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## Adaptive Switching Circuits



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Aciapiz'e or "Learning" systems can automatically modify their own structures to ortimize performanre based on past expertences. The system designer "teaches" by showing the system examples of iaplu sicrialo or patterms and simultaneously what he would like the output to be for each input. The syatem in turn orgenizes itself to comply as wall as possible with the wishes of the designer.

An griaptive pattern classification machine (called "Adallne", for adaptive linear) hac bcen dcrisca to illustrate adaptive behaviur aud artificial learning. During a training phase, crudc geometric patteria are fed to the machine by setting the toggle swithes in a $4 \times{ }^{\prime} 4$ input array. Setting another toggle switch tells the machine whether the desired output for the particular input pattern $18+1$ or -1 . All input patterns are classisied into two categories. The system learns a little from each pattern and accordingly experiences a design change. After training, the machine can be used to classify the original patterns and notsy (distort.ed) versionn of these patterns.

A statistical theory has been developed which relates the competence of the classifier to the amount of experience had (number of patterns "seen" in adapting). Imperfect syotem adjustment results from small-sample-size experience. The misadjustment, a dimensionless quantitative measure of the quality of adaption, is defined as the ratio of the increase in probability of error of a system adapted to a smail number of patterns to the probability of error of a "best-adapted" system (adapted to an arhitrarily large number of patterns). Treating the classifier as a roughly quantized sampled-data system, a statistical theory of aaption developed for adaptive sampled-data systems has been utilized to cerive a formula fur misadjustment,

$$
M=\frac{n+1}{N}
$$

The number of input lines is $(n+1)$, and the number of patterns seen in adapting is N. Thic formula leads to a bas:ic "rule of thumb" for adaptive classifiers: The number of patterns requirca to train an adaptive alassifier is equal to several times the number of bits per patters. This
ruie applies wlthout regard to patterns and noise characteristics. Experimental rviden:e is presented.

The pattern slassifier is actually an adaptive awitching circuit having a set of binary inputs and a binary outpist. The signal on each input line is either +1 or -1 according to the setting of the individ::nl pattiern switch. The sixteen input signals are linearly combined and then quantized. The wefghts (wich could be positive or negative) are determineed by an array ol polentlometer settings.

Iterative gradient methods are used during the training phase to find the potentioneter settings that minimize the number of classification. errors. A simple procedure has been devised which does not require actual measurement of gradient, and which guarantees convergence and permits control of rate of convergence. Alaline can usuaily adapt after seeing ten to tirenty patterns and ran easily distinquish a dozen different basic patterns.

As a generlc form of switching functions, Adallne is not completely seneral. All-possible-potentiometer-settings allows the realization of the "linearly separated truth functions", a subclass of all switching functions. Although this subclass is restricted, it is a useful class, and, most important, it is a searchable class (the best within the class can be found without trying all possibilities). Networks of Adalines overcome this restriction and are far more general, yet present adaption problems of no greater difficulty than those of single Adalines.

At present the purely mechanicel adaption process is accomplished by manual potentiometer-setting. A means of automating this is being developed which makes use of multi-aperture ferromagnetic devices. Solid-state adaptive logical elements will result that should ultimately be suiteble to be microminiaturized. Networks of such elements would be very effective in pattern recognition systems, information storage and retrieval-by-classification syatems, and self-repairing logical and computing systems.

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## I. INTRODUCTION

The modern science of switching theory began with work by Shannon ${ }^{1}$ in 1938. The field has developed rapidly since then, and at present a wealth of literature exists ${ }^{2}$ concerning the analysis and synthesis of logical networks which might range from simple interlock systems to tclephone switching systems to large-scale digital computing systems.

A key idea in switching theory is that the performance requirements of any logicai system can be completely specified by a boolean function expressing output conditions in terms of input conditions, and that the algebraic symbole in the boolean function are readily identifiable with simple storage and "and" - "or" elements. The problem of simplification of networks for most economic realization is reducei to a probiem of algebraic simplification of boolean functions, a task which is more easily accomplished by human designers than reduction-by-inspection of logical networks themselves.

An example illustrating the use of switching theory is that of the design of an interlock system for the control of traffic in a railroad switch yard. The first step is the preparation of a "truth table", an exhaustive listing of ali input possibilities (the positions of all incoming and outgoing trains), and what the desired system output should be (what the desired control signals should be) for each input situation. The next step is the construction of the boclean function, and the following steps are algebraic reduction and design of the logical control system.

The design of a traffic control system is an example wherein the truth table must be followed precisely and reliably. Errors would be destructive.

The design of the arithmetic element of a digital computer is another example wherein the truth table must be followed precisely. There are other situations in which some errors are inevitable, however, and here errors are usually costly but not catastrophic. These situations caम 1 for statistically optimum switching circuits. A common performance objective is the minimization of the average number of errors. An example is that of prediction of the next bit in a correlated stochastic
binary number sequence. The predictor output is to be a logical combin. ation of a finite number of previoue input nequenco hits. An uptimum system is a sequential switching circuit that predicts with a minimum number of errors.

Suppose that a record of the binary sequence is printed on tape and cut up Into pieces (with indication of the positive direction of time preserved), say $2 弓$ bita long. Place all pleces whore the most recen 5 event is ONE in one pile and the remainder in another pile. Delote the must recent bit on each piece of tape. If the statistical scheme could be discovered by which the pieces of tape are slassified, this would lead to a prediciifu ocheme. IL is upparenl that predietion is a certains aind of classification.

Assuming statistical regularity, a reasonable way to priseed might be to form a truth table, and let the data from each piece of tape be an entry in the table. It might be expected that with the data of 100 pieces of tape, a fairly good predictor could be developed. The truth table would have only 100 entries however, out of a total of $2^{24}$. The "best" way to 1111 in the remainder of the truth table depends upon the nature of the sequence statistics and the error cost criteria. Filling in the table is a difficult and a crucial part of the problem. Even if the truth table were filled in, however, the designer woild have the difficult task of realizing a logical network to satisfy a truth table with $2^{24}$ entries.

An approach to such problems is taken in this paper which does not require an explicit use of the truth table. The design objective is the minimization of the average number of errors, rather than a minimization of the number of locical components used. The nature of the logical elements is quite unconventional. The system design procedure is adaptive, and is based upon an iterstive search process. Performance feedback is used to achieve automatic system synthesis, i.e., the selection of the "best" system from a restricted but useful class of possibilitics. Tue designer "trains" the system to give the correct responses by "showing" it examples of inputs and respective desired outputs. The more examples "seen", the better is the system performance. System competence will be directily and quantitatively related to amount of experience.

In Fig. 1 , a combinatorial logical circult is shown wich is a typicul element in the ojaptive awitching circuits to be considered. This element bears some resemhlance to a "neuron" model introduced by von Neuman ${ }^{3}$, whence the name.


FIG. 1... AN adjustable negron.

The binary input signals on the individual lines have values of +1 or -1 , rather than the usual values of 1 or 0 . Within the neuron, a inear combination of the ingut signals is formed. The weights are the gains $a_{1}, a_{2}, \ldots$, which could have both positive and negative values. The output signal is +1 if this weighted sum is greater than a certain threshold, and -1 otherrise. The threshold level is determined by the setting of $a_{0}$, whose input is permanently connected to $a+1$ source. Varying $a_{0}$ varies a constant added to the linear combination of ingut signals.

For fixed gain settings, each of the $2^{5}$ possible input combinatings would cause either a +1 or -1 output. Thus, all possible inputs are classified into two categories. The input-output relationship is determined by choice of the gains $a_{0}, \ldots a_{5}$. In the adaptive neuron, these gains are set during line "training" procedure.

In Leiernl, there ure $2^{2^{\prime \prime}}$ different input-output rrilutionsbips or truth funcilond by whici the l.ive luyul variables can be mapped into the nincle oulput variable. Only a subset of these, the linearly separater trith fanctions ${ }^{4}$, can bu realized by sll possible choices of the gaine of the ncuron of Fig. 1. Although this subset is not all-inclusive*, It is a usceul subset, and it is "searchable", l.e., the "best" function In many practical ciccs can be found iteratively without trying all functions within the subset.

Application of this neurcn in adaptive pattern ciassiflers was first made by Mattson. ${ }^{5,5}$ he has shown that complete generality in choice of switching function coulc be had by combining these neurons. He devised an iterative digital computer routine for finding the best set of a's for the classification of noisy geometric patterns. An iteisive procedure having similar objectives has been devised by these authors and is dessribed in the next section. The latter procedure is quite simple to implement, and car he analjzed by statistical methods that have already been developed for the analysis of adaptive sampled data systems.

## III. AN ADAPIIVE PATIERN CLASSIFIER

An adaptive pattern classification machine (called "Adaline", for adaptive linear) has been constructed for the purpose of illustrating adaptive behavior and artificial learning. A photograph of this machine, which is about the size of a lunch pail, is show in fig. 2.

During a training phase, crude geometric patterns are fed to the machine by setting the toggle switches in the $4 \times 4$ input switch array. Setting another toggle switch (the reference switch) tells the machine whether the desired output for the particular input pattern is +1 or -1 . The system lcams a little from each pattern and accoriingly experiences a design change. The machine's total experience is stored in the values of the welghts $a_{0} \ldots a_{16^{\circ}}$. The machine can be trained on undistorted noise-free patterns by repeating them over and over until the iterative search process converges, or it can be trained on a sequence of noisy

[^0]
patterns on a one-pass basis such that the iterative process co: $\therefore$ : jes statisticaliy. Combinations of these methods can be accommodated simultaneously. After +raining, the machine can be used to classify the original patterns and noisy or distorted veriions of these patterns.

A block schematic of Adaline is shown in Fig. 3. In the actial machine, the quantizer is not built in as a device but is accomplished by the operator in viewing the output meter. Different quantizers (2-level, 3-level, 4-level) are realized by using the appropriate mater scales (sec fig. 2). Adaline can be used to classify patterrus into several eategories by using multi-levci quantizerc and by following exactly the same adapting procedure.

The following is a description of the iterative searching routine. A pattern is fed to the machine, and the reference switch is set to correspond to the desired output. The error (see Fig. 3) is then read (by switching the reference switch; the error voltage appears on the meter, rather than the neuron output voltage). All gains including the level are to be changed by the same absolute magnitude, such that the error is brought to zero. This is accomplished by changing each gain


FIG. 3.--SChEmatic of adaline.
(which could be positive or negative) in the direction which will diminish the error by in amount which reduces the error magnitude by $1 / 17$. The 17 gains may to variged in any scquence, and after all changes are made, the error foi the present input pattern is zero. Switching the reference back, the meter reads exactly the desired output. The next pattern, ind it.e desired output, is presenter and the error is read. The same anjustment, routine is followed and the error is brought to zero. If the filrit patteri. were reupplied ai this point, the error would be cmall but not necessarily :cro. More patterns ure inserted in like manner. Convergence is indicated by small errors (before niaption), with gmoll fluctuations about a stable root mean-square value. The iterative routine is purely mechanical, and requires no thought on the part cf the operator. Electronic automation of this proscdure will be discussed below.

The results of a typical عiaption on six noiseless patterns is given in Figs. It and 5. The patterns were selected in a random sequence, and were classified into 3 categories. Each $T$ was to be mapped to +60 on the meter dial, each $G$ to 0 , and each $F$ to -60 . As a measure of performance, after each adaptation, all six patterns were read in (without adaptation) and six errors were read. The sum of their squares denoted by $\Sigma \mathrm{se}^{2}$ was computed and plotted. Figure 5 shows the learning curve for the case in which all gains were initially zero.

## IV. STATISTICAL THEORY OF ADAPTION FOR SAMPLED-DATA STSTTEMS

This section is a summary of the portions of Widrow's statistical theory of adaption for sampled-data systems ${ }^{7,8}$ that is useful in the analysis of adaptive switching circuits.

Consider the general linear sampled-data system former of a tapped delay line, shown in Fig. 6. This system is intended to be a statistical predictor. The present output sample $g(m)$ is a linear comhination of present and past input samples, and is intended to approximate as closely as possible the next input sample $f(m+1)$. The constants in this linear combination are $h_{1}, h_{2}, h_{3}$, etc., the predictor impulse-response samples; or the gaing gesociated with the delay-line taps. Their choice constitates


FIG. 4..-PATTERNS FOR CLASSIFICATION EXPERIMENT.


Fig. 5.-. adaptive-element performance curve.


FIG. 6...AN ADJUSTABLE SAMPLED-DATA PREDICTOR.
the adjustable part of the predictor design. They may be adjusted in the following manner. Apply a mean aquare reading meter to $\epsilon(\pi)$, the difference between the present input and the delayed prediction. This weter will measure mean square error in prediction. Adjust $h_{1}, h_{2}, h_{3}, \ldots$, until the meter reading is minimized.

The problem of adjusting the $h$ 's is not trivial, because their effects upon performance interact. Suppuse that the predictor has only two impulses on its impulse response, $h_{1}$ and $h_{2}$. The mean square error for any setting of $h_{1}$ and $h_{2}$ can be readily derived:

$$
\begin{align*}
& \epsilon(m)=f(m)-h_{1} r(m-1)-h_{2} f(m-2) \\
& \epsilon^{\bar{L}}(m)=\varnothing_{f f}(0) h_{1}^{2}+\varnothing_{f f}(0) h_{2}^{2}-2 \varnothing_{f f}(1) h_{1}-2 \varnothing_{f f}(2) h_{2} \\
&+2 \varnothing_{f f}(1) h_{1} h_{2}+\varnothing_{f f}(0) \tag{1}
\end{align*}
$$

The discrete autocorrelation function of the input is $\varnothing_{f f}(j)$.
The mean square error given by equations (1) is what the mean square metar would read if it were to average over very large sample size. The mean square error is a parabolic function of the predictor adjustments $h_{0}$ and $h_{1}$, and, in general, can easily be shown to be a quadratic function of such adjustments, regardiess of how many there are.

The optimum n-impulse predictor can be derived analytically by setting the partial derivatives of $\overline{\epsilon^{2}}$ of equation (1) equal to zero. This is the discrete analogue of Wiener's optimization ${ }^{9}$ of continuous filters.

Finding the optimun systeu experimentally is the came as finding a minimum of a puraboloic. In $n$ dimensions. This could be done marualiy iy having a human operator read the meter and sct the adjustment, or 1 t. could he done automatically by making use of any one of several iterative grodient methods for sirface-searching, as devised by numerical analysts. When either or thes schemes is employed, an adaptive system results that consiccis essentially of a "worker" and a "boss". The worker in this case predicts, whereas the boss has the job of adjusting the worker.

Figure $i$ is $n$ bicnk-diagram representation or such a basic adaptive unit. The boss continually seeks a better worker by trial and error experimentation with the structure of the worker. Adaption is a multidimensional performance feeduack process. The "error" signal in the feedback control sense is the gradient of mean square errow with respect to adjustment.

Many of the commonly used gradient methods search surfaces for stationary points by making changes in the independent variables (starting with an initial guess) in proportion to measured partial derivatives to obtain the next guess, and so forth. These methods give rise to geometrlc (exponential) decays in the independent variables as they approack a stationary point for second-degree or quadratic surfaces. One-dimensional surface-searching is illustrated in Fig. 8.

The surface being explored in Fig. 8 is given by Eq. (2). The Pirst and second derivatives are given by Eq. (3) and (4).

$$
\begin{align*}
& y=a(x-b)^{2}+c  \tag{2}\\
& \frac{a y}{d x}=2 a(x-b)  \tag{3}\\
& \frac{d^{2} y}{d x^{2}}=2 a \tag{4}
\end{align*}
$$

A sampled-date feedback model of the iterative process is shown in Fig. 8b. Each time a guess in $x$ is to be made, the derivative is measured physically whereas in the model it is formed as a quantity proportional to $x$ (according to Eq. 3).


Fig. 7.--AN ADAPTIVE PREDICTOR.


FIG. 8. --ONE-DIMENSIONAL SURFACE SEARCHING.

The numerical sequence at the point $x(1)$ tiegins with the initial乃uess and proceeds as a sampled transtent that relaxes geometrichily toward the stationary point, exactly like the sequence of euesses in the surface exploration.

The flow-graph can be reduced, and the transfar function from any point to any other point can thus be found. The resulting charecteristic equation is

$$
-(2 a k+1) z^{-1}+1=0
$$

The iterative process is stable when

$$
\begin{equation*}
0>k>-\frac{1}{a} \tag{5}
\end{equation*}
$$

In order to choose the "loop gain" $k$ to get a specific transient decay ratc, one would have to measure the second derivative ( $2 a$ ) at some yoint on the curve. Transients decay completely in one step when

$$
k=-\frac{1}{2 a}
$$

Derivatives are measured in the actual adaptive system by varying the $h$ 's $u$ y fixed increments and subtracting meusured values of mean square error based on a sample size of $N$ samples. "Noise" in the measurements of the mean square error surface due to small sample size cause nolsy derivative measurements. These noises enter the adaption process, as indicated in Fig. 80, and cause noisy system adjustments. The larger the sample size taiken per derivative measurement, the less is the noise. The slower the adaptation, the more precise it is. The facter the adaptation, the more nolsy (and poor) are the adjustments.

Consider that the adaptive model has only a single adjustment. A plot of mean square error versus $h_{1}$ for this simplest system would be a parabola, analogous to the parabola of Fig. 8. Noise in the aystem adjustment causes loss in steady-state performance. It in useful to define a dimensionless parameter $M$ the "misadjustment", as the ratio of the mean increase in mean square error to the minimum mean square error. It is a measure of how the system performs on the average, after adapting transients have died out, compared with the fixed optimum system. Witn resard to the curve of Fig. 8,

$$
\begin{equation*}
M=\frac{\bar{v}-\ddot{c}}{c} \tag{6}
\end{equation*}
$$

Varlance $\ln \mathrm{x}$ nbout the optimum value causes the average or $y$ to be greater than the mirimum value $c$. The increase in $\bar{y}$ equals the variance in $x$ multiplied by $a$, as can te seen from Eq. (2).

More detailed derivations of misad,justment formulas covering several different methods of surface searciing and derivative measurement are preserted in Refs. 7 and 8. The particular Cormulas which can be applied to tre analysis of adaptive ewltching circuits are the following.

When derivatives are measured by data repeating, l.e., wher the same system irput data is applied for both the $N$ "forward" and the $N$ "backward" measurements of mean. square error, the misad justment in given by

$$
\begin{equation*}
M=\frac{1}{2(\mathbb{N} \tau)} \tag{7}
\end{equation*}
$$

$\tau$ is the time constant of the iterative process of Fig. 8, and is enual te $-(1 / 2 a k)$. A unit time constant means that the adjustment error decreases ,y a factor $1 / e$ per iteraticis cycle. Equation (7) is conservative, and appreciably so only for small values of $\tau$, less than 1 . In the limiting case of one-step adaption. $\tau=0$ and the appropriate misadjustment formula is

$$
\begin{equation*}
M=\frac{1}{N} \tag{8}
\end{equation*}
$$

In deriving Formulas (7) and (8), it has been assumed that the error samples are gaussian distributed, with zero mear, and are uncorrelated. It can be shown that these results are highiy insensitive to this distribution density shape, and are appreciably affested by correlation only when it exceers 0.8 .

It is interesting to note that the quality of sdaption depends only on the number of samples "seen" by the system in adapting. When Eq. (7) applies, the ( $N \tau$ ) product determines the misadjuslment. This product is equal to the number of samples seen per time constant of adaptation. If it may be cuisidered that lransients die out within two time conctants, then the misadjustment equals the reciprocal of the number of samples that elapse in adapting to a strag change in process. This statement is ouviously the case when Eq. (8) applies.

The expressions (7) and (8) are based on the supposition that fresh data is brought in for each cycle of iteration. If the system adapts on q fixed body of $N$ error samples, either by adapting with the one-step procedure and stopping, or by repeating the same data from iteration cycle to iteration cycle for several time constants and then atopping, the $w d-$ adjustment is given by Formula (8).

When there are minteractlag adjustments instead of just 0.1e, Expressions (7) and (8) may be gencrallzed by multiplicution by m. Milti. dimensional one-step surface searching may be accomplished by Newton's method. Multi-step searching may be conveniently achieved by mpans of the method of steepest descent (making changes in adjustwent in the direction of the surface gradienl and in profortion to its maonitude) or by the Southwell relaxation method (cyclic adjustment for minima, one coordinate at a time).

## V. STATISTICAL THEORY OF ADAPTION FOR THE ADAPTIVF NETRON ELENENT

The error signal measured and used in adaption of the neuron of Fig. 1 is the difference between the desired output and the sum before quantization. This error is indicated by $\in$ in Pig. 9. The actual neuron error, indicated by $\epsilon_{n}$ in fig. 7 , is the difference between the neuron output and the desired output.

The objective of adaption is the following. Given a collection of input patterns and the associated desired outputs, find the best set of weights $a_{0}, a_{1}, \ldots a_{m}$ to minimize the mean square of the neuron error, $\epsilon_{n}^{2}$. Individual neuron errors could only have the values of $+2,0$, and -2 with a two-level quantizer. Minimization of $\epsilon_{n}^{2}$ is therefore equivalent to minimizing the average number of neuron errort.

The simple adaption procedure described in this paper minimizes $\overline{\epsilon^{2}}$ rather than $\epsilon_{n}^{2}$. The measured error $\epsilon$ has zero mean (a consequance of the minimization of $\overline{\epsilon^{2}}$ ) and will be assumed to be gaussian-distributed. By uaking use of certain geometric arguments or by using a statistical theory of ampiltide quantization, ${ }^{10}$ it $\frac{c a n}{2}$ be shown that $\bar{\epsilon}_{n}^{2}$ is a monotonic function of $\overline{\epsilon^{\bar{c}}}$, and that minimization of $\overline{\epsilon^{2}}$ is equivalent to minimization of $\overline{\epsilon_{n}^{2}}$

and to minimization of the probability of neuron error.* The ratio of these mean squares his been calculated and is plotted in Fig. 9 as a function of the neuron error probability (which is $\epsilon_{n}^{2} / 4$ ).

Given any collection of input patterns and the associated desired outputs, the measured mean square error $\overline{\varepsilon^{2}}$ must be a precisely parabolis: function of the gain settings, $a_{0}, \ldots a_{n}$. Let the $k^{\text {th }}$ pattern be indicated as the vector $S(k)=s_{1}(k), s_{2}(k), \ldots s_{n}(k)$. The $s^{\prime} s$ have values of +1 or -1 , and represent the $n$ input components numbered $\ln$ a fixed manner. The kth error 18

$$
\begin{equation*}
E(k)-a(k)-a_{0}-a_{1} s_{1}(k)-a_{2} B_{2}(k)-\ldots-a_{n} s_{n}(k) \tag{9}
\end{equation*}
$$

For simplicity, let the neuron have only two input lines and a level control. The square of the error is accordingly

$$
\begin{align*}
\epsilon^{2}(k)=d^{2}(k) & +\varepsilon_{0}^{2}+s_{1}^{2}(k) a_{1}^{2}+s_{2}^{2}(k) a_{2}^{2} \\
& -2 d(k) a_{0}-2 d(k) s_{1}(k) a_{1}-2 d(k) s_{2}(k) a_{2} \\
& +2 a_{1}(k) a_{0} a_{1}+2 s_{2}(k) a_{0} a_{2}+2 s_{1}(k) s_{2}(k) a_{1} a_{2} \tag{10}
\end{align*}
$$

The mean square error averaged over $k$ is

$$
\begin{align*}
& \overline{\epsilon^{2}}=a_{0}^{2}+\varnothing\left(s_{1}, s_{1}\right) a_{1}^{2}+\not\left(s_{2}, s_{2}\right) a_{2}^{2}-\bar{d} a_{0} \\
& -2 \not \partial\left(d, s_{1}\right) a_{1}-2 \not \partial\left(d, s_{2}\right) a_{2}+2 \bar{s}_{1} a_{0} a_{1}+2 \bar{s}_{2} a_{0} A_{2} \\
& +2 \not\left(B_{1}, B_{2}\right) a_{1} a_{2}+\varnothing(d, d) \tag{11}
\end{align*}
$$

The $\varnothing$ 's are spatial correlations. $\varnothing\left(s_{1}, s_{2}\right)=\overline{B_{1} B_{2}}$, etc. Note that $\phi\left(s_{j}, s_{j}\right)=\overline{\mathbf{B}_{j} \mathbf{s}_{j}}=1$.

Adjusting the a's to minimize $\overline{\epsilon^{2}}$ is equivalent to searching a parabolic stochastic surface (having as many dimensions as there are a's) for a minimum. How well this surface can be searched will be limited by sample size, i.e., by the number of patterns seen in the searching process.

[^1]The method of searching that has proven most useful is the method of steepert descent. Vector adjuatment, changes are made in the direction of the graicitit The change made in a $a_{0}$ io proportional to the partial derivative of $\overline{c^{c}}$ with respect to $a_{0}$, etc. The partial derivatives are determined at one point, then the changes in all adjustments rac wide s!multalnously. This completes one iterative cycle. The process is then repeated. Transients decay along each ad Juating coordirate in relaxation towurd the etationary point. They conclet of eum of geometric sequence comporenti (there are as many natural "frequencies" as the number of adjustments, as can be seen from generalization of the slow graph of Fig. (U-- see Ref. 9). If the proportionality constant $k$ betiveen partial derivative ard size of change is made sufficiently small., transients will be stable. Just how big this constant could be for stabie searching depends upon the surface characteristics (i.e., upon pattern characteristics). It can be shown, however, that when all sernnd partial derivetives are equal (differentiation of Eq. 11 shows them all to have the value 2), the melhod of steepest descent will be stable when the proportionalit.' constant $k$ is less than the reciprocal of the second partial derivative. It car. alsc be shown that wher $k$ is small, transients can be well represented as being of a single time constant. This time constant is somewhat sensitive to the specific pattern information, but generally turns out to equal 1/2k.

When partial derivatives are measured by averaging over only a few pat.terns each iteration cycle, the measurements will be noisy, and t.ansients will be noisy exponentials. Stability and time constant will remain dependent on $k$ and the properties of the large-sample-size mean-square-error surface.

The method of adaption that has been used requires an extremely small sample size per iteration cycle, namely one pattern. One-pattern-at-a-time adaption has the advantages that derivatives are extremely easy to measure and that no storage is required within the adaptive machinery except for the gain values (which contain the past experience of the neuron).

The square of the error for a single pattern (the mean square error for a sumple size of one) is given by Eq. (10). The partial derivatives are

$$
\begin{align*}
& \left.\frac{\partial \epsilon^{2}(k)}{\partial a_{0}}-1-2 d i(i)_{1}+i a_{0}+i s_{1}(k) a_{1}+2 s_{p}(k) a_{2}\right] \\
& \frac{\partial \epsilon^{2}(k)}{\partial a_{1}}=s_{1}(k)\left[-2 d(k)+2 a_{0}+2 s_{1}(k) a_{1}+2 s_{i}(k) a_{2}\right] \\
& \left.\frac{\partial \epsilon^{2}(k)}{\partial a_{2}}=s_{2}(k)\left[-2 d(k)+2 a_{0}+2 s_{1}(k) a_{1}+2 s_{2} i k\right) a_{2}\right] \tag{12}
\end{align*}
$$

Comparison of the Eqs. (12) with Eq. (9) shows that the derivatives are simply related to the measured error, and suggeat thas the derivatives could be measured wichout squaring and averaging and without actual differentiation. The jth parilal derivative is given by

$$
\begin{equation*}
\frac{\partial \epsilon^{2}(k)}{\partial a_{j}}=-2 s_{j}(k) \quad \epsilon(k) \tag{13}
\end{equation*}
$$

It follows that all derivailves have the same magnitude, and have signs determined by the error sign and the respective input signal signs. Application of the method of steepest descent requires that all gain changes in a given iteration cycle have the same magnitude and the appropriate sign. Each gain change reduces the error magnitude by the same amount. The procedure described in Sec. C for bringing $\epsilon(k)$ to zero with each successive input pattern gives the constant $k$ a value of $1 / 2(n+1)$. From the previous discussion we see that the time constant of the iterative process is therefore $\tau=(n+1)$ patterns. On the $4 x 4$ Adaline, there are $n=16$ input line gains plus a level control. Therefore, the time constant should be roughly 17 patterns (for verification, see the learning curve of Fig. 5). The search procedure could be readily modified to speed up or slon down the adaption process. For exampie, bringing the error $\epsilon(k)$ to half its value rather than to zero with each input pattern haives k and doubles $\tau$.

The statistical theory of adapiion for sampied-data systems is based on search of mutidimensional stochastic parabolic surfaces for stationary points. The misad sustment, a dimensionless measure of how well a system will adapt, is defined as the ratio of the mean incresse in mean square error (due to searching the surfice with saidl-sample-size data) to the
minimum mean square error (a performance reference that could only be achieved with yerfect knowledge of input process statistics). The misadjustment Fomnulas (7) and (8) apply directly to the adaptive neuron.

The misadjustment formulas give the per unit increase in measured mear square error as a result of adapting on a finite number of patterns. Since the ratio of probubility of neuron error to the mean square error $\overline{\epsilon^{2}}$ is essentially constint over a wide range of error probabilities (Fig.9), the misadjustment as expressed by Formulas (7) and (8) may be interoreted in terms of the ratio of the increase in error probahility to the minimum error probability.

If adiaption is accomplisher by injection of a fresh pattern each iterution cycic, the meun values of the guins will converge, after adapting transients have died out, on the beat set of values ivr large sample size. The actual gain settings will experience random excursions about these values, and the resulting misadjustment, as derized from Eq. (7) is

$$
\begin{equation*}
M=\frac{(n+1)}{2 t} \tag{14}
\end{equation*}
$$

Following the procedure oi bringing $\epsilon(\mathrm{k})$ to zero each iteration cycle, the misadjustment is

$$
\begin{equation*}
M=\frac{(n+1)}{2 \tau}=\frac{(n+1)}{2(n+1)}=\frac{1}{2} \tag{15}
\end{equation*}
$$

If adaption is accomplished by taking a fixed collection of $N$ patterns and repeating them over and over for several time constants (where the time constant is long, several times $N$ ), the gains will stabilize on the best set of values for the $N$ patterns. In general, these gains will not be the best for the large collection of patterns that the N yatterns were abstracted from. Making use of Eq. (8), the misadjustment is

$$
\begin{equation*}
M=\frac{(n+1)}{N} \tag{16}
\end{equation*}
$$

An extensive series of simulation studies has been made to test the validity of the misad,fustment Formulas (14) and (15). These tests have shcm that the formulas are highly accurate over a very wide range of pattern and noise characteristics. A description of a typical experiment and its results is given : $\because:$ Fig. 10.

$$
\begin{aligned}
& x \rightarrow+1 \\
& \\
& \bullet \bullet
\end{aligned}
$$

$$
T \rightarrow-1
$$

$$
8 \bullet 8 \bullet \text { eg eg eg e }
$$

$$
8 \quad 8 \quad 6 \quad 0 \quad 6
$$

$$
\begin{array}{lllll}
88^{\bullet} & 0 & 0 & 0 & 0 \\
0 & 0 & 0
\end{array}
$$

$$
8^{\bullet} \quad \theta^{\bullet} \quad 8 \quad 8
$$

$$
8_{0} \bullet \bullet \quad \bullet \quad \bullet 8
$$

$10 \%$ Noise


Best neuron makes 12 errors out of 100


FIG. 10..-EAPERIMENTAL ADAPTION ON 10 NOISY $3 \times 3$ PATTERNS.

Noisy $3 \times 3$ matterns were cenerated oy randomly indectinp, errors in ten percent of the positions of the four "pure" patterns, $X, T, C, i$. These ratterus, s'own in Fig. 10, are ordered for convenience in checking. They were fed manualiy to Adaline and chosen randomly by looking up their identi:'ication numbers ir. a rurdom number table. The X's (numbired irom left tu righi, up to down) were qumbered 1 to 25, the T's were 25 to 50 , the C's were 50 to 75 , and the $J$ 's were 75 to 100.
'The best system was arrived at by slow precise ailaption on the full body of 100 noisy patterns, repeating the" over and over several times. l'his system was sole to classify tice patterns us desired, except for twelve errors out of the 100 total. The gains were then set to zero and ten patterns wexc chosen at random. The best system for the ten seiecied patterns was arrived at by slow adaption on these patterns, repeatirg them over and over several times. The resulting system was then tested ou the full body of 100 patterns, and 25 classification errors out of 100 were made. This number of errors was more than twice that made by the best syscem adapted on 100 patterns. The misadjustment was 108 parcent. This small-sample-size adaptation experiment was repeated three more times, and the misadjustments that resulted, in order, were 58 per cent, 67 per cent and $: 33$ per cent. Since $N=10$ patterns and $n=9$ input lines, the theoretical misadjustment was

$$
M=\frac{n+1}{N}=100 \text { yer sent }
$$

An averuge laken over the four experiments gives a measured misadjustment of 91.5 per cent.

The adaptive classifier can adapt after seelng remarkably few pat. terns. A misad.justment of 20 per cent should be accept able in most applications. Tc achieve this, all one has to do is supply the adaptive classifier with a number of patterns equal to five times the number of input lines, regardless of how noisy the patterns are and how difisicult the "pure" patterns are to separate. Although the misadjustment formilas nave been derived for the specific classifier consisting of a aingle adaptive neuron, it is suspected that the following "rule of thumb" will apply fairly well to all adaptive classifiers: the number of patterns required to train an acaptive classifier is equal to several times the nunber of bits per pattern.
output. The read-out oscillator provides the a-c current to operate the read-out wiles. The reud-oul amplifier converts the a-c output signals to a d-c output signal. The summer ( $\Sigma$ ), computes the error $\epsilon(k)$. The error sign cir :uit computes sgn $\epsilon(k)$. The error magnitude circuit provides a sisnal which blocks the "and" gate when the magnitude of the error falls below a preset level, thus preventing the operation of the adapt...drive circuit. Therefore, when the "Adapt" signal is applied, the adapt-drive circuit is repeatedly energized until the error falls below the preset level. The delay cirenit controls the amount of time between energizations of the adapt-drive circuit. This time must be long enough to allow the error to reach its new value after each energization.

When networks of neurons are used, it is possible that a single set of driving circuits could be employed to actuate all of the adaptive neurons. At present, this is not practicsl for large netiorks because of the power levels required. The MAD elements shown in Fig. 1.4 are quite large, and might ultimately be able to be made much smaller, perhaps in the form of thin films. It should ultimately be possible to mass produce large networks of adaptive microelectronic logical elements. Power levels should be low, space and waight requirements and cost should be low. These neurons should be thought of and treated as new kinds of circuit elements, adaptive logical components.
VIII. APPLICATIONS FOR ADAPIIVE IOGICAL CIRCUIT ELEMENTS.

The field of application of digital systems may be classified into two broad categories, fixed systemis and adaptive systems. The structure of the fixed system is completely determined by the designer, while the adantive system is designed to have both fixed and adjustable portions. The latter system has the ability to automatically modify its adjustable parts by trial and error experience in order to optimize performance (this is perforvance feedback). Fixed systems are by far the most common at present. Adaptive systems have received intensive study during the past several years, and some practical applications are being made in automatic control and in the recognition (classification) of pattern information.

Both sets of patterns were fed to two Adalines simultaneousiy and perfect adaption was ;ossible. The adaption procedure was the following: if the desired output. for a given input pattern applied to both machines was -1 , then both machines were adapted in the usual. manner to ensure this; if the desired output was +1 , the machine with the smallest measured error E was assigned to adapt. to give a +1 output while the other machine remained unchanged. If either or both machines gave outputs of +1 , the patitern was ciassified as +1 . If buth wachines gave -l outpuits, the pattern was classified as -1 .

This procedure assigns specific "responsibility" to the neuron that cen most easily assume it. If, at the beginning of adaption, a given neuron takes responsibility for producing a +1 with a certain input pattern, it will invariably take this responsibility each time the pattern is applied during training. Notice that it is not necessary for a teacher to assign responsibility. The combination does this automatically and requires only input patterns and the associated desired outputs, like the single neuron.

More complicated problems can be well solved by combinations of many neurons. Their inputs are connected in parallel while their outputs are connected to an OR element. The only new requirement is that of the job assigner, which is simple to implement. Such combinations greatly increase the generality of the classification scheme, and the ease of adaption is comparable to that of a single neuron. A theory of adsption for these combinations has yet to be completed. Preliminary considerations indicate that the misadjustment formulas will apply without appreciable change when combinations of neurons adapt on noisy nonlinearly separable patterns.

Various classification problems could be solved simultaneously by multiplexing neurons or combinations of neurons. One neuron might be trainca to decide whether the man in a given picture does or does not have a green tie, while another neuron or combination could be trained to decide whether or not the man has a checkered shirt. Each neuron or combination has its own output line, and each is fed the appropriate desired ou'bput signal during training. The input signals are common to all neurons. In this manner, it is possible to form adaptive classifiers that can separate with gracut accuracy large quantities of complicated patterns into many output sategories. All that is needed is la rge quantities of adaptive neurons.

The structure of the neuron described in this report and its adaption procedure is sufficiently simple that an effort is under way lo develop a physical device which is an all-electronic fully automatic Adaline. The objective is i self-contained device, like the one sketched in Fig. 11, that has a signal input line, a "desired output" input line (actuated during training only), an output line, and a power supply. The devile itself should be suitable fur mass production, should contiain few parts, should be rcliable, and probably should consist of solid-state compnents. -


FIG. ll.,-ELECTRONIC AUTOMATICALLY•ADATED NEURON.

To have such an adaptive neuron, it is necessary to be able to store the gain values, which could be positive or negative, in such manner that these values could be changed electronically.

Present efforts have been based on the use of multi-aperture magnetic cores (MAD elements ${ }^{\text {ll }}$ ). The special characteristics of these cores permit muitilevel storage with continuous, non-destructive read out. In addition, the stored levels are casily changed by small controlled amountc, with the direction of the change being determined by logic performed by the MAD element.

Figure 12 shows a block diagran for an electronic adaptive element, which realizes the adaptiol, technique described previously. The MAD element array conlalns magnetic cores and wire only; Fig. 13 shows how


FIG. 12.--BLOCK DIAGNAM, ELECTAONIC ADAPTIVE ELEMENT.


FIG. 13.-. mad element - windimgs fon use in electmonic adaptive elements.
each core is wifed. A photograph of the first experimental array for a jx5 Adaline 18 fhow in Flg. 24. Thls array performs the foliowing functions:

1. Storage ot the gaino and the d-e level (the $a_{1}^{\prime} s$ ). Thin storage is passive, l.e., the information is nct last, in the event of power raflure. There is one MAD element for each ghin and one for the d-c level. Thi's for $m \times n$ patterns $m n+1$ MAD elements are required.
2. Continuous computation of the $s u m a_{0}+\underset{i}{a_{1} \beta_{1}}(k)$ for the pattern connected to the input. The sum appears as two u-c algnuls, one appearing acrosis each of the read-out wires. The signsul across cne of these wires corresponds to the sum of those terms for which the $s_{i}(k)$ is negative; the other corresponds to the sum of the d-c level and those terms for which the $s_{1}(k)$ is positive. [Each read-out wire carries an a-c current. The voltage drop per core on a read-out wire is a linear function of the value of the gain stored in that core, provided that the aperture through which the wire passes is not blocked by energizing the block winding of that aperture. If blocked, the voltage drop is very umall. Thus, the sumuation is accomplished by energizing the "Block ( - )" winding of the $i^{\text {th }}$ core when $s_{1}(k)$ is negative and energizing the "Block $(+)$ " winding when $s_{i}(k)$ is positive.]
3. Computation of the adaption change $b_{a_{1}}$ in the gain $a_{1}$. Earh of these changes is proportional to the product $s_{1}(k) \operatorname{sgn}[\epsilon(k)]$. [The change In the stored level of the core is accomplished by applying the proper signal to the "Adapt Drive" wire. With the proper adapt-drive waveform, the direction of the change may be reversed by applying a d-c bias to one of the "Inpiat" windings. Input 1 is energized when both $s_{1}(k)$ and $\epsilon(k)$ are positive; Input 2 is energized when both are negative. For $s_{i}(k)$ and $\epsilon(k)$ of opposite sign, no current is applied to either input.] All of the changes $6 a_{i}$ are of the same magnitude. To reduce the orror to approximately zero, the Adapt Drive wire is snergized a sufficient number of times. The d-c winding on the MAD element carries a d-c bias current. This current may be removed between adapt-drive signals, but must be appilied during the adapt-irive signal.

The peripheral circuitiry supplies the necessary signals to the MAD element array, and converte the a-c reid-out signals to a more useful d-c


Fig. 14.--EXPERIMENTAL AUTOMATIC ADALINE.
output. The read-out oscillator provides the a-c current to operate the read-oui wires. The reud-oul amplifier converts the a-c output signals to a d-c output signal. The summer ( $\Sigma$ ), computes the error $\epsilon(k)$. The error sien cir ruit computes sgn $\epsilon(k)$. The error magnitude circuit provides a signal thich blocks the "and" gate when the magnitude of the error falls below a preset level, thus preventing the operation of the adapt...drive circuit. Therefore, when the "Adapt" signal is applied, the adapt-drive circuit is repeatedly energized until the error falls below the preset level. The de? gy cirenit controls the amount of time between energizations of the adapt-drive circuit. This time must be long enough to allow the error to reach its new value after each energization.

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Both fixed and adaptive systems may be realized by programning general purpose digital computers. These computers on the other hand are realized of conventional logical components (flip-flops, gates, etc.), but may be realized of networks of adaptive neurons. All details of organization, design, and construction of computers must be completely planned in lie present day scheme of things. If a computer were buill of adaptive neurons, details of structure could be imparted by the designer by training (showing it examples of what he would like it to do) rather than by direct designing. This design concept becomes more significant as size and complexity of digital systems increase. The demands of modern technology are such that le.rger and more complex digital systems are continually being contemplated, and in step with this, progress in microelectronics makes such systems physically and economically possible.

The problem of reliability is greatly aggravated by increase in size and complexity. Significant steps in improving the reliability of digital systens have been made, notably with the introduction of the magpetic-core memory, and the use of high-speed switching transistors as active logical elements. Although the reliability of indiviauil components has constantly increased, the requirement in numbers of components has increased in many cases far more rapidly. It is not expected that mass-produced microminiature components will ever be perfectly reliable, yet they will be usable in large systems. The problem is to devise new systems techniques to achieve reliable over-all operation where systems consist of large numbers of interacting imperfect components.

Errors caused by computer component failure are, in general, more deleterious to a fixed systen. In the event of a failure, the adaptive system will adjust whatever remaine adjustable to do the "best" job with the intact parts. As long as the adaption mechanism is reliable, system relia.bility is inherently increased.

Shannon and Moore ${ }^{12}$ and von Neumann ${ }^{3}$ have proposed schemes for making reliable fixed digital systems from unreliable components by using redundancy. Another method, using adaptive logic, is hereby proposed for improving system reliaitility.

The reliability of a system whose purpose is non-adaptive may be increased by combining adaption and redundancy. Consider a multiplex

Consisulinf of three machlnes oolving the same problem with the same input data. Let. the output of ench machine we a eingle binary number, expres. sed $n s+1$ of -1 . If these machines were perfectly rellable, their outputs would alwaje ugree. If not, then von Neumann proposed that the majority should mile. The neuron chown in Fla. 1 with $a_{0}$ set to zerc, and the other gains sel to +1 would sive a ma,jority output. Each machine rins equal vote. Unequal vote (higher vote going to the more rel iable machine) is possibie by making the u's adjustable, and causing these adjustmento Lo be mode automaticaily to optimi 7 : performance. The adaptive note taker is identical to the adaptive neuron. The vote taker can be trained by periodically injecting a certain input when the desired output is known.
von iveumann'c majority rule vote taker will give the correct outcome when the majorily ic correct. The adaptive vote taker could ideally give the correct outcome with only a fingle correct machine by gi:ing it a heavy rote and attenuating the votes of the unreliable machines. This is In effect an adaptive routing procedure for informition flow, and allows systems in a small measure to be self-healing.

The effectiveness of the adantive vote taker is being evaluated by William Pierce in a doctoral thesis research at Stanford University. It has bcen shown that the effective multiplex factor can be greatly insreased by uduption (particularly where the machines are fairly unreliable), und that system iffe expectance can also be greatly increased by adaption and redundancy. This work will be lescribed in a Stanford University technical report.

When adaptive reuron elements become available in large quantities, adaptive logical and computing systems will probably be organized quite differently from the way moderr computing systems are organized. The organizations of two related adaptive system types will be considered, that of adaptive pattern ciassifiers and of adaptive problem-solving machines.

The realization schemes utilized by Clark and Farley, ${ }^{13}$ Rosenblatt, ${ }^{14}$ and Mattson 5,6 for adaptive pattern classifiers made use of digital simulation. The approach suggested by this work is that adaptive pattern classifiere be constructed of networks of adaptive neuron elements.

One of the mosl promising areta of reseurch la compater eystem theory is that of problem-sclif:\% muchines, theorem-proving machines, und urtiElcialıy "inte!.1lgen*" machines. the earliest prononents of thic research vere Turing," and Shamon. ${ }^{i 6}$ Their auggestions were successfully put to proctice by Newell, Simon, unc: Shaw, ${ }^{17}$ b; Samuel, ${ }^{18}$ and by otheris. Proh!emsolvin: has been reparded us . multistage derisior process which begins With ar initial stalus and ends with a goal status. Each change in status rosults fror the relection of $a$ certain move from a collection of possibilltics which are "lecial" according to the rules of the game. Since the number of chains of moves increases approximately exponentially with the length of the chains, exnaustively trying all chains in search of a gonl is not practical, even for simple problems.

The appronch taker by Samuel in his checker-playing simuations to reduce the number of chains to be testad was two-fold. The langth of the chain: was limited to be sumewhere between ten and thirty moves ahead (a "ply" of 10 to 30 ), and cince most chains would not terminate by reachinc coais, a system of status evaluation was developed so that the various chains could be numerically compared. The second method of reducing the number of chains to be tested was to rheck against games stored in the memory. If un identical situation was encountered previousiy, certaln evaluations have alragdy been made ard need not be repeated. This use of stored games was called "rote learning". A procedure for making une status evaluation system aduptive was called "generalized learning". Both of these lparning methods could be used simultanecusly.

The rote learning portion of the over-ell procedure could be made to be much more powerful if it were possible to extract from the memory previous situations that are similar (are not necessarily identical) to the cirrent, situation. Far less experience and storage would be needed to reack a siven level of competence of play. Similar means that the previous eftuntion is in the same subclass with the current situation. A classification scheme would be needed to establish similarities in checker sitnations. The structure of this classifier would have to be formed from experience.

An automatic problem-solving computer should have a memory system from which information could be extracted i:conrding to classification
sulher Lifin iy address number. The extent of classification hefore storlnis should be slight (e.g., is the pattern of checkers or of chess?), and a consistent scheme for the arrargement of the pattern ifts should be established before storing. Final classifisation should be done within the meriory ltself. Eoch storage register should contain an Adol.jue or a network of sdalines.

A requesl from a "central control" for a certaln type of information is ront to every register in the momory oimultaneousiy. This hus che effect if retting the adjustments of all the Adalines. Ony y the registers whose clnssifiers respord properly (e.g., give +1 outputs) answer the request, and traisinit their information back to tne "central control".

Very sophisticated learning procedures would beccme possible if one hus sich recall-by-association parallel-access memery systems. The simplicity of Adaline and the projress being made in microelectronics gives a. strong indication that euch memory systems will come into existence in the not too distant future.

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[^0]:    It becomes a vanishingly small. fraction of all possible awitching functions as the number of inputs gets large.

[^1]:    The probability of neuron error is minimized by the aduption procedure subject to the restriction ticct, $\bar{\xi}=0$. This does not preclude the possibility that the error probability could be even less with neuron adjustmenta that will not cause $\bar{c}$ to be zcro.

