Adjacent Cell Interference in FH-MFSK Cellular Mobile Radio System

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Abstract—**The effect of adjacent cell interference in cellular mobile system using FH-MFSK transmission is evaluated quantitatively. The performance of base to mobile communication in the system is analyzed, assuming perfect synchronization between users in all the cells. Analysis of the system employing no power control shows that the number of simultaneous users possible at average bit error probability P_b of less than 1×10^{-3} is reduced greatly from the corresponding figure for the isolated cell (which is about 170). It is then shown that a simple power control strategy could reduce the adjacent cell interference significantly. A reasonable knowledge of the distribution of users within a cell allows the optimization of the receiver threshold with respect to distance from the base. With this optimization, each cell could accomodate $\gtrsim 115$ users at $P_b < 10^{-3}$, the exact figure being dependent on the user distribution. The power control also helps to reduce the average power transmitted from a base.**

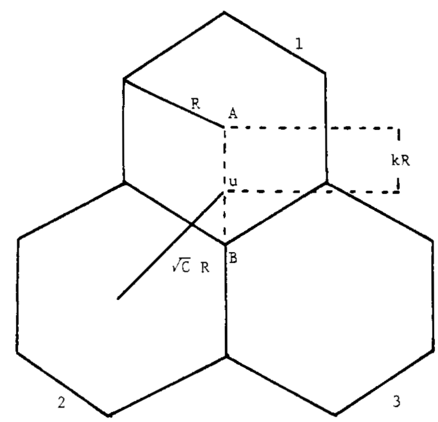

Fig. 1. Three-cell geometry.

I. INTRODUCTION

FOLLOWING the suggestion of FH-MFSK for mobile radio [1], many analyses of the system have appeared [2]-[4]. For such a scheme, the number of simultaneous users that could be handled at a specific bit error rate is known to decrease when more cells are operating nearby. However, no quantitative assessment has been made so far. Here, we analyze a cellular mobile radio system employing FH-MFSK modulation [1] with respect to the adjacent cell interference. We assume perfect synchronization between all the users in all the cells and random address assignment for a user in a cell. The analysis is more true in base to mobile transmission where near perfect synchronization is possible. Also, the power control strategy considered is exclusive to base to mobile transmission, and hence the following analysis is applicable to such a transmission.

Section II discusses the system in which each cell in a three-cell cellular system operates with constant power transmission (i.e., no power control). In Section III we analyze the effect of adjacent cell interference in a three-cell cellular system employing power control. Section IV extends the results of Section III to a more general cellular system with six nearest neighbors. This is followed by discussion and conclusion.

II. ANALYSIS OF A THREE-CELL CELLULAR SYSTEM WITHOUT POWER CONTROL

Consider a user u moving along the line AB in a cell of a three-cell system (Fig. 1). The propagation delay difference due to the difference in distances kR and $\sqrt{C}R$ (for $0.7 < k < 1$) is assumed small compared to one slot duration $\tau \cong 13\ \mu$s. This is true for cells of smaller size. Because of this, the orthogonality of received tones is approximatey valid.[1] The user is worst-positioned in the sense that he/she is subject to equal interference from both the nearby cells 2 and 3. Assuming M users are operating in each cell, we calculate the average probability of bit error P_b for the user u. We say average probability because the expression for P_b arrived at is by averaging all the probability of bit errors resulting from an ensemble of all possible random address assignment. By looking at Fig. 1, we can calculate the following. The signal to noise ratio SN_1 at u due to a tone from base 1 is

$$SN_1 = 10^{(S - 40 \log k)},$$

where S is the average SNR in decibels when u is in the cell corner. The signal to noise ratio at u due to a tone from base 2 or base 3 is

$$SN_2 = 10^{(S - 20 \log ((1.5-k)^2 + 0.75))}$$

and b is the normalized threshold (normalized with respect to noise at the receiver).

Consider the spurious row of user u. The probability that a tone corresponding to an entry in a spurious row is being transmitted by base 1 or 2 or 3 is

$$Pu = 1 - (1 - 2^{-K})^M, \tag{1}$$

where K is the number of bits per transmitted word and M is the number of users in any cell.

Manuscript received April 19, 1982; revised December 20, 1982. This work was supported by AFOSR Grant 77-3277. This paper was presented at the Global Communications Conference, Bal Harbor, FL, November 29-December 3, 1982.

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[1] More about the effect of propagation delay difference is discussed in the next section.

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Define the following. The probability of false alarm PF is

$$\mathrm{PF} = e^{-b^2/2}. \tag{2}$$

The probability of creating an entry conditioned on the fact that base 1 transmits the tone corresponding to that entry is

$$\mathrm{PE}_1 = \exp(-b^2/2(1 + \mathrm{SN}_1)). \tag{3}$$

The probability of creating an entry conditioned on the fact that base 2 or 3, but not both, transmits the tone corresponding to that entry is

$$\mathrm{PE}_2 = \exp(-b^2/2(1 + \mathrm{SN}_2)). \tag{4}$$

Similarly,[2]

$$\mathrm{PE}_3 = \exp(-b^2/2(1 + \mathrm{SN}_1 + \mathrm{SN}_2)) \tag{5}$$

$$\mathrm{PE}_4 = \exp(-b^2/2(1 + 2\,\mathrm{SN}_2)) \tag{6}$$

$$\mathrm{PE}_5 = \exp(-b^2/2(1 + \mathrm{SN}_1 + 2\,\mathrm{SN}_2)) \tag{7}$$

$$\overline{Pu} = 1 - Pu. \tag{8}$$

Then the unconditional probability of creating an entry in the spurious row, which we call the probability of insertion PI, can be calculated as

$$\begin{aligned}\mathrm{PI} = \mathrm{PF}\,\overline{Pu}^3 + Pu\,\overline{Pu}^2\,\mathrm{PE}_1 + 2Pu\,\overline{Pu}^2\,\mathrm{PE}_2 \\ + 2\overline{Pu}\,Pu^2\,\mathrm{PE}_3 + \overline{Pu}\,Pu^2\,\mathrm{PE}_4 + Pu^3\,\mathrm{PE}_5.\end{aligned} \tag{9}$$

Next, consider the correct row of user u. Proceeding along similar lines, the probability of an entry in the correct row of user u is

$$P_C = \overline{Pu}^2\,\mathrm{PE}_1 + 2Pu\,\overline{Pu}\,\mathrm{PE}_3 + Pu^2\,\mathrm{PE}_5. \tag{10}$$

The probability of i entries in the correct row is

$$P_C(i) = \binom{L}{i} P_C{}^i (1 - P_C)^{L-i}. \tag{11}$$

Using (9) and (11) for PI and $P_C(i)$, respectively, and using [1, table 1], we can calculate P_b. With P_b maintained at a value less that 1×10^{-3}, it is seen that increasing SN_1 beyond a certain value does not give a significant return in accomodating more simultaneous users M. Hence we assume a constant power transmission such that the farthest user in a cell has $S = 25$ dB at the receiver. Fig. 2 shows the number of users that could be accomodated at $P_b < 10^{-3}$ for various distances of the worst-positioned receiver from the base, i.e., for various kR, $k = 0.7, \cdots, 1.0$. For each k, the threshold b is optimized to minimize P_b. Fig. 2 shows that, when the worst-positioned receiver is at the cell corner, only 58 users could be accomodated at $P_b < 10^{-3}$ and only 64 users when the worst-positioned receiver is at three-fourths the distance from the base to the cell corner. Because of the poor performance of the above system, the need for power control is apparent.

[2] At the noncoherent detector, the variances of the interfering signals add together in both in-phase and quadrature phase components.

III. ADJACENT CELL INTERFERENCE IN A THREE-CELL CELLULAR SYSTEM WITH POWER CONTROL

In this section, we calculate the probability of bit error rate P_b for a user u in cell 1 (Fig. 1), taking into account the effect of adjacent cell interference, assuming each cell operates with power control. Before proceeding to calculate P_b, we must consider 1) the power control strategy and 2) the propagation delay difference in the arrival of tones from the bases at the user and its effect on synchronization and nonorthogonal interference.

1) Power Control Strategy: Consider a set of tones to be transmitted from base 1 during a slot period. If more than one user wants to transmit a particular frequency tone in a time slot, then the control unit at the base determines the distance of the farthest user requiring the frequency tone and adjusts the transmitted power accordingly, so that the farthest receiver receives a fixed average SNR (say $10 \times S$ dB). We assume that this is the power control strategy being employed by the system.

2) Effect of Propagation Delay Difference: As in [1] we assume that noncoherent detection is employed to detect on/off keyed tones. Due to multipath effect, a transmitted tone from base 1 arrives at user u through different paths at different times. The difference in the arrival times has a probability distribution, and the delay spread is a measure of spread in the arrival times [6]. The delay spread varies from place to place, usually ranging from 0.1 μs up to 2 μs. Since this is much less than τ, it is possible to assume that the received waveform $r_1(t)$ is approximately given by

$$r_1(t) = a_1 x\,(t - \tau_{p_1}) e^{j2\pi(f_0 t - \theta_1)}, \quad 0 < t - \tau_{p_1} < \tau \tag{12}$$

where $x(t)$ is related to the transmitted wave $s(t)$ as

$$s(t) = x(t) e^{j2\pi f_0 t}, \quad 0 < t < \tau. \tag{13}$$

Here τ_{p1} represents the propagation delay corresponding to a line of sight component, the random variable a_1 is Rayleigh distributed, and the phase θ_1 is uniformly distributed.

In this analysis, we assume that the tones are transmitted synchronously from the bases. However, the tones from bases 1 and 2 (or 3) will arrive at different times depending on the position of the user u. Assuming that the same tone is transmitted from bases 1 and 2 during a slot interval, the corresponding received waveforms at the receiver would be

$$r_i(t) = a_i x(t - \tau_{p_i}) e^{j2\pi(f_i t - \theta_i)}, \quad i = 1, 2. \tag{14}$$

Here, a_i are independent and identically distributed. Similar comments apply to θ_i. The difference $(\tau_{p2} - \tau_{p1})$ is of interest, and one can easily see by looking at Fig. 1 that

$$0 < (\tau_{p_1} - \tau_{p_2}) < \sqrt{3}(R/v) \tag{15}$$

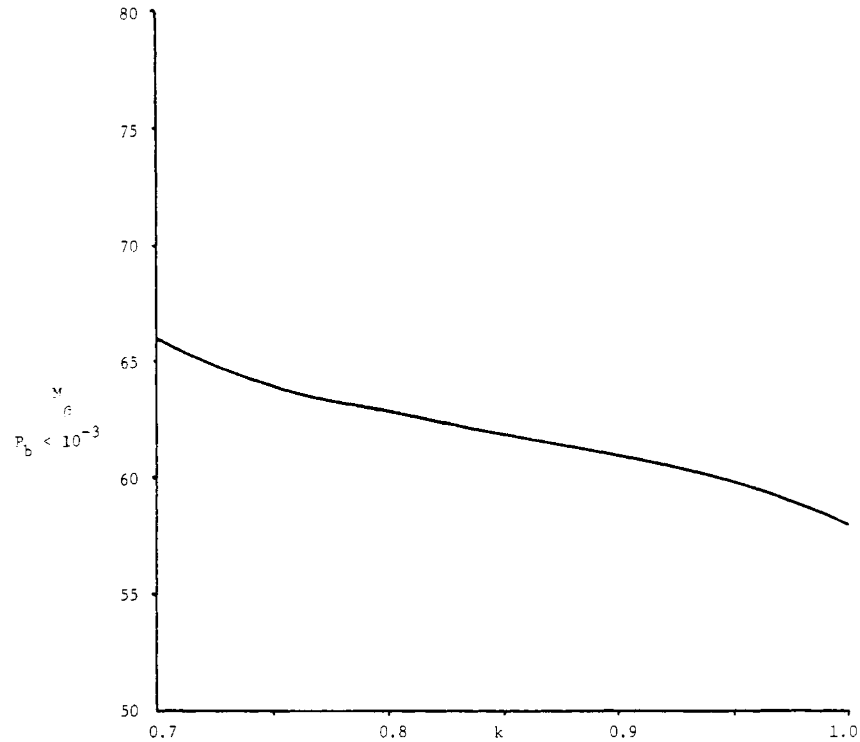

Fig. 2. Number of users at $P_b < 10^{-3}$ versus distance of user u from base 1.

where v is the velocity of light. When R is 9 km, the difference is bounded by 52 μs, which is about a four-slot duration.

With power control, the average received power in a slot of the received matrix of u due to transmission from base 1 can be expected to be higher than the interference power from adjacent cells. Also, the receiver u knows the radial distance from the base 1 and hence has an approximate knowledge of τ_{p1}. For these two reasons, it can be assumed that the receiver could synchronize the tones from base 1 with reasonable accuracy.

Consider the ith tone detector at receiver u. Assuming that the base 2 transmits the ith tone at the $(j\tau)$th instant (j an integer), the tone will arrive at u at the $((j + m)\tau + a\tau + \tau_{p1})$th instant, where m is an integer taking values 0, 1, 2, $\cdots$, $0 < a < 1$, and $(m\tau + a\tau)$ is the propagation delay difference $(\tau_{p2} - \tau_{p1})$. The interfering tone from base 2 will cause interference over a duration of $(1 - a)\tau$ s in the mth slot, with the arrival slot of tone from base 1 counted as zero. Also, a spill-over interference of duration $a\tau$ s will occur in the $(m + 1)$th slot. Thus an interfering tone from cell 2 is likely to cause interference in two adjacent slots. However, because of non-coherent detection, the sum of the interfering power in the adjacent slots is the same as that of an interfering tone over a full slot. Moreover, if the ith tone has been transmitted by base 2 during the $(j - 1)\,\tau$th instant, then the $(m - 1)$th slot will have interference over $(1 - a)\tau$ s and the mth slot over $a\tau$ s. Hence on an average, the effect of the ith tone interference from base 2 at the ith tone detector in u can be calculated assuming interference over a full slot.

When $(\tau_{p2} - \tau_{p1}) = 0 \bmod \tau$ approximate orthogonality of the jth tone with the ith carrier is fully justified, and hence the ith detector output will be negligible.

With $A \sin [(\omega_0 + (2\pi/\tau)i)t]$ denoting the carrier used in the ith detector, the in-phase output due to the jth tone from base 2 is

$$y_i^j = \int_0^{a\tau} A \sin\left(\omega_0 + \frac{2\pi}{\tau} i\right)t \cdot a_2 \sin\left(\left(\omega_0 + \frac{2\pi}{\tau} j\right)t + \theta\right) dt. \tag{16}$$

Let

$$a_2 \cos\theta \sim \text{normal}\,(0, \sigma^2)$$

$$a_2 \sin\theta \sim \text{normal}\,(0, \sigma^2).$$

Then it can be seen that y_i^j conditioned on $a\tau$ is also normally distributed, and

$$\operatorname{var}[y_i^j/a\tau] = \frac{\frac{A^2\sigma^2}{4}}{\pi^2(i-j)^2} \sin^2(\pi(i-j)a). \tag{17}$$

It can also be seen that

$$\operatorname{var}[y_i^i] = \frac{A^2\sigma^2}{4}. \tag{18}$$

Therefore, only the tones adjacent to the ith tone interfere significantly with the interference power being less than $(1/\pi^2(i - j)^2)$ of that of a possible ith tone. With more than 100 users operating in each cell, the probability of an ith tone being present at the input to the detector due to users is more than 0.7. Clearly, it is possible to conceive two situations:

1) ith tone present along with other tones,
2) ith tone not present.

In case 1) neglecting the nonorthogonal interference (17) implies that we are underestimating the probability of detection slightly, and hence we have a pessimistic assessment. On the contrary, in situation 2) the nonorthogonal interference is neglected compared to the noise at the receiver, which can at best be an approximation. With the above observations in mind, we shall neglect the effect of nonorthogonality in the following analysis.

Computation of P_b

In order to compute P_b, we first compute SN_0, the average signal to noise power ratio in the correct row of u due to users in the same cell; IN, the average interference to noise power ratio in the receiver of user u due to users in cell 2 or 3; and IN_0, the average interference to noise power ratio in the spurious row of u due to users in the same cell.

1) Computation of SN_0: Consider the time-frequency slot $((i, j), i \in (1 \cdots L), j \in (0, 1, \cdots, 2^K - 1))$ which is detected and transformed as an entry in the correct row of the decoded matrix of user u. Corresponding to this ith slot, the jth tone will be transmitted by the base. Because of the power control, the transmitted power in this frequency tone depends on the distance of the farthest user requiring the same tone in the ith slot. It is shown in the Appendix that the probability of more than four users in a cell requiring the transmission of the same tone in the same slot is negligibly small, and hence we need to consider at most four users in deciding the transmitted power. The calculation of transmitted power is also tied up with the distribution of users within a cell. We assume that the users are distributed uniformly in a cell with respect to distance from the base. If the base station is situated in a highly concentrated downtown area, we could expect the concentration of users to be near the base. However, as will be seen later, the optimum threshold in the receiver depends on the user distribution, and hence a reasonable knowledge of user distribution within a cell is necessary for best results.

Let u_n be the distance of the farthest user for an n sample user space creating an (i, j)th entry in the transmitted matrix. With

$$f_u(u) \sim \text{uniform}\,(A1 \times R, A2 \times R)\, A1 \times R \leqslant u \leqslant A2 \times R, \tag{19}$$

the maximum order statistic u_n has the distribution

$$f_{u_n}(X) = \frac{n}{(A2 \times R - A1 \times R)^n}\,(X - A1 \times R)^{n-1} \cdot A1 \times R \leqslant X \leqslant A2 \times R. \tag{20}$$

Normalizing u_n with respect to cell radius R, i.e., $u_n = R \times l_n$,

$$f_{l_n}(l) = \frac{n}{(A2 - A1)^n}\,(l - A1)^{n-1} A1 \leqslant l \leqslant A2. \tag{21}$$

We shall assume hereafter that $A1 = 0.05$, $A2 = 0.9$, and a uniform distribution of users with respect to the radial distance as well as the angular position. This geometry very closely approximates the hexagonal structure. With the worst-positioned receiver u at distance kR from the base, we have

$$P_n = \Pr[u_n < kR] = \Pr[l_n < k].$$

Hence

$$P_n = \left(\frac{k - A1}{A2 - A1}\right)^n \tag{22}$$

$$\overline{P_n} = 1 - P_n. \tag{23}$$

Then the average signal to noise ratio SN_0 can be evaluated as follows. Let

p_r $= 2^{-K}$,
$10 \times S$ average SNR in decibels at the farthest receiver,
M number of users operating in a cell (assumed same for all cells).

Then

$$\mathrm{SN}_0 = 10^S \left\{ (1 - p_r)^{M-1} + \sum_{n=1}^{4} \binom{M-1}{n} p_r{}^n \cdot (1 - p_r)^{M-1-n} [P_n + \overline{P_n} I_{son}/k^4] \right\}, \tag{24}$$

where I_{son} is the fourth moment (about origin) of l_n,

$$\begin{aligned} I_{son} &= \int_{A1}^{A2} l^4 \frac{n}{(A2 - A1)^n} (l - A1)^{n-1}\, dl \\ &= \left\{ \left(\frac{n}{n+4}\right)(A2 - A1)^4 + A1^4 + 6A1^2(A2 - A1)^2 \cdot \left(\frac{n}{n+2}\right) + 4A1^3\left(\frac{n}{n+1}\right)(A2 - A1) + 4A1\left(\frac{n}{n+3}\right)(A2 - A1)^3 \right\}. \end{aligned} \tag{25}$$

2) Computation of IN: Notice that the average interference to noise ratio IN is the same for either the entry in a spurious row or the entry in the correct row of user u. Considering the geometry in Fig. 1 and defining the distance squared between the base 2 or base 3 to the user u as CR^2, we have

$$C = (1.5 - k)^2 + 0.75. \tag{26}$$

Then IN can be calculated as follows:

$$\mathrm{IN} = 10^S \sum_{n=1}^{4} \binom{M}{n} p_r{}^n (1 - p_r)^{M-n} I_{In}. \tag{27}$$

where

$$I_{In} = \int_{A1}^{A2} (l^2/C)^2 \frac{n}{(A2-A1)^n} (l-A1)^{n-1} \, dl$$

or

$$I_{In} = \frac{1}{C^2} I_{son}. \tag{28}$$

3) Computation of IN_0: Proceeding along the same lines, we have

$$\mathrm{IN}_0 = 10^S \sum_{n=1}^{4} \binom{M-1}{n} p_r^n (1-p_r)^{M-1-n} I_{son}/k^4. \tag{29}$$

Using (22)-(29) calculating the average P_b is then possible. Define the following, as in the previous section.

The probability of false alarm is $\mathrm{PF} = e^{-b^2/2}$:

$$\mathrm{PE}_1 = \exp(-b^2/2(1+\mathrm{IN}_0)) \tag{30}$$

$$\mathrm{PE}_2 = \exp(-b^2/2(1+\mathrm{IN})) \tag{31}$$

$$\mathrm{PE}_3 = \exp(-b^2/2(1+\mathrm{IN}_0+\mathrm{IN})) \tag{32}$$

$$\mathrm{PE}_4 = \exp(-b^2/2(1+2\mathrm{IN})) \tag{33}$$

$$\mathrm{PE}_5 = \exp(-b^2/2(1+\mathrm{IN}_0+2\mathrm{IN})) \tag{34}$$

$$Pu_1 = 1-(1-2^{-K})^{M-1} \tag{35}$$

$$Pu_2 = 1-(1-2^{-K})^{M} \tag{36}$$

$$\overline{Pu}_1 = 1 - Pu_1 \tag{37}$$

$$\overline{Pu}_2 = 1 - Pu_2. \tag{38}$$

Then PI is given by

$$\mathrm{PI} = \overline{Pu}_1 \overline{Pu}_2^{\,2}\, \mathrm{PF} + Pu_1 \overline{Pu}_2^{\,2}\, \mathrm{PE}_1 + 2Pu_2 \overline{Pu}_2 \overline{Pu}_1\, \mathrm{PE}_2 + 2\overline{Pu}_2 Pu_1 Pu_2\, \mathrm{PE}_3 + \overline{Pu}_1 Pu_2^{\,2}\, \mathrm{PE}_4 + Pu_2^{\,2} Pu_1\, \mathrm{PE}_5. \tag{39}$$

Similarly, P_C is given by

$$P_C = \overline{Pu}_2^{\,2}\, \mathrm{PE}_6 + 2Pu_2 \overline{Pu}_2\, \mathrm{PE}_7 + Pu_2^{\,2}\, \mathrm{PE}_8, \tag{40}$$

where

$$\mathrm{PE}_6 = \exp(-b^2/2(1+\mathrm{SN}_0)) \tag{41}$$

$$\mathrm{PE}_7 = \exp(-b^2/2(1+\mathrm{SN}_0+\mathrm{IN})) \tag{42}$$

$$\mathrm{PE}_8 = \exp(-b^2/2(1+\mathrm{SN}_0+2\,\mathrm{IN})). \tag{43}$$

Then

$$P_C(i) = \binom{L}{i} P_C^{\,i} (1-P_C)^{L-i}. \tag{44}$$

3) Evaluation and Discussion of the Behavior of P_b: With PI and $P_C(i)$ as specified in (39) and (44) and with [1, Table I], we can now calculate P_b. Fig. 3 shows P_b versus k for three different values of $10 \times S$ and for few values of M. In each case, the threshold b in the receiver is optimized. That is, the value of b (nearest to 0.5 or 1 or 2 depending on the value of $10 \times S$ of 15 dB or 25 dB or 35 dB) that minimizes P_b is assumed. The following observations are made by looking at Fig. 3.

1) Increasing $10 \times S$ beyond 25 dB does not give rise to any significant reduction in P_b.
2) P_b depends both on the values of k and on M.
3) P_b peaks for some value near $k \cong 0.5$ and falls off on either side.

This last peculiar phenomenon, observation 3), wherein a mobile at the cell corner operates with much less P_b compared to a mobile near half the distance towards the base can be explained like this. Consider $S = 2.5$. Near the base station $\mathrm{SN}_0 > 30$ dB and $\mathrm{IN} < 7$ dB. It is known that an increase of transmitted power in an isolated cell decreases P_b, and with the same phenomenon happening here, P_b is decreased as the user moves towards the base. Towards the boundary of the cell, $\mathrm{SN}_0 \to 25$ dB, $\mathrm{IN}_0 < 19$ dB, and IN is $\lesssim 12$ dB. Since IN contributes to both the spurious and correct row equally and since IN_0 is significantly smaller than SN_0, a reduction in P_b occurs. At about halfway toward the boundary, $\mathrm{IN} < 10$ dB, $\mathrm{SN} \cong \mathrm{IN}_0 \cong 25$ dB and we get an increased P_b.

With the assumption of power control, the base station knows the radial distance kR of user u, and hence by transmitting this information to the mobiles the threshold can be optimized with respect to the distance. Requiring further the knowledge of M in each cell at the mobiles seems overly demanding. Hence we assume that the optimization of the threshold with respect to distance is done by assuming that the maximum number of users allowed is operating in each cell (say 125, as assumed in the following). Even though this threshold is suboptimal when the number of users operating in a cell is less than 125, the resulting P_b is less than 2×10^{-3}, and hence such a scheme is acceptable. Accordingly, with $10 \times S = 25$ dB and b optimized at each k, we get the set of curves P_b versus M for various k as shown in Fig. 4. The result shows that about 118 users can operate in each cell in a three-cell configuration at $P_b \lesssim 10^{-3}$. Compared to an isolated channel case, this is a reduction of 30 percent in spectral efficiency.

IV. ADJACENT CELL INTERFERENCE IN A GENERAL CELLULAR SYSTEM

Fig. 5 shows the general cellular structure employed in mobile radio. We consider only the nearest six neighbors in the interference analysis, since other cells contribute negligibly small interference power. The computation in the previous section shows that with $S = 2.5$, IN is typically < 12 dB, whereas IN_0 and SN_0 are more than 20 dB, thereby implying that the interference from the adjacent cell is small compared to the interference from the same cell. Therefore, we expect that the six adjacent cell configuration should perform nearly as well as a three-cell system. Indeed, this is the case as the

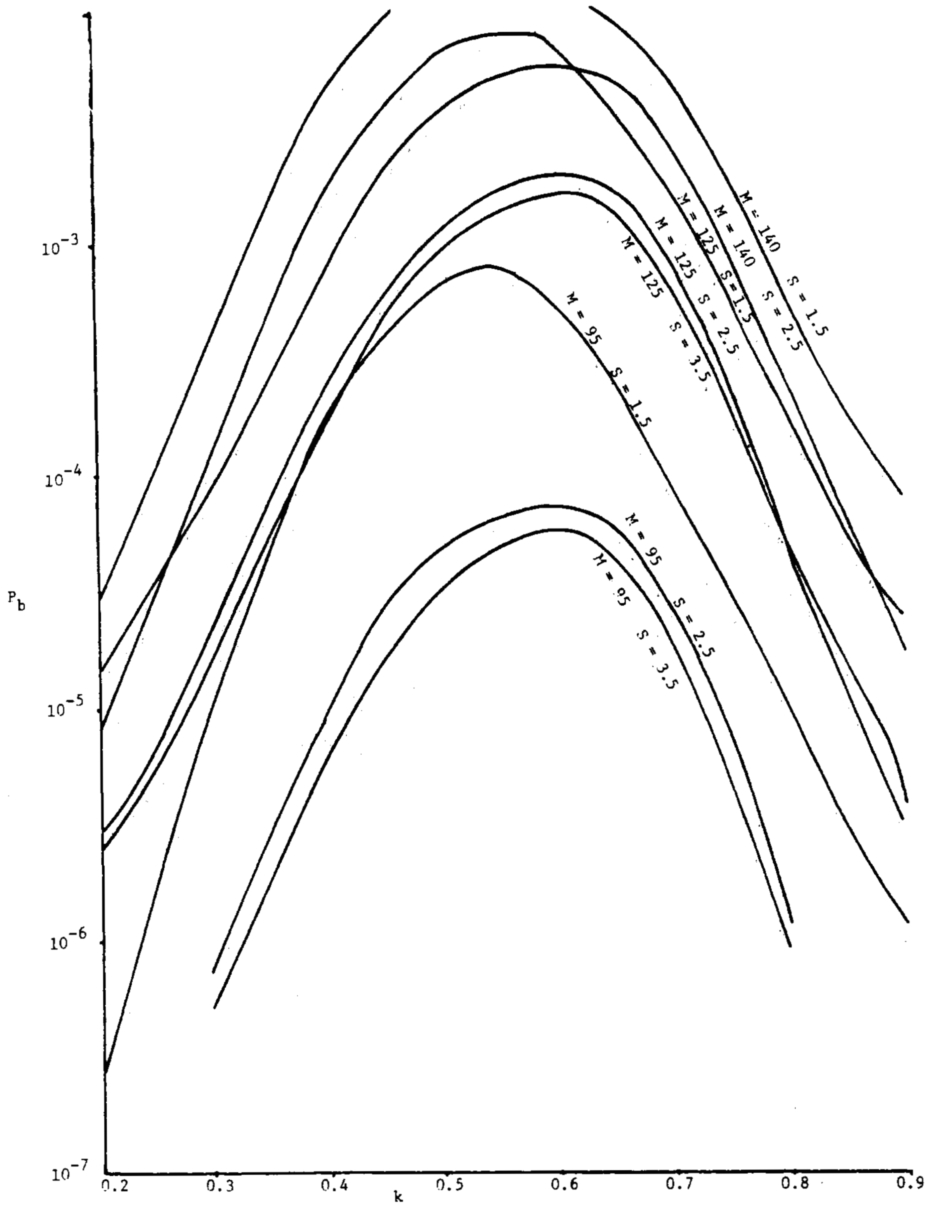

Fig. 3. Probability of bit error versus distance of user u from base 1.

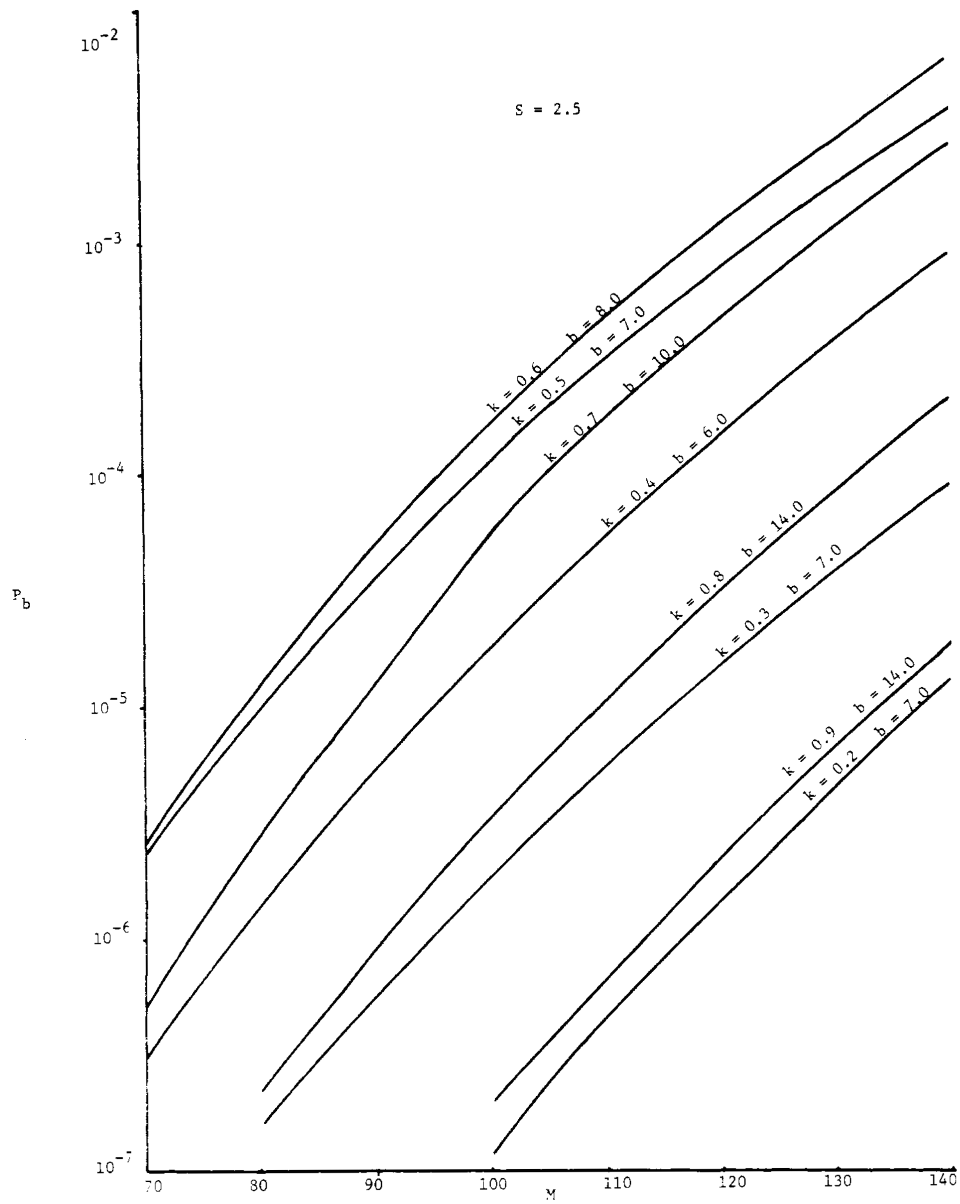

Fig. 4. Probability of bit error versus number of users in each cell.

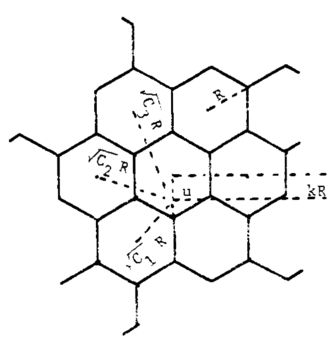

Fig. 5. General cellular structure.

computations had shown. The principle employed in deriving the equations for the evaluation of P_b is the same as the one in the previous section. Since the equations are many but are not difficult to reach they are not presented here.

One can notice from Fig. 5 that we are considering three groups of adjacent cells, each group having two cells, with base stations at distances $\sqrt{C_i}R$, $i = 1, 2, 3$, from the receiver u. Here C_i is given by

$$C_1 = (1.5 - k)^2 + 0.75 \tag{45}$$

$$C_2 = k^2 + 3 \tag{46}$$

$$C_3 = (1.5 + k)^2 + 0.75. \tag{47}$$

Fig. 6 shows P_b versus k for different values of $10 \times S$ and for few values of M. The same pattern as seen in Fig. 3 is noticed. Applying the optimization procedure discussed in Section III for fixing the threshold, we get Fig. 7, showing P_b versus M for various k's. One could observe the closeness of Figs. 7 and 4, supporting our earlier conclusion.

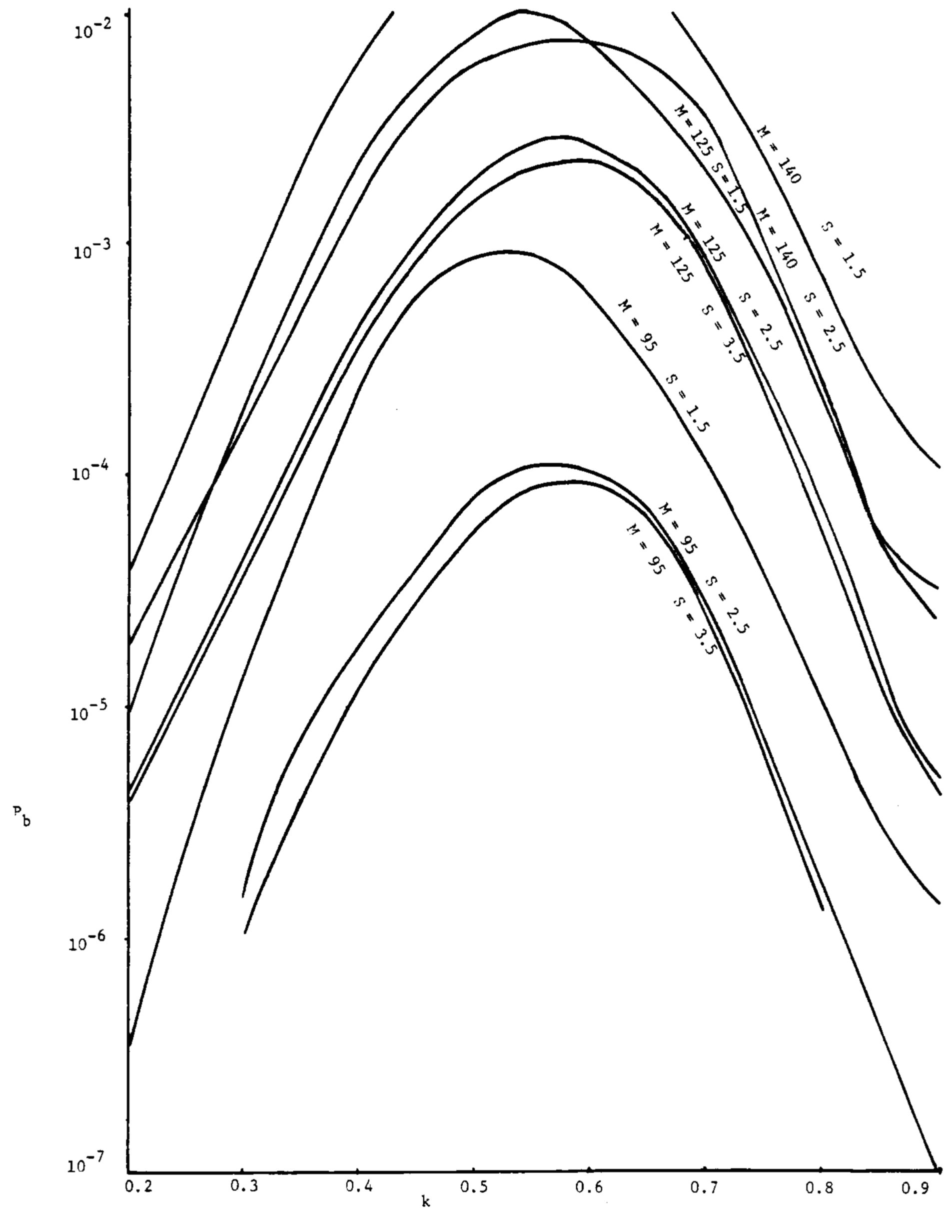

Fig. 6. Probability of bit error versus distance of user u from base 1.

CONCLUSION

A simple power control scheme has been evaluated to reduce the effect of adjacent cell interference on the performance of FH-MFSK cellular mobile radio systems. When the users are distributed uniformly within a cell, the results show spectral efficiency reduction of about 32 percent compared to an isolated cell. The best results can be achieved only with an approximate knowledge of user distribution within a cell. The effect of signal strength attenuation with respect to distance on the performance has also been investigated. When the signal strength varies inversely as the cube of distance, about 102 users could be accomodated at $P_b < 10^{-3}$, with the assumed uniform distribution of users within a cell. This results in spectral efficiency reduction of 40 percent compared to an isolated cell. A certain distribution of users might lead to a better performance than the one arrived at here. For example, when the users are distributed beta (3, 5) with respect to the normalized distance variable k, about 140 users can be accomodated in each cell at $P_b < 10^{-3}$.

We have obtained these results with the assumption that the nonorthogonal interference due to the differences in the arrival times of tones from different base stations can be neglected. In reality, the nonorthogonal interference is expected to cause some additional degradation in the performance. Whereas one can always overbound the effect of this interference, this would only lead to more pessimistic analysis. Also, we considered exclusively base to mobile communication. It is known that the reverse transmission would experience much poorer performance because of multipath delay spread. As yet, no precise evaluation on the performance of mobile to base transmission link is available. Finally, it is expected that, for asynchronous FH systems, the distribution of users and the number of users in a cell could influence P_b to a greater extent [5].

APPENDIX

We mentioned in Section III that the probability of more than four users transmitting the same frequency tone j in an

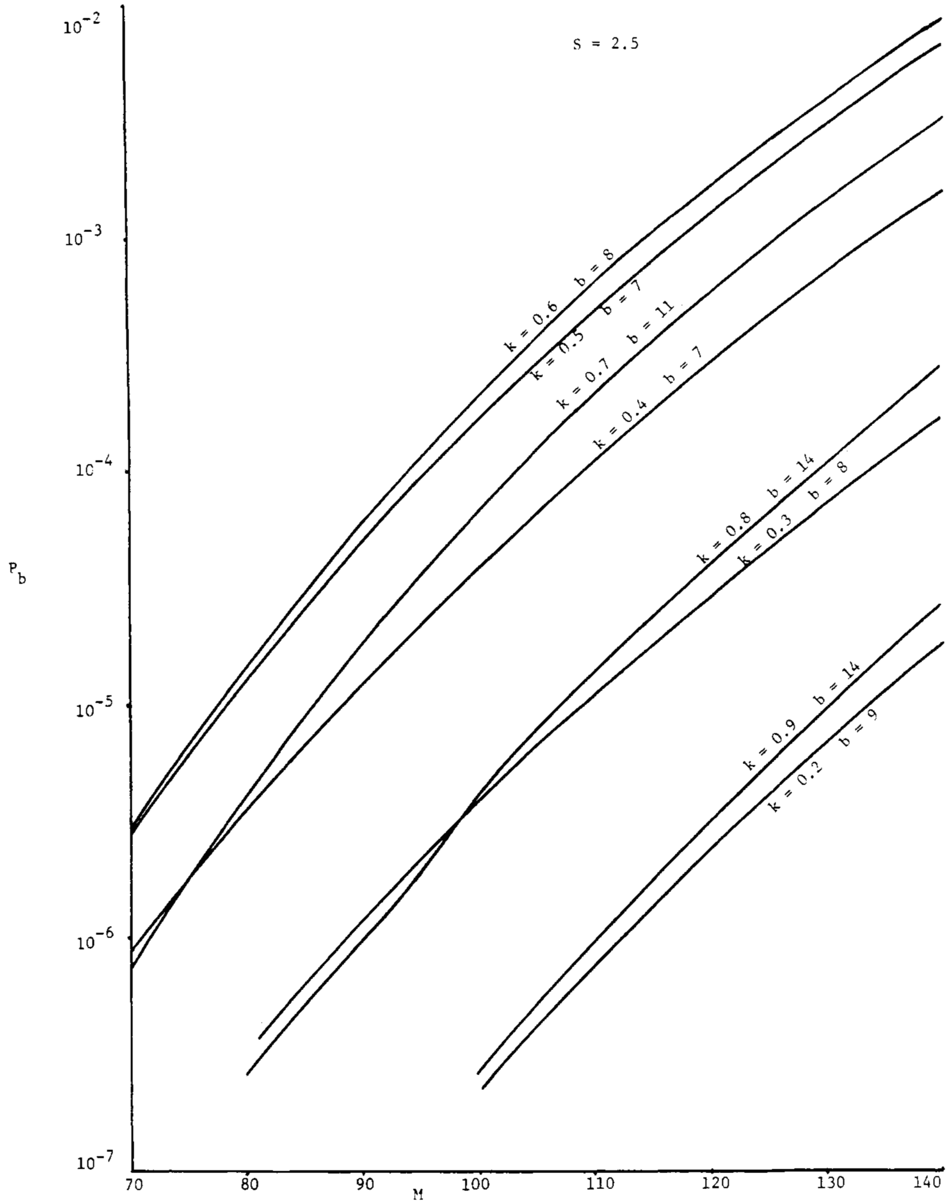

Fig. 7. Probability of bit error versus number of users in each cell.

ith time slot is negligibly small, and neglecting these probabilities in the calculations of power ratios IN, SN_0, and IN_0 does not introduce any serious error. We qualify that statement by considering M to be between 50 to 200 and by calculating

$$P = \binom{M}{n} p_r^n (1 - p_r)^{M-n},$$

$$p_r = 2^{-K} = 2^{-8},$$

and I_{son} as in (25).

Whereas I_{son} slowly increases towards a constant value as $n \to \infty$, the value of P decreases towards zero for large n, and hence the product also goes toward zero for large n. Table I shows I_{son} and the product $P \times I_{son}$ for various values of M of interest. Since only the product $P \times I_{son}$ determines the contribution to IN, SN_0, and IN_0, and since this value is very small for $n \geqslant 4$ compared to values for $n \cong 1, 2$, we are justified in neglecting the terms beyond $n = 4$.

TABLE I

		$P \times I_{son}$		
n	I_{son}	M = 50	M = 125	M = 200
1	0.1389	0.0223	0.0416	0.0496
2	0.2288	0.0035	0.0167	0.0319
3	0.2919	2.9×10^{-4}	0.0034	0.0106
4	0.3387	1.7×10^{-5}	4.9×10^{-4}	0.0024
5	0.3748	1×10^{-6}	5.3×10^{-5}	4.1×10^{-4}
6	0.4035	-	5×10^{-6}	5.8×10^{-5}
7	0.4269	-	-	7×10^{-6}
8	0.4463	-	-	1×10^{-6}

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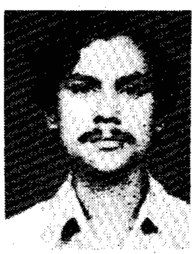

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