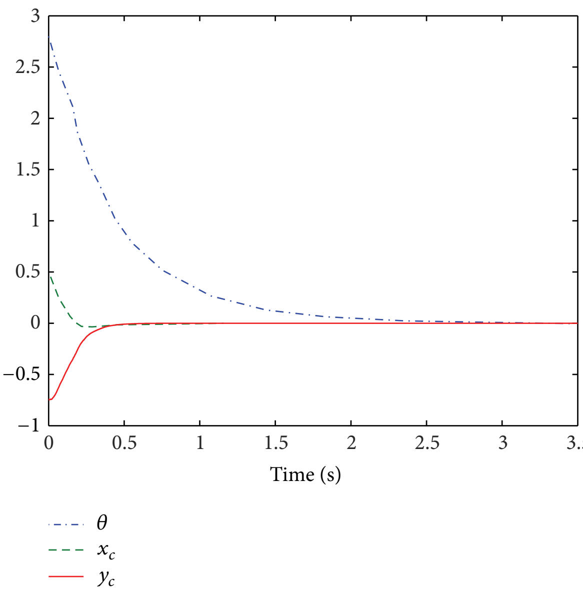

Figure 1: The responses of states θ, x_c, and y_c with respect to time.

Proof. From conditions in Theorem 16, it is easy to see that constants $\lambda > 0$, $\bar{c}_1 > 0$, and $c_2 > 0$. So, $\mathscr{L}V$ in (43) becomes the same form as (3.19) in [21]. Using (43) and Lemma 4, Theorem 16 can be proved. □

4. Switching Control Stability

In Section 2, the case of $x_0(t_0) \neq 0$ is discussed. We design controllers u_0 and u for systems (13a) and (13b) as in (17) and (40), respectively. Now we turn to the case of $x_0(t_0) = 0$. When the initial $x_0(t_0) = 0$, one can choose an open loop control $u_0 = -u_0^* \neq 0$ to drive the state x_0 away from zero in a limited time.

In fact, when we choose an open loop control $u_0 = -u_0^* \neq 0$, system (13a) will be in the following form:

$$
dx_0 = -u_0^* dt + p_1^* \omega_2(x_0)\, dB. \tag{44}
$$

For a given constant $l > 0$, define a stopping time $\tau_l = \inf\{t : t \ge t_0, |x_0(t)| \ge l\}$. With the similar analysis in Section V in [22], we have $P(\tau_l - t_0 \ge T) \le l/\mathrm{T}u_0^*$, which means that $P(\tau_l = \infty) = 0$ for any $l > 0$. Letting $t_s^* = \tau_l$, it is easy to see that

$$
\left|x_0\left(t_s^*\right)\right| = \left|x_0\left(\tau_l\right)\right| = l \neq 0. \tag{45}
$$

So, there exists $t_s^* > 0$, such that $x_0(t_s^*) \neq 0$. After that, at the time $t = t_s^*$, we switch the control inputs u_0 and u to (17) and (40) in $t \in [t_s^*, +\infty)$, respectively.

Theorem 17. *If Assumptions 9 and 10 hold, one can apply the following switching control procedure to system* (11)*:*

(i) *when the initial state belongs to*

$$
\left\{ \left(\theta\left(t_0\right), x_c\left(t_0\right), y_c\left(t_0\right)\right) \in \mathbb{R}^3 \mid \theta\left(t_0\right) \neq 0 \right\}, \tag{46}
$$

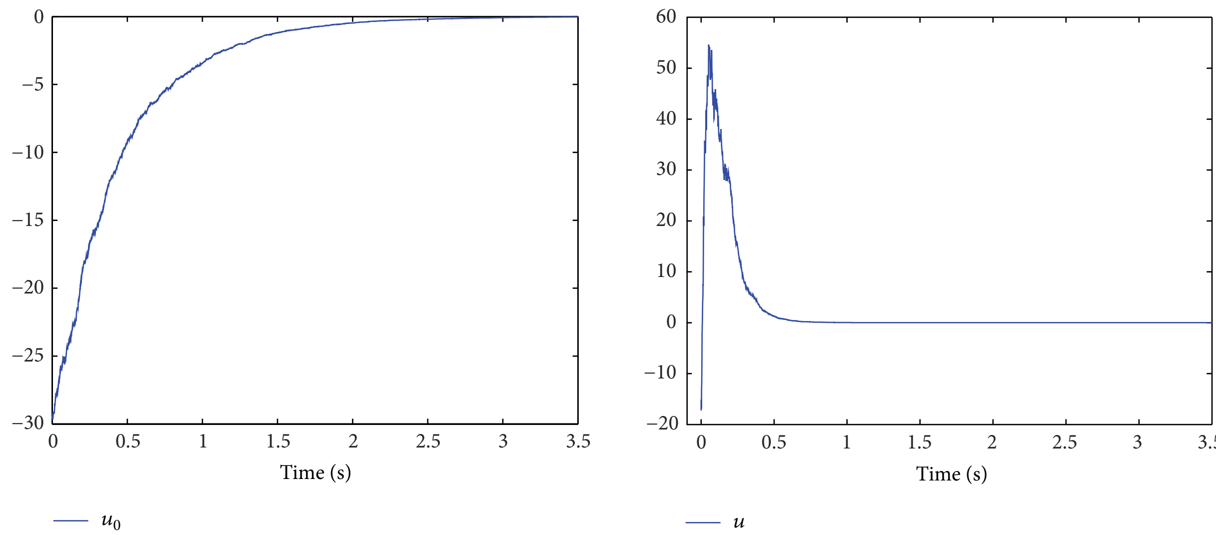

FIGURE 2: The responses of controllers u_0 and u with respect to time.

(ii) *when the initial state belongs to*

$$\left\{(\theta(t_0), x_c(t_0), y_c(t_0)) \in \mathbb{R}^3 \mid \theta(t_0) = 0\right\}. \quad (47)$$

One designs control inputs u_0 and u in form (17) *and* (40), *respectively; for $t \in [t_0, t_s^*)$, one can choose the control law $u_0 = -u_0^* \neq 0$ and $u = u^*$; for $t \in [t_s^*, +\infty)$, at the time $t = t_s^*$, one switches the control inputs u_0 and u to* (17) *and* (40), *respectively.*

Then, for any initial condition in the state space, the states of system (11) *are asymptotically regulated to zero in probability.*

Proof. Firstly, we consider the case that the initial state belongs to

$$\left\{(\theta(t_0), x_c(t_0), y_c(t_0)) \in \mathbb{R}^3 \mid \theta(t_0) \neq 0\right\}. \quad (48)$$

From Theorems 12 and 16, for the closed-loop system composed by (13a), (17), (24), and (40), states x_0 and $z(t)$ are regulated to zero in probability, $\widehat{\Theta}(t)$ is bounded in probability, and $P\{\lim_{t\to\infty}(|x_0(t)| + |z(t)|) = 0\} = 1$. This implies that states x_0 and $z(t)$ are globally asymptotically regulated to zero in probability and bounded in probability. As a result of (23), one gets that the states x_0, x_1, and x_2 of closed-loop system composed by (11), (17), and (40) asymptotically converge to zero in probability and all bounded in probability. By orthogonal transformation (12), one can obtain that the states θ, x_c, and y_c of closed-loop system composed by (11), (17), and (40) are asymptotically stabilized in probability.

Secondly, when the initial state belongs to

$$\left\{(\theta(t_0), x_c(t_0), y_c(t_0)) \in \mathbb{R}^3 \mid \theta(t_0) = 0\right\}, \quad (49)$$

we use the constant control $u_0 = -u_0^* \neq 0$ in order to drive x_0 far away from the origin, which guarantees that all the signals are bounded in probability during $[t_0, t_s^*)$. Then, in view of $x_0(t_s^*) \neq 0$, the switching control strategy is applied to system (11) at the time instant $t_s^* > 0$. This completes the proof. □

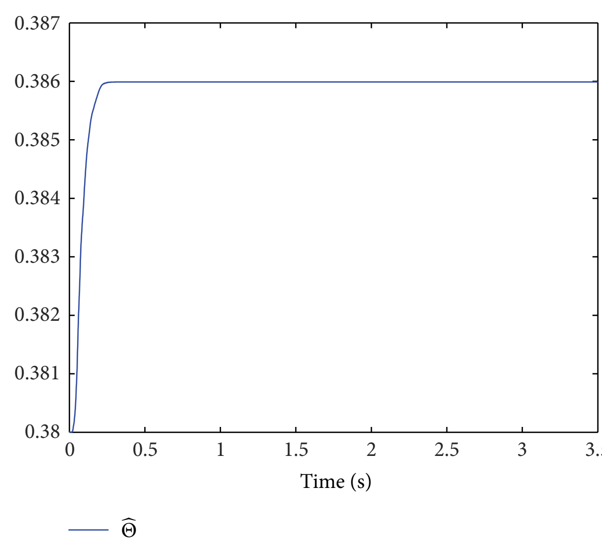

FIGURE 3: The response of estimate parameter $\widehat{\Theta}$ with respect to time.

5. A Simulation Example

Consider the system (11) with $\omega_2 = 0.5\theta$ and $v_2 = x_2 - (3.5x_1/u_0)$. In simulation, one can choose $p_1^* = 0.2$, $p_2^* = 0.1$, $e = 0.5$, $p_{\max} = 0.25$, $d = 1$, $c_1 = 5.5$, $c_2 = 0.2$, $\eta_0 = 10.5$, $m_1 = 0.5$, $m_2 = 1$, and $m_3 = 3$ and the initial values $\theta(0) = 2.8$, $x_c(0) = -0.678$, $y_c(0) = 0.528$, and $\widehat{\Theta}(0) = 0.38$. Figures 1, 2, and 3 give the responses of the closed-loop system consisting of (11), (17), and (40).

From Figure 1, it is easy to see that the states θ, x_c, and y_c are asymptotically regulated to zero in probability in spite of the stochastic disturbances. As shown in Figure 2, the control inputs u_0 and u are convergent to a small neighborhood of zero asymptotically. Figure 3 indicates that the estimated parameter $\widehat{\Theta}$ is bounded.

6. Conclusions

In this paper, we extend the nonholonomic mobile robots with unknown parameters to the stochastic case. Based on backstepping technique, adaptive state-feedback stabilizing controllers are designed for stochastic nonholonomic mobile robots with unknown parameters. A switching control strategy for the original system is given, which guarantees that the states of closed-loop system are asymptotically stabilized at the zero equilibrium point in probability.

There exist some problems to be discussed, for example, how to design the controller for the dynamic stochastic nonholonomic systems with unknown parameters.

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