

# AdS/CFT and Light-Front QCD

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## Abstract

The AdS/CFT correspondence between string theory in AdS space and conformal field theories in physical space-time leads to an analytic, semi-classical model for strongly-coupled QCD which has scale invariance and dimensional counting at short distances and color confinement at large distances. The AdS/CFT correspondence also provides insights into the inherently nonperturbative aspects of QCD such as the orbital and radial spectra of hadrons and the form of hadronic wavefunctions. In particular, we show that there is an exact correspondence between the fifth-dimensional coordinate of AdS space  $z$  and a specific impact variable  $\zeta$  which measures the separation of the quark and gluonic constituents within the hadron in ordinary space-time. This connection leads to AdS/CFT predictions for the analytic form of the frame-independent light-front wavefunctions (LFWFs) of mesons and baryons, the fundamental entities which encode hadron properties. The LFWFs in turn predict decay constants and spin correlations, as well as dynamical quantities such as form factors, structure functions, generalized parton distributions, and exclusive scattering amplitudes. Relativistic light-front equations in ordinary space-time are found which reproduce the results obtained using the fifth-dimensional theory and have remarkable algebraic structures and integrability properties. As specific examples we describe the behavior of the pion form factor in the space and time-like regions and determine the Dirac nucleon form factors in the space-like region. An extension to nonzero quark mass is used to determine hadronic distribution amplitudes of all mesons, heavy and light. We compare our results with the moments of the distribution amplitudes which have recently been computed from lattice gauge theory.

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# 1 Introduction

Quantum Chromodynamics, the Yang-Mills local gauge field theory of  $SU(3)_C$  color symmetry provides a fundamental description of hadron and nuclear physics in terms of quark and gluon degrees of freedom. Yet, because of its strong coupling nature, it has been difficult to find analytic solutions to QCD or to make precise predictions outside of its perturbative domain. An important theoretical goal is thus to find an initial approximation to QCD which is both analytically tractable and which can be systematically improved. For example, in quantum electrodynamics, the Coulombic Schrödinger and Dirac equations provide quite accurate first approximations to atomic bound state problems, which can then be systematically improved using the Bethe-Salpeter formalism and correcting for quantum fluctuations, such as the Lamb Shift and vacuum polarization.

It was originally believed that the AdS/CFT mathematical correspondence could only be applied to strictly conformal theories, such as  $\mathcal{N} = 4$  supersymmetric Yang-Mills gauge theory. Conformal symmetry is broken in physical QCD by quantum effects and quark masses. There are indications, however both from theory and phenomenology, that the QCD coupling is slowly varying at small momentum transfer. In these lectures we shall discuss how conformal symmetry, plus a simple ansatz for color confinement, provides a remarkably accurate first approximation for QCD.

The essential element for the application of AdS/CFT to hadron physics is the indication that the QCD coupling  $\alpha_s(Q^2)$  becomes large and constant in the low momentum domain  $Q \leq 1$  GeV/c, thus providing a window where conformal symmetry can be applied. Solutions of the Dyson-Schwinger equations for the three-gluon and four-gluon couplings [1, 2, 3, 4, 5, 6, 7] and phenomenological studies [8, 9, 10] of QCD couplings based on physical observables such as  $\tau$  decay [11] and the Bjorken sum rule [12], show that the QCD  $\beta$  function vanishes and  $\alpha_s(Q^2)$  become constant at small virtuality; *i.e.*, effective charges develop an “infrared fixed point.” Recent lattice simulations [13, 14] and nonperturbative analyses [15] have also indicated an infrared fixed point for QCD. One can understand this physically [16]: in a confining theory where gluons have an effective mass [17] or maximal wavelength, all vacuum polarization corrections to the gluon self-energy decouple at long wavelength; thus an infrared fixed point appears to be a natural consequence of confinement. Furthermore, if one considers a semi-classical approximation to QCD with massless quarks and without particle creation or absorption, then the resulting  $\beta$  function is zero, the coupling is constant, and the approximate theory is scale and conformal invariant [18, 19], allowing the mathematical tools of conformal symmetry to be applied. One can use conformal symmetry as a *template*, systematically correcting for its nonzero  $\beta$  function as well as higher-twist effects.

One of the key consequences of conformal invariance are the dimensional counting rules [20, 21]. The leading power fall-off of a hard exclusive process follows from the conformal scaling of the underlying hard-scattering amplitude:  $T_H \sim 1/Q^{n-4}$ , where

$n$  is the total number of fields (quarks, leptons, or gauge fields) participating in the hard scattering. Thus the reaction is dominated by subprocesses and Fock states involving the minimum number of interacting fields. In the case of  $2 \rightarrow 2$  scattering processes, this implies  $d\sigma/dt(AB \rightarrow CD) = F_{AB \rightarrow CD}(t/s)/s^{n-2}$ , where  $n = N_A + N_B + N_C + N_D$  and  $n_H$  is the minimum number of constituents of  $H$ . The near-constancy of the effective QCD coupling helps explain the empirical success of dimensional counting rules for the near-conformal power law fall-off of form factors and fixed angle scaling [22]. For example, one sees the onset of perturbative QCD scaling behavior even for exclusive nuclear amplitudes such as deuteron photodisintegration, here  $n = 1 + 6 + 3 + 3 = 13$ ,  $s^{11}d\sigma/dt(\gamma d \rightarrow pn) \sim \text{constant}$  at fixed CM angle.

In the case of hard exclusive reactions [23], the virtuality of the gluons exchanged in the underlying QCD process is typically much less than the momentum transfer scale  $Q$ , as several gluons share the total momentum transfer. Since the coupling is probed in the conformal window, this kinematic feature can explain why the measured proton Dirac form factor scales as  $Q^4 F_1(Q^2) \simeq \text{const}$  up to  $Q^2 < 35 \text{ GeV}^2$  [24] with little sign of the logarithmic running of the QCD coupling. Thus conformal symmetry can be a useful first approximant even for physical QCD. The measured deuteron form factor also appears to follow the leading-twist QCD predictions at large momentum transfers in the few GeV region [25, 26, 27].

Recently the Hall A collaboration at Jefferson Laboratory [28] has reported a significant exception to the general empirical success of dimensional counting in fixed CM angle Compton scattering  $d\sigma/dt(\gamma p \rightarrow \gamma p) \sim F(\theta_{CM})/s^8$ , instead of the predicted  $1/s^6$  scaling. However, the hadron form factor  $R_V(T)$ , which multiplies the  $\gamma q \rightarrow \gamma q$  amplitude is found by Hall-A to scale as  $1/t^2$ , in agreement with the PQCD and AdS/CFT prediction. In addition the timelike two-photon process  $\gamma\gamma \rightarrow p\bar{p}$  appears to satisfy dimensional counting [29, 30].

Our main tool for implementing conformal symmetry will be the use of Anti-de Sitter (AdS<sub>5</sub>) space in five dimensions which provides a mathematical realization of the group  $SO(4, 2)$ , the group of Poincare' plus conformal transformations. The AdS metric is

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad (1)$$

which is invariant under scale changes of the coordinate in the fifth dimension  $z \rightarrow \lambda z$  and  $x_\mu \rightarrow \lambda x_\mu$ . Thus one can match scale transformations of the theory in 3+1 physical space-time to scale transformations in the fifth dimension  $z$ . The isomorphism of the group of Poincare' and conformal transformations  $SO(4, 2)$  to the group of isometries of Anti-de Sitter space underlies the AdS/CFT correspondence [31] between string states defined on the 5-dimensional Anti-de Sitter (AdS) space-time and conformal field theories in physical space-time [32, 33]. In particular, we shall show that there is an exact correspondence between the fifth-dimensional coordinate of AdS space  $z$  and a specific impact variable  $\zeta$  which measures the separation of the quark and gluonic constituents within the hadron in ordinary space-time. This connection

leads to AdS/CFT predictions for the analytic form of the frame-independent light-front wavefunctions (LFWFs) of mesons and baryons, the fundamental entities which encode hadron properties. The LFWFs in turn predict decay constants and spin correlations, as well as dynamical quantities such as form factors, structure functions, generalized parton distributions, and exclusive scattering amplitudes.

Scale-changes in the physical  $3 + 1$  world can thus be represented by studying dynamics in a mathematical fifth dimension with the  $\text{AdS}_5$  metric. Different values of the holographic variable  $z$  determine the scale of the invariant separation between the partonic constituents. This is illustrated in Fig. 1. Hard scattering processes occur in the small- $z$  ultraviolet (UV) region of AdS space. In particular, the  $Q \rightarrow \infty$  zero separation limit corresponds to the  $z \rightarrow 0$  asymptotic boundary, where the QCD Lagrangian is defined.

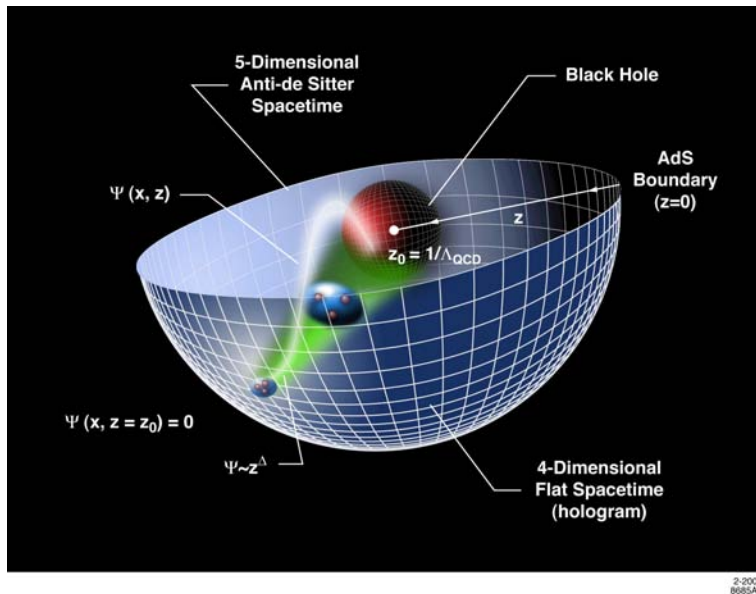


Figure 1: Artist's conception of AdS/CFT. The evolution of the proton at different length scales is mapped into the compact  $\text{AdS}_5$  dimension  $z$ . Dirichlet bag-like boundary condition,  $\Psi(z)|_{z=z_0} = 0$ , is imposed at the confinement radius  $z = z_0 = 1/\Lambda_{\text{QCD}}$ , thus limiting interquark separations.

As shown by Polchinski and Strassler [34], one can simulate confinement by imposing boundary conditions in the holographic variable  $z$ . The infrared (IR) cut-off at  $z_0 = 1/\Lambda_{\text{QCD}}$  breaks conformal invariance, allowing the introduction of the QCD mass scale and a spectrum of particle states. In the hard wall model [34] a cut-off is placed at a finite value  $z_0 = 1/\Lambda_{\text{QCD}}$  and the spectrum of states is linear in the radial and angular momentum quantum numbers:  $\mathcal{M} \sim 2n + L$ . In the soft wall model a smooth infrared cutoff is chosen to model confinement and reproduce the usual Regge behavior  $\mathcal{M}^2 \sim n + L$  [35]. The resulting models, although *ad hoc*, provide

a simple semi-classical approximation to QCD which has both constituent counting rule behavior at short distances and confinement at large distances.

It is thus natural, as a useful first approximation, to use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS space, into the modes propagating inside AdS. Under conformal transformations the interpolating operators transform according to their twist, and consequently the AdS isometries map the twist scaling dimensions into the AdS modes [36]. A physical hadron in four-dimensional Minkowski space has four-momentum  $P_\mu$  and invariant mass given by  $P_\mu P^\mu = \mathcal{M}^2$ . The physical states in AdS<sub>5</sub> space are represented by normalizable “string” modes  $\Phi_P(x, z) \sim e^{-iP \cdot x} \Phi(z)$ , with plane waves along the Poincaré coordinates and a profile function  $\Phi(z)$  along the holographic coordinate  $z$ , as illustrated in Fig. 1. For small- $z$ ,  $\Phi$  scales as  $\Phi \sim z^\Delta$ , where  $\Delta$  is the conformal dimension of the string state, the same dimension of the interpolating operator  $\mathcal{O}$  which creates a hadron out of the vacuum [34],  $\langle P|\mathcal{O}|0\rangle \neq 0$ . The scale dependence of each string mode  $\Phi(x, z)$  is thus determined by matching its behavior at  $z \rightarrow 0$  with the scaling dimension of the corresponding hadronic state at short distances  $x^2 \rightarrow 0$ . Changes in length scale are mapped to evolution in the holographic variable  $z$ . The string mode in  $z$  thus represents the extension of the hadron wave function into the fifth dimension. The eigenvalues of normalizable modes in AdS give the hadronic spectrum. AdS modes represent also the probability amplitude for the distribution of quarks and gluons at a given scale. There are also non-normalizable modes which are related to external currents: they propagate into the AdS interior and couple to boundary QCD interpolating operators [32, 33].

Following the approach described above, a limited set of operators is introduced to construct phenomenological viable five-dimensional dual holographic models. This simple prescription, which has been described as a “bottom-up” approach, has been successful in obtaining general properties of scattering amplitudes of hadronic bound states at strong coupling [34, 36, 37, 38, 39, 40], the low-lying hadron spectra [35, 41, 42, 43, 44, 45, 46, 47, 48, 49], hadron couplings and chiral symmetry breaking [41, 50, 51, 52, 53], quark potentials in confining backgrounds [54, 55], a description of weak hadron decays [56] and euclidean correlation functions [57]. Geometry back-reaction in AdS may also be relevant to the infrared physics [58] and wall dynamics [59]. The gauge theory/gravity duality also provides a convenient framework for the description of deep inelastic scattering structure functions at small  $x$  [60, 61, 62], a unified description of hard and soft pomeron physics [63] and gluon scattering amplitudes at strong coupling [64].

In the top-down approach, one introduces higher dimensional branes to the AdS<sub>5</sub>  $\times$  S<sup>5</sup> background [65] in order to have a theory of flavor. One can obtain models with massive quarks in the fundamental representation, compute the hadronic spectrum, and describe chiral symmetry breaking in the context of higher dimensional brane constructs [65, 66, 67, 68, 69]. However, a theory dual to QCD is unknown, and this “top-down” approach is difficult to extend beyond theories exceedingly constrained

by their symmetries [70].

As we shall discuss, there is a remarkable mapping between the AdS description of hadrons and the Hamiltonian formulation of QCD in physical space-time quantized on the light front. The light-front wavefunctions of bound states in QCD are relativistic and frame-independent generalizations of the familiar Schrödinger wavefunctions of atomic physics, but they are determined at fixed light-cone time  $\tau = t + z/c$ —the “front form” advocated by Dirac [71]—rather than at fixed ordinary time  $t$ . The light-front wavefunctions of a hadron are independent of the momentum of the hadron, and they are thus boost invariant; Wigner transformations and Melosh rotations are not required. The light-front formalism for gauge theories in light-cone gauge is particularly useful in that there are no ghosts and one has a direct physical interpretation of orbital angular momentum.

An important feature of light-front quantization is the fact that it provides exact formulas to write down matrix elements as a sum of bilinear forms, which can be mapped into their AdS/CFT counterparts in the semi-classical approximation. One can thus obtain not only an accurate description of the hadron spectrum for light quarks, but also a remarkably simple but realistic model of the valence wavefunctions of mesons, baryons, and glueballs. In terms of light front coordinates  $x^\pm = x^0 \pm x^3$  the AdS metric is

$$ds^2 = \frac{R^2}{z^2} (dx^+ dx^- - d\mathbf{x}_\perp^2 - dz^2). \quad (2)$$

At fixed light-front time  $x^+ = 0$ , the metric depends only on the transverse  $\mathbf{x}_\perp$  and the holographic variable  $z$ . Thus we can find an exact correspondence between the fifth-dimensional coordinate of anti-de Sitter space  $z$  and a specific impact variable  $\zeta$  in the light-front formalism. The new variable  $\zeta$  measures the separation of the constituents within the hadron in ordinary space-time. The amplitude  $\Phi(z)$  describing the hadronic state in AdS<sub>5</sub> can then be precisely mapped to the light-front wavefunctions  $\psi_{n/h}$  of hadrons in physical space-time [45], thus providing a relativistic description of hadrons in QCD at the amplitude level. This connection allows one to compute the analytic form [45] of the light-front wavefunctions of mesons and baryons. AdS/CFT also provides a non-perturbative derivation of dimensional counting rules for the power-law fall-off of form factors and exclusive scattering amplitudes at large momentum transfer. The AdS/CFT approach thus leads to a model of hadrons which has both confinement at large distances and the conformal scaling properties which reproduce dimensional counting rules for hard exclusive reactions.

## 2 Gauge/Gravity Semiclassical Correspondence

The formal statement of the duality between a gravity theory on  $(d + 1)$ -dimensional Anti-de Sitter AdS <sub>$d+1$</sub>  space and the strong coupling limit of a conformal field theory (CFT) on the  $d$ -dimensional asymptotic boundary of AdS <sub>$d+1$</sub>  at  $z = 0$  is expressed in

terms of the  $d + 1$  partition function for a field  $\Phi(x, z)$  propagating in the bulk

$$Z_{grav}[\Phi(x, z)] = e^{iS_{eff}[\Phi]} = \int \mathcal{D}[\Phi] e^{iS[\Phi]}, \quad (3)$$

where  $S_{eff}$  is the effective action of the  $\text{AdS}_{d+1}$  theory, and the  $d$ -dimensional generating functional of the conformal field theory in presence of an external source  $\Phi_0(x)$ ,

$$Z_{CFT}[\Phi_0(x)] = e^{iW_{CFT}[\Phi_0]} = \left\langle \exp \left( i \int d^d x \Phi_0(x) \mathcal{O}(x) \right) \right\rangle. \quad (4)$$

The functional  $W_{CFT}$  is the generator of connected Green's functions of the boundary theory and  $\mathcal{O}(x)$  is a QCD local interpolating operator. The precise relation of the gravity theory on AdS space to the conformal field theory at its boundary is [32, 33]

$$Z_{grav}[\Phi(x, z)|_{z=0} = \Phi_0(x)] = Z_{CFT}[\Phi_0], \quad (5)$$

where the partition function (3) on  $\text{AdS}_{d+1}$  is integrated over all possible configurations  $\Phi$  in the bulk which approach its boundary value  $\Phi_0$ . If we neglect the contributions from the non-classical configurations to the gravity partition function, then the generator  $W_{CFT}$  of connected Green's functions of the four-dimensional gauge theory (4) is precisely equal to the classical (on-shell) gravity action (3)

$$W_{CFT}[\phi_0] = S_{eff}[\Phi(x, z)|_{z=0} = \Phi_0(x)]_{\text{on-shell}}, \quad (6)$$

evaluated in terms of the classical solution to the bulk equation of motion. This defines the semiclassical approximation to the conformal field theory. In the limit  $z \rightarrow 0$ , the independent solutions behave as

$$\Phi(z, x) \rightarrow z^\Delta \Phi_+(x) + z^{d-\Delta} \Phi_-(x), \quad (7)$$

where  $\Delta$  is the conformal dimension. The non-normalizable solution  $\Phi_-$  is the boundary value of the bulk field  $\Phi$  which couples to a QCD gauge invariant operator  $\mathcal{O}$  in the  $z \rightarrow 0$  asymptotic boundary, thus  $\Phi_- = \Phi_0$ . The normalizable solution  $\Phi_+(x)$  is the response function and corresponds to the physical states [72]. The interpolating operators  $\mathcal{O}$  of the boundary conformal theory are constructed from local gauge-invariant products of quark and gluon fields and their covariant derivatives, taken at the same point in four-dimensional space-time in the  $x^2 \rightarrow 0$  limit. Their conformal twist-dimensions are matched to the scaling behavior of the AdS fields in the limit  $z \rightarrow 0$  and are thus encoded into the propagation of the modes inside AdS space.

## 2.1 AdS Wave Equations

AdS coordinates are the Minkowski coordinates  $x^\mu$  and  $z$ , the holographic coordinate, which we label  $x^\ell = (x^\mu, z)$ . The metric of the full space-time is  $ds^2 = g_{\ell m} dx^\ell dx^m$ ,

where  $g_{\ell m} = (R^2/z^2)\eta_{\ell m}$ , and  $\eta_{\ell m}$  has diagonal components  $(1, -1, \dots, -1)$ . Unless stated otherwise, 5-dimensional fields are represented by capital letters such as  $\Phi$  and  $\Psi$ . Holographic fields in 4-dimensional Minkowski space are represented by  $\phi$  and  $\psi$  and constituent quark and gluon fields by  $q$  and  $G$ . We begin by writing the action for scalar modes on  $\text{AdS}_{d+1}$ . We consider a quadratic action of a free field propagating in the AdS background

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} [g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2], \quad (8)$$

where  $\sqrt{g} \rightarrow (R/z)^{d+1}$  in the conformal limit and  $\mu$  is a fifth dimensional mass. Taking the variation of (8) we find the equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} (\sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi) + \mu^2 \Phi = 0. \quad (9)$$

Integrating by parts and using the equation of motion, the bulk contribution to the action vanishes, and one is left with a non-vanishing surface term in the ultraviolet boundary

$$S = \frac{R^{d-1}}{2} \lim_{z \rightarrow 0} \int d^d x \frac{1}{z^{d-1}} \Phi \partial_z \Phi, \quad (10)$$

which can be identified with the boundary functional  $W_{CFT}$ . Substituting the leading dependence (7) of  $\Phi$  near  $z = 0$  in the ultraviolet surface action (10) and using the functional relation  $\delta W_{CFT}/\delta \Phi_0 = \delta S_{\text{eff}}/\delta \Phi_0$ , it follows that  $\Phi_+(x)$  is related to the expectation values of  $\mathcal{O}$  in the presence of the source  $\Phi_0$  [72]:  $\langle 0|\mathcal{O}(x)|0\rangle_{\Phi_0} \sim \Phi_+(x)$ . The exact relation depends on the normalization of the fields used [73]. The field  $\Phi_+$  thus acts as a classical field, and it is the boundary limit of the normalizable string solution which propagates in the bulk.

Factoring out the dependence of the hadronic modes along the Poincaré coordinates  $x^\mu$ ,  $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$  in (9), we find the effective AdS wave equation for the scalar string mode  $\Phi(z)$

$$[z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0. \quad (11)$$

The eigenvalues of (11) are the hadronic invariant mass states  $P_\mu P^\mu = \mathcal{M}^2$  and the fifth-dimensional mass is related to the conformal dimension  $(\mu R)^2 = \Delta(\Delta - 4)$ . Stable solutions satisfy the condition  $(\mu R)^2 \geq -d^2/4$ , according to the Breitenlohner-Freedman bound [74].

Higher spin- $S$  bosonic modes in AdS are described by a set of  $S+1$  coupled differential equations [75]. Each hadronic state of integer spin  $S$ ,  $S \leq 2$ , is dual to a normalizable string mode  $\Phi_P(x, z)_{\mu_1 \mu_2 \dots \mu_S}^S = \epsilon_{\mu_1 \mu_2 \dots \mu_S} e^{-iP \cdot x} \Phi_S(z)$ , with four-momentum  $P_\mu$  and spin polarization indices along the 3+1 physical coordinates. For string modes with all the polarization indices along the Poincaré coordinates, the coupled differential wave equations for a spin- $S$  bosonic mode reduce to the homogeneous



equation [75]

$$[z^2 \partial_z^2 - (d-1-2S)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi_S(z) = 0, \quad (12)$$

with  $(\mu R)^2 = (\Delta - S)(\Delta - d + S)$ . We expect to avoid large anomalous dimensions associated with  $S > 2$  since modes with  $S \leq 2$  do not couple to stringy excitations.

### 3 The Holographic Light-Front Hamiltonian and Schrödinger Equation

We shall show in Sect. 5 how the string amplitude  $\Phi(z)$  can be mapped to the light-front wave functions of hadrons in physical space-time [45]. In fact, we find an exact correspondence between the holographic variable  $z$  and an impact variable  $\zeta$  which measures the transverse separation of the constituents within a hadron, we can identify  $\zeta = z$ . The mapping of  $z$  from AdS space to  $\zeta$  in the LF space allows the equations of motion in AdS space to be recast in the form of a light-front Hamiltonian equation [76]

$$H_{LF} |\phi\rangle = \mathcal{M}^2 |\phi\rangle, \quad (13)$$

a remarkable result which maps AdS/CFT solutions to light-front equations in physical 3+1 space-time. By substituting  $\phi(\zeta) = \zeta^{-3/2} \Phi(\zeta)$ , in the AdS scalar wave equation (11) for  $d = 4$ , we find an effective Schrödinger equation as a function of the weighted impact variable  $\zeta$

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta), \quad (14)$$

with the conformal potential  $V(\zeta) \rightarrow -(1 - 4L^2)/4\zeta^2$ , an effective two-particle light-front radial equation for mesons [16, 45]. Its eigenmodes determine the hadronic mass spectrum. We have written above  $(\mu R)^2 = -4 + L^2$ . The holographic hadronic light-front wave functions  $\phi(\zeta) = \langle \zeta | \phi \rangle$  are normalized according to

$$\langle \phi | \phi \rangle = \int d\zeta |\langle \zeta | \phi \rangle|^2 = 1, \quad (15)$$

and represent the probability amplitude to find  $n$ -partons at transverse impact separation  $\zeta = z$ . Its eigenvalues are set by the boundary conditions at  $\phi(z = 1/\Lambda_{\text{QCD}}) = 0$  and are given in terms of the roots of Bessel functions:  $\mathcal{M}_{L,k} = \beta_{L,k} \Lambda_{\text{QCD}}$ . The normalizable modes are

$$\phi_{L,k}(\zeta) = \frac{\sqrt{2} \Lambda_{\text{QCD}}}{J_{1+L}(\beta_{L,k})} \sqrt{\zeta} J_L(\zeta \beta_{L,k} \Lambda_{\text{QCD}}) \theta(\zeta \leq \Lambda_{\text{QCD}}^{-1}). \quad (16)$$

The lowest stable state  $L = 0$  is determined by the Breitenlohner-Freedman bound [74]. Higher excitations are matched to the small  $z$  asymptotic behavior of

each string mode to the corresponding conformal dimension of the boundary operators of each hadronic state. The effective wave equation (14) is a relativistic light-front equation defined at  $x^+ = 0$ . The AdS metric  $ds^2$  (2) is invariant if  $\mathbf{x}_\perp^2 \rightarrow \lambda^2 \mathbf{x}_\perp^2$  and  $z \rightarrow \lambda z$  at equal light-front time  $x^+ = 0$ . The Casimir operator for the rotation group  $SO(2)$  in the transverse light-front plane is  $L^2$ . This shows the natural holographic connection to the light front.

For higher spin bosonic modes we can also recast the wave equation AdS (12) into its light-front form (13). Using the substitution  $\phi_S(\zeta) = \zeta^{-3/2+S} \Phi_S(\zeta)$ ,  $\zeta = z$ , we find a LF Schrödinger equation identical to (14) with  $\phi \rightarrow \phi_S$ , provided that  $(\mu R)^2 = -(2-S)^2 + \nu^2$ . Stable solutions satisfy a generalized Breitenlohner-Freedman bound  $(\mu R)^2 \geq -(d-2S)^2/4$ , and thus the lowest stable state has scaling dimensions  $\Delta = 2$ , independent of  $S$ . The fundamental LF equation of AdS/CFT has the appearance of a Schrödinger equation, but it is relativistic, covariant, and analytically tractable.

The pseudoscalar meson interpolating operator  $\mathcal{O}_{2+L} = \bar{q}\gamma_5 D_{\{\ell_1} \cdots D_{\ell_m\}} q$ , written in terms of the symmetrized product of covariant derivatives  $D$  with total internal space-time orbital momentum  $L = \sum_{i=1}^m \ell_i$ , is a twist-two, dimension  $3 + L$  operator with scaling behavior determined by its twist-dimension  $2 + L$ . Likewise the vector-meson operator  $\mathcal{O}_{2+L}^\mu = \bar{q}\gamma^\mu D_{\{\ell_1} \cdots D_{\ell_m\}} q$  has scaling dimension  $2 + L$ . The scaling behavior of the scalar and vector AdS modes is precisely the scaling required to match the scaling dimension of the local pseudoscalar and vector-meson interpolating operators. The light meson spectrum is compared in Figure 2 with the experimental values.

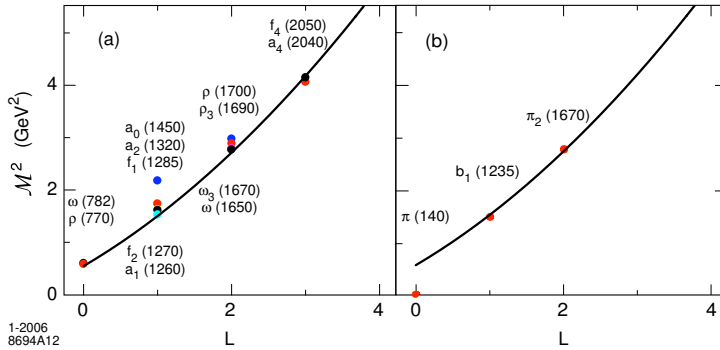


Figure 2: Light meson orbital states for  $\Lambda_{\text{QCD}} = 0.32 \text{ GeV}$ : (a) vector mesons and (b) pseudoscalar mesons. The data are from [77].

### 3.1 Integrability of AdS/CFT Equations

The integrability methods of [78] find a remarkable application in the AdS/CFT correspondence. Integrability follows if the equations describing a physical model can be factorized in terms of linear operators. These ladder operators generate all the

eigenfunctions once the lowest mass eigenfunction is known. In holographic QCD, the conformally invariant 3 + 1 light-front differential equations can be expressed in terms of ladder operators and their solutions can then be expressed in terms of analytical functions. In the conformal limit the ladder algebra for bosonic ( $B$ ) or fermionic ( $F$ ) modes is given in terms of the operator ( $\Gamma^B = 1$ ,  $\Gamma^F = \gamma_5$ )

$$\Pi_\nu^{B,F}(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \Gamma^{B,F} \right), \quad (17)$$

and its adjoint

$$\Pi_\nu^{B,F}(\zeta)^\dagger = -i \left( \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} \Gamma^{B,F} \right), \quad (18)$$

with commutation relations

$$[\Pi_\nu^{B,F}(\zeta), \Pi_\nu^{B,F}(\zeta)^\dagger] = \frac{2\nu + 1}{\zeta^2} \Gamma^{B,F}. \quad (19)$$

For bosonic modes the Hamiltonian is written as a bilinear form:  $H_{LC}^{B,F} = \Pi_\nu^{B,F\dagger} \Pi_\nu^{B,F}$ . For  $\nu^2 \geq 0$  the Hamiltonian is positive definite

$$\langle \phi | H_{LC}^\nu | \phi \rangle = \int d\zeta |\Pi_\nu \phi(z)|^2 \geq 0, \quad (20)$$

and its eigenvalues are positive:  $\mathcal{M}^2 \geq 0$ . For  $\nu^2 < 0$  the Hamiltonian is not bounded from below. The critical value of the potential corresponds to  $\nu = 0$  with potential  $V_{crit}(\zeta) = 1/4\zeta^2$ . LF quantum-mechanical stability conditions are thus equivalent to the stability conditions which follows from the Breitenlohner-Freedman stability bound [74]. Higher orbital states are constructed from the  $L$ -th application of the raising operator  $a^\dagger = -i\Pi^B$  on the ground state  $|L\rangle \sim (a^\dagger)^L|0\rangle$ . In the  $\zeta$  light-front coordinate representation

$$\langle \zeta | L \rangle \sim \sqrt{\zeta} (-\zeta)^L \left( \frac{1}{\zeta} \frac{d}{d\zeta} \right)^L J_0(\zeta \mathcal{M}) \sim \sqrt{\zeta} J_L(\zeta \mathcal{M}). \quad (21)$$

In the fermionic case the eigenmodes also satisfy a first order LF Dirac equation as will be shown in Sect. 4.

### 3.2 Soft-Wall Holographic Model

The predicted mass spectrum in the truncated space hard-wall (HW) model is linear  $M \propto L + 2n$  at high orbital angular momentum  $L$ , in contrast to the quadratic dependence  $M^2 \propto L + n$  in the usual Regge parameterization. It has been shown recently that by choosing a specific profile for a non-constant dilaton, the usual Regge dependence can be obtained [35]. This procedure retains conformal AdS metrics (1) while

introducing a smooth cutoff which depends on the profile of a dilaton background field  $\varphi$

$$S = \int d^4x dz \sqrt{g} e^{-\varphi(z)} \mathcal{L}, \quad (22)$$

where  $\varphi$  is a function of the holographic coordinate  $z$  which vanishes in the ultraviolet limit  $z \rightarrow 0$ . The IR hard-wall or truncated space holographic model corresponds to a constant dilaton field  $\varphi(z) = \varphi_0$  in the confining region,  $z \leq 1/\Lambda_{\text{QCD}}$ , and to very large values elsewhere:  $\varphi(z) \rightarrow \infty$  for  $z > 1/\Lambda_{\text{QCD}}$ . The introduction of a soft cutoff avoids the ambiguities in the choice of boundary conditions at the infrared wall. A convenient choice [35] for the background field with usual Regge behavior is  $\varphi(z) = \kappa^2 z^2$ . The resulting wave equations are equivalent to the radial equation of a two-dimensional oscillator, previously found in the context of mode propagation on  $\text{AdS}_5 \times S^5$ , in the light-cone formulation of Type II supergravity [79]. Also, equivalent results follow from the introduction of a gaussian warp factor in the AdS metric for the particular case of massless vector modes propagating in the distorted metric [80]. A different approach to the soft-wall (SW) consists in the non-conformal extension of the algebraic expressions found in the previous section to obtain directly the corresponding holographic LF wave equations. This method is particularly useful to extend the non-conformal results to the fermionic sector where the corresponding linear wave equations become exactly solvable. The extended generators are given in terms of the matrix-valued operator  $\Pi$  and its adjoint  $\Pi^\dagger$  ( $\Gamma^B = 1$ ,  $\Gamma^F = \gamma_5$ )

$$\Pi_\nu^{B,F}(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \Gamma^{B,F} - \kappa^2 \zeta \Gamma^{B,F} \right), \quad (23)$$

$$\Pi_\nu^{B,F}(\zeta)^\dagger = -i \left( \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} \Gamma^{B,F} + \kappa^2 \zeta \Gamma^{B,F} \right), \quad (24)$$

with commutation relations

$$[\Pi_\nu^{B,F}(\zeta), \Pi_\nu^{B,F}(\zeta)^\dagger] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \Gamma^{B,F}. \quad (25)$$

An account of the extended algebraic holographic model and a possible supersymmetric connection between the bosonic and fermionic operators used in the holographic construction will be described elsewhere.

## 4 Baryonic Spectra in AdS/QCD

The holographic model based on truncated AdS space can be used to obtain the hadronic spectrum of light quark  $q\bar{q}$ ,  $qqq$  and  $gg$  bound states. Specific hadrons are identified by the correspondence of the AdS amplitude with the twist dimension of the interpolating operator for the hadron's valence Fock state, including its orbital

angular momentum excitations. Bosonic modes with conformal dimension  $2 + L$  are dual to the interpolating operator  $\mathcal{O}_{\tau+L}$  with  $\tau = 2$ . For fermionic modes  $\tau = 3$ .

As an example, we will outline here the analysis of the baryon spectrum in AdS/CFT. The action for massive fermionic modes on AdS $_{d+1}$  is

$$S[\bar{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g} \bar{\Psi} (i\Gamma^\ell D_\ell - \mu) \Psi, \quad (26)$$

with the equation of motion

$$\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0. \quad (27)$$

Upon the substitution  $\Psi(x, z) = e^{-iP \cdot x} z^2 \psi(z)$ ,  $z \rightarrow \zeta$ , we find the light-front Dirac equation

$$(\alpha \Pi^F(\zeta) - \mathcal{M}) \psi(\zeta) = 0, \quad (28)$$

where the generator  $\Pi^F$  is given by (17) and  $i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$  in the Weyl representation. The solution is

$$\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_-], \quad (29)$$

with  $\gamma_5 u_\pm = u_\pm$ . A discrete four-dimensional spectrum follows when we impose the boundary condition  $\psi_\pm(\zeta = 1/\Lambda_{\text{QCD}}) = 0$ :  $\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{\text{QCD}}$ ,  $\mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{\text{QCD}}$ , with a scale-independent mass ratio [44].

Figure 3(a) shows the predicted orbital spectrum of the nucleon states and Fig. 3(b) the  $\Delta$  orbital resonances. The spin-3/2 trajectories are determined from the corresponding Rarita-Schwinger equation. The solution of the spin-3/2 for polarization along Minkowski coordinates,  $\psi_\mu$ , is identical to the spin-1/2 solution. The data for the baryon spectra are from [77]. The internal parity of states is determined from the SU(6) spin-flavor symmetry. Since only one parameter, the QCD mass scale  $\Lambda_{\text{QCD}}$  is introduced, the agreement with the pattern of physical states is remarkable. In particular, the ratio of  $\Delta$  to nucleon trajectories is determined by the ratio of zeros of Bessel functions.

We can solve the LF Dirac equation (28) with the non-conformal extended generator  $\Pi^F$  given by (23). The solutions to the Dirac equation are

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2), \quad (30)$$

$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2). \quad (31)$$

with eigenvalues  $\mathcal{M}^2 = 4\kappa^2(n + \nu + 1)$ . Comparing with usual Dirac equation in AdS space we find

$$\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R + V(z) \right] \Psi(x^\ell) = 0. \quad (32)$$

with  $V(z) = \kappa^2 z$ . Thus for fermions the ‘‘soft-wall’’ corresponds to fermion modes propagating in AdS conformal metrics in presence of a linear confining potential.

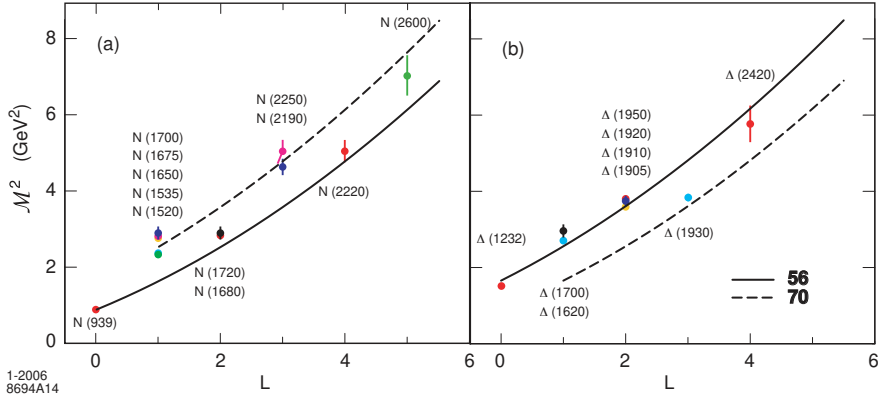


Figure 3: Predictions for the light baryon orbital spectrum for  $\Lambda_{QCD} = 0.25$  GeV. The **56** trajectory corresponds to  $L$  even  $P = +$ , and the **70** to  $L$  odd  $P = -$  states.

## 5 Hadronic Form Factors in AdS/QCD

The AdS/QCD correspondence is particularly relevant for the description of hadronic form factors, since it incorporates the connection between the twist of the hadron to the fall-off of its current matrix elements, as well as essential aspects of vector meson dominance. It also provides a convenient framework for analytically continuing the space-like results to the time-like region. Recent applications to the electromagnetic [81, 82, 83, 84, 85, 86, 87, 88] and gravitational [89] form factors of hadrons have followed from the original work described in [60, 90].

### 5.1 Meson Form Factors

In AdS/CFT, the hadronic matrix element for the electromagnetic current has the form of a convolution of the string modes for the initial and final hadrons with the external electromagnetic source which propagates inside AdS. We discuss first the truncated space or hard wall [34] holographic model, where quark and gluons as well as the external electromagnetic current propagate freely into the AdS interior according to the AdS metric. Assuming minimal coupling the form factor has the form [60, 90]

$$ig_5 \int d^4x dz \sqrt{g} A^\ell(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_\ell \Phi_P(x, z), \quad (33)$$

where  $g_5$  is a five-dimensional effective coupling constant and  $\Phi_P(x, z)$  is a normalizable mode representing a hadronic state,  $\Phi_P(x, z) \sim e^{-iP \cdot x} \Phi(z)$ , with hadronic invariant mass given by  $P_\mu P^\mu = \mathcal{M}^2$ . We consider the propagation inside AdS space of an electromagnetic probe polarized along Minkowski coordinates ( $Q^2 = -q^2 > 0$ )  $A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q^2, z)$ ,  $A_z = 0$ , where  $J(Q^2, z)$  has the value 1 at zero momentum transfer, since we are normalizing the bulk solutions to the total charge

operator, and as boundary limit the external current  $A_\mu(x, z \rightarrow 0) = \epsilon_\mu e^{-iQ \cdot x}$ . Thus  $J(Q^2 = 0, z) = J(Q^2, z = 0) = 1$ .

The propagation of the external current inside AdS space is described by the wave equation

$$[z^2 \partial_z^2 - z \partial_z - z^2 Q^2] J(Q^2, z) = 0, \quad (34)$$

with the solution  $J(Q^2, z) = zQK_1(zQ)$ . Substituting the normalizable mode  $\Phi(x, z)$  in (33) and extracting an overall delta function from momentum conservation at the vertex, we find the matrix element  $\langle P' | J^\mu(0) | P \rangle = 2(P + P')^\mu F(Q^2)$ , with

$$F(Q^2) = R^3 \int \frac{dz}{z^3} \Phi(z) J(Q^2, z) \Phi(z). \quad (35)$$

The form factor in AdS is thus represented as the overlap of the normalizable modes dual to the incoming and outgoing hadrons,  $\Phi_P$  and  $\Phi_{P'}$ , with the non-normalizable mode,  $J(Q^2, z)$ , dual to the external source [60]. Since  $K_n(x) \sim \sqrt{\pi/2x} e^{-x}$  for large  $x$ , it follows that the external electromagnetic field is suppressed inside the AdS cavity for large  $Q$ . At small  $z$  the string modes scale as  $\Phi \sim z^\Delta$ . At large enough  $Q$ , the important contribution to (35) is from the region near  $z \sim 1/Q$ :  $F(Q^2) \rightarrow (1/Q^2)^{\Delta-1}$ , and the ultraviolet point-like behavior [91] responsible for the power law scaling [20, 21] is recovered. This is a remarkable consequence of truncating AdS space since we are describing the coupling of an electromagnetic current to an extended mode, and instead of soft collision amplitudes characteristic of strings, hard point-like ultraviolet behavior is found [34].

The form factor in AdS space in presence of the dilaton background  $\varphi = \kappa^2 z^2$  has the additional term  $e^{-\kappa^2 z^2}$  in the metric

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q^2, z) \Phi(z). \quad (36)$$

Since the non-normalizable modes also couple to the dilaton field, we must study the solutions of the modified wave equation describing the propagation in AdS space of an electromagnetic probe. The solution is [84, 85]

$$J_\kappa(Q^2, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right), \quad (37)$$

where  $U(a, b, c)$  is the confluent hypergeometric function with the integral representation  $\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt$ . In the large  $Q^2$  limit,  $Q^2 \gg 4\kappa^2$  we find that  $J_\kappa(Q, z) \rightarrow zQK_1(zQ)$ . Thus, for large transverse momentum the current decouples from the dilaton background.

We can compute the pion form factor from the AdS expressions (35) and (36) for the hadronic string modes  $\Phi_\pi$  in the hard-wall (HW)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2}J_1(\beta_{0,1})} z^2 J_0(z\beta_{0,1}\Lambda_{QCD}), \quad (38)$$

and soft-wall (SW) model

$$\Phi_{\pi}^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2, \quad (39)$$

respectively. For the soft wall model the results for form factors can be expressed analytically. For integer twist  $\tau = n$  the form factor is expressed as a  $N - 1$  product of poles, corresponding to the first  $n - 1$  states along the vector meson trajectory [85]. Since the pion mode couples to a twist-two boundary interpolating operator which creates a two-component hadronic bound state, the form factor is given in the SW model by a simple monopole form. In Fig. 4, we plot the product  $Q^2 F_{\pi}(Q^2)$  for the soft and hard-wall holographic models. When the results for the pion form factor are

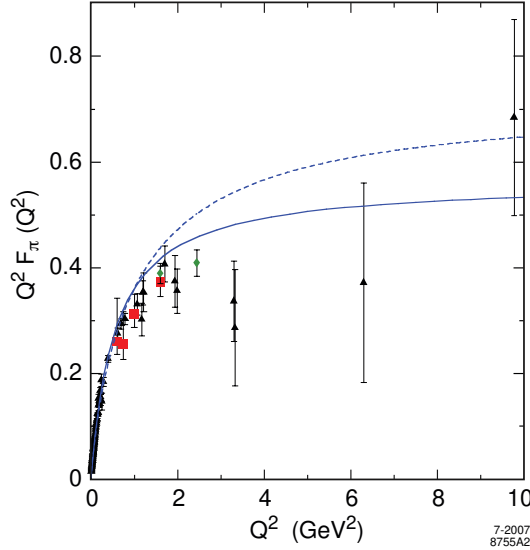


Figure 4: Space-like scaling behavior for  $Q^2 F_{\pi}(Q^2)$  as a function of  $Q^2 = -q^2$ . The continuous line is the prediction of the soft-wall model for  $\kappa = 0.375$  GeV. The dashed line is the prediction of the hard-model for  $\Lambda_{QCD} = 0.22$  GeV. The black triangles are from the data compilation of Baldini *et al.* [92], and the red boxes and cobalt green diamonds are JLAB data [93].

analytically continued to the time-like region,  $q^2 \rightarrow -q^2$  we obtain the results shown in Figure 5 for  $\log(|F_{\pi}(q^2)|)$  in the SW model. The monopole form of the SW model exhibits a pole at the  $\rho$  mass and reproduces well the  $\rho$  peak with  $M_{\rho} = 4\kappa^2 = 750$  MeV. In the strongly coupled semiclassical gauge/gravity limit hadrons have zero widths and are stable. The form factor accounts for the scaling behavior in the space-like region, but it does not give rise to the additional structure found in the time-like region since the  $\rho$  pole saturates 100% of the monopole form.



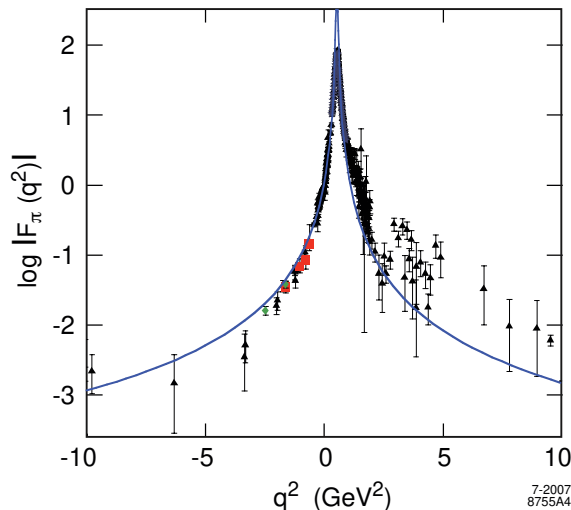


Figure 5: Space and time-like behavior of the pion form factor  $\log(|F_\pi(q^2)|)$  as a function of  $q^2$  for  $\kappa = 0.375$  GeV in the soft-wall model. The black (triangle) is from the data compilation of Baldini *et al.* [92], and the red (box) and cobalt green diamonds are JLAB data [93].

## 5.2 The Nucleon Dirac Form Factors

As an example of a twist  $\tau = 3$  fall-off we compute the spin non-flip nucleon form factor in the soft wall model. Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ R^4 \int \frac{dz}{z^4} e^{-\kappa^2 z^2} J_\kappa(Q, z) |\Psi_+(z)|^2, \quad (40)$$

$$F_-(Q^2) = g_- R^4 \int \frac{dz}{z^4} e^{-\kappa^2 z^2} J_\kappa(Q, z) |\Psi_-(z)|^2, \quad (41)$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory. We choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\Psi_+$  and  $\Psi_-$  correspond to nucleons with total angular momentum  $J^z = +1/2$  and  $-1/2$ . For the  $SU(6)$  spin-flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} e^{-\kappa^2 z^2} J_\kappa(Q, z) |\Psi_+(z)|^2, \quad (42)$$

$$F_1^n(Q^2) = -\frac{1}{3} R^4 \int \frac{dz}{z^4} e^{-\kappa^2 z^2} J_\kappa(Q, z) [|\Psi_+(z)|^2 - |\Psi_-(z)|^2], \quad (43)$$

where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ . The bulk-to-boundary propagator  $J_\kappa(Q, z)$  is the solution (37) of the AdS wave equation for the external electromagnetic current, and the plus and minus components of the twist 3 nucleon mode in the SW model are

$$\Psi_+(z) = \frac{\sqrt{2}\kappa^2}{R^2} z^{7/2}, \quad \Psi_-(z) = \frac{\kappa^3}{R^2} z^{9/2}. \quad (44)$$

For the SW model the results for  $Q^4 F_1^p(Q^2)$  and  $Q^4 F_1^n(Q^2)$  follow from the analytic form for the form factors for any  $\tau$  given in Appendix D of reference [85] and are shown in Figure 6.

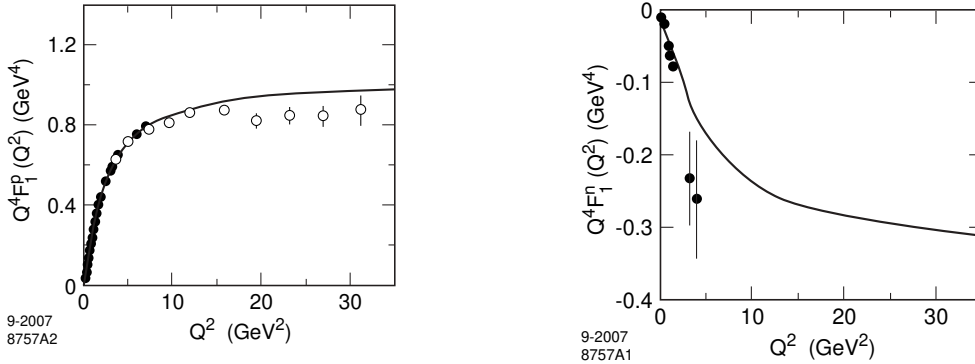


Figure 6: Predictions for  $Q^4 F_1^p(Q^2)$  and  $Q^4 F_1^n(Q^2)$  in the soft wall model for  $\kappa = 0.424$  GeV. The data compilation is from Diehl [94].

## 6 The Light-Front Fock Representation

The light-front expansion of any hadronic system is constructed by quantizing quantum chromodynamics at fixed light-cone time [71]  $\tau = t + z/c$ . In terms of the hadron four-momentum  $P = (P^+, P^-, \mathbf{P}_\perp)$ ,  $P^\pm = P^0 \pm P^3$ , the light-cone Lorentz invariant Hamiltonian for the composite system,  $H_{LF}^{QCD} = P^- P^+ - \mathbf{P}_\perp^2$ , has eigenvalues given in terms of the eigenmass  $\mathcal{M}$  squared corresponding to the mass spectrum of the color-singlet states in QCD [76].

The hadron wavefunction is an eigenstate of the total momentum  $P^+$  and  $\mathbf{P}_\perp$  and the longitudinal spin projection  $S_z$ , and is normalized according to

$$\langle \psi_h(P^+, \mathbf{P}_\perp, S_z) | \psi_h(P'^+, \mathbf{P}'_\perp, S'_z) \rangle = 2P^+ (2\pi)^3 \delta_{S_z, S'_z} \delta(P^+ - P'^+) \delta^{(2)}(\mathbf{P}_\perp - \mathbf{P}'_\perp). \quad (45)$$

The momentum generators  $P^+$  and  $\mathbf{P}_\perp$  are kinematical; *i.e.*, they are independent of the interactions. The LF time evolution operator  $P^- = i \frac{d}{d\tau}$  can be derived directly from the QCD Lagrangian in the light-cone gauge  $A^+ = 0$ . In principle, the complete set of bound states and scattering eigensolutions of  $H_{LF}$  can be obtained by solving the light-front Heisenberg equation  $H_{LF} |\psi_h\rangle = \mathcal{M}_h^2 |\psi_h\rangle$ , where  $|\psi_h\rangle$  is an expansion in multi-particle Fock eigenstates  $\{|n\rangle\}$  of the free LF Hamiltonian:  $|\psi_h\rangle = \sum_n \psi_{n/h} |n\rangle$ . The LF Heisenberg equation has in fact been solved for QCD(1+1) and a number of other theories using the discretized light-cone quantization method [95]. The light-cone gauge has the advantage that all gluon degrees

of freedom have physical polarization and positive metric. In addition, orbital angular momentum has a simple physical interpretation in this representation. The light-front wavefunctions (LFWFs)  $\psi_{n/h}$  provide a *frame-independent* representation of a hadron which relates its quark and gluon degrees of freedom to their asymptotic hadronic state.

Each hadronic eigenstate  $|\psi_h\rangle$  is expanded in a Fock-state complete basis of non-interacting  $n$ -particle states  $|n\rangle$  with an infinite number of components

$$|\psi_h(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n, \lambda_i} \int [dx_i] [d^2\mathbf{k}_{\perp i}] \psi_{n/h}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \frac{1}{\sqrt{x_i}} |n : x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle, \quad (46)$$

where the sum begins with the valence state; e.g.,  $n \geq 2$  for mesons. The coefficients of the Fock expansion

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \langle n : x_i, \mathbf{k}_{\perp i}, \lambda_i | \psi_h \rangle, \quad (47)$$

are independent of the total momentum  $P^+$  and  $\mathbf{P}_\perp$  of the hadron and depend only on the relative partonic coordinates, the longitudinal momentum fraction  $x_i = k_i^+/P^+$ , the relative transverse momentum  $\mathbf{k}_{\perp i}$ , and  $\lambda_i$ , the projection of the constituent's spin along the  $z$  direction. Thus, given the Fock-projection (47), the wavefunction of a hadron is determined in any frame. The amplitudes  $\psi_{n/h}$  represent the probability amplitudes to find on-mass-shell constituents  $i$  with longitudinal momentum  $x_i P^+$ , transverse momentum  $x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}$ , helicity  $\lambda_i$  and invariant mass

$$\mathcal{M}_n^2 = \sum_{i=1}^n k_i^\mu k_{i\mu} = \sum_{i=1}^n \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}, \quad (48)$$

in the hadron  $h$ . Momentum conservation requires  $\sum_{i=1}^n x_i = 1$  and  $\sum_{i=1}^n \mathbf{k}_{\perp i} = 0$ . In addition, each light front wavefunction  $\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$  obeys the angular momentum sum rule [96]  $J^z = \sum_{i=1}^n S_i^z + \sum_{i=1}^{n-1} L_i^z$ , where  $S_i^z = \lambda_i$  and the  $n-1$  orbital angular momenta have the operator form  $L_i^z = -i \left( \frac{\partial}{\partial k_i^x} k_i^y - \frac{\partial}{\partial k_i^y} k_i^x \right)$ . It should be emphasized that the assignment of quark and gluon spin and orbital angular momentum of a hadron is a gauge-dependent concept. The LF framework in light-cone gauge  $A^+ = 0$  provides a physical definition since there are no gauge field ghosts and the gluon has spin-projection  $J^z = \pm 1$ ; moreover, it is frame-independent.

The LFWFs are normalized according to

$$\sum_n \int [dx_i] [d^2\mathbf{k}_{\perp i}] |\psi_{n/h}(x_i, \mathbf{k}_{\perp i})|^2 = 1, \quad (49)$$

where the measure of the constituents phase-space momentum integration is

$$\int [dx_i] \equiv \prod_{i=1}^n \int dx_i \delta\left(1 - \sum_{j=1}^n x_j\right), \quad (50)$$

$$\int [d^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \int \frac{d^2\mathbf{k}_{\perp i}}{2(2\pi)^3} (16\pi^3) \delta^{(2)}\left(\sum_{j=1}^n \mathbf{k}_{\perp j}\right), \quad (51)$$

for the normalization given by (45). The spin indices have been suppressed.

Given the light-front wavefunctions  $\psi_{n/h}$ , one can compute a large range of hadron observables. For example, the valence and sea quark and gluon distributions which are measured in deep inelastic lepton scattering are defined from the squares of the LFWFs summed over all Fock states  $n$ . Form factors, exclusive weak transition amplitudes [97] such as  $B \rightarrow \ell\nu\pi$ , and the generalized parton distributions [98] measured in deeply virtual Compton scattering are (assuming the ‘‘handbag’’ approximation) overlaps of the initial and final LFWFs with  $n = n'$  and  $n = n' + 2$ . In the case of deeply virtual meson production such as  $\gamma^*p \rightarrow \pi X$  and  $\gamma^*p \rightarrow \rho p$ , the meson enters the amplitude directly through its LFWF. In inclusive reactions such as electron-positron annihilation to jets, the hadronic light-front wavefunctions are the amplitudes which control the coalescence of comoving quarks and gluons into hadrons. Thus one can study hadronization at the amplitude level. Light-front wavefunctions also control higher-twist contributions to inclusive and semi-inclusive reactions [99, 100].

The gauge-invariant distribution amplitude  $\phi_H(x_i, Q)$  defined from the integral over the transverse momenta  $\mathbf{k}_{\perp i}^2 \leq Q^2$  of the valence (smallest  $n$ ) Fock state provides a fundamental measure of the hadron at the amplitude level [101, 102]; they are the nonperturbative input to the factorized form of hard exclusive amplitudes and exclusive heavy hadron decays in perturbative QCD. The resulting distributions obey the DGLAP and ERBL evolution equations as a function of the maximal invariant mass, thus providing a physical factorization scheme [23]. In each case, the derived quantities satisfy the appropriate operator product expansions, sum rules, and evolution equations. However, at large  $x$  where the struck quark is far-off shell, DGLAP evolution is quenched [103], so that the fall-off of the DIS cross sections in  $Q^2$  satisfies inclusive-exclusive duality at fixed  $W^2$ .

The holographic mapping of hadronic LFWFs to AdS string modes is most transparent when one uses the impact parameter space representation. The total position coordinate of a hadron or its transverse center of momentum  $\mathbf{R}_{\perp}$ , is defined in terms of the energy momentum tensor  $T^{\mu\nu}$

$$\mathbf{R}_{\perp} = \frac{1}{P^+} \int dx^- \int d^2\mathbf{x}_{\perp} T^{++} \mathbf{x}_{\perp}. \quad (52)$$

In terms of partonic transverse coordinates  $x_i\mathbf{r}_{\perp i} = x_i\mathbf{R}_{\perp} + \mathbf{b}_{\perp i}$ , where the  $\mathbf{r}_{\perp i}$  are the physical transverse position coordinates and  $\mathbf{b}_{\perp i}$  frame independent internal coordinates, conjugate to the relative coordinates  $\mathbf{k}_{\perp i}$ . Thus,  $\sum_i \mathbf{b}_{\perp i} = 0$  and  $\mathbf{R}_{\perp} = \sum_i x_i\mathbf{r}_{\perp i}$ . The LFWFs  $\psi_n(x_j, \mathbf{k}_{\perp j})$  can be expanded in terms of the  $n - 1$  independent transverse coordinates  $\mathbf{b}_{\perp j}$ ,  $j = 1, 2, \dots, n - 1$

$$\psi_n(x_j, \mathbf{k}_{\perp j}) = (4\pi)^{(n-1)/2} \exp\left(i \sum_{j=1}^{n-1} \mathbf{b}_{\perp j} \cdot \mathbf{k}_{\perp j}\right) \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j}). \quad (53)$$

The normalization is defined by

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \left| \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j}) \right|^2 = 1. \quad (54)$$

## 6.1 The Form Factor in Light-Front QCD

One of the important advantages of the light-front formalism is that current matrix elements can be represented without approximation as overlaps of light-front wavefunctions. In the case of the elastic space-like form factors, the matrix element of the  $J^+$  current only couples Fock states with the same number of constituents. It is convenient to choose the light-front frame coordinates

$$\begin{aligned} P &= (P^+, P^-, \mathbf{P}_\perp) = \left( P^+, \frac{M^2}{P^+}, \vec{0}_\perp \right), \\ q &= (q^+, q^-, \mathbf{q}_\perp) = \left( 0, \frac{-q^2}{P^+}, \mathbf{q}_\perp \right), \end{aligned} \quad (55)$$

where  $q^2 = -Q^2 = -2P \cdot q = -\mathbf{q}_\perp^2$  is the space-like four momentum squared transferred to the composite system. The electromagnetic form factor of a meson is defined in terms of the hadronic amplitude of the electromagnetic current evaluated at light-cone time  $x^+ = 0$ :  $\langle P' | J^+(0) | P \rangle = 2(P + P')^+ F(Q^2)$ , where  $P' = P + q$  and  $F(0) = 1$ . If the charged parton  $n$  is the active constituent struck by the current, and the quarks  $i = 1, 2, \dots, n - 1$  are spectators, then the Drell-Yan West formula [104, 105, 106] in impact space is

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \exp\left(i \mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) \left| \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j}) \right|^2, \quad (56)$$

corresponding to a change of transverse momenta  $x_j \mathbf{q}_\perp$  for each of the  $n - 1$  spectators. This is a convenient form for comparison with AdS results, since the form factor is expressed in terms of the product of light-front wave functions with identical variables.

## 7 Light-Front /AdS Duality

We can now establish an explicit connection between the AdS/CFT and the LF formulae. To make more transparent the holographic connection between AdS<sub>5</sub> and the conformal quantum field theory defined at its asymptotic  $z \rightarrow 0$  boundary, it is convenient to use the AdS metric (2) in terms of light front coordinates  $x^\pm = x^0 \pm x^3$ . It is also useful to express (56) in terms of an effective single particle transverse distribution  $\tilde{\rho}$  [45]

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta), \quad (57)$$

where we have introduced the variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|, \quad (58)$$

representing the  $x$ -weighted transverse impact coordinate of the spectator system. On the other hand the form factor in AdS space (35) is represented as the overlap in the fifth dimension coordinate  $z$  of the normalizable modes dual to the incoming and outgoing hadrons,  $\Phi_P$  and  $\Phi_{P'}$ , with the non-normalizable source mode,  $J(Q, z) = zQK_1(zQ)$ . If we compare (57) in impact space with the expression for the form factor in AdS space (35) for arbitrary values of  $Q$  using the identity

$$\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q), \quad (59)$$

then we can identify the spectator density function appearing in the light-front formalism with the corresponding AdS density

$$\tilde{\rho}(x, \zeta) = \frac{R^3}{2\pi} \frac{x}{1-x} \frac{|\Phi(\zeta)|^2}{\zeta^4}. \quad (60)$$

Equation (60) gives a precise relation between string modes  $\Phi(\zeta)$  in AdS<sub>5</sub> and the QCD transverse charge density  $\tilde{\rho}(x, \zeta)$ . The variable  $\zeta$  represents a measure of the transverse separation between point-like constituents, and it is also the holographic variable  $z$  characterizing the string scale in AdS. Consequently the AdS string mode  $\Phi(z)$  can be regarded as the probability amplitude to find  $n$  partons at transverse impact separation  $\zeta = z$ . Furthermore, its eigenmodes determine the hadronic spectrum [45]. In the case of a two-parton constituent bound state, the correspondence between the string amplitude  $\Phi(z)$  and the light-front wave function  $\tilde{\psi}(x, \mathbf{b})$  is expressed in closed form [45]

$$\left| \tilde{\psi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi(\zeta)|^2}{\zeta^4}, \quad (61)$$

where  $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$ . Here  $\mathbf{b}_{\perp}$  is the impact separation conjugate to  $\mathbf{k}_{\perp}$ .

Hadron form factors can thus be predicted from the overlap integral of string modes propagating in AdS space with the boundary electromagnetic source which probes the AdS interior, or equivalently by using the Drell-Yan-West formula in physical space-time. If both quantities represent the same physical observable for any value of the transfer momentum  $q^2$ , an exact correspondence can be established between the string modes  $\Phi$  in fifth-dimensional AdS space and the light-front wavefunctions of hadrons  $\psi_{n/h}$  in 3+1 spacetime [45]. One can thus use holography to map the functional form of the string modes  $\Phi(z)$  in AdS space to the light front wavefunctions in

physical space time by identifying  $z$  with the transverse variable  $\zeta = \sqrt{x/(1-x)}|\vec{\eta}_\perp|$ . Here  $\vec{\eta}_\perp = \sum_{i=1}^{n-1} x_i \mathbf{b}_{\perp i}$  is the weighted impact separation, summed over the impact separation of the spectator constituents. The leading large- $Q^2$  behavior of form factors in AdS/QCD arises from small  $\zeta \sim 1/Q$ , corresponding to small transverse separation. The form factor of a hadron at large  $Q^2$  thus arises from the small  $z$  kinematic domain in AdS space. According to the AdS/CFT duality, this corresponds to small distances  $x^\mu x_\mu \sim 1/Q^2$  in physical space-time, the domain where the current matrix elements are controlled by the conformal twist-dimension,  $\Delta$ , of the hadron's interpolating operator. In the case of the front form, where  $x^+ = 0$ , this corresponds to small transverse separation  $x^\mu x_\mu = -\mathbf{x}_\perp^2$ .

As we have shown, the eigenvalues of the effective light-front equation provide a good description of the meson and baryon spectra for light quarks, and its eigensolutions provide a remarkably simple but realistic model of their valence wavefunctions. The resulting normalized light-front wavefunctions for the truncated space model are

$$\tilde{\psi}_{L,k}(x, \zeta) = B_{L,k} \sqrt{x(1-x)} J_L(\zeta \beta_{L,k} \Lambda_{\text{QCD}}) \theta(\zeta \leq \Lambda_{\text{QCD}}^{-1}), \quad (62)$$

where  $B_{L,k} = \pi^{-\frac{1}{2}} \Lambda_{\text{QCD}} / J_{1+L}(\beta_{L,k})$ . The results display confinement at large interquark separation and conformal symmetry at short distances, thus reproducing dimensional counting rules for hard exclusive processes. We have also derived analogous equations for baryons composed of massless quarks using a LF Dirac matrix representation for the baryon system. Most important, the eigensolutions of the AdS/CFT equation can be mapped to light-front equations of the hadrons in physical space-time, thus providing an elegant description of the light hadrons at the amplitude level. The meson LFWF is illustrated in Fig.7.

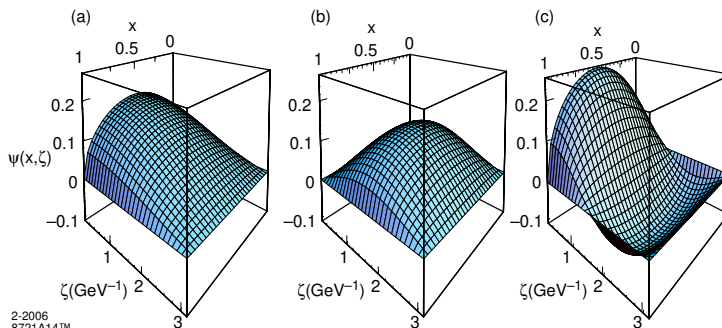


Figure 7: AdS/QCD Predictions for the light-front wavefunctions of a meson in the hard-wall model: (a)  $n = 0, L = 0$ , (b)  $n = 0, L = 1$ , (c)  $n = 1, L = 0$ .

## 7.1 Light-Front Mapping in the Soft-Wall Model

As discussed above, in the soft-wall model the current decouples from the dilaton field at large  $Q^2$  and we recover our previous scaling results for the ultraviolet behavior of

matrix elements. To obtain the corresponding basis set of LFWFs we compare the DYW expression of the form factor (57) with the AdS form factor (36) for large values of  $Q$ . Thus, in the large  $Q$  limit we can identify the light-front spectator density with the corresponding AdS density

$$\tilde{\rho}(x, \zeta) = \frac{R^3}{2\pi} \frac{x}{1-x} e^{-\kappa^2 \zeta^2} \frac{|\Phi(\zeta)|^2}{\zeta^4}. \quad (63)$$

When summed over all Fock states the Drell-Yan-West formula gives an exact result. The formula describes the coupling of the free electromagnetic current to the elementary constituents in the interaction representation. In the presence of a dilaton field in AdS space, or in the case where the electromagnetic probe propagates in modified confining AdS metrics, the electromagnetic AdS mode is no longer dual to a the free quark current, but dual to a dressed current, *i.e.*, a hadronic electromagnetic current including virtual  $\bar{q}q$  pairs and thus confined. Consequently, at finite values of the momentum transfer  $Q^2$  our simple identification discussed above has to be reinterpreted since we are comparing states in different representations: the interaction representation in light-cone QCD versus the Heisenberg representation in AdS. However both quantities should represent the same observables. We thus expect that the modified mapping corresponds to the presence of higher Fock states in the hadron.

## 8 Holographic Light-Front Wave Functions and Distribution Amplitudes of Flavored Mesons

As we have shown above, holographic light-front wave functions (LFWFs) of hadronic bound states follow from the mapping to physical space-time of string modes  $\Phi(z)$  in AdS<sub>5</sub> space [45]. For a two-parton bound state the mapping connects the transverse impact variable  $\zeta$ , the invariant separation between point-like constituents, identified with the holographic variable  $z$ ,  $\zeta^2 = z^2 = x(1-x)\mathbf{b}_\perp^2$ , where  $\mathbf{b}_\perp$  is the internal transverse position coordinate and  $x$  is the quark momentum fraction. In the soft-wall holographic model, the pion LFWF in impact space in the limit of massless constituents has the simple form [85]

$$\tilde{\psi}(x, \mathbf{b}_\perp)_{\bar{q}q/\pi} = \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2}. \quad (64)$$

The LFWF (64) can also be regarded as as the solution of a transverse oscillator in the light-front plane [85]. The LFWF in  $\mathbf{k}_\perp$  space is the Fourier transform

$$\psi(x, \mathbf{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{\mathbf{k}_\perp^2}{2\kappa^2 x(1-x)}}. \quad (65)$$

A simple generalization of the LFWF (65) for massive quarks follows from the assumption that the momentum space LFWF is a function of the invariant off-energy



shell quantity

$$\mathcal{M}^2 - \mathcal{E} = \sum_{i=1}^n \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i}. \quad (66)$$

Thus the holographic soft-wall LFWF ansatz for a meson bound state with massive constituents

$$\psi(x, \mathbf{k}_{\perp}) \sim \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2} \left( \frac{\mathbf{k}_{\perp}^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right)}. \quad (67)$$

The Fourier transform of (67) is the impact space LFWF

$$\tilde{\psi}(x, \mathbf{b}_{\perp}) \sim \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2}\kappa^2 x(1-x) \mathbf{b}_{\perp}^2 - \frac{1}{\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]}. \quad (68)$$

Impact space holographic LFWFs for the  $\pi$ ,  $K$ ,  $D$ ,  $\eta_c$ ,  $B$  and  $\eta_b$  mesons are depicted in Fig. 8.

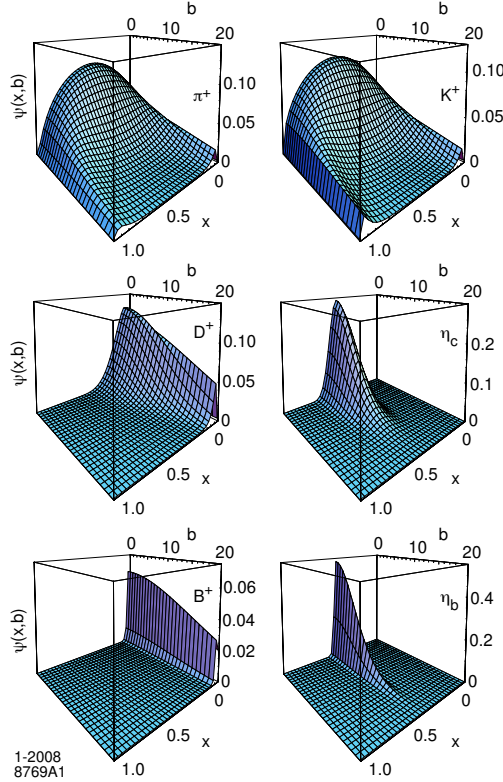


Figure 8: Two-parton flavored meson holographic LFWF  $\psi(x, \mathbf{b}_{\perp})$ :  $|\pi^+\rangle = |u\bar{d}\rangle$ ,  $|K^+\rangle = |u\bar{s}\rangle$ ,  $|D^+\rangle = |c\bar{d}\rangle$ ,  $|\eta_c\rangle = |c\bar{c}\rangle$ ,  $|B^+\rangle = |u\bar{b}\rangle$  and  $|\eta_b\rangle = |b\bar{b}\rangle$ . Values for the quark masses used are  $m_u = 2$  MeV,  $m_d = 5$  MeV,  $m_s = 95$  MeV,  $m_c = 1.25$  GeV and  $m_b = 4.2$  GeV. The value of  $\kappa = 0.375$  GeV is from the pion form factor [85].

The non-perturbative input to hard exclusive processes and heavy hadron decays can be computed in terms of gauge invariant hadronic distribution amplitudes (DAs), which describe the momentum-fraction distribution of partons at zero transverse impact distance in a Fock state with a fixed number of constituents, and thus they involve current or Lagrangian quark masses in the light-front wave function. The meson DA is computed from the transverse integral of the valence quark light-front wavefunction in the light-cone gauge [23]

$$\phi_M(x, Q) = \int^{\mathbf{k}_\perp^2 < Q^2} \frac{d^2 \mathbf{k}_\perp}{16\pi^3} \psi_M(x, \mathbf{k}_\perp), \quad (69)$$

and thus  $\phi(x) \equiv \phi(x, Q \rightarrow \infty) \rightarrow \tilde{\psi}(x, \mathbf{b}_\perp \rightarrow 0)/\sqrt{4\pi}$ . From (68) we obtain the holographic distribution amplitude  $\phi(x)$

$$\phi_M(x) \sim \frac{\kappa}{2\pi} \sqrt{x(1-x)} e^{-\frac{1}{2\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]}, \quad (70)$$

in the soft wall model. Predictions for the first and second moment of the meson distribution amplitude

$$\langle \xi^N \rangle_M = \frac{\int_{-1}^1 \xi^N \phi_M(\xi)}{\int_{-1}^1 \phi_M(\xi)}, \quad (71)$$

and comparisons with available lattice computations are given on Table 8 .

Table 1: Predictions for first and second moment of meson DA (top) and comparisons with available lattice results (bottom). Values of quark masses and  $\kappa$  as in Fig. 8.

$M$	$\langle \xi \rangle_M$	$\langle \xi^2 \rangle_M$
$\pi$		0.25
$K$	$0.04 \pm 0.02^a$	$0.235 \pm 0.005^a$
$D$	0.71	0.54
$\eta_c$		0.02
$B$	0.96	0.91
$\eta_b$		0.002
$\pi$		$0.28 \pm 0.03^b$
$K$	$0.029 \pm 0.002^b$	$0.27 \pm 0.02^b$
$\pi$		$0.269 \pm 0.039^c$
$K$	$0.0272 \pm 0.0005^c$	$0.260 \pm 0.006^c$

<sup>a</sup>The results correspond to  $m_s = 65 \pm 25$  MeV from [77].

<sup>b</sup>Lattice results from Ref. [107]

<sup>c</sup>Lattice results from Ref. [108]

It is interesting to note that the pion distribution amplitude predicted by AdS/QCD has a quite different  $x$ -behavior than the asymptotic distribution amplitude predicted

from the PQCD evolution [101] of the pion distribution amplitude. In the chiral limit, the AdS distribution amplitude  $\phi_{\text{AdS}}(x) \sim \sqrt{x(1-x)}$  gives for the second moment  $\langle \xi^2 \rangle_{\text{AdS}} \rightarrow 1/4$ , compared with the asymptotic value  $\langle \xi^2 \rangle_{\text{PQCD}} \rightarrow 1/5$  from the PQCD asymptotic DA  $\phi_{\text{PQCD}}(x) \sim x(1-x)$ . This observation appears to be consistent with the results of the Fermilab diffractive dijet experiment [109] which shows a broader  $x$  distribution for the dijets at small transverse momentum  $k_{\perp} \leq 1$  GeV. The broader shape of the pion distribution increases the magnitude of the leading twist perturbative QCD prediction for the pion form factor by a factor of 16/9 compared to the prediction based on the asymptotic form, bringing the PQCD prediction close to the empirical pion form factor [110].

Since they are complete and orthonormal, the AdS/CFT model wavefunctions can be used as an initial ansatz for a variational treatment or the basis states for the diagonalization of the light-front QCD Hamiltonian  $H_{LF}^{QCD}$  [76]. Even if one restricts the proton basis to  $|uud\rangle$ ,  $|uudgg\rangle$ , and  $|uudq\bar{q}\rangle$  Fock states, the resulting eigensolution will contain the effects of gluon exchange, the lowest order contribution to the QCD running coupling, intrinsic gluons and sea quarks.

## 9 Conclusions

One of the key difficulties in studies of quantum chromodynamics has been the absence of an analytic first approximation to the theory which not only can reproduce the hadronic spectrum, but also provides a good description of hadron wavefunctions. The AdS/CFT correspondence provides an elegant semi-classical approximation to QCD, which incorporates both color confinement and the conformal short-distance behavior appropriate for a theory with an infrared fixed point. Since the hadronic solutions are controlled by their twist dimension  $z^T$  at small  $z$ , one also reproduces dimensional counting rules for hard exclusive processes. The AdS/CFT approach leads to a model of hadrons which has both confinement at large distances and the conformal scaling properties which reproduce dimensional counting rules for hard exclusive reactions. The fundamental equations of AdS/CFT for mesons have the appearance of a radial Schrödinger Coulomb equation, but they are relativistic, covariant, and analytically tractable. The eigenvalues of the AdS/CFT equations provide a good description of the meson and baryon spectra for light quarks [44, 81, 111, 112], and its eigensolutions provide a remarkably simple but realistic model of their valence wavefunctions. One can also derive analogous equations for baryons composed of massless quarks using a Dirac matrix representation for the baryon system [16].

The lowest stable state of the AdS equations are determined by the Breitenlohner-Freedman bound [74]. We can model confinement by imposing Dirichlet boundary conditions at  $\phi(z = 1/\Lambda_{\text{QCD}}) = 0$ . The eigenvalues are then given in terms of the roots of the Bessel functions:  $\mathcal{M}_{L,k} = \beta_{L,k}\Lambda_{\text{QCD}}$ . Alternatively, one can introduce a dilaton field [35] which provides a confinement potential  $-\kappa^2\zeta^2$  to the effective potential  $V(\zeta)$ . The resulting hadron spectra are given by linear Regge trajectories in

the square of the hadron masses  $\mathcal{M}^2$ , characteristic of the Nambu string model. The AdS/CFT equations are integrable, and thus the radial and orbital excitations can be obtained from ladder operators [16]. We have found that the equations describing the propagation of light-front eigenmodes in 3+1 space possess remarkable algebraic structures. We have also shown that a simple extension of the conformal algebraic structure is equivalent to the soft-wall model. For fermionic modes it corresponds to a linear confining potential in the holographic variable  $z$ .

In this work we have shown that the eigensolutions  $\phi_H(z)$  of the AdS/CFT equations in the fifth dimension  $z$  have a remarkable mapping to the light-front wavefunctions  $\psi_H(x_i, \mathbf{b}_{\perp i})$ , the hadronic amplitudes which describe the valence constituents of hadrons in physical space time, but at fixed light-cone time  $\tau = t + z/c = 0$ . Similarly, the AdS/CFT equations for hadrons can be mapped to equivalent light-front equations. The correspondence of AdS/CFT amplitudes to the QCD wavefunctions in light-front coordinates in physical space-time provides an exact holographic mapping at all energy scales between string modes in AdS space and hadronic boundary states. Most important, the eigensolutions of the AdS/CFT equation can be mapped to light-front equations of the hadrons in physical space-time, thus providing an elegant description of the light hadrons at the amplitude level.

The mapping of AdS/CFT string modes to light-front wave functions thus provides a remarkable analytic first approximation to QCD. Since they are complete and orthonormal, the AdS/CFT model wavefunctions can also be used as a basis for the diagonalization of the full light-front QCD Hamiltonian, thus systematically improving the AdS/CFT approximation.

We have also shown the correspondence between the expressions for current matrix elements in AdS/CFT with the corresponding expressions for form factors as given in the light-front formalism. In first approximation, where one takes the current propagating in a non-confining background, one obtains the Drell-Yan West formula for valence Fock states, corresponding to the interaction picture of the light-front theory. Hadron form factors can thus be directly predicted from the overlap integrals in AdS space or equivalently by using the Drell-Yan-West formula in physical space-time. The form factor at high  $Q^2$  receives its main contributions from small  $\zeta \sim 1/Q$ , corresponding to small  $\vec{b}_{\perp} = \mathcal{O}(1/Q)$  and  $1 - x = \mathcal{O}(1/Q)$ . We have also shown how to improve this approximation by studying the propagation of non-normalizable solutions representing the electromagnetic current in a modified AdS confining metric, or equivalently in a dilaton background. This improvement in the description of the current corresponds in the light-front to multiple hadronic Fock states. The introduction of the confined current implies that the timelike form factors of hadrons will be mediated by vector mesons, including radial excitations. The wavefunction of the normalizable vector meson states  $\mathcal{A}(z)$  appearing in the spectral decomposition of the Green's function, which is dual to the non-normalizable photon propagation mode in AdS, is twist-3 [85]. This is the expected result for even parity axial mesons in QCD, or  $L = 1$  odd parity vector mesons composed of a scalar squark and anti-

squarks. In the case of quark-antiquark states, one also expects to find  $C = -1$ , twist-2 meson solutions for the zero helicity component of the  $\rho$  with  $S = 1$  and  $L = 0$ , which is supposed to give a dominant contribution to the  $\rho$  form factor.

We have applied our formulation to both the spacelike and timelike pion form factor. The description of the pion form factor in the spacelike domain is in good agreement with experiment in both confinement models, the hard and the soft wall holographic models. In the soft wall model the time-like pion form factor exhibits a pole at the  $\rho$  mass with zero width since hadrons are stable in this theory. If one introduces a width, the height of the  $\rho$  pole is in reasonable agreement with experiment. The space-like Dirac form factor for the proton is also very well reproduced by the double-pole analytic expression given in Appendix D of Ref. [85] for the case  $N = 3$ .

The deeply virtual Compton amplitude in the handbag approximation can be expressed as overlap of light-front wavefunctions [98]. The deeply virtual Compton amplitudes can be Fourier transformed to  $b_\perp$  and  $\sigma = x^-P^+/2$  space providing new insights into QCD distributions [113, 114, 115, 116, 117]. The distributions in the light-front direction  $\sigma$  typically display diffraction patterns arising from the interference of the initial and final state LFWFs [116, 118]. All of these processes can provide detailed tests of the AdS/CFT LFWFs predictions.

The phenomenology of the AdS/CFT model is just beginning, but it can be anticipated that it will have extensive applications to QCD phenomena. For example, the model LFWFs obtained from AdS/QCD provide a basis for understanding hadron structure functions and fragmentation functions as well as higher-twist contributions to inclusive processes at the amplitude level; the same wavefunctions can describe hadron formation from the coalescence of co-moving quarks. The spin and orbital angular momentum correlations which underly single and double spin correlations are also described by the AdS/CFT eigensolutions. The AdS/CFT hadronic wavefunctions also provide predictions for the generalized parton distributions of hadrons and their weak decay amplitudes from first principles. The amplitudes relevant to diffractive reactions could also be computed. We also anticipate that the extension of the AdS/CFT formalism to heavy quarks will allow a great variety of heavy hadron phenomena to be analyzed from first principles.

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