IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received December 8, 2020, accepted December 23, 2020, date of publication December 28, 2020, date of current version January 7, 2021.

Digital Object Identifier 10.1109/ACCESS.2020.3047659

Adaptive Terminal Sliding Mode Control for Attitude and Position Tracking Control of Quadrotor UAVs in the Existence of External Disturbance

OMID MOFID[1], SALEH MOBAYEN[1], (Member, IEEE), AND WING-KWONG WONG[1,2]
[1]Future Technology Research Center, National Yunlin University of Science and Technology, Yunlin 64002, Taiwan, R.O.C.
[2]Department of Electronic Engineering, National Yunlin University of Science and Technology, Yunlin 64002, Taiwan, R.O.C.

Corresponding author: Saleh Mobayen (mobayens@yuntech.edu.tw)

ABSTRACT In this paper, a 6-Degrees-of-Freedom (6-DOF) Unmanned Aerial Vehicle (UAV) system with external disturbance corresponding to sensor failure is considered. The control method is presented in two parts. In the first part, the upper bound of external disturbance is known and a Proportional-Integral-Derivative (PID) Sliding Mode Control (SMC) technique is planned for maintaining the desired position in the finite time. Whereas, the upper bound of the external disturbance is considered unknown in the second part and the adaptive PID-SMC method is offered for stability and position tracking control of UAV systems. Using the Lyapunov stability notion, the offered control method proves that the quadrotor's states can be tracked and stabilized in the finite time. Moreover, for the approximation of unknown bound of the external disturbances which are entered in the quadrotor dynamic model at any moment, adaptive control laws have been applied. Finally, simulation outcomes are provided to display the efficiency of the recommended technique.

INDEX TERMS Quadrotor UAVs, finite-time stability, sliding mode control, external disturbance, adaptive control procedure.

I. INTRODUCTION

A. BACKGROUND AND MOTIVATIONS

Generally speaking, UAVs have attracted much attention during recent years and they are able to carry out various particular atmospheric skills, for instance, exploration of moving objects [1], [2], accumulating of traffic data [3], large-scale systems [4], examination of power transition lines [5], organizing military operations' supplies [6] and supplying first-aid kit in natural disasters [7]. On the whole, unmanned aerial vehicles can be applied in military and civilian operations [8]–[11]. In fact, there exist three problems in the controlling of unmanned aerial vehicles: (I) UAVs are multi-input multi-output systems; (II) UAVs have unknown parameters; (III) UAVs have time-varying states and delays [12]. Quadrotors are under-actuated control systems with 6-DOF and four independent control inputs [13]–[15]. In fact, translational and rotational motions in the quadrotor can be done by changing of speed of four rotors. It can be said that having a simpler configuration than helicopters is the main advantage of UAVs [16]. In addition, on the stability issue of quadrotors, non-linear parts and atmospheric perturbation should be considered [17], [18]. In general, the main goal of control of a quadrotor is attitude and altitude control which keeps the quadrotor in the set point [19], [20]. Therefore, some control approaches have been used for control of quadrotor including PID [21], [22], Linear Quadratic Regulator (LQR) [23], [24], feedback linearization control [25], [26], back-stepping control [27], [28], SMC [29], [30], and adaptive control [29], [31]. SMC scheme is accepted as an efficient instrument for planning a robust controller of high-order systems with nonlinear and uncertain components [32], [33]. The little sensitivity to the perturbations and uncertainty in the SMC scheme can reduce the accuracy of the model of system [34], [35]. SMC is classified via robustness to parametric variations and insensitivity to

The associate editor coordinating the review of this manuscript and approving it for publication was Mohammad Alshabi.

disturbances and it has been famous as a helpful approach to deal with uncertainties of the system [36]–[39]. While SMC fulfills solitary asymptotic stability and tracking, the Fast Terminal Sliding Mode Control (FTSMC) has provided a fast terminal sliding surface [40]. The main advantage of FTSMC is the finite time stability and tracking of system's trajectories [41], [42].

B. LITERATURE REVIEW

In [43], a new Radial Basis Function Neural Network (RBFNN) for multilateral telerobotic system in the presence of time-delay, exterior perturbations and uncertainty has been offered by applying the adaptive sliding mode control technique. In [44], the control of synchronization of redundant actuator and motion coordination of multi-axes for Dual Linear Motor Driven (DLMD) gantry has been proposed based on the contouring control technique by integrating both motion coordination between axes and synchronization of redundant actuators. In [45], in the aim of attitude and altitude tracking control of UAVs under perturbations and indeterminacy, a novel robust controller has been recommended and the stability verification has been done by Lyapunov concept. However, the tracking control is done asymptoticly. In [46], an adaptive back@hyphestepping SMC of attitude tracking of quadrotor has been recommended using Lyapunov theory in the existence of nonlinear components. However, the influence of external disturbances has been ignored in this method. In [47], TSMC plan based on the transfer function of tensor is offered. The proposed controller of [47] has been designed separately. At first, transfer function of Single-Input Single-Output (SISO) system has been presented. Secondly, TSMC for the offered system is designed. Though the impact of uncertainty has not been considered in this paper. In [48], an adaptive non-singular terminal sliding mode control under unknown dynamics has been presented for unmanned aerial vehicle. Also, the tracking control of position and attitude of quadrotor is investigated based on the Lyapunov theory. However, the finite-time convergence rate and the chattering phenomenon elimination are not considered in this article. A finite-time tracking control of quadrotor in the presence of uncertainty has been introduced in [49]. Although this article presents a valuable convergence rate, however the removal to the chattering phenomenon which reduces the performance of system has not been examined. In [50], a Nonsingular Terminal Sliding Mode Control (NTSMC) technique has been proposed for quadrotor control under state uncertainties and exterior perturbations. Moreover, the fault detection and isolation technique in the target of fault identification has been recommended in [50]. However, the adaptive procedure for estimation of the upper bound of the external disturbance has not been adopted and the chattering problem has not been investigated. In [51], the finite-time convergence for quadrotor based on the Global Terminal Sliding Mode Control (GTSMC) technique has been suggested for a quadrotor UAV; however, the effects of the external disturbances have been overlooked in this paper. In [52], feedback linearization scheme which converts nonlinear components to the fourth-order dynamics has been recommended for flight-tuning of quad-copters. Nevertheless, the stability is asymptotic which does not offer fast system performance. Also, SMC is employed for the control of dynamical inversion of feedback linearization's error in [52]. But, the adaptive procedure which can enhance the proficiency of the method is not adopted in the mentioned reference. In [53], in inner loop, a novel Hybrid Robust Three-axis Attitude Control (HRTAC) has been designed for the control of three-axis angular rotations. In the outer loop, linear quadratic controller has been planned for the control of rotation angles. In [54], PID controller and non-linear state feedback based on Linear Matrix Inequality (LMI) have been designed for the stability and tracking control of unmanned aerial vehicles. In [55], a simple model of quadrotor is applied and then, SMC is used for tracking control. In [56], a back-stepping control procedure has been designed for the stability control of physical dynamic model of unmanned aerial vehicle under sensor and actuator fault. In [57], trajectory tracking control of quadrotor has been designed in two subsystems. In the first subsystem, dual-loop integral SMC has been planned for tracking control of desired attitude and angular velocity. In the second subsystem, a global asymptotic controller for position tracking has been offered. A flight regulator of quadrotor based on the nonsingular TSMC has been suggested in the appearance of exterior disturbance and parameter uncertainty in the inner and outer control loops in [50]. In [58], the uncertainty compensator and SMC design based on the exponential convergence rate have been offered for quadrotor. But, in this article, the influence of external disturbances has been reduced. An incremental SMC based on the disturbance observer has been designed for UAVs with the aim of fault tolerant in [59]. A novel fractional-order back-stepping based on the SMC for tracking control of UAV has been presented in [60]. As stated by the previous authors' researches, no comprehensive work has been done in designing an adaptive PID sliding mode control method for the aim of finite time tracking of attitude and altitude of unmanned aerial vehicles. Consequently, the highest innovations of the recommended scheme of this article compared with the above-stated studies are the implementation of the adaptive PID sliding mode control scheme with fast and finite time convergence for UAVs and the alleviation of the chattering phenomenon in the control inputs.

C. CONTRIBUTIONS

In this article, the principal contributions are mentioned as below:

- A PID-SMC scheme with fast convergence is planned for the finite-time control of 6-DOF UAVs with bounded external disturbances.
- An adaptive PID-SMC technique is planned in the aim of quadrotor's tracking and stability control in the presence of the external perturbations with unknown bound of disturbances.

- Attitude stabilization and position tracking of quadrotor UAVs in the existence of external disturbances are designed.
- The suggested controller approach appropriately guarantees the finite time convergence of sliding mode around the switching surface.

D. PAPER ORGANIZATION

The following sections are organized in this article: a dynamical state-space model of 6-DOF quadrotor is presented in Sect. II. The problem explanation and some preparations such as external disturbances corresponding to sensor failure are provided in Sect. III. The PID-SMC method for fast stability and tracking of quadrotor against known bounded external disturbances is proposed in Sect. IV. An adaptive PID-SMC method in the aim of stability and tracking control of quadrotor by unknown bounded external disturbances is provided in Sect. V. Simulation outcomes are displayed in Sect. VI. To summarize the findings, conclusions are reported in Sect. VII.

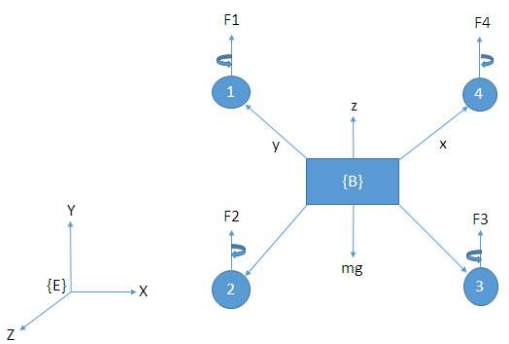

FIGURE 1. The structure of quadrotor with four motors.

II. DESCRIPTION OF UAV FLIGHT MODEL

An unmanned aerial quadrotor vehicle with 6-DOF with four rotors as illustrated in Fig.1 is considered in this section. On the whole, raising and reducing total thrust can cause every quadrotor to rotate and shift in six directions. A rigid cross-frame and symmetrical unmanned aerial vehicle has two pairs of rigid propellers (1, 3) and (2, 4) which rotate reversely. Also, thrust and drag forces are proportional to the square of the speed of propellers. For UAVs, there exist four types of movements and motions:

- raising and reducing four rotors' speed results in vertical motion,
- varying the speed of propellers (1, 3) causes roll and related lateral movement,
- modifying the speed of propellers (2, 4) creates pitch and related lateral movement,
- torque difference between (1, 3) and (2, 4) generates yaw rotation [61], [62].

Dynamical equations of quadrotor according to the Newton-Euler equations can be defined as [61]–[63]

$$\begin{cases} m\ddot{\xi} = F_f + F_d + F_g, \\ J\dot{\Omega} = -\Omega^T J\Omega + \Gamma_f - \Gamma_a - \Gamma_g, \end{cases} \tag{1}$$

where m and ξ denote system's mass and quadrotor's position relating to the inertia framework E, respectively. The matrix $J \in R^{3\times3}$ is recognized as quadrotor's inertia matrix regarding to the steady framework which is expressed as follows:

$$J = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}, \tag{2}$$

so that I_x, I_y and I_z demonstrate inertia amounts with regard to x, y, z axes. The expression Ω which is presented below describes the quadrotor's angular velocities:

$$\Omega = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}, \tag{3}$$

where ϕ, θ and ψ specify roll, pitch and yaw angles. Term F_f illustrates the consequence of forces which are produced from four propellers as follows:

$$F_f = \begin{bmatrix} \cos\phi\cos\psi\sin\theta + \sin\phi\sin\psi \\ \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi \\ \cos\phi\cos\theta \end{bmatrix} \sum_{i=1}^{4} F_i, \tag{4}$$

with $F_i = K_p w_i^2$. Now, K_p and w_i are the lift power factor and rotor's angular speed, respectively. The term F_d displays consequence of the powers which are created along x, y, z axes as follows:

$$F_d = \begin{bmatrix} -K_{fdx} & 0 & 0 \\ 0 & -K_{fdy} & 0 \\ 0 & 0 & -K_{fdz} \end{bmatrix} \dot{\xi}, \tag{5}$$

where K_{fdx}, K_{fdy} and K_{fdz} are positive translation drag constants. The gravity force F_g is expressed as

$$F_g = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix}, \tag{6}$$

so that g specifies gravity force. The term Γ_f is the quadrotor's moment respecting to the steady framework as:

$$\Gamma_f = \begin{bmatrix} d(F_3 - F_1) \\ d(F_4 - F_1) \\ C_d(w_1^2 - w_2^2 + w_3^2 - w_4^2) \end{bmatrix}, \tag{7}$$

so that d signifies the distance between center of quadrotor and propellers' rotation axis and C_d specifies the drag factor. The expression Γ_a shows torques of aerodynamic friction as

$$\Gamma_a = \begin{bmatrix} K_{fax} & 0 & 0 \\ 0 & K_{fay} & 0 \\ 0 & 0 & K_{faz} \end{bmatrix} \|\Omega\|^2, \tag{8}$$

so that K_{fax}, K_{fay}, K_{faz} specify aerodynamic friction factors. Expression Γ_g shows the resultant of torques because of effects of gyroscopic as

$$\Gamma_g = \sum_{i=1}^{4} \Omega^T J_r \begin{bmatrix} 0 \\ 0 \\ (-1)^{i+1} w_i \end{bmatrix}, \tag{9}$$

where J_r is the rotor inertia. The dynamic model of quadrotor is obtained as

$$\begin{aligned}
\ddot{x} &= \frac{1}{m}\left[-K_{fdx}\dot{x} + (\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi)u_1\right], \\
\ddot{y} &= \frac{1}{m}\left[-K_{fdy}\dot{y} + (\cos\phi \sin\theta \sin\psi - \sin\phi \cos\psi)u_1\right], \\
\ddot{z} &= \frac{1}{m}\left[-K_{fdz}\dot{z} + (\cos\phi \cos\theta)u_1\right] - g, \\
\ddot{\phi} &= \frac{1}{I_x}\left[(I_y - I_z)\dot{\psi}\dot{\theta} - K_{fax}\dot{\phi}^2 - J_r\bar{\Omega}\dot{\theta} + du_2\right], \\
\ddot{\theta} &= \frac{1}{I_y}\left[(I_z - I_x)\dot{\psi}\dot{\phi} - K_{fay}\dot{\theta}^2 + J_r\bar{\Omega}\dot{\phi} + du_3\right], \\
\ddot{\psi} &= \frac{1}{I_z}\left[(I_x - I_y)\dot{\phi}\dot{\theta} - K_{faz}\dot{\psi}^2 + C_D u_4\right],
\end{aligned} \tag{10}$$

where u_1, u_2, u_3 and u_4 are controller signals of system which correspond to the angular velocities of four propellers as

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ C_D & -C_D & C_D & -C_D \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix},$$
$$\bar{\Omega} = w_1 - w_2 + w_3 - w_4. \tag{11}$$

III. PROBLME DESCRIPTION AND SOME PRELIMINARIES

The dynamical equation of an UAV quadrotor with 6-DOF is expressed based on the state-space model. Then, the state-space model is divided into four subsystems and the desired vector is defined. To end, a Lemma which is related to the finite-time stability concept is introduced for the control objective.

Consider $X = [\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, z, \dot{z}]^T$, $U = [u_2, u_3, u_4, u_1]^T$ and $f_{si}(x,t) = [f_{s2}, f_{s3}, f_{s4}, f_{s1}]^T$ are the vectors of quadrotor's states, the control signals and sensor failure, respectively [64]. State-space formula of dynamical equation (10) can be considered as

$$\dot{X} = F(x) + G(x)U + f_{si}(x,t), \tag{12}$$

where $F(x)$ and $G(x)$ are

$$F(\mathrm{x}) = \begin{bmatrix} x_2 \\ a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 \\ x_4 \\ a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 \\ x_6 \\ a_7 x_2 x_4 + a_8 x_6^2 \\ x_8 \\ a_9 x_8 x_6 - g \end{bmatrix}, \tag{13}$$

$$G(\mathrm{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{\cos x_1 \cos x_3}{m} \end{bmatrix}, \tag{14}$$

with $a_1 = \frac{I_y - I_z}{I_x}$, $a_2 = -\frac{K_{fax}}{I_x}$, $a_3 = -\frac{J_r}{I_x}$, $a_4 = \frac{I_z - I_x}{I_y}$, $a_5 = -\frac{K_{fay}}{I_y}$, $a_6 = \frac{J_r}{I_y}$, $a_7 = \frac{I_x - I_y}{I_z}$, $a_8 = \frac{-K_{faz}}{I_z}$, $a_9 = \frac{-K_{fdz}}{m}$, $b_1 = \frac{d}{I_x}$, $b_2 = \frac{d}{I_y}$ and $b_3 = \frac{C_D}{I_z}$.

Now, the considered system in (12) is divided into four subsystems as follow:

$$\begin{aligned}
&\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 u_2 + f_{s2}, \end{cases} \\
&\begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 u_3 + f_{s3}, \end{cases} \\
&\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_4 + f_{s4}, \end{cases} \\
&\begin{cases} \dot{x}_7 = x_8 \\ \dot{x}_8 = a_9 x_8 x_6 - g + \dfrac{\cos x_1 \cos x_3}{m} u_1 + f_{s1}. \end{cases}
\end{aligned} \tag{15}$$

Hence, the state-space vector can be considered as $X = [X_1(t), X_2(t), X_3(t), X_4(t)]^T$, such that $X_1(t) = [x_1, x_2]^T$, $X_2(t) = [x_3, x_4]^T$, $X_3(t) = [x_5, x_6]^T$ and $X_4(t) = [x_7, x_8]^T$. Afterward, the desired vector can be defined as $X_d = [X_{1d}(t), X_{2d}(t), X_{3d}(t), X_{4d}(t)]^T$, such that $X_{1d}(t) = [x_{1d}, x_{2d}]^T$, $X_{2d}(t) = [x_{3d}, x_{4d}]^T$, $X_{3d}(t) = [x_{5d}, x_{6d}]^T$ and $X_{4d}(t) = [x_{7d}, x_{8d}]^T$ with $\dot{x}_{1d} = x_{2d}$, $\dot{x}_{3d} = x_{4d}$, $\dot{x}_{5d} = x_{6d}$ and $\dot{x}_{7d} = x_{8d}$, while x_{2d}, x_{4d}, x_{6d} and x_{8d} are differentiable functions of time.

IV. POSITION TRACKING OF QUADROTOR UAV

According to this section, firstly, the error signals between the real and desired trajectories of unmanned aerial vehicle are determined and then, the sliding surfaces are chosen suitably. Afterward, the suggested PID sliding manifold is defined. Finally, finite-time convergence errors and finite-time tracking action are studied based on Lyapunov concept. Consider the tracking errors as

$$E_i(t) = X_i(t) - X_{id}(t) = [e_i(t), \dot{e}_i(t)], \quad (\forall i = 1, 2, 3, 4) \tag{16}$$

where

$$\begin{cases} e_1(t) = x_1 - x_{1d} \\ \dot{e}_1(t) = x_2 - x_{2d} \end{cases} \quad \begin{cases} e_2(t) = x_3 - x_{3d} \\ \dot{e}_2(t) = x_4 - x_{4d} \end{cases}$$
$$\begin{cases} e_3(t) = x_5 - x_{5d} \\ \dot{e}_3(t) = x_6 - x_{6d} \end{cases} \quad \begin{cases} e_4(t) = x_7 - x_{7d} \\ \dot{e}_4(t) = x_8 - x_{8d} \end{cases} \tag{17}$$

Define the sliding surfaces as

$$S_i(t) = CE_i(t), \quad (\forall i = 1, \ldots, 4) \tag{18}$$

where $C = [c_1, c_2]$ are gain constants. Now, the suggested PID sliding surface is proposed as

$$\sigma_i(t) = k_p S_i(t) + k_i \int_0^t S_i(\tau) d\tau + k_d \dot{S}_i(t), \quad (\forall i = 1, 2, 3, 4) \tag{19}$$

where k_p, k_i and k_d denote the proportional, integral and derivative constants, respectively.

Theorem 1: Assume that the dynamic equations of quadrotor UAV are defined as (15) and the control inputs are designed as

$$\begin{aligned}
\dot{u}_2 = &-(c_2 b_1)^{-1} \Big\{ k_p(c_1 \dot{e}_1(t) + c_2(a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 \\
&+ b_1 u_2 - \dot{x}_{2d})) + k_i CE_1(t) + k_d(c_1(a_1 x_4 x_6 + a_2 x_2^2 \\
&+ a_3 \bar{\Omega} x_4 + b_1 u_2 - \dot{x}_{2d}) + c_2(a_1 \dot{x}_4 \dot{x}_6 + 2a_2 \dot{x}_2 x_2 \\
&+ a_3 \bar{\Omega} \dot{x}_4 - \ddot{x}_{2d})) + \kappa_1 sign(\sigma_1) |\sigma_1|^{\delta_1} + \nu_1 \sigma_1 \\
&+ \chi_1 sign(\sigma_1) \Big\}, \\
\dot{u}_3 = &-(c_2 b_2)^{-1} \Big\{ k_p(c_1 \dot{e}_2(t) + c_2(a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 \\
&+ b_2 u_3 - \dot{x}_{4d})) + k_i CE_2(t) + k_d(c_1(a_4 x_2 x_6 + a_5 x_4^2 \\
&+ a_6 \bar{\Omega} x_2 + b_2 u_3 - \dot{x}_{4d}) + c_2(a_4 \dot{x}_2 \dot{x}_6 + 2a_5 \dot{x}_4 x_4 \\
&+ a_6 \bar{\Omega} \dot{x}_2 - \ddot{x}_{4d})) + \kappa_2 sign(\sigma_2) |\sigma_2|^{\delta_2} + \nu_2 \sigma_2 \\
&+ \chi_2 sign(\sigma_2) \Big\}, \\
\dot{u}_4 = &-(c_2 b_3)^{-1} \Big\{ k_p(c_1 \dot{e}_3(t) + c_2(a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_4 - \dot{x}_{6d})) \\
&+ k_i CE_3(t) + k_d(c_1(a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_4 - \dot{x}_{6d}) \\
&+ c_2(a_7 \dot{x}_2 \dot{x}_4 + 2a_8 \dot{x}_6 x_6 - \ddot{x}_{6d})) + \kappa_3 sign(\sigma_3) |\sigma_3|^{\delta_3} \\
&+ \nu_3 \sigma_3 + \chi_3 sign(\sigma_3) \Big\}, \\
\dot{u}_1 = &-(c_2 \frac{cosx_1 cosx_3}{m})^{-1} \Big\{ k_p(c_1 \dot{e}_4(t) + c_2(a_9 x_8 \\
&- g + \frac{cosx_1 cosx_3}{m} u_1 - \dot{x}_{8d})) \\
&+ k_i CE_4(t) + k_d(c_1(a_9 x_8 - g + \frac{cosx_1 cosx_3}{m} u_1 - \dot{x}_{8d}) \\
&+ \kappa_4 sign(\sigma_4) |\sigma_4|^{\delta_4} + \nu_4 \sigma_4 + \chi_4 sign(\sigma_4) + c_2(a_9 \dot{x}_8 \\
&+ \frac{1}{m}[-\dot{x}_1 sinx_1 cosx_3 - \dot{x}_3 sinx_3 cosx_1] u_1 - \ddot{x}_{8d})) \Big\},
\end{aligned} \tag{20}$$

where κ_i and $\nu_i (\forall i = 1, \ldots, 4)$ are positive constants and $\chi_i (\forall i = 1, \ldots, 4)$ are scalar values which fulfill the subsequent condition:

$$|\Lambda_i| \leq \chi_i, \quad (\forall i = 1, \ldots, 4) \tag{21}$$

where $\Lambda_1 = (k_p c_2 + k_d c_1) f_{s2} + k_d c_2 \dot{f}_{s2}$, $\Lambda_2 = (k_p c_2 + k_d c_1) f_{s3} + k_d c_2 \dot{f}_{s3}$, $\Lambda_3 = (k_p c_2 + k_d c_1) f_{s4} + k_d c_2 \dot{f}_{s4}$ and $\Lambda_4 = (k_p c_2 + k_d c_1) f_{s1} + k_d c_2 \dot{f}_{s1}$.

Then, the switching surfaces (18) are stabilized exponentially and the PID sliding manifolds (1) converge to the equilibrium in the finite time. Therefore, the error signals (16) and (17) can converge to the origin in the finite time. Hence, the tracking control of quadrotor UAV under the external perturbations can be achieved completely.

Proof: The candidate Lyapunov function is formed as

$$V_i(t) = \frac{1}{2} \sigma_i^2(t). \quad (\forall i = 1, \ldots, 4) \tag{22}$$

Using the time derivative of the sliding surfaces (18), it can get

$$\dot{S}_i(t) = C\dot{E}_i(t), \quad (\forall i = 1, \ldots, 4) \tag{23}$$

By substituting (16) into (23), we obtain

$$\dot{S}_i(t) = [c_1, c_2] \begin{bmatrix} \dot{e}_i \\ \ddot{e}_i \end{bmatrix} = c_1 \dot{e}_i + c_2 \ddot{e}_i. \tag{24}$$

Now, by applying (17) to (2), the subsequent equations are achieved:

$$\begin{aligned}
\dot{S}_1(t) &= c_1 \dot{e}_1 + c_2 \ddot{e}_1 \\
&= c_1 \dot{e}_1 + c_2(a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 u_2 \\
&\quad + f_{s2} - \dot{x}_{2d}), \\
\dot{S}_2(t) &= c_1 \dot{e}_2 + c_2 \ddot{e}_2 \\
&= c_1 \dot{e}_2 + c_2(a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 u_3 \\
&\quad + f_{s3} - \dot{x}_{4d}), \\
\dot{S}_3(t) &= c_1 \dot{e}_3 + c_2 \ddot{e}_3 \\
&= c_1 \dot{e}_3 + c_2(a_7 x_2 x_4 + a_8 x_6^2 + b_3 u_4 + f_{s4} - \dot{x}_{6d}), \\
\dot{S}_4(t) &= c_1 \dot{e}_4 + c_2 \ddot{e}_4 \\
&= c_1 \dot{e}_4 + c_2(a_9 x_8 - g + \frac{cosx_1 cosx_3}{m} u_1 + f_{s1} - \dot{x}_{8d}).
\end{aligned} \tag{25}$$

Taking the time-derivative of (24), one obtains

$$\ddot{S}_i(t) = [c_1, c_2] \begin{bmatrix} \ddot{e}_i \\ e_i \end{bmatrix} = c_1 \ddot{e}_i + c_2 e_i. \tag{26}$$

where using (17), one finds

$$\begin{aligned}
\ddot{S}_1(t) &= c_1 \ddot{e}_1 + c_2 e_1 \\
&= c_1(a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 + b_1 u_2 + f_{s2} - \dot{x}_{2d}) \\
&\quad + c_2(a_1 \dot{x}_4 \dot{x}_6 + 2a_2 \dot{x}_2 x_2 + a_3 \bar{\Omega} \dot{x}_4 + b_1 \dot{u}_2 + \dot{f}_{s2} - \ddot{x}_{2d}), \\
\ddot{S}_2(t) &= c_1 \ddot{e}_2 + c_2 e_2 \\
&= c_1(a_4 x_2 x_6 + a_5 x_4^2 + a_6 \bar{\Omega} x_2 + b_2 u_3 + f_{s3} - \dot{x}_{4d}) \\
&\quad + c_2(a_4 \dot{x}_2 \dot{x}_6 + 2a_5 \dot{x}_4 x_4 + a_6 \bar{\Omega} \dot{x}_2 + b_2 \dot{u}_3 + \dot{f}_{s3} - \ddot{x}_{4d}), \\
\ddot{S}_3(t) &= c_1 \ddot{e}_3 + c_2 e_3 \\
&= c_1(a_7 x_2 x_4 + a_8 x_8^2 + b_3 u_4 + f_{s4} - \dot{x}_{6d}) \\
&\quad + c_2(a_7 \dot{x}_2 \dot{x}_4 + 2a_8 \dot{x}_6 x_6 + b_3 \dot{u}_4 + \dot{f}_{s4} - \ddot{x}_{6d}), \\
\ddot{S}_4(t) &= c_1 \ddot{e}_4 + c_2 e_4 \\
&= c_1(a_9 x_8 - g + \frac{cosx_1 cosx_3}{m} u_1 + f_{s1} - \dot{x}_{8d}) \\
&\quad + c_2(a_9 \dot{x}_8 + \frac{1}{m}[-\dot{x}_1 sinx_1 cosx_3 - \dot{x}_3 sinx_3 cosx_1] u_1 \\
&\quad + \frac{cosx_1 cosx_3}{m} \dot{u}_1 + \dot{f}_{s1} - \ddot{x}_{8d}).
\end{aligned} \tag{27}$$

Afterward, differentiating (19) with respect to time, one can obtain

$$\dot{\sigma}_i(t) = k_p\dot{S}_i(t) + k_iS_i(t) + k_d\ddot{S}_i(t), \quad (\forall i = 1, 2, 3, 4) \tag{28}$$

Now, by substituting (18), (25) and (27) into (28), it is realized that

$$\begin{aligned}
\dot{\sigma}_1(t) &= k_p(c_1\dot{e}_1 + c_2(a_1x_4x_6 + a_2x_2^2 + a_3\bar{\Omega}x_4 + b_1u_2 \\
&\quad + f_{s2} - \dot{x}_{2d})) + k_iCE_1(t) + k_d(c_1(a_1x_4x_6 + a_2x_2^2 \\
&\quad + a_3\bar{\Omega}x_4 + b_1u_2 + f_{s2} - \dot{x}_{2d}) + c_2(a_1\dot{x}_4\dot{x}_6 \\
&\quad + 2a_2\dot{x}_2x_2 + a_3\bar{\Omega}\dot{x}_4 + b_1\dot{u}_2 + \dot{f}_{s2} - \ddot{x}_{2d})),\\
\dot{\sigma}_2(t) &= k_p(c_1\dot{e}_2 + c_2(a_4x_2x_6 + a_5x_4^2 + a_6\bar{\Omega}x_2 + b_2u_3 \\
&\quad + f_{s3} - \dot{x}_{4d})) + k_iCE_2(t) + k_d(c_1(a_4x_2x_6 + a_5x_4^2 \\
&\quad + a_6\bar{\Omega}x_2 + b_2u_3 + f_{s3} - \dot{x}_{4d}) + c_2(a_4\dot{x}_2\dot{x}_6 \\
&\quad + 2a_5\dot{x}_4x_4 + a_6\bar{\Omega}\dot{x}_2 + b_2\dot{u}_3 + \dot{f}_{s3} - \ddot{x}_{4d})),\\
\dot{\sigma}_3(t) &= k_p(c_1\dot{e}_3 + c_2(a_7x_2x_4 + a_8x_6^2 + b_3u_4 + f_{s4} - \dot{x}_{6d})) \\
&\quad + k_iCE_3(t) + k_d(c_1(a_7x_2x_4 + a_8x_6^2 + b_3u_4 + f_{s4} - \dot{x}_{6d}) \\
&\quad + c_2(a_7\dot{x}_2\dot{x}_4 + 2a_8\dot{x}_6x_6 + b_3\dot{u}_4 + \dot{f}_{s4} - \ddot{x}_{6d})),\\
\dot{\sigma}_4(t) &= k_p(c_1\dot{e}_4 + c_2(a_9x_8 - g + \frac{cosx_1cosx_3}{m}u_1 + f_{s1} - \dot{x}_{8d})) \\
&\quad + k_iCE_4(t) + k_d(c_1(a_9x_8 - g + \frac{cosx_1cosx_3}{m}u_1 \\
&\quad + f_{s1} - \dot{x}_{8d}) + c_2(a_9\dot{x}_8 + \frac{1}{m}[-\dot{x}_1sinx_1cosx_3 \\
&\quad - \dot{x}_3sinx_3cosx_1]u_1 + \frac{cosx_1cosx_3}{m}\dot{u}_1 + \dot{f}_{s1} - \ddot{x}_{8d})).
\end{aligned} \tag{29}$$

Therefore, using (22) and (29), time-derivatives of the Lyapunov functions are calculated as

$$\begin{aligned}
\dot{V}_1(t) &= \sigma_1(t)\dot{\sigma}_1(t) \\
&= \sigma_1(t)[k_p(c_1\dot{e}_1 + c_2(a_1x_4x_6 + a_2x_2^2 + a_3\bar{\Omega}x_4 \\
&\quad + b_1u_2 + f_{s2} - \dot{x}_{2d})) + k_iCE_1(t) + k_d(c_1(a_1x_4x_6 \\
&\quad + a_2x_2^2 + a_3\bar{\Omega}x_4 + b_1u_2 + f_{s2} - \dot{x}_{2d}) \\
&\quad + c_2(a_1\dot{x}_4\dot{x}_6 + 2a_2\dot{x}_2x_2 + a_3\bar{\Omega}\dot{x}_4 + b_1\dot{u}_2 + \dot{f}_{s2} \\
&\quad - \ddot{x}_{2d}))],\\
\dot{V}_2(t) &= \sigma_2(t)\dot{\sigma}_2(t) \\
&= \sigma_2(t)[k_p(c_1\dot{e}_2 + c_2(a_4x_2x_6 + a_5x_4^2 + a_6\bar{\Omega}x_2 \\
&\quad + b_2u_3 + f_{s3} - \dot{x}_{4d})) + k_iCE_2(t) + k_d(c_1(a_4x_2x_6 \\
&\quad + a_5x_4^2 + a_6\bar{\Omega}x_2 + b_2u_3 + f_{s3} - \dot{x}_{4d}) \\
&\quad + c_2(a_4\dot{x}_2\dot{x}_6 + 2a_5\dot{x}_4x_4 + a_6\bar{\Omega}\dot{x}_2 + b_2\dot{u}_3 + \dot{f}_{s3} \\
&\quad - \ddot{x}_{4d}))],\\
\dot{V}_3(t) &= \sigma_3(t)\dot{\sigma}_3(t) \\
&= \sigma_3(t)[k_p(c_1\dot{e}_3 + c_2(a_7x_2x_4 + a_8x_6^2 + b_3u_4 + f_{s4} - \dot{x}_{6d})) \\
&\quad + k_iCE_3(t) + k_d(c_1(a_7x_2x_4 + a_8x_6^2 + b_3u_4 + f_{s4} - \dot{x}_{6d}) \\
&\quad + c_2(a_7\dot{x}_2\dot{x}_4 + 2a_8\dot{x}_6x_6 + b_3\dot{u}_4 + \dot{f}_{s4} - \ddot{x}_{6d}))],\\
\dot{V}_4(t) &= \sigma_4(t)\dot{\sigma}_4(t) = \sigma_4(t)[k_p(c_1\dot{e}_4 + c_2(a_9x_8 \\
&\quad - g + \frac{cosx_1cosx_3}{m}u_1 + f_{s1} - \dot{x}_{8d})) + k_iCE_4(t) \\
&\quad + k_d(c_1(a_9x_8 - g + \frac{cosx_1cosx_3}{m}u_1 + f_{s1} - \dot{x}_{8d}) \\
&\quad + c_2(a_9\dot{x}_8 + \frac{1}{m}[-\dot{x}_1sinx_1cosx_3 - \dot{x}_3sinx_3cosx_1]u_1 \\
&\quad + \frac{cosx_1cosx_3}{m}\dot{u}_1 + \dot{f}_{s1} - \ddot{x}_{8d}].
\end{aligned} \tag{30}$$

Substituting the control laws (20) into (30), one gets

$$\begin{aligned}
\dot{V}_i(t) = -\sigma_i(t)\kappa_isign(\sigma_i)\left|\sigma_i\right|^{\delta_i} - \sigma_i(t)\nu_i\sigma_i \\
- \sigma_i(t)\chi_isign(\sigma_i) + \sigma_i(t)\Lambda_i
\end{aligned} \tag{31}$$

By considering the condition (21), it yields

$$\begin{aligned}
\dot{V}_i(t) &\leq -\kappa_i\left|\sigma_i\right|^{\delta_i+1} - \nu_i\left|\sigma_i\right|^2 \\
&= -\rho_iV_i(\sigma) - \rho_jV_i^{\bar{\delta}_i}(\sigma), \\
&\quad (\forall i = 1, \ldots, 4 \& \forall j = 5, \ldots, 8)
\end{aligned} \tag{32}$$

where $\rho_i = 2\nu_i > 0$, $\rho_j = 2\kappa_i > 0$ and $\bar{\delta}_i = (\delta_i + 1)/2$. This finalizes the proof of theorem.

Remark 1: The sliding manifolds $\sigma_i(\forall i = 1, \ldots, 4)$ are converged to the origin in the finite time using the discontinuous control inputs $\dot{u}_i(\forall i = 1, \ldots, 4)$. Therefore, by taking integrations of $\dot{u}_i$ which are calculated as (20), the control signals $u_i(\forall i = 1, \ldots, 4)$ are obtained and the chattering action is removed.

V. ADAPTIVE POSITION TRACKING OF QUADROTOR UAV

It can be said that, at most of the time, the upper bound of external disturbance is unknown. So, an adaptive method is employed to approximate the bound of disturbance. The design procedure of the adaptive parameter-tuning scheme is expressed in the following theorem.

Assumption 1 [65], [66]: The initial conditions satisfy the inequality $\hat{\chi}_i(0) \geqq 0\ (\forall i = 1, \ldots, 4)$.

Theorem 2: Consider that the external disturbances f_{si} and $\dot{f}_{si}(\forall i = 1, \ldots, 4)$ are unknown but bounded while χ_i's in (21) are some unknown positive coefficients. Let the PID sliding manifold be expressed as (19). So, the tuning parameters $\hat{\chi}_i$ are adopted to approximate χ_i by the subsequent adaptation laws:

$$\dot{\hat{\chi}}_i = \Theta_i\left|\sigma_i(t)\right|(\forall i = 1, \ldots, 4) \tag{33}$$

where $\Theta_i(\forall i = 1, \ldots, 4)$ are the positive constants. Thus, the control inputs are obtained as

$$\begin{aligned}
\dot{u}_2 &= -(c_2b_1)^{-1}\Big\{k_p(c_1\dot{e}_1 + c_2(a_1x_4x_6 + a_2x_2^2 + a_3\bar{\Omega}x_4 \\
&\quad + b_1u_2 - \dot{x}_{2d})) + k_iCE_1(t) + k_d(c_1(a_1x_4x_6 + a_2x_2^2 \\
&\quad + a_3\bar{\Omega}x_4 + b_1u_2 - \dot{x}_{2d}) + c_2(a_1\dot{x}_4\dot{x}_6 + 2a_2\dot{x}_2x_2 \\
&\quad + a_3\bar{\Omega}\dot{x}_4 - \ddot{x}_{2d})) + \kappa_1sign(\sigma_1)\left|\sigma_1\right|^{\delta_1} + \nu_1\sigma_1 \\
&\quad + \hat{\chi}_1sign(\sigma_1)\Big\},\\
\dot{u}_3 &= -(c_2b_2)^{-1}\Big\{k_p(c_1\dot{e}_2 + c_2(a_4x_2x_6 + a_5x_4^2 + a_6\bar{\Omega}x_2 \\
&\quad + b_2u_3 - \dot{x}_{4d})) + k_iCE_2(t) + k_d(c_1(a_4x_2x_6 + a_5x_4^2 \\
&\quad + a_6\bar{\Omega}x_2 + b_2u_3 - \dot{x}_{4d}) + c_2(a_4\dot{x}_2\dot{x}_6 + 2a_5\dot{x}_4x_4 \\
&\quad + a_6\bar{\Omega}\dot{x}_2 - \ddot{x}_{4d})) + \kappa_2sign(\sigma_2)\left|\sigma_2\right|^{\delta_2} + \nu_2\sigma_2 \\
&\quad + \hat{\chi}_1sign(\sigma_2)\Big\},
\end{aligned}$$

$$\begin{aligned}\dot{u}_4 &= -(c_2 b_3)^{-1}\Big\{k_p(c_1\dot{e}_3+c_2(a_7x_2x_4+a_8x_6^2+b_3u_4-\dot{x}_{6d}))\\&\quad+k_iCE_3(t)+k_d(c_1(a_7x_2x_4+a_8x_6^2+b_3u_4-\dot{x}_{6d})\\&\quad+c_2(a_7\dot{x}_2\dot{x}_4+2a_8\dot{x}_6x_6-\ddot{x}_{6d}))+\kappa_3 sign(\sigma_3)\,|\sigma_3|^{\delta_3}\\&\quad+\nu_3\sigma_3+\hat{\chi}_3 sign(\sigma_3)\Big\},\\ \dot{u}_1 &= -(c_2\frac{cosx_1cosx_3}{m})^{-1}k_p(c_1\dot{e}_4+c_2(a_9x_8\\&\quad-g+\frac{cosx_1cosx_3}{m}u_1-\dot{x}_{8d}))+k_iCE_4(t)\\&\quad+k_d(c_1(a_9x_8-g+\frac{cosx_1cosx_3}{m}u_1-\dot{x}_{8d})\\&\quad+c_2(a_9\dot{x}_8+\frac{1}{m}[-\dot{x}_1sinx_1cosx_3-\dot{x}_3sinx_3cosx_1]u_1\\&\quad-\ddot{x}_{8d})+\nu_4\sigma_4+\kappa_4 sign(\sigma_4)\,|\sigma_4|^{\delta_4}+\hat{\chi}_4 sign(\sigma_4)\}.\end{aligned} \tag{34}$$

Therefore, the offered PID sliding manifolds can reach the origin with an exponential convergence rate. Thus, the tracking control of quadrotor UAV under the external perturbations can be achieved completely.

Proof: The Lyapunov function is formed as

$$V_i(t)=\frac{1}{2}\mu_i\tilde{\chi}_i^2+\frac{1}{2}\sigma_i^2(t),\quad (\forall i=1,\ldots,4) \tag{35}$$

where μ_i is a positive constant which fulfills $\mu_i<\frac{1}{\Theta_i}$. With respect to time-derivative of the Lyapunov function (35), it obtains

$$\dot{V}_i(t)=\mu_i\dot{\tilde{\chi}}_i\tilde{\chi}_i+\sigma_i(t)\dot{\sigma}_i(t) \tag{36}$$

Considering $\tilde{\chi}_i=\hat{\chi}_i-\chi_i$ and substituting (28) into (36), we obtain

$$\dot{V}_i(t)=\mu_i\dot{\hat{\chi}}_i(\hat{\chi}_i-\chi_i)+\sigma_i(t)(k_p\dot{S}_i(t)+k_iS_i(t)+k_d\ddot{S}_i(t)). \tag{37}$$

Now, the adaptation laws (33) are substituted into (37) where using (18), (25) and (27), we have

$$\begin{aligned}\dot{V}_1(t) &= \mu_1\Theta_1\left(\hat{\chi}_1-\chi_1\right)|\sigma_1(t)|+\sigma_1(t)\left(k_p\dot{S}_1(t)\right.\\&\quad\left.+k_iS_1(t)+k_d2(t)\right)\\&= \mu_1\Theta_1(\chi_1-\hat{\chi}_1)\,|\sigma_1(t)|+\sigma_1^T(t)[k_p(c_1\dot{e}_1\\&\quad+c_2(a_1x_4x_6+a_2x_2^2+a_3\bar{\Omega}x_4\\&\quad+b_1u_2+f_{s2}-\dot{x}_{2d}))+k_iCE_1(t)+k_d(c_1(a_1x_4x_6\\&\quad+a_2x_2^2+a_3\bar{\Omega}x_4+b_1u_2+f_{s2}-\dot{x}_{2d})\\&\quad+c_2(a_1\dot{x}_4\dot{x}_6+2a_2\dot{x}_2x_2+a_3\bar{\Omega}\dot{x}_4+b_1\dot{u}_2\\&\quad+\dot{f}_{s2}-\ddot{x}_{2d}))],\\ \dot{V}_2(t) &= \mu_2\Theta_2\left(\hat{\chi}_2-\chi_2\right)|\sigma_2(t)|+\sigma_2(t)\left(k_p\dot{S}_2(t)\right.\\&\quad\left.+k_iS_2(t)+k_d\ddot{S}_2(t)\right)\\&= \mu_2\Theta_2(\hat{\chi}_2-\chi_2)\,|\sigma_2(t)|+\sigma_2(t)[k_p(c_1\dot{e}_2\\&\quad+c_2(a_4x_2x_6+a_5x_4^2+a_6\bar{\Omega}x_2+b_2u_3+f_{s3}-\dot{x}_{4d}))\\&\quad+k_iCE_2(t)+k_d(c_1(a_4x_2x_6+a_5x_4^2+a_6\bar{\Omega}x_2\\&\quad+b_2u_3+f_{s3}-\dot{x}_{4d})+c_2(a_4\dot{x}_2\dot{x}_6+2a_5\dot{x}_4x_4\\&\quad+a_6\bar{\Omega}\dot{x}_2+b_2\dot{u}_3+\dot{f}_{s3}-\ddot{x}_{4d}))],\end{aligned}$$

$$\begin{aligned}\dot{V}_3(t) &= \mu_3\Theta_3\left(\hat{\chi}_3-\chi_3\right)|\sigma_3(t)|+\sigma_3(t)\left(k_p\dot{S}_3(t)\right.\\&\quad\left.+k_iS_3(t)+k_d\ddot{S}_3(t)\right)\\&= \mu_3\Theta_3(\hat{\chi}_3-\chi_3)\,|\sigma_3(t)|+\sigma_3(t)[k_p(c_1\dot{e}_3\\&\quad+c_2(a_7x_2x_4+a_8x_6^2+b_3u_4+f_{s4}-\dot{x}_{6d})))\\&\quad+k_iCE_3(t)+k_d(c_1(a_7x_2x_4+a_8x_6^2+b_3u_4\\&\quad+f_{s4}-\dot{x}_{6d})+c_2(a_7\dot{x}_2\dot{x}_4+2a_8\dot{x}_6x_6+b_3\dot{u}_4\\&\quad+\dot{f}_{s4}-\ddot{x}_{6d}))],\\ \dot{V}_4(t) &= \mu_4\Theta_4(\hat{\chi}_4-\chi_4)\,|\sigma_4(t)|+\sigma_4(t)(k_p\dot{S}_4(t)+k_iS_4(t)\\&\quad+k_d\ddot{S}_4(t))\\&= \mu_4\Theta_4(\hat{\chi}_4-\chi_4)\,|\sigma_4(t)|+\sigma_4(t)[k_p(c_1\dot{e}_4+c_2(a_9x_8\\&\quad-g+\frac{cosx_1cosx_3}{m}u_1+f_{s1}-\dot{x}_{8d}))\\&\quad+k_iCE_4(t)+k_d(c_1(a_9x_8-g+\frac{cosx_1cosx_3}{m}u_1\\&\quad+f_{s1}-\dot{x}_{8d})+c_2(a_9\dot{x}_8+\frac{1}{m}[-\dot{x}_1sinx_1cosx_3\\&\quad-\dot{x}_3sinx_3cosx_1]u_1+\frac{cosx_1cosx_3}{m}\dot{u}_1+\dot{f}_{s1}-\ddot{x}_{8d}))].\end{aligned} \tag{38}$$

Then, using the control inputs (34), the same expressions in equation (38) are removed, then one finds:

$$\begin{aligned}\dot{V}_i(t) &= \mu_i\Theta_i(\hat{\chi}_i-\chi_i)\,|\sigma_i(t)|+\sigma_i^T(t)(\kappa_i sign(\sigma_i(t))\,|\sigma_i(t)|^{\delta_i}\\&\quad-\nu_i\sigma_i(t)-\hat{\chi}_i sign(\sigma_i(t))+\Lambda_i)\\&\le \mu_i\Theta_i(\hat{\chi}_i-\chi_i)\,|\sigma_i(t)|-\kappa_i\,|\sigma_i|^{\delta_i+1}-\nu_i|\sigma_i(t)|^2\\&\quad-\hat{\chi}_i|\sigma_i(t)|+|\Lambda_i||\sigma_i(t)|,\end{aligned} \tag{39}$$

Since the expression $-\kappa_i\,|\sigma_i|^{\delta_i+1}-\nu_i|\sigma_i(t)|^2$ is negative, it can be removed from the above inequality. Hence, one obtains:

$$\dot{V}_i(t)\le\mu_i\Theta_i(\hat{\chi}_i-\chi_i)\,|\sigma_i(t)|-\hat{\chi}_i|\sigma_i(t)|+|\Lambda_i||\sigma_i(t)|, \tag{40}$$

Now, by adding and subtracting the term $\chi_i|\sigma_i(t)|$, one can achieve:

$$\begin{aligned}\dot{V}_i(t) &\le \mu_i\Theta_i(\hat{\chi}_i-\chi_i)\,|\sigma_i(t)|-\hat{\chi}_i|\sigma_i(t)|+|\Lambda_i||\sigma_i(t)|\\&\quad+\chi_i|\sigma_i(t)|-\chi_i|\sigma_i(t)|\end{aligned} \tag{41}$$

After some calculations, we have:

$$\dot{V}_i(t)\le-(1-\mu_i\Theta_i)\left(\hat{\chi}_i-\chi_i\right)|\sigma_i(t)|-(\chi_i-|\Lambda_i|)\,|\sigma_i(t)|. \tag{42}$$

If the conditions $|\Lambda_i|<\chi_i$ and $\mu_i\Theta_i<1$ are satisfied, then, by considering the Lyapunov function (35), Eq. (4) can be represented as follows:

$$\dot{V}_i(t)\le-\sqrt{\frac{2}{\mu_i}}(1-\mu_i\Theta_i)\frac{\tilde{\chi}_i}{\sqrt{\frac{2}{\mu_i}}}\,|\sigma_i(t)|-\sqrt{2}(\chi_i-|\Lambda_i|)\frac{|\sigma_i(t)|}{\sqrt{2}} \tag{43}$$

Now, by considering a new parameter as $\Upsilon_i=\min\{\sqrt{\frac{2}{\mu_i}}(1-\mu_i\Theta_i)\,|\sigma_i(t)|\,,\sqrt{2}(\chi_i-|\Lambda_i|)\}>0$, we have

$$\dot{V}_i(t)\le-\Upsilon_i(\frac{\tilde{\chi}_i}{\sqrt{\frac{2}{\mu_i}}}+\frac{|\sigma_i(t)|}{\sqrt{2}})=-\Upsilon_iV_i^{\frac{1}{2}}(t), \tag{44}$$

The proof of this theorem is completed. □

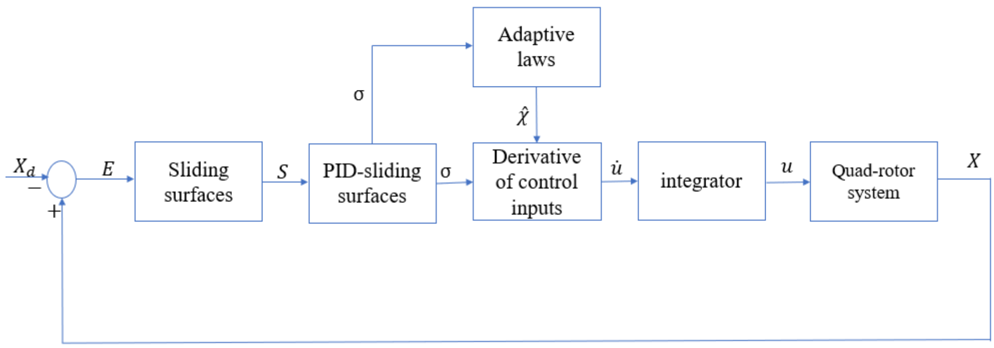

FIGURE 2. Block diagram of the proposed control technique.

TABLE 1. Quadrotor parameters [61], [62].

Parameter	Value	Parameter	Value
K_{fax} ($N/rad/s$)	5.5670e-4	m (Kg)	0.486
K_{fay} ($N/rad/s$)	5.5670e-4	d (m)	0.25
K_{faz} ($N/rad/s$)	6.3540e-4	I_x ($N.m/rad/s^2$)	3.8278e-3
K_{fdx} ($N/m/s$)	5.5670e-4	I_y ($N.m/rad/s^2$)	3.8278e-3
K_{fdy} ($N/m/s$)	5.5670e-4	I_z ($N.m/rad/s^2$)	7.6566e-3
K_{fdz} ($N/m/s$)	6.3540e-4	C_d ($N.m/rad/s$)	3.2320e-2
K_p ($N.m/rad/s$)	2.9842e-3	J_r ($N.m/rad/s^2$)	2.8385e-5

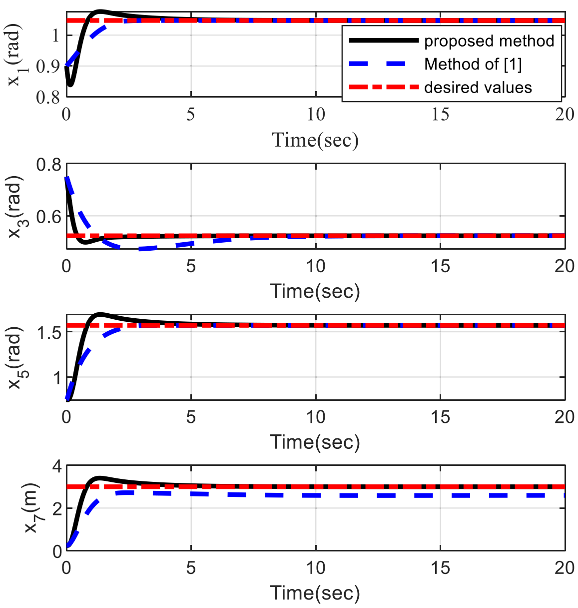

FIGURE 3. Tracking of state trajectories in the presence of known bounded disturbances.

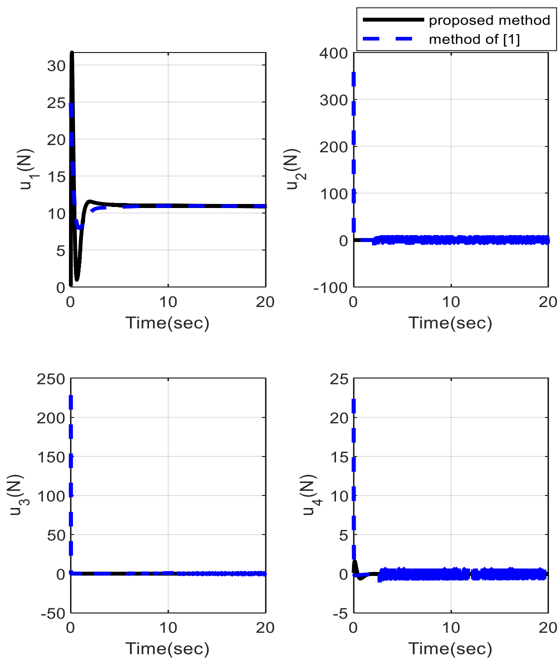

FIGURE 4. Time responses of control inputs under known bounded external disturbances.

VI. SIMULATION RESULTS

The block diagram of the proposed control technique has been depicted in Fig.2. The planned control process has been considered in two parts: in the first part, the upper bounds of the external disturbances are known, but in the second part, the bounds of external disturbances are unknown. To address the unknown bounds, an adaptive-regulator technique is offered to estimate the external disturbance. In both parts, the simulation outcomes are developed with realistic values according to Table 1. Also, the simulation results are compared with the results of the control method of [1]. The sliding surface in this article is defined as $s(t) = k_p e(t) + k_i \int_0^t e(w)\, dW + k_d \dot{e}$. The initial conditions are given as $X(0) = [0.9, -0.9, 0.75, -1, 0.75, -0.1, 0.25, -0.1]$. The initial quantities of adaptive controller can be put as $\hat{\chi}(0) = 0.5(\forall i = 1, \ldots, 4)$. The design parameters are selected as $C = [c_1, c_2] = [5, 0.6]$, $k_p = 4$, $k_i = 2$, $k_d = 1$, $\nu_1 = \nu_2 = 0.3$, $\nu_3 = \nu_4 = 0.15$, $\kappa_i = 15(\forall i = 1, \ldots, 4)$, $\delta_i = 3/5(\forall i = 1, \ldots, 4)$ and $\Theta_i = 0.05(\forall i = 1, \ldots, 4)$. It is noted that the parameters of the control strategy have been obtained by trial and error approach. Moreover, the desired quantities are taken as $\phi_d = \frac{\pi}{3}$, $\theta_d = \frac{\pi}{6}$, $\psi_d = \frac{\pi}{2}$ and $z_d = 3$. The external disturbances are taken as $f_{s1} = 0.1x_7 \sin(t)$, $f_{s2} = 0.1x_1 \cos(t)$, $f_{s3} = 0.1x_3 \sin(2t)$ and $f_{s4} = 0.1x_5 \cos(2t)$.

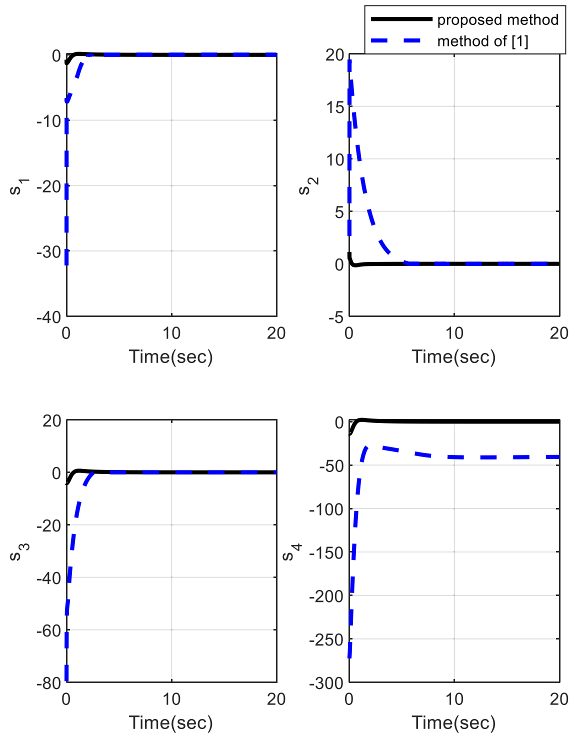

FIGURE 5. Sliding surfaces under known bounded external disturbances.

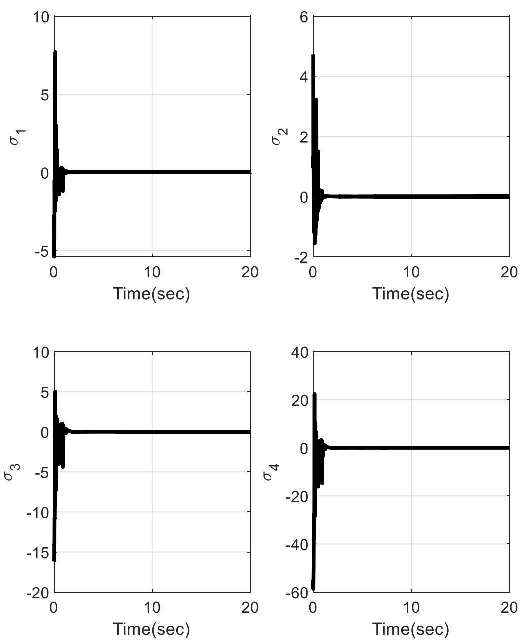

FIGURE 6. PID Sliding manifolds under external disturbances with known bounds.

Fig.3 displays the time history of the states of quadrotor UAV system with known bounded external disturbances. It can be found from Fig.3 that using the proposed control method, the tracking purpose is fulfilled appropriately.

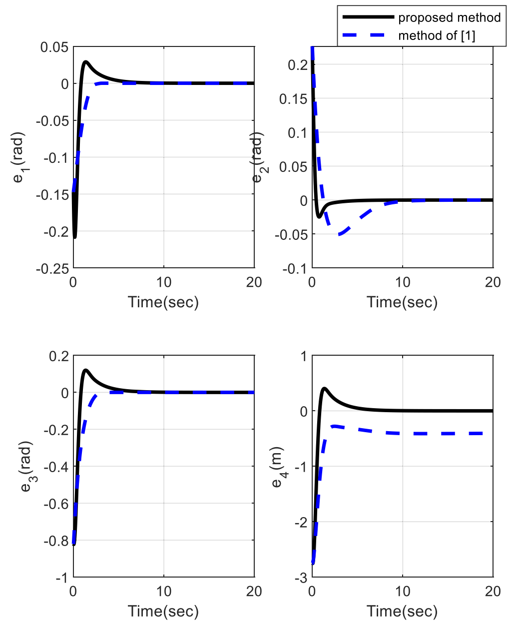

FIGURE 7. Time responses of the errors under known bounded external disturbances.

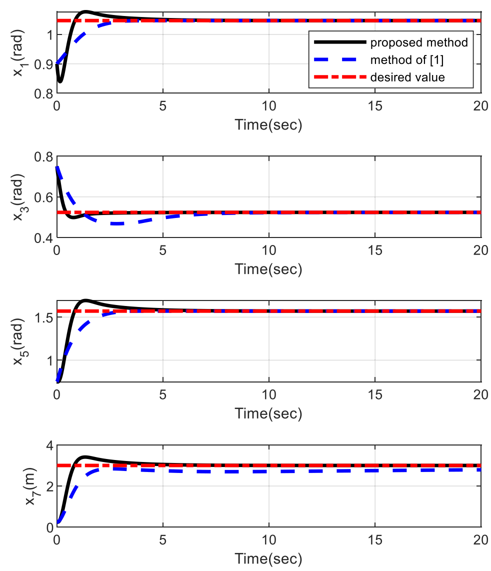

FIGURE 8. Time trajectories of system states in the presence of unknown bounded disturbances.

It can be perceived from Fig.4 that in the control signals with proper amplitudes, there is no chattering under the known bounded external disturbances.

Fig.5 shows the time trajectories of the sliding surfaces, which demonstrates the exponential convergence to zero.

Fig.6 displays the time responses of the PID-sliding manifolds $\sigma_i(t)$, which confirms that σ_i converges to zero in the finite time.

The time histories of errors defined in (16) and (1) are shown in Fig.7, which demonstrates the convergence of the error signals to zero properly.

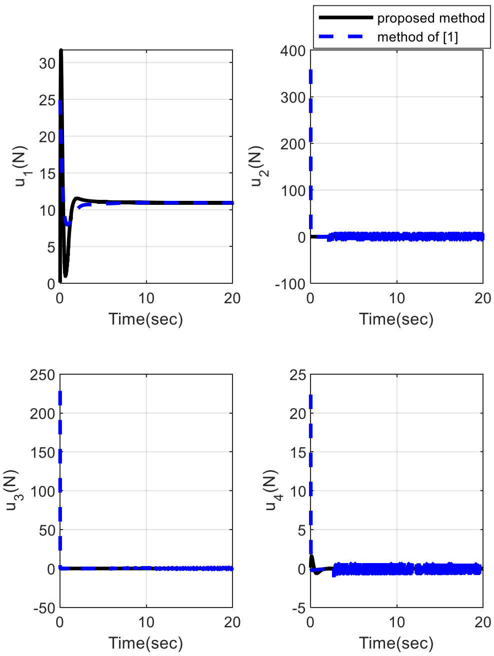

FIGURE 9. Time responses of control inputs in the presence of unknown bounded external disturbances.

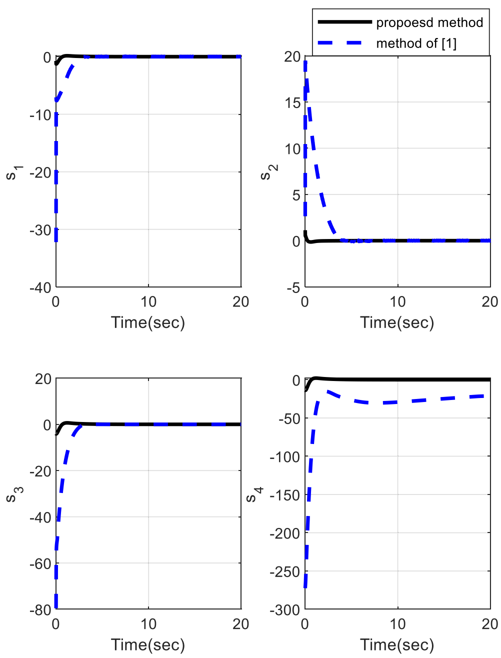

FIGURE 10. Sliding surfaces in the existence ofunknown bounded disturbances.

For the second part, as mentioned before, the simulation results are presented in the sense of unknown bounded disturbances. Fig.8 demonstrates the time histories of the quadrotor states with unknown bounded disturbances. It is confirmed from Fig.8 that the tracking purpose is achieved suitably.

It can be gotten from Fig.9 that in the control signals with proper amplitudes, there is no chattering in the presence of unknown bounded disturbances.

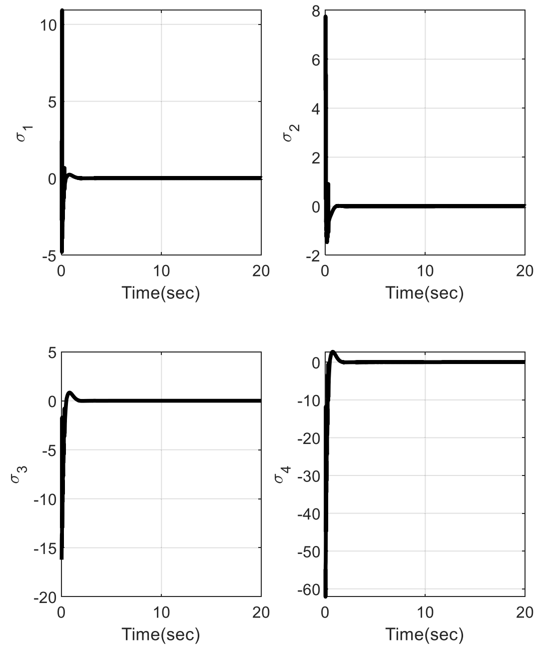

FIGURE 11. PID Sliding surfaces in the existence of unknown bounded disturbances.

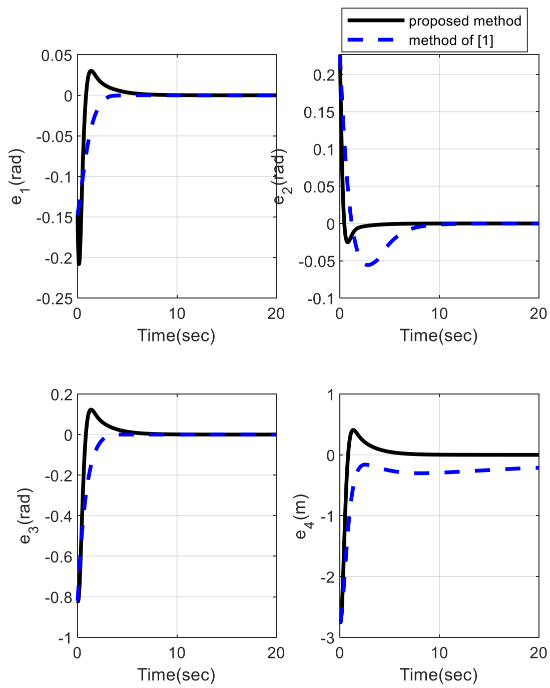

FIGURE 12. Time responses of the errors in the presence of unknown bounded external disturbances.

Fig.10 shows the sliding surfaces $s_i(t)$ which illustrates that $s_i(t)$ converges to the origin exponentially under unknown bounded external disturbances.

Fig.11 shows PID sliding manifold $\sigma_i(t)$, which proves that $\sigma_i(t)$ converges to zero suitably under unknown bounded external disturbances.

Time histories of the error signals are exhibited in Fig.12, which confirms the exponential convergence of errors to the origin.

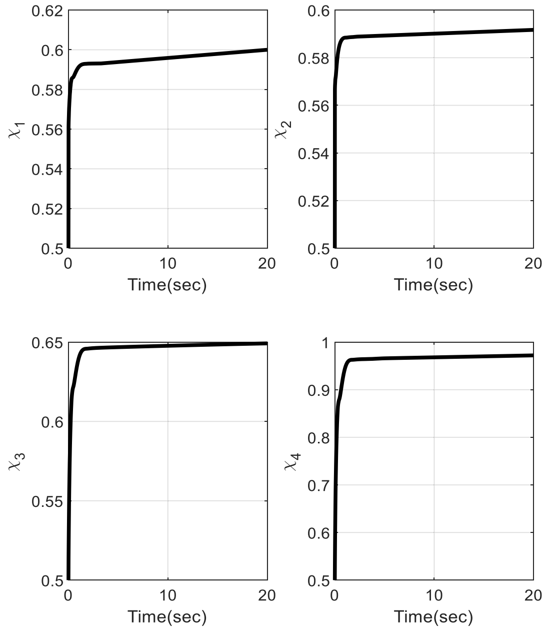

FIGURE 13. Adaptive-tuners $\hat{\chi}_i (i = 1, \ldots, 4)$.

The adaptation laws are presented in Fig.13, which demonstrates the approximation of uncertain parameters appropriately.

The above simulation results show that the planned controller based on the PID-SMC scheme and adaptive PID-SMC technique is useful for tracking control of unmanned aerial vehicle system. Additionally, As a result, it can be found from comparison that the sliding surfaces of the proposed method converge to zero faster than the surfaces of the method of [1]. Moreover, the control inputs in our proposed technique have suitable amplitude without chattering, but the control signals of the method of [1] have high amplitude with noticeable chattering problem which reduce the system performance.

VII. CONCLUSION

In this paper, the dynamical state-space equation of an unmanned aerial vehicle with six degree-of-freedom is set up according to the Newton-Euler formulization. Then, the external disturbance related to sensor failure and aerodynamic perturbation is entered in the dynamic model. The control procedure is done in two various steps. At the first step, a PID-SMC scheme is proposed in the aim of the finite-time stability and tracking control of UAV system with 6-DOF in the presence of exterior disturbances with known bounds. In the second step, the upper bound of the external disturbance has been considered unknown; thus, an adaptive PID-SMC is proposed to estimate the external disturbances. Additionally, a PID sliding surface has been specified and the stability of the system has been proved based on the Lyapunov stability concept. Simulations are provided to demonstrate the effectiveness of the recommended approach. The design problem of finite time tracker for a practical quadrotor UAV in the presence of time-delays in the control inputs will be studied in our future research work.

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OMID MOFID was born in Saveh, Iran, in October 1992. He received the B.S. degree in mathematical science and applications from the University of Tafresh, Tafresh, Iran, in 2015, and the M.Sc. degree in control engineering from the University of Zanjan. As an undergraduate, he worked at the control research laboratory. He is focusing on development and implementation of sliding mode control and adaptive control techniques on quad-rotor UAV systems. His research interests include mobile robots, adaptive control, sliding mode control, and aerospace vehicles.

SALEH MOBAYEN (Member, IEEE) received the B.Sc. and M.Sc. degrees in control engineering from the University of Tabriz, Tabriz, Iran, in 2007 and 2009, respectively, and the Ph.D. degree in control engineering from Tarbiat Modares University, Tehran, Iran, in January 2013. From February 2013 to December 2018, he was an Assistant Professor and a Faculty Member with the Department of Electrical Engineering, University of Zanjan, Zanjan, Iran. Since December 2018, he has been an Associate Professor of control engineering with the Department of Electrical Engineering, University of Zanjan. He is currently with the Future Technology Research Center, National Yunlin University of Science and Technology, as an Associate Professor. His research interests include control theory, sliding mode control, robust tracking, non-holonomic robots, and chaotic systems.

WING-KWONG WONG received the B.Sc. degree in computer science from the University of Toronto, Canada, in 1983, and the Ph.D. degree in computer science from The University of Texas at Austin, USA, in 1992. Since August 1993, he has been an Associate Professor with the Department of Electronic Engineering, National Yunlin University of Science and Technology (YunTech), Taiwan. He is currently the Director of the Future Technology Research Center, YunTech. He acknowledges the support of the Ministry of Science and Technology, Taiwan under Contract MOST 108-2511-H-224-007-MY3. His research interests include robotics, machine learning, the IoT, and e-learning.