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# ADHESIVE-BONDED SCARF AND STEPPED-LAP JOINTS

TECHNICAL REPORT

by

L. J. HART-SMITH

Prepared under Contract NAS1-11234  
Douglas Aircraft Company  
McDonnell Douglas Corporation  
3855 Lakewood Blvd  
Long Beach, California 90846



January 1973

for

Langley Research Center  
Hampton, Virginia 23366

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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## ABSTRACT

Continuum mechanics solutions are derived for the static load-carrying capacity of scarf and stepped-lap adhesive-bonded joints. The analyses account for adhesive plasticity and adherend stiffness imbalance and thermal mismatch. The scarf joint solutions include a simple algebraic formula which serves as a close lower bound, within a small fraction of a per cent of the true answer for most practical geometries and materials. The scarf joint solutions are believed to be the first such results ever obtained for dissimilar adherends. Digital computer programs have been developed and, for the stepped-lap joints, the critical adherend and adhesive stresses are computed for each step. The scarf joint solutions exhibit grossly different behavior from that for double-lap joints for long overlaps inasmuch as that the potential bond shear strength continues to increase with indefinitely long overlaps on the scarf joints. The stepped-lap joint solutions exhibit some characteristics of both the scarf and double-lap joints. The stepped-lap computer program handles arbitrary (different) step lengths and thicknesses and the solutions obtained have clarified potentially weak design details and the remedies. Indeed, the program has been used effectively to optimize the joint proportions.

## KEYWORD DESCRIPTORS

Bonded Joints	Scarf Joints
Adhesive Stresses and Strains	Stepped-Lap Joints
Adherend Stiffness Imbalance	Static Strength
Adherend Thermal Mismatch	Elastic-Plastic Formulation
Computer Analysis Programs	Advanced Composite Joints

## FOREWORD

This report was prepared by the Douglas Aircraft Company, McDonnell Douglas Corporation, Long Beach, California under the terms of Contract NAS1-11234. One summary report (NASA CR 2218) and four technical reports (NASA CR 112235, -6, -7, and -8) cover the work, which was performed between November 1971 and January 1973. The program was sponsored by the National Aeronautics and Space Administration's Langley Research Center, Hampton, Virginia. Dr. M. F. Card and Mr. H. G. Bush were the Contracting Agency's Technical Monitors.

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## SYMBOLS

$A_0, \dots, A_n$	= Coefficients of power series for shear stress distribution in adhesive layer
$a, c$	= Extents of plastic stress state in adhesive at ends of bonded joint (in.)
$b$	= Extent of elastic trough in adhesive (in.)
$C, D$	= Integration constants
CTHERM	= Non-dimensionalized adherend thermal mismatch coefficient
$d$	= Length of elastic zone in adhesive bond (in.)
$E$	= Young's modulus (longitudinal) for adherend (psi)
ETR	= Adherend extensional stiffness ratio
$F_{..}$	= Adherend allowable (or ultimate) stress (psi)
$G$	= Adhesive shear modulus for elastic-plastic representation (psi)
$l$	= Overlap (length of bond) (in.)
$P$	= Applied direct load on entire joint (lb in. / in.)
SGNLD	= Distinguisher between tensile and compressive shear loads
$T$	= Direct stress resultants in adherends (lb / in.)
$\Delta T$	= Temperature change ( $T_{\text{operating}} - T_{\text{cure}}$ )
$t$	= Thickness of adherend (in.)
$x$	= Axial (longitudinal) coordinate parallel to direction of load
$\alpha$	= Coefficient of thermal expansion (/°F)
$\gamma$	= Adhesive shear strain
$\gamma_e$	= Elastic adhesive shear strain
$\gamma_p$	= Plastic adhesive shear strain
$\delta$	= Axial (longitudinal) displacement of adherend (in.)
$\zeta, \xi, \chi$	= Non-dimensionalized axial coordinates (different origin and/or sense from $x$ )

- $\eta$  = Thickness of adhesive layer (in.)
- $\theta$  = Scarf angle (small) ( $^{\circ}$ )
- $\lambda$  = Exponent of elastic shear stress distribution (in. $^{-1}$ )
- $\nu$  = Poisson's ratio for adherend(s)
- $\tau$  = Adhesive shear stress (psi)
- $\tau_{av}$  = Average adhesive shear stress (psi)
- $\tau_p$  = Plastic (maximum) adhesive shear stress (psi)
- $\phi$  =  $x/l$  = Non-dimensionalized coordinate

#### SUBSCRIPTS

- a,c = Adhesive (cement)
- e,p = Elastic and plastic values
- i,o = Inner and outer adherends of symmetric bonded joint
- 1,2 = Different adherends at each end of joint
- 1,2,...n = Power series counter



## SUMMARY

It has long been known that bonded scarf joints have a higher efficiency than uniform lap joints and that the latter are limited in strength and unsuitable for joining thicker sections. What has not been well understood until recently is that, in the bonding together of dissimilar adherends in a scarf joint, any adherend stiffness imbalance or thermal mismatch imposes a limitation on the joint efficiency. As a consequence the adhesive layer is not (essentially) uniformly stressed along its length as it is for a scarf joint between identical adherends. One objective of this report is to analyze and quantify these limitations on efficiency of unbalanced scarf joints. In doing so, adhesive plasticity is accounted for by the Douglas elastic-plastic model which has been demonstrated to be effective for uniform lap joints. One dominant characteristic deduced for scarf joints is that for long overlaps, regardless of any adhesive ductility and/or adherend thermal mismatch, the ratio of the average adhesive shear stress to the peak adhesive shear stress is equal to the lower ratio ( $<1$ ) of the adherend extensional stiffnesses. The governing differential equations do not possess an explicit solution in terms of standard functions, so a series solution was employed. Even so, an algebraic expression was deduced for a lower bound which proved to be so close to the more precise solutions that it could be employed directly for practically all realistic joint proportions. Severe adverse effects of adherend thermal mismatch are confined to a specific overlap range. The effects decrease asymptotically to zero for very short or very long overlaps.

Stepped-lap joints represent a cross between scarf joints and uniform lap joints. The stepped-lap joint overcomes the upper limit on joint strength of uniform lap joints but retains the severe adhesive strain concentration at the end of each step. One advantage of stepped-lap joints over scarf joints is that the alignment and fit is far less critical when there are joints on more than a single interface. Another is that it is more suitable for boron-epoxy laminates than is a scarf joint because of the thick brittle filaments. This is particularly important for the titanium edge members frequently used in conjunction with boron-epoxy panels. Because the graphite fibers are so much thinner and more flexible than boron filaments, the former can take advantage of the higher efficiency of the scarf joint.

Digital computer FORTRAN IV programs are included for the iterative solutions necessary for these problems. The scarf joint solutions are in terms of non-dimensionalized parameters. The stepped-lap joint program is dimensional and permits each step to be varied independently so as to be able to identify and improve the most critical detail(s) of the joint. One key factor in the design of stepped-lap joints is that the bond load transfer is concentrated at the end of the joint from which the softer (less stiff) adherend extends. Consequently, it is necessary to restrict the length of the end step of the stiffer adherend to prevent it from being overloaded. Another characteristic of stepped-lap joints identified by the analyses is that the end three steps of the more critical end dominate the internal load distribution and effectively determine the load capacity. The steps at the less critical end are found to have practically no effect on the load capacity.

## 1. INTRODUCTION

It is generally recognized that, in the bonding together of thick sections, the use of either scarf or stepped-lap joints is mandatory if an acceptable structural efficiency is to be realized. References (1) and (2) explain how, for uniform lap joints, the maximum possible joint efficiency decreases with increasing thickness (extensional stiffness) of the members being bonded together. The objective of this report is to apply the elastic-plastic adhesive analysis techniques developed in References (1) and (2) to the scarf and stepped-lap joints. The approach used remains that of continuum mechanics rather than finite elements. The governing differential equations were relatively straightforward to set up but, in most cases, specific closed-form solutions could not be derived. Severe numerical accuracy problems had to be overcome in developing the FORTRAN IV digital computer programs employed and this phase of the work represented by far the bulk of the investigation. The computer programs are listed in the Appendices and representative non-dimensionalized solutions are illustrated to show the effect of the governing scarf joint parameters. Specific solutions are presented for stepped-lap joints.

This scarf joint analysis is concerned with the non-uniform adhesive shear stresses necessarily associated with the bonding together of dissimilar adherends. It is well-known that the stresses are uniform if the adherends are identical. It has only recently begun to be appreciated that the adhesive shear stresses are markedly non-uniform if the adherends are dissimilar. Indeed, the literature contains very few references to this problem. The mechanism whereby these non-uniform stresses are developed is illustrated in Figure 1 for the case of thermal mismatch between stiffness-balanced adherends. The first publication on scarf joints between dissimilar adherends appears to be that of Lubkin [Reference (3)] who, in 1957 sought the particular scarf angle associated with uniform adhesive stress for a particular ratio of adherend elastic moduli. He omitted consideration of any adherend thermal dissimilarity. Unfortunately the predictions of his equation [10] are such as to indicate the appropriate scarf angle  $\theta$  is so great (typically in excess of 45 degrees) as to be of no practical interest for bonding aerospace materials together. For realistic adhesives and adherend materials, the scarf angle should be restricted to less than 4 degrees in order for the potential bond

strength to exceed the adherend strength(s). Working independently, in 1971, the present author [Reference (4)] and Erdogan and Ratwani [Reference (5)] demonstrated by calculation the non-uniform adhesive shear stress associated with scarf joints between dissimilar adherends. The former work was based on a perfectly-plastic adhesive analysis, while the latter derived from a linearly-elastic formulation. Consequently neither afforded a complete solution but both demonstrated clearly that the adhesive load transfer is concentrated at that end of the joint from which the softer adherend extends. The present solution utilizes an elastic-plastic adhesive model with linearly elastic adherends and accounts for adherend stiffness and thermal imbalances. Eccentricities in the load path are excluded and, in keeping with common design practice, the scarf angle is considered to be so small that adhesive tension (or compression) stresses may be neglected in comparison with the shear stresses.

In 1968, an elastic finite-element analysis of scarf joints was performed by Richards [see Reference (6)]. Boron/epoxy-to-boron/epoxy and boron/epoxy-to-aluminum joints were analyzed. Thermal effects were neglected. In the former case, relatively small (<4%) stress concentrations were identified in the vicinity of the ends of the scarf. Their existence had not been demonstrated prior to that investigation. In the latter case a markedly non-uniform stress distribution was deduced, with significantly more load being transferred to and from the 0° plies in the laminate than occurred with the  $\pm 45^\circ$  plies. This is to be expected in view of the much lower modulus of the cross plies.

While the mathematical complexity of equations governing the scarf joint has restricted the number of solutions obtained, a number of investigations of the stepped-lap adhesive-bonded joint have been performed. Finite-element elastic solutions are reported in References (5) to (9) but none of these include any thermal mismatch effects. Reference (10) included adhesive and adherend non-linear behavior in the analysis but, for the stepped-lap joint, encountered convergence difficulties at high load levels. Grimes, Calcote, Wah, et al [Reference (10)] also performed non-linear iterative theoretical analyses of double-lap, single-lap and stepped-lap joints which they compared with their discrete element analyses, showing good agreement for the first two. They also formulated the scarf joint equations (see their Appendix A) in greater detail than is done here, but were unable to solve them. Corvelli and Saleme

[Reference (11)] developed analysis techniques for bonded joints which included analytical solutions for stepped-lap joints, but in a less comprehensive form than presented here.

Past attempts to include non-linear adhesive behavior in the analytical solutions have centered around the Ramberg-Osgood representation which has a smooth continuous characteristic. This has precluded the derivation of any explicit closed-form solutions. The present author had earlier derived such solutions for double- and single-lap adhesive-bonded joints using an elastic-plastic adhesive formulation [see References (12) and (13)]. These showed that the adhesive shear strain energy per unit bond area was the necessary and sufficient adhesive characteristic governing the potential bond shear strength. The precise shape of the stress-strain curve appeared to be unimportant. This belief was further reinforced in Reference (1) by the derivation of precisely the same potential bond shear-strength for any arbitrary bi-elastic adhesive characteristic having the same strain energy and failure stress and strain. In addition, the author's elastic-plastic solution was in good agreement with the discrete element solutions by Teodosiadis [Reference (14)], who represented the adhesive and interlaminar shear characteristics by six straight segments. The success of this elastic-plastic adhesive approach in these simpler problems led to the decision to apply the same techniques to the scarf and stepped-lap joints in this report.

The adhesive-bonded stepped-lap joint is of practical interest principally because of extensive use in the bonding of boron-epoxy to titanium edge members. The boron filaments are too thick (0.005 inch), and too hard to machine, to be as suitable for the more efficient scarf joints as the very thin graphite fibers are. In practice the stepped-lap joint contains a large number of small steps and closely approximates the behavior of the equivalent scarf joint. The only difference is marked for very brittle (high-temperature) adhesives and is the adhesive shear stress (and strain) concentrations at the ends of each step, particularly at the outermost steps. It transpired that peel stresses imposed more severe limitations for thick double- and single-lap joints than did the adhesive shear stresses [see References (1) and (2)]. In actual design practice for scarf and stepped-lap joints, the slope is small and the end step is

invariably thin so there is no way for severe peel stresses to develop. For any unusual stepped-lap joint, with a thick outer end step, the analysis in Reference (1) can be employed to assess any potential peel problem.

This report considers in turn elastic and elastic-plastic analyses of scarf and stepped-lap joints and discusses parametric effects and design procedures. The digital computer programs prepared from the analyses are recorded in the Appendices, along with brief instructions for their use.

## 2. ELASTIC ANALYSIS OF SCARF JOINTS

Figure 2 depicts the geometry and nomenclature for the analysis of a non-eccentric bonded scarf joint. The diagram serves for both the elastic and elastic-plastic solutions. In the former case, the plastic adhesive zones should be considered removed. That is, set  $a = c \equiv 0$  and  $b = \ell$ . The scarf angle  $\theta$  is considered so small that  $\cos\theta \approx 1$  and  $\theta \approx 0$ . In other words, the effect of adhesive peel stresses is omitted from consideration. This is quite legitimate for the small scarf angles associated with practical aerospace materials.

The conditions of horizontal equilibrium for a differential element  $dx$  within the joint are

$$\frac{dT_1}{dx} + \tau = 0 \quad , \quad \frac{dT_2}{dx} - \tau = 0 \quad . \quad (1)$$

The stress-strain relations for the adherend materials, accounting for thermo-elastic effects, yield

$$\frac{d\delta_1}{dx} = \frac{T_1}{(Et)_1} + \alpha_1 \Delta T \quad , \quad \frac{d\delta_2}{dx} = \frac{T_2}{(Et)_2} + \alpha_2 \Delta T \quad , \quad (2)$$

in which the adherend thicknesses, as a function of the axial coordinate  $x$  are

$$(Et)_1 = E_1 t_1 \left(1 - \frac{x}{\ell}\right) \quad , \quad (Et)_2 = E_2 t_2 \left(\frac{x}{\ell}\right) \quad . \quad (3)$$

The adhesive shear strain is taken to be uniform across the thickness of the bond. That is

$$\gamma = (\delta_2 - \delta_1)/\eta \quad . \quad (4)$$

The elastic adhesive shear stress follows as

$$\tau = G\gamma = G(\delta_2 - \delta_1)/\eta \quad . \quad (5)$$

In solving these equations it is desirable to non-dimensionalize the solution with respect to the peak adhesive shear stress  $\tau_p$  and the bond overlap. Thus, introducing the non-dimensionalized axial co-ordinate

$$\phi = x/\ell \quad , \quad (6)$$

a series solution is sought, having the form

$$\frac{\tau}{\tau_p} = \sum_1^{\infty} A_n \phi^{n-1} . \quad (7)$$

We define the adherend 1 end of the joint as critical so that

$$A_1 \equiv 1 , \quad (8)$$

if necessary by interchange of the identifying subscripts 1 and 2. While a single non-linear differential equation has been derived from the equations above, it cannot be solved directly. This is why a series solution is employed here and, in this case, it is more straightforward to work in terms of the equations above than the derivative governing equation.

The solution proceeds from equation (7). Substitution into equation (1) yields, for the adherend forces per unit width,

$$T_1 = \tau_{av} \ell - \tau_p \ell \sum_1^{\infty} \frac{A_n}{n} \phi^n , \quad T_2 = \tau_p \ell \sum_1^{\infty} \frac{A_n}{n} \phi^n . \quad (9)$$

Now equation (5) is differentiated.

$$\frac{d(\tau / \tau_p)}{d\phi} = \frac{G}{\tau_p \eta} \left[ \frac{d\delta_2}{d\phi} - \frac{d\delta_1}{d\phi} \right] . \quad (10)$$

Substitution of the series (7) and (9), with the aid of equations (2), leads to the solution

$$\sum_1^{\infty} (n-1) A_n \phi^{(n-2)} = \frac{G\ell}{\tau_p \eta} \left\{ (\alpha_2 - \alpha_1) \Delta T + \frac{\tau_p \ell}{E_2 t_2} \sum_1^{\infty} \frac{A_n}{n} \phi^{(n-1)} - \frac{\tau_{av} \ell}{E_1 t_1 (1 - \phi)} + \frac{\tau_p \ell}{E_1 t_1 (1 - \phi)} \sum_1^{\infty} \frac{A_n}{n} \phi^n \right\} . \quad (11)$$



Multiplication throughout by  $(1 - \phi)$  converts the equation directly into a form suitable for solution by recurrence relations.

$$(1 - \phi) \sum_1^{\infty} (n-1)A_n \phi^{(n-2)} = \frac{G\ell}{\tau_p \eta} (\alpha_2 - \alpha_1) \Delta T (1 - \phi) - \frac{G\ell^2 \tau_{av}}{\eta E_1 t_1 \tau_p} + \frac{G\ell^2}{\eta} \left[ \frac{(1 - \phi)}{E_2 t_2} \sum_1^{\infty} \frac{A_n}{n} \phi^{(n-1)} + \frac{1}{E_1 t_1} \sum_1^{\infty} \frac{A_n}{n} \phi^n \right] . \quad (12)$$

In order to give the solution the greatest coverage with the minimum number of independent variables, certain non-dimensional parameters are introduced. The non-dimensionalized overlap is given by the square root of

$$(\lambda\ell)^2 = \frac{G\ell^2}{\eta} \left[ \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right] = \frac{\tau_p \ell^2}{\eta \gamma_e} \left[ \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right] , \quad (13)$$

the non-dimensionalized thermal mismatch term is

$$CTHERM(1) = \frac{\lambda(\alpha_2 - \alpha_1) \Delta T}{\tau_p \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right)} , \quad CTHERM(2) = -CTHERM(1) , \quad (14)$$

and the adherend stiffness ratio is

$$ETR(1) = E_1 t_1 / E_2 t_2 , \quad ETR(2) = E_2 t_2 / E_1 t_1 . \quad (15)$$

It is interesting to note that precisely the same variables govern the double-lap joint [see Reference (1)]. Equation (12) then becomes

$$\sum_1^{\infty} [(n-1)A_n - (n-2)A_{n-1}] \phi^{(n-2)} = (\lambda\ell) \times CTHERM(1) \times (1 - \phi) - \frac{(\lambda\ell)^2}{[1 + ETR(1)]} \times \frac{\tau_{av}}{\tau_p} + \frac{(\lambda\ell)^2 ETR(1)}{[1 + ETR(1)]} \sum_1^{\infty} \frac{A_n}{n} \phi^{(n-1)} + \frac{(\lambda\ell)^2 [1 - ETR(1)]}{[1 + ETR(1)]} \sum_1^{\infty} \frac{A_n}{n} \phi^n . \quad (16)$$

By rearranging the limits of the series it follows that

$$\sum_1^{\infty} [(n+1)A_{n+2} - nA_{n+1}] \phi^n = (\lambda\ell) \times \text{CTHERM}(1) \times (1 - \phi) - \frac{(\lambda\ell)^2}{[1 + \text{ETR}(1)]} \times \frac{\tau_{av}}{\tau_p} + \frac{(\lambda\ell)^2 \text{ETR}(1)}{[1 + \text{ETR}(1)]} \sum_1^{\infty} \frac{A_{n+1}}{n-1} \phi^n + \frac{(\lambda\ell)^2 [1 - \text{ETR}(1)]}{[1 + \text{ETR}(1)]} \sum_1^{\infty} \frac{A_n}{n} \phi^n . \quad (17)$$

For large values of  $n$ , on setting to zero the coefficient of the term  $\phi^{n-2}$ , the recurrence relation is deduced as

$$A_n = \left\{ (n-2)A_{n-1} + (\lambda\ell)^2 \left[ \left( \frac{\text{ETR}(1)}{1 + \text{ETR}(1)} \right) \frac{A_{n-1}}{n-1} + \left( \frac{1 - \text{ETR}(1)}{1 + \text{ETR}(1)} \right) \frac{A_{n-2}}{n-2} \right] \right\} / (n-1) . \quad (18)$$

It remains now to establish the initial conditions by examining the coefficients of the  $\phi^0$  and  $\phi^1$  terms. From the coefficient of  $\phi^0$ ,

$$A_2 = (\lambda\ell) \text{CTHERM}(1) - (\lambda\ell)^2 \left[ \frac{1}{1 + \text{ETR}(1)} \right] \frac{\tau_{av}}{\tau_p} + (\lambda\ell)^2 \left[ \frac{\text{ETR}(1)}{1 + \text{ETR}(1)} \right] A_1 \quad (19)$$

while, from the coefficient of  $\phi^1$ ,

$$2A_3 - A_2 = -(\lambda\ell) \text{CTHERM}(1) + \frac{(\lambda\ell)^2}{[1 + \text{ETR}(1)]} \left\{ [1 - \text{ETR}(1)] A_1 + \text{ETR}(1) \frac{A_2}{2} \right\} . \quad (20)$$

It follows from equation (19) that, quite generally, for long overlaps (large values of  $\lambda\ell$ ),

$$\frac{\tau_{av}}{\tau_p} \rightarrow \text{ETR}(1) \leq 1 . \quad (\text{Interchange 1 and 2 if necessary.}) \quad (21)$$

This surprisingly simple result proves to dominate the entire behavior of bonded scarf joints, even for elastic-plastic adhesives. This equation demonstrates conclusively the importance of maintaining adherend stiffness balance whenever possible. When this is maintained, in the absence of any thermal mismatch, the adhesive is essentially uniformly stressed throughout the entire overlap of any length. The only minor exception is the local end effect identified by Richards in Reference (6).

Returning now to the solution, in terms of equations (18) to (20), it follows by integrating equation (7) that

$$\frac{\tau_{av}}{\tau_p} = \sum_{n=1}^{\infty} \frac{A_n}{n} . \quad (22)$$

In using this series it is necessary to employ two arbitrary constants to satisfy the boundary conditions. The first two are chosen. That is

$$\frac{\tau_{av}}{\tau_p} = A_1 \times \text{SIG}(3) + A_2 \times \text{SIG}(4) \quad (23)$$

and, because of equation (8),

$$\frac{\tau_{av}}{\tau_p} = \text{SIG}(3) + A_2 \times \text{SIG}(4) . \quad (24)$$

The summations SIG(3) and SIG(4) are the quantities formed by evaluating the coefficients in equation (22) by means of equations (20) and (18) after setting, in turn,

$$A_1 = 1 , \quad A_2 = 0 \quad \text{for SIG}(3) \quad (25)$$

and

$$A_1 = 0 , \quad A_2 = 1 \quad \text{for SIG}(4) . \quad (26)$$

The solution procedure employed in the FORTRAN IV digital computer program listed in Appendix A1 is as follows. The coefficient  $A_3$  for each set of initial values (25) and (26) is evaluated in terms of equation (20). Then a number of higher order coefficients are evaluated in turn through the recurrence relation (18), the same number being evaluated for SIG(3) as for SIG(4). The results of these summations are then substituted into equation (19) which takes on the form

$$A_2 \left\{ 1 + \frac{(\lambda l)^2}{[1 + \text{ETR}(1)]} \text{SIG}(4) \right\} = (\lambda l) \text{CTHERM}(1) + \frac{(\lambda l)^2 \text{ETR}(1)}{[1 + \text{ETR}(1)]} - \frac{(\lambda l)^2}{[1 + \text{ETR}(1)]} \text{SIG}(3) . \quad (27)$$

The unknown  $A_2$  is then to be evaluated and substituted into equation (19) re-arranged in the form

$$\frac{\tau_{av}}{\tau_p} = ETR(1) + \frac{[1 + ETR(1)] C_{THERM}(1)}{(\lambda\ell)} - \frac{[1 + ETR(1)]}{(\lambda\ell)^2} A_2 \quad (28)$$

This equation establishes the potential bond shear strength.

The detailed discussion of parametric effects is presented in Section 5 but certain features of the mathematics of the numerical solution merit elaboration at this stage. The most important feature is the decision to evaluate the terms  $A_n/n$  of the average stress series (22) directly rather than the quantities  $A_n$  of the series (7). To do so, equation (18) is re-organized to the form

$$\left(\frac{A_n}{n}\right) = \frac{(n-1)(n-2)\left(\frac{A_{n-1}}{n-1}\right) + (\lambda\ell)^2 \left[ \frac{ETR(1)}{1 + ETR(1)} \left(\frac{A_{n-1}}{n-1}\right) + \frac{1 - ETR(1)}{1 + ETR(1)} \left(\frac{A_{n-2}}{n-2}\right) \right]}{[n(n-1)]} \quad (29)$$

The reason for this is the factor  $(\lambda\ell)^2$  in equations (29) and (18). Because of this, for long overlaps, a much higher value of  $n$  is needed to reach negligible values of  $A_n$  from equation (18) than to reach negligible values of  $A_n/n$  from equation (29). Indeed, even with the use of equation (29) rather than equation (18) it remained impossible to compute reliable internal stress distributions for long overlap joints, even with as many as 50 terms of the shear stress series because of overflow in the computer. Such a computation is of little importance, however, since the critical location must be at one end or other of the joint. In spite of this problem, however, equation (27) converges rapidly, usually within the first five successive evaluations (for progressively increasing  $n$ ) of SIG(3) and SIG(4). The program in Appendix A1 used 20 terms. In addition to this, because  $A_2$  is divided by  $(\lambda\ell)^2$  in equation (28), an extremely reliable value of  $\tau_{av}/\tau_p$  can be computed readily. The program identifies the more critical end by the simple expedient of estimating the strength starting from each end of the joint and selecting the lower value. It is obvious that a computation of  $\tau_{av}/\tau_p > 1$  signifies simply that condition (8) was violated. A negative value of  $\tau_{av}/\tau_p$  indicates such severe thermal mismatch between adherends that the joint will break apart prior to application of any mechanical loads.

The computation of joint strength proceeding from the other end of the joint is effected by simply interchanging the subscripts 1 and 2 on all affected quantities. With regard to adherend stiffness imbalance alone, it is always possible to identify from equation (28) that the more critical end (1) is that for which  $ETR(1) \leq 1$ . The possible ambiguity arises as the result of the thermal mismatch terms. Since  $CTHERM(1)$  may be either negative or positive independently of whether  $ETR(1)$  is less than or greater than unity, severe thermal mismatch may nullify or even overpower any stiffness imbalance effects. This possibility is evidently greatest for short overlaps because of the factor  $(\lambda\lambda)$  in the denominator of the thermal term in equation (28). It follows that the critical end of the joint between given adherends may well change as the overlap changes and, indeed, such behavior was predicted by the computer program output.

Equations (1) and (2) have been set up for applied tensile loads in the adherends. In the event that the applied load is compressive, it can be seen with reference to Figure 2 that all quantities except the thermal strain terms will change sign. This implies that, in the absence of any thermal mismatch effects, the same end of the joint is critical for both tensile and compressive adherend loads and that the joint strength is the same. Rather than change the sign of all quantities with the exception of the thermal terms, the program merely changes the signs of  $CTHERM(1)$  and  $CTHERM(2)$  to account for compressive loading rather than tensile loading. It should be noted that, as a consequence, the opposite end of the same joint may be critical for a reversed load and that the strength may not be the same if there is also stiffness imbalance between the adherends. Likewise, just as for double-lap joints, if the thermal mismatch terms nullify any stiffness imbalance effects for one load direction, they must aggravate the stress concentrations for a load in the reverse direction. By analogy with the double-lap joint analyses in Reference (1), the case of in-plane shear loading is covered by the analysis above replacing  $E_1$  and  $E_2$  in equation (2) and those equations based on it by the shear moduli  $G_1$  and  $G_2$  and neglecting the thermal affects which induce bond stresses at right angles to those of concern for mechanical in-plane shear loads except at the sides of the joint. The direct adherend forces  $T_1$  and  $T_2$  are replaced by shear forces  $S_1$  and  $S_2$  per unit length. A more precise representation of thermal effects for in-plane shear loading would necessarily require a two-dimensional

analysis rather than the one-dimensional solution above and the justification for doing so is minimized by the small amount of adhesive plasticity that even the real brittle adhesives exhibit.

### 3. ELASTIC-PLASTIC ANALYSIS OF SCARF JOINTS

The preceding elastic analysis covers essentially the most difficult formula-  
tive portions of the elastic-plastic scarf joint analysis. New numerical  
difficulties of major proportions were encountered in the generation of speci-  
fic answers by the computer program, but the plastic part of the analysis is  
straightforward. The necessary additional geometry and nomenclature are iden-  
tified in Figure 2. Equations (1), (2) and (4) continue to apply, with the  
substitutions

$$dx = l d\xi = l d\chi = l d\zeta \quad (30)$$

as appropriate. Equation (5) is supplemented by the relation

$$\tau = \tau_p \quad \text{for } 0 \leq \xi \leq a \quad \text{and} \quad 0 \leq \zeta \leq c . \quad (31)$$

The relations (4) for the adherend stiffnesses are replaced by

$$(Et)_1 = E_1 t_1 (1 - \xi) = E_1 t_1 \left(1 - \frac{a}{l} - \chi\right) = E_1 t_1 \left(1 - \frac{a}{l} - \frac{b}{l} - \zeta\right) \quad (32)$$

and

$$(Et)_2 = E_2 t_2 \xi = E_2 t_2 \left(\frac{a}{l} + \chi\right) = E_2 t_2 \left(\frac{a}{l} + \frac{b}{l} + \zeta\right) . \quad (33)$$

In the elastic zone, the location of which has yet to be determined, the same  
power series solution is sought:

$$\frac{\tau}{\tau_p} = \sum_1^{\infty} A_n \chi^{(n-1)} \quad \text{or} \quad \frac{\gamma}{\gamma_e} = \sum_1^{\infty} A_n \chi^{(n-1)} , \quad (34)$$

again with  $A_1 = 1$  by definition of adherend 1 as the more highly loaded end of  
the joint.

In the left adhesive plastic zone of the joint illustrated in Figure 2, the  
adherend forces per unit width follow from equations (1) and (31) as

$$T_1 = \tau_{av} l - \tau_p l \xi \quad \text{and} \quad T_2 = \tau_p l \xi . \quad (35)$$

Substitution into equation (4), making use of equations (2), yields

$$\gamma = \frac{\delta_2 - \delta_1}{\eta} = \frac{1}{\eta} \left[ (\alpha_2 - \alpha_1) \Delta T \ell \xi + \int_0^\xi \frac{T_2 \ell d\xi}{(Et)_2} - \int_0^\xi \frac{T_1 \ell d\xi}{(Et)_1} \right], \quad (36)$$

$$\gamma = \frac{1}{\eta} \left[ (\alpha_2 - \alpha_1) \Delta T \ell \xi + \frac{\tau_p \ell^2}{E_2 t_2} \xi + \int_0^\xi \frac{(\tau_p - \tau_{av}) \ell^2}{(E_1 t_1)(1 - \xi)} d\xi - \frac{\tau_p \ell^2}{E_1 t_1} \xi \right] + C, \quad (37)$$

$$\gamma = \frac{1}{\eta} (\alpha_2 - \alpha_1) \Delta T \ell \xi + \tau_p \ell^2 \left[ \frac{1}{E_2 t_2} - \frac{1}{E_1 t_1} \right] \xi - \frac{(\tau_p - \tau_{av}) \ell^2}{E_1 t_1} \ln(1 - \xi) + C. \quad (38)$$

The appropriate boundary conditions are that

$$\gamma = \gamma_e + \gamma_p \quad \text{at } \xi = 0 \quad (39)$$

and

$$\gamma = \gamma_e \quad \text{at } \xi = a/\ell. \quad (40)$$

Consequently, from equations (39) and (38),

$$C = (\gamma_e + \gamma_p) \quad (41)$$

so that, from equations (40) and (38),

$$\gamma_p = -\frac{1}{\eta} \left[ (\alpha_2 - \alpha_1) \Delta T \ell \left( \frac{a}{\ell} \right) + \tau_p \ell^2 \left( \frac{1}{E_2 t_2} - \frac{1}{E_1 t_1} \right) \left( \frac{a}{\ell} \right) - \frac{(\tau_p - \tau_{av}) \ell^2}{E_1 t_1} \ln \left( 1 - \frac{a}{\ell} \right) \right]. \quad (42)$$

Equation (42) may be non-dimensionalized by use of the quantities in equations (13) to (15). It then adopts the form

$$\left( \frac{\gamma_p}{\gamma_e} \right) = -(\lambda \ell) \text{CTHERM}(1) \left( \frac{a}{\ell} \right) + (\lambda \ell)^2 \left( \frac{a}{\ell} \right) \frac{1 - \text{ETR}(1)}{1 + \text{ETR}(1)} + (\lambda \ell)^2 \frac{[1 - (\tau_{av}/\tau_p)]}{[1 + \text{ETR}(1)]} \ln \left( 1 - \frac{a}{\ell} \right). \quad (43)$$

In solving for the joint strength it is necessary to maintain continuity at the transition ( $\xi = a/\ell$ ) from plastic to elastic adhesive behavior. The continuity of adherend stresses requires that there be no change in  $d\gamma/dx$ .

From equations (4) and (2)

$$\frac{d\gamma}{dx} = \frac{1}{\eta} \left[ (\alpha_2 - \alpha_1) \Delta T + \frac{T_2}{(Et)_2} - \frac{T_1}{(Et)_1} \right] \quad (44)$$



or, in non-dimensionalized form, for the plastic side of the transition

$$\left. \frac{d(\gamma/\gamma_e)}{d\xi} \right|_{\xi = a/l} = (\lambda l) \text{CTHERM}(1) - (\lambda l)^2 \frac{1 - \text{ETR}(1)}{1 + \text{ETR}(1)} + \frac{(\lambda l)^2 [1 - (\tau_{av}/\tau_p)]}{[1 + \text{ETR}(1)][1 - (a/l)]} \quad (45)$$

For the elastic side, equation (34) requires that

$$\left. \frac{d(\gamma/\gamma_e)}{d\chi} \right|_{\chi = 0} = A_2 \quad (46)$$

Since  $A_1 \equiv 1$ , the elastic stress distribution can now be evaluated by a recurrence formula, just as in Section 2.

Under certain combinations of stiffness and thermal mismatch between adherends there will be no second plastic adhesive shear stress zone at the far end of the joint while under others there will be. In the former case, the evaluation of the elastic adhesive shear stress at  $\chi = 1 - (a/l)$  by means of the series (34) will lead to a result  $\tau_{\text{end}}/\tau_p \leq 1$ . A value of this ratio greater than unity indicates a need for evaluating the affects of the presence of a second plastic adhesive zone, at the far end of the joint. Referring again to Figure 2, the adherend forces per unit width are evaluated through equations (1) and (31) as

$$T_1 = \tau_p l \left( \frac{c}{l} - \zeta \right), \quad T_2 = \tau_{av} l - \tau_p l \left( \frac{c}{l} - \zeta \right) \quad (47)$$

Substitution of equations (47) and (33) into equation (44) leads to the expression

$$\frac{d(\gamma/\gamma_e)}{d\zeta} = \frac{G}{\tau_p \eta} (\alpha_2 - \alpha_1) \Delta T l - \frac{\tau_p l^2}{E_1 t_1} + \frac{\tau_p l^2}{E_2 t_2} - \frac{(\tau_p - \tau_{av}) l^2}{(E_2 t_2) [1 - \frac{c}{l} + \zeta]} \quad (48)$$

The transition relation at  $\zeta = 0$  follows as

$$\left. \frac{d(\gamma/\gamma_e)}{d\gamma} \right|_{\zeta = 0} = (\lambda l) \text{CTHERM}(1) - (\lambda l)^2 \left[ \frac{1 - \text{ETR}(1)}{1 + \text{ETR}(1)} \right] - (\lambda l)^2 \frac{[1 - (\tau_{av}/\tau_p)] \text{ETR}(1)}{[1 + \text{ETR}(1)] (1 - \frac{c}{l})} \quad (49)$$

Equation (48) may be integrated once, yielding

$$\left(\frac{\gamma}{\gamma_e}\right) = (\lambda\ell) \text{CTHERM}(1) \zeta - (\lambda\ell)^2 \frac{1 - \text{ETR}(1)}{1 + \text{ETR}(1)} \zeta - (\lambda\ell)^2 \frac{[1 - (\tau_{av}/\tau_p)]\text{ETR}(1)}{[1 + \text{ETR}(1)]} \ln\left(1 - \frac{c}{\ell} + \zeta\right) + C, \quad (50)$$

in which, since

$$\gamma = \gamma_e \quad \text{at} \quad \zeta = 0, \quad (51)$$

$$1 = -(\lambda\ell)^2 \frac{[1 - (\tau_{av}/\tau_p)]\text{ETR}(1)}{[1 + \text{ETR}(1)]} \ln\left(1 - \frac{c}{\ell}\right) + C. \quad (52)$$

Signifying by  $\gamma_{\max}$  the peak adhesive shear strain at the less critical (by definition) right hand end of the joint,

$$\left(\frac{\gamma_p}{\gamma_e}\right) > \frac{\gamma_{\max}}{\gamma_e} = (\lambda\ell) \text{CTHERM}(1) \left(\frac{c}{\ell}\right) - (\lambda\ell)^2 \frac{[1 - \text{ETR}(1)]}{[1 + \text{ETR}(1)]} \left(\frac{c}{\ell}\right) + (\lambda\ell)^2 \frac{[1 - (\tau_{av}/\tau_p)]\text{ETR}(1)}{[1 + \text{ETR}(1)]} \ln\left(1 - \frac{c}{\ell}\right). \quad (53)$$

A comparison of equations (43) and (53) shows complete consistency upon interchanging subscripts 1 and 2. While equation (53) could be employed to identify whether the left or right hand end of the joint in Figure 2 is more critical once the extent of the second plastic zone ( $c/\ell$ ) had been established, there is an inherent numerical difficulty in the step by step computation of the strength by the procedure outlined above. It was explained in Section 2 that, for the perfectly-elastic adhesive, only the average adhesive shear stress could be computed and not the stress distribution as a function of position along the joint. In the computer program in Appendix A3, the only reason why it proved possible to evaluate the extent of the elastic trough, for long overlaps, was the factor  $[1 - (a/\ell) - (c/\ell)]^{n-1} < 1$  in equation (34). At high values of  $n$ , this very small term was able to overpower the influence of the  $(\lambda\ell)^2$  factor in the numerator of the recurrence formula (18). This numerical accuracy problem prevented the reliable evaluation of  $d(\gamma/\gamma_e)/d\chi$  at  $\chi = (b/\ell)$  to match boundary conditions at the transition of the second plastic adhesive zone. Consequently an iterative solution had to be employed to evaluate the maximum possible

extent of the elastic trough.

Referring to equations (45) and (46), it can be seen that, in the iterative solution process, the second term of the elastic adhesive shear stress series  $A_2$  depends on the preceding estimates of both  $(a/l)$  and  $(\tau_{av}/\tau_p)$ . In the early development of the digital computer program for elastic-plastic scarf joints insurmountable convergence difficulties were encountered if the initial estimates for  $(\tau_{av}/\tau_p)$  and  $(a/l)$  were not sufficiently close to the true values. This difficulty was eventually overcome by the following technique. Equation (43) was re-arranged to read

$$\left(\frac{\tau_{av}}{\tau_p}\right) = 1 - \frac{\left\{ \frac{[1 + ETR(1)](\gamma_p)}{(\lambda l)^2} \left(\frac{\gamma_p}{\gamma_e}\right) + \left[ \frac{[1 + ETR(1)]CTHERM(1)}{(\lambda l)} - [1 - ETR(1)] \right] \left(\frac{a}{l}\right) \right\}}{\ln[1 - (a/l)]} \quad (54)$$

This can be differentiated with respect to  $(a/l)$  so that

$$d(\tau_{av}/\tau_p) / d(a/l) = 0 \quad \text{when}$$

$$\left(1 - \frac{a}{l}\right) \ln \left(1 - \frac{a}{l}\right) = \frac{\left\{ \frac{[1 + ETR(1)](\gamma_p)}{(\lambda l)^2} \left(\frac{\gamma_p}{\gamma_e}\right) + \left[ \frac{[1 + ETR(1)]CTHERM(1)}{(\lambda l)} - [1 - ETR(1)] \right] \left(\frac{a}{l}\right) \right\}}{[1 - ETR(1)] - \frac{[1 + ETR(1)]CTHERM(1)}{(\lambda l)}} \quad (55)$$

Substitution of equation (55) into equation (54) yields, for the minimum (stationary) value of  $(\tau_{av}/\tau_p)$

$$\begin{aligned} \frac{\tau_{av}}{\tau_p} &= ETR(1) + \frac{[1 + ETR(1)]CTHERM(1)}{(\lambda l)} \\ &+ \frac{a}{l} \left\{ [1 - ETR(1)] - \frac{[1 + ETR(1)]CTHERM(1)}{(\lambda l)} \right\} \quad (56) \end{aligned}$$

This is evidently consistent with the elastic solution  $(a/l) = 0$  for large overlaps and, upon subsequent comparison with the more precisely estimated joint strengths, proved to be an extremely close lower bound for all cases of practical interest. It is significantly conservative only for very short overlaps [small values of  $(\lambda l)$ ] or very brittle adhesives [very small values of  $(\gamma_p/\gamma_e)$ ]. The adhesive shear strain capacity  $\gamma_p$  is involved in equation (56) implicitly through the extent  $(a/l)$  of the plastic zone. Equation (55)

is solved by iteration to evaluate  $(a/l)$  and the result substituted into equation (56) or (54). Appendix A2 contains a listing of the FORTRAN IV digital computer program employed to solve equations (55) and (54), together with sample outputs and brief user instructions. The iteration technique eventually adopted proved to be quite convergent, after other re-arrangements of equation (55) demonstrated strongly divergent characteristics.

This program in Appendix A2 served to provide the initial estimates of  $(a/l)$  and  $(\tau_{av}/\tau_p)$  in the more precise solution listed in Appendix A3. The sequence of variables used in the solution is  $(a/l)$ ,  $(\tau_{av}/\tau_p)$  and  $(c/l)$  after which  $(\tau_{av}/\tau_p)$  is recomputed and the estimate of  $(a/l)$  adjusted until convergence is attained. In those cases in which the critical end is not evident by inspection, the potential bond shear strength is computed from each end of the joint and the lower value adopted. Brief user instructions and sample outputs are included in Appendix A3.

The analyses above for scarf joints pertain to adhesive shear stresses and it is demonstrated that a small enough scarf angle can always be found to transfer the full adherend strength through the bond with an adequate margin. There is, of course, a potential problem with the adherend strength(s) if the scarf angle is too small. Specifically, one adherend will fail if the scarf angle  $\theta$  is so small that

$$\theta < \tau_p / F_u , \quad (57)$$

(where  $F_u$  is the ultimate adherend stress in tension, compression, or shear, as appropriate) at the more critical end of the joint (identified by the adhesive shear stress analysis). Should this situation arise, the solution is to decrease the adherend stiffness imbalance across the joint by local reinforcement of the softer adherend. It is evident from equation (17) that this potential problem of breaking off the tip of (usually) the stiffer adherend is more likely to arise with the brittle adhesives (higher values of peak adhesive shear stress  $\tau_p$ ) than with ductile adhesives. This is one important reason for preferring to effect the load transfer with a shorter overlap of ductile adhesive than with a longer overlap of brittle adhesive. The extreme case of making the overlap so extremely long that the peak adhesive shear

stress actually developed is restricted to a small fraction of its capacity when adherend failure occurs outside the joint has theoretical appeal only, frequently being quite impractical.

#### 4. DISCUSSION OF PARAMETRIC EFFECTS

Representative solutions from Sections 2 and 3 for unbalanced bonded scarf joints are illustrated in Figures 3 through 7. Figures 3 and 4 show the separate effects of adherend stiffness and thermal mismatch, respectively, on the elastic joint strength. The deviations from unity in the  $(\tau_{av}/\tau_p)$  ratio, for a given overlap  $(\lambda\ell)$ , are proportional to the individual imbalances. The effect of stiffness imbalance is a smooth decrease from a fully-efficient bond  $(\tau_{av} = \tau_p)$  to a less efficient bond  $(\tau_{av} < \tau_p)$  asymptoting towards the solution given in equation (21). This diagram, more than any other, characterizes the dominant feature of the scarf joint behavior. This is that the potential bond strength continues to increase indefinitely with increasing overlap. This is in marked contrast to the behavior of uniform lap joints [References (1) and (2)], which develop maximum strengths which remain effectively constant beyond intermediate overlaps. The effect of this characteristic on the potential bond strength of scarf joints is that, by making the scarf angle sufficiently small, one can always design a joint in which the potential bond strength exceeds the adherend strength by any specified factor. This is amply demonstrated by curve D in Figure 4. While adherend stiffness and thermal mismatch combine to decrease the bond efficiency below the unit value of curve A, the bond strength for long overlaps ends up being proportional to the overlap. As a consequence of this characteristic, the elastic adhesive shear stresses play a far more important role in the strength of scarf joints than they do in the case of uniform lap joints. Nevertheless, it would be erroneous to conclude that one could always design an unbalanced scarf joint within the capabilities of an elastic adhesive. The limiting problem is that, as the scarf angle becomes very small, there is a strong probability of breaking off the tip of the stiffer adherend. While not as acute a design detail problem as its counterpart for stepped-lap joints, this feature restricts the scarf angle to exceed the value

$$\theta = \text{ARCTAN}(\tau_p/F_u) \quad (58)$$

in which  $F_u$  is the adherend ultimate strength (in tension, compression, or shear, as appropriate for the applied load).

The effect of adherend thermal mismatch on the potential bond strength of scarf joints is shown in Figure 4. It is clear that the effects are insignificant for very short and very long overlaps, being significant only for those

overlaps of practical interest. The effects are maximum at  $(\lambda\ell) = 2$  for all values of the thermal mismatch coefficient  $C_{THERM}$ .

Figure 5 shows the interaction between adherend stiffness and thermal mismatch. Curves B, D and E represent one set of solutions, with curve B showing the effect of stiffness imbalance alone. Curve D adds the influence of compounding thermal mismatch as well. Curve E demonstrates the behavior of self-cancelling adherend imbalances at  $(\lambda\ell) = 3$ . For values of  $(\lambda\ell)$  less than 3, the thermal mismatch effects dominate over those arising from stiffness mismatch and the more critical end of the joint is reversed. Curves A, C and F form another set showing how, for severe adherend thermal mismatch, there is a range of overlaps for which the residual thermal stresses are so severe that the joint will split apart without the application of any mechanical loads. Quite unlike the behavior of uniform lap joints [References (1) and (2)], this problem can be eliminated completely by sufficient extension of the overlap.

Just as is the case for uniform lap joints adhesive plasticity can increase the potential bond shear strength. The extent of this strength increase is shown in Figures 6 and 7 for stiffness and thermal mismatch, respectively. For each amount of adhesive plastic shear strain, there is an associated overlap below which the bond can be uniformly stressed. For indefinitely large overlaps the asymptotic solution (21) again holds, masking completely the influence of any adhesive plasticity. In the overlaps of practical interest, the actual amount of adhesive plasticity available from real structural adhesives can improve the potential joint strength greatly. One benefit of using a ductile adhesive of moderately high peak shear stress rather than a brittle adhesive of very high peak shear stress is that the joint is better able to withstand the variation in joint load which inevitably occurs as the result of manufacturing imperfections and non-uniform load distribution. Another benefit is that the problem of breaking off the tip of the adherend at the more critically loaded end [see equation (58)] is greatly alleviated. If the tip of the stronger adherend were allowed to be broken off, this would impose an effective net area loss on the cross-section of the weaker adherend.

## 5. ELASTIC ANALYSIS OF STEPPED-LAP JOINTS

The analysis for the strength of stepped-lap adhesive-bonded joints contains features of both the uniform lap joints [References (1) and (2)] and the scarf joint above. Peel stress problems are ignored on the grounds that the outermost end steps are invariably thin enough (in good design practice) not to induce significant peel stresses in the adhesive. Likewise, the small eccentricity in the load path has been ignored in the interests of obtaining a useful uncomplicated design tool.

A representative idealized stepped-lap joint is shown in Figure 8, along with the sign convention and nomenclature necessary for the analysis. Just as for the scarf joint analysis, the same diagram serves also for the elastic-plastic analysis, so it contains information not necessary for the elastic analysis. This begins with the equilibrium equations for a differential element of one of the steps.

$$\frac{dT_o}{dx} + 2\tau = 0 \quad , \quad \frac{dT_i}{dx} - 2\tau = 0 \quad . \quad (59)$$

Here the subscripts  $o$  and  $i$  refer to the "outer" and "inner" adherends, respectively, and the factors 2 in equations (59) account for the two bond surfaces surrounding the inner adherend. Consequently the adherend thicknesses  $t_o$  and  $t_i$  refer to the total cross-section and the forces  $T_o$  and  $T_i$  do likewise. The nature of the solution is such that it is, on occasions, necessary to interchange the subscripts  $o$  and  $i$  mathematically. The thermo-elastic relations for the adherends are

$$\frac{d\delta_o}{dx} = \frac{T_o}{E_o t_o} + \alpha_o \Delta T \quad , \quad \frac{d\delta_i}{dx} = \frac{T_i}{E_i t_i} + \alpha_i \Delta T \quad . \quad (60)$$

The adhesive shear strain, for tensile lap shear loading, is

$$\gamma = (\delta_i - \delta_o) / \eta \quad . \quad (61)$$

while the elastic adhesive shear stress is related to the shear strain by the relation

$$\tau = G\gamma = G(\delta_i - \delta_o) / \eta \quad . \quad (62)$$



The solution proceeds just as in Reference (1).

$$\frac{d\tau}{dx} = \frac{G}{\eta} \left[ \frac{d\delta_i}{dx} - \frac{d\delta_o}{dx} \right] = \frac{G}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right] \quad (63)$$

$$\frac{d^2\tau}{dx^2} = \frac{G}{\eta} \left[ \frac{2}{E_i t_i} + \frac{2}{E_o t_o} \right] \tau = \lambda^2 \tau \quad (64)$$

The solution of equation (64) is

$$\tau = A \cosh(\lambda x) + B \sinh(\lambda x) \quad (65)$$

where the integration constants A and B are to be determined by boundary conditions for each step. Substitution of equation into equation (59) yields

$$T_o = T_{o_{ref}} - 2 \frac{A}{\lambda} \sinh(\lambda x) - 2 \frac{B}{\lambda} [\cosh(\lambda x) - 1] \quad (66)$$

and

$$T_i = T_{i_{ref}} + 2 \frac{A}{\lambda} \sinh(\lambda x) + 2 \frac{B}{\lambda} [\cosh(\lambda x) - 1] \quad (67)$$

The values of  $T_{o_{ref}}$  and  $T_{i_{ref}}$  depend upon the origin of x adopted. In the solution it proves convenient to adopt the start of each step as the origin for that step. Integrating again, by means of equations (69),

$$\delta_o = \delta_{o_{ref}} + \alpha_o \Delta T x + \frac{1}{E_o t_o} \left[ T_{o_{ref}} x - 2 \frac{A}{\lambda^2} \cosh(\lambda x) - 2 \frac{B}{\lambda^2} [\sinh(\lambda x) - (\lambda x)] \right] \quad (68)$$

and

$$\delta_i = \delta_{i_{ref}} + \alpha_i \Delta T x + \frac{1}{E_i t_i} \left[ T_{i_{ref}} x + 2 \frac{A}{\lambda^2} \cosh(\lambda x) + 2 \frac{B}{\lambda^2} [\sinh(\lambda x) - (\lambda x)] \right] \quad (69)$$

In the FORTRAN IV digital computer program, listed in Appendix A4, used to solve the equations above for the elastic stepped-lap joint, the technique of solution is as follows. The solution proceeds, one joint step at a time starting with assumed values of the load and initial adhesive shear strain (or stress). The latter is set at the maximum adhesive allowable and remains so unless it is computed that the peak adhesive shear strain is greater elsewhere (most probably at the other end of the joint) in which case the initial strain is reduced as much as necessary to avoid exceeding the allowable. The key

equation in the solution is equation (65). The integration constant A is evaluated as the specified (or subsequently computed) adhesive shear stress at the start of the step under consideration.

$$A = \tau_{x=0} \quad (70)$$

The other constant B derives from equation (63), also evaluated at the start of that step. That is

$$\frac{d\tau}{dx} = A\lambda \sinh(\lambda x) + B\lambda \cosh(\lambda x) = \frac{G}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right] \quad (71)$$

so that at  $x = 0$

$$B = \frac{G}{\eta\lambda} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{x=0} \quad (72)$$

The values of  $\tau$ ,  $T_o$ ,  $T_i$ ,  $\delta_o$  and  $\delta_i$  at the end of that step then follow from equations (65), (66), (67), (68), (69) and (62), respectively. If, after one complete set of computations, the load computed to be transferred out of the far end of the joint does not match that assumed to act at the near (starting) end, the initial estimate is adjusted until the two quantities do match. At that stage, a check is made throughout the joint, step by step, to identify the most critical adhesive and adherend locations. If any negative margins are identified, the load and peak adhesive shear stress are reduced as much as is necessary to eliminate them.

While the formulation of the equations and analysis scheme above is quite straightforward, the actual numerical solution of the problem proved to be quite difficult. Even with double precision it was almost invariably impossible to compute values for all steps of the joint in a single pass, even if the initial conditions (load and peak adhesive shear stress) were precisely correct to 16 significant figures. A change of 1 in the 16th significant digit of an initial condition would frequently effect a change by a factor of up to  $\pm 10^7$  in a quantity computed in the fourth or fifth step. This was not the result of a poorly conditioned mathematical formulation. It follows directly from strong physical characteristics of stepped-lap joints. It is the nature of stepped-lap joints, be they bonded or bolted, that any non-uniformities in the load transfer are dominated by the geometry and materials of the end three

steps. What happens in between has only negligible effect on the critical loads which almost invariably occur at one end or other of the joint. Likewise, in a uniform lap joint, practically all the load is transferred through the end three (rows of) bolts or through a narrow effective end zone of adhesive. Because of this characteristic the initial coding of the equations led to a highly accurate estimate of the load (assuming that the adhesive was critical at one end of the joint) but was unable to compute the internal loads and check on the adherend strength margin. The technique finally employed for dealing with this problem took advantage of the seemingly undesirable characteristics and is summarized as follows. By printing out intermediate computations it became clear that, if the initial load estimate on a given step was too high (even if only minutely), on the step just before computations for a subsequent step caused overflows and underflows in the computer the computations would diverge in a characteristic way, precisely the opposite of that for an initial underestimate of that load. Therefore upper and lower bounds were placed on the load estimate and the trial load was taken as the average of these. If the trial load was found to be too high, it served as the new upper bound and, were it too low, it was used to raise the lower bound. This technique was found to bring the upper and lower bounds into precise agreement rapidly. Once this had occurred the computations for the start of that step were frozen and the solution proceeded to perturb each successive step in turn, using the same convergence check above, until the load transferred out of the far end of the joint precisely equalled that input at the near end. Then a check is made, at the ends of each step, on the adhesive and adherend stresses to ensure that neither exceeds the allowable. Due allowance is made for the sign of the quantities involved. In the absence of any thermal mismatch this last operation of checking on the allowables can be performed by simple linear scaling. However, if there is any adherend thermal mismatch present, this adjustment must be performed by iteration since, as is evident from equation (62), the thermal stress terms do not scale in proportion to the adhesive and adherend stresses. A necessary check on the accuracy of the numerical processes has been accomplished by checking that precisely the same solution is obtained regardless of whether the computations commence at the more critically loaded end of the joint or at the other end.

In view of the numerical problems encountered with this analytical solution, it stands to reason that they will have their counterpart in any finite-element solution. Very fine grids would be needed in the high stress gradient areas.

## 6. ELASTIC-PLASTIC ANALYSIS OF STEPPED-LAP JOINTS

In addition to the equations of Section 5 for the perfectly elastic analysis of stepped-lap joints, the elastic-plastic analysis requires, instead of equation (62), that

$$\tau = \tau_p \quad \text{for } \gamma \geq \gamma_e, \quad (73)$$

and

$$\tau = G\gamma \quad \text{for } \gamma \leq \gamma_e. \quad (74)$$

The elastic-plastic solution is best carried out in terms of the adhesive shear strains rather than the shear stresses. In the plastic adhesive zones, from equations (61) and (60),

$$\frac{d\gamma}{dx} = \frac{1}{\eta} \left[ \frac{d\delta_i}{dx} - \frac{d\delta_o}{dx} \right] = \frac{1}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right] \quad (75)$$

whence, from equations (59)

$$\frac{d^2\gamma}{dx^2} = \frac{2}{\eta} \left[ \frac{1}{E_i t_i} + \frac{1}{E_o t_o} \right] \tau_p = \frac{\lambda^2}{G} \tau_p = \text{constant}. \quad (76)$$

Therefore, in the plastic zone,

$$\gamma = \frac{\lambda^2}{2G} \tau_p x^2 + Cx + D \quad (77)$$

and

$$T_o = T_{o_{ref}} - 2\tau_p x, \quad T_i = T_{i_{ref}} + 2\tau_p x \quad (78)$$

while

$$\left. \begin{aligned} \delta_o &= \delta_{o_{ref}} + \alpha_o \Delta T x + \frac{1}{E_o t_o} \left[ T_{o_{ref}} x - \tau_p x^2 \right] \\ \delta_i &= \delta_{i_{ref}} + \alpha_i \Delta T x + \frac{1}{E_i t_i} \left[ T_{i_{ref}} x + \tau_p x^2 \right] \end{aligned} \right\} \quad (79)$$

In equation (77),  $D$  is set equal to  $\gamma$  at the start of any step, since a new zero for  $x$  is chosen at that location for each step. The other constant  $C$  follows from equations (75) and (77). Thus

$$C = \left. \frac{d\gamma}{dx} \right|_{x=0} = \frac{1}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{x=0} \quad (80)$$

Very few individual steps of stepped-lap joints have fully-plastic adhesive throughout the entire joint. Any adhesive plasticity is frequently confined to the end(s) of the step(s). Therefore, in performing an elastic-plastic analysis of a stepped-lap joint, it is necessary to be able to compute the extent of the plastic zones. Therefore, beginning at the left hand end of the step element shown in Figure 8 and assuming a sufficiently high load intensity for the adhesive to be in the plastic state, the first computation is that of the maximum possible extent of the plastic zone. This is then compared with the actual extent of the step. If necessary, a second computation is performed of the maximum possible extent of the elastic trough in that same step. Starting from equation (77) with  $\gamma = \gamma_{ref}$  at  $x = 0$ ,

$$\gamma = \frac{\lambda^2}{2G} \tau_p x^2 + Cx + \gamma_{ref} \quad (81)$$

where the constant  $C$  is given by equation (80). It is necessary to find the lesser value of  $x$  for which  $\gamma = \gamma_e$ . Equation (81) is re-arranged to read

$$\frac{\lambda^2 \tau_p}{2G} x_p^2 + Cx_p + (\gamma_{ref} - \gamma_e) = 0 \quad (82)$$

so that the maximum extent of plastic adhesive zone is given by

$$x_p = -C \pm \sqrt{C^2 - 2\lambda^2 \tau_p (\gamma_{ref} - \gamma_e)} \quad (83)$$

Now, since  $C = d\gamma/dx < 0$  at  $x = 0$  the minus sign in front of the radical holds. Once  $x_p$  has been computed, it is compared with the step length  $\ell_{step}$ . If  $x_p > \ell_{step}$ , that particular step is fully-plastic throughout and the values of the various quantities at the far end of the step are evaluated from equations (73) to (80). Should  $x_p$  be less than  $\ell_{step}$ , the difference is examined elastically, to see whether it remains elastic throughout or becomes plastic again at the far end. For  $x_p < \ell_{step}$ , the values of the various stresses, strains,

displacements and forces are evaluated in terms of equations (73) to (79) and the subscripts  $pe$  serve to identify the plastic-to-elastic transition. Likewise  $ep$  identifies the possible elastic-to-plastic transition at the far end of the joint. It is necessary that  $d\gamma/dx$  be maintained at these transitions, as is evident from equation (75). The maximum possible extent of elastic trough must be deduced from equation (65). In doing so, it is mathematically far simpler to shift the  $x$  origin to the middle of the elastic trough (of extent  $2x_e$ ) so that

$$\tau = \tau_p \cosh(\lambda x) / \cosh(\lambda x_e) \quad (84)$$

At the  $pe$  transition ( $x = -x_e$ ) equation (62) requires that

$$\frac{d\tau}{dx} = \frac{G}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{pe} = -\tau_p \lambda \tanh(\lambda x_e) \quad (85)$$

so that the elastic trough could extend, if  $l_{step}$  were great enough, a distance

$$2\lambda x_e = \tanh^{-1} \left\{ -\frac{1}{\lambda \eta \gamma_e} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{pe} \right\} . \quad (86)$$

By use of known formulas for hyperbolic functions in terms of exponentials and the interrelation between exponential and logarithmic functions, the solution (85) is more conveniently expressed as

$$2x_e = \frac{1}{\lambda} \ln \left\{ \frac{1 - \frac{1}{\lambda \eta \gamma_e} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{pe}}{1 + \frac{1}{\lambda \eta \gamma_e} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{pe}} \right\} . \quad (87)$$

In the event that  $x_e$  does not extend beyond the far (right hand) end of the step being analyzed, it is necessary to compute the load transferred between the adherends throughout the elastic trough. In doing so, it is quite simple to take the value of  $2x_e$  from equation (87) and substitute it back into equations (65) to (72) for the standard elastic analysis of the preceding section. Should the elastic trough not extend to the far end of the step under analysis, equations (73) to (80) are employed for the plastic zone to the end of the step.

Equation (77) now becomes

$$\gamma = \frac{\lambda^2 \tau_p}{2G} x^2 + Cx + \gamma_{ep} \quad (88)$$

with

$$\tau = \tau_p \quad \text{for } x > x_{ep} \quad (89)$$

The constant  $c$  in equation (88) is evaluated in terms of equation (75)

$$c = \left. \frac{d\gamma}{dx} \right|_{ep} = \frac{1}{\eta} \left[ \frac{T_i}{E_i t_i} - \frac{T_o}{E_o t_o} + (\alpha_i - \alpha_o) \Delta T \right]_{ep} \quad (90)$$

In the last steps of the joint at the far end, the adhesive may be fully plastic throughout in which case, in equation (87),  $\gamma_{ep}$  should be replaced by  $\gamma_{ref}$ . Likewise, in those steps, near the middle of the joint, in which the adhesive shear strains are so small as not to reach the plastic state at either end of the step, the step will be elastic throughout and equations (65) to (72) are employed in the analysis. Towards the far end of the joint there may be a step which starts elastically and becomes plastic. In this case the actual extent of elastic behavior is determined by iteration, using equations (65) to (72) with a cut off (either positive or negative) on the shear stress.

If it should transpire that, at the end of the step,  $\gamma$  exceeds  $(\gamma_e + \gamma_p)$  or  $T_i$  or  $T_o$  exceed their respective allowables, this does not cause any analytical difficulty. An iterative procedure is employed in the analysis to reduce the external load and initial adhesive strain whenever necessary. While this does not represent any analytical difficulty, one should recognize that exceeding the allowables on an inner step can occur only as the result of poor detail design. The improvement of such details can increase the potential joint strength.

No new numerical difficulties were encountered in the program listed in Appendix A5 for the elastic-plastic analysis of stepped-lap joints which did not have a direct counterpart in the perfectly elastic analysis. However, the logic associated with keeping track of the locations of the transitions between elastic and plastic adhesive behavior, and vice versa, as they moved with each successive iteration posed a formidable problem. One small computational



problem was that, if the load estimate at some early stage in the iteration sequence was too far removed from the correct value, the computer would predict physically unrealizable large negative shear strains in the adhesive. A special set of instructions was prepared for this quirk.

The computer program, as basically written, checks simultaneously for the allowable adherend and adhesive strengths at the most critical locations in each step. Since stepped-lap joints are frequently more critical in the adherend than in the adhesive, a special feature has been added to increase greatly all adherend strengths artificially in order to print out also the potential adhesive bond strength and confirm that it exceeds the adherend strength by an adequate margin.

The analysis above is presented for the case of tensile lap shear loads being positive and the sign convention is in accordance. The computer programs have been so coded that, by a single input for the variable SGNLD, the respective solutions for tensile shear loading (SGNLD = +1) and compressive shear loading (SGNLD = -1) can be printed out. In the event that there are simultaneous stiffness and thermal mismatches between the adherends, the joint strength will not be the same for each load sense. Such a situation is common in the bonding of titanium edge members to boron-epoxy panels.

## 7. DISCUSSION OF DESIGN OF STEPPED-LAP JOINTS

The digital computer programs developed above to analyze stepped-lap joints can serve also as a useful design tool. Three clear dominant joint characteristics have been confirmed by studies with this program. The first is that the joint load capacity is defined by the end three steps at the more critical end of the joint. If other steps have a significant influence it will be adverse and be due to poor design detailing. The second is that, once the joint is essentially well-designed, quite major changes can be made to other than the critical end three steps without any significant impact on the joint strength. Third is that, in a well-designed joint, it is the very end step that is likely to precipitate joint failure unless its length is restricted in the design process. The necessary restriction is that the product of maximum adhesive shear stress and total bond area on the end step must not exceed the product of adherend material allowable and, cross section of the end step. Consequently, a ductile adhesive with higher strain energy provides stronger joints than a brittle adhesive with higher peak stress but less strain energy. It should be noted also that minimizing adherend stiffness imbalance increases the potential bond shear strength.

Mathematically speaking, the stepped-lap family of joints represent perturbations about the scarf joint solution. These perturbations become progressively greater as the number of steps decreases until the stepped-lap solution reduces to a single-lap joint for one step. Stepped-lap joints with only two or three steps are usually confined to thin adherends for which the potential bond shear strength is far in excess of the adherend(s) strength. In such cases the added strain concentrations in the bond due to the step discontinuities are not very important. Most applications of stepped-lap joints contain a large number of steps and, with a ductile adhesive softening the most severe of the adhesive stress spikes, the behavior very closely approaches that of the scarf joint. For this reason, preliminary design of practical stepped-lap joints by means of the scarf joint solution appears to be quite realistic. In doing so, however, one should exercise caution with regard to the critical end step of the adherend. The stepped-lap joint analysis, and practical experience, have identified the end step of the stiffer adherend as a prime candidate for the most critical design detail. If the extensional modulus of a composite adherend is

significantly less than that of a metal adherend to which it is bonded, most of the shear load transfer will be concentrated at the composite end of the joint with the probable result that tip fracture of the stiffer adherend will occur. One simple remedy to this potential difficulty is to be found in the concept of the dual-slope scarf joint illustrated in the upper part of Figure 9. In this joint, in order to protect the tip of the adherend, the scarf angle  $\theta_1$  is set to exceed

$$\theta_{1\min} = \tau_p / F_u \quad (91)$$

in which  $\tau_p$  is the peak adhesive shear stress and  $F_u$  is the appropriate adherend allowable stress in tension, compression, or shear as dictated by the nature of the applied load. The next step in the preliminary design process is to estimate the total scarf length necessary to effect the transfer of the entire load  $P$ . A reasonable approximation to this is given by the approximation

$$P = \left( \frac{\tau_{av}}{\tau_p} \right) \tau_p \ell \approx \left[ \frac{E_1 t_1}{E_2 t_2} \right] \tau_p \ell \quad (92)$$

for the asymptotic scarf joint solution for very long overlaps, whence

$$\ell \approx \frac{P}{\tau_p} \left[ \frac{E_2 t_2}{E_1 t_1} \right] \quad (93)$$

The optimum location of the transition from scarf angle  $\theta_1$  to  $\theta_2$  can then be determined by trial and error using the stepped-lap joint computer program developed in Section 6. As a preliminary guide, it is suggested that one third of the total thickness be tried. The conversion of this conceptual scarf joint design into a practical stepped-lap joint is illustrated schematically in the lower part of Figure 9. It should be noted that the steps are thinner in the more critical load transfer region, and at the extreme opposite end for a single step to minimize potential peel stress problems. Normally peel stresses will not be a problem with stepped-lap joints for practical design configurations but the double-lap joint analysis can serve as a check if appropriate. The larger step sizes in the lightly loaded area effect an economy of fabrication which offsets the greater expense of proper detailing in the more critical areas.

For reasons evident from the discussion above, the dual-slope scarf joint has merits in its own right as well as for a model for approximate stepped-lap joint analysis. The steepening of the scarf angle at one end is particularly important for the brittle adhesives for which  $\tau_p$  is much higher than for the ductile adhesives. This greater importance follows from equation (91).

One characteristic of the internal stress distribution within stepped-lap bonded joints is directly traceable to double-lap joint phenomena and has no counterpart in scarf joint behavior. This characteristic is that, once each or any step is sufficiently long to contain a fully-developed elastic trough in the adhesive shear-stress distribution, an increase in that step length does not alter the joint shear strength. Indeed, as confirmed by application of the computer programs A4EF and A4EG, the internal adherend and adhesive stresses at the ends of each and every step are invariant with respect to such step length increases, whether one, some, or all of the step lengths are increased. That this should be so follows directly from the governing equations for each step of the joint. These are precisely the same as for an unbalanced double-lap joint, the shear strength of which is independent of overlap beyond some value. The impact of this phenomenon on the design of stepped-lap bonded joints is that, if analysis indicates inadequate bond strength and the overlap is already reasonably great, no further increase in step lengths can accomplish an improvement in joint strength. It is necessary to increase the number of steps and decrease the incremental step thickness.

The technique of refining the preliminary analysis developed by the rules above is as follows. An analysis is performed, and the limiting (critical) detail identified. If this is the strength of the end step of the stiffer adherend, the appropriate procedure is to decrease this length and increase the length of the other steps. A halving of the step thickness increment and doubling of the number of steps at the more critical end of the joint will also help. This situation can be identified by a solution in which the maximum adhesive shear strain developed is less than the allowable. In rare instances it may not be the very end but one or two steps inside which are critical. The procedure for improving the joint strength is the same. Reduce the length of the critical steps and increase the others. In doing so, it should be remembered that any fully-elastic step will not transfer much more load even if its length is

increased. Furthermore, if the adhesive shear stresses at each end of the step are less than their plastic value, increasing the step length indefinitely will not introduce a plastic zone. If the adhesive shear strain is predicted to be the limiting feature rather than the adherend strength, the joint strength may be improved by increasing the number of joint steps. In doing so, steps at one end of the joint will tend to become critical and length increases in the remaining (elastic) steps will continue to increase the joint strength, but at a decreasing rate. The behavior of bonded scarf joints (Figure 6) serves to explain this approach. Since the average bond stress on a scarf joint approaches a fixed fraction of the maximum bond shear stress, an overlap sufficiently long can always be found to develop a potential strength 50 percent in excess of the adherend strength. The only inherent difficulty in this approach is the care needed not to exceed the adherend allowables near the more critical end of the joint. One may look upon an optimally designed stepped-lap joint as an approximation to a dual slope scarf joint with a small angle at the less critical end to build up the total load transferred and a steeper angle (still small) at the more critical end to prevent breaking off the tip of the adherend.

In the presence of adherend thermal mismatch (advanced composite-to-metal for example), a reversal of load direction can reverse the more critical and less critical ends of the joint. Therefore it is necessary in such cases to design for both the maximum tensile shear and compressive shear loads to be applied.

Figures 10 to 12 illustrate solutions obtained to stepped-lap bonded joint analyses using the computer programs above. The joint is drawn to scale in Figures 10 and 11 and the material properties can be found in the sample printout included in the Appendix. The brittle and ductile adhesives referred to are, respectively, Narmco Metlbond 329 and Hysol EA951 which have the shear characteristics illustrated in Figure 13. The elastic solutions in Figure 10 show dramatically the sharp spikes in the shear stress distribution at the ends of each step. These spikes, separated by relatively lightly-loaded troughs, represent the influence of the uniform thickness steps. It is evident also from Figure 10 that the ductile adhesive, with its lower modulus and higher elastic shear strain carries slightly more load elastically than does the brittle adhesive. Figures 10 to 12 omit the influence of thermal mismatch between adherends and, had this been included, the elastic strength disparity in

Figure 10 would have been very pronounced in favor of the ductile adhesive for a tensile shear loading. Figure 11 shows the computed bond shear stress distributions, corresponding with Figure 10, when the adhesive properties are modified to account for plasticity. As is to be expected from the adhesive characteristics in Figure 13, this modification does not increase the joint strength of the brittle adhesive sufficiently for the bond to be stronger than the weaker adherend. The ductile adhesive, on the other hand, is computed to have a potential bond strength nearly as great as the strength of the titanium outside the joint. Actually, by the time the adhesive has used up only 15 percent of its total shear strain capacity, the load level is so high as to cause the end (thin) titanium step to yield, as shown in the middle illustration of Figure 11. The ductile adhesive has a considerable strength margin over the composite adherend. Figure 12 demonstrates how the theory identifies the end metal step as being prone to fatigue failure, even though the end step had been shortened to alleviate the problem. In the static load case the theory predicts that, once the titanium has yielded locally, as shown in the second illustration of Figure 12, the load level will increase until failure occurs in the composite at the end of the joint, as shown in the fourth illustration. Figures 11 and 12 depict only the most critical conditions within each step because the computer program does not normally output a continuous solution. The adhesive shear stress distribution throughout the lightly loaded regions is not crucial to the design/analysis cycle. For illustrative purposes one can easily artificially divide each step into a number of short segments in order to avoid adding another computation sequence to the programs. This has been demonstrated to be free from convergence problems (as confirmed by Figure 10) but, naturally, takes more computer time.

The following table enumerates a number of solutions obtained with the stepped-lap joint computer programs above. The effects of thermal stresses are included, as also is the influence of the direction of the applied load. Of interest is the way in which the adherend thermal and stiffness imbalances compound to decrease the joint strengths for tensile loading while they counteract each other for the compressive loading. The failure modes predicted are identified by the comment codes 1 through 5 which are explained at the foot of the table. All cases except those for optimized step lengths have the joint geometry shown in Figure 10. In optimizing the joint proportions, the computer program

STRENGTHS OF STEPPED-LAP ADHESIVE-BONDED JOINTS

JOINTS OF TITANIUM TO ISOTROPIC HTS GRAPHITE-EPOXY  
 TITANIUM 0.25 IN. THICK GRAPHITE-EPOXY 0.264 IN. THICK

FAILURE LOADS (LBS/INCH)

ADHESIVE	$\Delta T$	0° TENSION & COMPRESSION	-280°F TENSION	-280°F COMPRESSION	OPTIMIZED STEP LENGTHS		-400°F TENSION	-400°F COMPRESSION
					-280°F TENSION	-280°F COMPRESSION		
					EPON 951 PURELY-ELASTIC ELASTIC-PLASTIC POTENTIAL BOND STRENGTH COMMENTS	7829 14430 28362 1, 2		
METLBOND 329 PURELY-ELASTIC ELASTIC-PLASTIC COMMENTS	6764 13505 3	3812 10555 3	8552 16457 3			2547 9290 3	6152 17720 3	

- COMMENT LEGEND :
1. TITANIUM YIELDS ON END (THIN) STEP
  2. FAILURE IN COMPOSITE OUTSIDE JOINT AT 18216 LB/IN.
  3. FAILURE IN ADHESIVE AT COMPOSITE END OF JOINT
  4. FAILURE IN COMPOSITE ONE STEP IN FROM TITANIUM END OF JOINT
  5. FAILURE IN COMPOSITE ONE STEP IN FROM COMPOSITE END
- $\Delta T$  = OPERATING TEMPERATURE - CURE TEMPERATURE OF ADHESIVE

was used to identify the most critical location and the step lengths were modified by hand for re-analysis until the minimum tensile and compressive joint strengths matched the composite adherend strength. This took only two iterations to achieve the results shown and this feature is one of the more beneficial merits of the complete internal joint analysis.

Figure 14 illustrates the bond shear stress distributions for both ductile and brittle adhesives. A comparison is effected between a joint of uniform step lengths, at left, and that with optimized lengths, at right. A small loss in elastic joint strength is incurred by shortening the end steps (and some of this could be recovered by increasing the lengths of the other steps to compensate) but the problem of yielding the end titanium step has been eliminated for the ductile adhesive. It is interesting to observe that the brittle adhesive had insufficient strain energy in shear for the problem to arise. Another important phenomenon revealed is that the ductile adhesive uses up only about a third of its ultimate shear strain capacity in breaking the composite adherend just outside the joint. This leaves a generous margin for dealing with

stress concentration due to irregularities in load intensity or bond thickness across the width of the joint. Because of these ever-present considerations, the brittle adhesive should not be expected to develop the full predicted joint strength over each inch of a wide panel. Failure would be initiated by a local effect and then be propagated rapidly.

Figure 14 omits consideration of thermal effects in order not to complicate the comparisons made. Figure 15 includes these effects for both tensile and compressive shear loading with the ductile adhesive. This figure compares the performance of the preliminary design (Figures 10 and 11) with the optimized design. Improvements in ultimate compressive strength and tensile fatigue load capacity are demonstrated.



## 8. CONCLUSION

This report presents elastic and elastic-plastic analysis methods for adhesive-bonded scarf and stepped-lap joints. The solutions obtained are analytic in form and the necessary digital computer FORTRAN IV programs are listed in the Appendices. These solutions are believed to be the first for such joints which account for adhesive plasticity. They include also the effects of adherend stiffness- and thermal-mismatch. While the precise solutions require iterative numerical solutions, explicit algebraic formulas are derived for a close lower-bound on the strength of scarf joints. The dominant characteristic of scarf joints is that, for long overlaps, the average bond stress asymptotes towards a fixed fraction of the peak bond stress, that fraction equalling the lesser ratio of adherend extensional stiffnesses. Unlike uniform lap joints, which reach a definite strength limit which cannot be exceeded by using longer overlaps, the potential bond strength of scarf joints increases indefinitely with overlap so that a design can always be devised in which the failure is forced to occur outside the joint. In using this approach, however, it is necessary also to check on the adherend stresses at the tip of the stiffer adherend to ensure that the scarf angle is not too small. Stepped-lap joints exhibit some characteristics of both the scarf joint and uniform double-lap joints. Those steps near the middle of a stepped-lap joint carry significantly more load than that transferred in the corresponding area of a uniform lap joint but the load so transferred is usually not a major contribution. Most of the load transfer is effected through the end three steps at one or both ends of the joint, depending on the nature of the adherend imbalances and the direction of the load. Within each step, since the governing equations are precisely the same as for an unbalanced double-lap joint, it is found that no further load can be transferred once the overlap has exceeded a determinable value. In other words, unlike scarf joints, the potential shear strength of stepped-lap joints cannot be increased indefinitely by increasing the overlap(s). The appropriate procedure is to employ more steps of finer thickness increments in order to augment the load capacity. Because scarf and stepped-lap joints can efficiently transfer load between thicker adherends than is possible with uniform double-lap joints, the latter are restricted to thinner sections in practical applications.

The inclusion of adhesive plasticity in the analysis has a marked effect on the

strength predictions. On the other hand, the elastic adhesive stresses play a far more important role in the behavior of scarf and stepped-lap joints than they do for uniform lap joints. The inclusion in the analyses of thermal mismatch effects permits their application to the bonding of titanium to the advanced composite laminates and explains how the joint strength changes with the load direction in such a situation.

The elastic-plastic analysis of the internal stresses within stepped-lap bonded joints provides sufficient information for the joint proportions to be optimized. Analyses should be performed for each load direction and at the extremes of the environmental temperature range, taking due account of material property changes with temperature, in the optimization sequence.

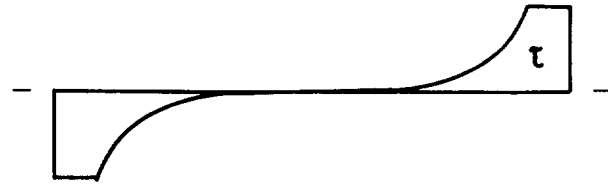
## REFERENCES

1. Hart-Smith, L. J., "Adhesive-Bonded Double-Lap Joints," Douglas Aircraft Company, NASA Langley Contract NAS1-11234, Report No. NASA CR 112235, January 1973.
2. Hart-Smith, L. J., "Adhesive-Bonded Single-Lap Joints," Douglas Aircraft Company, NASA Langley Contract NAS1-11234, Report No. NASA CR 112236, January 1973.
3. Lubkin, J. L., "A Theory of Elastic Scarf Joints," J. Appl. Mech. 24, 255-260, June 1957.
4. Sumida, P. T., Hart-Smith, L. J., Pride, R. A., and Iilg, W., "Filamentary Composite Reinforcement of Metal Structures," Douglas Aircraft Company, NASA Contract NAS1-9953, SPI 28th Annual Western Conference Proceedings, 74-90, May 1971.
5. Erdogan, F., and Ratwani, M., "Stress Distribution in Bonded Joints," J. Composite Materials 5, 378-393, July 1971.
6. Lehman, G. M. et al, "Investigation of Joints and Cutouts in Advanced Fibrous Composites for Aircraft Structures," Douglas Aircraft Company, AFFDL Contract F33615-67-C-1582, Third Quarterly Progress Report DAD-61566, January 1968.
7. Lehman, G. M. et al, "Investigation of Joints and Cutouts in Advanced Fibrous Composites for Aircraft Structures," Douglas Aircraft Company, AFFDL Contract F33615-67-C-1582, Fourth Quarterly Progress Report DAC-61593, April 1968.
8. Lehman, G. M. and Hawley, A. V., "Investigation of Joints in Advanced Fibrous Composites for Aircraft Structures," Douglas Aircraft Company, AFFDL Contract F33615-67-C-1582, Technical Report AFFDL-TR-69-43, Volume 1, June 1963.

9. Dickson, J. N., Hsu, T. M and McKinney, J. M., "Development of an Understanding of the Fatigue Phenomena of Bonded and Bolted Joints in Advanced Filamentary Composite Materials," Lockheed-Georgia Company, AFFDL Contract F33615-70-C-1302, Technical Report AFFDL-TR-72-64, Volume 1, June 1972. (See also Fourth Quarterly Interim Technical Report, May 1971).
10. Grimes G. C., Calcote, L. R., Wah, T., et al, "The Development of Non-linear Analysis Methods for Bonded Joints in Advanced Filamentary Composite Structures," Southwest Research Institute, AFFDL Contract F33615-69C-1641, Technical Report AFFDL-TR-72-97, September 1972. (See also Research and Development Interim Technical Reports No's III, February 1970, IV, May 1970, and V, March 1971).
11. Corvelli, N. and Saleme, E., "Analysis of Bonded Joints," Grumman Aerospace Corporation, Report No. ADR 02-01-70.1, July 1970.
12. Hart-Smith, L. J., "The Strength of Adhesive-Bonded Double-Lap Joints," Douglas Aircraft Company, IRAD Technical Report No. MDC-J0367, November 1969.
13. Hart-Smith, L. J., "The Strength of Adhesive-Bonded Single-Lap Joints," Douglas Aircraft Company, IRAD Technical Report No. MDC-J0742, April 1970.
14. Teodosiadis, R., "Plastic Analysis of Bonded Composite Lap Joints," Douglas Aircraft Company, IRAD Report No. DAC-67836, May 1969.



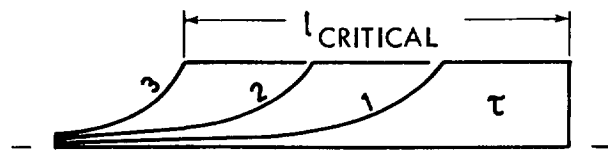
SCARF JOINT BETWEEN DISSIMILAR MATERIALS ( $\alpha_1 > \alpha_2$ ,  $E_1 t_1 = E_2 t_2$ )



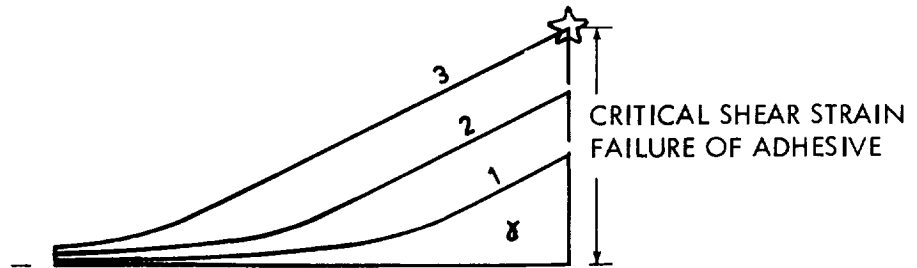
RESIDUAL ADHESIVE SHEAR STRESS



RESIDUAL ADHESIVE SHEAR STRAIN



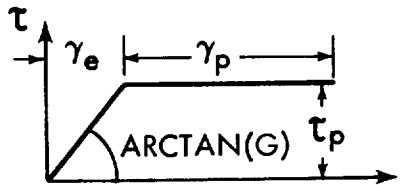
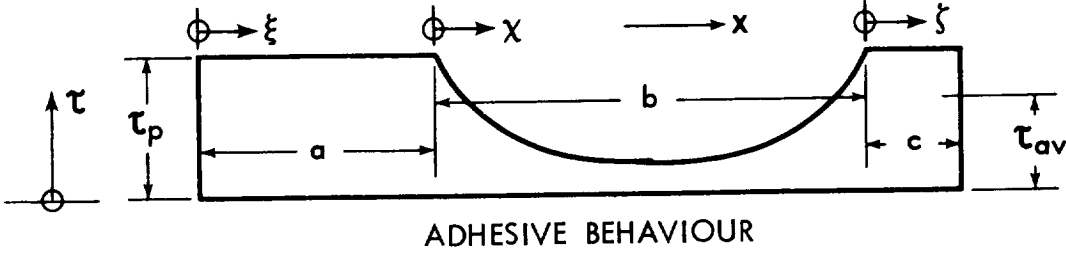
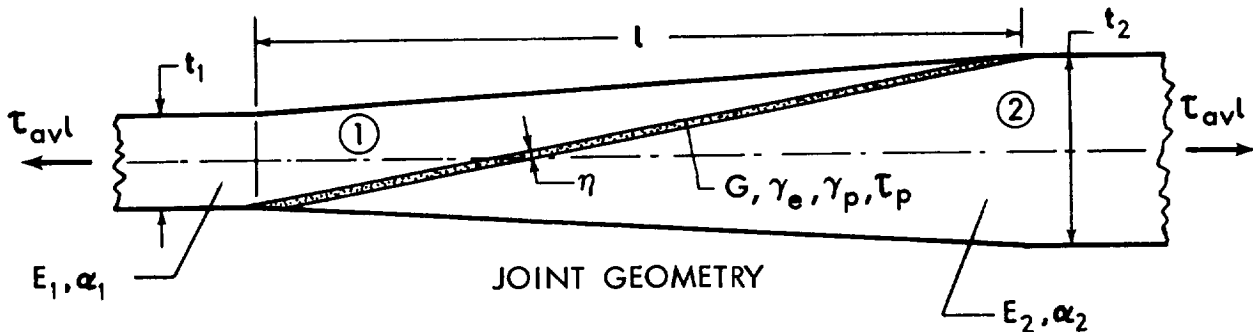
ADHESIVE SHEAR STRESSES  
UNDER PROGRESSIVELY INCREASING TENSILE LOADS



CORRESPONDING ADHESIVE SHEAR STRAINS

(NOTE THAT THE STRESSES AND STRAINS FOR PARTIAL LOADING, ABOVE, WOULD ALSO INDICATE THE BEHAVIOR UNDER STIFFNESS IMBALANCE IF  $\alpha_1 = \alpha_2$  AND  $E_1 t_1 > E_2 t_2$  BUT, WHEREAS THE CRITICAL END REVERSES WITH LOAD DIRECTION FOR THERMAL IMBALANCE, IT REMAINS THE SAME FOR STIFFNESS IMBALANCE.)

FIGURE 1. EXPLANATION OF NON-UNIFORM ADHESIVE SHEAR STRESSES IN BONDED SCARF JOINTS BETWEEN DISSIMILAR ADHERENDS



ADHESIVE SHEAR CHARACTERISTIC

$$(\lambda)^2 = \frac{G}{\eta} \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right) l^2$$

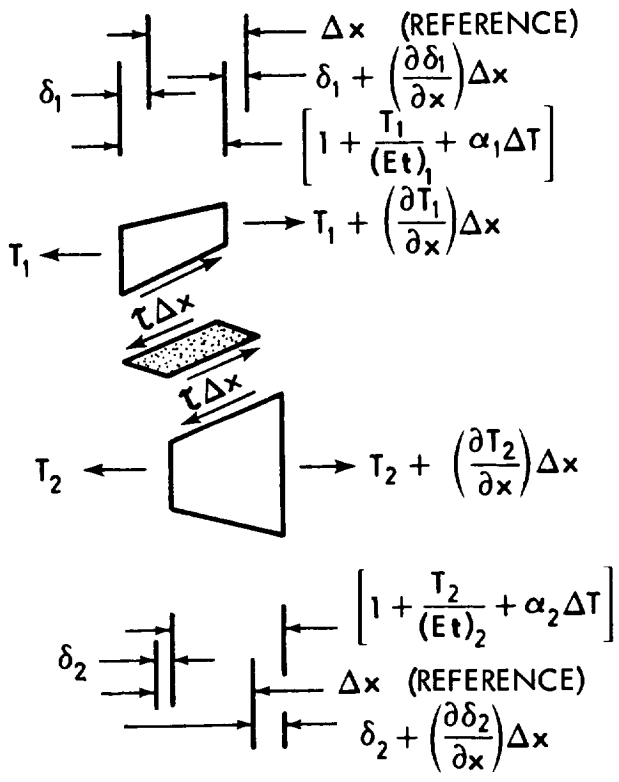
$$ETR(1) = E_1 t_1 / E_2 t_2$$

$$ETR(2) = E_2 t_2 / E_1 t_1$$

$$CTHERM(1) = \frac{(\alpha_2 - \alpha_1) \Delta T \lambda}{\tau_p \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right)}$$

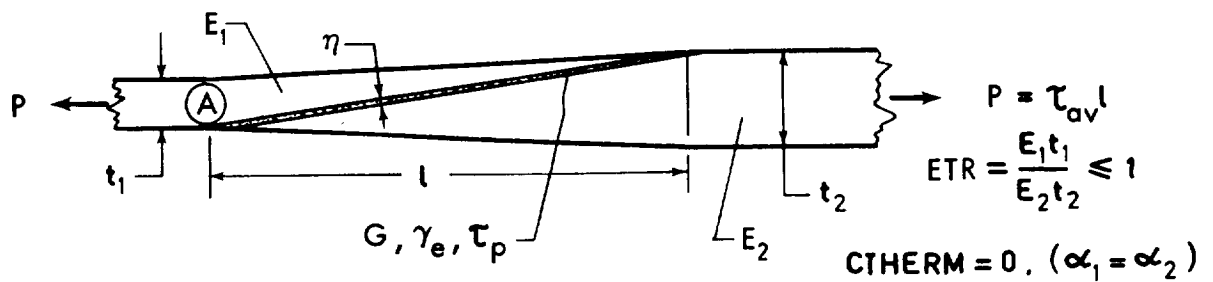
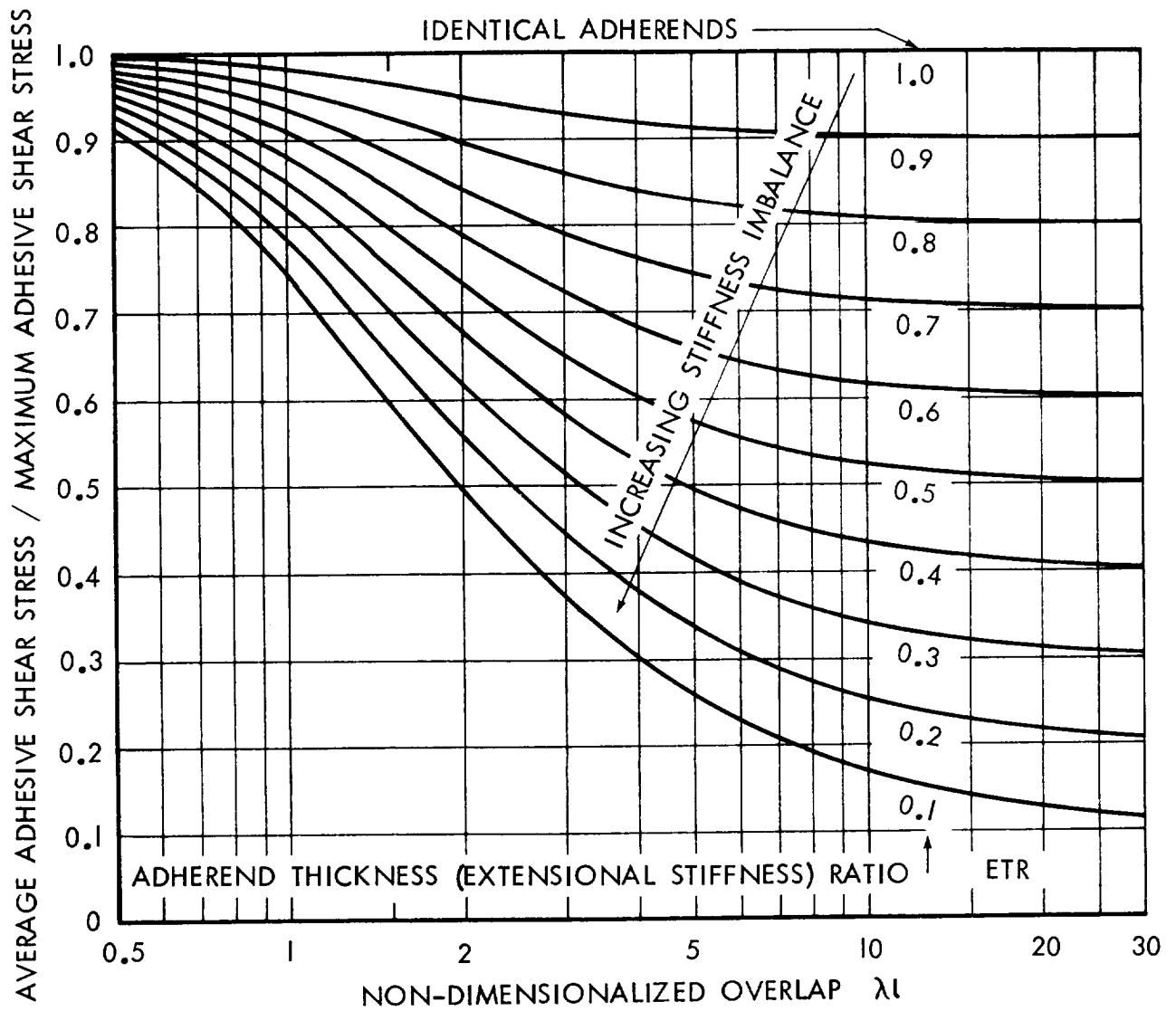
$$CTHERM(2) = \frac{(\alpha_1 - \alpha_2) \Delta T \lambda}{\tau_p \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right)}$$

NON-DIMENSIONALIZED JOINT PARAMETERS



DISPLACEMENTS AND ELEMENT LOADS

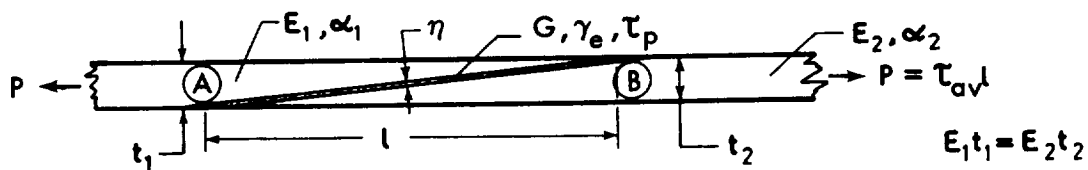
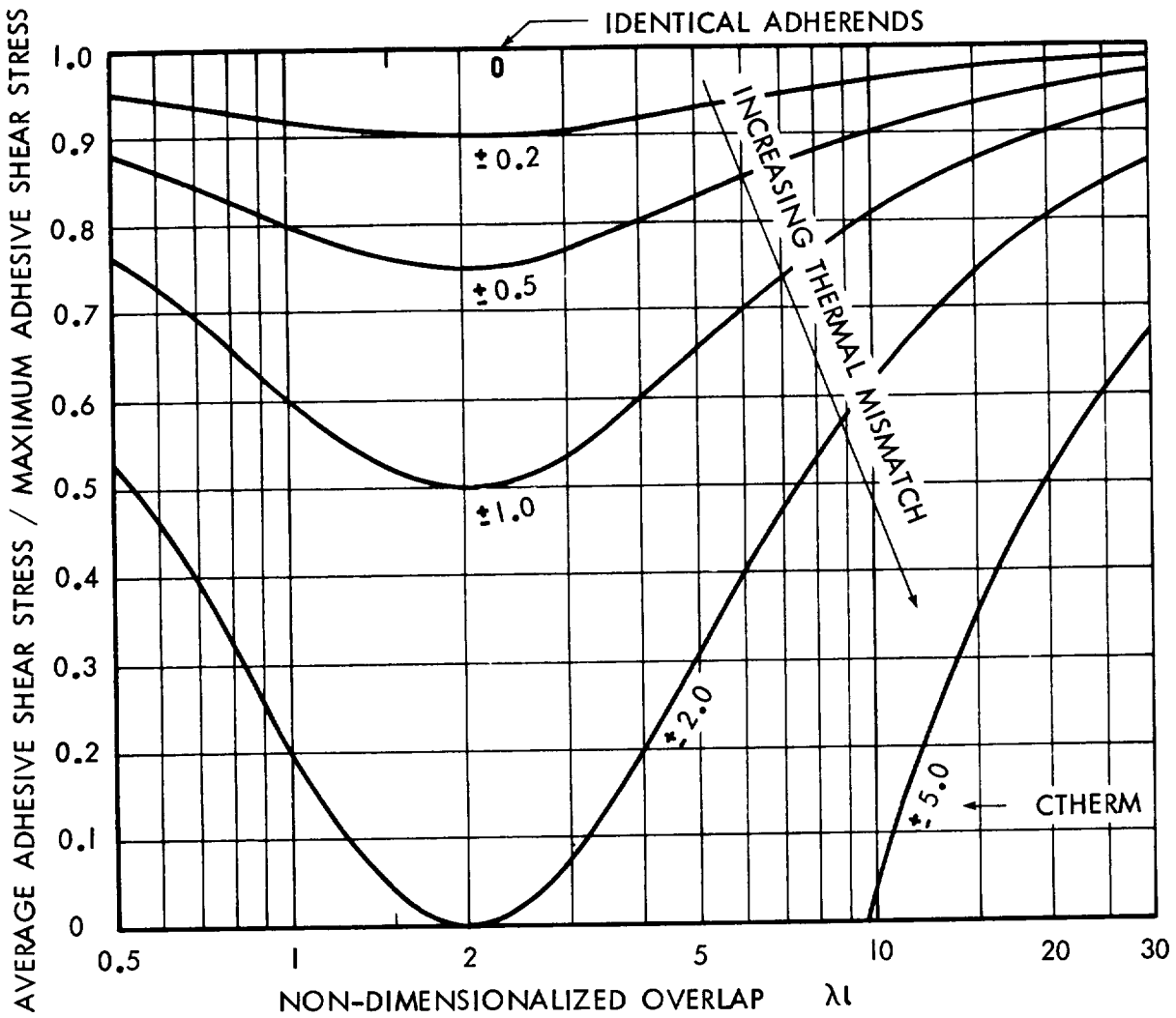
FIGURE 2. NOTATION AND GEOMETRY FOR ADHESIVE-BONDED SCARF JOINT ANALYSIS



$$\lambda^2 = \frac{G}{\eta} \left[ \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right]$$

LOCATION A IS CRITICAL FOR BOTH POSITIVE (TENSILE LAP-SHEAR) AND NEGATIVE (COMPRESSIVE LAP-SHEAR) VALUES OF LOAD P

FIGURE 3. EFFECT OF ADHEREND STIFFNESS IMBALANCE ON ELASTIC STRENGTH OF BONDED SCARF JOINTS



$$\lambda^2 = \frac{G}{\eta} \left[ \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right] \quad C_{THERM} = \frac{(\alpha_2 - \alpha_1) \Delta T \lambda}{\tau_p \left( \frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right)}, \quad \Delta T = T_{operating} - T_{stress\ free}$$

- LOCATION A CRITICAL FOR  $C_{THERM} < 0$  AND  $P > 0$
- LOCATION A CRITICAL FOR  $C_{THERM} > 0$  AND  $P < 0$
- LOCATION B CRITICAL FOR  $C_{THERM} < 0$  AND  $P < 0$
- LOCATION B CRITICAL FOR  $C_{THERM} > 0$  AND  $P > 0$

FIGURE 4. EFFECT OF ADHEREND THERMAL MISMATCH ON ELASTIC STRENGTH OF BONDED SCARF JOINTS



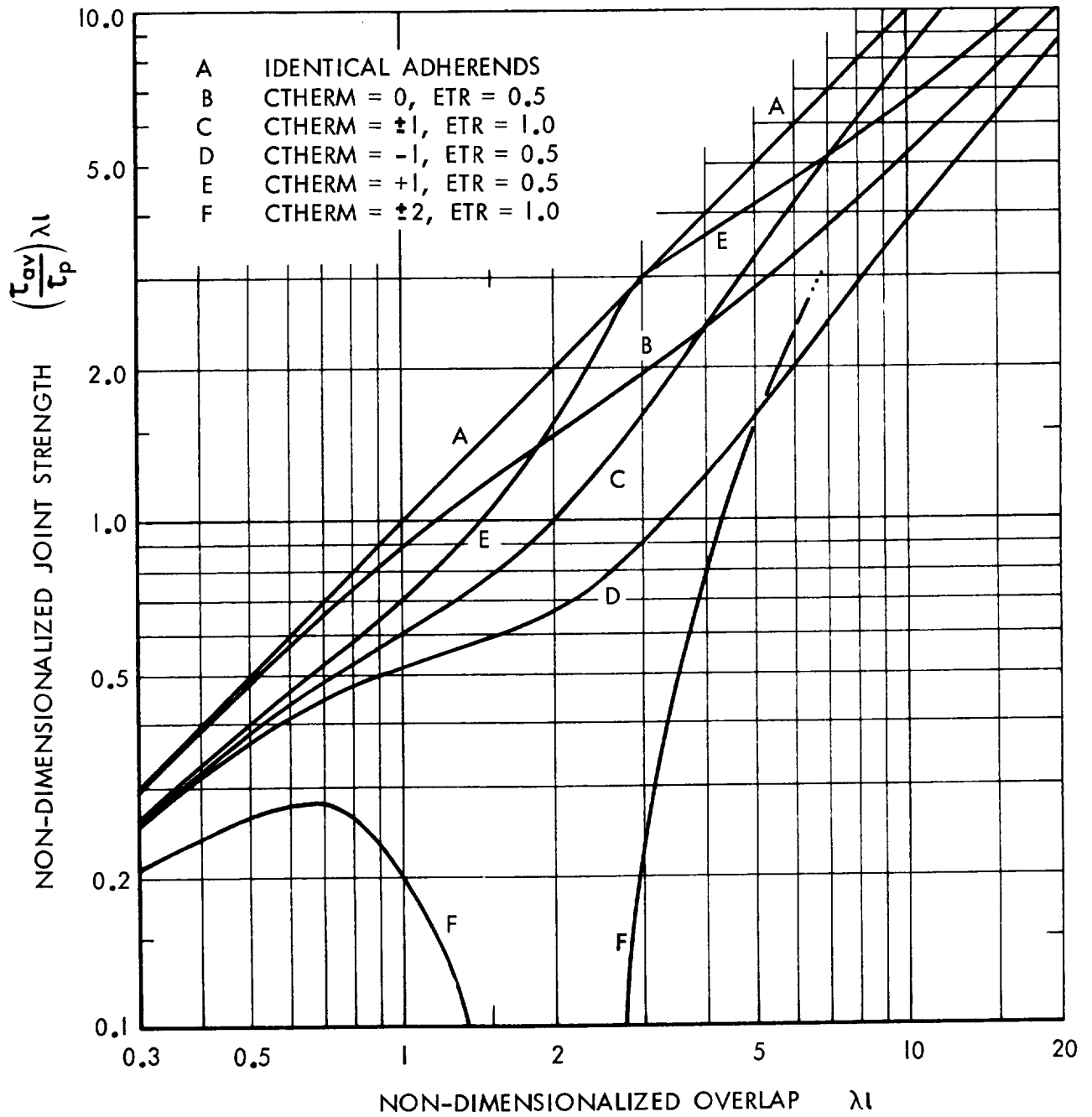
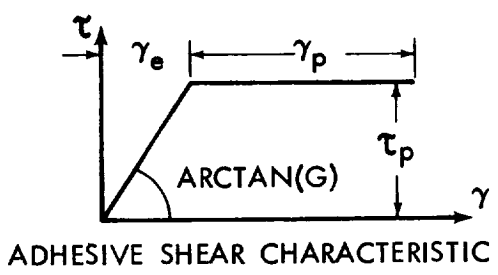
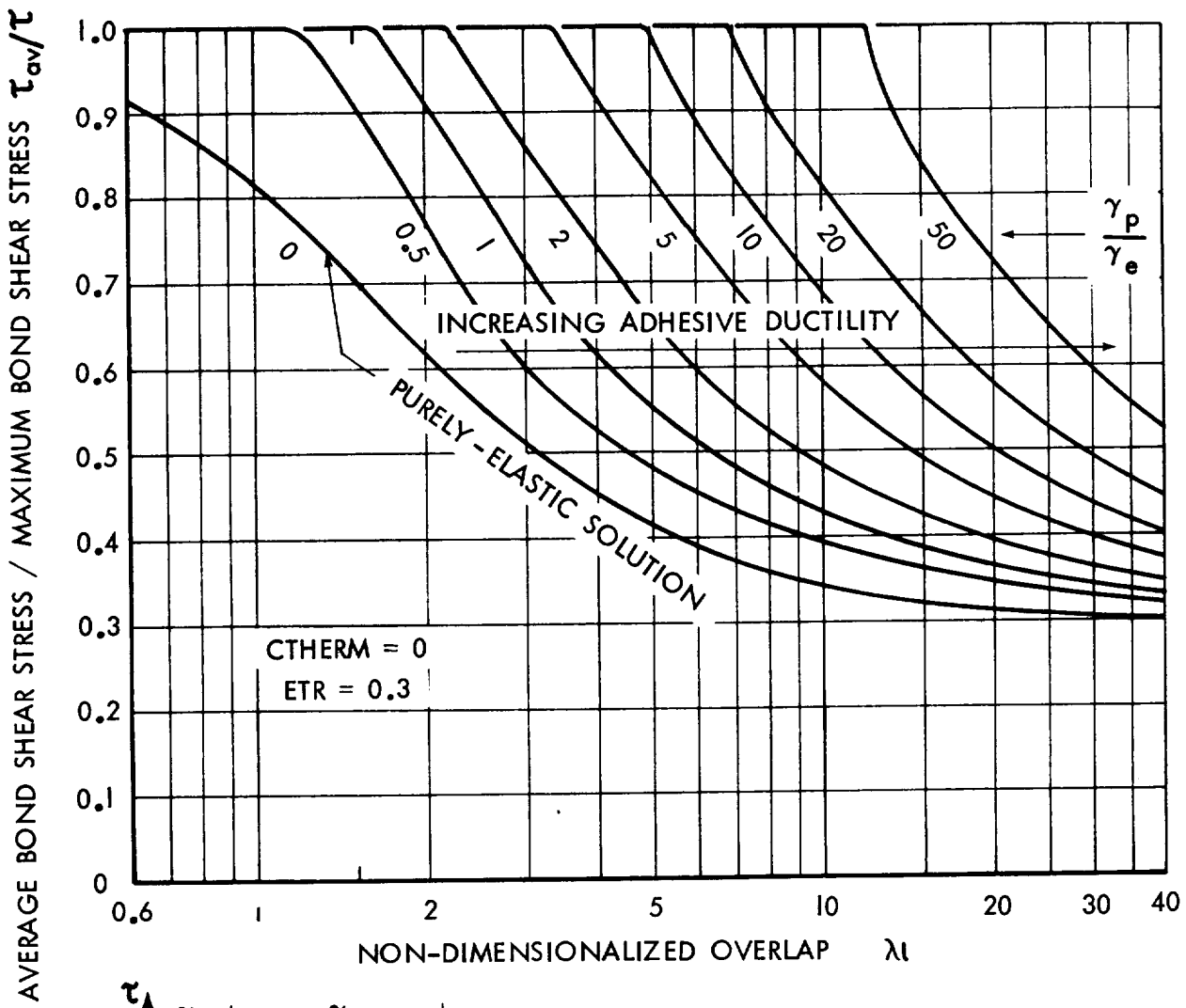


FIGURE 5. INTERACTION OF ADHEREND STIFFNESS AND THERMAL IMBALANCES FOR ELASTIC BONDED SCARF JOINTS



(See Figures 2, 3 and 4 for notation)

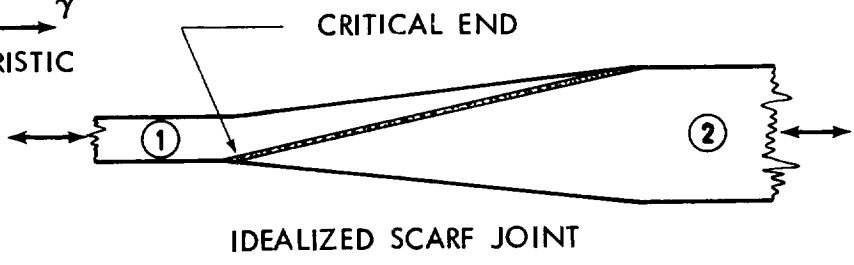
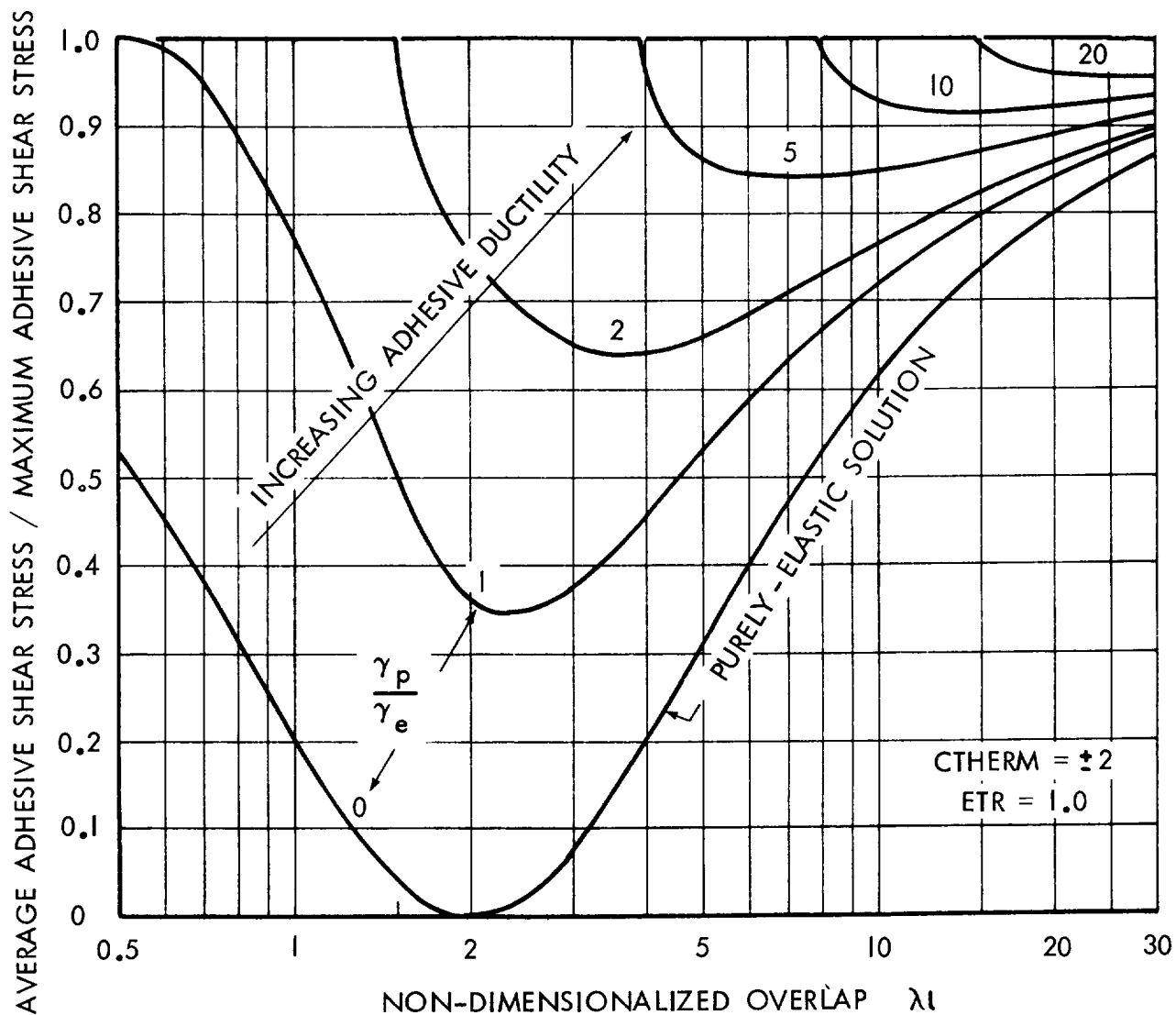
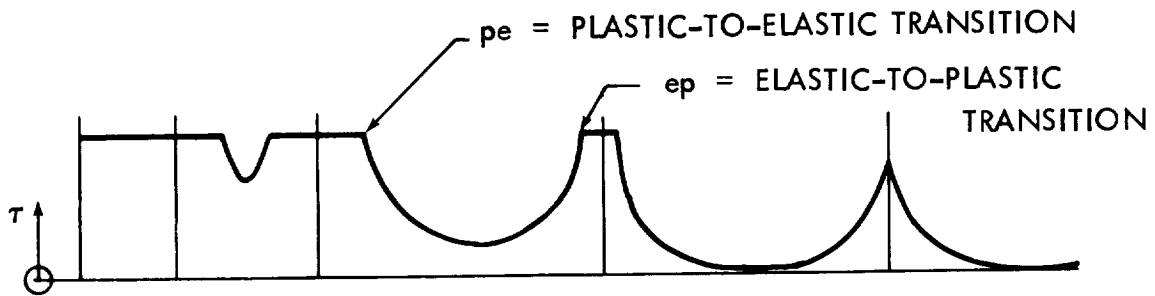
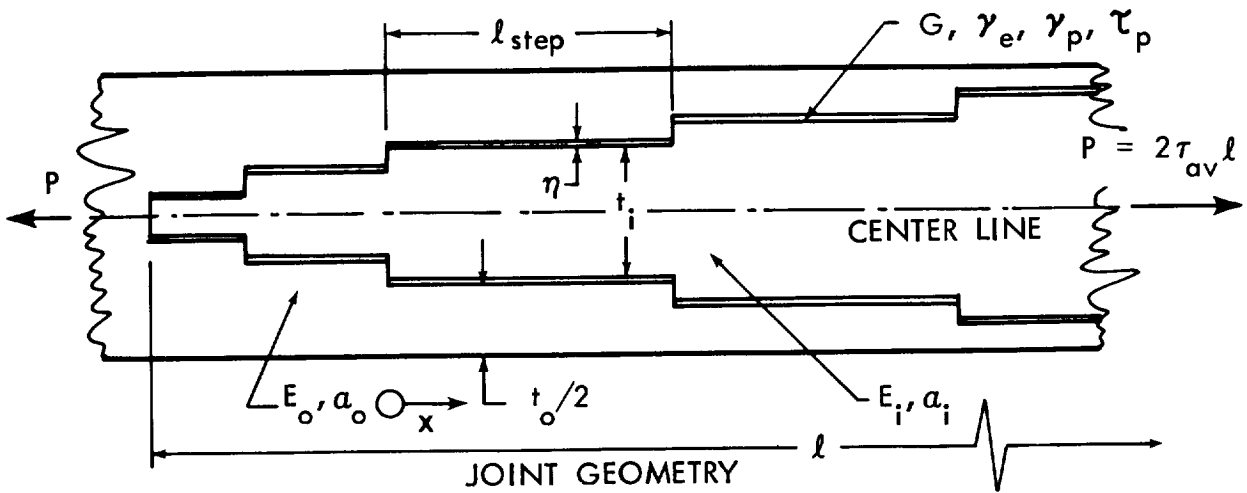


FIGURE 6. EFFECT OF ADHESIVE PLASTICITY IN REDUCING STRENGTH LOSS DUE TO ADHEREND STIFFNESS IMBALANCE FOR BONDED SCARF JOINTS

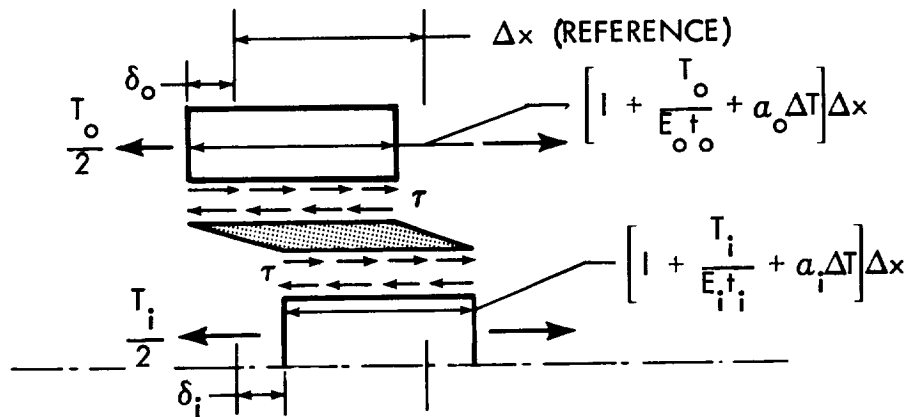


(See Figures 2, 3 and 4 for notation)

FIGURE 7. EFFECT OF ADHESIVE PLASTICITY IN REDUCING STRENGTH LOSS DUE TO ADHEREND THERMAL MISMATCH FOR BONDED SCARF JOINTS

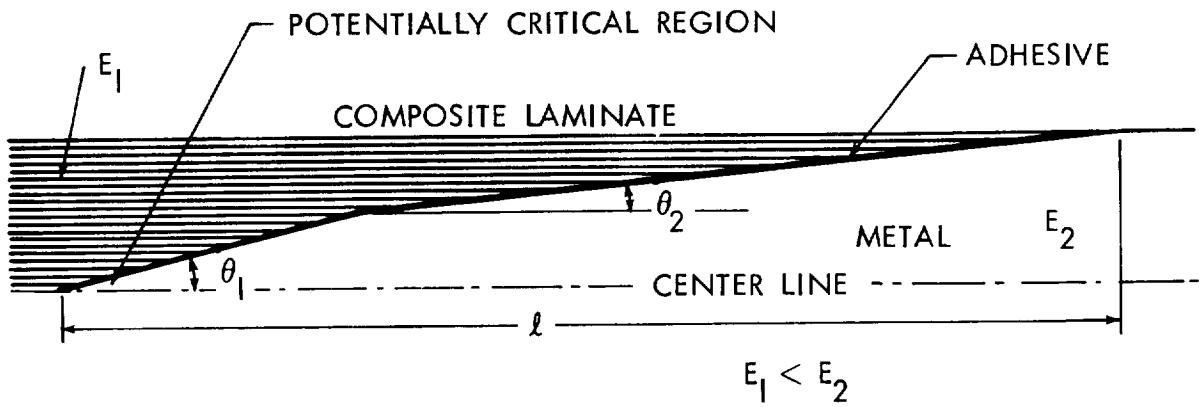


ADHESIVE SHEAR STRESS DISTRIBUTION

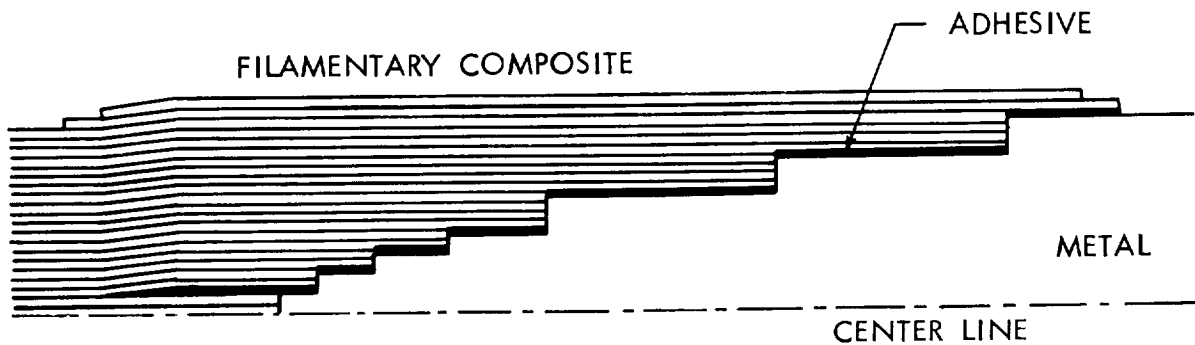


DISPLACEMENTS AND ELEMENT LOADS

FIGURE 8. NOTATION AND GEOMETRY FOR ADHESIVE-BONDED STEPPED-LAP JOINT ANALYSIS



(A) . IDEALIZED DUAL-SLOPE SCARF JOINT TO PROTECT TIP OF STIFFER ADHEREND



(B) . REPRESENTATION OF OPTIMIZED STEP PROPORTIONS FOR STEPPED-LAP JOINT WITH MODULUS OF COMPOSITE LAMINATE LESS THAN THAT OF METAL ADHEREND

FIGURE 9. PRACTICAL PROPORTIONING OF STEPPED-LAP JOINTS TO PROTECT AGAINST FATIGUE FAILURES AT TIP OF METAL ADHEREND

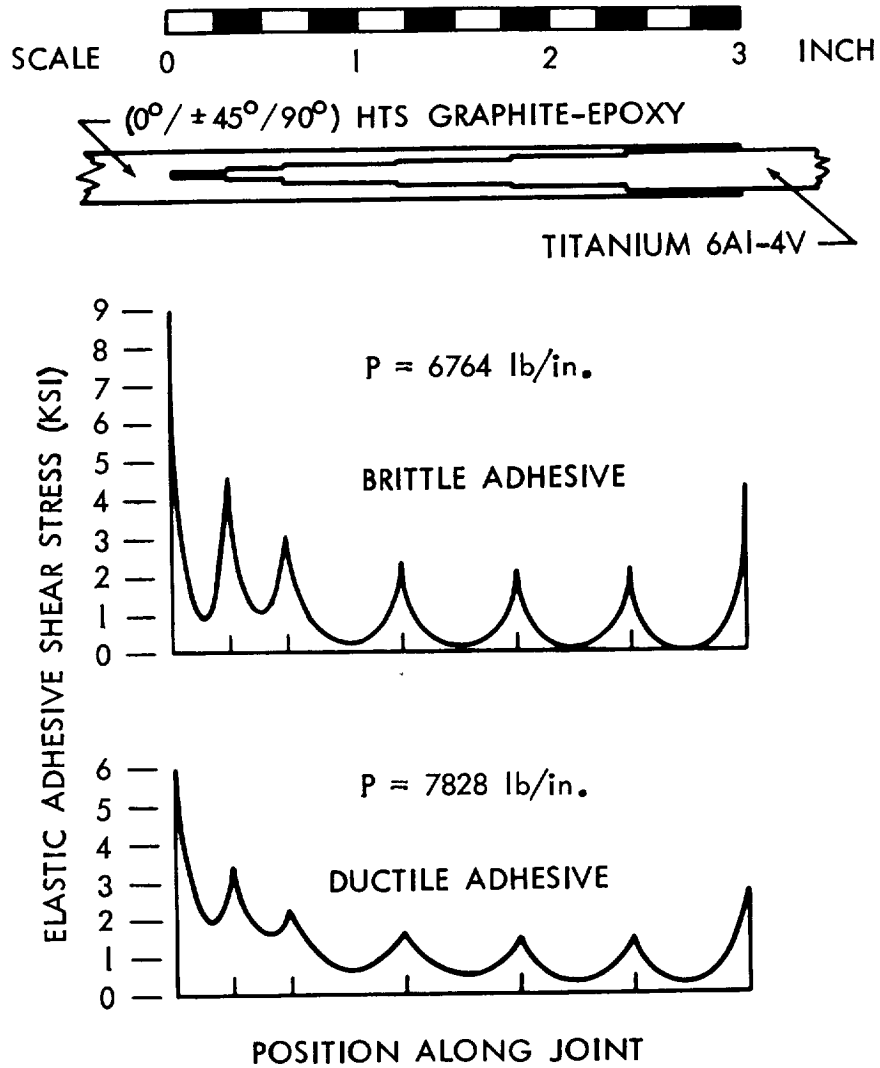


FIGURE 10. ELASTIC SHEAR STRESS DISTRIBUTIONS FOR BRITTLE AND DUCTILE ADHESIVES IN BONDED STEPPED-LAP JOINTS

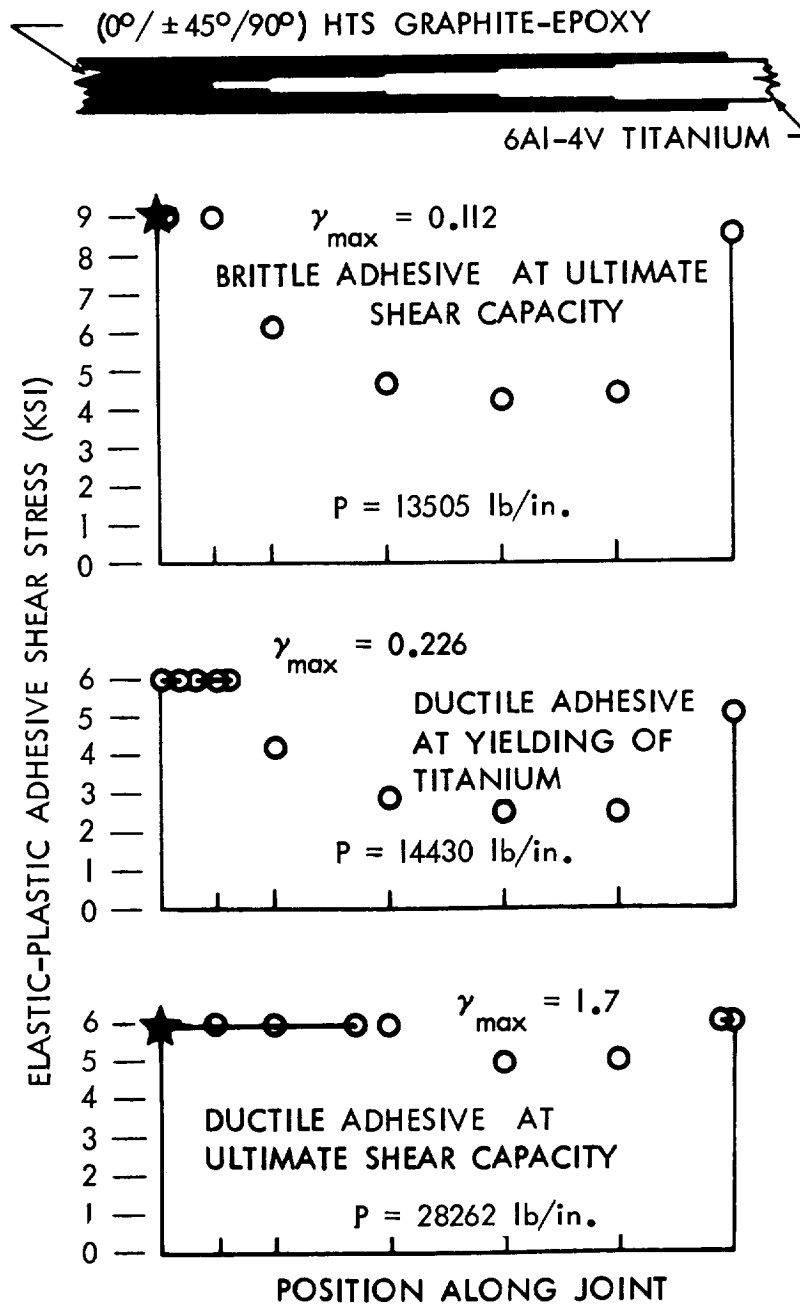


FIGURE II. ELASTIC-PLASTIC SHEAR STRESS DISTRIBUTIONS FOR BRITTLE AND DUCTILE ADHESIVES IN BONDED STEPPED-LAP JOINTS

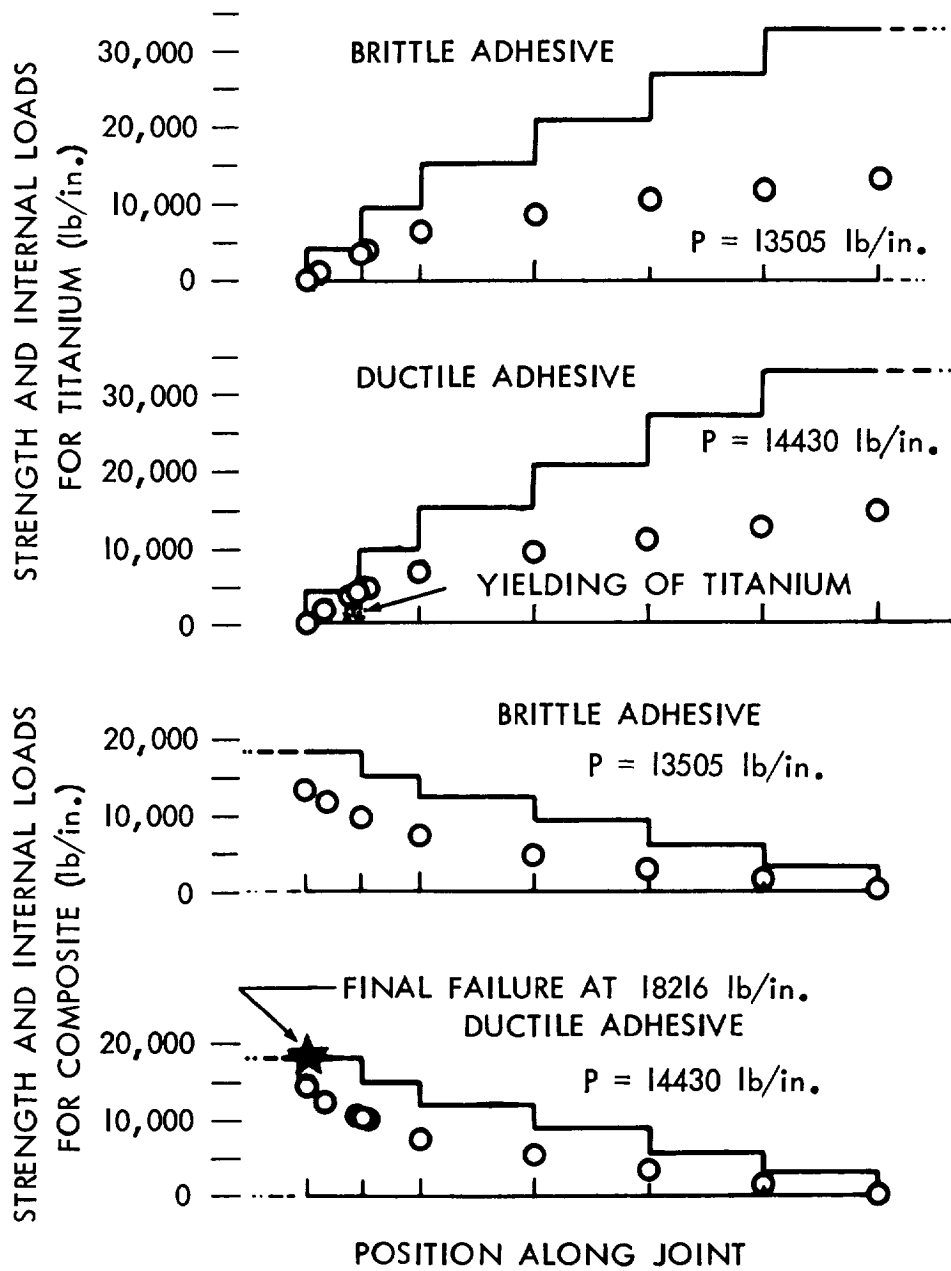


FIGURE 12. ADHEREND STRENGTHS AND INTERNAL LOADS FOR BONDED STEPPED-LAP JOINTS



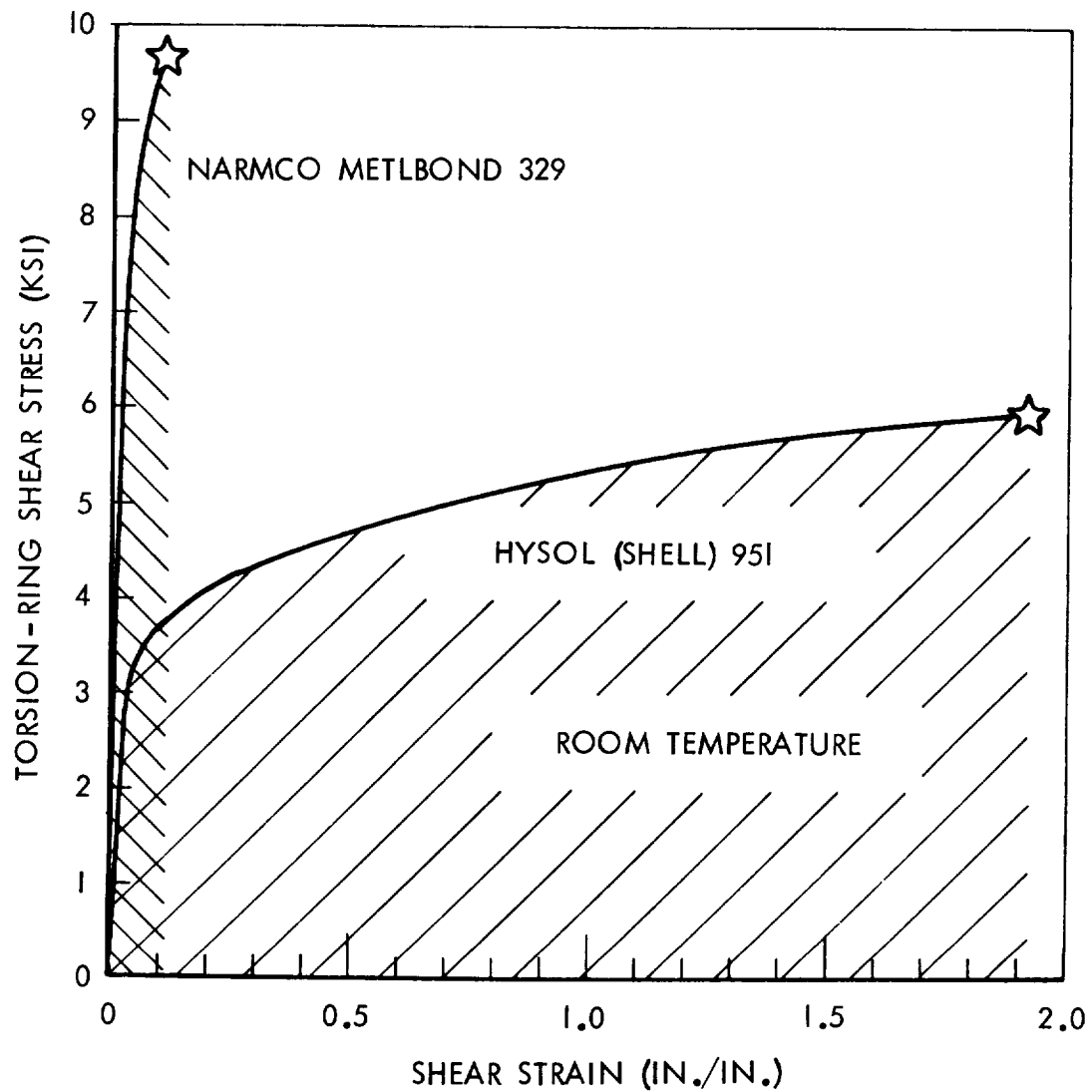


FIGURE 13. COMPARISON OF SHEAR STRESS-STRAIN CHARACTERISTICS FOR BRITTLE AND DUCTILE ADHESIVES

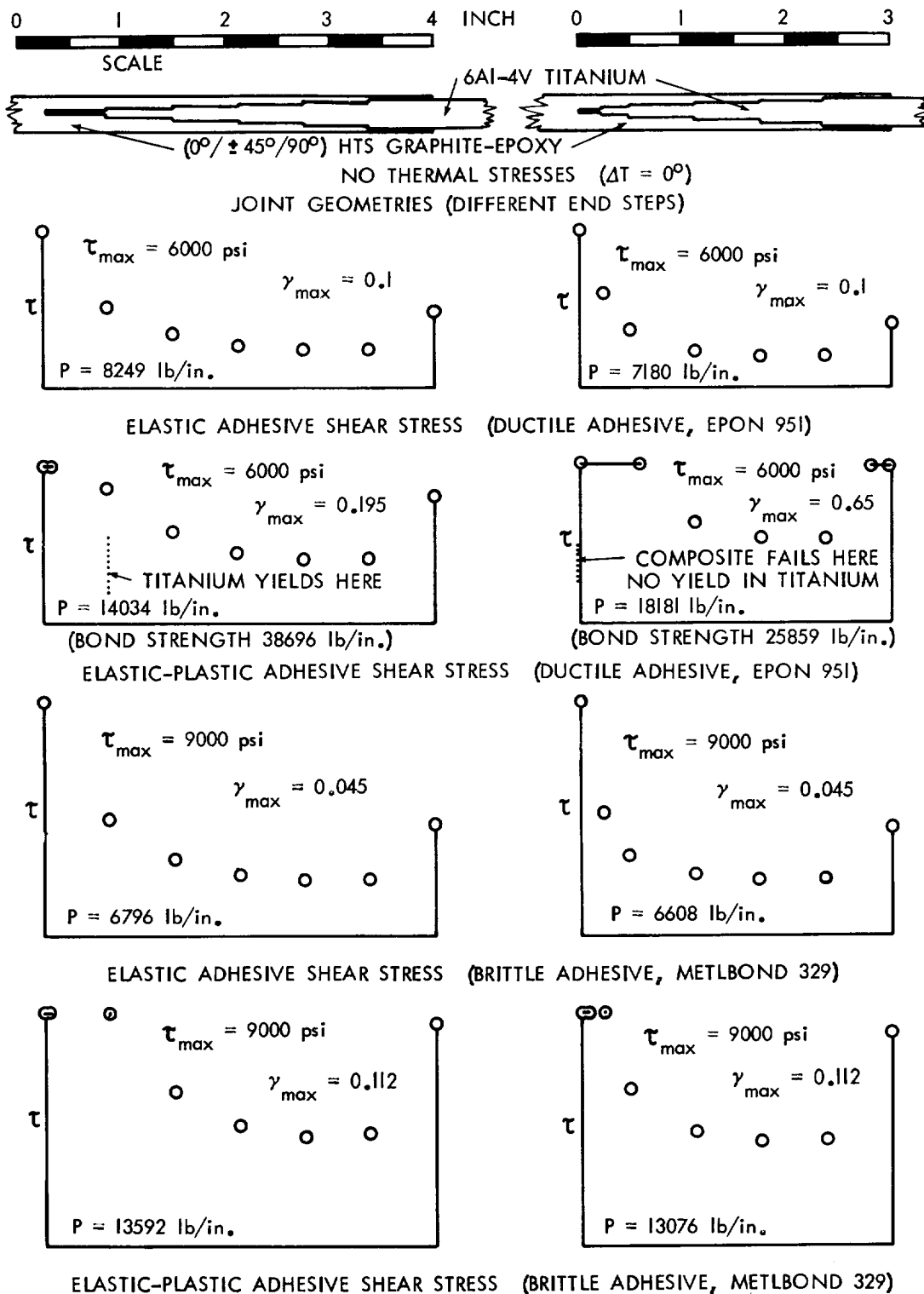
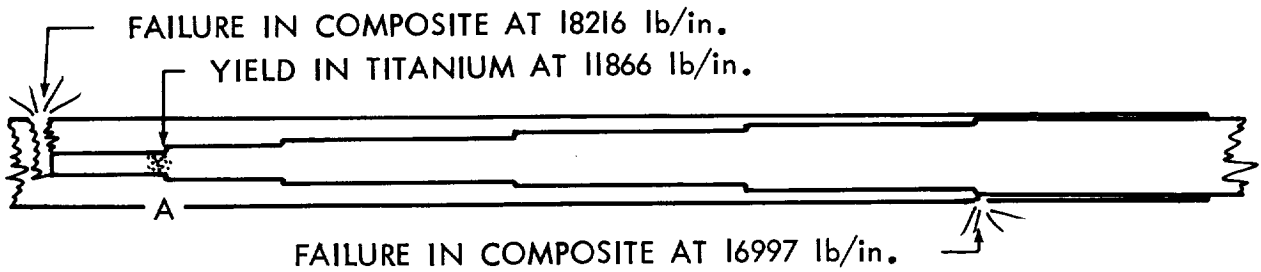


FIGURE 14. COMPARISON BETWEEN STEPPED-LAP JOINTS WITH UNIFORM STEP LENGTHS AND WITH OPTIMIZED STEP LENGTHS

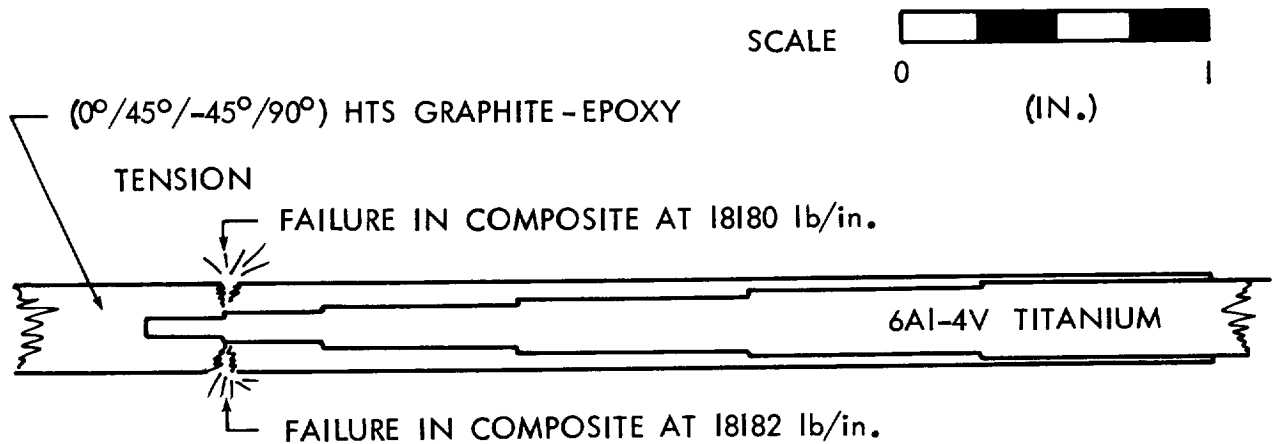
TENSION



COMPRESSION

NOTE THAT TITANIUM END STEPS WERE ALREADY SHORTENED DURING PRELIMINARY DESIGN. WITH UNIFORM STEPS 0.75 IN. LONG THROUGHOUT, PREMATURE FATIGUE FAILURE WOULD OCCUR AT A, FOLLOWED BY FAILURE OF COMPOSITE AT THE SAME (REDUCED) SECTION.

(A) PRELIMINARY DESIGN



COMPRESSION

NO YIELDING OF TITANIUM

(B) OPTIMIZED DESIGN

DUCTILE ADHESIVE CURED AT 350°F.  
STRENGTHS CALCULATED AT ROOM TEMPERATURE.  
STRENGTH OF COMPOSITE ADHEREND OUTSIDE JOINT = 18216 lb/in.  
POTENTIAL BOND SHEAR STRENGTH WOULD EXCEED 23257 lb/in. IN EVERY CASE SHOWN IF ADHERENDS WERE SUFFICIENTLY STRONG.

FIGURE 15. OPTIMIZATION OF DETAILS IN STEPPED-LAP BONDED JOINTS

## APPENDICES

### A.1 Computer Program A4EC For Elastic Strength of Bonded Scarf Joints

The FORTRAN IV digital computer program associated with the analysis in Section 2 is listed below. This program has been checked out thoroughly and sample solutions are illustrated in Section 4. Only shear stresses are considered, with the peel stresses neglected in accordance with the very small scarf angles used in practice. As discussed in Section 2, there are severe convergence problems associated with the series solution to this problem. While the average shear stresses computed are considered very reliable, no computation sequence for the stress distribution was found which was considered sufficiently accurate over the far end of the joint ( $x/l \approx 1$ ). The peak shear stress is located correctly by program A4EC at one end or other of the overlap. The only real need for a shear stress distribution is as an intermediate step in the computation of the internal adherend stresses. Since the convergence of the series was enhanced greatly by prior integration into the contributions to the average shear stress, it is recommended that any attempt to pursue the adherend stress distribution should proceed along similar lines. The adhesive shear stress distribution series can be integrated mathematically so that a more tractable series solution is obtained for the adherend stresses. The first two terms follow from the average shear stress solution and the subsequent ones would derive from recurrence formulae. The condition under which a need for such information could arise is the possible breaking off of the thin tip of the stiffer adherend for a very small scarf angle. Such a situation is unlikely for perfectly elastic adhesives because the shear stress drops off very rapidly away from the ends. A simpler procedure is available for the elastic-plastic adhesive.

The format of the input data necessary to operate the A4EC computer program is as follows:

CARD 1:

FORMAT (415)

IMAX = Number of thermal mismatch coefficients. IMAX .LE. 20.

JMAX = Number of non-dimensionalized overlaps. JMAX .LE. 40.  
(Note that this is one more than the number of overlaps to be read in. The limiting case of OL(1)=0 is set by the program.)

KMAX = Number of adherend stiffness imbalances. KMAX .LE. 10.  
(Note that this controls the number of columns of answers printed across the page and cannot be increased indefinitely.)

NMAX = Number of terms in power series. 10 .LE. NMAX .LE. 50.  
(Note NMAX = 20 is recommended.)

CARDS 2, 2A, 2B, etc.:

FORMAT (12F6.2)

OL(J)= Non-dimensionalized overlaps. Number restricted to 40 by dimension statement. (Note that OL(J) must be read in in ascending order and that OL(2), which is the first entry on card 2, must not exceed 0.5 because of internal computations. OL(1) = 0 is set by program as limiting case.) Values of OL(J) exceeding 50 are impractically large.

CARDS 3, 3A, 3B, etc.:

FORMAT (10F5.2)

ETR(K)=Adherend stiffness ratios  $(E_1 t_1)/(E_2 t_2)$ . Number restricted to 10 by dimension statement. (Subscripts 1 and 2 must be identified so that 0 .LT. ETR .LE. 1. Array should be read in in ascending or descending order.)

CARDS 4, 4A, 4B, etc.:

FORMAT (10F7.3)

CTHERM(I) = Adherend thermal mismatch coefficients in non-dimensionalized form. Number restricted to 20 by dimension statement. (Note that equal and opposite values must be read in consecutively to account for the difference between tensile and compressive application of the shear load. Values of up to  $\pm 5$  are sufficient for the available range of adhesives. Greater values are usually associated with failure of the joint under residual thermal stresses alone.)

The complete listing follows, along with sample output pages. The output tables come in pairs with the ratio of the average to maximum adhesive shear stress ( $\tau_{av}/\tau_p$ ) and the non-dimensionalized joint strength ( $\tau_{av}/\tau_p$ )( $\lambda\ell$ ) as functions of the adherend extensional stiffness ratio  $ETR = E_1t_1/E_2t_2 \leq 1$  horizontally and the non-dimensionalized joint overlap  $\lambda\ell = \sqrt{\frac{G}{\eta} \left( \frac{1}{E_1t_1} + \frac{1}{E_2t_2} \right) \ell^2}$  vertically.

Each table is prepared for a single value of thermal mismatch coefficient

$$CTHERM = \frac{(\alpha_2 - \alpha_1)\Delta T\lambda}{\tau_p \left( \frac{1}{E_1t_1} + \frac{1}{E_2t_2} \right)}$$

and equal and opposite values are treated in turn to cover both tensile and compressive shear loadings.

```

CDECK A4FC
C ELASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
C NON-DIMENSIONALIZED FORMULATION
C AVERAGE STRESSES
C STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
C DATA PRESENTATION FOR TENSILE SHEAR LOADING
C CHANGE SIGN OF CTERM TO USE FOR COMPRESSIVE SHEAR LOADS
C DIMENSION OL(J), ETR(K), CTERM(I), A(N,2), TRATIO(N,2),
C 1 TAUAVG(J,K), ICRTND(J,K), STRGTH(J,K), SIG(N,2),
C DIMENSION OL(40), ETR(10), CTERM(20), A(50,2), SIG(50,2),
C 1 TRATIO(50,2), TAUAVG(40,10), ICRTND(40,10), STRGTH(40,10)
C READ IN ARRAY SIZES
C READ (5,10) IMAX, JMAX, KMAX, NMAX
C 10 FORMAT (4I5)
C IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, NMAX .LE. 50 .AND. .GE. 10
C READ IN NON-DIMENSIONALIZED OVERLAP ARRAY
C OL(1) = 0.
C OL(2) MUST BE .LE. 0.5 BECAUSE OF A SUBSEQUENT TREND CHECK
C OL(J) MUST BE IN ASCENDING ORDER
C READ (5,20) (OL(J), J = 2, JMAX)
C 20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS 1 AND 2 SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
C READ (5,30) (ETR(K), K = 1, KMAX)
C 30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
C READ (5,40) (CTERM(I), I = 1, IMAX)
C 40 FORMAT (10F7.3)
C PRINT OUT INPUT DATA
C WRITE (6,50) IMAX, JMAX, KMAX, NMAX
C 50 FORMAT (1H1, 9H IMAX = , I2, 9H JMAX = , I2, 9H KMAX = , I2,
C 1 9H NMAX = , I2)
C WRITE (6,60) (OL(J), J = 1, JMAX)
C 60 FORMAT (10H OVERLAPS/, 12F6.2)
C WRITE (6,70) (ETR(K), K = 1, KMAX)
C 70 FORMAT (22H STIFFNESS IMBALANCES/, 10F5.2)
C WRITE (6,80) (CTERM(I), I = 1, IMAX)
C 80 FORMAT (20H THERMAL MISMATCHES/, 10F7.3)
C SET UNIFORM STRESS FOR ZERO OVERLAP
C DO 90 K = 1, KMAX
C TAUAVG(1,K) = 1.
C 90 STRGTH(1,K) = 0.
C START OF COMPUTATION DO LOOPS
C DO 310 I = 1, IMAX
C DO 180 K = 1, KMAX
C DO 180 J = 2, JMAX
C ESTABLISH ADHEREND 1 END OF JOINT AS REFERENCE
C SUBSEQUENTLY CHECK WHETHER ADHEREND 1 END OR ADHEREND 2 END IS CRITICAL
C NCRTND = 1
C THERMC = CTERM(I)
C VR = ETR(K)
C IF ((VR .NE. 1.) .OR. (THERMC .NE. 0.)) GO TO 100
C SET UNIFORM STRESS FOR BALANCED JOINTS
C TAUAVG(J,K) = 1.
C STRGTH(J,K) = OL(J)
C ICRTND(J,K) = 0
C GO TO 180
C 100 V1 = 1. + VR
C V2 = VR / V1
C V3 = (1. - VR) / V1
C OLAP = OL(J)
C OLAP2 = OLAP * OLAP
C COMPUTE INITIAL TERMS OF SERIES, ASSUMING A(1,1)=A(2,2)=1. & A(1,2)=A(
C A(1,1) = 1.
C A(2,1) = 0.
C A(3,1) = (OLAP/6.) * (-THERMC + V3*OLAP)
C A(1,2) = 0.
C A(2,2) = 0.5
C A(3,2) = (OLAP2 / 6.) * ( V2/2. ) + 1./6.
C COMPUTE NMAX TERMS OF AVERAGE STRESS POWER SERIES
C DO 110 N = 4, NMAX
C DO 110 M = 1, 2
C 110 A(N,M) = ( ((N-2)*(N-1) + OLAP2 * V2) * A(N-1,M)
C + V3 * OLAP2 * A(N-2,M) ) / (N*(N-1))
C COMPUTE A2 THROUGH RAPID CONVERGENCE OF AVERAGE STRESS
C NOTE THAT INDIVIDUAL TERMS IN DISTRIBUTION DO NOT CONVERGE AS RAPIDLY
C SIG(3,1) = 1. + A(3,1)
C SIG(3,2) = 0.5 + A(3,2)
C DO 120 N = 4, NMAX
C SIG(N,1) = SIG(N-1,1) + A(N,1)
C 120 SIG(N,2) = SIG(N-1,2) + A(N,2)
C COMPUTE A2(NMAX)
C A2SAVE = (THERMC/OLAP + V2 - (SIG(NMAX,1)/V1) ) /
C ( (SIG(NMAX,2)/V1) + (1./OLAP2) )
C COMPUTE AVERAGE SHEAR STRESS IN BOND
C TRATIO(J,NCRTND) = V1 * (V2 + THERMC/OLAP - A2SAVE/OLAP2)
C CHECK WHICH END OF JOINT IS CRITICAL
C IF (NCRTND .EQ. 2) GO TO 130

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A4EC0010
A4EC0020
A4EC0030
A4EC0040
A4EC0050
A4EC0060
A4EC0070
A4EC0080
A4EC0090
A4EC0100
A4EC0110
A4EC0120
A4EC0130
A4EC0140
A4EC0150
A4EC0160
A4EC0170
A4EC0180
A4EC0190
A4EC0200
A4EC0210
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A4EC0830
A4EC0840
A4EC0850
A4EC0860
A4EC0870
A4EC0880

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IF (THERMC .LT. 0.) GO TO 130
C IF ADHEREND 2 END IS CRITICAL INTERCHANGE 1 AND 2 AND RECOMPUTE
  NCRTND = 2
  VR = 1. / VR
  THERMC = - THERMC
  GO TO 100
130 CONTINUE
C IDENTIFY AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS & CRITICAL END
C NOTE THAT INITIAL SELECTION CRITERIA ASSUME ONE END OF JOINT OR
C 1 OTHER IS CRITICAL AND PRECLUDE POSSIBILITY OF MAXIMUM STRESS IN
C 2 MIDDLE. SUBSEQUENT SEPARATE CHECK ON THIS CONDITION.
  IF (CTHERM(I) .GT. 0.) GO TO 140
C BOTH IMBALANCES WILL INEVITABLY COMPOUND FOR C THERM(I) .LT. 0.
C 1 SINCE ETR(K) .LE. 1.. HENCE NCRTND = 1
  ICRTND(J,K) = 1
  IF (TRATIO(J,1) .GE. 0.) GO TO 160
  IF (TRATIO(J,1) .LT. 0.) GO TO 150
140 CONTINUE
C IDENTIFY MORE POWERFUL IMBALANCE FOR NULLIFYING BEHAVIOUR (NCRTND = 1)
C 1 C THERM(I) .GT. 0. .AND. ETR(K) .LE. 1.
C COVER SITUATION WHERE THERMAL IMBALANCE DOMINATES OVER STIFFNESS
C 1 IMBALANCE. NOTE NEED OL(2) .LE. 0.5 FOR THIS CHECK.
  ICRTND(J,K) = 2
  IF ((TRATIO(2,1) .GT. 1.) .AND. (TRATIO(J,2) .LT. 0.)) GO TO 150
  IF ((TRATIO(2,1) .GT. 1.) .AND. (TRATIO(J,2) .LE. 1.)) GO TO 170
C CHECK IF TWO IMBALANCES PRECISELY CANCEL
  ICRTND(J,K) = 0
  IF ((TRATIO(J,1) .EQ. 1.) .AND. (TRATIO(J,2) .EQ. 1.)) GO TO 160
C CHECK IF STIFFNESS IMBALANCE DOMINATES
  ICRTND(J,K) = 1
  IF ((TRATIO(J,1) .LE. 1.) .AND. (TRATIO(J,1) .GE. 0.) .AND.
  1 (TRATIO(J,2) .GT. 1.)) GO TO 160
C ALL POSSIBILITIES FOR EITHER END CRITICAL CHECKED OUT
C ONLY POSSIBILITY REMAINING IS THAT TAUJMAX IS IN MIDDLE OF JOINT
C NOTE THAT THIS PHENOMENON ARISES ONLY FOR JOINTS BROKEN WITHOUT LOAD
C 1 WHEN THE LOAD IN THE OPPOSITE SENSE IS EXAMINED.
C COMBINATION OF SEVERE THERMAL MISMATCH AND EXCESSIVE LENGTH IS NECESSARY
C IDENTIFY FAILURE CASES BY ASTERISKS
  TAUAVG(J,K) = 100.
  STRGTH(J,K) = 1000.
  ICRTND(J,K) = 10
  GO TO 180
C ZERO STRENGTH ATTAINED
150 TAUAVG(J,K) = 0.
  STRGTH(J,K) = 0.
  GO TO 180
C ADHEREND 1 END OF JOINT CRITICAL
160 TAUAVG(J,K) = TRATIO(J,1)
  STRGTH(J,K) = TAUAVG(J,K) * OL(J)
  GO TO 180
C ADHEREND 2 END OF JOINT CRITICAL
170 TAUAVG(J,K) = TRATIO(J,2)
  STRGTH(J,K) = TAUAVG(J,K) * OL(J)
180 CONTINUE
C IDENTIFY CRITICAL END OF JOINT FOR ZERO OVERLAP
  DO 190 K = 1, KMAX
190 ICRTND(1,K) = ICRTND(2,K)
C HENCE NEED OL(2) .LE. 0.2
  IF (CTHERM(I) .NE. 0.) GO TO 210
C PRINT OUT SPECIAL HEADING FOR ZERO THERMAL MISMATCH BETWEEN ADHERENDS
  WRITE (6,200) (ETR(K), K = 1, KMAX)
200 FORMAT (1H1,10(/),31X,48HADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)/,
  39X,31HNON-DIMENSIONALIZED FORMULATION//,
  2 38X,33HZERO THERMAL MISMATCH COEFFICIENT//,
  3 68X,28H0 = BOTH ENDS EQUALLY LOADED/, 20X,72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS , 1 = SOFT ET END CRITICAL/,
  5 68X,25H2 = STIFF ET END CRITICAL//,
  6 9H SCALED, 31X,39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
  7 7H L/T/, 7H RATIO/, 1H+, 4X, 10F10.1//)
  GO TO 230
210 THERMC = - C THERM(I)
C PRINT OUT HEADING
  WRITE (6,220) C THERM(I), THERMC, (ETR(K), K = 1, KMAX)
220 FORMAT (1H1,10(/),31X,48HADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)/,
  39X,31HNON-DIMENSIONALIZED FORMULATION//, 17X,31HTHERMAL MISMATCH COEFFICIENT = , F6.3, 17H FOR TENSION, = , F6.3, 16H
  3FOR COMPRESSION//,68X,28H0 = BOTH ENDS EQUALLY LOADED/,
  4 20X,72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,
  51 = SOFT ET END CRITICAL/, 68X,25H2 = STIFF ET END CRITICAL//,
  6 9H SCALED, 31X,39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
  7 7H L/T/, 7H RATIO/, 1H+, 4X, 10F10.1//)
230 CONTINUE
C PRINT OUT TABULATIONS OF AVERAGE BOND STRESSES
  DO 250 J = 1, JMAX
  WRITE (6,240) OL(J), ((TAUAVG(J,K), ICRTND(J,K)), K = 1, KMAX)
240 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, 11, 1X))
250 CONTINUE
  IF (CTHERM(I) .NE. 0.) GO TO 270

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A4EC0890
A4EC0900
A4EC0910
A4EC0920
A4EC0930
A4EC0940
A4EC0950
A4EC0960
A4EC0970
A4EC0980
A4EC0990
A4EC1000
A4EC1010
A4EC1020
A4EC1030
A4EC1040
A4EC1050
A4EC1060
A4EC1070
A4EC1080
A4EC1090
A4EC1100
A4EC1110
A4EC1120
A4EC1130
A4EC1140
A4EC1150
A4EC1160
A4EC1170
A4EC1180
A4EC1190
A4EC1200
A4EC1210
A4EC1220
A4EC1230
A4EC1240
A4EC1250
A4EC1260
A4EC1270
A4EC1280
A4EC1290
A4EC1300
A4EC1310
A4EC1320
A4EC1330
A4EC1340
A4EC1350
A4EC1360
A4EC1370
A4EC1380
A4EC1390
A4EC1400
A4EC1410
A4EC1420
A4EC1430
A4EC1440
A4EC1450
A4EC1460
A4EC1470
A4EC1480
A4EC1490
A4EC1500
A4EC1510
A4EC1520
A4EC1530
A4EC1540
A4EC1550
A4EC1560
A4EC1570
A4EC1580
A4EC1590
A4EC1600
A4EC1610
A4EC1620
A4EC1630
A4EC1640
A4EC1650
A4EC1660
A4EC1670
A4EC1680
A4EC1690
A4EC1700
A4EC1710
A4EC1720
A4EC1730
A4EC1740
A4EC1750
A4EC1760

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C PRINT OUT SPECIAL HEADING FOR ZERO THERMAL MISMATCH BETWEEN ADHERENDS A4EC1770
  WRITE (6,260) (ETR(K), K = 1, KMAX) A4EC1780
260 FORMAT (1H1,10(/),31X,48HADHESIVE-BONDED SCARF JOINTS (ELASTIC AA4EC1790
  ANALYSIS)/,30X,31HNON-DIMENSIONALIZED FORMULATION//, A4EC1800
  2 38X,33HZERO THERMAL MISMATCH COEFFICIENT//, A4EC1810
  3 68X,28H0 = BOTH ENDS EQUALLY LOADED/,20X,72HNON-DIMENSIONALIZA A4EC1820
  4ED STRENGTH , 1 = SOFT ET END CRITICAL/, A4EC1830
  5 68X,25H2 = STIFF ET END CRITICAL//, A4EC1840
  6 9H SCALED,31X,39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/, A4EC1850
  7 7H L/T/,7H RATIO/,1H+,4X,10F10.1//) A4EC1860
  GO TO 290 A4EC1870
C PRINT OUT HEADINGS A4EC1880
270 WRITE (6,280) CTERM(I), THERMC, (ETR(K), K = 1, KMAX) A4EC1890
280 FORMAT (1H1,10(/),31X,48HADHESIVE-BONDED SCARF JOINTS (ELASTIC AA4EC1900
  ANALYSIS)/,39X,31HNON-DIMENSIONALIZED FORMULATION//,17X,31HTHERA A4EC1910
  2MAL MISMATCH COEFFICIENT = ,F6.3,17H FOR TENSION, = ,F6.3,16H A4EC1920
  3FOR COMPRESSION//,68X,28H0 = BOTH ENDS EQUALLY LOADED/, A4EC1930
  4 20X,72HNON-DIMENSIONALIZED STRENGTH , A4EC1940
  51 = SOFT ET END CRITICAL/,68X,25H2 = STIFF ET END CRITICAL//, A4EC1950
  6 9H SCALED,31X,39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/, A4EC1960
  7 7H L/T/,7H RATIO/,1H+,4X,10F10.1//) A4EC1970
290 CONTINUE A4EC1980
C PRINT OUT TABULATIONS OF NON-DIMENSIONALIZED STRENGTHS A4EC1990
  DO 310 J = 1, JMAX A4EC2000
  WRITE (6,300) OL(J), ((STRGTH(J,K), ICRTND(J,K)), K = 1, KMAX) A4EC2010
300 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, 11, 1X)) A4EC2020
310 CONTINUE A4EC2030
  WRITE (6,320) A4EC2040
320 FORMAT (1H1, 18H PROGRAM COMPLETED/) A4EC2050
  STOP A4EC2060
  END A4EC2070

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ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED STRENGTH

0 = BOTH ENDS EQUALLY LOADED  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.1832	0.1826	0.1822	0.1818	0.1814	0.1811	0.1808	0.1806	0.1804	0.1802
0.50	0.4245	0.4161	0.4093	0.4035	0.3987	0.3945	0.3909	0.3877	0.3849	0.3823
1.00	0.9012	0.8301	0.7765	0.7347	0.7012	0.6736	0.6507	0.6312	0.6145	0.6000
1.20	1.1853	1.0527	0.9576	0.8861	0.8304	0.7857	0.7491	0.7186	0.6927	0.6706
1.50	1.3881	1.5000	1.3016	1.1615	1.0571	0.9765	0.9123	0.8601	0.8166	0.7800
1.70	1.4858	1.6219	1.5915	1.3857	1.2372	1.1251	1.0375	0.9671	0.9093	0.8611
2.00	1.6072	1.7784	1.9453	1.7899	1.5561	1.3850	1.2575	1.1516	1.0685	1.0000
2.50	1.7635	1.9994	2.2144	2.4369	2.2136	1.9151	1.6943	1.5244	1.3898	1.2875
3.00	1.8939	2.1618	2.4140	2.7205	3.0000	2.5462	2.2171	1.9676	1.7723	1.6154
4.00	2.0664	2.4416	2.8242	3.2124	3.6006	4.0000	4.4262	2.9971	2.6647	2.4000
5.00	2.2084	2.6771	3.1597	3.6525	4.1531	4.6596	4.7517	4.1334	3.6556	3.2759
6.00	2.3299	2.8909	3.4722	4.0681	4.6748	5.2897	5.9108	5.3214	4.6966	4.2000
8.00	2.5422	3.2882	4.0670	4.8679	5.6841	6.5112	7.3463	7.7637	6.8473	6.1176
10.00	2.7354	3.5639	4.6469	5.6528	6.6766	7.7127	8.7574	9.8034	9.0370	8.0769
12.00	2.9203	4.7443	5.2233	6.4342	7.6663	8.9080	10.1598	11.4179	11.2424	10.0541
15.00	3.1914	4.6063	6.0890	7.6074	9.1463	10.6979	12.2578	13.4232	14.5630	13.0349
17.00	3.3708	4.9923	6.6683	8.3915	10.1353	11.8913	13.6551	15.4243	16.7807	15.0273
20.00	3.6401	5.5435	7.5412	9.5710	11.6208	13.6822	15.7538	17.8243	19.9010	18.0198
25.00	4.0920	6.5035	9.0051	11.5442	14.1019	16.6699	19.2443	21.8227	24.0400	23.0127
30.00	4.5493	7.4657	10.4775	13.5241	16.5877	19.6606	22.7339	25.8208	28.9051	28.0088
35.00	5.0119	8.4371	11.9563	15.5087	19.0739	22.0533	26.2346	29.8190	33.4055	33.0065
40.00	5.4792	9.4129	13.4390	17.4966	21.5684	24.5475	29.7310	33.8173	37.9055	38.0050
45.00	5.9507	10.3930	14.9252	19.4869	24.0615	26.8428	33.2231	37.8159	42.4054	43.0039
50.00	6.4257	11.3763	16.4134	21.4739	26.5559	31.6390	36.7257	41.8147	46.9052	48.0032

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS

0 = BOTH ENDS EQUALLY LOADED  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	0.91633	0.91323	0.91084	0.90883	0.90707	0.90554	0.90420	0.90331	0.90194	0.90099
0.50	0.84893	0.83225	0.81855	0.80798	0.79735	0.78898	0.78171	0.77534	0.76971	0.76470
1.00	0.90116	0.83009	0.77653	0.73471	0.70116	0.67365	0.65067	0.63120	0.61449	0.60000
1.20	0.98775	0.87725	0.79401	0.73842	0.69196	0.65475	0.62426	0.59833	0.57729	0.55882
1.50	0.92538	1.00000	0.86776	0.77430	0.71677	0.65101	0.60823	0.57337	0.54442	0.52000
1.70	0.87400	0.95406	0.93616	0.81511	0.72779	0.66184	0.61028	0.56888	0.53491	0.50653
2.00	0.80360	0.89920	0.97265	0.89497	0.77306	0.69251	0.62723	0.57541	0.53426	0.50000
2.50	0.70542	0.79593	0.88575	0.97474	1.00000	0.76604	0.67771	0.60976	0.55599	0.51219
3.00	0.62796	0.72061	0.81366	0.90685	1.00000	0.34875	0.73902	0.65588	0.59078	0.53846
4.00	0.51660	0.61040	0.70606	0.80309	0.90116	1.00000	0.85654	0.74928	0.66619	0.60000
5.00	0.44169	0.53543	0.63194	0.73050	0.83261	0.93192	1.00000	0.82669	0.73113	0.65517
6.00	0.38831	0.48181	0.57870	0.67802	0.77914	0.88161	0.98514	1.00000	0.88690	0.78277
8.00	0.31778	0.41102	0.50838	0.60849	0.71152	0.81390	0.91328	1.00000	0.97046	0.70000
10.00	0.27354	0.36694	0.46459	0.56528	0.66766	0.77127	0.87574	0.98034	0.85591	0.76471
12.00	0.24336	0.33702	0.43528	0.53611	0.63873	0.74233	0.84665	0.95149	0.93030	0.86764
15.00	0.21276	0.30709	0.40593	0.50752	0.60976	0.71319	0.81718	0.92155	0.97086	0.86900
17.00	0.19828	0.29308	0.39226	0.49352	0.59619	0.69949	0.80324	0.90731	0.98710	0.88396
20.00	0.18200	0.27748	0.37756	0.47855	0.58104	0.68411	0.78754	0.89121	0.99505	0.90099
25.00	0.16368	0.26014	0.36020	0.46177	0.56408	0.66680	0.76977	0.87291	0.97616	0.92051
30.00	0.15164	0.24989	0.34975	0.45030	0.55292	0.65535	0.75796	0.86069	0.96350	0.93363
35.00	0.14320	0.24106	0.34160	0.44311	0.54505	0.64724	0.74956	0.85197	0.95444	0.94304
40.00	0.13598	0.23532	0.33597	0.43742	0.53921	0.64119	0.74328	0.84543	0.94764	0.95012
45.00	0.13224	0.23095	0.33167	0.43304	0.53470	0.63651	0.73840	0.84035	0.94234	0.95564
50.00	0.12851	0.22753	0.32828	0.42958	0.53112	0.63278	0.73451	0.83629	0.93810	0.96006

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED STRENGTH

0 = BOTH ENDS EQUALLY LOADED  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.1773	0.1778	0.1783	0.1788	0.1793	0.1798	0.1803	0.1808	0.1813	0.1818
0.50	0.3464	0.3527	0.3581	0.3629	0.3671	0.3708	0.3741	0.3771	0.3799	0.3823
1.00	0.4249	0.4516	0.4759	0.4981	0.5185	0.5374	0.5548	0.5710	0.5867	0.6000
1.50	0.4206	0.4567	0.4901	0.5213	0.5503	0.5775	0.6030	0.6269	0.6494	0.6706
2.00	0.4031	0.4535	0.5015	0.5472	0.5907	0.6321	0.6717	0.7094	0.7455	0.7800
2.50	0.3999	0.4534	0.5086	0.5647	0.6187	0.6708	0.7210	0.7694	0.8161	0.8611
3.00	0.3727	0.4498	0.5234	0.5964	0.6678	0.7375	0.8055	0.8719	0.9367	1.0000
3.50	0.3547	0.4593	0.5642	0.6688	0.7729	0.8763	0.9788	1.0804	1.1810	1.2805
4.00	0.3492	0.4855	0.6241	0.7644	0.9056	1.0475	1.1896	1.3317	1.4737	1.6154
4.50	0.3648	0.5715	0.7862	1.0070	1.2324	1.4614	1.6932	1.9286	2.1674	2.4090
5.00	0.4022	0.5861	0.8848	1.1794	1.6123	1.9355	2.2661	2.5996	2.9364	3.2759
6.00	0.4521	0.6181	1.2058	1.6093	2.0245	2.4485	2.8793	3.3155	3.7560	4.2000
8.00	0.5738	1.1135	1.5890	2.2886	2.9051	3.4485	4.0170	4.6151	5.2463	6.0000
10.00	0.7123	1.4343	2.2052	3.0066	3.7473	4.4745	5.1859	5.8869	6.5769	8.0000
12.00	0.8610	1.7712	2.7417	3.7417	4.8338	6.2195	7.5691	8.9279	10.2929	13.0349
15.00	1.0293	2.2966	3.4906	4.9906	6.8162	9.0635	11.5233	14.2911	17.3663	20.8000
17.00	1.5104	3.2662	4.9342	6.8733	9.2733	12.1297	15.4966	19.3570	23.3866	28.0000
25.00	1.9426	4.1423	6.4380	8.7739	11.6073	15.4792	19.8703	24.8479	29.6185	38.0000
30.00	2.3874	5.0948	7.9009	10.7451	13.6073	18.4792	23.8413	29.2324	34.6185	43.0000
35.00	2.8417	6.0583	9.3735	12.7243	16.0913	19.4670	22.8479	26.2324	34.1155	38.0000
40.00	3.3031	7.0295	10.8525	14.7086	18.5795	22.4581	26.3413	29.2324	38.6133	43.0000
45.00	3.7703	8.0053	12.3360	16.6964	21.0703	25.4512	29.8362	34.2239	38.6133	43.0000
50.00	4.2420	8.9872	13.8226	18.6866	23.5630	28.4458	33.3323	39.7212	43.1116	48.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS

0 = BOTH ENDS EQUALLY LOADED  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	0.88632	0.88900	0.89129	0.89325	0.89496	0.89646	0.89778	0.89897	0.90000	0.90099
0.50	0.69288	0.70537	0.71621	0.72571	0.73410	0.74157	0.74828	0.75428	0.75973	0.76470
1.00	0.42491	0.45156	0.47586	0.49810	0.51854	0.53737	0.55480	0.57096	0.58598	0.60000
1.50	0.35053	0.38056	0.40344	0.42440	0.44384	0.46126	0.47749	0.49224	0.50516	0.51822
2.00	0.26871	0.30236	0.33435	0.36479	0.39378	0.42141	0.44777	0.47294	0.49699	0.52000
2.50	0.22938	0.26442	0.29917	0.33217	0.36396	0.39459	0.42411	0.45257	0.48003	0.50653
3.00	0.19637	0.23442	0.26171	0.28821	0.31388	0.33873	0.36275	0.38596	0.40837	0.43000
3.50	0.14187	0.18374	0.22567	0.26752	0.30916	0.35051	0.39153	0.43216	0.47239	0.51219
4.00	0.11640	0.15182	0.19084	0.23479	0.28187	0.32915	0.37652	0.42390	0.47123	0.51856
4.50	0.09121	0.12287	0.15655	0.19517	0.23810	0.28534	0.33279	0.38036	0.42799	0.47566
5.00	0.08043	0.10722	0.13695	0.16987	0.20617	0.24584	0.28798	0.33252	0.37942	0.42728
6.00	0.07536	0.10335	0.13307	0.16582	0.20282	0.24311	0.28688	0.33416	0.38304	0.43341
8.00	0.07173	0.10118	0.13112	0.16306	0.20006	0.24033	0.28410	0.33137	0.38024	0.43061
10.00	0.07123	0.10143	0.13143	0.16343	0.20043	0.24070	0.28447	0.33174	0.38111	0.43148
12.00	0.07175	0.10176	0.13176	0.16376	0.20076	0.24103	0.28480	0.33207	0.38144	0.43181
15.00	0.07309	0.10310	0.13310	0.16510	0.20210	0.24237	0.28614	0.33341	0.38278	0.43314
17.00	0.07407	0.10408	0.13408	0.16608	0.20308	0.24335	0.28712	0.33465	0.38402	0.43447
20.00	0.07552	0.10553	0.13553	0.16753	0.20453	0.24431	0.28808	0.33599	0.38536	0.43580
25.00	0.07770	0.10771	0.13771	0.16971	0.20651	0.24617	0.29014	0.33836	0.38754	0.43798
30.00	0.07958	0.10959	0.13959	0.17159	0.20849	0.24803	0.29200	0.34051	0.38969	0.43986
35.00	0.08119	0.11120	0.14120	0.17320	0.21039	0.24969	0.29397	0.34249	0.39157	0.44174
40.00	0.08258	0.11258	0.14258	0.17458	0.21228	0.25158	0.29585	0.34477	0.39385	0.44362
45.00	0.08378	0.11378	0.14378	0.17578	0.21418	0.25348	0.29813	0.34705	0.39613	0.44550
50.00	0.08484	0.11494	0.14494	0.17694	0.21604	0.25534	0.29941	0.34933	0.39841	0.44738

## A.2 Computer Program A4ED For Lower Bound Elastic-Plastic Strength of Bonded Scarf Joints

This FORTRAN IV digital computer program covers a simple efficient approximate solution for the elastic-plastic strength of most bonded scarf joints of practical proportions and materials. Its development was needed as a sufficiently close starting point for convergence to proceed in the more precise program A4EE. It transpired, on examination of the equivalent results computed by A4EE that the quicker computations of A4ED were satisfactory as final answers provided that (1) and adhesive non-linear behavior was not negligible, i.e., that  $\gamma_p/\gamma_e > 0.5$ , (2) the thermal mismatch coefficient is not too high, i.e., that  $C_{THERM} < 2$ , and (3) that the stiffness mismatch between adherends be not too great, i.e., that  $0.2 \leq ETR \leq 1$ .

The input data for program A4ED is precisely the same as for program A4EE with the exception that  $\gamma_p/\gamma_e$  for the adhesive cannot be equal to zero for A4EE. In other words, perfectly elastic adhesive behavior must be excluded from A4EE. On the other hand, the values computed by A4ED for zero adhesive plasticity are unduly conservative.

A listing of the program and sample outputs follow.

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CDECK A4FD
C ELASTIC-PLASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
C LOWER BOUND ANALYSIS PROVIDED WHICH IS ACCURATE FOR DESIGN
C NON-DIMENSIONALIZED AVERAGE SHEAR STRESSES COMPUTED
C NON-DIMENSIONALIZED JOINT STRENGTHS COMPUTED
C RANGE OF ADHESIVE DUCTILITIES INCLUDED
C RANGES OF ADHEREND STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
C DATA PRESENTATION FOR TENSILE SHEAR LOADING
C CHANGE SIGN OF CTERM TO USE FOR COMPRESSIVE SHEAR LOADS
C SET CTERM .EQ. 0. AND REPLACE ADHEREND ET'S WITH GT'S FOR IN-PLANF
C 1 (EDGEWISE) SHEAR LOADING
C
C DIMENSION DL(J), ETR(K), CTERM(I), GPOVGE(L), A(N), TRATIO(J,NCRTND),
C 1 TAUAVG(J,K), ICRTND(J,K), STRGTH(J,K), THERMC(NCRTND),
C 2 VR(NCRTND), VU(NCRTND), VL(NCRTND), OLTRNT(NCRTND),
C 3 OLTRNC(NCRTND), TRANSL(K)
C DIMENSION OL(40), ETR(10), CTERM(20), GPOVGE(20), A(50),
C 1 TRATIO(40,2), TAUAVG(40,10), STRGTH(40,10), ICRTND(40,10),
C 2 THERMC(2), VR(2), VU(2), VL(2), OLTRNT(2), OLTRNC(2), TRANSL(10)
C
C READ IN INPUT DATA
C READ IN ARRAY SIZES
C READ (5,10) IMAX, JMAX, KMAX, LMAX, NMAX
C 10 FORMAT (5I5)
C IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, LMAX .LE. 20,
C 1 NMAX .LE. 50 .AND. .GE. 10.
C READ IN NON-DIMENSIONALIZED OVERLAP ARRAY
C OL(1) = 0.
C OL(J) MUST BE IN ASCENDING ORDER
C OL(2) MUST BE LESS THAN 0.2 FOR IDENTIFICATION OF CRITICAL END
C 1 OF JOINT OF ZERO OVERLAP (LIMITING CASE)
C OL(J) .LT. 100. FOR COMPATIBILITY WITH FORMAT STATEMENTS 470 & 590
C READ (5,20) (OL(J), J = 2, JMAX)
C NOTE JMAX ONE MORE THAN INPUT VALUES ON CARD(S)
C 20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
C STIFFNESS RATIOS SHOULD BE IN ASCENDING OR DESCENDING ORDER
C ETR(K) SHOULD INCLUDE VALUE 1. BUT MUST EXCLUDE VALUE 0.
C READ (5,30) (ETR(K), K = 1, KMAX)
C 30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
C CTERM .PROPNL. (ALPHA(2)-ALPHA(1))*(OPERATING TEMP. - CURE TEMP.)
C NEED CTERM(I) ARRAY TO CONTAIN BOTH POSITIVE AND NEGATIVE VALUES
C 1 TO COVER BOTH TENSILE AND COMPRESSIVE LOADS
C READ (5,40) (CTERM(I), I = 1, IMAX)
C 40 FORMAT (10F7.3)
C READ IN PLASTIC-TO-ELASTIC STRAIN RATIO ARRAY
C GPOVGE(L) MUST BE .GT. 0. FOR ELASTIC-PLASTIC ANALYSIS
C PURELY ELASTIC SOLUTION OBTAINED FROM SEPARATE PROCEDURE
C READ (5,50) (GPOVGE(L), L = 1, LMAX)
C 50 FORMAT (14F5.2)
C
C PRINT OUT INPUT DATA
C WRITE (6,60) IMAX, JMAX, KMAX, LMAX, NMAX
C 60 FORMAT (1H1, 9H IMAX = ,I2, 9H JMAX = ,I2, 9H KMAX = ,I2,
C 1 9H LMAX = ,I2, 9H NMAX = ,I2)
C WRITE (6,70)
C 70 FORMAT (10H OVERLAPS)
C WRITE (6,80) (OL(J), J = 1, JMAX)
C 80 FORMAT (12F6.2)
C WRITE (6,90)
C 90 FORMAT (22H STIFFNESS IMBALANCES)
C WRITE (6,100) (ETR(K), K = 1, KMAX)
C 100 FORMAT (14F5.2)
C WRITE (6,110)
C 110 FORMAT (20H THERMAL MISMATCHES)
C WRITE (6,120) (CTERM(I), I = 1, IMAX)
C 120 FORMAT (10F7.3)
C WRITE (6,130)
C 130 FORMAT (34H PLASTIC TO ELASTIC STRAIN RATIOS)
C WRITE (6,140) (GPOVGE(L), L = 1, LMAX)
C 140 FORMAT (14F5.1)
C
C START COMPUTATIONAL DD LOOPS
C DD 620 L = 1, LMAX
C GAMMAR = GPOVGE(L)
C ENSURE EXCLUSION OF NEGATIVE PLASTICITY IN ADHESIVE (ERROR IN DATA)
C IF (GAMMAR .LT. 0.) GO TO 620
C DD 620 I = 1, IMAX
C THERMC(1) = CTERM(I)
C THERMC(2) = - THERMC(1)
C DD 350 K = 1, KMAX
C VR(1) = ETR(K)
C VR(2) = 1. / VR(1)
C VU(1) = 1. - VR(1)
C VL(1) = 1. + VR(1)
C VU(2) = 1. - VR(2)

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      VL(2) = 1. + VR(2)
C
C SPECIAL PROCEDURE FOR PURELY ELASTIC ADHESIVE JOINT
  IF (GAMMAR .GT. 0.) GO TO 160
C SET ZERO TRANSITIONAL LENGTH FOR PURELY-ELASTIC JOINTS
  TRANSL(K) = 0.
  DO 150 J = 2, JMAX
    OLAP = OL(J)
  DO 150 NCRTND = 1, 2
    150 TRATIO(J,NCRTND) = 1. - VU(NCRTND) + VL(NCRTND)*THERMC(NCRTND)/OLAP
  GO TO 270
C
C ESTABLISH TRANSITIONAL OVERLAPS FROM FULLY-PLASTIC TO ELASTIC-PLASTIC
C 1 BEHAVIOUR AS REFERENCE LENGTH FOR START OF ITERATIONS
C SPECIAL PROCEDURE FOR LESS THAN COMPLETELY UNBALANCED JOINTS
  160 IF ( (THERMC(1) .EQ. 0.) .AND. (VR(1) .EQ. 1.) ) GO TO 170
    IF (THERMC(1) .EQ. 0.) GO TO 190
    IF (VR(1) .EQ. 1.) GO TO 190
C IF NONE OF THESE, JOINT CONTAINS BOTH IMBALANCES
  GO TO 200
C SET INFINITE TRANSITIONAL OVERLAP FOR IDENTICAL ADHESIVES
  170 OLTRNT(1) = 1000000.
    OLTRNT(2) = 1000000.
C
C OLTRNC(1) = 1000000.
C OLTRNC(2) = 1000000.
  GO TO 210
C SET TRANSITIONAL OVERLAPS FOR STIFFNESS IMBALANCE ONLY
C IN THE ABSENCE OF THERMAL MISMATCH, SAME END IS CRITICAL FOR BOTH
C 1 TENSILE SHEAR AND COMPRESSIVE SHEAR LOADING
  180 IF (VU(1) .GT. 0.) OLTRNT(1) = SQRT(GAMMAR*VL(1)/VU(1))
    IF (VU(1) .LE. 0.) OLTRNT(1) = 1000000.
    IF (VU(2) .GT. 0.) OLTRNT(2) = SQRT(GAMMAR*VL(2)/VU(2))
    IF (VU(2) .LE. 0.) OLTRNT(2) = 1000000.
C
C IF (VU(1) .GT. 0.) OLTRNC(1) = SQRT(GAMMAR*VL(1)/VU(1))
C IF (VU(1) .LE. 0.) OLTRNC(1) = 1000000.
C IF (VU(2) .GT. 0.) OLTRNC(2) = SQRT(GAMMAR*VL(2)/VU(2))
C IF (VU(2) .LE. 0.) OLTRNC(2) = 1000000.
  GO TO 210
C SET TRANSITIONAL OVERLAPS FOR THERMAL MISMATCH ONLY
  190 IF (THERMC(1) .LT. 0.) OLTRNT(1) = -GAMMAR/THERMC(1)
    IF (THERMC(1) .GE. 0.) OLTRNT(1) = 1000000.
    IF (THERMC(2) .LT. 0.) OLTRNT(2) = -GAMMAR/THERMC(2)
    IF (THERMC(2) .GE. 0.) OLTRNT(2) = 1000000.
C
C IF (THERMC(1) .LT. 0.) OLTRNC(1) = GAMMAR/THERMC(1)
C IF (THERMC(1) .GE. 0.) OLTRNC(1) = 1000000.
C IF (THERMC(2) .LT. 0.) OLTRNC(2) = GAMMAR/THERMC(2)
C IF (THERMC(2) .GE. 0.) OLTRNC(2) = 1000000.
  GO TO 210
C 200 CONTINUE
C STANDARD PROCEDURE FOR COMPLETELY UNBALANCED JOINTS
  V1 = THERMC(1) * VL(1) / (2. * VU(1))
  V2 = THERMC(2) * VL(2) / (2. * VU(2))
  V3 = V1*V1 + GAMMAR*VL(1)/VU(1)
  V4 = V2*V2 + GAMMAR*VL(2)/VU(2)
C ESTABLISH TRANSITIONAL OVERLAPS BELOW WHICH JOINT IS FULLY PLASTIC
C NEXT FOUR STATEMENTS APPLY FOR TENSILE SHEAR LOADING
  IF (V3 .GE. 0.) OLTRNT(1) = V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V3 .LT. 0.) .OR. (OLTRNT(1) .LE. 0.) ) OLTRNT(1) = 1000000.
  IF (V4 .GE. 0.) OLTRNT(2) = V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V4 .LT. 0.) .OR. (OLTRNT(2) .LE. 0.) ) OLTRNT(2) = 1000000.
C IF ICRTND .EQ. 2 FOR SHORT OVERLAPS, OLTRNT(1) WILL BE COMPUTED VERY
C 1 LARGE, AND VICE VERSA
C THIS IS PHYSICALLY REALISTIC AND DOES NOT LEAD TO IMPOSSIBLE COMPUTING
C IF BOTH V3 AND V4 ARE POSITIVE, EITHER OLTRNT(1) OR OLTRNT(2) WILL BE
C 1 COMPUTED NEGATIVE. NEED TO PREVENT COMPUTATIONS BASED ON THIS
C 2 UNREAL SITUATION. HENCE CHECKS ABOVE AND BELOW
C NEXT FOUR STATEMENTS WOULD APPLY FOR COMPRESSIVE SHEAR LOADING
  IF (V3 .GE. 0.) OLTRNC(1) = -V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V3 .LT. 0.) .OR. (OLTRNC(1) .LE. 0.) ) OLTRNC(1) = 1000000.
  IF (V4 .GE. 0.) OLTRNC(2) = -V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V4 .LT. 0.) .OR. (OLTRNC(2) .LE. 0.) ) OLTRNC(2) = 1000000.
C
C 210 DO 260 NCRTND = 1, 2
C SET UNIFORM STRESS FOR SHORT OVERLAPS
  DO 220 J = 2, JMAX
    JSAVE = J

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C IF NOT, JOINT IS FULLY PLASTIC
  IF (OL(J) .GT. OLTRNT(NCRTND)) GO TO 230
  220 TRATIO(J,NCRTND) = 1.
  IF (JSAVE .EQ. JMAX) GO TO 260
C
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
  230 DO 250 J = JSAVE, JMAX
    OLAP = OL(J)
    OLAP2 = OLAP * OLAP
C COMPUTE AOVERL FOR MINIMUM VALUE OF TAVOTP BY ITERATION
C SET INITIAL ESTIMATE OF EXTENT OF PLASTIC ZONE FROM TRANSITIONAL OLAP
    AOVERL = OLTRNT(NCRTND) / OLAP
    DO 240 N = 1, NMAX
      ARMDR = 1. - AOVERL
      AOVERL = -ARMDR * ALOG(ARMDR) + (GAMMAR / ((VU(NCRTND)/VL(NCRTND))*
      1 OLAP2 - THERMC(NCRTND)*OLAP))
      IF (AOVERL .GT. 0.999) AOVERL = 0.999
      IF (AOVERL .LT. 0.001) AOVERL = 0.001
    240 CONTINUE
    TRATIO(J,NCRTND) = 1.
    IF (AOVERL .GT. 0.9999) GO TO 250
C COMPUTE CORRESPONDING AVERAGE SHEAR STRESS
    TRATIO(J,NCRTND) = 1. - (VL(NCRTND)*GAMMAR/OLAP2 + (VL(NCRTND)*
    1 THERMC(NCRTND)/OLAP - VU(NCRTND)) * AOVERL) / ALOG(ARMDR)
  250 CONTINUE
  260 CONTINUE
C
C VALUES COMPUTED ARE NOW STORED IN TRATIO(J,NCRTND)
  270 DO 340 J = 2, JMAX
    OLAP = OL(J)
    TAU1 = TRATIO(J,1)
    TAU2 = TRATIO(J,2)
    IF ((TAU1 .LT. 1.) .OR. (TAU2 .LT. 1.)) GO TO 280
C IF SO, JOINT IS NOT FULLY PLASTIC
C IF NOT, IDENTIFY CRITICAL END OF JOINT FROM SHEAR STRAIN GRADIENT
    GRADNT = THERMC(1) - OLAP*VU(1)/VL(1)
    IF (GRADNT .LT. 0.) ICRTND(J,K) = 1
    IF (GRADNT .EQ. 0.) ICRTND(J,K) = 0
    IF (GRADNT .GT. 0.) ICRTND(J,K) = 2
    TAUAVG(J,K) = 1.
    STRGTH(J,K) = OLAP
C TRANSITIONAL OVERLAPS ALREADY COMPUTED FOR ELASTIC ADHESIVE
C BYPASS RECOMPUTATION. THIS APPLIES TO ELASTIC-PLASTIC ADHESIVES
    IF (GAMMAR .EQ. 0.) GO TO 340
    MCRTND = ICRTND(J,K)
    IF (MCRTND .EQ. 0) MCRTND = 1
    TRANSL(K) = OLTRNT(MCRTND)
    GO TO 340
  280 DIFFNC = TAU1 - TAU2
C IF DIFFNC .LT. 0., NCRTND .EQ. 1
C IF DIFFNC .EQ. 0., NCRTND .EQ. 0
C IF DIFFNC .GT. 0., NCRTND .EQ. 2
    IF (DIFFNC) 290, 300, 310
C ADHEREND (1) END OF JOINT CRITICAL
  290 TAUAVG(J,K) = TAU1
    STRGTH(J,K) = TAU1 * OLAP
    ICRTND(J,K) = 1
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
    IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
    GO TO 320
C BOTH ENDS OF JOINT EQUALLY CRITICAL FROM NULLIFYING (OR ZERO)
C 1 ADHEREND IMBALANCES
  300 TAUAVG(J,K) = TAU1
    STRGTH(J,K) = TAU1 * OLAP
    ICRTND(J,K) = 0
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
    IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
    GO TO 320
C ADHEREND (2) END OF JOINT CRITICAL
  310 TAUAVG(J,K) = TAU2
    STRGTH(J,K) = TAU2 * OLAP
    ICRTND(J,K) = 2
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
    IF (J .EQ. 2) TRANSL(K) = OLTRNT(2)
C COVER CASES OF ZERO OR NEGATIVE ESTIMATED STRENGTHS
  320 IF (TAUAVG(J,K) .GT. 0.) GO TO 330
C IF NOT, JOINT HAS BROKEN DUE TO THERMAL STRESSES WITHOUT EXTERNAL LOAD
    TAUAVG(J,K) = 0.
    STRGTH(J,K) = 0.
    GO TO 340
  330 IF (TAUAVG(J,K) .LE. 1.) GO TO 340
C IF NOT, THERE HAS BEEN A COMPUTATIONAL MISTAKE
C PRINT ASTERISKS TO IDENTIFY ERROR
C RERUN WITH GREATER VALUE OF NMAX
    TAUAVG(J,K) = 100.
    STRGTH(J,K) = 1000.
  340 CONTINUE
  350 CONTINUE

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A4ED1770
A4ED1780
A4ED1790
A4ED1800
A4ED1810
A4ED1820
A4ED1830
A4ED1840
A4ED1850
A4ED1860
A4ED1870
A4ED1880
A4ED1890
A4ED1900
A4ED1910
A4ED1920
A4ED1930
A4ED1940
A4ED1950
A4ED1960
A4ED1970
A4ED1980
A4ED1990
A4ED2000
A4ED2010
A4ED2020
A4ED2030
A4ED2040
A4ED2050
A4ED2060
A4ED2070
A4ED2080
A4ED2090
A4ED2100
A4ED2110
A4ED2120
A4ED2130
A4ED2140
A4ED2150
A4ED2160
A4ED2170
A4ED2180
A4ED2190
A4ED2200
A4ED2210
A4ED2220
A4ED2230
A4ED2240
A4ED2250
A4ED2260
A4ED2270
A4ED2280
A4ED2290
A4ED2300
A4ED2310
A4ED2320
A4ED2330
A4ED2340
A4ED2350
A4ED2360
A4ED2370
A4ED2380
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A4ED2400
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A4ED2440
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A4ED2470
A4ED2480
A4ED2490
A4ED2500
A4ED2510
A4ED2520
A4ED2530
A4ED2540
A4ED2550
A4ED2560
A4ED2570
A4ED2580
A4ED2590
A4ED2600
A4ED2610
A4ED2620
A4ED2630
A4ED2640

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C SET UNIFORM STRESS FOR ZERO OVERLAP
DO 360 K = 1, KMAX
TAUAVG(1,K) = 1.
STRGTH(1,K) = 0.
360 ICRTND(1,K) = ICRTND(2,K)
HENCE NEED FOR DL(2) TO BE SMALL ENOUGH TO BE LESS THAN THAT AT WHICH
1 NCRTND CHANGES
END OF COMPUTATIONS. START PRINTING OUT OF TABULATED RESULTS
PRINT OUT AVERAGE STRESS HEADING
WRITE (6,370)
370 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS (ELAS
ITIC-PLASTIC ANALYSIS)/,
2 39X, 31HNON-DIMENSIONALIZED FORMULATION/)
IF (GAMMAR.NE. 0.) GO TO 390
WRITE (6,380)
380 FORMAT (1H0, 42X, 23HPURELY ELASTIC ADHESIVE)
GO TO 410
390 WRITE (6,400) GAMMAR
400 FFORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO
1 = , F5.2)
410 IF (CTHERM(1).NE. 0.) GO TO 430
WRITE (6,420)
420 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)
GO TO 450
430 WRITE (6,440) THERMC(1), THERMC(2)
440 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,
1 17H FOR TENSION, = , F6.3, 16H FOR COMPRESSION)
450 WRITE (6,460) (ETR(K), K = 1, KMAX)
460 FORMAT( 1H0, 67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,
1 72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS , 1 = SOFT ET EA
2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,
3 8H0 SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
4 7H L/T/, 7H RATIO, F7.1, 9F10.1/, 1H )
C WRITE OUT TABULATIONS OF AVERAGE BOND STRESSES
DO 480 J = 1, JMAX
WRITE (6,470) OL(J), ((TAUAVG(J,K), ICRTND(J,K))), K = 1, KMAX)
470 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, 11, 1X))
480 CONTINUE
C PRINT OUT JOINT STRENGTH HEADING
WRITE (6,490)
490 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS (ELAS
ITIC-PLASTIC ANALYSIS)/,
2 39X, 31HNON-DIMENSIONALIZED FORMULATION/)
IF (GAMMAR.NE. 0.) GO TO 510
WRITE (6,500)
500 FORMAT (1H0, 42X, 23HPURELY ELASTIC ADHESIVE)
GO TO 530
510 WRITE (6,520) GAMMAR
520 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO
1 = , F5.2)
530 IF (CTHERM(1).NE. 0.) GO TO 550
WRITE (6,540)
540 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)
GO TO 570
550 WRITE (6,560) THERMC(1), THERMC(2)
560 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,
1 17H FOR TENSION, = , F6.3, 16H FOR COMPRESSION)
570 WRITE (6,580) (ETR(K), K = 1, KMAX)
580 FORMAT( 1H0, 67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,
1 72HNON-DIMENSIONALIZED JOINT STRENGTH , 1 = SOFT ET E
2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/,
3 8H0 SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
4 7H L/T/, 7H RATIO, F7.1, 9F10.1/, 1H )
C WRITE OUT TABULATIONS OF JOINT STRENGTHS
DO 600 J = 1, JMAX
WRITE (6,590) OL(J), ((STRGTH(J,K), ICRTND(J,K))), K = 1, KMAX)
590 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, 11, 1X))
600 CONTINUE
C WRITE OUT TRANSITIONAL JOINT STRENGTHS
WRITE (6,610) (TRANSL(K), K = 1, KMAX)
610 FORMAT (8H0 TRANSL, 1X, 10(F7.4, 3X))
620 CONTINUE
C
WRITE (6,630)
630 FORMAT (1H1, 18H PROGRAM COMPLETED)
STOP
END

```

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A4ED2650
A4ED2660
A4ED2670
A4ED2680
A4ED2690
A4ED2700
A4ED2710
A4ED2720
A4ED2730
A4ED2740
A4ED2750
A4ED2760
A4ED2770
A4ED2780
A4ED2790
A4ED2800
A4ED2810
A4ED2820
A4ED2830
A4ED2840
A4ED2850
A4ED2860
A4ED2870
A4ED2880
A4ED2890
A4ED2900
A4ED2910
A4ED2920
A4ED2930
A4ED2940
A4ED2950
A4ED2960
A4ED2970
A4ED2980
A4ED2990
A4ED3000
A4ED3010
A4ED3020
A4ED3030
A4ED3040
A4ED3050
A4ED3060
A4ED3070
A4ED3080
A4ED3090
A4ED3100
A4ED3110
A4ED3120
A4ED3130
A4ED3140
A4ED3150
A4ED3160
A4ED3170
A4ED3180
A4ED3190
A4ED3200
A4ED3210
A4ED3220
A4ED3230
A4ED3240
A4ED3250
A4ED3260
A4ED3270
A4ED3280
A4ED3290
A4ED3300
A4ED3310
A4ED3320
A4ED3330
A4ED3340
A4ED3350
A4ED3360
A4ED3370
A4ED3380
A4ED3390
A4ED3400
A4ED3410
A4ED3420
A4ED3430
A4ED3440
A4ED3450

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ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH , 0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT FT END CRITICAL  
2 = STIFF FT END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.50	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.20	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
1.50	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.70	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000
2.00	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
2.50	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000
3.00	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
4.00	3.6044	3.9980	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
5.00	3.9361	4.7495	4.7374	4.9984	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
6.00	4.1817	4.7235	5.2105	5.6279	5.9985	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
8.00	4.5565	5.3297	6.0557	6.7756	7.3222	7.9859	8.0000	8.0000	8.0000	8.0000	8.0000
10.00	4.8582	5.8486	6.7955	7.6917	8.5245	9.2490	9.9987	10.0000	10.0000	10.0000	10.0000
12.00	5.1248	6.3263	7.4952	8.5945	9.5526	10.6237	11.4915	12.0000	12.0000	12.0000	12.0000
15.00	5.4976	7.0078	8.4752	9.9527	11.2764	12.5802	13.7802	14.7760	15.0000	15.0000	15.0000
17.00	5.7213	7.4394	9.1166	10.7510	12.3334	13.8491	15.2483	16.5094	17.0000	17.0000	17.0000
20.00	6.0556	8.0732	10.0626	12.0045	13.9765	14.8093	16.0690	17.1924	18.0000	18.0000	18.0000
25.00	6.5942	9.1229	11.6144	14.0647	16.4470	17.7951	18.6371	19.2839	20.0000	20.0000	20.0000
30.00	7.1195	10.1731	13.1484	16.0647	19.0133	21.0943	21.6371	22.8391	24.0000	24.0000	24.0000
35.00	7.6363	11.1731	14.6723	18.1044	21.5662	24.0034	24.6371	25.8391	28.0000	28.0000	28.0000
40.00	8.1490	12.1881	16.1907	20.1535	24.0707	27.0324	27.7201	28.8159	32.0000	32.0000	32.0000
45.00	8.6574	13.1993	17.7044	22.1696	26.5896	30.0948	30.2474	30.4220	36.0000	36.0000	36.0000
50.00	9.1630	14.2075	19.2150	24.1825	29.1048	33.0927	33.7691	33.4570	40.0000	40.0000	40.0000
TRANSL	3.1576	3.5895	4.1142	4.7761	5.6533	6.8990	8.8633	12.5777	23.1106	5.0000	

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS , 0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT FT END CRITICAL  
2 = STIFF FT END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
3.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.00	0.90110	0.99959	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.00	0.69695	0.86990	0.86442	0.93799	0.99975	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
6.00	0.54956	0.66521	0.75496	0.84070	0.91527	0.99924	1.00000	1.00000	1.00000	1.00000	1.00000
8.00	0.48592	0.58486	0.67955	0.76917	0.85245	0.92690	0.99987	1.00000	1.00000	1.00000	1.00000
10.00	0.42707	0.52719	0.62395	0.71654	0.80439	0.88572	0.95580	1.00000	1.00000	1.00000	1.00000
12.00	0.36604	0.46685	0.56502	0.66018	0.75176	0.83868	0.91863	0.98507	1.00000	1.00000	1.00000
15.00	0.33655	0.43755	0.53627	0.63241	0.72549	0.81455	0.89914	0.97114	1.00000	1.00000	1.00000
17.00	0.30278	0.40391	0.50313	0.60023	0.69483	0.78625	0.87321	0.95737	1.00000	1.00000	1.00000
20.00	0.26377	0.36491	0.46458	0.56259	0.65368	0.75237	0.84274	0.92773	1.00000	1.00000	1.00000
25.00	0.23732	0.33841	0.43829	0.53681	0.63378	0.72992	0.82124	0.90046	1.00000	1.00000	1.00000
30.00	0.21820	0.31929	0.41822	0.51807	0.61561	0.71153	0.80528	0.89563	1.00000	1.00000	1.00000
35.00	0.20372	0.30470	0.40322	0.50304	0.60177	0.69931	0.79300	0.88484	1.00000	1.00000	1.00000
40.00	0.19239	0.29332	0.39343	0.49266	0.59089	0.68788	0.78328	0.87620	1.00000	1.00000	1.00000
45.00	0.18326	0.28415	0.38430	0.48365	0.58210	0.67945	0.77533	0.86914	1.00000	1.00000	1.00000
50.00											

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH  
0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT FT END CRITICAL  
2 = STIFF FT END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.50	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.20	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
1.50	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.70	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000
2.00	1.9971	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
2.50	2.4966	2.4969	2.4966	2.4971	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000
3.00	2.9952	2.9958	2.9956	2.9968	2.9972	2.9972	2.9972	2.9972	2.9972	2.9972
4.00	3.9922	3.9927	3.9939	3.9978	3.9978	3.9978	3.9978	3.9978	3.9978	3.9978
5.00	4.9877	4.9877	4.9877	4.9877	4.9877	4.9877	4.9877	4.9877	4.9877	4.9877
6.00	5.9803	5.9803	5.9803	5.9803	5.9803	5.9803	5.9803	5.9803	5.9803	5.9803
8.00	7.9658	7.9658	7.9658	7.9658	7.9658	7.9658	7.9658	7.9658	7.9658	7.9658
10.00	9.9484	9.9484	9.9484	9.9484	9.9484	9.9484	9.9484	9.9484	9.9484	9.9484
12.00	11.9281	11.9281	11.9281	11.9281	11.9281	11.9281	11.9281	11.9281	11.9281	11.9281
15.00	15.8957	15.8957	15.8957	15.8957	15.8957	15.8957	15.8957	15.8957	15.8957	15.8957
17.00	19.8498	19.8498	19.8498	19.8498	19.8498	19.8498	19.8498	19.8498	19.8498	19.8498
20.00	23.7904	23.7904	23.7904	23.7904	23.7904	23.7904	23.7904	23.7904	23.7904	23.7904
25.00	29.7229	29.7229	29.7229	29.7229	29.7229	29.7229	29.7229	29.7229	29.7229	29.7229
30.00	35.6466	35.6466	35.6466	35.6466	35.6466	35.6466	35.6466	35.6466	35.6466	35.6466
35.00	41.5617	41.5617	41.5617	41.5617	41.5617	41.5617	41.5617	41.5617	41.5617	41.5617
40.00	47.4682	47.4682	47.4682	47.4682	47.4682	47.4682	47.4682	47.4682	47.4682	47.4682
45.00	53.3661	53.3661	53.3661	53.3661	53.3661	53.3661	53.3661	53.3661	53.3661	53.3661
50.00	59.2554	59.2554	59.2554	59.2554	59.2554	59.2554	59.2554	59.2554	59.2554	59.2554
TRANSL	1.9354	2.0895	2.2570	2.4427	2.6533	2.8990	3.1967	3.5777	4.1107	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS  
0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT FT END CRITICAL  
2 = STIFF FT END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.00	0.99855	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.50	0.99866	0.99377	0.99378	0.99884	0.99884	0.99884	0.99884	0.99884	0.99884	0.99884
3.00	0.99077	0.98177	0.98177	0.98177	0.98177	0.98177	0.98177	0.98177	0.98177	0.98177
4.00	0.98070	0.97162	0.97162	0.97162	0.97162	0.97162	0.97162	0.97162	0.97162	0.97162
5.00	0.96900	0.96091	0.96091	0.96091	0.96091	0.96091	0.96091	0.96091	0.96091	0.96091
6.00	0.95484	0.94860	0.94860	0.94860	0.94860	0.94860	0.94860	0.94860	0.94860	0.94860
8.00	0.93504	0.93148	0.93148	0.93148	0.93148	0.93148	0.93148	0.93148	0.93148	0.93148
10.00	0.90443	0.90160	0.90160	0.90160	0.90160	0.90160	0.90160	0.90160	0.90160	0.90160
12.00	0.87051	0.86959	0.86959	0.86959	0.86959	0.86959	0.86959	0.86959	0.86959	0.86959
15.00	0.82650	0.82756	0.82756	0.82756	0.82756	0.82756	0.82756	0.82756	0.82756	0.82756
17.00	0.78246	0.78249	0.78249	0.78249	0.78249	0.78249	0.78249	0.78249	0.78249	0.78249
20.00	0.73831	0.73831	0.73831	0.73831	0.73831	0.73831	0.73831	0.73831	0.73831	0.73831
25.00	0.68189	0.68189	0.68189	0.68189	0.68189	0.68189	0.68189	0.68189	0.68189	0.68189
30.00	0.61815	0.61815	0.61815	0.61815	0.61815	0.61815	0.61815	0.61815	0.61815	0.61815
35.00	0.54825	0.54825	0.54825	0.54825	0.54825	0.54825	0.54825	0.54825	0.54825	0.54825
40.00	0.46480	0.46480	0.46480	0.46480	0.46480	0.46480	0.46480	0.46480	0.46480	0.46480
45.00	0.36815	0.36815	0.36815	0.36815	0.36815	0.36815	0.36815	0.36815	0.36815	0.36815
50.00	0.26198	0.26198	0.26198	0.26198	0.26198	0.26198	0.26198	0.26198	0.26198	0.26198

### A.3 Computer Program A4EE For Elastic-Plastic Strength of Bonded Scarf Joints

This FORTRAN IV digital computer program provides for the precise series solution for the average shear stress on bonded scarf joints with small scarf angles. It accounts for adherend stiffness and thermal imbalance as well as adhesive plasticity. The governing analysis is presented in Sections 3 and 4. This program A4EE will not handle perfectly elastic adhesives for which the program A4EC was developed. Severe convergence difficulties were encountered in the development of the numerical program. This contributed to the omission of a solution for the adherend and adhesive shear stress distributions. Whether or not the adherend allowable stresses are exceeded can be determined simply by evaluating the ratio of the adhesive peak shear stress to the adherend allowable direct stress. If this ratio exceeds the tangent of the scarf angle, the scarf angle is too small and the tip will either break off or be yielded depending on the nature of the adherend material.

The input data required to operate program A4EE is as follows.

CARD 1:

FORMAT (515)

IMAX = Number of thermal mismatch coefficients. IMAX .LE. 20.

JMAX = Number of non-dimensionalized overlaps. JMAX .LE. 40.

(Note that this is one more than the number of overlaps to be read in. The limiting case of  $OL(1)=0$  is set by the program.)

KMAX = Number of adherend stiffness imbalances. KMAX .LE. 10.

(Note that this controls the number of answers printed across the page and cannot be increased indefinitely for a single pass through the program.)

LMAX = Number of plastic-to-elastic adhesive shear strain ratios.

LMAX .LE. 20.

NMAX = Number of terms in power series. 10 .LE. NMAX .LE. 50.

(Note NMAX = 20 is recommended.)

CARDS 2, 2A, 2B, etc.:

FORMAT (12F6.2)

OL(J) = Non-dimensionalized overlaps. Number restricted to 40 by dimension statement. (Note that OL(J) must be read in in ascending order and that OL(2), which is the first entry on card 2, must not exceed 0.5 because of internal computations. OL(1) = 0 is set by the program as a limiting case.) Values of OL(J) exceeding 50 are impractically large.

CARDS 3, 3A, 3B, etc.:

FORMAT (10F5.2)

ETR(K) = Adherend stiffness ratios  $(E_1 t_1)/(E_2 t_2)$ .

Number of values restricted to 10 by dimension statement.

(Subscripts 1 and 2 must be identified such that  $0 < \text{ETR}(K) \leq 1$ .

Array should be read in in ascending or descending order.)

CARDS 4, 4A, 4B, etc.:

FORMAT (10F7.3)

CTHERM(I) = Adherend thermal mismatch coefficients in non-dimensionalized form. Number restricted to 20 by dimension statement. (Note that equal and opposite values must be read in consecutively to account for the difference between tensile and compressive application of the shear load. Values up to  $\pm 5$  are sufficient for the available range of adhesives and adherends. Greater values are usually associated with failure of the joint under residual thermal stresses alone.)

CARDS 5, 5A, 5B, etc.:

FORMAT (14F5.2)

GPOVGE(L) = Ratio of adhesive plastic-to-elastic strain ratios. Number of entries restricted to 20 by dimension statement. (Value of zero, for elastic case, is rejected by program A4EE to prevent breakdown of the computational sequence, but accepted by A4ED.)

A complete listing and sample outputs follow. The output tables come in pairs with the ratio of the average to maximum adhesive shear stress ( $\tau_{av}/\tau_p$ ) and the non-dimensionalized joint strength ( $\tau_{av}/\tau_p$ )( $\lambda\ell$ ) as functions of the adherend extensional stiffness ratio  $E_{TR} = E_1t_1/E_2t_2 \leq 1$  horizontally and the non-dimensionalized joint overlap  $\lambda\ell = \sqrt{\frac{G}{\eta} \left( \frac{1}{E_1t_1} + \frac{1}{E_2t_2} \right)} \ell^2$  vertically. Each table is prepared for a single value of thermal mismatch coefficient

$$C_{THERM} = \frac{(\alpha_2 - \alpha_1)\Delta T\lambda}{\tau_p \left( \frac{1}{E_1t_1} + \frac{1}{E_2t_2} \right)}$$

and equal and opposite values are treated in turn to

cover both tensile and compressive shear loadings. Each table is prepared for a single value of the plastic-to-elastic adhesive shear strain ratio  $\gamma_p/\gamma_e$ . The quantity **TRANSL** listed at the foot of each column of the non-dimensionalized strength table defines the transitional overlap at which the adhesive behavior changes from fully-plastic to elastic-plastic.

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CDECK A4EE
ELASTIC-PLASTIC ANALYSIS OF UNBALANCED SCARF JOINTS
PRECISE SOLUTION, NOT LOWER BOUND
NON-DIMENSIONALIZED AVERAGE SHEAR STRESSES COMPUTED
NON-DIMENSIONALIZED JOINT STRENGTHS COMPUTED
RANGE OF ADHESIVE DUCTILITIES INCLUDED
RANGES OF ADHEREND STIFFNESS AND THERMAL IMBALANCES ACCOUNTED FOR
DATA PRESENTATION FOR TENSILE SHEAR LOADING
CHANGE SIGN OF CTERM TO USE FOR COMPRESSIVE SHEAR LOADS
SET CTERM .EQ. 0. AND REPLACE ADHEREND ET'S WITH GT'S FOR IN-PLANE
1 (EDGEWISE) SHEAR LOADING
DIMENSION OL(J), ETR(K), CTERM(I), GPOVGE(L), TRATIO(J,NCRTND),
1 A(N), TAUAVG(J,K), STRGTH(J,K), ICRTND(J,K), THERMC(NCRTND),
2 VR(NCRTND), VU(NCRTND), VL(NCRTND), OLTRNT(NCRTND),
3 OLTRNC(NCRTND), TRANSL(K), TAUEND(N1), SAVOTP(M), TAVOTP(M),
4 BOVERL(N1)
DIMENSION OL(40), ETR(10), CTERM(20), GPOVGE(20), A(50),
1 TRATIO(40,2), TAUAVG(40,10), STRGTH(40,10), ICRTND(40,10),
2 THERMC(2), VR(2), VU(2), VL(2), OLTRNT(2), OLTRNC(2), TRANSL(10),
3 TAUEND(50), SAVOTP(50), TAVOTP(50), BOVERL(50)
C
C READ IN INPUT DATA
C READ IN ARRAY SIZES
C READ (5,10) IMAX, JMAX, KMAX, LMAX, NMAX
10 FORMAT (5I5)
M IMAX .LE. 20, JMAX .LE. 40, KMAX .LE. 10, LMAX .LE. 20,
C 1 NMAX .LE. 50 .AND. .GE. 10.
C READ IN NON-DIMENSIONALIZED OVERLAP APRAY
C OL(J) = 0.
C OL(J) MUST BE IN ASCENDING ORDER
C JL(2) MUST BE LESS THAN 0.2 FOR IDENTIFICATION OF CRITICAL END
1 OF JOINT OF ZERO OVERLAP (LIMITING CASE)
C OL(J) .LT. 100. FOR COMPATIBILITY WITH FORMAT STATEMENTS 550 & 640
C READ (5,20) (OL(J), J = 2, JMAX)
C NOTE JMAX ONE MORE THAN INPUT VALUES ON CARD(S)
20 FORMAT (12F6.2)
C READ IN STIFFNESS IMBALANCE ARRAY
C IDENTIFY ADHERENDS SUCH THAT ETR(K) = (ET)1/(ET)2 .LE. 1.
C STIFFNESS RATIOS SHOULD BE IN ASCENDING OR DESCENDING ORDER
C ETR(K) SHOULD INCLUDE VALUE 1. BUT MUST EXCLUDE VALUE 0.
C READ (5,30) (ETR(K), K = 1, KMAX)
30 FORMAT (10F5.2)
C READ IN NON-DIMENSIONALIZED THERMAL MISMATCH COEFFICIENTS
C CTERM .PROPNL. (ALPHA(2)-ALPHA(1))*(OPERATING TEMP. - CURE TEMP.)
C NEED CTERM(I) ARRAY TO CONTAIN BOTH POSITIVE AND NEGATIVE VALUFS
1 TO COVER BOTH TENSILE AND COMPRESSIVE LOADS
C READ (5,40) (CTERM(I), I = 1, IMAX)
40 FORMAT (10F7.3)
C READ IN PLASTIC-TO-ELASTIC STRAIN RATIO ARRAY
C GPOVGE(L) MUST BE .GT. 0. FOR ELASTIC-PLASTIC ANALYSIS
C PURELY ELASTIC SOLUTION OBTAINED FROM SEPARATE PROCEDURE
C READ (5,50) (GPOVGE(L), L = 1, LMAX)
50 FORMAT (14F5.2)
C
C PRINT OUT INPUT DATA
C WRITE (6,60) IMAX, JMAX, KMAX, LMAX, NMAX
60 FORMAT (1H1, 9H IMAX = ,I2, 9H JMAX = ,I2, 9H KMAX = ,I2,
1 9H LMAX = ,I2, 9H NMAX = ,I2)
C WRITE (6,70)
70 FORMAT (10H OVERLAPS)
C WRITE (6,80) (OL(J), J = 1, JMAX)
80 FORMAT (12F6.2)
C WRITE (6,90)
90 FORMAT (22H STIFFNESS IMBALANCES)
C WRITE (6,100) (ETR(K), K = 1, KMAX)
100 FORMAT (14F5.2)
C WRITE (6,110)
110 FORMAT (20H THERMAL MISMATCHES)
C WRITE (6,120) (CTERM(I), I = 1, IMAX)
120 FORMAT (10F7.3)
C WRITE (6,130)
130 FORMAT (34H PLASTIC TO ELASTIC STRAIN RATIOS)
C WRITE (6,140) (GPOVGE(L), L = 1, LMAX)
140 FORMAT (14F5.1)
C
C STORE CONSTANTS
ANMAX = NMAX
A(1) = 1.
TAUEND(1) = 1.
C
C START COMPUTATIONAL DO LOOPS
DO 670 L = 1, LMAX
GAMMAR = GPOVGE(L)
C ENSURE EXCLUSION OF NEGATIVE PLASTICITY IN ADHESIVE (ERROR IN DATA)
C EXCLUDE PURELY-ELASTIC ADHESIVE. SFPARATE PROGRAM NEEDED
IF (GAMMAR .LE. 0.) GO TO 670
DO 670 I = 1, IMAX

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A4EE0010
A4EE0020
A4EE0030
A4EE0040
A4EE0050
A4EE0060
A4EE0070
A4EE0080
A4EE0090
A4EE0100
A4EE0110
A4EE0120
A4EE0130
A4EE0140
A4EE0150
A4EE0160
A4EE0170
A4EE0180
A4EE0190
A4EE0200
A4EE0210
A4EE0220
A4EE0230
A4EE0240
A4EE0250
A4EE0260
A4EE0270
A4EE0280
A4EE0290
A4EE0300
A4EE0310
A4EE0320
A4EE0330
A4EE0340
A4EE0350
A4EE0360
A4EE0370
A4EE0380
A4EE0390
A4EE0400
A4EE0410
A4EE0420
A4EE0430
A4EE0440
A4EE0450
A4EE0460
A4EE0470
A4EE0480
A4EE0490
A4EE0500
A4EE0510
A4EE0520
A4EE0530
A4EE0540
A4EE0550
A4EE0560
A4EE0570
A4EE0580
A4EE0590
A4EE0600
A4EE0610
A4EE0620
A4EE0630
A4EE0640
A4EE0650
A4EE0660
A4EE0670
A4EE0680
A4EE0690
A4EE0700
A4EE0710
A4EE0720
A4EE0730
A4EE0740
A4EE0750
A4EE0760
A4EE0770
A4EE0780
A4EE0790
A4EE0800
A4EE0810
A4EE0820
A4EE0830
A4EE0840
A4EE0850
A4EE0860
A4EE0870
A4EE0880

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THERMC(1) = CHERM(I)
THERMC(2) = - THERMC(1)
DO 460 K = 1, KMAX
  VR(1) = ETR(K)
  VR(2) = 1. / VR(1)
  VU(1) = 1. - VR(1)
  VL(1) = 1. + VR(1)
  VU(2) = 1. - VR(2)
  VL(2) = 1. + VR(2)
C ESTABLISH TRANSITIONAL OVERLAPS FROM FULLY-PLASTIC TO ELASTIC-PLASTIC
C 1 BEHAVIOUR AS REFERENCE LENGTH FOR START OF ITERATIONS
C SPECIAL PROCEDURE FOR LESS THAN COMPLETELY UNBALANCED JOINTS
  IF ( (THERMC(1) .EQ. 0.) .AND. (VR(1) .EQ. 1.) ) GO TO 150
  IF (THERMC(1) .EQ. 0.) GO TO 160
  IF (VR(1) .EQ. 1.) GO TO 170
C IF NONE OF THESE, JOINT CONTAINS BOTH IMBALANCES
  GO TO 190
C SET INFINITE TRANSITIONAL OVERLAP FOR IDENTICAL ADHERENDS
  150 OLTRNT(1) = 1000000.
  OLTRNT(2) = 1000000.
C OLTRNC(1) = 1000000.
  OLTRNC(2) = 1000000.
  GO TO 190
C SET TRANSITIONAL OVERLAPS FOR STIFFNESS IMBALANCE ONLY
C IN THE ABSENCE OF THERMAL MISMATCH, SAME END IS CRITICAL FOR BOTH
C 1 TENSILE SHEAR AND COMPRESSIVE SHEAR LOADING
  160 IF (VU(1) .GT. 0.) OLTRNT(1) = SQRT(GAMMAR*VL(1)/VU(1))
  IF (VU(1) .LE. 0.) OLTRNT(1) = 1000000.
  IF (VU(2) .GT. 0.) OLTRNT(2) = SQRT(GAMMAR*VL(2)/VU(2))
  IF (VU(2) .LE. 0.) OLTRNT(2) = 1000000.
C OLTRNC(1) = SQRT(GAMMAR*VL(1)/VU(1))
  OLTRNC(1) = 1000000.
  OLTRNC(2) = SQRT(GAMMAR*VL(2)/VU(2))
  OLTRNC(2) = 1000000.
  GO TO 190
C SET TRANSITIONAL OVERLAPS FOR THERMAL MISMATCH ONLY
  170 IF (THERMC(1) .LT. 0.) OLTRNT(1) = -GAMMAR/THERMC(1)
  IF (THERMC(1) .GE. 0.) OLTRNT(1) = 1000000.
  IF (THERMC(2) .LT. 0.) OLTRNT(2) = -GAMMAR/THERMC(2)
  IF (THERMC(2) .GE. 0.) OLTRNT(2) = 1000000.
C OLTRNC(1) = GAMMAR/THERMC(1)
  OLTRNC(1) = 1000000.
  OLTRNC(2) = GAMMAR/THERMC(2)
  OLTRNC(2) = 1000000.
  GO TO 190
C 180 CONTINUE
C STANDARD PROCEDURE FOR COMPLETELY UNBALANCED JOINTS
  V1 = THERMC(1) * VL(1) / (2. * VU(1))
  V2 = THERMC(2) * VL(2) / (2. * VU(2))
  V3 = V1*V1 + GAMMAR*VL(1)/VU(1)
  V4 = V2*V2 + GAMMAR*VL(2)/VU(2)
C ESTABLISH TRANSITIONAL OVERLAPS BELOW WHICH JOINT IS FULLY PLASTIC
C NEXT FOUR STATEMENTS APPLY FOR TENSILE SHEAR LOADING
  IF (V3 .GE. 0.) OLTRNT(1) = V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V3 .LT. 0.) .OR. (OLTRNT(1) .LE. 0.) ) OLTRNT(1) = 1000000.
  IF (V4 .GE. 0.) OLTRNT(2) = V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V4 .LT. 0.) .OR. (OLTRNT(2) .LE. 0.) ) OLTRNT(2) = 1000000.
C IF ICRTND .EQ. 2 FOR SHORT OVERLAPS, OLTRNT(1) WILL BE COMPUTED VERY
C 1 LARGE, AND VICE VERSA
C THIS IS PHYSICALLY REALISTIC AND DOES NOT LEAD TO IMPOSSIBLE COMPUTING
C IF BOTH V3 AND V4 ARE POSITIVE, EITHER OLTRNT(1) OR OLTRNT(2) WILL BE
C 1 COMPUTED NEGATIVE. NEED TO PREVENT COMPUTATIONS BASED ON THIS
C 2 UNREAL SITUATION. HENCE CHECKS ABOVE AND BELOW
C NEXT FOUR STATEMENTS WOULD APPLY FOR COMPRESSIVE SHEAR LOADING
  IF (V3 .GE. 0.) OLTRNC(1) = -V1 + SQRT(V3)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V3 .LT. 0.) .OR. (OLTRNC(1) .LE. 0.) ) OLTRNC(1) = 1000000.
  IF (V4 .GE. 0.) OLTRNC(2) = -V2 + SQRT(V4)
C IF NOT, OTHER END OF JOINT CRITICAL
C OTHER END OF JOINT IDENTIFIED AS CRITICAL BY SHEAR STRAIN GRADIENT
C SET INFINITE TRANSITIONAL OVERLAP TO ACCOUNT FOR THIS
  IF ( (V4 .LT. 0.) .OR. (OLTRNC(2) .LE. 0.) ) OLTRNC(2) = 1000000.
C 190 DO 390 NCRTND = 1, 2
  THERM = THERMC(NCRTND)
  VRREF = VR(NCRTND)
  VUREF = VU(NCRTND)
  VLREF = VL(NCRTND)
C SET UNIFORM STRESS FOR SHORT OVERLAPS

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A4EE0890
A4EE0900
A4EE0910
A4EE0920
A4EE0930
A4EE0940
A4EE0950
A4EE0960
A4EE0970
A4EE0980
A4EE0990
A4EE1000
A4EE1010
A4EE1020
A4EE1030
A4EE1040
A4EE1050
A4EE1060
A4EE1070
A4EE1080
A4EE1090
A4EE1100
A4EE1110
A4EE1120
A4EE1130
A4EE1140
A4EE1150
A4EE1160
A4EE1170
A4EE1180
A4EE1190
A4EE1200
A4EE1210
A4EE1220
A4EE1230
A4EE1240
A4EE1250
A4EE1260
A4EE1270
A4EE1280
A4EE1290
A4EE1300
A4EE1310
A4EE1320
A4EE1330
A4EE1340
A4EE1350
A4EE1360
A4EE1370
A4EE1380
A4EE1390
A4EE1400
A4EE1410
A4EE1420
A4EE1430
A4EE1440
A4EE1450
A4EE1460
A4EE1470
A4EE1480
A4EE1490
A4EE1500
A4EE1510
A4EE1520
A4EE1530
A4EE1540
A4EE1550
A4EE1560
A4EE1570
A4EE1580
A4EE1590
A4EE1600
A4EE1610
A4EE1620
A4EE1630
A4EE1640
A4EE1650
A4EE1660
A4EE1670
A4EE1680
A4EE1690
A4EE1700
A4EE1710
A4EE1720
A4EE1730
A4EE1740
A4EE1750
A4EE1760

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DO 200 J = 2, JMAX
  JSAVE = J
  IF (OL(J) .GT. OLRNT(NCRTND)) GO TO 210
C IF NOT, JOINT IS FULLY PLASTIC
200 TRATIO(J,NCRTND) = 1.
  IF (JSAVE .EQ. JMAX) GO TO 380
C
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
210 DO 380 J = JSAVE, JMAX
  OLAP = OL(J)
  OLAP2 = OLAP * OLAP
C COMPUTE ADVERL FOR MINIMUM VALUE OF TAVOTP BY ITERATION
C SET INITIAL ESTIMATE OF EXTENT OF PLASTIC ZONE FROM TRANSITIONAL OLAP
  ADVERL = OLRNT(NCRTND) / OLAP
  N2MAX = 2 * NMAX
  DO 220 N = 1, N2MAX
  ARMDR = 1. - ADVERL
  ADVERL = -ARMDR * ALOG(ARMDR) + (GAMMAR / ((VUREF/VLREF)*OLAP2
  1 - THERM * OLAP))
  IF (ADVERL .GT. 0.9999) ADVERL = 0.9999
  IF (ADVERL .LT. 0.001) ADVERL = 0.001
220 CONTINUE
  TRATIO(J,NCRTND) = 1.
  IF (ADVERL .EQ. 0.9999) GO TO 230
C COMPUTE CORRESPONDING AVERAGE SHEAR STRESS
  TAUREF = 1. - (VLREF * GAMMAR / OLAP2 + (VLREF * THERM / OLAP - VUREF) *
  1 ADVERL) / ALOG(1. - ADVERL)
  TRATIO(J,NCRTND) = TAUREF
230 CONTINUE
  AREF = ADVERL * 0.999
C THE FACTOR IS TO PREVENT DIVERGENCE IN THE SERIES COEFFICIENTS
C SET MINIMUM POSSIBLE VALUE OF ADVERL, AT WHICH TAVOTP .EQ. 1.
  AMIN = GAMMAR / ((VUREF/VLREF)*OLAP2 - THERM*OLAP)
C TRUE EXTENT OF FIRST PLASTIC ZONE BOUNDED WITHIN AMIN AND AREF
  ADEL = (AREF - AMIN) / (ANMAX - 1.)
C MINIMUM VALUES OF TAVOTP ARE NOW COMPUTED. THESE APPROXIMATE THE TRUE
C 1 SOLUTIONS FOR ALL BUT SHORT OVERLAPS OR THICK ADHERENDS
C 2 IN CONJUNCTION WITH SEVERE ADHEREND MISMATCH AND/OR BRITTLE
C 3 ADHESIVES. REFINE ANSWER BY PRECISE SOLUTION IN POWER SERIES
C COMPUTE JOINT STRENGTH FOR ELASTIC-PLASTIC ADHESIVE BEHAVIOUR
  DO 360 M = 1, NMAX
  AM = M
  ADVERL = AREF - (AM - 1.) * ADEL
  ARMDR = 1. - ADVERL
C COMPUTE ASSOCIATED AVERAGE BOND STRESS
  TAVOTP(M) = 1. - (VLREF * GAMMAR / OLAP2 + (VLREF * THERM / OLAP - VUREF)
  1 * ADVERL) / ALOG(ARMDR)
C
C START COMPUTING ELASTIC STRESS SERIES
C 1
C 2 ESTABLISH A(2) AT START OF ELASTIC ZONE FROM CONTINUITY OF SHEAR
C 1 STRAINS IN ADHESIVE AT TRANSITION. THIS ENSURES ADHEREND STRESS
C 2 CONTINUITY
  A(2) = THERM * OLAP - OLAP2 * (VUREF - (1. - TAVOTP(M)) / ARMDR) / VLREF
C A(2) SHOULD BE .LT. 0. FOR ADVERL .LT. AREF
C A(2) SHOULD BE .EQ. 0. FOR ADVERL .EQ. AREF
C A(2) SHOULD BE .GT. 0. FOR ADVERL .GT. AREF
  A(3) = (A(2) - THERM * OLAP + OLAP2 * VUREF / VLREF) / (2. * ARMDR)
C CONVERT STRESS TERMS INTO AVERAGE STRESS TERMS BY DIVIDING BY N
  A(2) = A(2) / 2.
  A(3) = A(3) / 3.
C COMPUTE SUBSEQUENT TERMS FROM RECURRENCE FORMULA
  DO 240 N = 4, NMAX
  NSAVE = N
  AN = N
  A(N) = ((2. * ADVERL - 1.) * (AN - 2.) * (AN - 1.) * A(N - 1) +
  1 (AN - 3.) * (AN - 2.) * A(N - 2) + (OLAP2 / VLREF) *
  2 ((ADVERL * VUREF + VRREF) * A(N - 2) + VUREF * A(N - 3))) /
  3 (ADVERL * ARMDR * (AN - 1.) * AN)
  IF (ABS(A(N)) .LT. 1.E50) GO TO 240
C IF NOT, OVERFLOW IS IMMINENT, SO CUT DOWN ON NMAX
  GO TO 250
240 CONTINUE
  GO TO 270
250 DO 260 N = NSAVE, NMAX
260 A(N) = 0.
C ESTIMATE ELASTIC ADHESIVE STRESS AT OTHER END OF JOINT, OR IDENTIFY
C 1 EXISTENCE OF SECOND PLASTIC ADHESIVE ZONE, AS APPROPRIATE
C START BY ASSUMING NO SECOND PLASTIC ADHESIVE ZONE
270 COVERL = 0.
  TAUEND(1) = 1.
  TAUEND(NMAX) = 1.
  DO 280 N = 2, NMAX
  AN = N
280 TAUEND(NMAX) = TAUEND(NMAX) + A(N) * (ARMDR ** (N - 1)) * AN
  IF (TAUEND(NMAX) .LE. 1.) GO TO 310
C IF SO, ONLY THE ONE PLASTIC ZONE, AT THE NCRTND REFERENCE END
C IF NOT, HAVE IDENTIFIED EXISTENCE OF SECOND PLASTIC ZONE, AT OTHER ENDA

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C
C PROCEDURE FOR SECOND PLASTIC ZONE
C USE LINEAR INTERPOLATION TO ESTIMATE COVERL (EXTENT OF SECOND PLASTIC
C 1 ZONE)
  DELBOL = (1. - ADOVERL) / (ANMAX - 1.)
  BOVERL(1) = 1. - ADOVERL
  DO 300 N1 = 2, NMAX
    BOVERL(N1) = BOVERL(N1-1) + DELBOL
    TAUEND(N1) = 1.
  DO 290 N = 2, NMAX
    AN = N
    V = BOVERL(N1)**(N-1)
  290 TAUEND(N1) = TAUEND(N1) + A(N)*V*AN
C CHECK FOR CONVERGENCE OF COVERL
  IF ((1.0001.GT.TAUEND(N1)) .AND. (0.9999.LT.TAUEND(N1))) GO TO 310
C IF NOT, ITERATE ON COVERL
C COMPUTE VALUE OF COVERL NEEDED TO RESTRICT STRESSES TO ELASTIC LEVEL
  IF (TAUEND(N1) .LT. 1.) GO TO 300
C IF SO, ESTIMATE OF BOVERL IS INSUFFICIENT AND THAT OF COVERL EXCESSIVE
C IF NOT, CORRECT TRANSITION LIES BETWEEN N1 AND N1-1 LOCATIONS
  COVERL = 1. - ADOVERL - BOVERL(N1-1) -
  1 ((1. - TAUEND(N1-1)) / (TAUEND(N1) - TAUEND(N1-1)))
  GO TO 310
  300 CONTINUE
C DEFER CHECK ON WHETHER COVERL IS SO LARGE THAT CRITICAL END OF JOINT
C 1 IS AT OTHER END UNTIL AFTER CONVERGENCE OF ADOVERL IS ESTABLISHED
C
  310 SAVOTP(M) = 1.
  BOVL = 1. - ADOVERL - COVERL
C EVALUATE AVERAGE STRESS IN TERMS OF SERIES COEFFICIENTS
  DO 320 N = 2, NMAX
    AN = N
  320 SAVOTP(M) = SAVOTP(M) + A(N)*(BOVL**(N))
C
C CHECK ON CONVERGENCE OF ADOVERL
  IF (SAVOTP(M) .GT. 1.) GO TO 360
C IF SO, CANNOT HAVE CONVERGED YET
  IF ( (SAVOTP(M) .LT. TAVOTP(M)) .AND. (M .EQ. 1) ) GO TO 330
C IF SO, SOLUTION IS NUMERICALLY INDISTINGUISHIBLE FROM THE LOWER BOUND
C NEED BOTH M .EQ. 1 VALUES AND M .EQ. 2 VALUES FOR FIRST CHECK
  IF (M .EQ. 1) GO TO 360
C PROTECT AGAINST DIVISION BY ZERO
  IF ( (TAVOTP(M) .EQ. 0.) .AND. (SAVOTP(M) .EQ. 0.) ) GO TO 340
C IF SO, CONVERGENCE ESTABLISHED
  IF ( (SAVOTP(M) .LT. 0.00001) .AND. (SAVOTP(M) .GT. -0.00001) )
  1 RATIO = 1. + TAVOTP(M)
  IF ( (TAVOTP(M) .LT. 0.00001) .AND. (TAVOTP(M) .GT. -0.00001) )
  1 RATIO = 1. + SAVOTP(M)
C IF NONE OF THE ABOVE, NO FURTHER FAILURE CASES LEFT TO CHECK FOR
  RATIO = SAVOTP(M) / TAVOTP(M)
C CHECK ON CONVERGENCE OF JOINT STRENGTH PREDICTIONS
  IF ( (1.0001.GT.RATIO) .AND. (0.9999.LT.RATIO) ) GO TO 350
C IF SO, CONVERGENCE IS ESTABLISHED
C IF NOT, NEED TO RE-ESTIMATE ADOVERL
C USE LINEAR INTERPOLATION TO ESTIMATE ADOVERL (EXTENT OF FIRST PLASTIC
C 1 ZONE)
  IF (SAVOTP(M) .GT. TAVOTP(M)) GO TO 360
C IF SO, CONVERGENCE OF ADOVERL NOT YET ESTABLISHED
C IF NOT, CORRECT VALUE OF ADOVERL LIES BETWEEN M AND M-1 LOCATIONS
  TRATIO(J,NCRTND) = TAVOTP(M-1) + (TAVOTP(M) - TAVOTP(M-1)) *
  1 (1. - SAVOTP(M-1) / SAVOTP(M-1)) /
  2 (1. - (SAVOTP(M) - TAVOTP(M) + TAVOTP(M-1)) / SAVOTP(M-1))
  GO TO 370
  330 TRATIO(J,NCRTND) = TAUREF
  GO TO 370
  340 TRATIO(J,NCRTND) = 0.
  GO TO 370
  350 TRATIO(J,NCRTND) = TAVOTP(M)
  GO TO 370
  360 CONTINUE
C IF REFINEMENT HAS NOT CONVERGED, USE LOWER BOUND ESTIMATE
C TRATIO(J,NCRTND) = TAVOTP(1), AS SET EARLIER
C PROTECT AGAINST ACCUMULATED NUMERICAL ERRORS
C USE LOWER BOUND SOLUTION IF REFINEMENT RESULTS IN STILL LOWER VALUES
  370 IF (TRATIO(J,NCRTND) .LT. TAVOTP(1)) TRATIO(J,NCRTND) = TAVOTP(1)
  380 CONTINUE
C
C CONVERGENCE OF ADOVERL ESTABLISHED. RECORD AVERAGE SHEAR STRESS
C VALUES COMPUTED ARE NOW STORED IN TRATIO(J,NCRTND)
C NEED TO SELECT LOWER VALUE TO IDENTIFY CRITICAL END OF JOINT
  DO 450 J = 2, JMAX
    OLAP = OL(J)
    TAU1 = TRATIO(J,1)
    TAU2 = TRATIO(J,2)
    IF ( (TAU1 .LT. 1.) .OR. (TAU2 .LT. 1.) ) GO TO 390
C IF SO, JOINT IS NOT FULLY PLASTIC
C IF NOT, IDENTIFY CRITICAL END OF JOINT FROM SHEAR STRAIN GRADIENT
  GRADNT = THERMC(1) - OLAP*VU(1)/VL(1)

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A4EE2650
A4EE2660
A4EE2670
A4EE2680
A4EE2690
A4EE2700
A4EE2710
A4EE2720
A4EE2730
A4EE2740
A4EE2750
A4EE2760
A4EE2770
A4EE2780
A4EE2790
A4EE2800
A4EE2810
A4EE2820
A4EE2830
A4EE2840
A4EE2850
A4EE2860
A4EE2870
A4EE2880
A4EE2890
A4EE2900
A4EE2910
A4EE2920
A4EE2930
A4EE2940
A4EE2950
A4EE2960
A4EE2970
A4EE2980
A4EE2990
A4EE3000
A4EE3010
A4EE3020
A4EE3030
A4EE3040
A4EE3050
A4EE3060
A4EE3070
A4EE3080
A4EE3090
A4EE3100
A4EE3110
A4EE3120
A4EE3130
A4EE3140
A4EE3150
A4EE3160
A4EE3170
A4EE3180
A4EE3190
A4EE3200
A4EE3210
A4EE3220
A4EE3230
A4EE3240
A4EE3250
A4EE3260
A4EE3270
A4EE3280
A4EE3290
A4EE3300
A4EE3310
A4EE3320
A4EE3330
A4EE3340
A4EE3350
A4EE3360
A4EE3370
A4EE3380
A4EE3390
A4EE3400
A4EE3410
A4EE3420
A4EE3430
A4EE3440
A4EE3450
A4EE3460
A4EE3470
A4EE3480
A4EE3490
A4EE3500
A4EE3510
A4EE3520

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IF (GRADNT .LT. 0.) ICRTND(J,K) = 1
IF (GRADNT .EQ. 0.) ICRTND(J,K) = 0
IF (GRADNT .GT. 0.) ICRTND(J,K) = 2
TAUAVG(J,K) = 1.
STRGTH(J,K) = OLAP
MCRTND = ICRTND(J,K)
IF (MCRTND .EQ. 0) MCRTND = 1
TRANSL(K) = OLTRNT(MCRTND)
GO TO 450
390 DIFFNC = TAU1 - TAU2
C IF DIFFNC .LT. 0., MCRTND .EQ. 1
C IF DIFFNC .EQ. 0., MCRTND .EQ. 0
C IF DIFFNC .GT. 0., MCRTND .EQ. 2
IF (DIFFNC) 400, 410, 420
C ADHEREND (1) END OF JOINT CRITICAL
400 TAUAVG(J,K) = TAU1
STRGTH(J,K) = TAU1 * OLAP
ICRTND(J,K) = 1
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
GO TO 430
C BOTH ENDS OF JOINT EQUALLY CRITICAL FROM NULLIFYING (OR ZERO)
C 1 ADHEREND IMBALANCES
410 TAUAVG(J,K) = TAU1
STRGTH(J,K) = TAU1 * OLAP
ICRTND(J,K) = 0
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
IF (J .EQ. 2) TRANSL(K) = OLTRNT(1)
GO TO 430
C ADHEREND (2) END OF JOINT CRITICAL
420 TAUAVG(J,K) = TAU2
STRGTH(J,K) = TAU2 * OLAP
ICRTND(J,K) = 2
C COVER SITUATION WHERE TRANSITIONAL LENGTH IS LESS THAN OL(2)
IF (J .EQ. 2) TRANSL(K) = OLTRNT(2)
C COVER CASES OF ZERO OR NEGATIVE ESTIMATED STRENGTHS
430 IF (TAUAVG(J,K) .GT. 0.) GO TO 440
C IF NOT, JOINT HAS BROKEN DUE TO THERMAL STRESSES WITHOUT EXTERNAL LOAD
TAUAVG(J,K) = 0.
STRGTH(J,K) = 0.
GO TO 450
440 IF (TAUAVG(J,K) .LE. 1.) GO TO 450
C IF NOT, THERE HAS BEEN A COMPUTATIONAL MISTAKE
C PRINT ASTERISKS TO IDENTIFY ERROR
C RERUN WITH GREATER VALUE OF NMAX
TAUAVG(J,K) = 100.
STRGTH(J,K) = 1000.
450 CONTINUE
460 CONTINUE
C
C SET UNIFORM STRESS FOR ZERO OVERLAP
DO 470 K = 1, KMAX
TAUAVG(1,K) = 1.
STRGTH(1,K) = 0.
470 ICRTND(1,K) = ICRTND(2,K)
C HENCE NEED FOR OL(2) TO BE SMALL ENOUGH TO BE LESS THAN THAT AT WHICH
C 1 MCRTND CHANGES
C
C END OF COMPUTATIONS. START PRINTING OUT OF TABULATED RESULTS
C
C PRINT OUT AVERAGE STRESS HEADING
WRITE (6,490)
480 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS (ELAS
1 TIC-PLASTIC ANALYSIS)/,
2 39X, 31HNON-DIMENSIONALIZED FORMULATION/)
WRITE (6,490) GAMMAR
490 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO
1 = , F5.2)
IF (CTHERM(1) .NE. 0.) GO TO 510
WRITE (6,500)
500 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT)
GO TO 530
510 WRITE (6,520) THERMC(1), THERMC(2)
520 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3,
1 17H FOR TENSION, = , F6.3, 16H FOR COMPRESSION)
530 WRITE (6,540) (ETR(K), K = 1, KMAX)
540 FORMAT( 1H0, 67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X,
1 72HAVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS , 1 = SOFT ET
2ND CRITICAL/, 69X, 25H2 = STIFF ET END CRITICAL/,
3 4HO SCALED, 31X, 39HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/,
4 7H L/T/, 7H RATIO, F7.1, 9F10.1/, 1H )
C WRITE OUT TABULATIONS OF AVERAGE BOND STRESSES
DO 560 J = 1, JMAX
WRITE (6,550) OL(J), ((TAUAVG(J,K), ICRTND(J,K)), K = 1, KMAX)
550 FORMAT (1H , F6.2, 2X, 10(F7.5, 1X, 11, 1X))
560 CONTINUE
C
C PRINT OUT JOINT STRENGTH HEADING

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A4EE3530
A4EE3540
A4EE3550
A4EE3560
A4EE3570
A4EE3580
A4EE3590
A4EE3600
A4EE3610
A4EE3620
A4EE3630
A4EE3640
A4EE3650
A4EE3660
A4EE3670
A4EE3680
A4EE3690
A4EE3700
A4EE3710
A4EE3720
A4EE3730
A4EE3740
A4EE3750
A4EE3760
A4EE3770
A4EE3780
A4EE3790
A4EE3800
A4EE3810
A4EE3820
A4EE3830
A4EE3840
A4EE3850
A4EE3860
A4EE3870
A4EE3880
A4EE3890
A4EE3900
A4EE3910
A4EE3920
A4EE3930
A4EE3940
A4EE3950
A4EE3960
A4EE3970
A4EE3980
A4EE3990
A4EE4000
A4EE4010
A4EE4020
A4EE4030
A4EE4040
A4EE4050
A4EE4060
A4EE4070
A4EE4080
A4EE4090
A4EE4100
A4EE4110
A4EE4120
A4EE4130
A4EE4140
A4EE4150
A4EE4160
A4EE4170
A4EE4180
A4EE4190
A4EE4200
A4EE4210
A4EE4220
A4EE4230
A4EE4240
A4EE4250
A4EE4260
A4EE4270
A4EE4280
A4EE4290
A4EE4300
A4EE4310
A4EE4320
A4EE4330
A4EE4340
A4EE4350
A4EE4360
A4EE4370
A4EE4380
A4EE4390
A4EE4400

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WRITE (6,570)
570 FORMAT (1H1/, 5(1H0/), 27X, 56HADHESIVE-BONDED SCARF JOINTS (ELAS A4EE4410
1 TIC-PLASTIC ANALYSIS)/, A4EE4420
2 30X, 31HNON-DIMENSIONALIZED FORMULATION/) A4EE4430
WRITE (6,580) GAMMAR A4EE4440
580 FORMAT(1H0, 27X, 49HPLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO A4EE4450
1 = F5.2) A4EE4460
IF (CTHERM(1) .NE. 0.) GO TO 600 A4EE4470
WRITE (6,590) A4EE4480
590 FORMAT (1H , 37X, 33HZERO THERMAL MISMATCH COEFFICIENT) A4EE4490
GO TO 620 A4EE4500
600 WRITE (6,610) THERMC(1), THERMC(2) A4EE4510
610 FORMAT (1H , 16X, 31HTHERMAL MISMATCH COEFFICIENT = , F6.3, A4EE4520
1 17H FOR TENSION, = , F6.3, 16H FOR COMPRESSION) A4EE4530
620 WRITE (6,630) (ETR(K), K = 1, KMAX) A4EE4540
630 FORMAT( 1H0, 67X, 30H0 = BOTH ENDS EQUALLY CRITICAL/, 20X, A4EE4550
1 72HNON-DIMENSIONALIZED JOINT STRENGTH, 1 = SOFT ET EA4EE4560
2ND CRITICAL/, 68X, 25H2 = STIFF ET END CRITICAL/, A4EE4570
3 9H0 SCALED, 31X, 30HEXTENSIONAL STIFFNESS (THICKNESS) RATIO/, A4EE4580
4 7H L/T/, 7H RATIO, F7.1, 9F10.1/, 1H ) A4EE4590
C WRITE OUT TABULATIONS OF JOINT STRENGTHS A4EE4600
DO 650 J = 1, JMAX A4EE4610
WRITE (6,640) DL(J), ((STPGTH(J,K), ICRTND(J,K)), K = 1, KMAX) A4EE4620
640 FORMAT (1H , F6.2, 2X, 10(F7.4, 1X, 11, 1X)) A4EE4630
650 CONTINUE A4EE4640
C WRITE OUT TRANSITIONAL JOINT STRENGTHS A4EE4650
WRITE (6,660) (TRANSL(K), K = 1, KMAX) A4EE4660
660 FORMAT (8H0 TRANSL, 1X, 10(F7.4, 3X)) A4EE4670
670 CONTINUE A4EE4680
C A4EE4690
WRITE (6,680) A4EE4700
680 FORMAT (1H1, 18H PROGRAM COMPLETED) A4EE4710
STOP A4EE4720
END A4EE4730
A4EE4740

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ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH ,  
0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.50	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.20	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
1.50	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.70	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000
2.00	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
2.50	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000
3.00	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
4.00	3.6484	3.9998	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
5.00	4.0026	4.3926	4.7267	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
6.00	4.2598	4.7826	5.2518	5.6524	5.9999	6.0000	6.0000	6.0000	6.0000	6.0000
8.00	4.5876	5.3763	6.1055	6.7624	7.3451	7.9402	8.0000	8.0000	8.0000	8.0000
10.00	4.8582	5.8512	6.8081	7.7193	8.5517	9.2849	9.9997	10.0000	10.0000	10.0000
12.00	5.1248	6.3263	7.4862	8.6009	9.6666	10.6464	11.4904	12.0000	12.0000	12.0000
15.00	5.4906	7.0028	8.4752	9.9027	11.2764	12.5839	13.7902	14.7790	15.0000	15.0000
17.00	5.7213	7.4384	9.1166	10.7510	12.3334	13.8491	15.2743	16.5133	17.0000	17.0000
20.00	6.0557	8.0782	10.0626	12.0046	13.8965	15.7251	17.4641	19.0517	20.0000	20.0000
25.00	6.5943	9.1229	11.6144	14.0647	16.4670	18.8645	21.6371	27.2339	25.0000	25.0000
30.00	7.1200	10.1525	13.1488	16.1044	19.0134	21.8645	24.6371	31.3471	30.0000	30.0000
35.00	7.6382	11.1736	14.6729	18.1335	21.5462	24.9034	28.1948	35.3935	35.0000	35.0000
40.00	8.1518	12.1893	16.1911	20.1535	24.0706	27.9324	31.7201	39.4290	40.0000	40.0000
45.00	8.6644	13.2016	17.7052	22.1698	26.5896	30.9547	35.2474	43.4290	45.0000	45.0000
50.00	9.1709	14.2113	19.2164	24.1828	29.1047	33.9726	38.7690	47.4290	50.0000	50.0000
TRANSL	3.1576	3.5895	4.1142	4.7761	5.6533	6.8990	8.8633	12.5777	23.1106	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = 1.000 FOR TENSION, = -1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS ,  
0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
3.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
4.00	0.91211	0.99994	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
5.00	0.80052	0.87925	0.94534	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
6.00	0.70936	0.79709	0.87531	0.94206	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000
8.00	0.57345	0.67203	0.76319	0.84500	0.91814	0.99253	1.00000	1.00000	1.00000	1.00000
10.00	0.44582	0.54512	0.64081	0.71193	0.85517	0.92849	0.99997	1.00000	1.00000	1.00000
12.00	0.36604	0.52719	0.62385	0.71675	0.80555	0.88720	0.95753	1.00000	1.00000	1.00000
15.00	0.33655	0.46685	0.56502	0.66018	0.75176	0.83893	0.91935	0.98527	1.00000	1.00000
17.00	0.33655	0.43755	0.53627	0.63241	0.72549	0.81465	0.89949	0.97137	1.00000	1.00000
20.00	0.30278	0.40391	0.50313	0.60023	0.69483	0.78625	0.87321	0.95259	1.00000	1.00000
25.00	0.26377	0.36492	0.46458	0.56259	0.65868	0.75237	0.84276	0.92771	1.00000	1.00000
30.00	0.23733	0.33842	0.43829	0.53681	0.63378	0.72882	0.82124	0.90946	1.00000	1.00000
35.00	0.21823	0.31925	0.41923	0.51807	0.61561	0.71153	0.80528	0.89563	1.00000	1.00000
40.00	0.20380	0.30473	0.40478	0.50384	0.60177	0.69831	0.79300	0.88484	1.00000	1.00000
45.00	0.19250	0.29337	0.39345	0.49266	0.59088	0.68783	0.78328	0.87620	1.00000	1.00000
50.00	0.18342	0.28423	0.38433	0.48366	0.58209	0.67945	0.77538	0.86914	1.00000	1.00000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

NON-DIMENSIONALIZED JOINT STRENGTH , 0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.20	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000	0.2000
0.50	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.20	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000	1.2000
1.50	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000	1.5000
1.70	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000	1.7000
2.00	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
2.50	2.4992	2.4996	2.4999	2.4999	2.5000	2.5000	2.5000	2.5000	2.5000	2.5000
3.00	2.3553	2.5329	2.9991	2.9995	2.9999	3.0000	3.0000	3.0000	3.0000	3.0000
4.00	2.5137	2.7894	3.0434	3.2754	3.4845	3.8805	3.9996	3.9999	4.0000	4.0000
5.00	2.6328	3.0062	3.3544	3.6794	3.9806	4.2563	4.5041	4.6992	4.9998	5.0000
6.00	2.6965	3.1912	3.6503	4.0727	4.4663	4.8326	5.1691	5.4712	5.9999	5.9999
8.00	2.8421	3.5216	4.1808	4.8180	5.4241	5.9866	6.5054	6.9831	7.4109	7.9253
10.00	3.0443	3.9160	4.7601	5.5762	6.3663	7.1293	7.8520	8.5154	9.1169	9.6345
12.00	3.2462	4.3151	5.3556	6.3665	7.5553	8.8282	9.1993	10.0606	10.8436	11.5248
15.00	3.5475	4.9134	6.2500	7.5553	9.3202	10.0633	11.2543	12.3923	13.4594	14.3952
17.00	3.7478	5.3123	6.8469	8.3202	9.5437	11.2501	12.6351	13.9624	15.2136	16.3252
20.00	4.0481	5.9108	7.7433	9.5437	11.3090	13.0345	14.7123	16.3232	17.8594	19.2379
25.00	4.5482	6.9090	9.2397	11.5357	13.7965	16.0158	18.1851	20.2891	22.2970	24.1246
30.00	5.0482	7.9076	10.7355	13.5301	16.2877	19.0028	21.6660	24.2608	26.7525	29.0380
35.00	5.5482	8.9059	12.2311	15.5259	18.7812	21.9932	25.1519	28.2338	31.2190	33.9591
40.00	6.0482	9.9053	13.7313	17.5227	21.2763	24.9858	28.6411	32.2236	35.6929	38.9126
45.00	6.5482	10.9053	15.2299	19.5203	23.7723	27.9799	32.1325	36.2107	40.1718	43.8654
50.00	7.0494	11.9051	16.7289	21.5183	26.2692	30.9752	35.6255	40.2001	44.6546	48.8249
TRANS	1.9354	2.0895	2.2570	2.4427	2.6533	2.8990	3.1967	3.5777	4.1107	5.0000

ADHESIVE-BONDED SCARF JOINTS (ELASTIC-PLASTIC ANALYSIS)  
NON-DIMENSIONALIZED FORMULATION

PLASTIC TO ELASTIC ADHESIVE SHEAR STRAIN RATIO = 5.0  
THERMAL MISMATCH COEFFICIENT = -1.000 FOR TENSION, = 1.000 FOR COMPRESSION

AVERAGE SHEAR STRESS / MAXIMUM SHEAR STRESS , 0 = BOTH ENDS EQUALLY CRITICAL  
1 = SOFT ET END CRITICAL  
2 = STIFF ET END CRITICAL

SCALED L/T RATIO	EXTENSIONAL STIFFNESS (THICKNESS) RATIO									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
0.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.20	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.70	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.00	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2.50	0.99967	0.99984	0.99996	0.99998	0.99999	1.00000	1.00000	1.00000	1.00000	1.00000
3.00	0.78511	0.84430	0.99968	0.99984	0.99995	0.99999	0.99999	0.99998	1.00000	1.00000
4.00	0.62842	0.69735	0.76086	0.81985	0.87112	0.91702	0.95990	0.99998	0.99996	1.00000
5.00	0.52655	0.60124	0.67088	0.73587	0.79611	0.85127	0.90083	0.94187	0.99988	0.99998
6.00	0.44942	0.53186	0.60838	0.68225	0.74438	0.80544	0.86152	0.91187	0.96336	0.99966
8.00	0.35526	0.44920	0.52260	0.59762	0.67491	0.74833	0.81317	0.87289	0.92636	0.96345
10.00	0.30443	0.39160	0.47601	0.55762	0.63663	0.71293	0.78521	0.85154	0.91169	0.96040
12.00	0.27051	0.35959	0.44630	0.53054	0.61213	0.69083	0.76661	0.83838	0.90363	0.95968
15.00	0.23650	0.32756	0.41666	0.50372	0.58955	0.67388	0.75528	0.82615	0.89729	0.96031
17.00	0.22046	0.31249	0.40276	0.49119	0.57761	0.66177	0.74324	0.81646	0.89297	0.96189
20.00	0.20241	0.29554	0.38716	0.47718	0.56545	0.65172	0.73562	0.81646	0.89188	0.96498
25.00	0.18193	0.27636	0.36959	0.46143	0.55186	0.64063	0.72740	0.80869	0.89175	0.96793
30.00	0.16827	0.26359	0.35785	0.45100	0.54292	0.63363	0.72240	0.80869	0.89197	0.97055
35.00	0.15852	0.25447	0.34952	0.44360	0.53661	0.62838	0.71863	0.80869	0.89232	0.97282
40.00	0.15121	0.24764	0.34328	0.43707	0.53191	0.62464	0.71403	0.80559	0.89271	0.97479
45.00	0.14553	0.24234	0.33844	0.43378	0.52827	0.62178	0.71145	0.80468	0.89271	0.97479
50.00	0.14099	0.23810	0.33458	0.43037	0.52538	0.61950	0.71251	0.80400	0.89309	0.97650

#### A.4 Computer Program A4EF For Elastic Strength of Stepped-Lap Bonded Joints

The analysis in Section 5 has been prepared as the FORTRAN IV digital computer program A4EF. The program computes the elastic joint strength of any stepped-lap bonded joint and prints out the most critical adherend and adhesive stresses for each step of the joint. In order to obtain a more complete internal stress distribution, each step can be subdivided and a series of shorter steps input instead. The input data is printed out to supplement the solution output. Eccentricities are excluded from the joint and a symmetric two-sided bonded joint is analyzed in which the thicknesses of the two outer adherends are lumped together in evaluating the joint strengths. The reason for this is the greater utilization of the back-to-back stepped-lap joint than of the single-sided joint. A single-sided joint can be analyzed with this program in one of two ways. One can add a mirror image of the actual joint and halve the strength predicted for this joint of twice the actual thickness and twice the bond area or one can change certain factors of 2, identified in the listing, to 1 for single-sided joints. The program accounts for arbitrary combinations of adherend stiffness and thermal imbalances as well as non-uniform step thickness increments and step lengths. It has been used successfully in optimizing the joint proportions in order to maximize the joint strength.

A complete listing of the program A4EF follows after the input and output have been described.

CARD 1:

FORMAT (I2)

M = Number of configurations (each requiring a complete set of data)  
to be solved.

CARDS 2, 2A:

FORMAT (8F10.3)

TAUMAX =  $\tau_p$  = Peak adhesive shear stress.

G = Elastic adhesive shear modulus.

GAMMAX =  $\gamma_e + \gamma_p$  = Maximum adhesive shear strain. (This may be set less than  $\gamma_e$  to cover partial loads.)

GAMMAE =  $\gamma_e$  = Elastic adhesive shear strain.

ETA =  $\eta$  = Bond line thickness.

ALPHAO =  $\alpha_o$  = Coefficient of thermal expansion of outer adherend.

ALPHAI =  $\alpha_i$  = Coefficient of thermal expansion of inner adherend.

DELTMP =  $\Delta T = T_{\text{operating}} - T_{\text{stress-free}} \approx T_{\text{operating}} - T_{\text{cure}}$   
= Temperature differential.

SGNLD = +1 for tensile shear load, and  
= -1 for compressive shear load.

ANSTEP = Number of steps in the joint. This serves to control the number of adherend property cards read in.

CARDS 3, 3A, 3B, ..etc.., 3(N = ANSTEP + 1)

FORMAT (7F10.3)

THICKO(N) = Sum of thicknesses of outer adherends for nth step.

THICKI(N) = Thickness of nth step of inner adherend.

STEPL(N) = Length of nth step.

ETOTR(N) = Net extensional stiffness of outer adherends at nth step.

ETINR(N) = Extensional stiffness of inner adherend at nth step.

STROTR(N) = Net strength of outer adherends at nth step.

STRINR(N) = Strength of inner adherend at nth step.

The output is in tabular form with one row devoted to each step or step portion. Those entries not defined in the input description above are: TAU the adhesive shear stress, GAMMA the adhesive shear strain, DELTAO the displacement of the outer adherends, DELTAI the displacement of the inner adherend, with TOUTER and TINNER being the loads ( $\sigma_t$ ) in the outer and inner adherends, respectively.

The more accurate solution is obtained by starting the iterative solution from the more critically loaded end. Therefore, in those cases in which the a priori identification of the more critical end is not possible, the program outputs solutions from each end, and the second one is to be preferred. Such cases have been run and the computational procedure in double precision has been shown to be sufficiently accurate from either end. The need for this higher precision on IBM computers arises from the precision loss throughout the nested do loops in the iteration sequence. The greater number of significant digits employed by CDC machines has been found to obviate the need for this and the program can be modified to single-precision operation on CDC machines in a straightforward manner.



```

CDECK A4FF
C STEPPED-LAP ADHESIVE-BONDED-JOINTS
C PERFECTLY-ELASTIC SOLUTIONS
C JOINT ANALYSIS PROGRAM
C SOLUTION EXAMINES ADHESIVE SHEAR STRESS AND ADHEREND NORMAL (AXIAL)
C 1 STRESS BUT OMITTS CONSIDERATION OF ADHESIVE PEEL STRESS ON THE
C 2 GROUND THAT OUTER END STEP IS USUALLY SUFFICIENTLY THIN FOR
C 3 PEEL STRESS PROBLEMS NOT TO ARISE
C NOTE THAT CONVERGENCE PROBLEM IS ACUTE FOR STEPPED-LAP JOINTS, EVEN
C 1 WITH DOUBLE-PRECISION. STEPS TAKEN HERE TO CONSTRAIN TENDENCY
C 2 TO DIVERGE (BY FREEZING SOLUTION ONE STEP AT A TIME) HAVE BEEN
C 3 ADOPTED AFTER TRYING BOTH MORE AND LESS STRINGENT TECHNIQUES
C NOTE ALSO THAT CONVERGENCE DIFFICULTIES ARE PROBLEM DEPENDENT, BEING
C 1 MORE SEVERE FOR BRITTLE (HIGH MODULUS) ADHESIVES. LOW MODULUS
C 2 ADHESIVES PROVED AMENABLE TO A CONVERGENT SOLUTION IN ONLY A
C 3 SINGLE PASS STRAIGHT THROUGH THE JOINT FROM END TO END, IN
C 4 SINGLE-PRECISION, WITH ONLY A SMALL LOSS OF ACCURACY IN LATER
C 5 STEPS.
C THE UNDEPLYING DIFFICULTY IS ONE OF NUMERICAL ACCURACY LOSS IN THE
C 1 PRESENCE OF EXTREMELY HIGH ADHESIVE SHEAR STRESS GRADIENTS AT
C 2 BOTH ENDS OF EACH OF THE OUTER STEPS.
C NOTE THAT PROGRAM CANNOT HANDLE PROPERLY A JOINT WITH SUCH HIGH
C 1 RESIDUAL THERMAL STRESSES THAT IT BREAKS APART PRIOR TO
C 2 APPLICATION OF MECHANICAL LOADS. ANSWER FROM ONE END OF JOINT
C 4 WILL BE ZERO, BUT FROM OTHER END WILL BE LARGE AND POSITIVE.
C 5 ACTUALLY, THE LATTER ANSWER IS FOR A LOAD OF REVERSED SIGN, WITH
C 6 THE THERMAL STRESSES HELPING RATHER THAN HINDERING. SUCH A
C 7 SITUATION CAN BE SPOTTED IF THE SHEAR STRESS IN THE FORMER
C 8 SOLUTION IS NEGATIVE AT THE START OF THE JOINT
C DIMENSION OUTER(50), TINNER(50), GAMMA(50), TAU(50), DELTAO(50),
C 1 DELTAI(50), STEPL(50), THICKO(50), THICKI(50), ETOTR(50),
C 2 ETINR(50), STOTR(50), STRINR(50), STEP(50), THCKND(50),
C 3 THCKNI(50), FTOTR(50), FTINR(50), STRGTR(50), STRGNR(50)
C DOUBLE PRECISION TOUTER, TINNER, GAMMA, TAU, DELTAO, DELTAI,
C 1 TMAX, TMIN, TLOAD, A, B, C, D, E, F, ALAMDA, DELT, DELDT,
C 2 C1, C2, C3, C4, C5, V, STEP, THCKND, THCKNI, FTOUTR, FTINR,
C 3 STRGTR, STRGNR, TCHECK, STEPL, THICKO, THICKI, ETOTR, ETINR,
C 4 STOTR, STRINR, TAUUPR, TAUWLR
C READ (5,10) M
C 10 FORMAT (I2)
C M.FQ. NUMBER OF JOINT CONFIGURATIONS TO BE SOLVED
C READ IN MATERIAL PROPERTIES
C DO 390 MOUNT = 1, M
C NRVRS = 0
C JFLAG = 1
C JFLAG IDENTIFIES END OF JOINT FROM WHICH ANALYSIS COMMENCES
C READ (5,20) TAUWLR, G, GAMMAX, GAMMAF, ETA, ALPHAO, ALPHAI,
C 1 DELTMP, SGNLD, ANSTEP
C 20 FORMAT (8F10.3)
C NSTEPS = ANSTEP
C MSTEPS = NSTEPS + 1
C READ IN JOINT GEOMETRY
C READ (5,30) ((THICKO(N), THICKI(N), STEPL(N), ETOTR(N), ETINR(N),
C 1 STOTR(N), STRINR(N)), N = 1, MSTEPS)
C 30 FORMAT (7F10.3)
C CHECK ON CONSISTENCY OF ADHESIVE DATA
C VCHECK = G * GAMMAE
C R = TAUWLR / VCHECK
C IF ((1.001 .LT. R) .OR. (0.999 .GT. R)) GO TO 390
C IF (GAMMAX .GE. GAMMAE) GO TO 40
C IF NOT, REDUCE PEAK SHEAR STRESS TO LESS THAN MAXIMUM ELASTIC VALUE
C TAUWLR = G * GAMMAX
C SET UP RECURRING CONSTANTS
C 40 C1 = G / ETA
C C2 = 2. * C1
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
C 50 C3 = ALPHAO * DELTMP
C C4 = ALPHAI * DELTMP
C C5 = C4 - C3
C PRINT OUT INPUT DATA
C WRITE (6,60) ALPHAO, ALPHAI, DELTMP
C 60 FORMAT (1H1/, 5(1H0/), 11H INPUT DATA//, 10H ALPHAO = , F10.7,
C 1 13H (PER DEG. F), 3X, 9HALPHAI = , F10.7, 13H (PER DEG. F),
C 2 3X, 9HDELTMP = , F6.1, 9H (DEG. F)//,
C 3 1H, 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 4X,
C 4 6HSTOTR, 5X, 6HSTRINR, 5X, 5HETOTR, 5X, 5HETINR//)
C DO 80 N = 1, MSTEPS
C WRITE (6,70) N, STEPL(N), THICKO(N), THICKI(N), STOTR(N),
C 1 STRINR(N), ETOTR(N), ETINR(N)
C 70 FORMAT (1H, 4X, I2, 3(1X, F6.4), 2(1X, F10.1), 2(1X, F10.1))
C 80 CONTINUE
C ESTIMATE MAXIMUM POSSIBLE BOND CAPACITY FOR FULLY-PLASTIC ADHESIVE
C PROND = TAUWLR * OLAP * 2.
C NOTE FACTOR 2. INCLUDED FOR DOUBLE-SIDED JOINT
C REDUCE TO 1. IF JOINT HAS ONLY ONE SIDE BONDED

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A4FF0010
A4FF0020
A4FF0030
A4FF0040
A4FF0050
A4FF0060
A4FF0070
A4FF0080
A4FF0090
A4FF0100
A4FF0110
A4FF0120
A4FF0130
A4FF0140
A4FF0150
A4FF0160
A4FF0170
A4FF0180
A4FF0190
A4FF0200
A4FF0210
A4FF0220
A4FF0230
A4FF0240
A4FF0250
A4FF0260
A4FF0270
A4FF0280
A4FF0290
A4FF0300
A4FF0310
A4FF0320
A4FF0330
A4FF0340
A4FF0350
A4FF0360
A4FF0370
A4FF0380
A4FF0390
A4FF0400
A4FF0410
A4FF0420
A4FF0430
A4FF0440
A4FF0450
A4FF0460
A4FF0470
A4FF0480
A4FF0490
A4FF0500
A4FF0510
A4FF0520
A4FF0530
A4FF0540
A4FF0550
A4FF0560
A4FF0570
A4FF0580
A4FF0590
A4FF0600
A4FF0610
A4FF0620
A4FF0630
A4FF0640
A4FF0650
A4FF0660
A4FF0670
A4FF0680
A4FF0690
A4FF0700
A4FF0710
A4FF0720
A4FF0730
A4FF0740
A4FF0750
A4FF0760
A4FF0770
A4FF0780
A4FF0790
A4FF0800
A4FF0810
A4FF0820
A4FF0830
A4FF0840
A4FF0850
A4FF0860
A4FF0870
A4FF0880

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C PERFECTLY-ELASTIC BOND CAPACITY WILL BE CLOSER TO ASYMPTOTE OF SCARE 84FF0900
C 1 JOINT SOLUTION AND IS SIGNIFICANTLY LOWER THAN PLASTIC ESTIMATE 84FF0900
C ACTUAL LOAD CAPACITY MAY BE SIGNIFICANTLY LESS IF THERMAL MISMATCH 84FF0910
C 1 BETWEEN ADHERENDS IS SEVERE 84FF0920
C REDUCTION IN LOAD TO ACCOUNT FOR LIMITED ADHEREND STRENGTH IS 84FF0930
C 1 ACCOMPLISHED LATER IN PROGRAM 84FF0940
C PROVIDE OUTER LOOP TO ADJUST ADHESIVE PEAK SHEAR STRESS AT START OF 84FF0950
C 1 JOINT FOR CASES IN WHICH EITHER ADHESIVE IS MORE CRITICAL AT 84FF0960
C 2 OTHER END OF JOINT OR ADHERENDS ARE MORE CRITICAL THAN ADHESIVE. 84FF0970
C TAUUPP = 2. * TAU MAX 84FF0980
C TAUUWP = 0. 84FF0990
C NOTE THAT PROGRAM IS PREVENTED FROM HANDLING PROBLEM IN WHICH SHEAR 84FF1000
C 1 STRESS IN ADHESIVE REVERSES SIGN, WHEN COMPUTATIONS START FROM 84FF1010
C 2 THE LESS CRITICAL END. SOLUTION IS OBTAINABLE FROM OTHER END. 84FF1020
C NOTE ALSO THAT, IF THE MAXIMUM SHEAR STRESS AND APPLIED LOADS HAVE 84FF1030
C 1 OPPOSITE SIGNS, JOINT MUST BREAK APART UNDER RESIDUAL THERMAL 84FF1040
C 2 STRESS ALONE WITHOUT ANY EXTERNALLY APPLIED LOAD, SO NO CASES OF 84FF1050
C 3 REAL CONCERN ARE EXCLUDED BY THE RESTRICTION ABOVE 84FF1060
C DO 290 I = 1, 50 84FF1070
C TAU(I) = (TAUUPP + TAUUWP) / 2. 84FF1080
C IF (TAU(I) .GT. TAU MAX) TAU(I) = TAU MAX 84FF1090
C IF (I .EQ. 1) TAU(I) = TAU MAX 84FF1100
C SET INITIAL CONDITIONS 84FF1110
C GAMMA(I) = TAU(I) / G 84FF1120
C TOUTER(I) = 5. * PROND 84FF1130
C TINNER(I) = 0. 84FF1140
C DELTAQ(I) = 0. 84FF1150
C DELTAT(I) = SGNLD * GAMMA(I) * ETA 84FF1160
C TMAX = 10. * PROND 84FF1170
C TMIN = 0. 84FF1180
C TLOAD = 5. * PROND 84FF1190
C OPERATE ON THE LOAD LEVEL IN INTERMEDIATE LOOP 84FF1200
C LEAVE ADJUSTMENT OF TAU MAX FOR OUTER LOOP 84FF1210
C TCHECK = 0. 84FF1220
C DO 190 IFLAG = 1, NSTEPS 84FF1230
C SCHECK = 1000000000000000. 84FF1240
C DO 150 NCOUNT = 1, 100 84FF1250
C CONVERGENCE NEARLY ALWAYS OCCURRED BETWEEN 20 AND 30 CYCLES IN TEST 84FF1260
C 1 CASES, BUT THERE WERE SOME EXCEPTIONS 84FF1270
C INTERMEDIATE LOOP ADJUSTS LOAD LEVEL 84FF1280
C TLOAD = TOUTER(IFLAG) 84FF1290
C CHECK ON CONVERGENCE OF TOUTER(IFLAG) 84FF1300
C P = TOUTER(IFLAG) / SCHECK 84FF1310
C IF ( (1.000000001 .GT. P) .AND. (0.999999999 .LT. P) ) GO TO 160 84FF1320
C DO 100 N = IFLAG, NSTEPS 84FF1330
C INNER LOOP COMPUTES ELASTIC JOINT STRENGTH 84FF1340
C A = TAU(N) 84FF1350
C ALAMDA = DSORT(C2 * (1. / ETINR(N) + 1. / ETOTR(N))) 84FF1360
C B = (TINNER(N) / ETINR(N) - TOUTER(N) / ETOTR(N) + C5 * SGNLD) 84FF1370
C 1 * C1 / ALAMDA 84FF1380
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE 84FF1390
C C = STEPL(N) 84FF1400
C D = ALAMDA * C 84FF1410
C E = DSINH(D) 84FF1420
C F = DCOSH(D) 84FF1430
C TAU(N+1) = A * E + B * F 84FF1440
C DELT = (2. / ALAMDA) * (A * F + B * (E - 1.)) 84FF1450
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND. 84FF1460
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1. 84FF1470
C TOUTER(N+1) = TOUTER(N) - DELT 84FF1480
C TINNER(N+1) = TINNER(N) + DELT 84FF1490
C IF (N .EQ. NSTEPS) GO TO 90 84FF1500
C IF (TINNER(N+1) .LT. (-1. * TOUTER(1))) GO TO 130 84FF1510
C IF (TOUTER(N+1) .LT. (-1. * TOUTER(1))) GO TO 140 84FF1520
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL 84FF1530
C IF THE FACTOR -1 IS EITHER TOO LARGE OR TOO SMALL, CONVERGENCE FAILS 84FF1540
C 90 DELDT = (2. / (ALAMDA**2)) * (A * (E - 1.) + B * (F - D)) 84FF1550
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND. 84FF1560
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1. 84FF1570
C DELTAQ(N+1) = DELTAQ(N) + C3 * C + SGNLD * (TOUTER(N) * C - 84FF1580
C 1 DELDT) / ETOTR(N) 84FF1590
C DELTAT(N+1) = DELTAT(N) + C4 * C + SGNLD * (TINNER(N) * C + 84FF1600
C 1 DELDT) / ETINR(N) 84FF1610
C GAMMA(N+1) = TAU(N+1) / G 84FF1620
C 100 CONTINUE 84FF1630
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED 84FF1640
C R1 = TOUTER(1) / TINNER(MSTEPS) 84FF1650
C CHECK ALSO WHETHER OR NOT CONVERGENCE HAS BEEN OBTAINED 84FF1660
C R2 = TCHECK / TINNER(MSTEPS) 84FF1670
C IF ( (1.000001 .GT. R1) .AND. (0.999999 .LT. R1) .AND. 84FF1680
C 1 (1.000001 .GT. R2) .AND. (0.999999 .LT. R2) ) GO TO 200 84FF1690
C IF (TOUTER(1) .LT. TINNER(MSTEPS)) GO TO 110 84FF1700
C IF SO, LOAD ESTIMATE IS TOO LOW 84FF1710
C IF NOT, LOAD ESTIMATE IS TOO HIGH 84FF1720
C GO TO 120 84FF1730
C R1 IS UNSUITABLE FOR A CONVERGENCE CHECK BECAUSE NEGATIVE VALUES OF R1 84FF1740
C 1 REPRESENT TOO HIGH A LOAD ESTIMATE, JUST LIKE THOSE VALUES IN 84FF1750
C 2 EXCESS OF UNITY 84FF1760

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110 TMIN = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
TCHECK = TINNER(MSTEPS)
GO TO 150
120 TMAX = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
TCHECK = TINNER(MSTEPS)
GO TO 150
C NOTE THAT LABELS 26 AND 7 GOVERN FINE ADJUSTMENTS TO THE JOINT LOADS,
C 1 WHILE LABELS 27 AND 28 REPRESENT COARSE ADJUSTMENTS
130 TMAX = TOUTER(IFLAG)
SCHECK = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
GO TO 150
140 TMIN = TOUTER(IFLAG)
SCHECK = TOUTER(IFLAG)
C IF ADHEREND, RATHER THAN ADHESIVE, LIMITS JOINT STRENGTH, NEED TO
C 1 BOOST PLOAD IN PROPORTION TO TAUJMAX, EVEN IF IT MEANS EXCEEDING
C 2 ADHEREND STRENGTHS IN INTERMEDIATE COMPUTATIONS. CORRECTIONS
C 3 ARE APPLIED LATER
IF (TMIN .GT. TMAX) TMAX = 5. * TMAX
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
150 CONTINUE
IF (N .EQ. NSTEPS) GO TO 190
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C 1 BECAUSE OF NUMERICAL ACCURACY PROBLEMS. REMEDY IS TO FREEZE
C 2 EARLIER VALUES, WHICH HAVE CONVERGED AND SLIGHTLY PERTURB
C 3 INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
TMAX = TOUTER(1)
TMIN = -1. * TMAX
GO TO 190
160 ICOUNT = IFLAG + 1
IF (TOUTER(ICOUNT) .GT. 0.) GO TO 170
IF (TOUTER(ICOUNT) .LT. 0.) GO TO 180
TMAX = TOUTER(1) / 10.
TMIN = -1. * TMAX
GO TO 190
170 TMAX = 1.1 * TOUTER(ICOUNT)
TMIN = 0.9 * TOUTER(ICOUNT)
GO TO 190
180 TMIN = 0.9 * TOUTER(ICOUNT)
TMAX = 1.1 * TOUTER(ICOUNT)
C THE LIMITS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C 1 THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
190 CONTINUE
NRVPS = 1
C
200 IF (IC5 .GT. 0.000001) .OR. (IC5 .LT. -0.000001) GO TO 240
IF NOT, FIRST SOLUTION MAY BE SCALED IN THE ABSENCE OF ANY THERMAL
C 1 MISMATCH BETWEEN ADHERENDS
IF SO, SOLUTION MUST BE REFINED BY ITERATION, SINCE THERMAL STRESS
C 1 TERMS DO NOT SCALE LINEARLY, EVEN FOR ELASTIC ADHESIVE AND
C 2 ADHERENDS
APPLY SCALE FACTOR TO SOLUTION FOR ONLY ADHEREND STIFFNESS IMBALANCE
ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
C 1 WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE
PROGRAM ASSUMES ADHEREND ALLOWABLES HAVE SAME MAGNITUDE IN TENSION AS
C 1 IN COMPRESSION. DISTINCTION IS USUALLY UNIMPORTANT SINCE, IN
C 2 PRACTICAL JOINTS, RESIDUAL THERMAL STRESSES ARE UNLIKELY TO
C 3 BREAK ADHEREND(S) RATHER THAN ADHESIVE
RSCALE = TOUTER(1) / STOTP(1)
IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
RTAJMX = TAU(1) / TAUJMX
IF (RTAJMX .LT. 0.) RTAJMX = -1. * RTAJMX
DO 220 N = 2, MSTEPS
RINR = TINNER(N) / STRINR(N)
IF (RINR .LT. 0.) RINR = -1. * RINR
IF (RINR .GT. RSCALE) RSCALE = RINR
IF (N .EQ. MSTEPS) GO TO 210
ROTR = TOUTER(N) / STOTR(N)
IF (ROTR .LT. 0.) ROTR = -1. * ROTR
IF (ROTR .GT. RSCALE) RSCALE = ROTR
210 RTAU = TAU(N) / TAUJMX
IF (RTAU .LT. 0.) RTAU = -1. * RTAU
IF (RTAU .GT. RTAJMX) RTAJMX = RTAU
220 CONTINUE
RECTR = RSCALE
C RECTR IS PROPORTIONALITY CONSTANT GOVERNING ELASTIC SOLUTION
C IF RTAJMX .GT. RSCALE, ADHESIVE PLASTICITY CAN INCREASE STRENGTH
C USUALLY ADHESIVE IS CRITICAL AT ONE END OF JOINT OR OTHER, SO RTAJMX
C 1 .GT. 1, MAY WELL JUST SIGNIFY THAT FAR END OF JOINT IS CRITICAL
NOTE THAT PROGRAM ASSUMES THAT ANY INTERNAL ADHEREND STRESSES OF
C 1 REVERSED SIGN WITH RESPECT TO STRESS OUTSIDE THE JOINT ARE NOT
C 2 CRITICAL. IF THEY ARE, IT MEANS THAT THE JOINT WILL FAIL DUE
C 3 TO RESIDUAL THERMAL STRESSES ALONE WITHOUT ANY MECHANICAL LOADS
IF (RSCALE .LT. RTAJMX) RECTR = RTAJMX
DO 230 N = 1, MSTEPS

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A4FF1780
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A4FF1800
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A4FF1930
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A4FF1960
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A4FF1980
A4FF1990
A4FF2000
A4FF2010
A4FF2020
A4FF2030
A4FF2040
A4FF2050
A4FF2060
A4FF2070
A4FF2080
A4FF2090
A4FF2100
A4FF2110
A4FF2120
A4FF2130
A4FF2140
A4FF2150
A4FF2160
A4FF2170
A4FF2180
A4FF2190
A4FF2200
A4FF2210
A4FF2220
A4FF2230
A4FF2240
A4FF2250
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A4FF2270
A4FF2280
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A4FF2400
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A4FF2420
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A4FF2470
A4FF2480
A4FF2490
A4FF2500
A4FF2510
A4FF2520
A4FF2530
A4FF2540
A4FF2550
A4FF2560
A4FF2570
A4FF2580
A4FF2590
A4FF2600
A4FF2610
A4FF2620
A4FF2630
A4FF2640

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TOUTER(N) = TOUTER(N) / RECTR          A4FF2650
TINNER(N) = TINNER(N) / RECTR          A4FF2660
TAU(N) = TAU(N) / RECTR                A4FF2670
GAMMA(N) = GAMMA(N) / RECTR           A4FF2680
DELTAO(N) = DELTAO(N) / RECTR         A4FF2690
230 DELTAI(N) = DELTAI(N) / RECTR      A4FF2700
GO TO 310                               A4FF2710
C                                         A4FF2720
C USE ITERATIVE SOLUTION WHEN ADHESION THERMAL MISMATCH IS PRESENT A4FF2730
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR A4FF2740
C 1 WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE. A4FF2750
240 RSCALE = TOUTER(1) / STROTR(1)      A4FF2760
IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE A4FF2770
RTAUMX = TAU(1) / TAU MAX              A4FF2780
IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX A4FF2790
DO 260 N = 2, MSTEPS                    A4FF2800
RINR = TINNER(N) / STRINP(N-1)         A4FF2810
C NEED TO COMPARE LOAD WITH STRENGTH ON THIN SIDE OF STEP. HENCE (N-1) A4FF2820
IF (RINP .LT. 0.) RINP = -1. * RINP    A4FF2830
IF (RINP .GT. RSCALE) RSCALE = RINP    A4FF2840
IF (N .EQ. MSTEPS) GO TO 250           A4FF2850
ROTR = TOUTER(N) / STROTR(N)           A4FF2860
IF (ROTR .LT. 0.) ROTR = -1. * ROTR    A4FF2870
IF (ROTR .GT. RSCALE) RSCALE = ROTR    A4FF2880
250 RTAU = TAU(N) / TAU MAX             A4FF2890
IF (RTAU .LT. 0.) RTAU = -1. * RTAU    A4FF2900
IF (RTAU .GT. RTAUMX) RTAUMX = RTAU    A4FF2910
260 CONTINUE                            A4FF2920
C CHECK ON CONVERGENCE                  A4FF2930
P = RTAUMX                              A4FF2940
IF (RTAUMX .LT. RSCALE) P = RSCALE     A4FF2950
IF ( (1.00001 .GT. P) .AND. (0.99999 .LT. R) ) GO TO 310 A4FF2960
V = 2. * TAU MAX                        A4FF2970
P = (V + TAUUPR) / (V + TAU LWR)        A4FF2980
IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 310 A4FF2990
C IF EITHER RTAUMX OR RSCALE .GT. UNITY, TAU(1) MUST BE DECREASED A4FF3000
IF ( (RTAUMX .GT. 1.00001) .OR. (RSCALE .GT. 1.00001) ) GO TO 270 A4FF3010
C IF BOTH RTAUMX AND RSCALE ARE .LT. UNITY, TAU(1) MUST BE INCREASED A4FF3020
IF ( (RTAUMX .LT. 0.99999) .AND. (RSCALE .LT. 0.99999) ) GO TO 280 A4FF3030
C IF NONE OF THE THREE CHECKS ABOVE IS MET, SOLUTION HAS FAILED TO A4FF3040
C 1 CONVERGE WITHIN SPECIFIED NUMBER OF ITERATIONS. PRINT OUT ANSWER A4FF3050
GO TO 14                                 A4FF3060
270 IF (TAU(1) .GT. 0.) TAUUPR = TAU(1) A4FF3070
IF (TAU(1) .LT. 0.) TAU LWR = TAU(1)   A4FF3080
GO TO 290                                A4FF3090
280 IF (TAU(1) .GT. 0.) TAU LWR = TAU(1) A4FF3100
IF (TAU(1) .LT. 0.) TAUUPR = TAU(1)   A4FF3110
290 CONTINUE                            A4FF3120
C IF PROGRAM GOES BEYOND PRECEDING CONTINUE STATEMENT, SOLUTION HAS NOT A4FF3130
C 1 CONVERGED                            A4FF3140
WRITE (6,300)                            A4FF3150
300 FORMAT (1H1, 18H DIVERGENT SOLUTION) A4FF3160
C                                         A4FF3170
C PRINT OUT RESULTS OF ELASTIC COMPUTATIONS A4FF3180
310 WRITE (6,320) TOUTER(1), TAU MAX, SG NLD, DELTMP A4FF3190
320 FORMAT (1H1/, 5(1H/), A4FF3200
1 39H ELASTIC JOINT STRENGTH, PLOAD (LBS) = , F10.1/, A4FF3210
2 49H ALLOWABLE ADHESIVE SHEAR STRESS, TAU MAX (PSI) = , F8.1/, A4FF3220
3 1H, 8HSG NLD = , F4.1, 54H SG NLD = +1 FOR TENSILE SHEAR AND -1 A4FF3230
4 FOR COMPRESSIVE/, 36H TEMPERATURE DIFFERENTIAL (DEG F) = , F6.1/ A4FF3240
5, 1H, 5X, 1HN, 2X, 5HSTEPL, 1X, 6HTHICKO, 1X, 6HTHICKI, 3X, A4FF3250
6 3HTAU, 4X, 5HGAMMA, 1X, 6HDELTAO, 1X, 6HDELTAI, 5X, 6HTOUTER, A4FF3260
7 5X, 6HSTROTP, 5X, 6HTINNER, 5X, 6HSTRINP/1 A4FF3270
DO 340 N = 1, MSTEPS                    A4FF3280
WRITE (6,330) N, STEPL(N), THICKO(N), THICKI(N), TAU(N), GAMMA(N), A4FF3290
1 DELTAO(N), DELTAI(N), TOUTER(N), STROTP(N), TINNER(N), STRINP(N) A4FF3300
2) A4FF3310
330 FORMAT (1H , 4X, 12, 1X, F6.4, 1X, F6.4, 1X, F6.4, 1X, F7.1, 1X, A4FF3320
1 F6.3, 1X, F6.4, 1X, F6.4, 1X, F10.1, 1X, F10.1, 1X, F10.1, 1X, A4FF3330
2 F10.1) A4FF3340
340 CONTINUE                            A4FF3350
350 CONTINUE                            A4FF3360
C                                         A4FF3370
C RECOMPUTE SOLUTION FROM OTHER END OF JOINT, IF APPROPRIATE A4FF3380
C NOTE THAT, IF COMPUTER PRINTS OUT TWO SOLUTIONS TO A GIVEN PROBLEM BY A4FF3390
C 1 REVERSING ENDS AND RE-ANALYZING, IT IS BECAUSE THE FIRST FAILED A4FF3400
C 2 TO CONVERGE, EVEN IF THE ANSWERS PRINTED SEEM TO SUGGEST A4FF3410
C 3 OTHERWISE, THE SECOND SOLUTION IS TO BE PREFERRED, PARTICULARLY A4FF3420
C 4 IF IT STARTS AT THAT END OF THE JOINT AT WHICH THE ADHESIVE A4FF3430
C 5 SHEAR STRESS IS AT ITS HIGHEST. A4FF3440
C IDENTIFY CRITICAL END OF JOINT A4FF3450
C AVOID REVERSING ENDS BACK AGAIN A4FF3460
IF (JFLAG .EQ. 2) GO TO 390             A4FF3470
IF (NRVRS .EQ. 1) GO TO 360            A4FF3480
C IF SO, SOLUTION HAS FAILED TO CONVERGE, SO TRY AGAIN FROM OTHER END A4FF3490
C ACCURACY AT FAR END OF JOINT MAY BE POOR IF FAR END IS CRITICAL A4FF3500
IF ( (TAU(MSTEPS) .LE. TAU(1)) .AND. (TAU(MSTEPS) .GE. A4FF3510
1 (-1. * TAU(1))) ) GO TO 390          A4FF3520

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C IF, AT FAR END OF JOINT, TAU(MSTEPS) .GT. TAU(1) AT NEAR END,
C 1 FAILURE TO CONVERGE MAY BE SIMPLY THE RESULT OF THE FAR END
C 2 OF THE JOINT BEING MORE CRITICAL THAN THE STARTING (NEAR) END
C REVERSE DATA AND REANALYZE
360 DO 370 N = 1, MSTEPS
STEP(N) = STEPL(N)
THCKNO(N) = THICKO(N)
THCKNI(N) = THICKI(N)
ETOTR(N) = ETOTR(N)
ETINR(N) = ETINR(N)
STRGTR(N) = STRGTR(N)
370 STRGNR(N) = STRINR(N)
DO 380 N = 1, NSTEPS
STEPL(N) = STEP(MSTEPS - N)
THICKO(N) = THCKNI(MSTEPS - N)
THICKI(N) = THCKNO(MSTEPS - N)
ETOTR(N) = ETINR(MSTEPS - N)
ETINR(N) = ETOTR(MSTEPS - N)
STRGTR(N) = STRGNR(MSTEPS - N)
STRINR(N) = STRGTR(MSTEPS - N)
380 STEPL(MSTEPS) = STEP(MSTEPS)
THICKO(MSTEPS) = 0.
THICKI(MSTEPS) = THCKNO(1)
ETOTR(MSTEPS) = 0.
ETINR(MSTEPS) = ETOTR(1)
STRGTR(MSTEPS) = 0.
STRINR(MSTEPS) = STRGTR(1)
V = ALPHA0
ALPHA0 = ALPHA1
ALPHA1 = V
JFLAG = 2
NRVRS = 0
GO TO 50
390 CONTINUE
STOP
END

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A4FF3530
A4FF3540
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A4FF3560
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A4FF3580
A4FF3590
A4FF3600
A4FF3610
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A4FF3630
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A4FF3790
A4FF3800
A4FF3810
A4FF3820
A4FF3830
A4FF3840
A4FF3850
A4FF3860
A4FF3870
A4FF3880

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INPUT DATA

ALPHA = 0.0 (PER DEG. F) ALPHA = 0.000050 (PER DEG. F) DELTMP = 0.0 (DEG. F)

N	STEPL	THICK	THICKI	STOTR	STRNR	ETOTR	ETINR
1	0.0750	0.2640	0.0000	18216.0	3900.0	2112000.0	480000.0
2	0.0750	0.2640	0.0000	18216.0	3900.0	2112000.0	480000.0
3	0.0750	0.2640	0.0000	18216.0	3900.0	2112000.0	480000.0
4	0.0750	0.2640	0.0000	18216.0	3900.0	2112000.0	480000.0
5	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
6	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
7	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
8	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
9	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
10	0.0750	0.2640	0.0000	15180.0	9620.0	1760000.0	1184000.0
11	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
12	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
13	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
14	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
15	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
16	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
17	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
18	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
19	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
20	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
21	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
22	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
23	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
24	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
25	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
26	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
27	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
28	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
29	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
30	0.1250	0.1760	0.0000	12144.0	15340.0	1400000.0	1888000.0
31	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 7828.0  
 ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 5000.0  
 N = 1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = 0.0

N	STEPL	THICK	THICKI	TAU	GAMMA	DELTA D	DELTA I	TOUTER	STOTR	TINNER	STRNR
1	0.0750	0.2640	0.0000	6700.0	0.100	0.000	0.0005	7828.0	18216.0	0.0	3900.0
2	0.0750	0.2640	0.0000	3534.7	0.039	0.000	0.0006	7132.9	18216.0	695.2	3900.0
3	0.0750	0.2640	0.0000	2945.7	0.037	0.000	0.0007	6705.6	18216.0	1122.4	3900.0
4	0.0750	0.2640	0.0000	2245.9	0.037	0.000	0.0009	6394.6	18216.0	1433.5	3900.0
5	0.0750	0.2640	0.0000	3249.0	0.056	0.000	0.0012	6089.3	18216.0	1738.7	3900.0
6	0.0750	0.2640	0.0000	2359.1	0.039	0.000	0.0016	5681.3	15180.0	2146.6	9620.0
7	0.0750	0.2640	0.0000	1826.4	0.030	0.000	0.0018	5259.9	15180.0	2568.0	9620.0
8	0.0750	0.2640	0.0000	1647.6	0.027	0.000	0.0020	4957.9	15180.0	2877.1	9620.0
9	0.0750	0.2640	0.0000	1788.0	0.033	0.000	0.0022	4694.4	15180.0	3133.5	9620.0
10	0.0750	0.2640	0.0000	1788.0	0.033	0.000	0.0024	4444.8	12144.0	3387.8	9620.0
11	0.1250	0.1760	0.0000	2274.9	0.038	0.000	0.0027	4140.5	12144.0	3687.7	15340.0
12	0.1250	0.1760	0.0000	1225.1	0.013	0.000	0.0029	3719.5	12144.0	4108.2	15340.0
13	0.1250	0.1760	0.0000	680.6	0.013	0.000	0.0032	3479.6	12144.0	4348.8	15340.0
14	0.1250	0.1760	0.0000	545.4	0.011	0.000	0.0035	3305.1	12144.0	4522.7	15340.0
15	0.1250	0.1760	0.0000	1421.7	0.027	0.000	0.0039	3112.4	12144.0	4715.6	15340.0
16	0.1250	0.1760	0.0000	855.5	0.014	0.000	0.0041	2807.4	9108.0	5020.6	21060.0
17	0.1250	0.1760	0.0000	545.4	0.009	0.000	0.0044	2511.4	9108.0	5316.6	21060.0
18	0.1250	0.1760	0.0000	519.8	0.009	0.000	0.0046	2343.3	9108.0	5484.7	21060.0
19	0.1250	0.1760	0.0000	745.0	0.013	0.000	0.0049	2215.4	9108.0	5612.6	21060.0
20	0.1250	0.1760	0.0000	1408.6	0.023	0.000	0.0052	2061.2	9108.0	5766.9	21060.0
21	0.1250	0.1760	0.0000	674.8	0.011	0.000	0.0054	1800.0	6072.0	5920.7	26780.0
22	0.1250	0.1760	0.0000	401.3	0.007	0.000	0.0057	1553.3	6072.0	6027.5	26780.0
23	0.1250	0.1760	0.0000	401.3	0.007	0.000	0.0059	1425.3	6072.0	6127.8	26780.0
24	0.1250	0.1760	0.0000	675.4	0.011	0.000	0.0054	1330.0	6072.0	6225.8	26780.0
25	0.1250	0.1760	0.0000	1410.0	0.024	0.000	0.0054	1202.2	6072.0	6319.0	26780.0
26	0.1250	0.1760	0.0000	513.3	0.006	0.000	0.0057	1085.7	3036.0	6402.7	26780.0
27	0.1250	0.1760	0.0000	377.4	0.003	0.000	0.0059	985.2	3036.0	6498.0	26780.0
28	0.1250	0.1760	0.0000	377.4	0.003	0.000	0.0061	905.7	3036.0	6584.8	26780.0
29	0.1250	0.1760	0.0000	352.6	0.003	0.000	0.0071	845.7	3036.0	6675.8	26780.0
30	0.1250	0.1760	0.0000	352.6	0.003	0.000	0.0073	782.3	3036.0	6769.0	26780.0
31	0.0000	0.0000	0.0000	2739.5	0.046	0.000	0.0075	421.3	0.0	7828.0	32500.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 6764.2  
 ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 9000.0  
 N = 1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = 0.0

N	STEPL	THICK	THICKI	TAU	GAMMA	DELTA D	DELTA I	TOUTER	STOTR	TINNER	STRNR
1	0.0750	0.2640	0.0000	9000.0	0.145	0.000	0.0002	6764.2	18216.0	0.0	3900.0
2	0.0750	0.2640	0.0000	3132.7	0.016	0.000	0.0003	5932.4	18216.0	831.8	3900.0
3	0.0750	0.2640	0.0000	1888.9	0.006	0.000	0.0005	5633.1	18216.0	1131.1	3900.0
4	0.0750	0.2640	0.0000	1673.5	0.004	0.000	0.0006	5487.7	18216.0	1276.5	3900.0
5	0.0750	0.2640	0.0000	4574.4	0.023	0.000	0.0009	5311.9	18216.0	1452.5	3900.0
6	0.0750	0.2640	0.0000	2158.4	0.008	0.000	0.0011	4883.6	15180.0	1880.6	9620.0
7	0.0750	0.2640	0.0000	1198.9	0.011	0.000	0.0012	4403.8	15180.0	2369.4	9620.0
8	0.0750	0.2640	0.0000	1016.1	0.006	0.000	0.0014	4165.3	15180.0	2568.0	9620.0
9	0.0750	0.2640	0.0000	1524.2	0.009	0.000	0.0016	4008.2	15180.0	2877.1	9620.0
10	0.0750	0.2640	0.0000	894.4	0.004	0.000	0.0019	3827.1	12144.0	2937.1	9620.0
11	0.1250	0.1760	0.0000	3553.7	0.019	0.000	0.0020	3662.6	12144.0	3263.3	15340.0
12	0.1250	0.1760	0.0000	1198.9	0.004	0.000	0.0022	3397.9	12144.0	3701.6	15340.0
13	0.1250	0.1760	0.0000	309.9	0.001	0.000	0.0024	3229.9	12144.0	3835.2	15340.0
14	0.1250	0.1760	0.0000	376.1	0.004	0.000	0.0027	2864.8	12144.0	3999.4	15340.0
15	0.1250	0.1760	0.0000	2370.1	0.012	0.000	0.0032	2757.0	12144.0	4007.2	15340.0
16	0.1250	0.1760	0.0000	623.3	0.003	0.000	0.0034	2415.9	9108.0	4348.3	21060.0
17	0.1250	0.1760	0.0000	563.3	0.001	0.000	0.0034	2081.8	9108.0	4682.4	21060.0
18	0.1250	0.1760	0.0000	208.0	0.001	0.000	0.0039	1984.2	9108.0	4780.0	21060.0
19	0.1250	0.1760	0.0000	594.7	0.003	0.000	0.0041	1936.7	9108.0	4827.7	21060.0
20	0.1250	0.1760	0.0000	2114.0	0.011	0.000	0.0044	1848.3	9108.0	4916.6	21060.0
21	0.1250	0.1760	0.0000	492.0	0.002	0.000	0.0046	1550.0	6072.0	5020.6	26780.0
22	0.1250	0.1760	0.0000	138.6	0.001	0.000	0.0048	1272.4	6072.0	5316.6	26780.0
23	0.1250	0.1760	0.0000	138.6	0.001	0.000	0.0048	1175.3	6072.0	5484.7	26780.0
24	0.1250	0.1760	0.0000	140.0	0.001	0.000	0.0050	1105.3	6072.0	5584.8	26780.0
25	0.1250	0.1760	0.0000	2164.4	0.009	0.000	0.0052	1085.7	6072.0	5657.6	26780.0
26	0.1250	0.1760	0.0000	324.4	0.001	0.000	0.0054	822.2	3036.0	5941.9	32500.0
27	0.1250	0.1760	0.0000	2164.4	0.009	0.000	0.0056	585.5	3036.0	6178.8	32500.0
28	0.1250	0.1760	0.0000	324.4	0.001	0.000	0.0058	551.0	3036.0	6213.2	32500.0
29	0.1250	0.1760	0.0000	90.3	0.000	0.000	0.0060	537.1	3036.0	6227.1	32500.0
30	0.1250	0.1760	0.0000	673.4	0.003	0.000	0.0062	470.6	3036.0	6293.6	32500.0
31	0.0000	0.0000	0.0000	4301.8	0.022	0.000	0.0064	-0.0	0.0	6764.2	32500.0

#### A.5 Computer Program A4EG For Elastic-Plastic Strength of Stepped-Lap Bonded Joints

The elastic-plastic strength of stepped-lap joints is covered by the analysis in Section 6. The digital computer program A4EG has been prepared as a design tool for the analysis of such joints. By printing out detailed internal stresses, the program can serve to aid in design improvement by changing the joint proportions in such a manner as to reduce the load transfer in the more critical regions and to increase it in those less severely loaded areas.

In addition to those features of the elastic solution A4EF, this elastic-plastic program A4EG seeks the existence and extent of any plastic adhesive zones within any step or step portion. The convergence of the nested iterative do loops is complicated by the addition of an extra loop accounting for the maximum adhesive shear strain. This is only rarely a known quantity for ductile adhesives because the end step of the stiffer adherend is usually the most critical detail.

A complete listing of the program A4EG follows. Precisely the same input data is used as for program A4EF and the output format is the same except inasmuch as A4EG prints out separate elastic and elastic-plastic solutions.

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CDECK A4FG
STEPPED-LAP ADHESIVE-BONDED-JOINTS
ELASTIC-PLASTIC SOLUTIONS
JOINT ANALYSIS PROGRAM
PROGRAM CAN BE USED TO OPTIMIZE JOINT DESIGN PROPORTIONS
SOLUTION EXAMINES ADHESIVE SHEAR STRESS AND ADHEREND NORMAL (AXIAL)
1 STRESS BUT OMITTS CONSIDERATION OF ADHESIVE PEEL STRESS ON THE
2 GROUND THAT OUTER END STEP IS USUALLY SUFFICIENTLY THIN FOR
3 PEEL STRESS PROBLEMS NOT TO ARISE
NOTE THAT CONVERGENCE PROBLEM IS ACUTE FOR STEPPED-LAP JOINTS, EVEN
1 WITH DOUBLE-PRECISION. STEPS TAKEN HERE TO CONSTRAIN TENDENCY
2 TO DIVERGE (BY FREEZING SOLUTION ONE STEP AT A TIME) HAVE BEEN
3 ADOPTED AFTER TRYING BOTH MORE AND LESS STRINGENT TECHNIQUES
NOTE ALSO THAT CONVERGENCE DIFFICULTIES ARE PROBLEM DEPENDENT, BEING
1 MORE SEVERE FOR BRITTLE (HIGH MODULUS) ADHESIVES. LOW MODULUS
2 ADHESIVES PROVED AMENABLE TO A CONVERGENT SOLUTION IN ONLY A
3 SINGLE PASS STRAIGHT THROUGH THE JOINT FROM END TO END, IN
4 SINGLE-PRECISION, WITH ONLY A SMALL LOSS OF ACCURACY IN LATER
5 STEPS.
THE UNDERLYING DIFFICULTY IS ONE OF NUMERICAL ACCURACY LOSS IN THE
1 PRESENCE OF EXTREMELY HIGH ADHESIVE SHEAR STRESS GRADIENTS AT
2 BOTH ENDS OF EACH OF THE OUTER STEPS.
PROGRAM HAS BEEN ADAPTED TO RUN ON CDC COMPUTERS IN SINGLE PRECISION
1 BUT ONLY WORKS INTERMITTENTLY IN SINGLE PRECISION ON IBM MACHINES
1 DIMENSION TOUTER(150), TINNER(150), GAMMA(150), TAU(150),
1 DELTAO(150), DELTAI(150), STEPL(50), THICKO(50), THICKI(50),
2 ETOTR(50), ETINR(50), STOTR(50), STRINR(50), STEP(150),
3 THCKNO(150), THCKNI(150), FTOUTR(150), FTINNR(150), STRGTR(150),
4 STRGNR(150)
1 DOUBLE PRECISION TOUTER, TINNER, GAMMA, TAU, DELTAO, DELTAI,
1 TMAX, TMIN, A, B, C, D, E, F, ALAMDA, DELT, DELDT, C1, C2, C3,
2 C4, C5, C7, C10, V, V4, V5, V6, V7, V8, V9, V10, STEP, TCHECK,
3 THCKNO, THCKNI, ETOUTR, ETINNR, STRGTR, STRGNR, STEPL, THICKO,
4 THICKI, ETOTR, ETINR, STOTR, STRINR, TAUUPR, TAULWR, GAMUPR,
5 GAMLWR, FLSTR, XP, EL, FLMAX, ELMIN
1 READ (5,10) M
10 FORMAT (I2)
C M.EQ. NUMBER OF JOINT CONFIGURATIONS TO BE SOLVED
C READ IN MATERIAL PROPERTIES
DO 820 MOUNT = 1, M
NRVRS = 0
JFLAG = 1
C JFLAG IDENTIFIES END OF JOINT FROM WHICH ANALYSIS COMMENCES
1 READ (5,20) TAUMAX, G, GAMMAX, GAMMAE, ETA, ALPHAO, ALPHAI,
1 DELTMP, SGND, ANSTEP
20 FORMAT (8F10.3)
NSTEPS = ANSTEP
MSTEPS = NSTEPS + 1
LMAX = 3 * NSTEPS
MCHECK = LMAX + 1
C READ IN JOINT GEOMETRY
1 READ (5,30) ((THICKO(N), THICKI(N), STEPL(N), ETOTR(N), ETINR(N),
1 STOTR(N), STRINR(N)), N = 1, MSTEPS)
30 FORMAT (7F10.3)
C CHECK ON CONSISTENCY OF ADHESIVE DATA
VCHECK = G * GAMMAE
R = TAUMAX / VCHECK
IF ((1.001 * LT. R) .OR. (0.999 * GT. R)) GO TO 820
IF (GAMMAE .GE. GAMMAX) GO TO 40
C IF NOT, REDUCE PFAK SHEAR STRESS TO LESS THAN MAXIMUM ELASTIC VALUE
TAUMAX = G * GAMMAX
GAMMAE = GAMMAX
C SUM LAP LENGTHS
40 OLAP = STEPL(1)
DO 50 N = 2, NSTEPS
50 OLAP = OLAP + STEPL(N)
C A CHECK ON THE CONSTANCY OF THE TOTAL THICKNESS OF THE STEPPED-LAP JOINT
1 NT ADHERENDS IS NOT PROVIDED BECAUSE STRONGER JOINTS ARE OBTAINED
2 BY MATCHING THE ADHEREND EXTENSIONAL STIFFNESSES AT THE ENDS OF
1 THE JOINT AND MAINTAINING THIS TOTAL APPROXIMATELY CONSTANT THROUGHOUT
1 GHOUT THE LENGTH OF THE JOINT. THE OMISSION OF A CHECK ON THE
1 ADHEREND THICKNESSES MAKES THE PROGRAM MORE VERSATILE.
C SET UP RECURRING CONSTANTS
40 C1 = G / ETA
C2 = 2. * C1
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
C7 = -1. * GAMMAE
C10 = -1. * TAUMAX
60 C3 = ALPHAO * DELTMP
C4 = ALPHAI * DELTMP
C5 = C4 - C3
C PREPARE FOR SEPARATE COMPUTATION OF POTENTIAL ADHESIVE SHEAR STRENGTH
1 ON ASSUMPTION OF ACTUAL ADHEREND STIFFNESSES & INFINITE STRENGTH
KADHSV = 0
C PRINT OUT INPUT DATA

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A4FG0010
A4FG0020
A4FG0030
A4FG0040
A4FG0050
A4FG0060
A4FG0070
A4FG0080
A4FG0090
A4FG0100
A4FG0110
A4FG0120
A4FG0130
A4FG0140
A4FG0150
A4FG0160
A4FG0170
A4FG0180
A4FG0190
A4FG0200
A4FG0210
A4FG0220
A4FG0230
A4FG0240
A4FG0250
A4FG0260
A4FG0270
A4FG0280
A4FG0290
A4FG0300
A4FG0310
A4FG0320
A4FG0330
A4FG0340
A4FG0350
A4FG0360
A4FG0370
A4FG0380
A4FG0390
A4FG0400
A4FG0410
A4FG0420
A4FG0430
A4FG0440
A4FG0450
A4FG0460
A4FG0470
A4FG0480
A4FG0490
A4FG0500
A4FG0510
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A4FG0530
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A4FG0550
A4FG0560
A4FG0570
A4FG0580
A4FG0590
A4FG0600
A4FG0610
A4FG0620
A4FG0630
A4FG0640
A4FG0650
A4FG0660
A4FG0670
A4FG0680
A4FG0690
A4FG0700
A4FG0710
A4FG0720
A4FG0730
A4FG0740
A4FG0750
A4FG0760
A4FG0770
A4FG0780
A4FG0790
A4FG0800
A4FG0810
A4FG0820
A4FG0830
A4FG0840
A4FG0850
A4FG0860
A4FG0870
A4FG0880

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70 WRITE (6,80) ALPHA0, ALPHA1, DELTMP
80 FORMAT (1H1/, 5(1H0/), 11H INPUT DATA//, 10H ALPHA0 = , F10.7,
1 13H (PER DEG. F), 3X, 9HALPHA1 = , F10.7, 13H (PER DEG. F),
2 3X, 9HDELTMP = , F6.1, 9H (DEG. F)//,
3 1H, 5X, 1H, 2X, 5HSTEPL, 1X, 6HTHICK0, 1X, 6HTHICK1, 4X,
4 6HSTROTR, 5X, 6HSTRINR, 5X, 5HETOTR, 5X, 5HETINR//)
DO 100 N = 1, MSTEPS
WRITE (6,90) N, STEPL(N), THICK0(N), THICK1(N), STROTR(N),
1 STRINR(N), ETOTR(N), ETINR(N)
90 FORMAT (1H , 4X, I2, 3(1X, F6.4), 2(1X, F10.1), 2(1X, F10.1))
100 CONTINUE
IF (KADHSV .EQ. 1) GO TO 410
C
C START WITH ELASTIC ANALYSIS
C PROCEED TO ELASTIC-PLASTIC ANALYSIS ONLY IF ADHESIVE IS MORE CRITICAL
C 1 THAN ADHEREND(S)
C NEED ELASTIC SOLUTION TO IDENTIFY CRITICAL END FOR ELASTIC-PLASTIC
C 1 SOLUTION WHEN BOTH THERMAL AND STIFFNESS ADHEREND MISMATCHES
C 2 ARE PRESENT
C ESTIMATE MAXIMUM POSSIBLE BOND CAPACITY FOR FULLY-PLASTIC ADHESIVE
PROND = TAUMAX * OLAP * 2.
C NOTE FACTOR 2, INCLUDED FOR DOUBLE-SIDED JOINT
REDUCE TO 1. IF JOINT HAS ONLY ONE SIDE BONDED
C PERFECTLY-PLASTIC BOND CAPACITY WILL BE CLOSER TO ASYMPTOTE OF SCARF
C 1 JOINT SOLUTION AND IS SIGNIFICANTLY LOWER THAN PLASTIC ESTIMATE
C SCARF JOINT STRENGTH ESTIMATE WOULD BE THE LESSER OF PROND = 2.*TAUMAX
C 1 *OLAP*(F1T1)/(E2T2) AND PROND = 2.*TAUMAX*OLAP*(E2T2)/(F1T1)
C NOTE, HOWEVER, THAT STEPPED-LAP JOINTS EXHIBIT CHARACTERISTICS OF
C 1 DOUBLE-LAP JOINTS TO THE EXTENT THAT THE LOAD TRANSFERRED ON ANY
C 2 ONE STEP IS INDEPENDENT OF THAT STEP LENGTH ONCE THE LENGTH
C 3 EXCEEDS A TRANSITIONAL VALUE. LIKEWISE, THE TOTAL LOAD TRANSFER
C 4 BECOMES INDEPENDENT OF EACH AND EVERY (LONG) STEP IN THE JOINT
C ACTUAL LOAD CAPACITY MAY BE SIGNIFICANTLY LESS IF THERMAL MISMATCH
C 1 BETWEEN ADHERENDS IS SEVERE
C REDUCTION IN LOAD TO ACCOUNT FOR LIMITED ADHEREND STRENGTH IS
C 1 ACCOMPLISHED LATER IN PROGRAM
C PROVIDE OUTER LOOP TO ADJUST ADHESIVE PEAK SHEAR STRAIN AT START OF
C 1 JOINT FOR CASES IN WHICH EITHER ADHESIVE IS MORE CRITICAL AT
C 2 OTHER END OF JOINT OR ADHERENDS ARE MORE CRITICAL THAN ADHESIVE.
TAUUPR = 2. * TAUMAX
TAULWR = 2.
C NOTE THAT PROGRAM IS PREVENTED FROM HANDLING PROBLEM IN WHICH SHEAR
C 1 STRESS IN ADHESIVE REVERSES SIGN, WHEN COMPUTATIONS START FROM
C 2 THE LESS CRITICAL END. SOLUTION IS OBTAINABLE FROM OTHER END.
C NOTE ALSO THAT, IF THE MAXIMUM SHEAR STRESS AND APPLIED LOADS HAVE
C 1 OPPOSITE SIGNS, JOINT MUST BREAK APART UNDER RESIDUAL THERMAL
C 2 STRESS ALONE WITHOUT ANY EXTERNALLY APPLIED LOAD, SO NO CASES OF
C 3 REAL CONCERN ARE EXCLUDED BY THE RESTRICTION ABOVE
DO 310 I = 1, 50
TAU(I) = (TAUUPR + TAULWR) / 2.
IF (TAU(I) .GT. TAUMAX) TAU(I) = TAUMAX
IF (I .EQ. 1) TAU(I) = TAUMAX
C SET INITIAL CONDITIONS
GAMMA(I) = TAU(I) / G
TOUTER(I) = 5. * PROND
TINNER(I) = 0.
DELTA(I) = 0.
DELTAI(I) = SGNLD * GAMMA(I) * ETA
TMAX = 10. * PROND
TMIN = 0.
C OPERATE ON THE LOAD LEVEL IN INTERMEDIATE LOOP
C LEAVE ADJUSTMENT OF TAUMAX FOR OUTER LOOP
TCHECK = 0.
DO 210 IFLAG = 1, NSTEPS
SCHECK = 1000000000000000000.
DO 170 NCOUNT = 1, 100
C CONVERGENCE NEARLY ALWAYS OCCURRED BETWEEN 20 AND 30 CYCLES IN TEST
C 1 CASES, BUT THERE WERE SOME EXCEPTIONS
C INTERMEDIATE LOOP ADJUSTS LOAD LEVEL
VIO = -1. * TOUTER(I)
C CHECK ON CONVERGENCE OF TOUTER(IFLAG)
R = TOUTER(IFLAG) / SCHECK
IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 180
DO 120 N = IFLAG, NSTEPS
C INNER LOOP COMPUTES ELASTIC JOINT STRENGTH
A = TAU(N)
ALAMDA = DSORT(C2 * (1. / ETINR(N) + 1. / ETOTR(N)))
B = (TINNER(N) / ETINR(N) - TOUTER(N) / ETOTR(N) + C5 * SGNLD)
1 * C1 / ALAMDA
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE
C = STEPL(N)
D = ALAMDA * C
E = DSINH(D)
F = DCOSH(D)
TAU(N+1) = A * F + B * F
DELT = (2. / ALAMDA) * (A * E + B * (F - 1.))
C FACTOR 2, ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.

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TOUTER(N+1) = TOUTER(N) - DELT
TINNER(N+1) = TINNER(N) + DELT
IF (N.EQ.NSTEPS) GO TO 110
IF (TAU(N+1).LT.C1) GO TO 150
IF (TOUTER(N+1).LT.V1) GO TO 160
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
C IF THE FACTOR -1. IS TOO LARGE OR TOO SMALL, CONVERGENCE FAILS
110 DELDT = (2. / (ALAMDA**2)) * (A * (F - 1.) + R * (F - D))
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
DELTAQ(N+1) = DELTAQ(N) + C3 * C + SGNLD * (TOUTER(N) * C -
1 DELDLT) / FTOTR(N)
DELTAI(N+1) = DELTAI(N) + C4 * C + SGNLD * (TINNER(N) * C +
1 DELDLT) / FTINR(N)
GAMMA(N+1) = TAU(N+1) / G
120 CONTINUE
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED
R1 = TOUTER(1) / TINNER(MSTEPS)
C CHECK ALSO WHETHER OR NOT CONVERGENCE HAS BEEN OBTAINED
R2 = TCHECK / TINNER(MSTEPS)
IF ( (1.000001.GT. R1) .AND. (0.999999.LT. R1) .AND.
1 (1.000001.GT. R2) .AND. (0.999999.LT. R2) ) GO TO 220
C IF SO, LOAD ESTIMATE IS TOO LOW
C IF NOT, LOAD ESTIMATE IS TOO HIGH
GO TO 140
C R1 IS UNSUITABLE FOR A CONVERGENCE CHECK BECAUSE NEGATIVE VALUES OF R1
1 REPRESENT TOO HIGH A LOAD ESTIMATE, JUST LIKE THOSE VALUES IN
2 EXCESS OF UNITY
130 TMIN = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
TCHECK = TINNER(MSTEPS)
GO TO 170
140 TMAX = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
TCHECK = TINNER(MSTEPS)
GO TO 170
C NOTE THAT LABELS 26 AND 7 GOVERN FINE ADJUSTMENTS TO THE JOINT LOADS,
C 1 WHILE LABELS 27 AND 28 REPRESENT COARSE ADJUSTMENTS
150 TMAX = TOUTER(IFLAG)
SCHECK = TOUTER(IFLAG)
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
GO TO 170
160 TMIN = TOUTER(IFLAG)
SCHECK = TOUTER(IFLAG)
C IF ADHEREND, RATHER THAN ADHESIVE, LIMITS JOINT STRENGTH, NEED TO
C 1 BOOST PLOAD IN PROPORTION TO TAU MAX, EVEN IF IT MEANS EXCEEDING
C 2 ADHEREND STRENGTHS IN INTERMEDIATE COMPUTATIONS. CORRECTIONS
C 3 ARE APPLIED LATER
IF (TMIN.GE. TMAX) TMAX = 5. * TMAX
TOUTER(IFLAG) = (TMIN + TMAX) / 2.
170 CONTINUE
IF (N.EQ.NSTEPS) GO TO 210
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C 1 BECAUSE OF NUMERICAL ACCURACY PROBLEMS. REMEDY IS TO FREEZE
C 2 EARLIER VALUES, WHICH HAVE CONVERGED AND SLIGHTLY PERTURB
C 3 INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
TMAX = TOUTER(1)
TMIN = -1. * TMAX
GO TO 210
180 ICOUNT = IFLAG + 1
IF (TOUTER(ICOUNT).GT. 0.) GO TO 190
IF (TOUTER(ICOUNT).LT. 0.) GO TO 200
TMAX = TOUTER(1) / 10.
TMIN = -1. * TMAX
GO TO 210
190 TMAX = 1.1 * TOUTER(ICOUNT)
TMIN = 0.9 * TOUTER(ICOUNT)
GO TO 210
200 TMIN = 0.9 * TOUTER(ICOUNT)
TMAX = 1.1 * TOUTER(ICOUNT)
C THE BOUNDS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
210 CONTINUE
NRVRS = 1
C
220 IF ( (C5.GT. 0.000001) .OR. (C5.LT. -0.000001) ) GO TO 260
IF NOT, FIRST SOLUTION MAY BE SCALED IN THE ABSENCE OF ANY THERMAL
1 MISMATCH BETWEEN ADHERENDS
IF SO, SOLUTION MUST BE REFINED BY ITERATION, SINCE THERMAL STRESS
1 TERMS DO NOT SCALE LINEARLY, EVEN FOR ELASTIC ADHESIVE AND
2 ADHERENDS
APPLY SCALE FACTOR TO SOLUTION FOR ONLY ADHEREND STIFFNESS IMBALANCE
ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
1 WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE
C PROGRAM ASSUMES ADHEREND ALLOWABLES HAVE SAME MAGNITUDE IN TENSION AS
1 IN COMPRESSION. DISTINCTION IS USUALLY UNIMPORTANT SINCE, IN

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A4FG1770
A4FG1780
A4FG1790
A4FG1800
A4FG1810
A4FG1820
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A4FG1840
A4FG1850
A4FG1860
A4FG1870
A4FG1880
A4FG1890
A4FG1900
A4FG1910
A4FG1920
A4FG1930
A4FG1940
A4FG1950
A4FG1960
A4FG1970
A4FG1980
A4FG1990
A4FG2000
A4FG2010
A4FG2020
A4FG2030
A4FG2040
A4FG2050
A4FG2060
A4FG2070
A4FG2080
A4FG2090
A4FG2100
A4FG2110
A4FG2120
A4FG2130
A4FG2140
A4FG2150
A4FG2160
A4FG2170
A4FG2180
A4FG2190
A4FG2200
A4FG2210
A4FG2220
A4FG2230
A4FG2240
A4FG2250
A4FG2260
A4FG2270
A4FG2280
A4FG2290
A4FG2300
A4FG2310
A4FG2320
A4FG2330
A4FG2340
A4FG2350
A4FG2360
A4FG2370
A4FG2380
A4FG2390
A4FG2400
A4FG2410
A4FG2420
A4FG2430
A4FG2440
A4FG2450
A4FG2460
A4FG2470
A4FG2480
A4FG2490
A4FG2500
A4FG2510
A4FG2520
A4FG2530
A4FG2540
A4FG2550
A4FG2560
A4FG2570
A4FG2580
A4FG2590
A4FG2600
A4FG2610
A4FG2620
A4FG2630
A4FG2640

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C 2 PRACTICAL JOINTS, RESIDUAL THERMAL STRESSES ARE UNLIKELY TO
C 3 BREAK ADHEREND(S) RATHER THAN ADHESIVE
RSCALE = TOUTER(1) / STROTP(1)
IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
RTAUMX = TAU(1) / TAUMAX
IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX
DO 240 N = 2, MSTEPS
RINP = TINNER(N) / STRINP(N-1)
C NEED STRENGTH ON THIN SIDE OF STEP, HENCE THE (N-1) IN COMPARISON
IF (RINP .LT. 0.) RINP = -1. * RINP
IF (RINP .GT. RSCALE) RSCALE = RINP
IF (N.EQ. MSTEPS) GO TO 230
ROTR = TOUTER(N) / STROTR(N)
IF (ROTR .LT. 0.) ROTR = -1. * ROTR
IF (ROTR .GT. RSCALE) RSCALE = ROTR
230 RTAU = TAU(N) / TAUMAX
IF (RTAU .LT. 0.) RTAU = -1. * RTAU
IF (RTAU .GT. RTAUMX) RTAUMX = RTAU
240 CONTINUE
RECTR = RSCALE
C RECTR IS PROPORTIONALITY CONSTANT GOVERNING ELASTIC SOLUTION
C IF RTAUMX .GT. RSCALE, ADHESIVE PLASTICITY CAN INCREASE STRENGTH
C USUALLY ADHESIVE IS CRITICAL AT ONE END OF JOINT OR OTHER, SO RTAUMX
C 1 .GT. 1. MAY WELL JUST SIGNIFY THAT FAR END OF JOINT IS CRITICAL
C NOTE THAT PROGRAM ASSUMES THAT ANY INTERNAL ADHEREND STRESSES OF
C 1 REVERSED SIGN WITH RESPECT TO STRESS OUTSIDE THE JOINT ARE NOT
C 2 CRITICAL. IF THEY ARE, IT MEANS THAT THE JOINT WILL FAIL DUE
C 3 TO RESIDUAL THERMAL STRESSES ALONE WITHOUT ANY MECHANICAL LOADS
IF (RSCALE .LT. RTAUMX) RECTR = RTAUMX
DO 250 N = 1, MSTEPS
TOUTER(N) = TOUTER(N) / RECTR
TINNER(N) = TINNER(N) / RECTR
TAU(N) = TAU(N) / RECTR
GAMMA(N) = GAMMA(N) / RECTR
DELTAQ(N) = DELTAQ(N) / RECTR
250 DELTAI(N) = DELTAI(N) / RECTR
GO TO 330

C USE ITERATIVE SOLUTION WHEN ADHEREND THERMAL MISMATCH IS PRESENT
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC ADHESIVE OR
C 1 WHETHER OTHER END OF JOINT IS MORE CRITICAL FOR ADHESIVE.
260 RSCALE = TOUTER(1) / STROTP(1)
IF (RSCALE .LT. 0.) RSCALE = -1. * RSCALE
RTAUMX = TAU(1) / TAUMAX
IF (RTAUMX .LT. 0.) RTAUMX = -1. * RTAUMX
DO 280 N = 2, MSTEPS
RINP = TINNER(N) / STRINP(N)
IF (RINP .GT. RSCALE) RSCALE = RINP
IF (N.EQ. MSTEPS) GO TO 270
ROTR = TOUTER(N) / STROTR(N)
IF (ROTR .GT. RSCALE) RSCALE = ROTR
270 RTAU = TAU(N) / TAUMAX
IF (RTAU .LT. 0.) RTAU = -1. * RTAU
IF (RTAU .GT. RTAUMX) RTAUMX = RTAU
280 CONTINUE
C CHECK ON CONVERGENCE
R = RTAUMX
IF (RTAUMX .LT. RSCALE) R = RSCALE
IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 330
V = 2. * TAUMAX
R = (V + TAUUPP) / (V + TAILWR)
IF ( (1.00001 .GT. R) .AND. (0.99999 .LT. R) ) GO TO 330
C IF EITHER RTAUMX OR RSCALE .GT. UNITY, TAU(1) MUST BE DECREASED
C IF ( RTAUMX .GT. 1.00001 ) .OR. ( RSCALE .GT. 1.00001 ) GO TO 290
C IF BOTH RTAUMX AND RSCALE ARE .LT. UNITY, TAU(1) MUST BE INCREASED
C IF ( RTAUMX .LT. 0.99999 ) .AND. ( RSCALE .LT. 0.99999 ) GO TO 300
C IF NEITHER CHECK IS MET, SOLUTION HAS CONVERGED
GO TO 330
290 TAUUPP = TAU(1)
GO TO 310
300 TAILWR = TAU(1)
310 CONTINUE
C IF PROGRAM GOES BEYOND PRECEDING CONTINUE STATEMENT, SOLUTION HAS NOT
C 1 CONVERGED
WRITE (6,320)
320 FORMAT (1H1, 18H DIVERGENT SOLUTION)

C PRINT OUT RESULTS OF ELASTIC COMPUTATIONS
330 WRITE (6,340) TOUTER(1), TAUMAX, SGNLD, DELTMP
340 FORMAT (1H1/, 5(1H0/),
1 39H ELASTIC JOINT STRENGTH, PLDAD (LPS) = , F10.1/,
2 49H ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = , F8.1/,
3 1H , 8HSGNLD = , F4.1, 54H SGNLD = +1 FOR TENSILE SHEAR AND -1A4FG3470
4 FOR COMPRESSIVE/, 36H TEMPERATURE DIFFERENTIAL (DEG F) = , F6.1//A4FG3480
5, 1H , 5X, 1HN, 2X, 5HSTEP, 1X, 6HTHICKO, 1X, 6HTHICKI, 3X,
6 3HTAU, 4X, 5HGAMMA, 1X, 6HDELTAQ, 1X, 6HDELTAI, 5X, 6HTOUTER,
7 5X, 6HSTROTR, 5X, 6HTINNER, 5X, 6HSTRINP//)
DO 360 N = 1, MSTEPS
A4FG2650
A4FG2660
A4EG2670
A4FG2680
A4FG2690
A4EG2700
A4FG2710
A4EG2720
A4EG2730
A4EG2740
A4FG2750
A4FG2760
A4FG2770
A4EG2780
A4EG2790
A4EG2800
A4FG2810
A4EG2820
A4EG2830
A4FG2840
A4EG2850
A4EG2860
A4EG2870
A4FG2880
A4FG2890
A4EG2900
A4FG2910
A4EG2920
A4EG2930
A4FG2940
A4EG2950
A4EG2960
A4EG2970
A4EG2980
A4FG2990
A4EG3000
A4FG3010
A4EG3020
A4EG3030
A4EG3040
A4FG3050
A4EG3060
A4EG3070
A4FG3080
A4EG3090
A4EG3100
A4EG3110
A4EG3120
A4FG3130
A4EG3140
A4FG3150
A4EG3160
A4FG3170
A4FG3180
A4EG3190
A4EG3200
A4EG3210
A4FG3220
A4FG3230
A4EG3240
A4EG3250
A4EG3260
A4EG3270
A4FG3280
A4EG3290
A4EG3300
A4FG3310
A4FG3320
A4FG3330
A4EG3340
A4EG3350
A4FG3360
A4EG3370
A4FG3380
A4FG3390
A4FG3400
A4FG3410
A4EG3420
A4FG3430
A4EG3440
A4EG3450
A4EG3460
A4FG3470
A4FG3480
A4FG3490
A4EG3500
A4FG3510
A4FG3520

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WRITE (6,350) N, STEPL(N), THCKO(N), THCKI(N), TAU(N), GAMMA(N), A4EG3530
1 DELTAO(N), DELTAI(N), TOUTER(N), STOTR(N), TINNER(N), STRINR(N) A4EG3540
2) A4EG3550
350 FORMAT (1H, 4X, I2, 1X, F6.4, 1X, F6.4, 1X, F6.4, 1X, F7.1, 1X, A4EG3560
1 F6.3, 1X, F6.4, 1X, F6.4, 1X, F10.1, 1X, F10.1, 1X, A4EG3570
2 F10.1) A4EG3580
360 CONTINUE A4EG3590
A4EG3600
C RECOMPUTE SOLUTION FROM OTHER END OF JOINT, IF APPROPRIATE A4EG3610
NOTE THAT, IF COMPUTER PRINTS OUT TWO SOLUTIONS TO A GIVEN PROBLEM BY A4EG3620
1 REVERSING ENDS AND RE-ANALYZING, IT IS BECAUSE THE FIRST FAILED A4EG3630
2 TO CONVERGE, EVEN IF THE ANSWERS PRINTED SEEM TO SUGGEST A4EG3640
3 OTHERWISE, THE SECOND SOLUTION IS TO BE PREFERRED, PARTICULARLY A4EG3650
4 IF IT STARTS AT THAT END OF THE JOINT AT WHICH THE ADHESIVE A4EG3660
5 SHEAR STRESS IS AT ITS HIGHEST. A4EG3670
C IDENTIFY CRITICAL END OF JOINT A4EG3680
C AVOID REVERSING ENDS BACK AGAIN A4EG3690
IF (JFLAG .EQ. 2) GO TO 400 A4EG3700
IF (NPVRS .EQ. 1) GO TO 370 A4EG3710
C IF SO, SOLUTION HAS FAILED TO CONVERGE, SO TRY AGAIN FROM OTHER END A4EG3720
C ACCURACY AT FAR END OF JOINT MAY BE POOR IF FAR END IS CRITICAL A4EG3730
IF ((TAU(MSTEPS) .LE. TAU(1)) .AND. (TAU(MSTEPS) .GE. A4EG3740
1 (-1. * TAU(1)))) GO TO 400 A4EG3750
C IF, AT FAR END OF JOINT, TAU(MSTEPS) .GT. TAU(1) AT NEAR END, A4EG3760
1 FAILURE TO CONVERGE MAY BE SIMPLY THE RESULT OF THE FAR END A4EG3770
2 OF THE JOINT BEING MORE CRITICAL THAN THE STARTING (NEAR) END A4EG3780
C REVERSE DATA AND REANALYZE A4EG3790
370 DO 380 N = 1, MSTEPS A4EG3800
STEP(N) = STEPL(N) A4EG3810
THCKO(N) = THCKO(N) A4EG3820
THCKI(N) = THCKI(N) A4EG3830
ETOUTR(N) = ETOUTR(N) A4EG3840
ETINNR(N) = ETINNR(N) A4EG3850
STOTR(N) = STOTR(N) A4EG3860
STRGTR(N) = STRGTR(N) A4EG3870
380 STRGNR(N) = STRINR(N) A4EG3880
DO 390 N = 1, MSTEPS A4EG3890
STEPL(N) = STEP(MSTEPS - N) A4EG3900
THCKO(N) = THCKO(MSTEPS - N) A4EG3910
THCKI(N) = THCKI(MSTEPS - N) A4EG3920
ETOUTR(N) = ETOUTR(MSTEPS - N) A4EG3930
ETINNR(N) = ETINNR(MSTEPS - N) A4EG3940
STOTR(N) = STOTR(MSTEPS - N) A4EG3950
STRGTR(N) = STRGTR(MSTEPS - N) A4EG3960
390 STEPL(MSTEPS) = STEP(MSTEPS) A4EG3970
THCKO(MSTEPS) = 0. A4EG3980
THCKI(MSTEPS) = THCKO(1) A4EG3990
ETOUTR(MSTEPS) = 0. A4EG4000
ETINNR(MSTEPS) = ETOUTR(1) A4EG4010
STOTR(MSTEPS) = 0. A4EG4020
STRGTR(MSTEPS) = STRGTR(1) A4EG4030
V = ALPHA0 A4EG4040
ALPHA0 = ALPHA1 A4EG4050
ALPHA1 = V A4EG4060
JFLAG = 2 A4EG4070
NPVRS = 0 A4EG4080
GO TO 60 A4EG4090
C BYPASS ELASTIC-PLASTIC COMPUTATIONS IF ADHERENDS ARE MORE CRITICAL A4EG4100
C 1 THAN ADHESIVE A4EG4110
400 IF (PSCALE .GE. 9TAUMX) GO TO 820 A4EG4120
C RECORD ELASTIC JOINT STRENGTH A4EG4130
ELSTR = TOUTER(1) A4EG4140
A4EG4150
C START ELASTIC-PLASTIC SOLUTION A4EG4160
C ELASTIC SOLUTION HAS IDENTIFIED CRITICAL END OF JOINT, AND REVERSED A4EG4170
C 1 ORDER OF DATA IF NECESSARY, SO THERE IS NO NEED FOR SUCH A4EG4180
C 2 CAPABILITY IN THE ELASTIC-PLASTIC SOLUTION A4EG4190
C ADD EXTRA LOCATIONS INSIDE STEPS TO ACCOUNT FOR POTENTIAL PLASTIC-TO- A4EG4200
C 1 ELASTIC AND ELASTIC-TO-PLASTIC TRANSITIONS IN ADHESIVE A4EG4210
410 DO 420 N = 1, MSTEPS A4EG4220
L = 3 * N A4EG4230
THCKNO(L-2) = THCKO(N) A4EG4240
THCKNO(L-1) = THCKO(N) A4EG4250
THCKNO(L) = THCKO(N) A4EG4260
THCKNI(L-2) = THCKI(N) A4EG4270
THCKNI(L-1) = THCKI(N) A4EG4280
THCKNI(L) = THCKI(N) A4EG4290
ETOUTR(L-2) = ETOUTR(N) A4EG4300
ETOUTR(L-1) = ETOUTR(N) A4EG4310
ETOUTR(L) = ETOUTR(N) A4EG4320
ETINNR(L-2) = ETINNR(N) A4EG4330
ETINNR(L-1) = ETINNR(N) A4EG4340
ETINNR(L) = ETINNR(N) A4EG4350
STOTR(L-2) = STOTR(N) A4EG4360
STOTR(L-1) = STOTR(N) A4EG4370
STOTR(L) = STOTR(N) A4EG4380
STRGNR(L-2) = STRINR(N) A4EG4390
STRGNR(L-1) = STRINR(N) A4EG4400

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420 STRGNR(L) = STRINR(N)
THCKNO(MCHECK) = 0.
THCKNI(MCHECK) = THICKI(MSTEPS)
ETOUTR(MCHECK) = 0.
FTINNR(MCHECK) = FTINR(MSTEPS)
STRGTR(MCHECK) = 0.
STRGNR(MCHECK) = STRINR(MSTEPS)
C USE OUTER LOOP TO ADJUST MAXIMUM ADHESIVE SHEAR STRAIN LEVEL
GAMUPR = GAMMAX
GAMLWR = GAMMAE
C GAMUPR AND GAMLWR SERVE AS BOUNDS ON SHEAR STRAIN ACTUALLY DEVELOPED
C SET UPPER AND LOWER BOUNDS ON STRENGTH
TUPPER = TAUJMAX * OLAP * 2.
C NOTE THAT THE FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF
C 1 ADHERENDS. IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
TLOWER = FLSTR
C TMAX .EQ. FULLY-PLASTIC JOINT STRENGTH
C TMIN .EQ. PERFECTLY-ELASTIC JOINT STRENGTH
ICHECK = 0
JCHECK = 10
C NEED TO SET DIFFERENT VALUES FOR ICHECK AND JCHECK WITH NEITHER
C 1 EQUAL TO EITHER 1 OR 2
DO 760 I = 1, 50
IF ( (KADHSV .EQ. 1) .AND. (I .GT. 1) ) GO TO 770
C THIS INSTRUCTION PRINTS OUT SOLUTION FOR POTENTIAL BOND SHEAR STRENGTH
TMAX = TUPPER
TMIN = TLOWER
GAMMA(1) = (GAMUPR + GAMLWR) / 2.
IF ( (KADHSV .EQ. 1) .OR. (I .EQ. 1) ) GO TO 440
IF (ICHECK .NE. JCHECK) GO TO 450
IF (ICHECK .EQ. 1) GO TO 430
C IF NOT, ICHECK .EQ. 2 AND LOAD HAS BEEN TOO HIGH FOR TWO CONSECUTIVE
C 1 ITERATIONS
GAMMA(1) = (GAMMA(1) + GAMLWR) / 2.
ICHECK = 2
GO TO 450
C IF ICHECK .EQ. 1, LOAD HAS BEEN TOO LOW FOR TWO CONSECUTIVE ITERATIONS
430 GAMMA(1) = (GAMMA(1) + GAMUPR) / 2.
ICHECK = 1
GO TO 450
440 GAMMA(1) = GAMMAX
C SET INITIAL CONDITIONS
450 TOUTER(1) = (TUPPER + TLOWER) / 2.
TINNER(1) = 0.
TAU(1) = TAUJMAX
DELTAQ(1) = 0.
DELTAI(1) = SGNLD * GAMMA(1) * ETA
DO 700 IFLAG = 1, NSTEPS
J1 = 3 * IFLAG - 2
J2 = J1 + 3
SCHECK = 10000000000000000.
DO 660 NCOUNT = 1, 50
C USUALLY 20 CYCLES OF ITERATION WERE SUFFICIENT AT THIS POINT
C MIDDLE LOOP ADJUSTS LOAD LEVEL
C CHECK ON CONVERGENCE OF TOUTER(J1)
P = TOUTER(J1) / SCHECK
IF ( (1.000000001 .GT. P) .AND. (0.999999999 .LT. P) ) GO TO 670
V11 = TOUTER(1)
V10 = -1. * V11
DO 600 N = IFLAG, NSTEPS
C INNERMOST LOOP COMPUTES JOINT STRENGTH
L = 3 * N - 2
STEP(L) = STEP(N)
STEP(L+1) = 0.
STEP(L+2) = 0.
C IF ADHESIVE IS NOT LOADED INTO PLASTIC ZONE IN LATER STEPS OF JOINT,
C 1 BYPASS SUCH COMPUTATIONS AND PROCEED TO PERFECTLY-ELASTIC SOLUTION
V8 = GAMMA(L)
IF ( (V8 .LT. GAMMAE) .AND. (V8 .GE. 07) ) GO TO 510
XP = 10000000000.
MFLAG = 0
C SOLVE FOR MAXIMUM POSSIBLE EXTENT OF PLASTIC ADHESIVE ZONE
C 1 AND COMPARE WITH STEP LENGTH
V4 = ETOUTR(N)
V5 = FTINR(N)
V6 = TOUTER(L)
V7 = TINNER(L)
V9 = STEP(L)
V = (1. / V4 + 1. / V5) / ETA
A = TAUJMAX * V
C IF BONDED ON ONE SIDE OF JOINT ONLY, DIVIDE A BY 2.
B = (05 * SGNLD - V6 / V4 + V7 / V5) / ETA
C NOTE THAT B SHOULD BE NEGATIVE AT AND NEAR MORE CRITICAL END OF JOINT
C IF NOT, SOLUTION IS PROCEEDING FROM WRONG END OF JOINT
C HENCE NEED FOR PRIOR ELASTIC SOLUTION TO IDENTIFY CRITICAL END
IF (V8 .LT. 0.) GO TO 460
C PROCEDURE FOR POSITIVE PLASTIC ADHESIVE SHEAR STRAINS
IF (B .GE. 0.) GO TO 470

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A4EG4410
A4EG4420
A4EG4430
A4EG4440
A4EG4450
A4EG4460
A4EG4470
A4EG4480
A4EG4490
A4EG4500
A4EG4510
A4EG4520
A4EG4530
A4EG4540
A4EG4550
A4EG4560
A4EG4570
A4EG4580
A4EG4590
A4EG4600
A4EG4610
A4EG4620
A4EG4630
A4EG4640
A4EG4650
A4EG4660
A4EG4670
A4EG4680
A4EG4690
A4EG4700
A4EG4710
A4EG4720
A4EG4730
A4EG4740
A4EG4750
A4EG4760
A4EG4770
A4EG4780
A4EG4790
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A4EG4890
A4EG4900
A4EG4910
A4EG4920
A4EG4930
A4EG4940
A4EG4950
A4EG4960
A4EG4970
A4EG4980
A4EG4990
A4EG5000
A4EG5010
A4EG5020
A4EG5030
A4EG5040
A4EG5050
A4EG5060
A4EG5070
A4EG5080
A4EG5090
A4EG5100
A4EG5110
A4EG5120
A4EG5130
A4EG5140
A4EG5150
A4EG5160
A4EG5170
A4EG5180
A4EG5190
A4EG5200
A4EG5210
A4EG5220
A4EG5230
A4EG5240
A4EG5250
A4EG5260
A4EG5270
A4EG5280

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C = VR - GAMMAE
D = R**2 - 4. * A * C
IF (D .LT. 0.) GO TO 470
C IF SO, PLASTIC ZONE IS UNBOUNDED, AS AT FAR END OF JOINT
XP = (-1. * B - DSQRT(D)) / (2. * A)
GO TO 470
C PROCEDURE FOR NEGATIVE PLASTIC ADHESIVE SHEAR STRAINS
460 IF (B .LE. 0.) GO TO 470
C = VR + GAMMAE
D = R**2 + 4. * A * C
IF (D .LT. 0.) GO TO 470
C IF SO, PLASTIC ZONE IS UNBOUNDED, AS AT FAR END OF JOINT
XP = (B - DSQRT(D)) / (2. * A)
470 IF (XP .GE. V9) GO TO 480
C IF SO, ADHESIVE IS FULLY-PLASTIC THROUGHOUT THAT STEP
C IF NOT, BREAK UP STEP INTO PLASTIC AND ELASTIC PORTIONS
MFLAG = 1
STEP(L) = XP
STEP(L+1) = V9 - XP
V9 = XP
C MAY HAVE TO DECREASE STEP(3*N-1) AND ADD TO STEP(3*N) LATER
C
C PROCEDURE FOR FULLY-PLASTIC STEP OR STEP PORTION
C THIS SERIES OF EQUATIONS HOLDS REGARDLESS OF SIGN OF SHEAR STRESS
C 1 GRADIENT AT START OF STEP
480 DELT = 2. * TAU(L) * V9
C THE USE OF TAU(L) INSTEAD OF TAUMAX COVERS REVERSAL OF SIGN
C FACTOR 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
L = L + 1
TOUTER(L) = V6 - DELT
TINNER(L) = V7 + DELT
GAMMA(L) = GAMMA(L-1) + R * V9 + TAU(L-1) * V * (V9**2)
TAU(L) = TAU(L-1)
DELTAO(L) = DELTAO(L-1) + C3 * V9 + SGNLD * (V6 * V9 - TAU(L) *
1 (V9**2) / 2.) / V4
DELTAI(L) = DELTAI(L-1) + C4 * V9 + SGNLD * (V7 * V9 + TAU(L) *
1 (V9**2) / 2.) / V5
C NOTE THAT USE OF TAU(L) INSTEAD OF TAUMAX AUTOMATICALLY ACCOUNTS FOR
C 1 SIGN OF ADHESIVE SHEAR STRESS
1 IF (MFLAG .EQ. 1) GO TO 490
C IF NOT, STEP IS PLASTIC THROUGHOUT
L1 = L + 1
L2 = L + 2
TOUTER(L1) = TOUTER(L)
TOUTER(L2) = TOUTER(L)
TINNER(L1) = TINNER(L)
TINNER(L2) = TINNER(L)
TAU(L1) = TAU(L)
TAU(L2) = TAU(L)
GAMMA(L1) = GAMMA(L)
GAMMA(L2) = GAMMA(L)
DELTAO(L1) = DELTAO(L)
DELTAO(L2) = DELTAO(L)
DELTAI(L1) = DELTAI(L)
DELTAI(L2) = DELTAI(L)
490 IF (N .EQ. NSTEPS) GO TO 500
IF (TOUTER(L) .LT. V10) GO TO 620
IF (TINNER(L) .LT. V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
500 IF (MFLAG .EQ. 1) GO TO 510
GO TO 590
C
C PROCEDURE FOR PERFECTLY-PLASTIC ZONE
C IDENTIFY WHETHER STEP IS ELASTIC-PLASTIC OR FULLY ELASTIC THROUGHOUT
510 K = L - 3 * N + 2
C K .EQ. 0 CORRESPONDS TO NO PLASTIC ZONE AT NEAR END OF JOINT
C SET INITIAL CONDITIONS AT START OF STEP
V4 = ETOTR(N)
V5 = ETINP(N)
V6 = TOUTER(L)
V7 = TINNER(L)
ALAMDA = DSQRT(C2 * (1. / V4 + 1. / V5))
MFLAG = 0
C COMPUTE VALUES AT FAR END OF ELASTIC ZONE
A = TAU(L)
B = (V7 / V5 - V6 / V4 + C5 * SGNLD) * C1 / ALAMDA
C NOTE THAT SGNLD SIGNIFIES WHETHER SHEAR LOAD IS TENSILE OR COMPRESSIVE
C = STEP(L)
D = ALAMDA * C
E = DSINH(D)
F = DCOSH(D)
TAU(L+1) = A * F + B * E
IF ( (TAU(L+1) .LE. TAUMAX) .AND. (TAU(L+1) .GE. C10) )
1 GO TO 540
C IF NOT, ELASTIC STEP SIZE IS EXCESSIVE. REDUCE BY ITERATION
ELMAX = C
ELMIN = 0.

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A4EG5290  
A4EG5300  
A4EG5310  
A4EG5320  
A4EG5330  
A4EG5340  
A4EG5350  
A4EG5360  
A4EG5370  
A4EG5380  
A4EG5390  
A4EG5400  
A4EG5410  
A4EG5420  
A4EG5430  
A4EG5440  
A4EG5450  
A4EG5460  
A4EG5470  
A4EG5480  
A4EG5490  
A4EG5500  
A4EG5510  
A4EG5520  
A4EG5530  
A4EG5540  
A4EG5550  
A4EG5560  
A4EG5570  
A4EG5580  
A4EG5590  
A4EG5600  
A4EG5610  
A4EG5620  
A4EG5630  
A4EG5640  
A4EG5650  
A4EG5660  
A4EG5670  
A4EG5680  
A4EG5690  
A4EG5700  
A4EG5710  
A4EG5720  
A4EG5730  
A4EG5740  
A4EG5750  
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A4EG6100  
A4EG6110  
A4EG6120  
A4EG6130  
A4EG6140  
A4EG6150  
A4EG6160

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DO 520 ICOUNT = 1, 100
FL = (FLMAX + FLMIN) / 2.
D = ALAMDA * FL
E = DSINH(D)
F = DCOSH(D)
TCHECK = A * E + B * F
IF ( (TCHECK .GT. TAUIMAX) .OR. (TCHECK .LT. C10) ) FLMAX = FL
IF ( (TCHECK .LT. TAUIMAX) .AND. (TCHECK .GT. C10) ) FLMIN = FL
R = FLMIN / FLMAX
IF ( (1.000000001 .GT. R) .AND. (0.999999999 .LT. R) ) GO TO 530
520 CONTINUE
530 STEP(L) = FL
STEP(L+1) = STEP(L+1) + C - FL
LELAG = 1
IF (TCHECK .GT. 0.) TAU(L+1) = TAUIMAX
IF (TCHECK .LT. 0.) TAU(L+1) = C10
540 DELT = (2. / ALAMDA) * (A * E + B * (F - 1.))
C FACTOR OF 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
L = L + 1
TOUTER(L) = V6 - DELT
TINNER(L) = V7 + DELT
DELDLT = (2. / (ALAMDA**2)) * (A * (F - 1.) + B * (F - D))
C FACTOR OF 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
DELTAO(L) = DELTAO(L-1) + C3 * EL + SGNLD * (V6 * EL - DELDLT) / V4
DELTAI(L) = DELTAI(L-1) + C4 * EL + SGNLD * (V7 * EL + DELDLT) / V5
GAMMA(L) = TAU(L) / G
IF (LELAG .EQ. 1) GO TO 550
C IF NOT, THERE IS NO (SECOND) PLASTIC ZONE AT FAR END OF STEP
L1 = L + 1
TOUTER(L1) = TOUTER(L)
TINNER(L1) = TINNER(L)
TAU(L1) = TAU(L)
GAMMA(L1) = GAMMA(L)
DELTAO(L1) = DELTAO(L)
DELTAI(L1) = DELTAI(L)
IF (K .EQ. 1) GO TO 550
C IF NOT, THERE WAS NO (FIRST) PLASTIC ZONE AT NEAR END OF JOINT
L2 = L + 2
TOUTER(L2) = TOUTER(L)
TINNER(L2) = TINNER(L)
TAU(L2) = TAU(L)
GAMMA(L2) = GAMMA(L)
DELTAO(L2) = DELTAO(L)
DELTAI(L2) = DELTAI(L)
550 IF (N .EQ. NSTEPS) GO TO 560
IF (TOUTER(L) .LT. V10) GO TO 620
IF (TINNER(L) .LT. V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
C IF V10 IS EITHER TOO LARGE OR TOO SMALL, CONVERGENCE FAILS
560 IF (LELAG .EQ. 1) GO TO 570
GO TO 590
C
C PROCEDURE FOR (SECOND) PLASTIC ZONE AT FAR END OF STEP
570 V9 = STEP(L)
DELT = 2. * TAU(L) * V9
C FACTOR OF 2. ACCOUNTS FOR BONDING ON BOTH SIDES OF INNER ADHEREND.
C 1 IF BONDED ON ONE SIDE ONLY, REDUCE TO 1.
C SET INITIAL CONDITIONS AT START OF STEP
V4 = ETOTR(N)
V5 = ETINR(N)
V6 = TOUTER(L)
V7 = TINNER(L)
L = L + 1
TOUTER(L) = V6 - DELT
TINNER(L) = V7 + DELT
A = (TAU(L-1) / ETA) * (1./V4 + 1./V5)
C NOTE THE USE OF TAU(L-1) INSTEAD OF TAUIMAX IN ORDER TO ACCOUNT
C 1 AUTOMATICALLY FOR THE SIGN OF THE SHEAR STRESS
C IF BONDED ON ONE SIDE OF JOINT ONLY, DIVIDE A BY 2.
B = (C5 * SGNLD - V6 / V4 + V7 / V5) / ETA
GAMMA(L) = GAMMA(L-1) + R * V9 + A * (V9**2)
TAU(L) = TAU(L-1)
DELTAO(L) = DELTAO(L-1) + C3 * V9 + SGNLD * (V6 * V9 - TAU(L) *
1 (V9**2) / 2.) / V4
DELTAI(L) = DELTAI(L-1) + C4 * V9 + SGNLD * (V7 * V9 + TAU(L) *
1 (V9**2) / 2.) / V5
C IF THERE HAS BEEN NO PLASTIC ZONE AT START OF STEP, TRANSFER VALUES
C 1 JUST COMPUTED ACROSS TO LAST SUBDIVISION IN STEP
C THIS IS NECESSARY TO PROVIDE INPUT DATA FOR START OF NEXT STEP
IF (K .EQ. 1) GO TO 580
L1 = L + 1
TOUTER(L1) = TOUTER(L)
TINNER(L1) = TINNER(L)
TAU(L1) = TAU(L)
GAMMA(L1) = GAMMA(L)
DELTAO(L1) = DELTAO(L)

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DELTA(I) = DELTA(I)
580 IF (N.EQ.NSTEPS) GO TO 630
IF (TOUTER(L).LT.V10) GO TO 620
IF (TINNER(L).LT.V10) GO TO 610
C NOTE THAT THESE CONVERGENCE CHECKS ARE CRITICAL
590 IF (N.EQ.NSTEPS) GO TO 630
600 CONTINUE
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO HIGH
610 TMAX = TOUTER(J1)
SCHECK = TOUTER(J1)
TOUTER(J1) = (TMAX + TMIN) / 2.
GO TO 660
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO LOW
620 TMIN = TOUTER(J1)
SCHECK = TOUTER(J1)
TOUTER(J1) = (TMAX + TMIN) / 2.
GO TO 660
C
C CHECK WHETHER OR NOT PRECISELY 100 PERCENT OF LOAD HAS TRANSFERRED
630 R1 = TOUTER(1) / TINNER(MCHECK)
P2 = TCHECK / TINNER(MCHECK)
IF ( (1.000001.GT.R1).AND.(0.999999.LT.R1).AND.
1 (1.000001.GT.P2).AND.(0.999999.LT.P2) ) GO TO 710
IF (TOUTER(1).LT.TINNER(MCHECK)) GO TO 640
C NOTE THAT, HERE, LOAD IS TAKEN TO BE POSITIVE WHETHER TENSILE OR NOT
C 1 FOR A NEGATIVE LOAD, PRECEDING INSTRUCTION SHOULD BE
C 2 INTERCHANGED WITH THE FOLLOWING ONE
C IF SO, LOAD ESTIMATE IS TOO LOW
C IF REVERSE HOLDS, LOAD ESTIMATE IS TOO HIGH
GO TO 650
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO LOW
640 TMIN = TOUTER(J1)
TOUTER(J1) = (TMAX + TMIN) / 2.
TCHECK = TINNER(MCHECK)
GO TO 660
C PROCEDURE FOR WHEN LOAD ESTIMATE IS TOO HIGH
650 TMAX = TOUTER(J1)
TOUTER(J1) = (TMAX + TMIN) / 2.
TCHECK = TINNER(MCHECK)
660 CONTINUE
C CONVERGENCE WILL NOT PROCEED TO FAR END OF JOINT IN SINGLE PASS
C 1 BECAUSE OF NUMERICAL ACCURACY PROBLEMS. REMEDY IS TO FREEZE
C 2 EARLIER VALUES, WHICH HAVE CONVERGED, AND TO PERTURB SLIGHTLY
C 3 INTERMEDIATE VALUES, AND TO CHECK FOR CONVERGENCE AT THE FAR END
TMAX = TUPPER
TMIN = -1. * TMAX
GO TO 700
670 IF (TOUTER(J2).GT.0.) GO TO 680
IF (TOUTER(J2).LT.0.) GO TO 690
TMAX = TOUTER(1) / 10.
TMIN = -1. * TMAX
GO TO 700
680 TMAX = 1.1 * TOUTER(J2)
TMIN = 0.9 * TOUTER(J2)
GO TO 700
690 TMAX = 0.9 * TOUTER(J2)
TMIN = 1.1 * TOUTER(J2)
C THE BOUNDS ABOVE ARE CRITICAL IN ENSURING CONVERGENCE
C THEY MUST BE NEITHER TOO LARGE NOR TOO SMALL
700 CONTINUE
C
C ASCERTAIN WHETHER INTERNAL LOADS ARE CRITICAL FOR ELASTIC-PLASTIC
C 1 ADHESIVE
710 PSCALE = TOUTER(1) / STRGTR(1)
IF (PSCALE.LT.0.) PSCALE = -1. * PSCALE
PGAMAX = GAMMA(1) / GAMMAX
IF (PGAMAX.LT.0.) PGAMAX = -1. * PGAMAX
DO 730 N = 2, MCHECK
RINR = TINNER(N) / STRGNR(N-1)
C AT EACH STEP TRANSITION NEED THE THINNER SECTION. HENCE THE (N-1)
IF (RINR.LT.0.) RINR = -1. * RINR
IF (RINR.GT.PSCALE) PSCALE = RINR
IF (N.EQ.MCHECK) GO TO 720
ROTR = TOUTER(N) / STRGTR(N)
IF (ROTR.LT.0.) ROTR = -1. * ROTR
IF (ROTR.GT.PSCALE) PSCALE = ROTR
720 RGAMX = GAMMA(N) / GAMMAX
IF (RGAMX.LT.0.) RGAMX = -1. * RGAMX
IF (RGAMX.GT.PGAMAX) PGAMAX = RGAMX
730 CONTINUE
C
C IF UPPER AND LOWER BOUNDS ON JOINT LOAD HAVE COALESCED,
C 1 NO MORE CONVERGENCE IS POSSIBLE. PRINT OUT RESULTS.
P = TUPPER / TLOWER
IF ( (1.000000001.GT.P).AND.(0.999999999.LT.P) ) GO TO 770
C ADJUST MAXIMUM ADHESIVE SHEAR STRAIN IF ADHEREND STRENGTH GOVERNS OVER
C 1 ADHESIVE STRENGTH CONSIDERATIONS
IF ( (RSCALE.GT.1.0001).OR.(RGAMAX.GT.1.0001) )

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A4EG7920

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INPUT DATA

ALPHA = 0.000050 (PER DEG. F) ALPHAI = 0.0 (PER DEG. F) DELTMP = -280.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.7500	0.2500	0.0440	32500.0	3036.0	4000000.0	352000.0
2	0.7500	0.2060	0.0880	26780.0	6072.0	3296000.0	704000.0
3	0.7500	0.1620	0.1320	21060.0	9108.0	2592000.0	1056000.0
4	0.7500	0.1180	0.1760	15340.0	12144.0	1888000.0	1408000.0
5	0.3750	0.0740	0.2200	9620.0	15180.0	1184000.0	1760000.0
6	0.3750	0.0300	0.2640	3900.0	18216.0	480000.0	2112000.0
7	*****	0.0	0.2640	0.0	18216.0	0.0	2112000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1  
 ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0  
 N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.7500	0.2500	0.0440	5696.0	0.095	0.0	-0.0005	10730.1	32500.0	0.0	3036.0
2	0.7500	0.2060	0.0880	2568.5	0.043	-0.0028	-0.0030	8817.0	26780.0	1913.1	6072.0
3	0.7500	0.1620	0.1320	2243.7	0.037	-0.0057	-0.0059	7344.6	21060.0	3385.5	9108.0
4	0.7500	0.1180	0.1760	2239.0	0.037	-0.0087	-0.0089	5804.3	15340.0	4925.8	12144.0
5	0.3750	0.0740	0.2200	2797.8	0.047	-0.0117	-0.0119	4018.4	9620.0	6711.8	15180.0
6	0.3750	0.0300	0.2640	3722.2	0.062	-0.0132	-0.0135	2232.4	3900.0	8497.8	18216.0
7	*****	0.0	0.2640	6000.0	0.100	-0.0147	-0.0152	-0.0	0.0	10730.1	18216.0

INPUT DATA

ALPHA = 0.0 (PER DEG. F) ALPHAI = 0.000050 (PER DEG. F) DELTMP = -280.0 (DEG. F)

N	STEPL	THICKO	THICKI	STROTR	STRINR	ETOTR	ETINR
1	0.3750	0.2640	0.0300	18216.0	3900.0	2112000.0	480000.0
2	0.3750	0.2200	0.0740	15180.0	6072.0	1760000.0	1184000.0
3	0.7500	0.1760	0.1180	12144.0	9108.0	1408000.0	1888000.0
4	0.7500	0.1320	0.1620	9108.0	12144.0	1056000.0	2592000.0
5	0.7500	0.0880	0.2060	6072.0	15180.0	704000.0	3296000.0
6	0.7500	0.0440	0.2500	3036.0	18216.0	352000.0	4000000.0
7	*****	0.0	0.2500	0.0	18216.0	0.0	4000000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1  
 ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0  
 N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	0.100	0.0	-0.0005	10730.1	18216.0	0.0	3900.0
2	0.3750	0.2200	0.0740	3722.2	0.062	-0.0117	-0.0120	8497.8	15180.0	2232.4	9620.0
3	0.7500	0.1760	0.1180	2797.8	0.047	-0.0033	-0.0035	6711.8	12144.0	4018.4	15340.0
4	0.7500	0.1320	0.1620	2239.0	0.037	-0.0064	-0.0066	4925.8	9108.0	5804.3	21060.0
5	0.7500	0.0880	0.2060	2433.7	0.037	-0.0093	-0.0095	3385.5	6072.0	7344.6	26780.0
6	0.7500	0.0440	0.2500	2568.5	0.043	-0.0122	-0.0124	1913.1	3036.0	8817.0	32500.0
7	*****	0.0	0.2500	5696.1	0.095	-0.0147	-0.0152	-0.0	0.0	10730.1	32500.0

ELASTIC-PLASTIC JOINT STRENGTH, PLOAD (LBS) = 16996.6  
 ALLOWABLE ADHESIVE SHEAR STRAIN, GAMMAX = 1.7  
 N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICKO	THICKI	TAU	GAMMA	DELTAO	DELTAI	TOUTER	STROTR	TINNER	STRINR
1	0.1238	0.2640	0.0300	6000.0	0.218	0.0	-0.0011	16996.6	18216.0	0.0	3900.0
2	0.2357	0.2640	0.0300	6000.0	0.100	-0.0010	-0.0014	15510.9	18216.0	1486.7	3900.0
3	0.0155	0.2640	0.0300	6000.0	0.100	-0.0026	-0.0030	13282.4	18216.0	3714.2	3900.0
4	0.0185	0.2200	0.0740	6000.0	0.110	-0.0047	-0.0031	13096.3	15180.0	3900.3	9620.0
5	0.3565	0.2200	0.0740	6000.0	0.109	-0.0077	-0.0071	12874.5	15180.0	4122.1	9620.0
6	0.0	0.2200	0.0740	4616.9	0.077	-0.0077	-0.0071	10040.8	15180.0	6955.3	9620.0
7	0.0	0.7500	0.1760	3483.0	0.058	-0.0120	-0.0114	10040.8	12144.0	6955.8	15340.0
8	0.0	0.1760	0.1180	3483.0	0.058	-0.0120	-0.0114	7168.7	12144.0	9827.9	15340.0
9	0.0	0.1760	0.1180	3483.0	0.058	-0.0120	-0.0114	7168.7	12144.0	9827.9	15340.0
10	0.7500	0.1320	0.1620	3483.0	0.058	-0.0120	-0.0114	7168.7	12144.0	9827.9	21060.0
11	0.0	0.1320	0.1620	3340.8	0.056	-0.0160	-0.0154	4824.1	9108.0	12172.6	21060.0
12	0.0	0.1320	0.1620	3340.8	0.056	-0.0160	-0.0154	4824.1	9108.0	12172.6	21060.0
13	0.7500	0.0880	0.2060	3340.8	0.056	-0.0198	-0.0193	4824.1	6072.0	14320.6	26780.0
14	0.0	0.0880	0.2060	3679.3	0.061	-0.0198	-0.0193	2676.1	6072.0	14320.6	26780.0
15	0.0	0.0880	0.2060	3679.3	0.061	-0.0198	-0.0193	2676.1	6072.0	14320.6	26780.0
16	0.0	0.0440	0.2500	3679.3	0.061	-0.0198	-0.0193	2676.1	3036.0	16358.6	32500.0
17	0.0365	0.0440	0.2500	6000.0	0.100	-0.0233	-0.0232	438.1	3036.0	16996.6	32500.0
18	0.0	0.0440	0.2500	6000.0	0.136	-0.0233	-0.0232	-0.0	3036.0	16996.6	32500.0
19	0.0	0.0	0.2500	6000.0	0.136	-0.0233	-0.0232	-0.0	0.0	16996.6	32500.0

INPUT DATA

ALPHA0 = 0.0 (PER DEG. F) ALPHA1 = 0.000050 (PER DEG. F) DELTNP = -28.0 (DEG. F)

N	STEPL	THICK0	THICK1	STROTR	STRINR	ETOTR	ETINR
1	0.3750	0.2640	0.0300	*****	*****	2112000.0	480000.0
2	0.3750	0.2200	0.0740	*****	*****	1760000.0	1184000.0
3	0.7500	0.1760	0.1180	*****	*****	1408000.0	1888000.0
4	0.7500	0.1320	0.1620	*****	*****	1056000.0	2592000.0
5	0.7500	0.0880	0.2060	*****	*****	704000.0	3296000.0
6	0.7500	0.0440	0.2500	*****	*****	352000.0	4000000.0
7	*****	0.0	0.2500	*****	*****	0.0	4000000.0

ELASTIC JOINT STRENGTH, PLOAD (LBS) = 10730.1  
 ALLOWABLE ADHESIVE SHEAR STRESS, TAUMAX (PSI) = 6000.0  
 N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICK0	THICK1	TAU	GAMMA	DELTA0	DELTA1	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	0.100	0.0	-0.0005	10730.1	*****	0.0	*****
2	0.3750	0.2200	0.0740	3722.2	0.062	-0.0017	-0.0020	8497.8	*****	2232.4	*****
3	0.7500	0.1760	0.1180	2797.8	0.047	-0.0033	-0.0035	6711.8	*****	4018.4	*****
4	0.7500	0.1320	0.1620	2239.0	0.037	-0.0064	-0.0066	4923.8	*****	5804.3	*****
5	0.7500	0.0880	0.2060	2243.7	0.037	-0.0093	-0.0095	3385.5	*****	7344.6	*****
6	0.7500	0.0440	0.2500	2568.5	0.043	-0.0122	-0.0124	1913.1	*****	8817.0	*****
7	*****	0.0	0.2500	5696.1	0.093	-0.0147	-0.0152	-0.0	*****	10730.1	*****

ELASTIC-PLASTIC JOINT STRENGTH, PLOAD (LBS) = 30568.5  
 ALLOWABLE ADHESIVE SHEAR STRAIN, GAMMAX = 1.7  
 N = -1.0 N = +1 FOR TENSILE SHEAR AND -1 FOR COMPRESSIVE  
 TEMPERATURE DIFFERENTIAL (DEG F) = -280.0

N	STEPL	THICK0	THICK1	TAU	GAMMA	DELTA0	DELTA1	TOUTER	STROTR	TINNER	STRINR
1	0.3750	0.2640	0.0300	6000.0	1.700	0.0	-0.0085	30568.5	*****	0.0	*****
2	0.0	0.2640	0.0300	6000.0	1.151	-0.0052	-0.0099	26068.0	*****	4500.0	*****
3	0.0	0.2640	0.0300	6000.0	1.151	-0.0052	-0.0099	26068.0	*****	4500.0	*****
4	0.3750	0.2200	0.0740	6000.0	1.151	-0.0052	-0.0099	26068.0	*****	4500.0	*****
5	0.0	0.2200	0.0740	6000.0	0.669	-0.0105	-0.0122	21568.5	*****	9000.0	*****
6	0.0	0.2200	0.0740	6000.0	0.669	-0.0105	-0.0122	21568.5	*****	9000.0	*****
7	0.7500	0.1760	0.1180	6000.0	0.669	-0.0105	-0.0122	21568.5	*****	9000.0	*****
8	0.0	0.1760	0.1180	6000.0	0.133	-0.0208	-0.0177	12568.5	*****	18000.0	*****
9	0.0	0.1760	0.1180	6000.0	0.133	-0.0208	-0.0177	12568.5	*****	18000.0	*****
10	0.0520	0.1320	0.1620	6000.0	0.100	-0.0214	-0.0182	11944.1	*****	18624.4	*****
11	0.6980	0.1320	0.1620	6000.0	0.100	-0.0214	-0.0182	11944.1	*****	18624.4	*****
12	0.0	0.1320	0.1620	5744.6	0.196	-0.0271	-0.0240	7948.2	*****	22620.3	*****
13	0.7476	0.0880	0.2060	5744.6	0.096	-0.0271	-0.0240	7948.2	*****	22620.3	*****
14	0.0024	0.0880	0.2060	6000.0	0.100	-0.0337	-0.0306	4326.7	*****	26213.1	*****
15	0.0	0.0880	0.2060	6000.0	0.100	-0.0337	-0.0306	4326.7	*****	26213.1	*****
16	0.0018	0.0440	0.2500	6000.0	0.100	-0.0337	-0.0306	4326.7	*****	26213.1	*****
17	0.6194	0.0440	0.2500	6000.0	0.100	-0.0337	-0.0306	4326.7	*****	26213.1	*****
18	0.1288	0.0440	0.2500	6000.0	0.100	-0.0389	-0.0358	1545.5	*****	29023.0	*****
19	0.0	0.0	0.2500	6000.0	0.271	-0.0393	-0.0369	0.0	0.0	30568.5	*****