

Adaptive testing of a deterministic implementation against a nondeterministic finite state machine

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abstract A number of authors have looked at the problem of deriving a checking experiment from a nondeterministic finite state machine that models the required behaviour of a system. We show that these methods can be extended if it is known that the implementation is equivalent to some (unknown) deterministic finite state machine.

When testing a deterministic implementation, the test output provides information about the implementation under test and can thus guide future testing. The use of an adaptive test process is thus proposed.

1 Introduction

Nondeterministic finite state machines (NFSMs) are used to model a number of types of computer system including communications protocols (Tanenbaum [1996]). If an NFSM model M , of the required behaviour, exists and some implementation I has been produced, it is important to verify I against M . Testing usually forms part of the verification process. When testing it is normal to assume that the *implementation under test (IUT)* I behaves like some (possibly nondeterministic) finite state machine M_I .

As nondeterminism can aid abstraction, many specifications are nondeterministic. Actual implementations are, however, often deterministic. The situation, in which M is nondeterministic and M_I is deterministic, is thus of interest.



An NFSM M is defined by a tuple (S, s_1, h, X, Y) in which S is a finite set of *states*, s_1 is the *initial state*, X is the finite *input alphabet*, Y is the finite *output alphabet* and h is the *state transition function*. Given an NFSM M , S_M shall denote the state set of M . When M receives an input value $x \in X$, while in state $s \in S$, a *transition* is executed producing an output value $y \in Y$ and moving M to some state $s' \in S$. The function h gives the possible transitions and has type $h : S \times X \rightarrow \mathcal{P}(S \times Y)$, where \mathcal{P} denotes the power set operator. Thus, if M receives input $x \in X$ while in state $s \in S$, the output $y \in Y$ and next state $s' \in S$ satisfy $(s', y) \in h(s, x)$.

An NFSM, that shall be denoted M_0 throughout this paper, is given in Figure 1. Here, $S = \{s_1, \dots, s_5\}$, $X = \{a, b\}$ and $Y = \{0, 1\}$. If b is input while M_0 is in state s_4 , either 0 is output and M_0 moves to state s_5 or 1 is output and M_0 moves to state s_2 . Thus $h(s_4, b) = \{(s_5, 0), (s_2, 1)\}$.

A *finite automaton (FA)* N is defined by a tuple $(S, s_1, \gamma, \Sigma, F)$, in which S is a finite set of *states*, $s_1 \in S$ is the *initial state*, γ is the *state transition function*, Σ is the finite *alphabet* and $F \subseteq S$ is the set of *final states*. The function γ takes a state $s \in S$ and a value $x \in \Sigma$ and gives a set of possible next states $\gamma(s, x)$. Let ϵ denote the empty sequence and, given a set Γ , Γ^* denote the set of strings consisting of elements of Γ . Thus $\epsilon \in \Gamma^*$. The transition function γ can be extended, to take values from Σ^* , giving the function γ^* defined by: $\gamma^*(s, \epsilon) = \{s\}$, $\gamma^*(s, yx) = \{s' \mid (\exists s'' \in \gamma^*(s, y) \bullet s' \in \gamma(s'', x))\}$ (where $x \in \Sigma$ and $y \in \Sigma^*$).

FA are used to define languages: a string $x \in \Sigma^*$ is *accepted* by N , and thus is part of the language defined by N , if and only if $\gamma^*(s_1, x) \cap F \neq \emptyset$. The FA N is deterministic if for each state $s \in S$ and input value $x \in \Sigma$, there is only one possible next state. Given a nondeterministic FA there is some deterministic FA that defines the same language. There are standard algorithms for finding such an equivalent deterministic FA (Rabin and Scott [1959]).

An NFSM M can be thought of as a FA $F(M)$ in which the elements of Σ are the input/output pairs and all the states are final states. The NFSM M is then characterized by the language defined by $F(M)$: the set of input/output sequences that M allows. This set shall be denoted $L(M)$.

If $L(M_I) \subseteq L(M)$, M_I is said to be a *reduction* of M and this is denoted $M_I \leq M$. One definition of conformance is that I *conforms* to M if $M_I \leq M$. Testing can thus be characterized as trying to determine whether I conforms to M . This definition of conformance matches the notion of correctness used in (Dick and Faivre [1993], Hierons [1997a]) and will be used throughout the



rest of the paper.

A transition is defined by its initial state, final state, input and output. Given a sequence t (possibly of length 1) of transitions, t^{in} will denote the input sequence from t , t^{out} will denote the output sequence produced by t , $start(t)$ will denote the initial state of t and $end(t)$ will denote the final state of t .

From the function h , the next state function h_1 and output function h_2 can be derived. Thus if M receives input x while in state s the next state is one of those contained in $h_1(s, x)$ and the output is one of the values from $h_2(s, x)$. The functions h , h_1 , and h_2 can be extended, in a similar manner to γ , to take a state and an input sequence giving functions h^* , h_1^* , and h_2^* respectively.

An NFSM M is *completely specified* if, for each $s \in S$ and $x \in X$, $|h(s, x)| \geq 1$ and M is *deterministic* if for each $s \in S$ and $x \in X$, $|h(s, x)| \leq 1$. If M is deterministic and completely specified the transitions are completely defined by a *next state function* δ and an *output function* λ . A *deterministic finite state machine* (DFSM) can thus be defined by the tuple $(S, s_1, \delta, \lambda, X, Y)$.

It should be noted that, unlike FA, there are NFSM for which there is no equivalent DFSM. Consider, for example, the response of M_0 to the input of a . There are two possible output values: 0 and 1. This behaviour cannot be represented by a DFSM.

For each $y \in Y$, $s \in S$, and $x \in X$, $h_y(s, x)$ will denote the set of possible next states if a transition is executed from s using input x and produces output y . Again, this can be extended to h_y^* . An NFSM M is said to be *observable* if for every $s \in S$, $x \in X$, and $y \in Y$, $|h_y(s, x)| \leq 1$. Consider, for example, the response of M_0 to the input value a while in state s_1 . Although there are two possible behaviours, they have different output values. Thus, when the output is known, there is only one possible next state. This reduces the uncertainty caused by nondeterminism.

An NFSM M is observable if and only if the corresponding FA $F(M)$ is deterministic. Clearly every DFSM is observable (the converse is not true). Any (completely specified) NFSM M is equivalent to some observable NFSM (*ONFSM*). Observability will be discussed further in Section 3.

An NFSM is said to be *connected* if for every ordered pair of states (s, s') there is some input sequence x such that $s' \in h_1^*(s, x)$. Two states s and s' are said to be *quasi-equivalent* ($s =_q s'$) if for every input sequence x , $h_1^*(s, x) = h_1^*(s', x)$. An NFSM M is said to be *reduced* if M is connected



and no two states of M are quasi-equivalent.

It is assumed throughout this paper that an implementation that behaves like some unknown DFSM M_I is being tested against some NFSM M that is reduced, connected, observable, and completely specified. Related work is discussed in Sections 2 and 3. In Section 4 the special case, when it is known that M_I conforms to M if and only if M_I is a submachine of M , is discussed and the results are generalized in Section 5. The use of adaptive processes is explored in Section 6 and finally conclusions are drawn.

2 Testing From a DFSM

Throughout this section it will be assumed that M is a reduced completely specified DFSM $(S, s_1, \delta, \lambda, X, Y)$ and I behaves like some unknown reduced DFSM $M_I = (S', s'_1, \delta_1, \lambda_1, X, Y)$. As M is deterministic, I conforms to M if and only if $L(M) = L(M_I)$. Thus, if I conforms to M , there is a one-to-one correspondence between the states of M and the states of M_I : M and M_I are isomorphic.

An input sequence x is said to *distinguish* two states s and s' of M if $\lambda^*(s, x) \neq \lambda^*(s', x)$. Similarly, an input sequence x *distinguishes* between M and M_I if $\lambda^*(s_1, x) \neq \lambda_1^*(s'_1, x)$. The states s and s' of M are *distinguishable* if there is some input sequence that distinguishes them. As M is reduced, each pair of states from M is distinguishable. A set of sequences W is said to *verify* a state s of M if for each state $s' \in S$, $s \neq s'$, there is some $w \in W$ that distinguishes s and s' . The set W is said to be a *characterizing set* for M if it verifies every state of M .

While a characterizing set is sufficient for state verification, two alternative approaches are commonly applied: using a distinguishing sequence or a set of unique input/output sequences (Sidhu and Leung [1988]). A *distinguishing sequence* for M is an input sequence D that verifies each state of M . Thus, if M has a distinguishing sequence D then $\{D\}$ is a characterizing set for M . A *unique input/output sequence* for a state s of M is an input sequence x that verifies s . While x can verify the state s of M , it need not be able to verify any other state of M .

Unfortunately, not every DFSM has either a distinguishing sequence or a unique input/output sequence for every state. Every completely specified reduced DFSM has a characterizing set (Chow [1978]).

Chow [1978] introduces the assumption that there is some known upper



bound m on the number of states of M_I . A *state cover* V is a tree in which each node corresponds to a state of M , the branches correspond to transitions in M , and every state of M has some corresponding node in V . Given a set A and a natural number k , let A^k denote the set of strings, from A^* , that have length at most k (note, this is not the standard definition of A^k). If V is a state cover and W a characterizing set for M then the test set $VX^{m-n+1}W$ will distinguish between M and any nonconforming DFMSM with no more than m states: this test is called a *checking experiment*.

A sequence of transitions from M is a *transition test* for a transition t from M if it consists of t followed by either a unique input/output sequence for the final state of t or a distinguishing sequence for M . A number of authors (e.g. Chow [1978], Aho et al. [1988], Yang and Ural [1990], Hierons [1996], Ural et al. [1997], and Hierons [1997b]) consider the case where M is deterministic and $m = n$. They produce a single test sequence that contains, for each transition t of M , a transition test. Most work is then based on trying to find the shortest test sequence with this property (e.g. Aho et al. [1988], Yang and Ural [1990], Hierons [1996], and Hierons [1997b]). Ural et al. [1997] instead assume there is a distinguishing sequence and find the shortest sequence that contains both a transition test, for each transition of M , and a set of subsequences that check the distinguishing sequence. This test is guaranteed to determine whether M_I conforms to M , as long as M_I has no more states than M , and is called a *checking sequence*.

3 Testing from an NFSM

3.1 Preliminaries

A number of problems are associated with testing nondeterministic systems. One problem is that it is impossible to be certain whether every possible response to a particular input sequence has been observed. Luo et al. [1994] add the test hypothesis, called the *complete testing assumption*, that there is some integer α such that if an input sequence x has been executed α times from some state s then it is guaranteed that every element of $h_2^*(s, x)$ has been produced. Thus, if a test set contains a number of input sequences, testing involves executing each α times.

Another problem is that, having executed an input sequence and observed the output sequence, there may be more than one current valid state for M .



A number of authors (e.g. Luo et al. [1994], Petrenko et al. [1994]) add the assumption that M is observable, and note that any unobservable NFSM can be converted into an equivalent observable NFSM. The advantage of observability is that, although M may be nondeterministic, the output of a transition defines the next state and thus eliminates one form of uncertainty. Thus, if an input sequence is executed from some state, by observing the output sequence the expected final state can be determined. As noted, it will be assumed that any NFSM considered is observable.

In testing it is common to assume that M has *reset capability*: there is some special input that will always move M to the initial state s_1 . This allows a set of input sequences to be executed: they are simply separated by resets. Throughout this paper it will be assumed that M_I has a *reliable reset*.

When considering NFSMs it is necessary to produce a new definition for a characterizing set. Petrenko et al. [1994] say that an input sequence x *r-distinguishes* states s and s' if $h_2^*(s, x) \cap h_2^*(s', x) = \emptyset$. Thus, in any implementation that conforms to M , x is guaranteed to distinguish between states that correspond to s and s' . If $h_2^*(s, x) \neq h_2^*(s', x)$ but $h_2^*(s, x) \cap h_2^*(s', x) \neq \emptyset$ then the execution of x , from two states of M_I corresponding to s and s' , may lead to the same output.

States s and s' being r-distinguishable shall be denoted $s \neq_r s'$, otherwise $s =_r s'$. An NFSM is said to be *r-reduced* if for every $s, s' \in S$, $s \neq_r s' \bullet s \neq_r s'$. A set of sequences W is a *characterizing set* if for all $s, s' \in S$, $s \neq_r s'$, there is some input sequence in W that r-distinguishes s and s' . It should be noted that it is only necessary to distinguish between states that are pairwise r-distinguishable. An NFSM that is not r-reduced is said to be *r-unreduced*.

Unfortunately the properties of being observable and r-reduced can conflict since, while any r-reduced unobservable NFSM M can be converted into an ONFSM M' , M' may not be r-reduced. This is the case in the example given in Figure 2. In fact, the following result shows that the states in M' correspond to sets of states in M : if any pair of these sets intersect (and M is completely specified) then M' is r-unreduced. The machine given in Figure 2 is an example of this: the initial state of M' corresponds to $\{s_1\}$, u_2 corresponds to $\{s_1, s_2\}$ and u_3 corresponds to $\{s_2\}$.

Lemma 1 *If M is completely specified, strongly connected, r-reduced, and unobservable and M' is a strongly connected ONFSM that is equivalent to M then each state of M' corresponds to a set of states from M .*



Proof

Let A_s denote the set of input/output sequences that move M' from its initial state to state s . Let S_{A_s} denote the set of final states allowed after the input/output sequences from A_s are executed from the initial state of M . As M' is observable, the input/output sequences in A_s have no other possible final state in M' . Thus the set of input/output sequences executable from s must correspond to the union of the sets of input/output sequences allowed from the states in S_{A_s} . Thus s corresponds to the state set S_{A_s} . \square

This result suggests that examples such as that given in Figure 2 may be common. In particular, if M' has more states than M there must be some intersection and thus M' will be r-unreduced. The example given in Figure 3 shows, however, that it is possible for M' to be r-reduced.

3.2 Test Generation

This section will describe results and algorithms, found in Yevtushenko et al. [1991] and Petrenko et al. [1994], for generating a checking experiment from a reduced ONFSM. An initial result given is that if M has n states and M_I has no more than m states then the input sequence X^{mn} is a checking sequence.

Both then look at conditions that allow this set to be reduced. They assume that there is a set V such that for each state s_i there is some input sequence $v_i \in V$ with the property that $h_2^*(s_1, v_i) = \{s_i\}$: the input sequence v_i is guaranteed to bring the NFSM to state s_i . The set V is called a *deterministic state cover*. The NFSM M_0 has, for example, a deterministic state cover $V = \{\epsilon, b, ba, baa, baaa\}$. They produce a test technique, in the presence of a deterministic state cover, that utilizes states being r-distinguishable, but does not require the NFSM to be r-reduced. For each state $s_i \in S$, the set W_i denotes the set of input sequences used to distinguish between s_i and each s_j such that $s_j \not\equiv_r s_i$.

Let u_1 denote the initial state of M_I . Yevtushenko et al. [1991] and Petrenko et al. [1994] consider a tree with root (s_1, u_1) and edges corresponding to input/output pairs that are allowed by both M and M_I . Then a node is a leaf if one of the following is the case:

1. The state pair has already appeared somewhere else in the tree as an intermediate node.



2. There is some input value such that M and M_I do not have matching transitions.

Then $M_I \leq M$ if and only if all the leaves are of type 1.

Let P_1, \dots, P_k denote the maximal sets of r-distinguished states from M and, for each $s_i \in S_M$, $Q(s_i)$ denote the set of states from S_{M_I} that agree with s_i on W_i . If M is r-reduced and $M_I \leq M$, there is only one such set: $P_1 = S_M$. By considering the possible pairs and using the fact that each state s_i is in some pair in V , the following result is obtained:

- if the states from some P_j are met $\sum_{s_i \in P_j} (|Q(s_i)| - 1) + 1$ times in some path after a sequence from V then a leaf must have been met.

Thus it is sufficient to stop a path when there is some P_j such that the path contains this number of instances of states from P_j . While the $Q(s_i)$ are not known, this expression is bounded above by $m - |P_j| + 1$.

The maximal sets of pairwise r-distinguishable states, P_1, \dots, P_k , are found and the test set is generated in the following manner (Yevtushenko et al. [1991], Petrenko et al. [1994]):

1. For each state s_i of M let v_i denote the input sequence in V that reaches s_i . For each state s_i a tree D_i , starting with s_i , is constructed. The nodes represent states of M , while the edges represent possible transitions. A node is a leaf if there is some P_j such that the states from P_j are met $(m - |P_j| + 1)$ times in total on the path to that node (not counting the root node).
2. Then, for an input/output sequence x/y in D_i with final state s_i , the test $v_i x W_i$ is included. The empty sequence is included in the set of sequences from D_i .
3. The test set is the set of all such input/output sequences.

Clearly the size of this test set depends on the number of states that are pairwise r-distinguished: as the number of r-distinguished states reduces, the size of the P_j reduces and thus the size of the test set increases. In Sections 4 and 5 \neq_r will be extended, thus potentially increasing the size of the P_j and thus reducing the size of the checking experiment produced.



4 Testing Deterministic Submachines

4.1 Deterministic Equivalence

An NFSM M' is a *submachine* of M if it is isomorphic to some NFSM M_S whose state set and transition set are subsets of those of M . It will be assumed throughout this section that if $M_I \leq M$ then M_I is a submachine of M . This condition will be weakened in Section 5.

The concept of state distinguishing can be extended in this case. A set A of input sequences is said to *d-distinguish* states s and s' of M if for every deterministic submachine M' of M , and corresponding states u and u' from M' , there is some $x \in A$ that distinguishes u and u' . If s and s' are d-distinguishable we write $s \neq_d s'$ and otherwise $s =_d s'$.

In M_0 there are states that are pairwise d-distinguishable but not r-distinguishable. The input sequence aaa will, for example, distinguish between the states corresponding to s_1 and s_2 in any deterministic submachine: from the state corresponding to s_2 it will produce output 001 while from the state corresponding to s_1 it will either produce 000 or 110.

In the unobservable case described in Figure 4, two input sequences are required: aa and ab . Each choice for the execution of a from s_1 will lead to a following transition that is not allowed after the execution of a from s_2 .

The algorithms given in Petrenko et al. [1994], that use $=_r$, can be applied using $=_d$ instead. As the use of $=_d$ can increase the size of the sets of pairwise distinguished states, it can reduce the size of the test set. Clearly, $s_i \neq_r s_j \Rightarrow s_i \neq_d s_j$, and thus d-distinguishability can never lead to longer tests than r-distinguishability.

4.2 Finding d-distinguishing sets

Suppose $s_i =_r s_j$ but s_i and s_j are not quasi-equivalent. Then there may be some set of input sequences that d-distinguished s_i and s_j . The obvious approach, to finding a d-distinguishing set, is to use a breadth first search of a tree starting with (s_i, s_j) . In this tree, edges represent input values. Each node represents the possible configurations, given the input so far, that are consistent with the same output having been produced from each state. Thus a node represents a set of tuples, where each tuple contains the corresponding states s'_i and s'_j and the deterministic choices that are required in order to allow M_I to move from s_i to s'_i and from s_j to s'_j producing the



same output sequence. Given input sequence x , let $c(x)$ denote the set of tuples corresponding to x and π_k denote the projection function that returns the k th element of a tuple.

A set of choices can be represented by a predicate p , which takes a deterministic submachine M' of M and returns true if and only if M' allows those choices. Thus a node contains a set of tuples of the form (s'_i, s'_j, p) . The form of p depends upon the representation of the choices.

What is required is one of:

1. Some input sequence x such that $c(x) = \emptyset$.
2. Some set of input sequences x_1, \dots, x_r such that for every p_1, \dots, p_r , $p_q \in \pi_3(c_q)$ for some $c_q \in c(x_q)$, and every deterministic submachine M' of M , $\neg(p_1(M') \wedge p_2(M') \wedge \dots \wedge p_r(M'))$.

In the first case, only one sequence is required, in the second a set of sequences is required. For pragmatic reasons, limits can be placed on the size of sets and sequences considered in the search: only those that can reduce the test effort are of interest.

5 Deterministic implementations that are not submachines

5.1 Extending deterministic distinguishing

Let the states of M be denoted s_1, \dots, s_n and let V denote a deterministic state cover of M . In order to generalize the notion of deterministically distinguishing states it is sufficient to consider all deterministic reductions of M that have no more than k states that cannot be reached by V . Given two states, s_i and s_j , we require a set of input sequences that is guaranteed to distinguish between any pair of corresponding states in the implementation.

Let $L(s)$ denote the set of input/output sequences allowed from the state s and $s \leq s'$ denote $L(s) \subseteq L(s')$. It is important to note that if $M_I \leq M$ then, for any reachable state $s \in S_{M_I}$, there is some $s' \in S_M$ such that $s \leq s'$.

We say that states s_i and s_j are *deterministically (V, k) distinguished* if there is a set of input sequences A such that, for every DFSM $M' = (U, u_1, \delta, \lambda, X, Y)$ that conforms to M and has no more than k states that are not reached by V :



- given $u_i, u_j \in U$ with $u_i \leq s_i$ and $u_j \leq s_j$ there is some input sequence $x \in A$ that distinguishes between u_i and u_j .

This is written $s_i \not\equiv_{(V,k)} s_j$ and otherwise $s_i \equiv_{(V,k)} s_j$. An NFSM M is said to be *deterministically* (V, k) *reduced* if for every $s_i, s_j \in S_M$, $s_i \not\equiv_{(V,k)} s_j$ and otherwise M is said to be *deterministically* (V, k) *unreduced*. Clearly, $\equiv_{(V,0)}$ is equivalent to \equiv_d and as $k \rightarrow \infty$, $\equiv_{(V,k)} \rightarrow \equiv_r$.

The NFSM M_0 given in Figure 1 has a deterministic state cover $V = \{\epsilon, b, ba, baa, baaa\}$, where ϵ denotes the empty sequence. Consider the input of aaa . If M_I is in a state $s \leq s_2$, the input of aaa leads to output 001. If $k = 0$ and aaa is input, while M_I is in a state $s \leq s_1$, either 000 or 110 can be output. If $k = 1$ and a is input while M_I is in a state $s \leq s_1$, M_I might output 0 and move to another (non-equivalent) state $s' \leq s_1$. The input of aaa , from s , then leads to the output of 011. Thus if aaa is input and M_I is in state s , if $s \leq s_2$ then M_I outputs 001 and if $s \leq s_1$ and $k = 1$ then the output sequence generated is one of: 000, 110, or 011. Thus, s_1 and s_2 are deterministically $(V, 1)$ distinguished by aaa . It is easy to check that $s_1 \equiv_{(V,1)} s_5$, $s_1 \equiv_{(V,1)} s_4$ and all other state pairs are r-distinguished.

5.2 Finding the value of k

If there is some upper bound m on the number of states of M_I , an upper bound can be found for k . Suppose S' denotes a maximal (in terms of size) set of pairwise r-distinguished states of M . Then, for any conforming implementation, each of these must have a corresponding separate state reached by V . Thus, an initial upper bound of $k = m - |S'|$ can be used. There may be further information, about the implementation, that can be used to reduce this.

The NFSM given in Figure 1 has maximal sets of pairwise r-distinguished states $P_1 = \{s_2, s_3, s_4, s_5\}$ and $P_2 = \{s_1, s_3\}$. If the value $m = n = 5$ is used, an upper bound of $k = 1$ is found. This information helps reduce the required test size as s_1 and s_2 are deterministically $(V, 1)$ distinguished and thus the set P_2 can be extended to $P'_2 = \{s_1, s_2, s_3\}$. The set P_1 is not affected and thus $P'_1 = P_1$.

In this case, the tree derived from each state has the property that each leaf represents meeting the states from P'_1 $m - |P'_1| + 1 = 2$ times or meeting the states from P'_2 $m - |P'_2| + 1 = 3$ times. The characterizing set is $W = \{aaa\}$. Input aaa produces output 000 or 011 or 110 from s_1 , 001 from s_2 ,



101 from s_3 , 011 from s_4 , and 110 from s_5 . This fails to distinguish s_1 from either s_4 or s_5 .

It is now possible to derive a test set. In this case every input sequence of length 2, from any state, will pass through the elements of some P'_i twice. As an example, we will consider the input sequence bb . From s_1 this will reach states s_2 and then s_1 , both of which are from P'_2 . Similarly, from s_2 it goes to s_1 and then s_2 . From s_3 it moves to s_4 and then s_5 or s_2 ; in each case both states are in P'_1 . From s_4 it passes through s_2 or s_5 to s_1 or s_2 respectively. In the first case both are in P'_2 and in the second both are in P'_1 . Finally, from s_5 it will reach s_2 and then s_1 which are both in P'_2 . The checking experiment thus contains the following:

$$\{\epsilon, b, ba, baa, baaa\} \{\epsilon, a, b, aa, ab, ba, bb\} \{aaa\}.$$

It is also necessary to consider sequences of length 3 from M_0 . Any sequence of length 3, such that the first two states it meets need not be from P'_1 , must be included. An example is the sequence aba from s_1 : this might go to s_1 , then s_2 and finally s_4 . It is easy to check that no sequences of length greater than 3 need be considered: every sequence of length 3 in M_0 meets either at least two states from P'_1 or three states from P'_2 . The sequences of length 3 that must be included (and thus, in the checking experiment, preceded by the corresponding v_i and followed by W) are:

1. From s_1 : any string starting with either a or bb .
2. From s_2 : any string starting with b .
3. From s_3 : none.
4. From s_4 : any string starting with bb .
5. From s_5 : any string starting with bb .

The checking sequence generated can be reduced by removing those sequences that are contained in the beginning of other sequences.



6 Adaptive Testing

6.1 The deterministic state cover

When input sequences are applied to M_I , the output provides information about M_I . Clearly, M_I has some deterministic state cover V . This can be developed by using a breadth first search, at each step simply executing candidate values. The output determines the (expected) next state and thus whether the expected next state is one already included in the tree. Given an input sequence x that provides a new expected state, and thus will be used in V , at this point x can be followed by the appropriate W_i .

There are a number of possible orders in which to execute the required sequences. One possibility is to initially execute W (or the corresponding set required for s_1). This provides information, about M_I , which can be used to provide part of V . The rest of V is developed in an adaptive manner. This search is continued until a deterministic state cover V has been found.

6.2 Testing Submachines

Suppose the set W has been produced for any deterministic FSM that is a submachine of M . The particular IUT, M_I , may have properties that mean that not all of these sequences are required. If two states s_i and s_j are deterministically distinguished by a set A then in M_I one input sequence $\alpha \in A$ will distinguish between the corresponding states. Once α has been found, A can be replaced by α , or some initial subsequence of α , in W . Similarly, some other sequences in W may not be required when testing M_I , and thus may be removed, and others may be shortened. Thus, once VW has been executed, each W_i can be replaced by some subset of (possibly shortened) sequences drawn from W_i .

6.3 General deterministic implementations

While it is still always possible to devise a state cover V for M that is implemented (and deterministic) in M_I , this may not reach all states in M_I . If some states of M_I may not be reached by executing V , it is not possible to reduce the size of W using information derived from the execution of VW .

As noted in Section 5, the value of k can be derived from an upper bound m on the number of states of M_I . Associated with W and M there is some



maximal (in terms of size) set of states S' that are pairwise distinguished by W , and thus a value of $k = m - |S'|$ can be used. The set S' is based on states that are guaranteed to be distinguished by W . Once VW has been executed, it may transpire that there is some larger set of states S'' reached by V that is pairwise distinguished, in M_I , by elements from W . The value $k = m - |S''|$ can then be used and this reduction in the value of k can potentially further increase the set of pairwise deterministically (V, k) distinguished states.

The knowledge, of the behaviour of certain instances of states from S , can also be used to directly reduce the size of the test sequence. This is because, given some P_j , there may be some state in M_I that can be used to extend P_j . This happens if there is an input/output sequence x/y whose final state in M is s_i for some $s_i \notin P_j$ but the corresponding state u in M_I has the property that for all $s \in P_j$, $u \notin Q(s)$. Thus it may be possible to extend a set P_j by some maximal (size) set \overline{P}_j of states from M_I such that the states in \overline{P}_j are pairwise distinguishable and each is pairwise distinguishable from each state in P_j . Then:

$$|\overline{P}_j| + \sum_{s_i \in P_j} |Q(s_i)| \leq m$$

Given P_j , this information gives the following upper bound on the number of occurrences of states from P_j in a test sequence:

$$m - |P_j| - |\overline{P}_j| + 1$$

As testing proceeds, these values can be updated.

An input sequence x can be seen as a route to some state of M_I . It is possible to update these values by considering the execution of W at the end of routes. As each sequence from D_i is followed by some W_i , this fits the test technique.

Suppose that, in the example, $W = \{aaa\}$ is initially executed from s_1 . If this were to produce output 000 then this instance of s_1 would be distinguishable from both s_4 and s_5 . Thus s_4 and s_5 can be included in \overline{P}_2 and s_1 can be included in \overline{P}_1 . Further, as $m = 5$ and M_I is reduced, the set $\{aaa\}$ must be a characterizing set for M_I . Thus the test set can be reduced to:

$$\{\epsilon, b, ba, baa, baaa\} \{\epsilon, a, b\} \{aaa\}$$

Again, sequences contained in the beginning of others can be removed.



7 Conclusions

The problem of testing a nondeterministic implementation against an NFSM has received much attention but there has been little work on testing from an NFSM when the IUT is known to be deterministic. When the IUT is deterministic, it is possible to generalize the notion of r-distinguishing states. When it is known that if the IUT conforms to M then it is a submachine of M , it is possible to d-distinguish states. When instead there is some upper bound k on the number of states in the IUT that are not reached by the deterministic state cover V , it is possible to consider (V, k) distinguishing states. In the case where $k = 0$, this reduces to d-distinguishing states.

When the IUT is deterministic, much can be learnt about the structure of the IUT during test execution. A test can thus be generated in an adaptive manner. This guarantees the existence of a deterministic state cover and allows the test set to be reduced as testing proceeds.

An interesting question, when applying adaptive testing, is how the test order than maximizes the expected reductions can be found. There is also the problem of limiting the search for deterministically (V, k) distinguishing sets. There may be no good upper bound on the size of these: instead limits can be placed on the size of sets that could reduce the total test effort.

8 References

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