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Adhesive contact of a conical punch on an elastic half-space (*)

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Résumé. — On montre que la théorie générale de Sneddon (1965) permet d'étudier les contacts de poinçons axisymétriques adhésifs. On donne la relation entre charge et pénétration, et la force d'adhérence pour un cône de révolution.

Abstract. — It is shown that the Sneddon general theory (1965) enables the contact of axisymmetric adhesive punches to be studied. The relation between load and penetration, and the adherence force for conical punch are given.

Sneddon [1] has derived a solution of the axisymmetric Boussinesq problem from which he deduced simple formulae for the depth of penetration δ of the tip of a punch of arbitrary profile, for the total load which must be applied to the punch to achieve this penetration, for distribution of pressure $\sigma_z(r, 0)$ under the punch, and for the displacement $u_z(r, 0)$ of the surface :

$$\delta = \int_0^1 \frac{f'(x)}{\sqrt{1 - x^2}} \,\mathrm{d}x + \frac{\pi}{2}\chi(1) \tag{1}$$

$$P = \frac{\pi a E}{1-\nu^2}\int_0^1 \chi(t)\,\mathrm{d}t = \frac{2\,Ea}{1-\nu^2}\left[\delta - \int_0^1 \frac{x f(x)}{\sqrt{1-x^2}}\,\mathrm{d}x\right] \tag{2}$$

$$\sigma_z(r, 0) = -\frac{E}{2\,a(1-\nu^2)}\left[\frac{\chi(1)}{\sqrt{1-\dfrac{r^2}{a^2}}} - \int_{r/a}^1 \frac{\chi'(t)}{\sqrt{t^2 - \dfrac{r^2}{a^2}}}\,\mathrm{d}t\right], \quad r < a \tag{3}$$

$$u_z(r, 0) = \int_0^1 \frac{\chi(t)}{\sqrt{\dfrac{r^2}{a^2} - t^2}}\,\mathrm{d}t\,, \quad r > a\,. \tag{4}$$

In these formulae, a is the radius of the contact, $z = f(x) = f(r/a)$ describes the profile of the punch (with $f(0) = 0$) and $\chi(t)$ is defined by

$$\chi(t) = \frac{2}{\pi}\left(\delta - t\int_0^t \frac{f'(x)}{\sqrt{t^2 - x^2}}\,\mathrm{d}x\right). \tag{5}$$

For punches with continuous profile, Sneddon lets $\chi(1) = 0$ in order to have a finite stress at the edge of the contact and uses this criterion to determine δ. For spherical or conical punches, the classical results are easily found.

The aim of this note is to show that the hypothesis $\chi(1) = 0$ is not imperative, and that $\chi(1) \neq 0$ allows us to describe adhesive contacts taking into account the Dupré energy of adhesion $w = \gamma_1 + \gamma_2 - \gamma_{12}$ of the facing solids (the γ_i and γ_{ij} are the surface and interfacial energies). Letting

$$K_{\mathrm{I}} = -\frac{E}{2(1 - \nu^2)}\sqrt{\frac{\pi}{a}}\cdot\chi(1) \tag{6}$$

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the stress σ_z and the discontinuity of displacement

$$[u_z] = f\left(\frac{r}{a}\right) - \delta + u_z(r, 0) \qquad (7)$$

at a distance ρ of the edge of the contact can be written

$$\sigma_z(a - \rho, 0) \sim \frac{K_I}{\sqrt{2\pi\rho}} \qquad (8)$$

$$[u_z(a + \rho)] \sim \frac{4(1 - \nu^2)}{E} \cdot K_I \sqrt{\frac{\rho}{2\pi}}. \qquad (9)$$

These formulae are found in fracture mechanics in mode I and plane deformation, and K_I is the stress intensity factor. They appear because the edge of the contact can be considered as a fracture tip that recedes or advances according as the load P increases or decreases. In this case the strain energy release rate is given by

$$G = \frac{1}{2} \cdot \frac{1 - \nu^2}{E} \cdot K_I^2 \qquad (10)$$

(the factor 1/2 arises because the punch is not deformable), and at equilibrium one has $G = w$. Let P_1, the apparent load which, if $\chi(1) = 0$ would give the same radius of contact a as observed under the load P with $\chi(1) \neq 0$. Then

$$P_1 - P = -\frac{\pi E a}{1 - \nu^2} \cdot \chi(1) = (4\pi a^3)^{1/2}.K_I \qquad (11)$$

i.e.

$$G = \frac{1 - \nu^2}{E} \cdot \frac{(P_1 - P)^2}{8\pi a^3}. \qquad (12)$$

For a spherical punch (radius R), taking the approximation

$$f(x) = \frac{a^2}{2R} x^2,$$

all the results of the Johnson, Kendall, Roberts [2] theory are directly found. The relations between their theory and fracture mechanics have been shown elsewhere [3].

In the case of a conical punch, with semi-angle $\frac{\pi}{2} - \beta$, one has

$$f(x) = (a.\mathrm{tg}\,\beta).x \qquad (13)$$

$$\chi(t) = \frac{2\delta}{\pi} - a.t.\mathrm{tg}\,\beta \qquad (14)$$

hence

$$\delta = \frac{\pi a}{2}\mathrm{tg}\,\beta - \left(\frac{1 - \nu^2}{E}\pi a w\right)^{1/2} \qquad (15)$$

$$P = \frac{2Ea}{1 - \nu^2}\left(\delta - \frac{\pi a}{4}\mathrm{tg}\,\beta\right) \qquad (16)$$

$$\sigma_z(r, 0) = \frac{P_1 - P}{2\pi a^2} \cdot \frac{1}{\sqrt{1 - \frac{r^2}{a^2}}} - \frac{P_1}{\pi a^2}\cosh^{-1}\frac{a}{r}, \quad r < a \qquad (17)$$

$$u_z(r, 0) = -\frac{P_1 - P}{a} \cdot \frac{1 - \nu^2}{\pi E}\sin^{-1}\frac{a}{r} + a\,\mathrm{tg}\,\beta\left[\sin^{-1}\frac{a}{r} - \frac{r}{a} + \sqrt{\frac{r^2}{a^2} - 1}\right], \quad r > a \qquad (18)$$

with

$$P_1 = \frac{\pi E}{2(1 - \nu^2)} a^2\,\mathrm{tg}\,\beta. \qquad (19)$$

It can be verified that the connection of the elastic half-space to the cone is tangential if $\chi(1) = 0$, and vertical if $\chi(1) \neq 0$ (geometry of fracture mechanics). The relation between P_1 and P is given, from equations (12) and (19), by

$$(x_1 - x)^4 = x_1^3 \qquad (20)$$

where $x_1 = P_1/P^*$ and $x = P/P^*$ with

$$P^* = \frac{1 - \nu^2}{\pi E} \cdot \left(\frac{8}{\mathrm{tg}\,\beta}\right)^3 . w^2. \qquad (21)$$

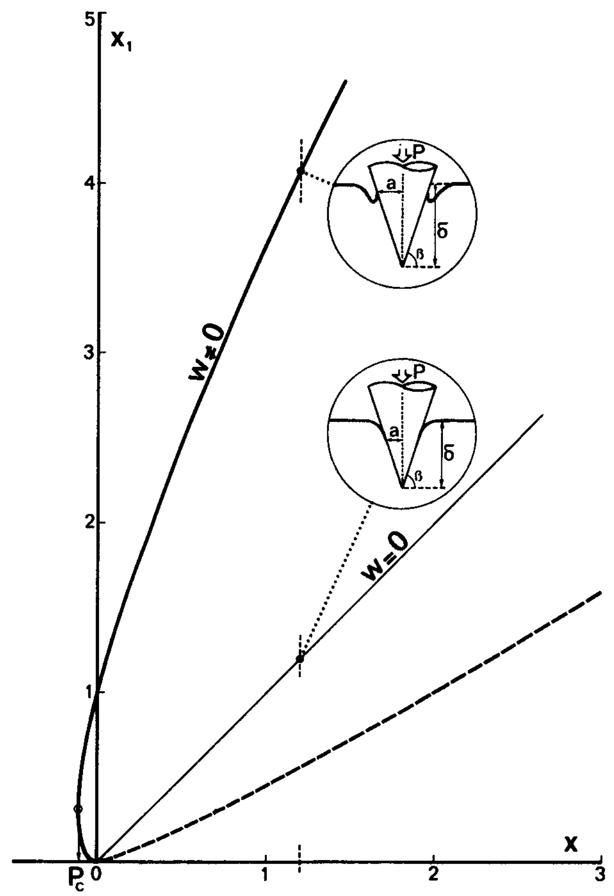

Fig. 1. — Apparent load P_1 *versus* applied load P, in reduced coordinates.

This relation is shown on figure 1. The radius of contact under zero load is

$$a = \frac{1 - \nu^2}{2\,\pi E}\left(\frac{8}{\operatorname{tg}\beta}\right)^2 w \qquad (22)$$

and the quasistatic force of adherence at fixed load (vertical tangent to the curve) is :

$$P_c = -\frac{54(1 - \nu^2)\,w^2}{\pi . E . \operatorname{tg}^3 \beta}. \qquad (23)$$

The stress distribution under the punch results from the superposition, on the same area, of the stresses due to a non adhesive conical punch under a load P_1, and the stresses due to a flat punch under the tensile load $P_1 - P$. This stress distribution is shown on figure 2, in reduced coordinates, for the case $P = 0$.

Stresses are infinite at the origin of coordinates, as for conventional conical punches; and infinite at $r = a$, as for conventional flat punches. Physically this means that a certain volume of material under the apex and at the periphery will flow plastically and hence reduce the high concentration of stress in that neighbourhood. If $\delta/a \ll 1$, i. e. for $\beta \ll 1$, the plastic flow will be confined to a very small region, and the elastic solution will have a wider field of application.

These results could also be obtained by a method similar to that in [2] by writing that the total energy of the system remains constant for a variation of contact area $dA = 2\,\pi a\,da$ at fixed load P.

This increase of the area of contact under the action of attractive molecular forces, certainly plays a non negligible role in the hardness of solids under light load.

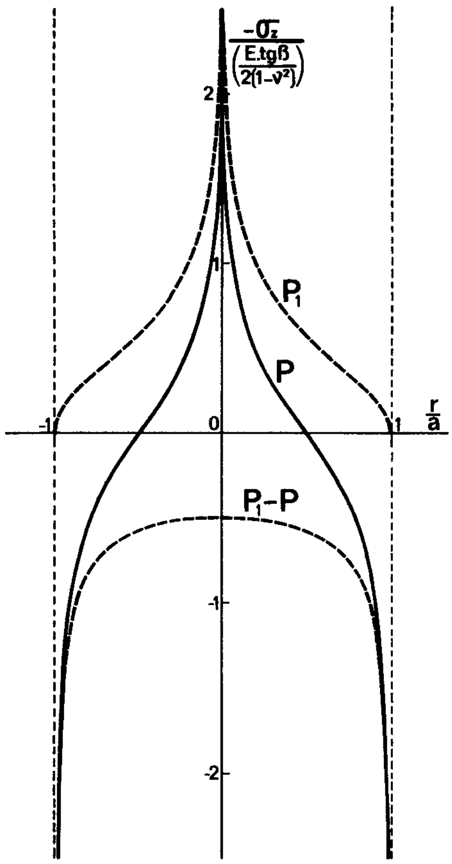

Fig. 2. — Stress distribution under a conical punch, in reduced coordinates.

References

[1] Sneddon, I. N., *Int. J. Eng. Sci.* 3 (1965) 47-57.

[2] Johnson, K. L., Kendall, K. and Roberts, A. D., *Proc. R. Soc.* **A 324** (1971) 301-313.

[3] Maugis, D. and Barquins, M., in *Adhesion and adsorption of polymers*, L. H. Lee ed. (Plenum Publ. Corp.) part A, 1980, p. 203-277.