Adjoint Equations and Analysis of Complex Systems

by

Guri I. Marchuk

Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russia



KLUWER ACADEMIC PUBLISHERS

DORDRECHT / BOSTON / LONDON

Table of Contents

AUIH	OR 5 PREFACE TO THE ENGLISH EDITION	13
INTRO	DDUCTION	1
PA	RT I. ADJOINT EQUATIONS AND PERTURBATION THEORY	7
СНАР	TER 1. Main and Adjoint Equations. Perturbation Theory	و
1.1.	Main and Adjoint Operators in Linear Problems.	
	Elements of Theory	g
1.2.	Adjoint Operators in Spectral Problems. Fourier Method	22
1.3.	Adjoint Equations and Functionals. Elements of Theory	28
1.4.	Adjoint Equations and Importance of Information	38
1.5.	Adjoint Equations and Perturbation Theory	
	for Linear Functionals	43
1.6.	Simple Nonlinear Problems	50
1.7.	Adjoint Equations for Non-Stationary Problems	59
1.8.	Adjoint Equations and Simple Inverse Problems	67
1.9.	Perturbation Theory	76
1.10.	Adjoint Equations. Perturbation Algorithms	86
1.11.	Perturbation Algorithms for Eigenvalue Problems	88
CHAP	TER 2. Simple Main and Adjoint Equations of Mathematical Physics	95
2.1.	Diffusion Equation	95
2.2.	Heat Conductivity Equation	104
2.3.	Oscillation Equation	113
CHAP	TER 3. Nonlinear Equations	123
3.1.	Nonlinear Equations and Adjoint Problems	124
3.2.	General Formulation of the Adjoint Problem	129
3.3.	Small Perturbation Theory	130
3.4.	Case of Problem with Perturbed Operator	138
3.5.	On the Higher-Order Perturbation Theory	145
3.6.	Other Approaches to Construction of Adjoint Operators	
	in Nonlinear Problems	152

CHAF	PTER 4. Inverse Problems and Adjoint Equations	164
4.1.	Basic Definitions and Examples	165
4.2.	•	
	with Constant Operators	173
4.3.	Inverse Evolutionary Problem with Operator	
	Depending on Time	180
4.4.	Statement of Inverse Problems on the Basis of Adjoint	405
4 5	Equation Methods and Perturbation Theory	187
4.5.	Formulation of Perturbation Theory	901
	for Complex Nonlinear Models	201
PA	ART II. PROBLEMS OF ENVIRONMENT AND OPTIMIZATION	
	METHODS ON THE BASIS OF ADJOINT EQUATIONS	207
CHAF	PTER 5. Analysis of Mathematical Models in Environmental Problems	212
5.1.	·	
0.1.	Uniqueness of Solution	213
5.2.	•	217
5.3.		221
5.4.	* *	230
5.5.		235
5.6.	The Structure and Modelling of Turbulent Motions	
	in the Atmosphere	237
5.7.	Adjoint Equation for a Simple Diffusion Equation	242
5.8.	General Case of Adjoint Problem for	
	Three-Dimensional Domain	253
5.9.	± • • • • • • • • • • • • • • • • • • •	261
5.10.	Adjoint Equation and Lagrange Identity	264
CHAF	PTER 6. Adjoint Equations, Optimization	269
6.1.	Statement of the Problem	269
6.2.	Adjoint Equations and Optimization Problem	276
6.3.	Multi-Criterial Optimization Problem	278
6.4.	Problem of Minimax	281
6.5.	Generalized Optimization Problem of Plant Location	282
6.6.	Some General Remarks	28 4
6.7.	Estimation of Biospherical Losses to Environment Pollution	
	by Industrial Emissions	289
6.8.	Economics of Natural Resources	293

CONTENTS

6.9.	Common Economic Criterion	296
6.10.	Mathematical Problems of Optimization of Emissions	
	at Operating Industrial Plants	301
6.11.	· · · · · · · · · · · · · · · · · · ·	307
6.12.	•	310
CHAI	PTER 7. Adjoint Equations and Models of General Circulation	
(11.11	of Atmosphere and Ocean	326
7.1.	Temperature Anomalies in the Atmosphere	328
7.2.	Temperature anomalies in the Atmosphere and Ocean	341
7.3.	Adjoint Functions of Atmosphere Dynamics	345
7.4.	Adjoint Equations for Baroclinic Atmosphere	351
7.5.	Baroclinic Model of the Atmosphere with a Heat Flux	
	from the Ocean	355
7.6.	Problems of Climate Change Sensitivity in Various Regions	
	of the World	360
7.7.	Application of the Sensitivity Theory to the Comparison	
	of Mathematical Models	381
CHAI	PTER 8. Adjoint Equations in Data Processing Problems	389
8.1.	Data Assimilation Problem for Evolutionary Equation	390
8.2.	Numerical Algorithm for Solving the Data Assimilation Problem	394
8.3.	Numerical Method of Solution of Geophysical Data Assimilation	
	Problem Using the Splitting Scheme	400
8.4.	Numerical Solution of Oceanic Data Initialization Problem	408
APPE	ENDIX I: Splitting Methods in the Solution of Global Problems	422
1.	Splitting Methods for Solution of Non-Stationary Problems	422
2.	Multicomponent Splitting of Problems	429
3.	An Example of Application: Two-Dimensional Problem	
1	with Constant Coefficients	437
ΔPDE	ENDIX II: Difference Analogue of Non-Stationary Heat Diffusion	
ALLI	Equation in Atmosphere and Ocean	441
	Equation in Atmosphere and Ocean	441
BIBLIOGRAPHY		
INDEX		464
****	42.	404