Adoption Timing of New Equipment with Another Innovation Anticipated

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***Abstract*—In dynamic environments, decisions about adopting or replacing new equipment or processes are influenced by the expectation of further innovations in technology. This paper examines the issue of the time at which an equipment (vintage one) currently in use is to be replaced with better equipment (vintage two), in the face of uncertain future availability of even better equipment (vintage three). We present an operational model useful in making adoption timing decisions and then present results that offer interesting insights into the impact of uncertainty and output expansion on the adoption time of vintage two. First, we characterize the optimal adoption time of vintage two in terms of the operating costs of various vintages, switching costs between vintages, and the hazard rate for the time of appearance of vintage three. We indicate the conditions under which it is optimal to follow a "now or never" or a "wait and adopt" policy with respect to adoption of vintage two. Second, it is shown that output expansion need not always spur the adoption of innovations. In fact, under certain conditions, output expansion may delay adoption of vintage two, even though vintage two has a higher operating cost savings per unit fixed cost than vintage three. We also provide interesting results on the impact of various costs on the adoption time of vintage two and consider the implications of our results for the supplier of vintages two and three. Finally, we indicate how important factors such as benefits of new technologies other than operating cost reductions, learning effects, changes in costs over time, and fixed operating costs can be incorporated in the model.**

***Index Terms*—Adoption of innovations, stochastic models, technological expectations, technology management.**

I. Introduction and Related Research

NEW technologies, in the form of either equipment or processes, bring about numerous benefits. They reduce production costs through labor savings and lower material usage and reject rates. They may also help increase revenues by producing output of better quality and greater variety. The decision to adopt a new process or equipment involves a tradeoff between the fixed adoption costs and benefits stated earlier. However, in environments with technological change, this decision is complicated by the uncertain economic life of the equipment, which depends upon the possibility of further innovations in equipment and processes.

For example, a bank, insurance company, or a university computer center may have to decide whether to adopt the next generation of mainframe or personal computers or upgrade of a software program. In the electronics industry, chip manufacturers have to decide whether and when to adopt the next generation of chip-manufacturing equipment. Manufacturing firms have to make similar decisions with regard to purchases of robots, machine tools, workstations, and factory simulation software. In healthcare, hospitals routinely have to decide whether to buy or upgrade expensive diagnostic imaging equipment. For instance, the cost of new magnetic resonance imaging equipment is substantial ($2–3 million) and investment required for upgrading the equipment can be significant (from $25 000 to $500 000).

Briefly, the problem posed here is as follows. Consider a firm that is currently using a particular equipment (or process), say vintage one (V1). A better equipment, say vintage two (V2), with lower operating costs has just become available. Another equipment, vintage three (V3), with lower operating costs than V2, is likely to appear at an uncertain future time. There is a fixed cost for replacing one vintage with another. How long, if at all, should the firm wait before adopting V2? What is the impact of output expansion, uncertainty, and various costs on the adoption timing decision? We seek an answer to these and related questions here. We also consider the implications of these results for the suppliers of these vintages.

We briefly review closely related works in the management science and economics literature that model adoption of new technology. Broadly, our approach is similar to that of Kamien and Schwartz [15] in considering three vintages with successively lower operating costs. Two important differences are that we consider switching costs between technologies and a nonstationary environment where demand and costs may change over time. Therefore, in contrast to [15], the decision is not a simple "now or never" choice, wherein V2 is either adopted as soon as it becomes available or never adopted. Balcer and Lippman [3] propose a model with a sequence of technological innovations that may or may not be adopted, with uncertainty both in the time and size of each discovery as well as the future pace of discovery. They provide several interesting conclusions about the impact of a sequence of uncertain technological innovations on the adoption timing decision. Here, we explore instead the impact on the adoption timing decision of factors such as costs and change in output levels in a model with uncertainty in the arrival of one innovation.

Monahan and Smunt [21] develop a model and run simulations to analyze the impact of uncertainty in both interest

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rates and the state of technology on the rate of technology acquisition. Some important differences are that while they consider a sequence of innovations, we allow demand to increase over time and focus on analytically evaluating the impact of different factors on adoption timing decisions. Reinganum [26] presents a game-theoretic approach to the diffusion of new technology which lowers a firm's operating costs and is available at lower prices over time due to technical progress. (Refer to [28] for other related economic models.)

Goldstein *et al*. [12] consider a machine-replacement problem with an expected technological breakthrough characterized by a constant hazard rate, i.e., the conditional probability of the breakthrough, given it has not occurred, is constant over time. They formulate a stationary dynamic programming model and develop a solution procedure, using the approach in Sethi and Chand [27], to determine the replacement timing of the current technology. Nair and Hopp [22] and Nair [23] consider more general models with nonstationary technological breakthroughs and develop algorithms based on efficient forecast horizon procedures. While Nair and Hopp [22] consider a two technology model, Nair [23] considers a model with a sequence of breakthroughs. These papers focus on developing procedures for determining the optimal replacement time and not on deriving insights into the impact of costs and other factors on the time of replacement. Fuller and Vickson [9] and Vickson [29] use optimal control theory to study the replacement rate of an old technology with a new one but do not consider the uncertain future appearance of a third technology.

While the above papers primarily deal with replacement issues, Klincewicz and Luss [17] consider the issue of when to install facilities of fixed capacity using current technology, given spare capacity of the old technology. Related works that explicitly model capacity aspects along with technology replacement that lowers operating and other costs under various scenarios are Cohen and Halperin [4], Hinomoto [14], and Li and Tirupati [18]. Gaimon [10] presents a dynamic game analysis to understand the impact of competitive forces on the deterministic replacement of old with new technology capacity. Gaimon [10, p. 410] mentions at the outset that "... firms also run the risk of making an enormous investment in technology that may soon become obsolete." Gaimon and Ho [11] present an analysis of capacity acquisition decisions in an environment characterized by both uncertainty as well as a competitive environment and use game theoretic methods to obtain useful insights.

Rajagopalan *et al*. [25] develop a dynamic programming model for making technology and capacity replacement decisions in an environment with a sequence of uncertain technological breakthroughs. Their focus is on determining optimal solution procedures while the focus in this paper is on obtaining qualitative insights. In particular, we analytically study the impact of uncertainty, demand, and costs on the optimal adoption time. Erlenkotter *et al*. [7] address the optimal timing for initiating a project to expand water supplies when a sudden and unexpected shift in consumption patterns is anticipated, assuming a constant hazard rate for the time of this shift. We do not make such an assumption because a constant hazard rate for the time of appearance of a technological innovation is quite restrictive.

The paper is organized as follows. In Section II, we explain the rationale for the model and describe the terms and assumptions used. We then present a mathematical formulation and obtain expressions for determining the optimal adoption time. Section III characterizes the optimal adoption time of V2 under different scenarios as a "now or never" or a "wait and adopt" policy. In Section IV, we study the impact on the optimal adoption time of changes in costs, demand, and uncertainty in the appearance of V3. We consider various model extensions in Section V and conclude in Section VI with a summary and areas for future research.

II. The Model

The firm is currently using V1 equipment, and new equipment (V2) with lower operating costs has just been introduced (at time zero). Better equipment (V3), with even lower operating costs, is anticipated in the near future. The uncertain time of appearance (ξ) of V3 has the probability density function $g(\xi)$ and cumulative distribution $G(\xi)$. Then, the conditional probability that V3 will appear at time ξ, given that it has not appeared until ξ, is given by $h(\xi) = g(\xi)/(1-G(\xi))$, referred to as the hazard rate. For example, if the probability function $g(.)$ has a gamma distribution with parameters α and β [8], then the hazard rate is decreasing, constant, or increasing, respectively, when α is less than one, equal to one, or greater than one. Thus, we model the time of appearance of V3 in a very general fashion. No further innovations are anticipated after the appearance of V3.

The total operating or production cost of the firm $c_i(x)$ is a function of the output level x and the vintage i used. We implicitly assume here that the only benefits from successive vintages are lower production costs. However, we show later how other benefits such as increase in demand for successive technologies can be incorporated in the model. There is a fixed cost F_{ij} for replacing vintage i with vintage j, comprising fixed purchase and installation costs for vintage j less salvage value for vintage i equipment disposed or conversion costs or upgrade costs. V2 and V3 may be offered by the same or competing suppliers. For instance, V2 and V3 may be two successive versions of a word-processing program such as WordPerfect offered by the same supplier. Alternatively, V2 and V3 may be two different word-processing programs such as WordPerfect and Word offered by competing suppliers.

The basic tradeoff considered here is as follows. Adopting V2 immediately reduces current production costs, but V3 may appear soon and the firm may regret having purchased V2 prematurely. Alternatively, the firm could delay adoption of V2 and wait for V3. But V3 may not appear soon and the firm would incur the higher production costs of V1. Hence, the key issue addressed is should V2 be adopted, and if so, how long should the firm wait (say until τ) before adopting V2 technology? V3 may however appear before this time τ and so V2 may never be adopted. A related issue is, if V2 is adopted and then V3 appears, when should V3 replace

V2? If V3 appears at time ξ ($\geq\tau$), V3 may not replace V2 immediately but at a later time T because the benefits from postponing the fixed cost expenditure F_{23} from ξ until T may be greater than the reduction in operating costs foregone by not replacing V2 with V3 until T.

We ignore potential innovations beyond V3. This is reasonable if it is difficult to forecast their arrival time and characteristics or if such innovations are so far into the future that they have a negligible impact on the current decision to adopt V2. For example, a firm deciding whether to replace Windows 3.1 (V1) with Windows-95 (V2) in 1996 may be influenced by the arrival date and characteristics of Windows 98 (V3) but not later vintages, say Windows 2001.

We assume here that the reduction in operating cost by adopting the new equipment is not significant enough to have an impact on the firm's pricing decisions. For example, adoption of a CT-scan equipment by a large hospital or a new computer system by a large bank may yield several benefits, but these may not be significant enough to influence the firm's pricing decisions. Also, to keep the exposition simple in this section, discussion of how the model can be extended to incorporate other factors such as learning, equipment maintenance, fixed operating costs, and changes over time in fixed and variable costs is postponed to a later section. We also do not explicitly consider capacity issues because the focus here is on understanding the impact of technological uncertainty and equipment costs on adoption timing decisions. The focus is not on the impact of demand/capacity imbalances on equipment acquisition decisions. In any case, equipment capacity may not be the constraining resource. Further, capacity costs can be incorporated by including an equivalent or amortized cost [20] in the cost term $c_i(.)$ and one-time technology conversion costs in F_{ij}.

The notation used in the rest of the paper is summarized next.

Parameters:

- ξ — time of appearance of V3 (a random variable);
- $g(\xi)$, $G(\xi)$ — probability density and cumulative probability density function, respectively, for the appearance of V3 at time ξ;
- $h(\xi)$ — conditional probability that V3 will appear at time ξ, given that it has not appeared until then (referred to as the hazard rate); $h(\xi) = g(\xi)/(1-G(\xi))$;
- q_t — output level of the firm at time t (q_0—output level at time zero);
- $c_i(x)$ — total operating costs at output level x using vintage i (1, 2, 3) equipment; $c_i(x)$ is increasing in x and $c_1(q_0) > c_2(q_0) > c_3(q_0)$;
- $c_i(x)$ — fixed cost of replacing vintage i (1, 2) with vintage j (2, 3) equipment, continuous-time discount rate (e^{-rt} is the corresponding discount factor).

Decision Variables:

- τ ($\geq$0) — optimal "threshold" time for replacement of V1 with V2; that is, V1 would be replaced with V2 at τ unless V3 has already appeared ($\xi < \tau$), in which case V3 will replace V1 and V2 will never be adopted;
- T ($\geq\tau$) — optimal "threshold" time for replacement of V2 with V3; that is, V2 would be replaced with V3 at time T if V3 has already appeared ($\xi \leq T$), otherwise such action may be delayed until ξ ($\xi > T$) when V3 appears. (If V3 appears when V1 is still in use, then T is not a relevant decision variable.)

Assumptions:

1) Output level q_t is assumed to be nondecreasing with time; this assumption, while strong, is reasonable for many firms in the industries mentioned earlier if short-term demand fluctuations are ignored and a smoothed demand profile is used.
2) $h(.)$, the hazard rate, is assumed to be a continuous function.
3) The difference in operating costs between successive vintages, $(c_i(x) - c_j(x))$, with $i < j$, is increasing in the output level x. That is, the marginal operating cost of an incremental unit of output is higher for vintage i than for vintage j. The end result of many technological developments has been the creation of capital intensive equipment with lower operating costs, especially at increasing output levels [2]. This is a weak condition satisfied by a wide variety of cost functions, including ones typically used in the economics literature. For example, the function $c(x) = ax^b$, where a (>0) and b (>0) may be unique to each vintage, satisfies this condition. Note that $c(x)$ could be concave or convex in this case.

Finally, we need the following conditions to establish a nontrivial model. First, if V1 is currently being used and V3 becomes available, it is optimal to replace V1 with V3 immediately. If this is not true, then the appearance of V3 is irrelevant to the optimal adoption time of V2 and the problem reduces to a simple net present value computation to determine when V2 should replace V1. Mathematically, this requires the following (see Appendix A):

$$c_1(q_0) - c_3(q_0) > rF_{23}. \tag{1}$$

Second, at any time t, if both V2 and V3 are available it is more profitable to adopt V3. If this were not true there is no decision, as it is optimal to adopt V2 immediately. This requires that the total cost of adopting V2 at time t and replacing it with V3 at T is greater than the cost of directly adopting V3 at time t and using it until T. So, for any t and T (>t), we have

$$\int_t^T c_2(q_z)e^{-rz}\,dz + F_{12}e^{-rt} + F_{23}e^{-rT} > \int_t^T c_3(q_z)e^{-rz}\,dz + F_{13}e^{-rt}. \tag{2}$$

We now present a discounted, infinite horizon, continuous time model for the above problem. From the above discussion, the model is simple yet rich enough to capture the key tradeoffs.

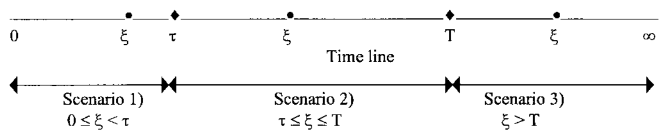

Fig. 1. Time of appearance of V3 (ξ): three possible scenarios.

Given the definitions of τ, T, and ξ, three mutually exclusive scenarios exist (see Fig. 1).

1) $0 \leq \xi < \tau$: V3 becomes available at some time $\xi < \tau$ and is adopted immediately. V1 is used until ξ and V3 thereafter. V2 is never adopted.
2) $\tau \leq \xi \leq T$: V3 becomes available at some time ξ between τ and T but is adopted only at T. V1 is used until time τ, V2 from τ to T, and V3 from T onwards.
3) $\xi > T$: V3 becomes available after T and is adopted immediately. V1 is used until τ, V2 from τ to ξ, and V3 thereafter.

The problem is to determine optimal τ and T that minimize expected total costs C (equal to the discounted sum of operating costs and fixed replacement costs), where

$$\begin{aligned} C = &\int_0^\tau \left[\int_0^\xi c_1(q_z) e^{-rz}\, dz \right. \\ &\quad \left. + \int_\xi^\infty c_3(q_z) e^{-rz}\, dz + F_{13} e^{-r\xi} \right] g(\xi)\, d\xi \\ &+ \int_\tau^T \left[\int_0^\tau c_1(q_z) e^{-rz}\, dz + \int_\tau^T c_2(q_z) e^{-rz}\, dz \right. \\ &\quad + \int_T^\infty c_3(q_z) e^{-rz}\, dz \\ &\quad \left. + F_{12} e^{-r\tau} + F_{23} e^{-rT} \right] g(\xi)\, d\xi \\ &+ \int_T^\infty \left[\int_0^\tau c_1(q_z) e^{-rz}\, dz + \int_\tau^\xi c_2(q_z) e^{-rz}\, dz \right. \\ &\quad + \int_\xi^\infty c_3(q_z) e^{-rz}\, dz \\ &\quad \left. + F_{12} e^{-r\tau} + F_{23} e^{-r\xi} \right] g(\xi)\, d\xi. \end{aligned}$$

The three outer integral terms correspond, respectively, to scenarios 1)–3) described earlier with the inner integral terms representing the nature and timing of replacement in each case. For illustration, consider the second set of terms which describe scenario 2) in which V3 appears at some time ξ between τ and T, which are the limits of the outer integration sign. Recall that, in this case, V1 is used from zero to τ, V2 from τ to T, and V3 from T onwards. Therefore, we have terms (inside the square brackets) for operating costs $c_1(.)$ from zero to τ, $c_2(.)$ from τ to T, and $c_2(.)$ from T until ∞. Also, a fixed cost F_{12} is spent at time τ for replacing V1 with V2, and F_{23} is incurred at T for replacing V2 with V3.

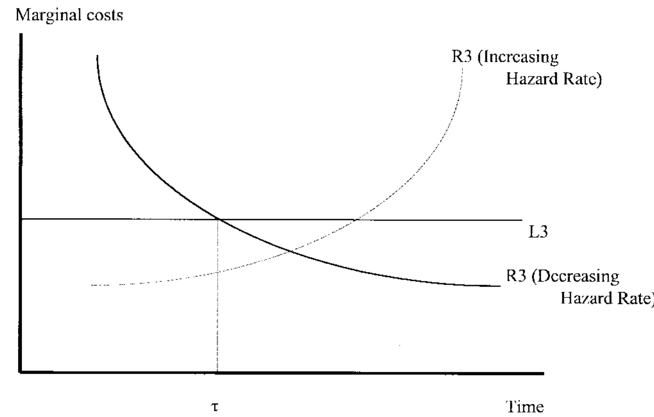

Fig. 2. Constant demand—impact of hazard rate.

III. Optimal Adoption Time

We now derive and discuss the conditions for determining the optimal adoption time of V2 and V3. The optimal adoption time of V2 and V3 can be obtained from the first order conditions $\partial C/\partial\tau = 0$ and $\partial C/\partial T = 0$, respectively. These two conditions reduce to (refer to Appendixes B and C for details of the derivation)

$$\begin{aligned} e^{-r\tau}&(c_1(q_\tau) - c_2(q_\tau) - rF_{12}) \\ &= h(\tau)\left(\int_\tau^T (c_2(q_z) - c_3(q_z)) e^{-r(z-\tau)}\, dz \right. \\ &\qquad \left. + F_{12} - F_{13} + F_{23} e^{-r(T-\tau)} \right) \end{aligned} \tag{3}$$

$$e^{-rT}(c_2(q_T) - c_3(q_T) - rF_{23}) \int_\tau^T g(\xi)\, d\xi = 0 \tag{4}$$

where $h(\tau)$ is the hazard rate at time τ. The expression on the left side in (3), referred to as L3 henceforth, is the discounted value of the net marginal savings from replacing V1 with V2 at time τ. The expression on the right in (3), referred to hereafter as R3, is the product of two terms: one is the likelihood of V3 appearing at $\tau(h(\tau))$ and the other is the discounted incremental cost of adopting V2 rather than V3 from τ to T. The interpretation for the cost expression in R3 (in big brackets) is clear from observing that it is simply the difference between the set of terms on the left and those on the right in (2). The marginal impact is therefore as follows. An increase in L3, with R3 unchanged, leads to earlier adoption of V2, and an increase in R3 leads to a delay in the adoption of V2 (see Fig. 2). An alternative interpretation of (3) is that, delaying the adoption of V2 for an instant results in a loss L3 and an expected gain R3.

The expression within brackets in (4) is similar to L3 in (3). In particular, it is optimal to replace V2 with V3 at T if $(c_2(q_T) - c_3(q_T) - rF_{23}) \geq 0$. There is no term corresponding to R3 in (4) as no new vintage is anticipated after the appearance of V3.

The optimal adoption timing policy under various scenarios, obtained using (3) and (4), is summarized in Proposition 1 (refer to part IV in the Appendix for the proof).

TABLE I

Output level q_t	**Optimal τ and T**		
	Change in hazard rate $h(.)$ over time		
	Decreasing	Increasing	Constant
Constant over time	$\tau \geq 0$: solve (3) $T = \tau$ or $T = \infty$	$\tau = \infty$ $T = \infty$	If L3 $\geq$ R3: $\tau = 0, T = \tau$ or ∞ If L3 $<$ R3: $\tau = \infty, T = \infty$
Increasing over time	τ ($\geq$0): solve (3) to obtain optimal τ. L3 should intersect R3 from below for τ to be a global optimum. T ($\geq\tau$): solve (4) to obtain optimal T.		

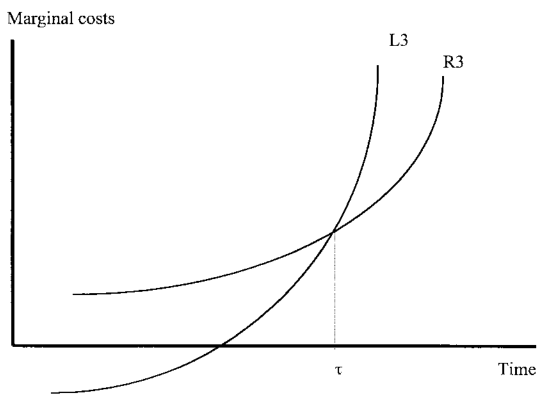

Fig. 3. Increasing output and hazard rate: V2 adopted at τ.

Proposition 1: The optimal adoption time of V2 and V3 is summarized in Table I.

We first consider the cases where output level is constant over time. If both output level and hazard rate are constant, then it is optimal to either adopt V2 now ($\tau = 0$) or never ($\tau = \infty$). If the hazard rate is increasing, V2 is never adopted ($\tau = \infty$) as V3 is increasingly likely to appear with the passage of time and so the incremental value of adopting V2 decreases with time. Clearly, if V2 is not adopted, replacement of V2 with V3 is not an issue.

If the hazard rate is decreasing, the optimal policy is to wait and adopt ($\tau \geq 0$). Referring to Fig. 2, an interior optimum for τ (i.e., $0 < \tau < \infty$) is possible only if the hazard rate is decreasing with time. So, when the likelihood of V3 appearing decreases with time, it may become worthwhile to adopt V2 after some time, even though it was not so at $\tau = 0$. This result is similar to that in Balcer and Lippman, where the current best technology (here V2) may be adopted after some delay rather than "now or never" only if the likelihood of further technological improvement diminishes with time.

However, unlike in Balcer and Lippman, we find that V2 may be adopted after some delay even if the hazard rate is increasing if the output level is increasing over time; in Proposition 1, note that $\tau \geq 0$ independent of the hazard rate trend. Even if the hazard rate is increasing over time, if the benefit from replacing V1 with V2 is increasing at a higher rate than the expected gain from replacing V1 with V3 (i.e., L3 intersects R3 from below as in Fig. 3), V2 may be adopted after some delay. This is in contrast to the constant output case, where V2 is never adopted if the hazard rate is strictly increasing over time.

Example: Suppose a hospital is planning to replace (upgrade) its current magnetic resonance imaging equipment (say MR1) with MR2 that has become recently available. For simplicity, we assume that the only benefit of MR2 over MR1 is its lower scan time, leading to lower variable costs. In reality, there may be other benefits such as better image resolution, greater range of applications, etc. Note, however, that better image resolution also results eventually in lower costs as the doctor spends less time and there is less likelihood of repeat scans. Suppose $c_i(q) = c_i q$, where c_i is the unit (per scan) variable operating cost, and c_1 = \$75 and c_2 = \$60. Demand is increasing over time (at a declining growth rate), given by $q_t = 1000 + 150 * t$, and the switching cost F_{12} = \$125 000. Another equipment (MR3) is expected in the future. Consider the case where the hazard rate is constant over time (i.e., $g(\xi)$ is exponential), with $h = 1/6$. That is, the expected time of appearance of MR3 is 6 years from now. The other costs are estimated to be: c_3 = \$40, F_{23} = \$350 000, and F_{13} = \$425 000, and the discount rate $r = 0.1$. Using (3) and (4), we find that $T = 5$ and $\tau = 1.11$. That is, MR2 will not be considered for adoption until 1.11 years have passed. If the hazard rate is increasing over time, given by $h(t) = 0.15 + 0.02 * t$, then $\tau = 3.265$. Note that there is a reasonable likelihood MR2 will be adopted, even though the hazard rate is increasing over time. □

It is useful to keep in mind while interpreting Proposition 1 and future results that the adoption of V2 is not a certain event. In fact, a delay in adopting V2 can also be interpreted as a decreasing likelihood that V2 will ever be adopted. Specifically, if τ is the optimal delay, then $G(\tau)$ is the cumulative probability (nondecreasing) that V3 has appeared by time τ. It also represents the probability that V2 will never be adopted. In the MR example, the probability MR2 will never be adopted is 0.168 in the constant hazard rate case. To a supplier selling both V2 and V3, any delay in the customer's adoption of V2 is unattractive as it implies a greater likelihood that V2 may never be adopted. The supplier would ideally like the customers to adopt all the vintages and may therefore look for ways to induce the customer to purchase V2 earlier. On the other hand, if V2 and V3 are offered by competing suppliers, delays in adopting V2 are attractive to the supplier of V3.

Recall that T is defined as a threshold time for the replacement of V2 with V3, assuming V2 has been adopted. Proposition 1 states that, if output is constant over time, then the adoption policy for V3 is always "now ($T = \tau$) or never ($T = \infty$)." The case $T = \infty$ implies that, having adopted V2, it will never be replaced by V3. This represents a scenario where V2 and V3 represent significant breakthroughs relative to V1, but V3 may be only a minor improvement over V2. That is, the operating cost savings from replacing V2 with V3 are not sufficient to justify the fixed cost of replacement. If V2 and V3 are offered by competing suppliers, the case $T = \infty$ has important implications for the supplier of V3 because once V2 has been adopted, V3 will not be adopted. Therefore, it is in the interest of the supplier of V3 to hasten its arrival. On the other hand, $T = \tau$ implies that, having adopted V2 at τ, V3 will be adopted (replacing V2) as soon as it appears. (Clearly, V3 did not appear until τ, otherwise V3 would have been adopted, preempting V2.) This represents the case where V3 is a significant improvement over V2 and is adopted as soon as it appears. One could also interpret $T = \infty$ as the case where the switching cost from V2 to V3 is high or the adoption of V2 cannibalizes V3 sales. Then $T = \tau$ is the case with low switching costs and no cannibalization.

IV. Parametric Analysis

In this section, we evaluate the impact on the optimal adoption time of V2 of changing problem parameters such as demand level, hazard rate, and costs.

1) Impact of Uncertainty in the Time of Appearance of V3: The impact of changes in the hazard rate on the optimal adoption time is immediately clear from (3) and so we consider it first. Using the implicit function theorem

$$\frac{\partial \tau}{\partial h} = -\frac{\partial}{\partial h}\left(\frac{\partial C}{\partial \tau}\right)\frac{1}{\partial^2 C/\partial \tau^2}.$$

Recall that $\partial^2 C/\partial \tau^2 > 0$ at the optimum. Then, differentiating the expression for $\partial C/\partial \tau$ in (3), with respect to the hazard rate and using (2), it is easy to see that $\partial(\partial C/\partial \tau)/\partial h < 0$ and so $\partial \tau/\partial h > 0$, i.e., an increase in the hazard rate results in a delay in the adoption of V2. This is as expected because increased likelihood of appearance of V3 forces the adopting firm to wait for V3 rather than commit itself to V2. In the example discussed in Section III, if h increases to 0.2, τ is 2.653, and the probability V2 will never be adopted is as high as 0.41.

There is strong empirical support for this result. Karlson [16] studied the adoption of basic oxygen furnaces (BOF) and large electric arc furnaces (LEF) by the U.S. steel industry from the late 1960's through the 1970's, with LEF being an alternative emerging technology (V3) to BOF (V2) in replacing open-hearth technology (V1). Data were gathered from 48 steel plants in 25 companies. It was found that firms generally delayed adoption of a technology until the rate of technical progress or the hazard rate had diminished. A study by Cainarca *et al.* [5] found that expectation of rapid technological innovation delayed the rapid diffusion of flexible automation in Italian metalworking firms.

Antonelli [1] studied the adoption of open-end spinning rotors, a major innovation in cotton spinning, by the textile industry in 26 countries over the years 1975–1985. In 1975, the technology in use was ring-frame or spindle technology (V1), which could be replaced by an open-end spinning machine (V2) at that time, but further innovation in open-end spinning rotor machines was anticipated (V3). The study found that in countries with a well-developed technological capability where many of the innovations were taking place, expectation of further improvements delayed adoption of open-end technology. In the early and mid-1980's, as the rate of technical progress or hazard rate diminished, firms began to adopt the innovations. On the other hand, in countries with lower technological capabilities and less diffusion of information about technological changes, there was no significant delay in adoption; it appears that lack of knowledge about impending improvements (in effect, a low estimate of the hazard rate) led to early and premature adoption of the open-end technology. Another plausible explanation for the earlier adoption is that textiles is an industry with good demand growth in these less-developed countries, unlike the more developed ones; recall from Proposition 1 that V2 may be adopted even if the hazard rate is increasing if demand is increasing over time.

2) Impact of a Shift in Demand (Output) Level: Kamien and Schwartz [15] concluded, on the basis of their model, that demand expansion leads to earlier adoption of new equipment. Intuition also suggests that a firm is more likely to adopt V2 when the firm's output is likely to expand, as the benefit from replacing V1 with V2 is greater at higher output levels. However, we show here that this need not always be true. First consider the case where $T = \tau$. Then the integral term in (3) vanishes and so

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = \frac{\partial c_1(q)}{\partial q} - \frac{\partial c_2(q)}{\partial q} > 0.$$

The inequality above follows from assumption 3). Therefore, in this case, output expansion leads unequivocally to earlier adoption of V2. This is to be expected because $T = \tau$ implies that V3 will be adopted as soon as it appears, independent of whether V2 was adopted or not. Demand expansion makes V2 even more attractive relative to V1 and, because adoption of V2 has no influence on whether and when V3 will be adopted, V2 is adopted earlier.

Now consider the general case with $T > \tau$. Here we assume cost is proportional to output levels, i.e., $c_i(q) = \hat{c}_i q^k (k > 0)$. This is a common assumption in the technology and capacity acquisition literature [19], [20] and it satisfies assumption 3). Let $\Delta \hat{c}_{ij} = (\hat{c}_i - \hat{c}_j)$, where $i > j$. Also, for reasons that will be clear shortly, define $\Delta S_{\tau T}$ as

$$\Delta S_{\tau T} = \Delta \hat{c}_{23}\left[\int_{\tau}^{T} (q_z - q_\tau)e^{-r(z-\tau)}\,dz + \int_{T}^{\infty} (q_T - q_\tau)e^{-r(z-\tau)}\,dz\right].$$

$\Delta S_{\tau T}$ represents the reduction in operating costs, from using V3 rather than V2 from τ onwards, corresponding to the increase in demand that occurs from τ to T. Then, we have the following counter-intuitive result on the impact of a shift in output on τ.

Proposition 2:

1) When $T > \tau$, demand is constant, and $c_i(q) = \hat{c}_i q^k$ $(k > 0)$, if $\Delta \hat{c}_{13}/F_{13} < (>)\, \Delta \hat{c}_{12}/F_{12}$, then V2 is adopted later (earlier) at a higher demand level.
2) When $T > \tau$, demand is increasing, and $c_i(q) = \hat{c}_i q$, i.e., cost is linear in output, if $\Delta \hat{c}_{13}/(F_{13} - \Delta S_{\tau T}) < (>)\, \Delta \hat{c}_{12}/F_{12}$, then V2 is adopted later (earlier) at a higher demand level.

Refer to Appendix E for the proof. When firms anticipate an increase in output level, it is reasonable to expect that they would be keen to quickly adopt equipment that reduces operating costs. But Proposition 2 indicates that this may not always be the best strategy when another technology is anticipated. In fact, the firm may want to delay adoption, despite a potential expansion in demand. Furthermore, the condition under which demand expansion leads to a delay in adoption of V2 is quite striking. In the constant demand case, observe that an output increase results in a delay in adopting V2 when the ratio of per unit operating cost savings to fixed costs for V2 ($\Delta\hat{c}_{12}/F_{12}$) is actually greater than that for V3 ($\Delta\hat{c}_{13}/F_{13}$). The constant demand case is important as it is similar to that considered in Kamien and Schwartz [15], where it was concluded that demand expansion spurs the adoption of new equipment.

Why does this occur? Since V3 has lower operating costs than V2, $\Delta\hat{c}_{13} > \Delta\hat{c}_{12}$. So, the condition in Proposition 2-1), $\Delta\hat{c}_{13}/F_{13} < \Delta\hat{c}_{12}/F_{12}$ is true only if $F_{12} < F_{13}$. From optimality condition (3), the smaller the value of F_{12} relative to F_{13}, the higher the hazard rate for the appearance of V3 at the optimum. While a higher demand level does make V2 attractive relative to V1, it also makes V3 more attractive as V3 has lower operating costs than V2. This factor, together with the higher likelihood of appearance of V3 $h(.)$, makes the adoption of V2 less attractive relative to waiting for V3. This translates into a delay in the adoption of V2.

The result in the increasing demand case is similar, though restricted to the case where cost is linear in output. The ratio $\Delta\hat{c}_{ij}/F_{ij}$ could also be interpreted as a "benefit-to-cost ratio" for replacing vintage i with vintage j (in fact, $\Delta\hat{c}_{ij}q/F_{ij}$ represents the payback period for replacing vintage i with vintage j, a measure commonly used to evaluate investments). When demand is increasing, the ratio $\Delta\hat{c}_{13}/(F_{13} - \Delta S_{\tau T})$ represents an "adjusted" benefit-to-cost ratio in comparing V2 and V3 as potential replacements to V1, since it also takes into account differences in cost savings between V2 and V3 from future demand growth. Again, demand increase delays adoption of V2 when the "adjusted" benefit-to-cost ratio for V2 is greater than that for V3.

Prior to the study by Karlson of the impact of technological expectations on adoption decisions in the steel industry, many researchers had claimed that large U.S. steel producers were lethargic in the adoption of the latest BOF technology. Karlson pointed out that this delay in adoption was not because they were lethargic but because better technology was on its way in the form of improved LEF technology, which they adopted later. Karlson also notes that demand for steel was booming in the late 1960's and early 1970's. Karlson concludes that the supposedly lethargic steel producers who, despite this increasing demand, delayed the replacement of their open-hearth furnaces and replaced them with large electric arc furnaces made an optimal choice. On the other hand, some firms such as Kaiser and McLouth that were early adopters of BOF technology have since closed their oxygen furnaces, and oxygen steel-making has gradually been abandoned since 1978. Thus, we observe both analytically and empirically that demand expansion need not spur adoption of a new technology, unlike in Kamien and Schwartz, but may delay its adoption. In fact, such a delay is more likely when $T > \tau$, i.e., when adoption of V2 (BOF) delays the adoption of V3 (LEF), as they are incompatible technologies. Thus, uncertainty in the time of appearance of V3 leads to unexpected results.

3) Impact of Costs: We now consider the impact of costs on the optimal adoption time of V2. Using optimality condition (3) and the implicit function theorem, it is straightforward to obtain the impact of fixed switching costs and variable operating costs on the optimal adoption time. The cost impact is summarized in the following result.

Proposition 3—Impact of Fixed Switching Cost: $\partial\tau/\partial F_{12} > 0$, $\partial\tau/\partial F_{13} < 0$, $\partial\tau/\partial F_{23} > 0$.

4) Impact of Variable Operating Cost: $\partial\tau/\partial c_3 < 0$, $\partial\tau/\partial c_2 > 0$, $\partial\tau/\partial c_1 < 0$.

The impact of F_{12} and F_{13} is as expected. Higher values of F_{12} and lower values of F_{13} make V2 less attractive relative to V3, leading to a delay in its adoption. However, it is interesting to note that higher values of F_{23} lead to greater delays in adopting V2. Suppose V2 and V3 are offered by competing suppliers. Our result implies that, when the switching costs F_{23} are higher, the supplier of V3 is more likely to take advantage of the potential for greater delay in adoption of V2 by pre-announcing the arrival of V3. This result is consistent with the conclusions of the study by Eliashberg and Robertson [6], where they observed that pre-announcing behavior is more common in environments with high switching costs. When V2 and V3 are offered by the same supplier, our results imply that the supplier will have to indicate clearly that F_{23} will be small to induce customers to adopt V2. In fact, this is commonly observed in several industries: 1) firms emphasize that upgrading equipment or software is easy and inexpensive and 2) firms provide incentives for future upgrades if current upgrades are adopted.

As expected, an increase in V2 operating costs $(c_2(.))$ delays the adoption of V2 and an increase in $c_3(.)$ leads to earlier adoption of V2. Consistent with these results, the study by Karlson in the steel industry found that delay in adoption of a technology (say V2) was negatively correlated with cost savings from the same technology (V2) and positively correlated with cost savings from the competing technology (V3).

We find that an increase in c_1 leads to earlier adoption of V2. This is interesting because, intuitively, an increase in c_1 makes both V2 and V3 equally attractive in terms of potential savings in operating costs. This result may help explain why firms at high levels of capacity utilization (with high current operating costs due to overtime, etc.) adopt the current best technology soon after it appears [24]. Recall that in the steel industry study [16] a few firms adopted BOF technology (V2) early despite the emerging LEF technology. Also, demand for steel was booming at this time (late 1960's). A plausible explanation for the decision to adopt BOF technology early may be that the pressure of increasing output resulted in increased operating costs $c_1(q)$. While other steel manufacturers also may have faced increasing demand, their current costs $c_1(q)$ may have been lower due to lower capacity utilization or more capital intensive or modern plants.

V. Model Extensions

We show how other benefits of new technologies and learning effects can be incorporated in the basic model presented in Section II and discuss the impact of these extensions on the results.

1) Other Benefits of New Technologies: As indicated in Section I, new technologies can result in other benefits such as better quality, greater product variety, or faster production. The bottom-line impact of these benefits has to come from lower costs, higher prices, or higher demand. While we considered a cost minimization objective so far, we present briefly the profit maximization case here. Let the total profit with technology $i\,(=1,2,3)$ be given by $\pi_i(q_{iz})$. This profit level is obtained by assuming that, using technology i at time z, the firm produces at a profit-maximizing output level q_{iz} at which its marginal production cost equals marginal revenue. Thus, output and profit levels at any instant are functions of the technology used and other factors (e.g., market growth), but they are independent of the adoption timing decision and are therefore considered exogenous to the model. Then, we can simply substitute $\pi_i(q_{iz})$ for $c_i(q_{iz})$ and develop a profit-maximization model with fixed switching costs. Appropriate changes to (2) and the other assumptions would be necessary. For instance, we would assume profits to be higher for the more recent technologies and profits to increase with output volume for all the technologies.

The model, analysis, and most of the results given earlier do not change in the profit-maximization case. For instance, the impact of changes in profits (instead of costs) on adoption time of V2 would be similar. However, Proposition 2 is applicable only under certain conditions because this result addresses the impact of output changes on the adoption time of V2. In the profit-maximization model, we noted earlier that output levels may not be the same for all technologies and so an equivalent analysis is not possible. However, if output levels are identical for all the technologies at any given time but prices vary with the technology (say because quality of output is different), a result similar to Proposition 2 can be obtained.

2) Learning Effects: Learning effects are often represented as a decrease in operating costs with increasing volume and this can be modeled through $c_i(.)$. For instance, we could represent unit costs declining over time or with cumulative output due to learning curve effects. Also, we could make operating costs $c_3(.)$ a function of whether V2 was adopted or not, which represents experience with prior vintages. If adopting V2 lowers $c_3(.)$, then V2 is more likely to be adopted. There is another interesting learning effect. Knowledge gained from adopting an innovation may result in lower costs of adopting succeeding innovations, and this can be modeled by making F_{13} larger than $(F_{12}+F_{23})$. It can be shown that this results in earlier or more likely adoption of V2. In fact, this is one of the typical strategies used by software firms to induce purchase of upgrades.

3) Changes in Cost over Time: Equipment purchase cost and operating cost may decrease or increase over time, and this can be modeled by making F_{ij} and $c_i(.)$ functions of time. To illustrate briefly, if fixed costs are decreasing over time, we could adopt the commonly used approach of multiplying F_{ij} by a term e^{-mt} [20], where m is the exponential rate of decrease. For example, the value of m for the cost per bit of dynamic random access memory chips has been estimated to be 0.49 [13]. This would change the adoption time, but not the key results or insights presented earlier.

4) Fixed Operating and Maintenance Costs: In many instances, there may be fixed costs for operating and maintaining a plant. For instance, maintenance, space, and insurance costs may often be independent of the firm's output rate and these costs may be different for different technologies. [Note that variable maintenance costs or other variable costs can be included in $c_i(.)$.] A fixed cost f_i can be included in the model by adding terms of the type $\int f_i e^{-rz}\,dz$. Again, this does not change the key results or insights.

VI. Summary and Conclusions

This paper has modeled an equipment adoption timing decision when better equipment is anticipated. The model has the advantage of being simple yet rich enough to capture the key tradeoffs and provide key insights. First, an innovation may be adopted after some delay, even though another innovation is anticipated with increasing likelihood, due to the impact of increasing demand and operating costs. Second, counter to prior results [15], demand expansion may lead to a delay in the adoption of the current best technology. Finally, the optimal delay in adopting V2 is directly related to the hazard rate for the time of appearance of V3. Evidence from related empirical studies appears to broadly support our results and point out the importance of factors such as demand growth, switching costs between innovations, and cost savings from an innovation on the adoption timing decision.

This paper does ignore some factors in modeling the adoption decision. For instance, output levels and price were assumed to be exogenous. Allowing these variables to be endogenous to the model and a function of the technology used and modeling the impact of competition on the adoption timing decision would be interesting extensions. Another important issue is equipment capacity. The fact that many firms do have equipment of different vintages at any one time indicates the interaction between capacity and technology acquisition decisions. Rajagopalan *et al.* [25] address this aspect, but they do not focus on deriving any managerial insights of the type found in this paper. Incorporating these factors to obtain useful managerial insights offers a challenging opportunity for future research.

Appendix A
Derivation of (1)

Suppose we are considering the replacement of V1 with V3 and no further innovations are anticipated. We then have a simple net present value problem. V3 will be adopted at time t and used forever if future reductions in operating costs by replacing V1 with V3 are greater than the fixed cost F_{13} (all costs discounted). We then have

$$\int_t^\infty (c_1(q_z)-c_3(q_z))e^{-rz}\,dz > F_{13}e^{-rt}. \tag{A1}$$

Given assumptions 1) and 3) in Section II

$$\int_t^\infty (c_1(q_z)-c_3(q_z))e^{-rz}\,dz \geq \int_t^\infty (c_1(q_t)-c_3(q_t))e^{-rz}\,dz = \frac{(c_1(q_t)-c_3(q_t))}{r}\,e^{-rt}$$

and so condition (A1) will be satisfied if $c_1(q_t) - c_3(q_t) > rF_{13}$, i.e., (1) is true.

APPENDIX B
DERIVATION OF NECESSARY CONDITION (3)

For notational ease, denote $c_i(q_z)e^{-rz}$ as $\overline{c}_i(z)$. Then, differentiating C with respect to τ

$$\begin{aligned}\frac{\partial C}{\partial \tau} &= 0\\ &= g(\tau)\left[\int_0^\tau \overline{c}_1(z)\,dz + \int_\tau^\infty \overline{c}_3(z)\,dz + F_{13}e^{-r\tau}\right]\\ &\quad + \int_\tau^T [\overline{c}_1(\tau) - \overline{c}_2(\tau) - rF_{12}e^{-r\tau}]g(\xi)\,d\xi\\ &\quad - g(\tau)\left[\int_0^\tau \overline{c}_1(z)\,dz + \int_\tau^T \overline{c}_2(z)\,dz + \int_T^\infty \overline{c}_3(z)\,dz\right.\\ &\qquad\qquad \left. + F_{12}e^{-r\tau} + F_{23}e^{-rT}\right]\\ &\quad + \int_T^\infty \left[\overline{c}_1(\tau) - \overline{c}_2(\tau) - rF_{12}e^{-r\tau}\right]g(\xi)\,d\xi.\end{aligned}$$

Deleting common terms and rearranging

$$\begin{aligned}&\left[\overline{c}_1(\tau) - \overline{c}_2(\tau) - rF_{12}e^{-r\tau}\right]\int_\tau^\infty g(\xi)\,d\xi\\ &- \left[\int_\tau^T (\overline{c}_2(z) - \overline{c}_3(z))\,dz + e^{-r\tau}(F_{12} - F_{13})\right.\\ &\qquad \left. + F_{23}e^{-rT}\right] g(\tau) = 0.\end{aligned}$$

Replacing $\overline{c}_i(z)$ with $c_i(q_z)e^{-rz}$ and rearranging

$$\begin{aligned}&e^{-r\tau}[c_1(q_\tau) - c_2(q_\tau) - rF_{12}] - \frac{g(\tau)}{1 - G(\tau)}\\ &\cdot \left[\int_\tau^T (c_2(q_z) - c_3(q_z))e^{-rz}\,dz + e^{-r\tau}(F_{12} - F_{13})\right.\\ &\qquad \left. + F_{23}e^{-rT}\right] = 0.\end{aligned}$$

Replacing $g(\tau)/(1 - G(\tau))$ by $h(\tau)$, the hazard rate at τ and rearranging, we get

$$\begin{aligned}&e^{-r\tau}\Bigg([c_1(q_\tau) - c_2(q_\tau) - rF_{12}] - h(\tau)\\ &\cdot \left[\int_\tau^T (c_2(q_z) - c_3(q_z))e^{-r(z-\tau)}\,dz + F_{12} - F_{13}\right.\\ &\qquad \left. + F_{23}e^{-r(T-\tau)}\right]\Bigg) = 0.\end{aligned}$$

APPENDIX C
DERIVATION OF (4)

Differentiating C with respect to T, and using $\overline{c}_i(z)$ instead of $c_i(q_z)e^{-rz}$, we have

$$\begin{aligned}\frac{\partial C}{\partial T} &= 0\\ &= g(T)\left[\int_0^\tau \overline{c}_1(z)\,dz + \int_\tau^T \overline{c}_2(z)\,dz + \int_T^\infty \overline{c}_3(z)\,dz\right.\\ &\qquad \left. + F_{12}e^{-r\tau} + F_{23}e^{-rT}\right]\\ &\quad + \left[\overline{c}_2(T) - \overline{c}_3(T) - rF_{23}e^{-rT}\right]\int_\tau^T g(\xi)\,d\xi\\ &\quad - g(T)\left[\int_0^\tau \overline{c}_1(z)\,dz + \int_\tau^T \overline{c}_2(z)\,dz\right.\\ &\qquad \left. + \int_T^\infty \overline{c}_3(z)\,dz + F_{12}e^{-r\tau} + F_{23}e^{-rT}\right].\end{aligned}$$

The first and third set of terms cancel, and returning to the original notation, we have

$$e^{-rT}(c_2(q_T) - c_3(q_T) - rF_{23})\int_\tau^T g(\xi)\,d\xi = 0. \quad (4)$$

APPENDIX D
PROOF OF PROPOSITION 1

A. Constant Demand Case

1) Optimal T: Let the demand be q. From (4), if $c_2(q) - c_3(q) - rF_{23} > 0$, then $\partial C/\partial T > 0$. Since the objective is to minimize C and we must have $T \geq \tau$, the optimal solution is $T = \tau$, i.e., V3 replaces V2 as soon as it appears. If $c_2(q) - c_3(q) - rF_{23} < 0$, then $\partial C/\partial T < 0$ and optimal $T = \infty$, i.e., V3 is never adopted. If $c_2(q) - c_3(q) - rF_{23} = 0$, any value of $T \in [0, \infty]$ is optimal.

2) Optimal τ: We have shown that when demand is constant, optimal T is equal to τ or ∞. Necessary condition (3) reduces to the following expressions in the two cases.

$T = \tau$:

$$\begin{aligned}\partial C/\partial \tau = e^{-r\tau}(&c_1(q) - c_2(q) - rF_{12}\\ &- h(\tau)(F_{12} - F_{13} + F_{23})).\end{aligned}$$

$T = \infty$:

$$\begin{aligned}\partial C/\partial \tau = e^{-r\tau}(&c_1(q) - c_2(q) - rF_{12}\\ &- h(\tau)((c_2(q) - c_3(q))/r + F_{12} - F_{13})).\end{aligned}$$

The following two sufficient conditions must be satisfied for an interior optimum τ ($\tau \neq 0, \infty$) to exist: 1) $\partial C/\partial \tau = 0$ at this τ value and 2) $\partial C/\partial \tau > 0$ to the right of this τ value. We now consider each of the hazard rate cases in turn.

3) Hazard Rate Is Constant over Time: Note that all the terms are independent of time (in both cases) except for the multiplier $e^{-r\tau}$ which is strictly positive for any finite value of τ. So, conditions 1) and 2) cannot both be satisfied and no interior optimum exists. In particular, if L3 > R3, then

$\partial C/\partial\tau > 0$, and so (recall that the objective is to minimize C) the optimal $\tau = 0$. If L3 < R3, then $\partial C/\partial\tau < 0$ and the optimal $\tau = \infty$.

4) Hazard Rate Is Increasing over Time: Let us express R3 as R3 $= h(\tau)B$. Then, $\partial C/\partial\tau = 0$ when L3 $=$ R3 $= h(\tau)B$. Since the hazard rate is increasing over time, $h(t) > h(\tau)$ for $t > \tau$, and so L3 $< h(t)B$. Therefore, $\partial C/\partial\tau < 0$ for $t > \tau$, violating condition 2). As the objective is to minimize, $\partial C/\partial\tau < 0$ implies that optimal $\tau = \infty$.

5) Hazard Rate Is Decreasing over Time: Then $h(t) < h(\tau)$ for $t > \tau$ and so conditions 1) and 2) may both be satisfied. Therefore, an interior optimum may exist.

B. Demand Increasing over Time

In this case, it is clear that (3) and (4) will have to be used to obtain the optimal τ and T. There is only one possibility that requires some discussion. Given a T that solves (4), there may not be any τ that satisfies (3) and $\tau \le T$. This implies that L3 < R3 at $\tau = T$. Then, the optimal procedure is to set $T = \tau$ in (3) and solve for τ. Necessary condition (4) is also satisfied, as the integral term in (4) vanishes when $T = \tau$.

1) Sufficient Conditions: We now show that the necessary conditions (3) and (4) are also sufficient to obtain a global interior optimum. First, consider (4) for $\partial C/\partial T$

$$\frac{\partial C}{\partial T} = e^{-rT}(c_2(q_T) - c_3(q_T) - rF_{23})\int_\tau^T g(\xi)\,d\xi.$$

While $C(.)$ is not necessarily convex in T, the term e^{-rT} and the integral term are always positive. Also, the term in square brackets is increasing due to assumptions 1) and 3) in Section II. Therefore, $\partial C/\partial T$ can change sign only once from negative to positive. Also, $\partial^2 C/\partial T^2 > 0$ at the optimum T because differentiating $\partial C/\partial T$ in (4) with respect to T we get

$$\partial^2 C/\partial T^2 = \partial c_2(q_\tau)/\partial T - \partial c_3(q_\tau)/\partial T > 0.$$

The last inequality follows from assumptions 1) and 3). In effect, we have shown that $C(.)$ is strongly quasi-convex for finite T.

The function $C(.)$ is not necessarily convex in τ either. However, as for $\partial C/\partial T$, we show that $\partial C/\partial\tau$ can change sign only once, from negative to positive. Restating the integral term in (3) with z replaced by $(z - \tau)$, we get

$$\begin{aligned}\frac{\partial C}{\partial\tau} = e^{-r\tau}\Bigg[&c_1(q_\tau) - c_2(q_\tau) - rF_{12} - h(\tau)\\ &\cdot\Bigg(\int_0^{T-\tau}(c_2(q_{z+\tau}) - c_3(q_{z+\tau}))e^{-rz}\,dz\\ &+F_{12} - F_{13} + F_{23}e^{-r(T-\tau)}\Bigg)\Bigg].\end{aligned}$$

The term $(c_1(q_\tau) - c_2(q_\tau))$ is increasing due to assumptions 1) and 3) in Section II. Recall that the set of terms within the inside brackets is B. From condition (2), $B > 0$. Also

$$\partial B/\partial\tau = (c_2(q_T) - c_3(q_T))e^{-r(T-\tau)} - rF_{23}e^{-r(T-\tau)} = 0.$$

The last equality follows from (4). Therefore, noting that $e^{-r\tau}$ is nonnegative, if the hazard rate is constant or decreasing, $\partial C/\partial\tau$ is increasing and can change sign only once. Further, at the optimum τ, $\partial^2 C/\partial\tau^2$ is given by

$$\begin{aligned}\frac{\partial^2 C}{\partial\tau^2} =& e^{-r\tau}\left[\frac{\partial c_1(q)}{\partial\tau} - \frac{\partial c_2(q)}{\partial\tau} - \frac{\partial h(\tau)}{\partial\tau}B - h(\tau)\frac{\partial B}{\partial\tau}\right]\\ &- r\frac{\partial C}{\partial\tau}.\end{aligned}\tag{A2}$$

First, $\partial C/\partial\tau = 0$ at the optimum τ and we showed earlier that $\partial B/\partial\tau = 0$. Also, $\partial c_1(q_\tau)/\partial\tau - \partial c_2(q_\tau)/\partial\tau > 0$ from assumptions 1) and 3) in Section II. Finally, if the hazard rate is constant or decreasing (i.e., $\partial h/\partial\tau \le 0$), it is clear from (A2) that $\partial^2 C/\partial\tau^2 > 0$ at the optimum τ. Thus, when the hazard rate is constant or decreasing, we have shown that the local optimum determined by (3) and (4) is also the global optimum.

Consider the case where the hazard rate is increasing over time. Using (A2), it follows that $\partial^2 C/\partial\tau^2 > 0$ and we have a local optimum at τ only if

$$\partial c_1(q_\tau)/\partial\tau - \partial c_2(q_\tau)/\partial\tau > (\partial h(\tau)/\partial\tau)B \tag{A3}$$

where B was defined earlier. Now, recall that $\partial B/\partial\tau = 0$ $h(.) \ge 0$ and is increasing, and also, $(c_1(q_\tau) - c_2(q_\tau))$ is increasing. Therefore, from (3) for $\partial C/\partial\tau$, there is at most one point (τ value) where $\partial C/\partial\tau$ may change sign, ensuring the strong quasi-convexity of $C(.)$ for finite τ. Alternatively, there is at most one τ value where L3 and R3 may intersect (refer to Fig. 3). If (A3) is satisfied at this τ value, i.e., L3 intersects R3 from below, then this τ represents a global optimum.

Finally, from (4), if $T > \tau$ but finite, the cost expression in (4) must be equal to zero; also, the cost expression in (4) is independent of τ. If $T = \tau$, the integral term vanishes and if $T = \infty$, the exponential term vanishes. So, in all cases, $\partial(\partial C/\partial T)/\partial\tau$ or $\partial^2 C/(\partial T\partial\tau)$ is equal to zero. Therefore, given our earlier results

$$\left(\partial^2 C/\partial T^2\right)\left(\partial^2 C/\partial\tau^2\right) - \partial^2 C/(\partial T\partial\tau) > 0$$

at the optimum, and this completes the proof.

APPENDIX E
PROOF OF PROPOSITION 2

1) Constant Demand and $c_i(q) = \hat{c}_i q^k$: When demand is constant (=q), $T = \tau$, or $T = \infty$. So, if T must be greater than τ, then $T = \infty$. Now, substituting $c_i(q) = \hat{c}_i q^k$ and $\Delta\hat{c}_{ij} = \hat{c}_i - \hat{c}_j$, (3) simplifies to

$$\begin{aligned}\frac{\partial C}{\partial\tau} =& (\Delta\hat{c}_{12}q^k - rF_{12}) - h(\tau)\left(\frac{\Delta c_{23}q^k}{r} + F_{12} - F_{13}\right)\\ =& 0.\end{aligned}\tag{A4}$$

Differentiating (A4) with respect to q, we have

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial\tau}\right) = kq^{k-1}\left(\Delta\hat{c}_{12} - \frac{h(\tau)\Delta\hat{c}_{23}}{r}\right).$$

Substituting for $h(\tau)$ from (A4)

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = kq^{k-1}\left(\Delta\hat{c}_{12} - \frac{\Delta\hat{c}_{23}}{r}\,\frac{(\Delta\hat{c}_{12}q^k - rF_{12})}{(\Delta\hat{c}_{23}q^k/r + F_{12} - F_{13})}\right).$$

Canceling common terms

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = kq^{k-1}\left(\frac{\Delta\hat{c}_{12}F_{12} - \Delta\hat{c}_{12}F_{13} + \Delta\hat{c}_{23}F_{12}}{\Delta\hat{c}_{23}q^k/r + F_{12} - F_{13}}\right).$$

Since $\Delta\hat{c}_{12} + \Delta\hat{c}_{23} = \Delta\hat{c}_{13}$

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = kq^{k-1}\left(\frac{\Delta\hat{c}_{13}F_{12} - \Delta\hat{c}_{12}F_{13}}{\Delta\hat{c}_{23}q^k/r + F_{12} - F_{13}}\right).$$

The denominator is strictly positive from specialization of (2) for constant demand. Therefore

$$\text{if } \Delta\hat{c}_{13}/F_{13} > (<)\, \Delta\hat{c}_{12}/F_{12}, \text{then } \partial(\partial C/\partial\tau)/\partial q > (<)\, 0.$$

As before, $\partial(\partial C/\partial\tau)/\partial q > (<)\, 0$ implies that $\partial\tau/\partial q < (>)\, 0$.

2) Increasing Demand and $c_i(q) = \hat{c}_i q$: Substituting $c_i(q) = \hat{c}_i q$ and $\Delta\hat{c}_{ij} = \hat{c}_i - \hat{c}_i$, (3) simplifies to

$$\frac{\partial C}{\partial \tau} = (\Delta\hat{c}_{12}q_\tau - rF_{12}) - h(\tau) \cdot\left(\Delta\hat{c}_{23}\int_\tau^T q_z e^{-r(z-\tau)}\,dz + F_{12} - F_{13} + F_{23}e^{-r(T-\tau)}\right). \tag{A5}$$

From (4), $\Delta\hat{c}_{23}q_T = rF_{23}$, and so we have

$$F_{23}e^{-r(T-\tau)} = \frac{\Delta\hat{c}_{23}q_T e^{-r(T-\tau)}}{r} = \Delta\hat{c}_{23}\int_T^\infty q_T e^{-r(z-\tau)}\,dz.$$

Substituting for $F_{23}e^{-r(T-\tau)}$ from the above expression in (A5), we get

$$\frac{\partial C}{\partial \tau} = (\Delta\hat{c}_{12}q_\tau - rF_{12}) - h(\tau) \cdot\left(\Delta\hat{c}_{23}\int_\tau^T q_z e^{-r(z-\tau)}\,dz + F_{12} - F_{13} + \Delta\hat{c}_{23}\int_T^\infty q_T e^{-r(z-\tau)}\,dz\right). \tag{A6}$$

Note that

$$\int_\tau^T q_z e^{-r(z-\tau)}\,dz = \int_\tau^\infty q_\tau e^{-r(z-\tau)}\,dz + \int_\tau^T [q_z - q_\tau]e^{-r(z-\tau)}\,dz - \int_T^\infty q_\tau e^{-r(z-\tau)}\,dz$$

and the first integral term on the right is equal to q_τ/r. Substituting the above expression in (A6) and rearranging terms, we obtain

$$\frac{\partial C}{\partial \tau} = (\Delta\hat{c}_{12}q_\tau - rF_{12}) - h(\tau) \cdot\left(\frac{\Delta\hat{c}_{23}q_\tau}{r} + F_{12} - F_{13} + \Delta\hat{c}_{23}\int_\tau^T [q_z - q_\tau] \cdot e^{-r(z-\tau)}\,dz + \Delta\hat{c}_{23}\int_T^\infty [q_T - q_\tau]e^{-r(z-\tau)}\,dz\right) = 0. \tag{A7}$$

Now, the last two terms within the large brackets is $\Delta S_{\tau T}$ (defined earlier). So

$$\frac{\partial C}{\partial \tau} = (\Delta\hat{c}_{12}q_\tau - rF_{12}) - h(\tau) \cdot\left(\frac{\Delta\hat{c}_{23}q_\tau}{r} + F_{12} - F_{13} + \Delta S_{\tau T}\right) = 0. \tag{A8}$$

Differentiating (A8) with respect to q and combining terms (note that $\partial\Delta S_{\tau T}/\partial q = 0$ because the derivative of the terms $(q_z - q_\tau)$ and $(q_T - q_\tau)$ with respect to q is equal to zero),

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = \left(\Delta\hat{c}_{12} - \frac{h(\tau)\Delta\hat{c}_{23}}{r}\right).$$

Substituting for $h(\tau)$ from (A8) in the above expression, we obtain

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = \left(\Delta\hat{c}_{12} - \frac{\Delta\hat{c}_{23}}{r}\,\frac{(\Delta\hat{c}_{12}q_\tau - rF_{12})}{(\Delta\hat{c}_{23}q_\tau/r + F_{12} - F_{13} + \Delta S_{\tau T})}\right).$$

Canceling common terms

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = \left(\frac{\Delta\hat{c}_{12}F_{12} + \Delta\hat{c}_{12}\Delta S_{\tau T} - \Delta\hat{c}_{12}F_{13} + \Delta\hat{c}_{23}F_{12}}{\Delta\hat{c}_{23}q_\tau/r + F_{12} - F_{13} + \Delta S_{\tau T}}\right).$$

Since $\Delta\hat{c}_{12} + \Delta\hat{c}_{23} = \Delta\hat{c}_{13}$

$$\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right) = \left(\frac{\Delta\hat{c}_{13}F_{12} - \Delta\hat{c}_{12}(F_{13} - \Delta S_{\tau T})}{\Delta\hat{c}_{23}q_\tau/r + F_{12} - F_{13} + \Delta S_{\tau T}}\right).$$

The denominator is strictly positive from (2). Therefore

$$\text{if } \Delta\hat{c}_{13}/(F_{13} - \Delta S_{\tau T}) > (<)\, \Delta\hat{c}_{12}/F_{12},$$
$$\text{then } \partial(\partial C/\partial\tau)/\partial q > (<)\, 0.$$

From the implicit function theorem

$$\frac{\partial\tau}{\partial q} = -\frac{\partial}{\partial q}\left(\frac{\partial C}{\partial \tau}\right)\frac{1}{\partial^2 C/\partial\tau^2}$$

$\partial^2 C/\partial\tau^2 > 0$ from the sufficient conditions; so, $\partial(\partial C/\partial\tau)/\partial q > (<)\, 0$ implies $\partial\tau/\partial q < (>)\, 0$.

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REFERENCES

[1] C. Antonelli, "The role of technological expectations in a mixed model of international diffusion of process innovations: The case of open-end spinning rotors," *Res. Policy*, vol. 18, pp. 273–288, 1989.

[2] R. A. Ayres, *The Next Industrial Revolution.* Cambridge, MA: Ballinger Publishing, 1984.

[3] Y. Balcer and S. A. Lippman, "Technological expectations and adoption of improved technology," *J. Econ. Theory*, vol. 34, no. 2, pp. 292–318, 1984.

[4] M. Cohen and R. A. Halperin, "Optimal technology choice in a dynamic-stochastic environment," *J. Oper. Manag.*, vol. 6, no. 3, pp. 317–331, 1986.

[5] G. C. Cainarca, M. G. Colombo, and S. Mariotti, "An evolutionary pattern of innovation diffusion: The case of flexible automation," *Res. Policy*, vol. 18, no. 1, pp. 59–86, 1989.

[6] J. Eliashberg and T. S. Robertson, "New product preannouncing behavior: A market signaling study," *J. Marketing Res.*, vol. 25, no. 3, pp. 282–292, 1988.

[7] D. Erlenkotter, S. P. Sethi, and N. Okada, "Planning for surprise: Water resources development under demand and supply uncertainty—I: The general model," *Manag. Sci.*, vol. 35, pp. 149–163, 1989.

[8] W. Feller, *An Introduction to Probability Theory and Its Applications.* New York: Wiley, 1957.

[9] J. D. Fuller and R. G. Vickson, "The optimal timing of new plants for oil from the Alberta tar sands," *Oper. Res.*, vol. 35, no. 5, pp. 704–715, 1987.

[10] C. Gaimon, "Dynamic game results of the acquisition of new technology," *Oper. Res.*, vol. 37, no. 3, pp. 410–425, 1989.

[11] C. Gaimon and J. Ho, "Uncertainty and the acquisition of capacity: A competitive analysis," *Comput. Oper. Res.*, vol. 21, no. 10, pp. 1073–1088, 1994.

[12] T. Goldstein, S. P. Ladany, and A. Mehrez, "A discounted machine replacement model with an expected future technological breakthrough," *Naval Res. Logistics*, vol. 35, pp. 209–220, 1988.

[13] J. O. Hilbrink, "Economic impact and technical change," *IEEE Trans. Eng. Manage.*, vol. 36, pp. 37–46, Feb. 1989.

[14] H. Hinomoto, "Capacity expansion with facilities under technological improvement," *Manag. Sci.*, vol. 11, pp. 581–592, 1965.

[15] M. I. Kamien and N. L. Schwartz, "Some economic consequences of anticipating technical advance," *Western Econ. J.*, vol. 10, pp. 123–138, 1972.

[16] S. H. Karlson, "Adoption of competing inventions by United States steel producers," *Rev. Econ. Statist.*, pp. 415–422, Aug. 1986.

[17] J. G. Klincewicz and H. Luss, "Optimal timing decisions for the introduction of new technologies," *Europ. J. Oper. Res.*, vol. 20, pp. 211–220, 1985.

[18] S. Li and D. Tirupati, "Dynamic capacity expansion problem with multiple products: Technology selection and timing of capacity additions," *Oper. Res.*, vol. 42, no. 5, pp. 958–976, 1994.

[19] M. B. Lieberman, "Market growth, economies of scale, and plant size in the chemical processing industries," *J. Indust. Econ.*, vol. 26, no. 2, pp. 175–191, 1987.

[20] H. Luss, "Operations research and capacity expansion problems: A survey," *Oper. Res.*, vol. 30, pp. 907–947, 1982.

[21] G. E. Monahan and T. L. Smunt, "Optimal acquisition of automated flexible manufacturing process," *Oper. Res.*, vol. 37, no. 2, pp. 288–300, 1989.

[22] S. Nair and W. Hopp, "A model for equipment replacement due to technological obsolescence," *Europ. J. Oper. Res.*, vol. 63, pp. 207–221, 1992.

[23] S. Nair, "Modeling strategic investment decisions under sequential technological change," *Manag. Sci.*, vol. 41, no. 2, pp. 282–297, 1995.

[24] H. Noori, *Managing the Dynamics of New Technology: Issues in Manufacturing Management.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[25] S. Rajagopalan, M. R. Singh, and T. E. Morton, "Capacity acquisition and replacement with uncertain technological breakthroughs," *Manag. Sci.*, vol. 44, no. 1, pp. 12–30, 1998.

[26] J. Reinganum, "On the diffusion of new technology: A game theoretic approach," *Rev. Econ. Studies*, vol. 48, pp. 395–405, 1981.

[27] S. P. Sethi and S. Chand, "Planning horizon procedures for machine replacement models," *Manag. Sci.*, vol. 25, pp. 140–151, 1979.

[28] P. Stoneman, *The Economic Analysis of Technological Change.* London: Oxford, 1983.

[29] R. G. Vickson, "Optimal conversion to a new production technique under learning," *IIE Trans.*, vol. 17, no. 2, pp. 175–181, 1985.

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