PHYSICAL REVIEW D **72,** 126001 (2005)

AdS strings with torsion: Noncomplex heterotic compactifications

Andrew R. Frey*
California Institute of Technology, Mail Code 452-48, Pasadena, California 91125, USA

Matthew Lippert†
University of Kentucky, Lexington, Kentucky 40506, USA
and University of Louisville, Louisville, Kentucky 40292, USA
(Received 7 September 2005; published 8 December 2005)

Combining the effects of fluxes and gaugino condensation in heterotic supergravity, we use a ten-dimensional approach to find a new class of four-dimensional supersymmetric AdS_4 compactifications on almost-Hermitian manifolds of $SU(3)$ structure. Computation of the torsion allows a classification of the internal geometry, which for a particular combination of fluxes and condensate, is nearly Kähler. We argue that all moduli are fixed, and we show that the Kähler potential and superpotential proposed in the literature yield the correct AdS_4 radius. In the nearly Kähler case, we are able to solve the H Bianchi identity using a nonstandard embedding. Finally, we point out subtleties in deriving the effective superpotential and understanding the heterotic supergravity in the presence of a gaugino condensate.

DOI: 10.1103/PhysRevD.72.126001 PACS numbers: 11.25.Mj, 04.65.+e

I. INTRODUCTION

Understanding how to fix the expectation values of moduli in compactifications of string theory has driven much of the progress made in discovering and categorizing string vacua over the last few years. The key points are that supergravity form flux can provide potentials for moduli and that compactification manifolds other than the traditional Calabi-Yau 3-fold (CY_3) simply have fewer moduli (for topological reasons). At tree level, these "flux vacua" are relatively tractable. For example, in some type IIB backgrounds, fluxes simply warp the CY_3, fix the complex structure moduli, and generate zero vacuum energy [1]. Similarly, in heterotic vacua with H flux, the compact manifold is non-Kähler and only the dilaton is unfixed [2–6]. In more general cases, classification of G structures has been employed to study the resulting geometries and particularly the topological torsions of the compactification manifolds.

As has been known for some time, nonperturbative effects, such as gaugino condensation in the 4D effective field theory, can also generate potentials for some moduli [7,8]. In fact, it has turned out that these nonperturbative effects happen to fix the moduli left unfixed by flux and torsion [9]. In the end, the vacuum can be supersymmetric with a negative cosmological constant; that is, they are anti-de Sitter (AdS_4) compactifications. Unfortunately, however, these AdS_4 compactifications, which have been studied in type II and heterotic string theory, are only well understood in the 4D effective field theory. A more complete picture of the 10D physics would be useful, for example, in resolving some disputes over the proper interpretation of effective field theory in these vacua [10–13].

In this paper, we investigate the 10D effects of gaugino condensation, focusing on heterotic flux compactifications because gauge degrees of freedom are conveniently described in the bulk supergravity. In particular, we will see the backreaction of both the flux and the condensate, making use of G structures to describe supersymmetric backgrounds. To our knowledge, this paper is the first use of G structures to study supergravity gaugino condensation. Therefore, we will be relatively explicit in our calculations. We find that gaugino condensation is consistent with supersymmetry in $AdS_4 \times X^6$ compactifications where X^6 has $SU(3)$ structure and belongs to a certain class of almost-Hermitian manifolds. We find our backgrounds to be consistent with the 4D superpotential combining the effects of flux and gaugino condensation given in [14], although we raise some questions about the derivation of the superpotential. For specific choices of fluxes and condensate, our compactification specializes to become nearly Kähler (NK), which leads to many simplifications. In the NK case, we are able to use a nonstandard embedding to solve the H flux Bianchi identity, which in turn fixes the compactification and AdS_4 scales.

Our paper is organized as follows. We begin in Sec. II with a brief overview of flux vacua in type II and heterotic string theories, including both nonperturbative effects and their description in terms of G structures. The review provides points of comparison to our work. Then, in Sec. III we use the heterotic supersymmetry variation equations to derive relations for the geometry in terms of general fluxes and gaugino condensates. We then use these relations to compute the components of the torsion and classify the compactification manifold. Section IV deals with the four-dimensional effective theory, discussing moduli fixing and the 10D uplift of the gaugino condensate. We also verify that the proposed superpotential and Kähler potential yield the correct vacuum energy. We do

*Electronic address: frey@theory.caltech.edu
†Electronic address: lippert@pa.uky.edu

 © 2005 The American Physical Society

find some subtleties related to the derivation of the superpotential, which we describe in some detail. Specializing our flux and condensate choices in Sec. V yields a NK compactification. After discussing the NK geometry, we revisit the now simpler and explicit gaugino condensate and moduli fixing and, in addition, solve the Bianchi identity. We end in Sec. VI with discussion and directions of future interest.

II. REVIEW

We begin by briefly reviewing the effects of flux and gaugino condensation in string theory compactifications. We will particularly highlight the parallels between the stories in the IIB and heterotic supergravities.

The inclusion of background fluxes in string compactifications is an old idea [2–6] which has more recently been applied very successfully to the moduli problem (see [9,15–18] for some of the most recent examples). Quantized Neveu-Schwarz–Neveu-Schwarz (NS) and Ramond-Ramond (RR) fluxes can be wrapped on cycles of the compactification manifold, inducing potentials for many moduli and deforming the background geometry. One can consider small additions of such fields perturbatively, ignoring their gravitational effects (e.g. [19,20]) and keeping the simpler CY_3 compactification. However, the full backreaction modifies the internal space in complicated but interesting ways so that it is no longer CY_3.

In type IIB, [1] showed that, under some constraints on the brane content, the compactification manifold remains conformally CY_3, even in the presence of NS and RR fluxes. This relatively mild backreaction makes the dimensional reduction somewhat easier to analyze. In the 4D $\mathcal{N} = 1$ effective theory, the fluxes induce a superpotential of the form [21]

$$W \sim \int_M G \wedge \Omega \tag{1}$$

where G, a combination of the RR and NS fluxes and axion dilaton, is coupled to the geometry through the holomorphic 3-form Ω. The equations of motion require G to be imaginary self-dual [supersymmetry further requires G to be (2,1)], and the associated effective potential generically fixes the dilaton and all the complex structure moduli.[1] However, the Kähler moduli, and, in particular, the volume modulus, remain unfixed, resulting in a no-scale Minkowski compactification. See [22] for a review.

Starting with these self-dual flux solutions in IIB, T dualizing twice and then S dualizing leads to solutions of either IIB or heterotic string theory (depending on the initial brane content) with only NS flux [23–25]. Similarly, solutions have been studied in type IIA [26–30] and M theory [31–39] where fluxes also generate torsion. While some connections between these many flux vacua are known [25,40–44], the chain of U dualities relating them remains to be worked out in entirety.

In the case of heterotic string theory, fluxes more dramatically affect the compactification geometry. Adding NS flux H generates torsion in addition to warping, as implied by the title of [2]. Imposing supersymmetry relates the flux to the complex structure J by

$$dJ \sim \star H. \tag{2}$$

Because J is closed for Kähler manifolds, the resulting supersymmetric compactification has non-Kähler geometry. The low-energy superpotential is of the form [40,45]

$$W \sim \int_M (H - idJ) \wedge \Omega. \tag{3}$$

As in the IIB self-dual flux case, the effective potential fixes the complex structure moduli. Because of the torsion, some of the Kähler moduli, including the volume modulus, are fixed as well [40,46]. From another point of view, rather than fixing moduli, the compactification geometry consistent with flux just does not have as many moduli. Here, the analog of the IIB volume modulus is the dilaton, which is absent from (3) and remains unfixed.

Supersymmetry of a non-Kähler compactification requires, rather than a special holonomy group, a reduced structure group for the tangent bundle. This occurs because the supersymmetry transformations yield a spinor invariant under the torsionful connection, which in turn defines a G structure. Equivalently, the spinor bilinears define a set of global, nonvanishing, G-invariant tensors. For $G \subset SO(n)$, these tensors include a metric g and an oriented volume ϵ. In addition, $G \subset U(m)$ means the manifold is equipped with an almost-Hermitian metric, an almost complex structure (ACS) J, and a holomorphic m-form Ω. As we will discuss in more detail in Sec. III C, the torsion is reflected in the derivatives of the invariant tensors. Six-dimensional non-Kähler manifolds with $SU(m)$ structure have been discussed in heterotic [26,47,48], types IIA [26–30], and IIB [49–52] strings. Compactifications of M theory using G_2 structures and $SU(m)$ structures [31–39] have also been studied.

The fact that the superpotentials (1) and (3) leave some moduli unfixed led [9] to reconsider nonperturbative potentials for the other moduli, following the arguments of [7,8]. In pure 4D $\mathcal{N} = 1$ non-Abelian gauge theory, the gauge field F becomes strongly coupled at an energy scale Λ and the gaugino condenses:

$$\langle \chi\chi \rangle \sim \Lambda^3 \sim M_{UV}^3 e^{-1/bg_{YM}^2}, \tag{4}$$

where M_{UV} is the UV cutoff scale, b is an $\mathcal{O}(1)$ one-loop determinant, and g_{YM} is the 4D gauge coupling. This

[1] Because the required calculations are prohibitively laborious, general arguments for fixing complex structure moduli are typically invoked. However, in [17] the fixing was demonstrated explicitly for an example with only three complex structure moduli.

condensate induces an effective superpotential of the form [53]

$$W \sim e^{-1/bg_{\rm YM}^2}. \tag{5}$$

In the self-dual flux IIB vacua described above, [9] showed that the superpotentials (1) and (5) can, in combination, freeze the remaining Kähler moduli, in particular, the volume modulus. Specifically, if the gaugino condensation occurs in a D7-brane gauge group, the gauge coupling depends on the (fixed) dilaton and the volume modulus. Therefore, the volume modulus is fixed as well.[2] Most relevantly for us, however, the combined flux and gaugino superpotentials lead to supersymmetric AdS_4 vacua [9,17,55]. In addition, metastable dS_4 minima have been constructed by adding branes [9] or otherwise breaking supersymmetry [56,57] to increase the vacuum energy.

In a heterotic CY_3 compactification, gaugino condensation on its own drives the 4D dilaton to the strong coupling region. However, when the condensate is balanced against H flux, the 4D dilaton is finite, and the vacuum is a no-scale Minkowski spacetime with broken supersymmetry [7,8]. Even though the cosmological constant vanishes, the superpotential $W \neq 0$. This solution is nontrivial, as the wrapped fluxes are quantized and fractional values are needed to match the exponentially small contribution of the condensate [58].[3] From a 10D perspective, $SU(3)$ holonomy of the CY_3 requires that the flux and the condensate be $(3,0)+(0,3)$ forms. More recently, AdS_4 and even de Sitter vacua have been shown to arise from gaugino condensation in the strong string coupling limit (heterotic M theory) [59–61]. Finally, [14] has, analogously to the IIB case, considered adding gaugino condensation to compactifications with both H flux and torsion.

III. CONDITIONS FOR SUPERSYMMETRIC AdS_4

In this section, we will examine the conditions for supersymmetry in heterotic supergravity in the presence of both H flux and gaugino condensation. We will find that gaugino condensation is actually consistent with supersymmetry in $AdS_4 \times X^6$ compactifications. In particular, we will study the backreaction of the condensate, using the G-structure formalism. Since this is the first use of G structures to study supergravity gaugino condensation, our calculations will be relatively explicit.

As it turns out, the H flux and gaugino condensate induce a significant backreaction on the geometry of the compact manifold X^6. In the well-studied [2,4–6,25,40,45,46,62] case of supersymmetric Minkowski compactifications, the H flux generates a torsion and the internal manifold ceases to be Kähler. We will see a similar but even more dramatic effect in the AdS_4 case here.

A. Ansatz and $SU(3)$ structure

To set our field conventions, we give here the string-frame action of the effective supergravity for the bosonic fields and gaugino [63][4]

$$S = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\Big\{R + 4\partial_M\phi\partial^M\phi - \frac{1}{2}\Big|H - \frac{1}{2}\Sigma\Big|^2 - \frac{\kappa_{10}^2}{2g_{10}^2}\,{\rm Tr}(F^2 + 4\bar\chi\Gamma^M D_M\chi)\Big\}, \tag{6}$$

where

$$\Sigma_{MNP} = \frac{\kappa_{10}^2}{g_{10}^2}\,{\rm Tr}\bar\chi\Gamma_{MNP}\chi \tag{7}$$

is the gaugino condensate, $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, and $\kappa_{10}^2/g_{10}^2 = \alpha'/4$ (see [64]). Index, form, and spinor conventions are given in Appendix A. For convenience, we will define $T = H - \frac{1}{2}\Sigma$. We will work in string frame with a metric ansatz of the form

$$ds^2 = e^{2A}\hat g_{\mu\nu}dx^\mu dx^\nu + g_{mn}dx^m dx^n \tag{8}$$

where $\hat g_{\mu\nu}$ is an AdS_4 metric of radius R and g_{mn} is the metric on the internal space X^6. We will at times factor out the volume modulus by writing $g_{mn} = e^{2u}\tilde g_{mn}$, where the volume of X^6 is $V_6 = e^{6u}(2\pi\sqrt{\alpha'})^6$ and $\tilde g_{mn}$ is a fiducial metric with volume $\tilde V_6 = (2\pi\sqrt{\alpha'})^6$. In the interest of preserving $SO(3,2)$ symmetry, we take the other fields neither to depend on nor have components in the AdS_4 directions.

The string-frame supersymmetric variations of the dilatino λ, gaugino χ, and gravitino ψ_M are

$$\delta\lambda = -\frac{1}{2}\partial_M\phi\Gamma^M\varepsilon + \frac{1}{24}H_{MNP}\Gamma^{MNP}\varepsilon + \frac{1}{96}\Sigma_{MNP}\Gamma^{MNP}\varepsilon, \tag{9}$$

$$\delta\chi = -\tfrac{1}{4}F_{MN}\Gamma^{MN}\varepsilon, \tag{10}$$

$$\delta\psi_M = \nabla_M\varepsilon - \frac{1}{8}H_{MNP}\Gamma^{NP}\varepsilon + \frac{1}{96}\Sigma_{NPQ}\Gamma^{NPQ}\Gamma_M\varepsilon. \tag{11}$$

The supersymmetry parameter ε is a 10D Majorana-Weyl spinor with positive chirality which we decompose into 4D and 6D positive chirality Weyl spinors, ζ (anticommuting) and η (commuting), as

[2]Instantonic D3 branes can introduce a similar nonperturbative superpotential, and leading order α' corrections additionally restore dependence of the potential on the volume modulus [54].

[3]In [58], one-loop corrections were used to fix the volume modulus. Alternatively, world sheet instantons, as with the IIB D3 instantons, can give a potential to Kähler moduli.

[4]Compared with [63], we have made the following rescalings: $\chi_{BdR} = \sqrt{2}\chi$, $\psi_{BdR} = \sqrt{2}\psi$, $H_{BdR} = H/3\sqrt{2}$, and $\phi_{BdR} = e^{2\phi/3}$.

$$\varepsilon = e^{A/2}(\zeta \otimes \eta + \zeta^* \otimes \eta^*), \tag{12}$$

where the warp factor is included for convenience of normalization. Because ε is Majorana, there is only one independent positive chirality 6D spinor, meaning the solution has an $SU(3)$ structure. We will see later that we can normalize $\eta^\dagger \eta \equiv 1$.

With that normalization, we can define the $SU(3)$ structure in terms of

$$J_m{}^n = -i\eta^\dagger \gamma_m{}^n \eta, \qquad \Omega_{mnp} = \eta^T \gamma_{mnp} \eta. \tag{13}$$

By the Fierz identities for η, J is an almost complex structure ($J^2 = -1$), and (with J written as a form)

$$J \wedge \Omega = 0, \qquad \Omega \wedge \bar{\Omega} = -\frac{4i}{3} J \wedge J \wedge J. \tag{14}$$

The $SU(3)$ structure determines the almost-Hermitian metric g_{mn} and volume form $\epsilon = (i/8)\Omega \wedge \bar{\Omega}$. Ω is $(3, 0)$ with respect to J and imaginary self-dual. See, for example, [65] for a review of $SU(n)$ structures. Henceforth, if we count holomorphic and antiholomorphic indices on a form, we do so with respect to J.

B. Algebraic relations

In supersymmetric vacua, the supersymmetric variations (9)–(11) are all required to vanish. We start by inserting (12) into the μ component of (11), noting that the covariant derivative is

$$\begin{aligned}\nabla_\mu \varepsilon &= \hat{\nabla}_\mu \varepsilon + \tfrac{1}{2}\Gamma_\mu{}^N \partial_N A \varepsilon \\ &= e^{A/2}[\gamma_\mu \zeta \otimes (-m\eta^* + \tfrac{1}{2}\partial_n A \gamma^n \eta) + \gamma_\mu \zeta^* \\ &\quad \otimes (-\bar{m}\eta - \tfrac{1}{2}\partial_n A \gamma^n \eta^*)],\end{aligned} \tag{15}$$

where m is proportional to the 4D superpotential and is related to the AdS_4 radius R and cosmological constant Λ by

$$|m| = \frac{1}{2R} = \frac{1}{2}\sqrt{\frac{|\Lambda|}{3}} \tag{16}$$

(in 4D string frame). The second equality of (15) follows from the AdS_4 covariant derivative $\hat{\nabla}_\mu \zeta = -\bar{m}\gamma_\mu \zeta^*$ for a Weyl Killing spinor (see [34,37]). Subdividing (11) by 4D (or 6D) chirality gives

$$\frac{1}{96}\Sigma_{mnp}\gamma^{mnp}\eta = \frac{1}{2}\partial_m A \gamma^m \eta - m\eta^*. \tag{17}$$

Similarly, the dilatino variation (9), combined with (17), becomes

$$\frac{1}{24}T_{mnp}\gamma^{mnp}\eta = \frac{1}{2}\partial_m(\phi - 3A)\gamma^m \eta + 3m\eta^*. \tag{18}$$

Now we can simplify in terms of the $SU(3)$ structure. Using (17), its adjoint, and some gamma matrix algebra, we compute

$$\frac{1}{16}\Sigma_{mnp}J^{np} = \frac{1}{96}\Sigma_{npq}\eta^\dagger \{\gamma_m, \gamma^{npq}\}\eta = \partial_n A J_m{}^n. \tag{19}$$

In terms of our form notation (A2), this is

$$J \lrcorner \Sigma = -8 dA \lrcorner J. \tag{20}$$

Furthermore, multiplying (17) by η^T, we find

$$\Omega \lrcorner \Sigma = -16m. \tag{21}$$

Similarly, using (18) we can compute

$$J \lrcorner T = 2d(3A - \phi) \lrcorner J, \tag{22}$$

$$\Omega \lrcorner T = 12m. \tag{23}$$

Using (B11), we have altogether

$$\begin{aligned}\Sigma &= -2(\bar{m}\Omega + m\bar{\Omega}) + \Sigma_0 - 4J \wedge (dA \lrcorner J), \\ T &= \tfrac{3}{2}(\bar{m}\Omega + m\bar{\Omega}) + T_0 + J \wedge (d(3A - \phi) \lrcorner J),\end{aligned} \tag{24}$$

where $J \lrcorner \Sigma_0 = \Omega \lrcorner \Sigma_0 = 0$ and similarly for T_0. Finally, the gaugino variation gives conditions which are familiar from Calabi-Yau compactifications [66]:

$$F \lrcorner \Omega = J \lrcorner F = 0. \tag{25}$$

With respect to the ACS, these conditions are respectively $F = F_{(1,1)}$ and the Donaldson-Uhlenbeck-Yau equation.

We should note that we have not imposed that $\Sigma = a\Omega + \bar{a}\,\bar{\Omega}$ as in the usual CY compactification. In particular, Eq. (20) is not identically zero. Instead, our condensate is intentionally general at this stage, and we will further address the dimensional reduction of the gaugino in Sec. IV B.

C. Differential relations and torsion

We return now to the gravitino variation. Multiplying (17) by γ_q and rearranging using some gamma matrix algebra, we get

$$\begin{aligned}\frac{1}{96}\Sigma_{mnp}\gamma^{mnp}\gamma_q \eta &= \frac{1}{16}\Sigma_{qmn}\gamma^{mn}\eta - \frac{1}{2}\partial_m A \eta \\ &\quad - \frac{1}{2}\partial_m A \gamma_q{}^m \eta + m\gamma_q \eta^*.\end{aligned} \tag{26}$$

Then we can write the internal components of (11) as

$$\nabla_m \eta = \tfrac{1}{2}\partial_n A \gamma_m{}^n \eta + \tfrac{1}{8}T_{mnp}\gamma^{np}\eta - m\gamma_m \eta^*. \tag{27}$$

Note that (27) implies[5] that we can set $\eta^\dagger \eta = 1$ constant. Therefore, the $SU(3)$ structure is properly normalized.

While the current form of (27) is useful for our calculations, it is also instructive to see that η is parallel with respect to a torsionful connection. Inserting $\eta^\dagger \eta = 1$, we can use the Fierz identity (B8) along with some gamma matrix algebra and the anti-self-duality of $\bar{\Omega}$ to write

[5] The last term does not contribute due to the antisymmetry of γ_m.

$$\bar{\Omega}_{mpq}\gamma^{pq}\eta = 8\gamma_m\eta^* \text{ and similarly } \Omega_{mpq}\gamma^{pq}\eta = 0. \quad (28)$$

Inserting these into (27), we find

$$\nabla_m\eta = \tfrac{1}{2}\partial_n A\gamma_m{}^n\eta + \tfrac{1}{8}\tau_{mnp}\gamma^{np}\eta, \quad (29)$$

where the (intrinsic) torsion is

$$\tau^m{}_{np} = T^m{}_{np} - \bar{m}\Omega^m{}_{np} - m\bar{\Omega}^m{}_{np}. \quad (30)$$

It is also important to interpret the spinor equations (27) and (29), starting with (27). Although the last term seems unusual, (27) is actually just a Killing spinor equation for a Weyl spinor, much like the Killing equation for a spinor in AdS_4. Here, though, we have a connection with contorsion $\kappa_{pmn} = 2g_{m[p}\partial_{n]}A - \frac{1}{2}T_{pmn}$. To write (27) in a canonical form, shift the contorsion into the covariant derivative and define $\eta' = e^{i\beta/2-i\pi/4}\eta + e^{-i\beta/2+i\pi/4}\eta^*$ where $m = e^{i\beta}|m|$. Then we have $\nabla_m\eta' = -i|m|\gamma_m\eta'$, the defining equation for a real Killing spinor [67]. On the other hand, (29) shows that η is parallel with respect to an alternative connection, one with contorsion $\kappa_{pmn} = 2g_{m[p}\partial_{n]}A - \frac{1}{2}\tau_{pmn}$. In the more familiar CY_3 compactification, the Killing spinor is parallel with respect to the Levi-Cività connection, meaning that the manifold has $SU(3)$ holonomy, not just $SU(3)$ structure.

To understand the relation between $SU(3)$ holonomy and structure, it is useful to examine the torsion in more detail.[6] What is important is not the contorsion itself (in fact, it is possible to remove the warp factor term from the contorsion by rescaling the internal metric to $g_{mn} = e^{2A}\bar{g}_{mn}$) but the intrinsic torsion, or simply the torsion. The torsion τ cannot be removed by any conformal rescaling, and it is actually a topological quantity. The torsion gives a topological obstruction to special holonomy (of the Levi-Cività connection) for X^6. Rather, after metric rescaling, it is the connection with torsion τ which has $SU(3)$ holonomy. We can see this obstruction directly in the $SU(3)$ structure. The torsion, thought of as an $su(3)$ valued one-form, can be decomposed into five irreducible $SU(3)$ modules W_i, which give dJ and $d\Omega$ as follows:

$$dJ = \frac{3i}{4}(W_1\bar{\Omega} - \bar{W}_1\Omega) + W_3 + J\wedge W_4, \quad (31)$$

$$d\Omega = W_1 J\wedge J + J\wedge W_2 + \Omega\wedge W_5. \quad (32)$$

Decomposing the torsion with respect to the ACS, dJ has (3, 0) and (0, 3) parts which give W_1 and $\bar{W}_1$ and a (2, 1) + (1, 2) part which can further be decomposed into a primitive part W_3 and a nonprimitive part $J\wedge W_4$. Similarly, $d\Omega$ has a (3, 1) part W_5 and a primitive (2, 2) part $J\wedge W_2$. The nonprimitive (2, 2) piece of $d\Omega$ redundantly gives W_1. Additionally, $3W_4 - 2W_5$ is conformally invariant.

[6]See Appendix C for the definition of the torsion compared to the contorsion.

What, then, are the derivatives of the $SU(3)$ structure tensors in our compactification? Using (13) and (29), we find that

$$\nabla_m J_n{}^p = -\partial_q A(g_{mn}J^{pq} - g^{pq}J_{mn} - \delta^p_m J_n{}^q + \delta^q_n J_m{}^p) + \tfrac{1}{2}\tau_{mn}{}^q J_q{}^p - \tfrac{1}{2}\tau_{mq}{}^p J_n{}^q, \quad (33)$$

showing that J is covariantly constant with the appropriate contorsion (once again, the ∂A terms can be removed by rescaling the metric). Since τ is the $SU(3)$ holonomy torsion, this is consistent with the fact that J is an $SU(3)$ singlet.

For computing dJ there is a more useful expression of ∇J, which we find starting from (27). After some gamma matrix algebra, we arrive at

$$\nabla_m J_n{}^p = -\partial_q A(g_{mn}J^{pq} - g^{pq}J_{mn} - \delta^p_m J_n{}^q + \delta^q_n J_m{}^p) + \tfrac{1}{2}T_{mn}{}^q J_q{}^p - \tfrac{1}{2}T_{mq}{}^p J_n{}^q - 2\,\mathrm{Im}(\bar{m}\Omega_{mn}{}^p). \quad (34)$$

We can relate (34) to (33) using the Fierz identity (B7), which yields

$$\Omega_{mn}{}^p = \frac{i}{2}\Omega_{mn}{}^q J_q{}^p - \frac{i}{2}\Omega_{mq}{}^p J_n{}^q. \quad (35)$$

[In fact, deriving (35) is one step in showing that Ω is (3, 0) with respect to J.] Continuing, we antisymmetrize to find

$$(dJ)_{mnp} = 6\partial_{[m}AJ_{np]} - 3T_{[mq}{}^q J_{p]q} - 6\,\mathrm{Im}(\bar{m}\Omega_{mnp}). \quad (36)$$

To substitute in for $T_{[mq}{}^q J_{p]q}$, we use (18) to write

$$-3T_{[mn}{}^q J_{p]q} = \frac{-i}{12}T_{mnp}\eta^\dagger[\gamma_{mnp},\gamma^{qrs}]\eta + \frac{1}{6}T^{qrs}\epsilon_{mnpqrs} = 6\partial_{[m}(\phi - 3A)J_{np]} + (\star T)_{mnp} + 12\,\mathrm{Im}(\bar{m}\Omega_{mnp}). \quad (37)$$

Plugging into (36), we find

$$dJ = 2d(\phi - 2A)\wedge J + \star T + 6\,\mathrm{Im}(\bar{m}\Omega). \quad (38)$$

Taking the Hodge dual of T from (24),

$$\star T = \frac{3i}{2}\bar{m}\Omega - \frac{3i}{2}m\bar{\Omega} + d(3A - \phi)\wedge J + \star T_0. \quad (39)$$

The expression for dJ becomes

$$dJ = \frac{3i}{2}(m\bar{\Omega} - \bar{m}\Omega) + \star T_0 + d(\phi - A)\wedge J. \quad (40)$$

Reading directly from (40), $W_1 = 2m$, $W_4 = d(\phi - A)$, and $W_3 = \star T_0$.

We can perform a similar calculation to find $d\Omega$. Some gamma matrix algebra yields

$$(d\Omega)_{mnpq} = 3(dA\wedge\Omega)_{mnpq} + 6T_{m[s}{}^s\Omega_{pq]s} + 8m\,\mathrm{Re}(\eta^\dagger\gamma_{mnpq}\eta). \quad (41)$$

Again, we use (18) to substitute in for $T_{m[s}{}^s\Omega_{pq]s}$ as

$$6T_{m[n}{}^{s}\Omega_{pq]s} = 2(d(\phi - 3A) \wedge \Omega)_{mnpq} - 6\,m\mathrm{Re}(\eta^\dagger \gamma_{mnpq}\eta). \tag{42}$$

Then

$$d\Omega = d(2\phi - 3A) \wedge \Omega + 2mJ \wedge J. \tag{43}$$

It is easy to confirm the result for W_1, and we see $W_5 = -d(2\phi - 3A)$. Finally, we can conclude that $W_2 = 0$, since there is no primitive piece of $d\Omega$.

To summarize, we have found the torsion classes of X^6 to be

$$W_1 = 2m, \qquad W_2 = 0, \qquad W_3 = \star T_0, \qquad W_4 = d(\phi - A), \quad \text{and} \quad W_5 = -d(2\phi - 3A). \tag{44}$$

D. Almost-Hermitian geometry

Manifolds with $U(m)$ structure, called almost-Hermitian manifolds, were classified [68] into 16 categories depending on which components W_1 to W_4 of the torsion are nonzero. For a manifold to be complex, both $W_1 = 0$ and $W_2 = 0$ are required. Simply put, on a complex manifold, dJ should have no $(3, 0) + (0, 3)$ parts, and $d\Omega$ should have no $(2, 2)$ parts (just by counting holomorphic indices). Some examples of almost-Hermitian manifolds are given in Table I along with their nonzero torsion components and whether or not they are complex.

It may be helpful to place some familiar examples in this context. If the first four torsion components are all zero, the manifold is Kähler. The vanishing of W_5 in addition signals that the manifold is Calabi-Yau. The non-Kähler compactifications considered in [2,4–6,25,40,45,46,62] are in fact Hermitian with $W_5 = -2W_4$, and they are conformally balanced [26,65]. Half-flat manifolds have also been of recent interest [29,42,47,49,69,70].

In our $\mathrm{AdS}_4 \times X^6$ examples, we have found that generically X^6 has nonzero W_1, W_3, W_4, and W_5. The internal manifold X^6 is therefore not complex, but is of the type $\mathcal{G}_1$ [68]; however, there are some interesting special cases. Let us start by taking $m = |m|e^{i\beta}$ and redefining $\Omega = ie^{i\beta}\Omega'$, so W_1 is purely imaginary when (31) and (32) are written in terms of Ω'. Then, for $J \lrcorner T = 0$, $W_4 = (2/3)W_5 = d\phi$, so X^6 is conformally half flat with $W_2 = 0$. If we additionally have $J \lrcorner \Sigma = 0$, $W_4 = W_5 = 0$, and X^6 is actually half flat.[7] Under the further condition that the primitive part of T (and therefore $\star T$) vanishes, then X^6 is conformally nearly Kähler (for $J \lrcorner \Sigma \neq 0$) or actually nearly Kähler ($J \lrcorner \Sigma = 0$). We will return in some detail to the case of NK X^6 in Sec. V.

TABLE I. Almost-Hermitian manifolds are classified by torsion.

Manifold name	W_1	W_2	W_3	W_4	W_5	Complex?
Hermitian	0	0	W_3	W_4	W_5	Y
Balanced	0	0	W_3	0	W_5	Y
Special Hermitian	0	0	W_3	0	0	Y
Kähler	0	0	0	0	W_5	Y
Calabi-Yau	0	0	0	0	0	Y
Nearly Kähler	W_1	0	0	0	0	N
Almost Kähler	0	W_2	0	0	0	N
Quasi-Kähler	W_1	W_2	0	0	0	N
Half flat	$\mathrm{Im}W_1$	$\mathrm{Im}W_2$	W_3	0	0	N
Semi-Kähler	W_1	W_2	W_3	0	0	N
$\mathcal{G}_1$	W_1	0	W_3	W_4	W_5	N
$\mathcal{G}_2$	0	W_2	W_3	W_4	W_5	N

IV. THE 4D EFFECTIVE THEORY

In this section, we will relate the supersymmetric backgrounds discussed above to the 4D effective field theory that arises from the compactification. First, we give a dictionary relating parameters of the 10D and 4D descriptions, naturally leading to a discussion of moduli fixing. Then we raise more subtle issues regarding the gaugino condensate. Finally, we give a consistency check for proposed Kähler and superpotentials, finding a puzzle for the superpotential.

A. Dictionary and moduli fixing

To understand the interplay between the 4D and 10D physics in our AdS_4 backgrounds, we should begin by understanding the dimensional reduction of the fields. Along the way, we will see how different moduli are fixed in different manners.

First off, the Einstein frame is defined with respect to the 4D part of the string-frame metric by $g_{E,\mu\nu} = e^{6u-2\phi}g_{\mu\nu}$, and the Planck mass is $m_P^2 = 1/\pi\alpha'$ (see Appendix D). Note that we are including the moduli expectation values in the rescaling, so they do not enter in the Plank mass. Then, converting to the Einstein frame, the action (6) for the gauge fields reduces to

$$S_F = -\frac{1}{4g_{\mathrm{YM}}^2}\int d^4x\sqrt{-g_E}\,\mathrm{Tr}F_{\mu\nu}F_E^{\mu\nu}, \qquad \frac{1}{g_{\mathrm{YM}}^2} = \frac{1}{4\pi}e^{6u-2\phi}, \tag{45}$$

where the subscript E on the gauge field denotes that the spacetime indices have been raised with the Einstein-frame metric.[8] The gauge theory is weakly coupled in the regime of large radius and/or weak coupling.

[7]Note that this is a different class of half-flat manifolds than that considered in [29].

[8]We are implicitly assuming that the moduli are frozen at their expectation values. Further, for simplicity, we are assuming that the dilaton and warp factor are constant over the compactification. This assumption does not actually change the results qualitatively.

The gaugino decomposes under dimensional reduction as

$$\chi = (\chi_6 \otimes \chi_4 + \chi_6^* \otimes \chi_4^*) + \chi_{\rm KK}, \tag{46}$$

where χ_4 carries all the gauge indices and χ_6 satisfies a zero mode equation (other polarizations are superpartners of other polarizations of the gauge field and are lumped with higher Kaluza-Klein modes in $\chi_{\rm KK}$). Using a Majorana spin flip identity (and dropping KK modes), we can reduce the gaugino action from (6) to

$$S_\chi = -\frac{2}{g_{\rm YM}^2}\int d^4x\sqrt{-g_E}\,{\rm Tr}\bar{\chi}_E\gamma_E^\mu D_{E,\mu}\chi_E, \qquad \chi_E = \exp\left[\frac{3\phi - 9u}{2}\right]\chi_4. \tag{47}$$

The important point is that the canonically normalized gaugino in the Einstein frame is rescaled from the dimensionally reduced gaugino given in (46). In fact, the rescaling is precisely the appropriate rescaling for a field of dimension $m^{3/2}$.[9] Therefore, if $\langle\overline{\chi_E^*}\chi_E\rangle \equiv \Lambda_E^3$ sets the scale of gaugino condensation in the Einstein frame, we see that mass rescaling simply tells us that $\Lambda^3 = \langle\overline{\chi_4^*}\chi_4\rangle$ is the equivalent mass scale in the string frame. From (46), we can therefore write

$$\begin{aligned}\Sigma_{mnp} &= \frac{\kappa_{10}^2}{g_{10}^2}[\langle{\rm Tr}\overline{\chi_4^*}\chi_4\rangle(\chi_6^T\gamma_{mnp}\chi_6) \\ &\quad - \langle{\rm Tr}\bar{\chi}_4\chi_4^*\rangle(\chi_6^\dagger\gamma_{mnp}\chi_6^*)] + \Sigma'_{mnp} \\ &= \frac{\alpha'}{4}[\Lambda^3(\chi_6^T\gamma_{mnp}\chi_6) - \bar{\Lambda}^3(\chi_6^\dagger\gamma_{mnp}\chi_6^*)] + \Sigma'_{mnp}.\end{aligned} \tag{48}$$

This is the relation between the 4D and 10D descriptions of the condensate (Σ' is the expectation value of other Kaluza-Klein modes, presumably generated by some other quantum effect). For now, let us assume that a significant portion of $\chi_6^T\gamma_{mnp}\chi_6$ lies along Ω; then we have $\alpha'\Lambda^3 \sim |m|$. The effective field theory description of gaugino condensation is consistent as long as the condensate scale is less than the Kaluza-Klein scale, $\Lambda \ll m_{\rm KK} = e^{-u}/\sqrt{\alpha'}$, which we find is valid as long as

$$3u + \tfrac{1}{2}\ln(\alpha'|m|^2) \leq 0. \tag{49}$$

Fortunately, (49) is satisfied in the large radius regime of the compactification as long as the AdS_4 radius is sufficiently large.

Now we are in position to see how the 4D dilaton modulus S and the radial modulus T are fixed, which we understand from the point of view of the effective field theory.[10] Analogously to CY_3 compactifications [7], the superpotential becomes

$$W \sim C(T) + Ae^{iaS}. \tag{50}$$

The effective potential freezes the vacuum expectation value of the dilaton. Similarly, because of the torsion dJ in the superpotential (3), we expect C to depend on T, so the supergravity is no longer no scale, and T will be fixed. Any complex-structure-like moduli will presumably be fixed by additional dependencies in C and A.

Simultaneously, in our supersymmetric vacuum, the vacuum value of W determines the (negative) cosmological constant. In the string frame, as we mentioned in Eq. (16), the superpotential therefore determines m, which is a derived value, like a modulus. To evaluate m, we note that in a supersymmetric AdS_4 vacuum all contributions to the superpotential should be of similar magnitude, which agrees with our 10D results (24) and (40). In fact, (21) and (24) set the condensate proportional to the AdS_4 scale m. Combining the expressions for the condensate scale gives an approximate equation giving the ϕ in terms of m and u. We find

$$m \sim \alpha'\Lambda^3 \sim \frac{e^{-3u}e^{iaS}}{\sqrt{\alpha'}}, \tag{51}$$

using the Kaluza-Klein mass as the ultraviolet cutoff of Eq. (4). Assuming that we can obtain the small coupling regime in the gauge theory, the AdS_4 is large and nearly flat, with a very tiny cosmological constant. Additionally, (51) implies that the effective field theory approximation is valid [see (49)].

Before we reinterpret some of the above in 10D language, we should also give a caveat. In checking the validity of the effective theory and in estimating the AdS_4 curvature, we have assumed that $\chi_6^T\gamma_{mnp}\chi_6 \sim \Omega_{mnp}$, which is true for CY_3 compactifications. However, as we will discuss in further detail in IV B below, in these non-Kähler examples the zero mode χ_6 may lead to other components.

We can also understand how most of the moduli are fixed from a 10D perspective. Starting with complex-structure-like moduli, we note that X^6 is not complex, and the torsion can greatly reduce the number of deformations of the almost complex structure. One argument that the remaining moduli are fixed is similar to that of [71] for CY_3 compactifications with flux and condensates: As in the CY_3 case, $H = A\Omega + \bar{A}\bar{\Omega} + \cdots$. On the CY_3, the $\cdots$ vanish, and the flux H takes only quantized values [71]. The complex structure moduli are therefore fixed to discrete values that allow H to lie in the integral cohomology. In our vacua, $dH \neq 0$, so the precise quantization condition is not known. However, we expect that there will be some quantization mechanism, which will again force Ω to take a compatible form and freeze the complex-structure-like moduli.

[9] Note that because it is a connection, the gauge field is not rescaled.

[10] The moduli are defined in Appendix D.

For the volume modulus u, the situation is similar to the case of the vanishing condensate, in which the radial modulus is fixed by flux via the torsion constraint (2). In our AdS_4 case, (2) is generalized to (40). Now u is fixed by the presence of nonzero W_3 as can be seen by the following scaling argument: Under the dilation $u \to u + c$ and $m \to e^{-c}m$,

$$g_{mn} \to e^{2c} g_{mn}, \qquad J_{mn} \to e^{2c} J_{mn}, \qquad \Omega_{mnp} \to e^{3c}\Omega_{mnp}. \tag{52}$$

However, we do not expect that the primitive T_0, and therefore W_3, should scale under the dilation; for example, if it is closed, H_0 should be an element of the integral cohomology. Therefore, under the scaling, $dJ \to e^{2c}\{\frac{3i}{2} \times (m\bar{\Omega} - \bar{m}\Omega) + d(\phi - A) \wedge J\} + (\star T)_0 \neq e^{2c}dJ$. The rescaled manifold is not a solution, and u is not a modulus; from the perspective of the 4D theory, it must develop a potential (possibly a very steep one). In fact, a similar scaling argument has been given in the absence of a gaugino condensate, see [25,40].

The H flux definition in terms of Chern-Simons forms (or equivalently its Bianchi identity) gives an alternate and comprehensive way to understand moduli fixing from the 10D perspective. As discussed in [40,62], H appears as torsion in the Lorentz-Chern-Simons term, so H is defined only implicitly. The key point is that solving for H also constrains u. In fact, in the most general case, there should be enough components of H to constrain additional complex-structure-like moduli, as well. Finally, because H has components proportional to m, we expect that m is constrained. In Sec. V D, we will show that, for the special case of a nearly Kähler compactification, the Bianchi identity simplifies considerably and can be solved using a nonstandard embedding to give an explicit value for m.

B. Gaugino condensate

We remind the reader that $\Sigma = a\Omega + \bar{a}\bar{\Omega}$ when we consider gaugino condensation in Calabi-Yau compactifications. However, we have so far allowed a more general form of Σ for two reasons. One is that we wish to leave open the possibility that 10D quantum effects may turn on different components of the gaugino condensate than the 4D effective theory. We will not say anything concrete about this possibility, but we return to this point in Sec. VI E. The other reason that we have left Σ general is that we do not expect the 4D condensate to lift to $\Sigma = a\Omega + \bar{a}\bar{\Omega}$ in a compactification with torsion. We will explain why here.

Let us begin by briefly considering the dimensional reduction of the gaugino. The Kaluza-Klein zero modes [given by χ_6 in (46)] satisfy the Dirac equation following from the action (6)

$$\left(\gamma^m D_m + \partial_m(2A - \phi)\gamma^m - \frac{1}{24}T_{mnp}\gamma^{mnp}\right)\chi = 0, \tag{53}$$

where we use the full spinor χ to include the full gauge structure. However, because the unbroken low-energy gauge group commutes with the background gauge fields A_m, the (adjoint) gaugino zero modes are neutral under the gauge background. Therefore, we can replace $D_m \to \nabla_m$ and $\chi \to \chi_6$ in (53).

It is straightforward to see that $\chi_6 = \eta$, the supersymmetry parameter, in the case of a CY_3 compactification. Simply put, on a CY_3 (with or without gaugino condensate and compensating H flux), the Dirac equation is $\gamma^m\nabla_m\chi_6 = 0$, while η is the unique covariantly constant spinor. Clearly, then, with the same normalization, $\chi_6 = \eta$. In that case, (48) gives $\Sigma = a\Omega + \bar{a}\bar{\Omega}$.

On the other hand, in our solutions, the gaugino Dirac equation (53) is generally not the same as the Dirac equation following from the supersymmetry equation (29). For example, if there is any primitive part of T, it appears in different proportions in the two Dirac equations, as do the warp factor and dilaton. In other words, χ_6 is not a singlet of the $SU(3)$ structure. So, in general, we do not expect the low-energy condensate to lift to $\Sigma = a\Omega + \bar{a}\bar{\Omega}$.

A very interesting question, then, is whether the background Σ is always the same as the uplift of the 4D gaugino condensate, or whether some other 10D quantum effects are responsible for some components of the background condensate Σ. That is, can Σ be identified with the condensate of the effective theory in our backgrounds? While we cannot find the gaugino zero mode in all backgrounds, we will see in Sec. V B that the answer is "yes" in some backgrounds, while the effective theory cannot account for all of the condensates in other backgrounds.

C. A check of potentials

Now we will present a consistency check of proposals for both the superpotential and Kähler potential. The appropriate generalization of the Gukov-Vafa-Witten superpotential for the heterotic theory has been argued [14,45,46] to include both the contributions of torsion and gluino condensation. Consistent with those calculations, we employ the ansatz

$$W = \frac{m_p^3}{\sqrt{4\pi}}\frac{1}{(2\pi\sqrt{\alpha'})^5}\int (H + bidJ + c\Sigma) \wedge \tilde{\Omega}. \tag{54}$$

The 3-form $\tilde{\Omega} = e^{-3u}\Omega$ corresponds to the fiducial metric $\tilde{g}_{mn}$.[11] To agree with [14,45,46] in our conventions, we should take $b = +1$.

The Kähler potential for a CY_3 compactification is given by

[11] See Appendix D, Eqs. (D4) and (D5), for normalizations.

$$\mathcal{K} = -3\ln(-i(T-\bar{T})) - \ln(-i(S-\bar{S})) - \ln\left(\frac{i}{(2\pi\sqrt{\alpha'})^6}\int \tilde{\Omega}\wedge\bar{\tilde{\Omega}}\right) + 3\log 2 \qquad (55)$$

in terms of the 4D superfields S and T and the rescaled holomorphic 3-form $\tilde{\Omega}$.[12] In terms of 10D variables, the Kähler potential is simply

$$\mathcal{K} = 2\phi - 12u - 4\log 2. \qquad (56)$$

Unfortunately, the moduli spaces of non-Kähler manifolds, even those with $SU(3)$ structure, are unknown, and (55) appears inapplicable. However, as argued in [47,48], we can perhaps think of X^6 as a deformation of a Calabi-Yau for which (55) is valid (this assertion is supported by direct calculation on half-flat manifolds [47]). Alternately, [48] further notes that for manifolds with a single Kähler modulus, the Kähler potential is sufficiently simple as to be universal. We will adopt (55) as an ansatz whose consistency is verified by the calculation of this section. For simplicity's sake, we are ignoring any possible effect from a warp factor.

The 4D effective potential in the Einstein frame is now given by [see (D3)]

$$V = m_P^{-2}e^{\mathcal{K}}\left(\sum_{a,b}\mathcal{K}^{a\bar{b}}D_aW\bar{D}_{\bar{b}}\bar{W} - 3|W|^2\right) \qquad (57)$$

where the sum is over all moduli a and b and $\mathcal{K}_{a\bar{b}} = \partial_a\bar{\partial}_{\bar{b}}\mathcal{K}$. Because we are considering a supersymmetric vacuum, the Kähler covariant derivatives vanish, leaving only the final term to give the cosmological constant. As a first check, we can confirm that the proposed potentials give the correct cosmological constant for our background.

Plugging from Eqs. (24) and (40) into the superpotential and using $\bar{\Omega}\wedge\tilde{\Omega} = 8ie^{3u}\tilde{\epsilon}$, we find immediately

$$W = i\frac{m_P^3}{\sqrt{4\pi}}(2\pi\sqrt{\alpha'})(4-12b-16c)e^{3u}m. \qquad (58)$$

Putting everything together, the absolute Einstein-frame cosmological constant is

$$\Lambda_E = -\frac{V}{m_P^2} = 3\pi m_P^2\alpha' e^{2\phi-6u}|1-3b-4c|^2|m|^2 = \frac{1}{4}e^{2\phi-6u}|1-3b-4c|^2\Lambda_s \qquad (59)$$

after using (16) and (D2), where Λ_s is the (absolute) cosmological constant in the string frame. The exponential of the moduli is precisely the necessary combination for conversion of a mass dimension 2 constant between frames, so the Kähler potential and superpotential are consistent if

$$1-3b-4c = \pm 2. \qquad (60)$$

[12]The 4D superfields are defined in Appendix D.

Some consistent solutions are

$$b=1, c=-1; \qquad b=1, c=0; \qquad b=-1, c=\tfrac{1}{2}; \qquad b=-1, c=\tfrac{3}{2}. \qquad (61)$$

We can also check the superpotential and Kähler potential by computing the supersymmetry variation of the Einstein-frame gravitino. For our metric ansatz (8), the gravitino with the canonical supersymmetry transformation in the Einstein frame is

$$\psi_{E,\mu} \equiv \Psi_\mu\otimes\eta + \Psi_\mu^*\otimes\eta^* = e^{(3u-\phi)/2}[\psi_\mu + \Gamma_\mu(\tfrac{1}{2}\Gamma^m\psi_m + \lambda)]. \qquad (62)$$

Here Γ^M is still the 10D string-frame Dirac matrix, and Ψ and Ψ^* are the positive and negative chirality spinors in 4D. This linear combination is the same one which diagonalizes the gravitino kinetic term. After some algebra, we find

$$\delta\Psi_\mu = e^{(3u-\phi)/2}\left[\nabla_\mu\zeta - \frac{1}{48}H_{mnp}\bar{\Omega}^{mnp}\gamma_\mu\zeta^* - \frac{1}{2}\eta^\dagger\gamma^m(\nabla_m\eta^*)\gamma_\mu\zeta^*\right]. \qquad (63)$$

Note that any contributions from Σ have *canceled* between the external gravitino variation and the dilatino variation. Using (28), we can rewrite the last term as

$$-\frac{1}{2}\eta^\dagger\gamma^m(\nabla_m\eta^*) = -\frac{1}{16}\eta^T\gamma_{np}\nabla_m\eta^*\bar{\Omega}^{mnp} = \frac{i}{48}(dJ)_{mnp}\bar{\Omega}^{mnp} + \frac{1}{16}\times(\nabla_m\eta^T)\gamma_{np}\eta^*\bar{\Omega}^{mnp}, \qquad (64)$$

and the last term vanishes, again because of (28). In the end, then,

$$\delta\Psi_\mu = e^{(3u-\phi)/2}\left[\nabla_\mu\zeta - \frac{1}{48}(H-idJ)_{mnp}\bar{\Omega}^{mnp}\gamma_\mu\zeta^*\right]. \qquad (65)$$

Again, all variables on the right-hand side of (65) are given in the 10D string frame. From [72], we know that the second term in the gravitino variation must be proportional to $e^{\mathcal{K}/2}\bar{W}$, so we find (up to the overall phase)

$$W = \frac{m_P^3}{\sqrt{4\pi}}\frac{1}{(2\pi\sqrt{\alpha'})^5}\int(H+idJ)\wedge\tilde{\Omega}, \qquad (66)$$

consistent with the second solution of (61).

Oddly, the superpotential derived in this way does not contain the gaugino condensate Σ, in contrast to the proposal of [14]. Our result is especially counterintuitive because the effective 4D field theory certainly gains a nonperturbative superpotential associated with gaugino condensation. Furthermore, our result (65), when evaluated in our AdS_4 backgrounds, gives $\nabla_\mu\zeta = -\bar{m}\gamma_\mu\zeta^*$, which is exactly the substitution we used in Eq. (15). In other words, (66), when evaluated on the background fields, gives exactly the value of the 4D superpotential we assumed at the start. (Unfortunately, we cannot compare directly to the 4D effective superpotential in terms of the gauge coupling because the exact coefficients of the appropriate terms are not known for that formula.) It would therefore seem inconsistent to add a nonperturbative superpotential. One possible resolution is that, in the "dictionary" between the 10D supergravity and 4D effective theory, H or dJ includes the nonperturbative part of the superpotential; we will return to this question in our discussion of future directions.

V. NEARLY KÄHLER COMPACTIFICATION

We have found in Sec. III that generically X^6 has nonzero W_1, W_3, W_4, and W_5 and is of type $\mathcal{G}_1$. However, by imposing certain conditions on the choice of fluxes and condensates, we can easily turn off all the torsions but W_1, making the manifold nearly Kähler. Nearly Kähler compactifications have been of recent interest in massive type IIA supergravity [28,73], M theory [36], and heterotic strings [48]. In this section, we will first list some properties of NK manifolds and relate them to our compactifications; in following subsections, we elaborate on our earlier discussions in the special case of an NK compactification.

A. Nearly Kähler geometry

Nearly Kähler manifolds, in many ways the simplest non-Kähler manifolds, have been studied by Gray [74–76] and have many interesting mathematical properties.[13] The defining property of NK $2n$ folds is weak $SU(n)$ holonomy, which appears in the $SU(3)$ holonomy of the torsional derivative. Among other curvature identities, every NK manifold in six dimensions is Einstein and has vanishing first Chern class [76]. In addition, the cone over any six-dimensional NK manifold will have holonomy G_2 [78–80].

Many examples of NK manifolds are known due to a theorem by Gray [81] that 3-symmetric spaces[14] have an NK metric and ACS (or are products of such manifolds). Furthermore, [81] showed that 3-symmetric spaces can be identified in a natural way with cosets of connected Lie groups and proceeded to classify such cosets. For example, the simplest of a NK manifold is the sphere $S^6 \simeq G_2/SU(3)$.

Another nice result in the mathematical literature is due to Grunewald [82], regarding the Killing spinor. Any spin manifold with a real Killing spinor, with respect to the Levi-Cività connection, is NK, and, conversely, any NK manifold has a real Killing spinor. In fact, following from (69), the Killing number is $|m|/2$. This point is important in ensuring that supersymmetry is truly preserved for an NK compactification with the appropriate constraints on the flux backgrounds (in other words, given an NK manifold and the appropriate algebraic relations for the flux and condensate, there does exist an appropriate supersymmetry transformation parameter on the manifold). Additionally, [67] has shown that manifolds with real Killing spinors are compact.

Let us now remind the reader of the appearance of NK manifolds in our supersymmetric backgrounds. In general, the supersymmetric vacua described in Sec. III C are not NK because W_3, W_4, and W_5 are nonvanishing. However, as mentioned in Sec. III D, we can set the primitive part $T_0 = 0$ (implying $H_0 = \Sigma_0/2$) to remove the W_3 torsion. Then choosing the nonprimitive $J \lrcorner T = 0$, we find that X^6 is conformally NK [meaning that $W_4 = (2/3)W_5$]. Finally, requiring $J \lrcorner \Sigma = 0$ also forces $W_4 = W_5 = 0$, so we have a truly nearly Kähler manifold. In the following, we will distinguish two cases: $\Sigma_0 = 0$, which we will simply call NK, and $\Sigma_0 \neq 0$, which we will denote NK′. It is useful also to simplify (for either case)

$$H = \tfrac{1}{2}(\bar{m}\Omega + m\bar{\Omega}), \tag{67}$$

$$\Sigma = -2(\bar{m}\Omega + m\bar{\Omega}) \quad \{+\Sigma_0 \text{ for NK}'\}, \tag{68}$$

$$\tau = \tfrac{1}{2}(\bar{m}\Omega + m\bar{\Omega}). \tag{69}$$

B. Gaugino condensation for NK

At the end of Sec. IV B, we asked if the background Σ can be identified with the gaugino condensate of the effective field theory. Although we do not know the general answer, we will show here that such an identification is consistent in NK compactifications.

Since $dA = d\phi = 0$ in the NK case, the gaugino zero mode Dirac equation (53) becomes the covariant Dirac equation for the connection with torsion $\tau_\chi = T/3 = (\bar{m}\Omega + m\bar{\Omega})/2$ with the specific form of T given by (67) and (68). But the supersymmetry parameter η is constant with respect to the $SU(3)$ structure torsion (69), which is the same. Therefore, the gaugino zero mode is $\chi_6 = \eta$. Then the uplifted 4D condensate (48) is just $\Sigma = \kappa_{10}^2/g_{10}^2(\langle\overline{\chi_4^*}\chi_4\rangle\Omega + \langle\bar{\chi}_4\chi_4^*\rangle\bar{\Omega})$, which is consistent with the 10D decomposition for NK compactifications. In other words, Σ can be generated completely by the condensate in the effective 4D theory. However, in the NK′ case, the additional background condensate Σ_0 is not of the correct form and must therefore be generated by some as yet unknown quantum effect.

[13]For a more recent work which reviews the properties of NK manifolds see [77].

[14]A 3-symmetric space has global isometries θ_p for each point p, where p is the fixed point of θ_p. Each $\theta_p^3 = 1$ and is holomorphic with respect to a canonically associated ACS.

C. Moduli fixing for NK

In the NK case, the moduli fixing is slightly different from the general case described in Sec. IVA. Because $W_3 = 0$, it would seem that u is now unfixed. However, we can use a scaling argument instead to fix the size of X^6 in terms of the AdS_4 scale m. For a dimensionless NK manifold of unit radius, the almost complex structure J_0 and $SU(3)$ structure Ω_0 are related by [83]

$$dJ_0 = \frac{3i}{2}(\bar{\Omega}_0 - \Omega_0), \qquad d\Omega_0 = 2J_0 \wedge J_0. \tag{70}$$

We have already computed the derivatives of J (40) and Ω (43), which in the NK case reduce to

$$dJ = \frac{3i}{2}(m\bar{\Omega} - \bar{m}\Omega), \qquad d\Omega = 2mJ \wedge J. \tag{71}$$

To compare the two formulas, we need to rescale the unit radius NK manifold, first by giving it dimensionful coordinates. Since our fiducial metric has volume $(2\pi)^6\alpha'^3$, we rescale the coordinates by $x \to 2\pi\sqrt{\alpha'}/\omega_6^{1/6}$, where ω_6 is the volume of the unit NK manifold.[15] Additionally, in rescaling $\tilde{g}_{mn} \to g_{mn}$, we must rescale the ACS and 3-form, so we eventually get

$$J = \frac{4\pi^2\alpha'}{\omega_6^{1/3}} e^{2u} J_0, \qquad \Omega = \frac{8\pi^3\alpha'^{3/2}}{\omega_6^{1/2}} e^{3u+i\beta}\Omega_0 \tag{72}$$

(including a possible phase β for the 3-form). Equations (70) and (71) match for

$$m = \frac{\omega_6^{1/6}}{2\pi\sqrt{\alpha'}} e^{-u+i\beta}. \tag{73}$$

So, for the NK solution, the AdS_4 and compactification scales are essentially the same (differing by a factor of order unity). The criterion (49) for the validity of effective field theory is therefore only satisfied in the NK case if

$$u \lesssim \frac{1}{2}\ln 2\pi - \frac{1}{12}\ln\omega_6, \tag{74}$$

which means the compactification is in the large radius regime only if ω_6 is sufficiently small. For example, for the case of S^6, ω_6 is small enough that $u \lesssim 1/2$ is valid. The direct relation between m and u also means that the NK manifold has no radial modulus, properly speaking. Because changing m changes the AdS_4 boundary conditions, u corresponds to a non-normalizable mode.

In the general case we found weakly coupled solutions with large u and exponentially large AdS_4 radius, but unfortunately, that is no longer possible for NK compactifications. Using (73) in (51), the equation for the dilaton now becomes

$$2u \sim -e^{6u-2\phi}, \tag{75}$$

which, in the most optimistic case, gives $u \sim 0$ and $g_{\mathrm{YM}}^2 \lesssim 1$.

Incidentally, [15,57] argued that the general superpotential ansatz (54) cannot be responsible for fixing complex structure moduli unless there is $(2,1)+(1,2)$ flux. However, our NK background should have no massless scalars. The resolution is that, due to the relation (71), there are no variations of Ω independent of variations of J. That is to say, NK manifolds have no "complex structure" moduli. In fact, [48] has already proposed that NK manifolds have no such moduli.

As we mentioned in the general analysis of moduli fixing (Sec. IVA), the H flux Bianchi identity also constrains the moduli. Since the NK $SU(3)$ structure is so simple, we can solve the Bianchi identity, which we discuss in the following section.

D. Bianchi identity

We have yet to consider the restrictions imposed by the Bianchi identity for H,

$$dH = \frac{\alpha'}{4}\left(\mathrm{Tr}R_- \wedge R_- - \frac{1}{30}\mathrm{Tr}F \wedge F\right) \tag{76}$$

where R_- is the Ricci two-form constructed using the torsion $\tau_- = -H$. Note that this is opposed to the curvature $R(\tau)$ constructed with the torsion τ associated with the $SU(3)$ structure given in Eq. (30).

An advantage of the NK case over our more general one (or even NK′) is that the Bianchi identity becomes sufficiently tractable that we can make explicit computations. In particular, a significant simplification results from the fact that, in the NK case, (67) and (69) imply $\tau_- = -(m\bar{\Omega} + \bar{m}\Omega)/2 = -\tau$.

In torsion-free CY_3 compactifications, the Bianchi identity is typically simplified by imposing the standard embedding of the spin connection into the gauge connection, canceling to two terms on the right-hand side of (76) to yield $dH = 0$ and breaking the gauge group from $E_8 \to E_6$ We could employ a similar tactic here, but since (67) and (70) gives

$$dH = 2|m|^2 J \wedge J, \tag{77}$$

setting $\mathrm{Tr}R_- \wedge R_- - \frac{1}{30}\mathrm{Tr}F \wedge F$ would mean $m = 0$, requiring a CY_3 compactification with no flux or condensate and a Minkowski spacetime. Instead we will try to embed a more general spin connection with torsion proportional to τ into the gauge connection.

We denote by R_ξ the Riemann curvature constructed using the torsion $\tau_\xi = \xi\tau = \xi(m\bar{\Omega} + \bar{m}\Omega)/2$. Using identities (C1) and (C3), we can relate R_ξ, of which $R_- = R_{-1}$ is a special case, to $R(\tau) = R_1$, which for consistency we will hereafter denote R_+:

[15] For example, for S^6, $\omega_6 = 16\pi^3/15$.

$$R_{\xi mnpq} = R_{+mnpq} - (\xi - 1)\nabla^{+}_{[p}\tau_{q]mn} + \frac{\xi - 1}{2}\tau_{mnr}\tau_{pq}{}^{r} + \frac{(\xi - 1)^2}{2}\tau^{r}{}_{n[q}\tau_{p]mr}. \quad (78)$$

Since Ω is an $SU(3)$ invariant tensor, the covariant derivative vanishes. Simplifying the other terms using (B6) gives

$$R_{\xi mnpq} = R_{+mnpq} + \frac{1}{2}|m|^2\Big[\frac{\xi^2 - 1}{4}(J \wedge J)_{mnpq} + \frac{(\xi - 1)(\xi - 3)}{2}J_{mn}J_{pq} - \frac{(\xi - 1)(\xi - 3)}{2} \times (g_{mp}g_{nq} - g_{mq}g_{np})\Big]. \quad (79)$$

We can then plug (79) into $\mathrm{Tr}R_\xi \wedge R_\xi$. With some extensive algebra, we obtain

$$\mathrm{Tr}\,R_\xi \wedge R_\xi = \mathrm{Tr}R_+ \wedge R_+ - 6(\xi - 1)^2|m|^4 J \wedge J. \quad (80)$$

In order to simplify cross terms in the expansion of the trace, we use (29) and (C4) to deduce that

$$R_{+mnpq}J^{pq} = 0, \qquad R_{+[mn}{}^{sr}(J \wedge J)_{pq]rs} = 4R_{+[mnpq]}. \quad (81)$$

From (79) with $\xi = 0$, we find $R_{+[mnpq]} = -\frac{1}{3}|m|^2(J \wedge J)_{mnpq}$.

Setting $\xi = -1$ in (80) enables us to substitute $\mathrm{Tr}R_+ \wedge R_+$ for $\mathrm{Tr}R_- \wedge R_-$ in the Bianchi identity (76). A natural idea would now be to embed the $SU(3)$ spin connection with torsion τ into the gauge connection. However, setting $\frac{1}{30}\mathrm{Tr}F \wedge F = \mathrm{Tr}R_+ \wedge R_+$ and employing (77) reduces (76) to

$$2|m|^2 J \wedge J = -24\alpha'|m|^4 J \wedge J, \quad (82)$$

whose only solution is an unacceptable $m = 0$.

Instead, we use (80) again to write

$$\mathrm{Tr}\,R_- \wedge R_- = \mathrm{Tr}R_\xi \wedge R_\xi + 6(\xi - 3)(\xi + 1)|m|^4 J \wedge J. \quad (83)$$

We impose a nonstandard embedding with $\frac{1}{30}\,\mathrm{Tr}F \wedge F = \mathrm{Tr}R_\xi \wedge R_\xi$. Any connection with torsion other than τ has holonomy $SO(6)$ rather than $SU(3)$, so this embedding breaks the gauge group $E_8 \to SO(10)$ or $SO(32) \to SO(26)$. The Bianchi identity (76) is now just a condition on m,

$$\alpha'|m|^2 = \frac{1}{3(\xi - 3)(\xi + 1)}, \quad (84)$$

which has positive solutions for $|m|$ when $\xi < -1$ or $\xi > 3$. To our knowledge, there seems to be no reason why ξ is constrained (within the allowed region), but, once it and m are chosen, changing ξ would be a non-normalizable mode in AdS_4 and therefore not a modulus. This is as we discussed in the previous subsection. In a sense, the AdS_4 radius determines the embedding of the spin connection into the gauge connection.

Unfortunately, this embedding also breaks supersymmetry[16] because, in general, $R_{\xi mnpq}J^{pq} \neq 0$ (the only exception is R_+). Therefore, in our embedding, $F \lrcorner J \neq 0$, which means that the gaugino variation does not vanish. There are three reasons why we are not concerned with this violation of supersymmetry. First, NK compactifications are necessarily toy models since (75) implies that the 4D theory is not weakly coupled. In addition, this supersymmetry breaking is formally a subleading effect in $m \lesssim 1$. Finally, it is not unreasonable to expect that a supersymmetric solution to the Bianchi identity will generally require an expansion in α' and the introduction of sources such as NS5 branes.

VI. DISCUSSION OF OPEN QUESTIONS

In this section, we will discuss open questions about supersymmetry and gaugino condensation in the heterotic theory.

A. Superpotential

In Sec. IV C, we realized that the superpotential of the 4D effective theory is still not completely understood. We can point out two related issues that, as yet, lack explanations.

First, our supersymmetric AdS_4 backgrounds are incompatible with a superpotential of the form $\int(T + idJ) \wedge \Omega$, even though that form is suggested by the 10D supergravity action [14]. In particular, we showed in (61) that such a superpotential leads to the wrong value of the cosmological constant. Additionally, reminding ourselves of CY_3 compactifications with gaugino condensates confirms that the superpotential cannot depend on H and Σ only through T. In those compactifications, at least to lowest order, the vacua are no scale and Minkowski, as can be seen in the effective theory. However, supersymmetry is broken by the condensate, which implies that the superpotential cannot vanish, $W \neq 0$. On the other hand, $T = 0$ in CY_3 compactifications. So, indeed, even though Σ enters the supergravity only through T, the effective superpotential has some alternate dependence on Σ.

More disturbing, perhaps, is our discovery that Σ does not seem to enter the superpotential at all, which we found by computing the Einstein-frame gravitino supersymmetry variation. Certainly, we know that the condensate generates a nonperturabative superpotential in the effective field theory. Perhaps this nonperturbative superpotential should not appear in our semiclassical treatment of the background, even though we explicitly left $\Sigma \neq 0$. On the other hand, adding any new contribution to the superpotential

[16] We thank N. Prezas and P. Manousselis for bringing this fact to our attention.

would conflict with our initial choice of the AdS_4 Killing spinor.

What is the real story? One possibility is that the dictionary from 10D variables to the 4D effective field theory is nontrivial in the presence of gaugino condensates. In other words, perhaps some of the H flux or torsion dJ contains the nonperturbative part of the superpotential. One way this idea could work out is that T is the fundamental flux parameter, rather than H. In that case, the superpotential would be written as $W \propto \int(T + \Sigma/2 + idJ) \wedge \Omega$, explicitly displaying the nonperturbative component. However, we think it more likely that the condensate affects the 10D supergravity in some more subtle way. For example, the gaugino kinetic term $\bar{\chi}\gamma^M D_M \chi$ should also acquire an expectation value when the 4D gaugino condenses. Specifically, making use of the decomposition (46) and the gaugino zero mode Dirac equation (53), we can see that

$$\begin{aligned}\langle \bar{\chi}\Gamma^M D_M \chi\rangle &= \frac{1}{24} T_{mnp}(\langle \bar{\chi}_4 \chi_4^* \rangle \chi_6^\dagger \gamma^{mnp} \chi_6^* \\ &\quad - \langle \overline{\chi_4^*} \chi_4 \rangle \chi_6^T \gamma^{mnp} \chi_6) \\ &= \frac{1}{4} T \lrcorner \Sigma \end{aligned} \tag{85}$$

for zero-momentum gaugini in AdS_4. Here, we assume that the entire condensate is generated in the 4D effective theory. Most likely, however, understanding all the effects of the condensate in 10D will require understanding the one-particle-irreducible (1PI) effective action of the supergravity.

B. Equations of motion and Bianchi identity

In Sec. III we set the supersymmetry variations to zero in order to find supersymmetric backgrounds. However, we have not explicitly shown that these backgrounds are solutions to the equations of motion; generally speaking, the supersymmetry conditions do not imply all of the equations of motion. The independent equations of motion must be imposed additionally to guarantee that the backgrounds are indeed solutions. For example, the heterotic supersymmetric background with constant dilaton presented in [14] was shown in [65] not to satisfy the H and F equations of motion. Similarly, in the context of massive IIA, [29] argued that both the form field equations of motion and Bianchi identity were additionally required for supersymmetric vacua to be solutions.

In the case considered here, the gaugino condensate makes the derivation of the equations of motion more subtle. In particular, the expectation value of the gaugino kinetic term should appear in the equations of motion. Again, it seems likely that the appropriate 10D equations of motion are given by the 1PI effective action of the supergravity in the presence of a condensate.

In addition to the equations of motion, prospective solutions must also satisfy the Bianchi identity. For the particular case of NK compactifications we were able to make considerable progress by relying on a generalization of the standard embedding. A similar calculation may be possible for the less tractable general case. However, it seems unlikely that the solution could be so simple. In general, we will not be able to employ this approach, and instead we may need to involve the gauge field in some more complicated way. Perhaps a series of solutions in powers of α' is the best that we can expect, as in [40,62].

C. Topology change

One can approach flux vacua from two different directions. We have taken the view that one looks for self-consistent combinations of compactification manifold, fluxes, condensate, and AdS_4 radius. The topological data and choice of flux will be consistent with only discrete values of continuous moduli, and not all discrete choices will necessarily yield consistent backgrounds. In particular, CY_3 compactifications with H flux are not supersymmetric; different topological data, including non-Kählerity (and noncomplexity if gaugino condensates are added), are required. In this view, therefore, we do not describe the fluxes as backreacting on a preexisting CY_3 geometry.

However, one could instead choose to begin with a particular flux-free CY_3 compactification and then turn on fluxes using appropriate branes as domain walls. This is the context in which the backreaction of fluxes can be made precise. In type II supergravity, [42] argued, in fact, that NS5-brane domain walls in CY_3 compactifications are mirror symmetric to topology-changing domain walls (presumably wrapped Kaluza-Klein monopoles), which in fact transform a CY_3 into a half-flat manifold.

In our case, the reader might think that topology change could occur via instantons, allowing the decay of non-supersymmetric Minkowski vacua into our AdS_4 vacua. However, including gravitational effects, tunneling could only occur if the Minkowski vacuum were lifted by loop or string effects to de Sitter, and the end state would be a big crunch universe rather than AdS_4 [84]. Nonetheless, it would be interesting to trace the connection between the Minkowski and the AdS_4 vacua.

D. Dualities

Another possible angle to explore is the existence of an AdS_4/CFT_3 duality. Beyond the standard AdS/CFT duality with compact spheres (e.g. $AdS_5 \times S^5$) [85], examples of such dualities are known for cases where the compactification manifolds are more complicated, such as the manifold $T^{1,1}$ [86]. While one would expect the AdS_4 compactifications studied here to have a $2 +$ 1-dimensional CFT dual, we have few clues as to what the dual theory would be. The 't Hooft coupling is given by the AdS_4 scale to be $\lambda \sim |e^{-4iaS}|$, but we cannot say much else. Some clue to the duality could be found by relating our backgrounds to the near-horizon limit of some 2-brane geometry and the IR limit of its world volume theory. For an NK compactification, we could perhaps use the fact that a cone with a NK base has holonomy G_2, as suggested in [28] in the type IIA context. Then our backgrounds would be the near-horizon limit of a 2-brane at the tip of such a G_2 cone. However, the heterotic string is lacking in 2-branes, so such a picture would likely arise via some duality.

Similarly, an important goal to pursue is to relate the many different types of flux compactifications in the various different theories to each other, forming a single coherent picture. Besides relating to heterotic M-theory solutions, heterotic flux vacua are U dual to much-studied flux vacua in IIA [18,26–29], IIB [1,17,49,51,52], and M theory [32,34–37]. Though in some examples these dualities have been made explicit [40], the general connections between all flux vacua have yet to be elucidated. In addition to dualities, dynamical transitions among vacua are possible. As noted above, certain vacua could be related to others by domain walls or tunneling. A thorough understanding of these connections would be a basis for a cartography of the landscape of flux vacua.

E. Future directions

In the analysis of Sec. III, rather than specifying the gaugino condensate from the outset, we deliberately worked with the most general case possible. We found the usual condensate $\Sigma \sim \Omega + \mathrm{c.c.}$ consistent with the NK compactification, but, in general, as seen from (20), Σ also has $(2,1)$ and $(1,2)$ components. As discussed in Sec. IV B, the usual condensation mechanism may generate these unusual components as a result of the non-Kähler geometry. However, there may be other, as yet unknown, mechanisms to generate such condensates, and we see no reason to exclude them *a priori*.

Following this type of reasoning, one could further generalize to consider the condensation of other, more exotic fermion bilinears. Gravitinos and dilatinos, along with other components of the gaugino, could conceivably condense through some unknown quantum effect. One could simply posit the existence of such a condensate and investigate its effect on the supergravity solution. However, we leave such explorations for future work. Of course, it will be necessary to understand how the 10D 1PI effective action is modified in the presence of general condensates, just as we have noted above in the relatively simpler case of gaugino condensation.

One could also investigate whether our AdS_4 compactifications can be lifted to dS_4. Along the lines of [9], one could add $\overline{NS5}$ branes to break supersymmetry and increase the vacuum energy. However, one would then need to be sure to stabilize the $\overline{NS5}$ moduli.

Further, the heterotic flux vacua discussed here should lift to heterotic M theory in the strong coupling limit. Heterotic M theory is attractive for phenomenological model building, and flux compactifications of 11 dimensions with nonperturbative effects have been extensively studied [87]. In addition to gaugino condensation on the E_8 branes, open M5 brane instantons are needed to stabilize the orbifold length. Furthermore, both AdS_4 solutions [59] and metastable dS_4 vacua [60,61] have been constructed in the context of heterotic M theory. In fact, our backgrounds are the perturbative description of the AdS_4 backgrounds in [59], but including the backreaction of the condensate and H flux.

VII. CONCLUSION

We have presented, from a primarily 10D perspective, a class of supersymmetric heterotic AdS_4 compactifications with both H flux and gaugino condensation. The effects combine to fix all the moduli and to yield a noncomplex internal geometry. In the general case, we found supersymmetric backgrounds at weak coupling and large internal volume with exponentially large AdS_4 radius. We also showed that proposed super- and Kähler potentials can reproduce the correct 4D cosmological constant, although there appear to be subtleties regarding the derivation of the superpotential via dimensional reduction which are not yet fully understood.

To elucidate the 10D geometry of the supersymmetric AdS_4 backgrounds, we used the G-structure formalism that has been used extensively for flux vacua. To our knowledge, this is its first application in the context of gaugino condensation. The form of the condensate was left intentionally general, so as to include the possibility of being generated by nonstandard, possibly 10D, effects. Furthermore, because we were not compactifying on a CY_3, a condensate resulting from the standard 4D mechanism does not necessarily take the standard form in any event.

For a particular choice of flux and condensate all the torsion classes but one vanished, and we found the internal manifold to be nearly Kähler, which greatly simplified the analysis. Here the condensate took its usual form, the internal and AdS_4 sizes were roughly equal, and we solved the Bianchi identity with a nonstandard embedding to give an explicit value for m.

Despite the progress made here, important issues remain, such as verifying the equations of motion and solving the Bianchi identity in the general case. The generation and effects of exotic gaugino or other fermion condensates pose interesting questions. More broadly, we have yet to really explore the connections, via dualities or domain walls, of the compactifications presented here to all the other extant flux vacua.

ACKNOWLEDGMENTS

We would like the thank Ben Freivogel, Sergei Gukov, Shamit Kachru, Pantelis Manouselis, Liam McAllister, Jeremy Michelson, Andrei Micu, Hirosi Ooguri, Nikolaos Prezas, Sergey Prokushkin, Michael Schulz, Al Shapere, Alessandro Tomasiello, and Chengang Zhou for helpful discussions. The work of A. R. F. was supported by the John A. McCone fellowship in theoretical physics at the California Institute of Technology. A. R. F. was also partially supprted from the DOE Grant No. DE-FG03-92-ER40701. The work of M. L. was supported by the DOE under Contract No. DE-FG001-00ER45832.

Note added.—During the final preparation of this work, [91] appeared, which also discusses heterotic compactifications with torsion, flux, and gaugino condensation. We work from a more ten-dimensional point of view.

APPENDIX A: FORM AND SPINOR CONVENTIONS

For our index conventions, we take upper case Latin for the full ten dimensions, lower case Greek for the four Poincaré invariant dimensions, and lower case Latin for the internal dimensions. Hats denote tangent space indices. We work in a signature in which timelike norms are negative.

Our differential form conventions are as follows:

$$\begin{aligned} \epsilon_{012\cdots(d-1)} &= +\sqrt{-g} \text{ for } d \text{ dimensions}, \\ T_{[M_1\cdots M_p]} &= \frac{1}{p!}(T_{M_1\cdots M_p} \pm \text{permutations}), \\ (\star T)_{M_1\cdots M_{d-p}} &= \frac{1}{p!}\epsilon_{M_1\cdots M_{d-p}}{}^{N_1\cdots N_p} T_{N_1\cdots N_p}, \\ T &= \frac{1}{p!} T_{M_1\cdots M_p} dx^{M_1}\cdots dx^{M_p}. \end{aligned} \tag{A1}$$

Wedges and exterior derivatives are defined consistently with those conventions. We also use the notation

$$(R \lrcorner S)_{N_1\cdots N_q} = \frac{1}{p!} R^{M_1\cdots M_p} S_{M_1\cdots M_p N_1\cdots N_q}, \tag{A2}$$

which is common in the G structure literature.

Gamma matrices in tangent space have the algebra $\{\Gamma^{\hat{M}}, \Gamma^{\hat{N}}\} = 2\eta^{\hat{M}\hat{N}}$. With these conventions, a Majorana basis is real and symmetric for spacelike indices and antisymmetric for time. Gammas can be converted to coordinate indices with the vielbein. We define $\Gamma^{M_1\cdots M_p} = \Gamma^{[M_1}\cdots\Gamma^{M_p]}$. The chirality is given by

$$\Gamma_{\widehat{(10)}} = \Gamma^{\hat{0}}\cdots\Gamma^{\hat{9}} = \frac{1}{10!}\epsilon_{M_1\cdots M_{10}}\Gamma^{M_1\cdots M_{10}}. \tag{A3}$$

We can decompose the Γ matrices as

$$\Gamma^\mu = \gamma^\mu \otimes 1, \qquad \Gamma^m = \gamma_{(\hat{4})} \otimes \gamma^m \tag{A4}$$

with 4D and 6D chirality $\gamma_{(\hat{4})} = -i\gamma^{\hat{0}}\cdots\gamma^{\hat{3}}$, $\gamma_{(\hat{6})} = i\gamma^{\hat{4}}\cdots\gamma^{\hat{9}}$. The γ^μ have the same symmetry and reality properties as Γ^M, while the γ^m are imaginary and antisymmetric.

APPENDIX B: GAMMA MATRIX AND $SU(3)$ STRUCTURE IDENTITIES

A comprehensive list of (anti)commutators appears in [88], although there is at least one typographical error. It is necessary to replace

$$[\gamma_{mnp}, \gamma^{rst}] = 2\gamma_{mnp}{}^{rst} - 36\delta^{[rs}_{[mn}\gamma_{p]}{}^{t]}. \tag{B1}$$

Contractions of gamma matrices are given by

$$\gamma^a\gamma_{m_1\cdots m_{2k}}\gamma_a = (d-4k)\gamma_{m_1\cdots m_{2k}}, \tag{B2}$$

$$\gamma^a\gamma_{m_1\cdots m_{2k+1}}\gamma_a = (4k-d+2)\gamma_{m_1\cdots m_{2k+1}}. \tag{B3}$$

Other useful identities are

$$\gamma_{mnp} = \frac{i}{6}\gamma_{(\hat{6})}\gamma^{qrs}\epsilon_{mnpqrs}, \qquad \gamma_{mnpq} = \frac{i}{2}\gamma_{(\hat{6})}\gamma^{rs}\epsilon_{mnpqrs}, \tag{B4}$$

$$\eta^\dagger\gamma_{mnpqrs}\eta = -i\epsilon_{mnpqrs} \tag{B5}$$

for positive chirality η. Using (B4), we can see that Ω as defined in (13) is self-dual, $\star\Omega = i\Omega$. Self-duality implies

$$\Omega^{mnp}\Omega_{mnp} = 0, \qquad \bar{\Omega}^{mnp}\Omega_{mnp} = 48 \tag{B6}$$

(when combined with the relation $\star\epsilon = 1$ for the associated volume form).

The Fierz identities that we use come from expanding in terms of the complete set of γ matrices. Specifically, we find

$$\eta\eta^\dagger = \frac{1}{8} - \frac{i}{16}J_{mn}\gamma^{mn} - \frac{i}{16}J_{mn}\gamma^{mn}\gamma_{(\hat{6})} + \frac{1}{8}\gamma_{(\hat{6})}, \tag{B7}$$

$$\eta\eta^T = -\frac{1}{48}\Omega_{mnp}\gamma^{mnp} \tag{B8}$$

for the normalized positive chirality spinor χ used in the text. This identity can be used to show that $J_m{}^n J_n{}^p = -\delta_m^p$ and also that

$$(J\wedge J)_{mnpq} = 6J_{[mn}J_{pq]} = \epsilon_{mnpq}{}^{rs}J_{rs} = 2(\star J)_{mnpq}. \tag{B9}$$

Other helpful identities which follow from self-duality of Ω and the Fierz identities are

$$\begin{aligned} \Omega^{mnr}\bar{\Omega}_{pqr} &= 4\delta^{mn}_{[pq]} - 4J^m{}_{[p}J^n{}_{q]} + 8i\delta^{[m}_{[p}J_{q]}{}^{n]}, \\ \Omega^{mnr}\Omega_{pqr} &= 0. \end{aligned} \tag{B10}$$

The first of (B10) is also given in [52].

We can also decompose any tensor with respect to the $SU(3)$ structure. We write a real 3-form R and complex 4-form S as

$$\begin{aligned} R &= \frac{3i}{2}\,\mathrm{Im}(\bar{R}_1\Omega) + R_3 + J\wedge R_4, \\ S &= S_1 J\wedge J + J\wedge S_2 + \Omega\wedge S_5. \end{aligned} \tag{B11}$$

We have labeled the components in a fashion consistent with the torsion modules W_i in Eqs. (31) and (32). Then we can invert (B11) to get

$$R_1 = -\frac{i}{6}\Omega \lrcorner R, \qquad R_{4,p} = \frac{1}{2}(J \lrcorner R)_p,$$
$$S_1 = \frac{1}{12}(J \wedge J) \lrcorner S, \qquad S_{5,p} = \frac{1}{24}(\bar{\Omega} \lrcorner S)_p \tag{B12}$$

as in [27,65,69]. R_3 and S_2 are primitive in the sense that $J \lrcorner R_3 = 0$ and $J \lrcorner S_2 = 0$.

APPENDIX C: CONTORSION AND INTRINSIC TORSION

Here we present a brief review of various definitions and identities involving torsions. These formulas and conventions can be found, for example, in [89,90].

The difference between a torsional connection $\bar{\Gamma}$ and the torsion-free Levi-Cività connection Γ is the contorsion tensor

$$\bar{\Gamma}^m{}_{np} - \Gamma^m_{np} = \kappa^m{}_{np}, \qquad \kappa_{mnp} = -\kappa_{pnm}, \tag{C1}$$

where the antisymmetry follows from metric compatibility. Then, because the vielbein must be covariantly constant with respect both to the torsionful and torsionless derivatives $\bar{\nabla}, \nabla$, the spin connection is shifted by

$$\begin{aligned}\bar{\omega}_m{}^{\hat{a}}{}_{\hat{b}} - \omega_m{}^{\hat{a}}{}_{\hat{b}} &= \kappa^p{}_{mn} e^{\hat{a}}_p e^n_{\hat{b}} \\ &= -\kappa_m{}^{\hat{a}}{}_{\hat{b}} \text{ (for } \kappa \text{ totally antisymmetric).}\end{aligned} \tag{C2}$$

The Riemann tensor is still given by the usual formulas

$$\bar{R}^m{}_{npq} = 2\partial_{[p}\bar{\Gamma}^m{}_{q]n} + 2\bar{\Gamma}^m{}_{[p|r|}\bar{\Gamma}^r{}_{q]n},$$
$$\bar{R}^{\hat{a}}{}_{\hat{b}} = d\bar{\omega}^{\hat{a}}{}_{\hat{b}} + \bar{\omega}^{\hat{a}}{}_{\hat{c}} \wedge \bar{\omega}^{\hat{c}}{}_{\hat{b}}. \tag{C3}$$

The (intrinsic) torsion τ is defined by $\bar{\nabla}_{[n}\bar{\nabla}_{p]}f = -(1/2)\tau^m{}_{np}\bar{\nabla}_m f$ for a scalar f and is related to the contorsion by $2\kappa^m_{[np]} = \tau^m{}_{np}$. The torsion τ is totally antisymmetric and modifies the usual relations

$$[\bar{\nabla}_m, \bar{\nabla}_n]v^p = \bar{R}^p{}_{qmn}v^q - \tau^q{}_{mn}\bar{\nabla}_q v^p,$$
$$[\bar{\nabla}_m, \bar{\nabla}_n]\psi = \tfrac{1}{4}\bar{R}_{mnpq}\gamma^{pq}\psi \tag{C4}$$

for vectors and spinors. Also, the torsion gives a topological obstruction to finding special holonomy with the Levi-Cività connection, as reviewed in Sec. III C.

APPENDIX D: SUPERGRAVITY POTENTIAL NORMALIZATIONS

Here we will describe the normalization of the 4D $\mathcal{N} = 1$ supergravity variables in the effective field theory description. We roughly follow [17]. After dimensional reduction on the metric $g_{mn} = e^{2u}\tilde{g}_{mn}$, we find a 4D string (or Jordan) frame action

$$S = \frac{\tilde{V}_6}{(2\pi)^7\alpha'^4}\int d^4x\sqrt{-g}e^{6u-2\phi}R(g) + \cdots, \tag{D1}$$

where $\tilde{g}_{mn}$ has volume $\tilde{V}_6$, and we have used the correct string theory value for the 10D gravitational coupling. Rescaling to the Einstein frame $g_{\mu\nu} = e^{2\phi-6u}g_{E,\mu\nu}$, we find

$$S = \int d^4x\sqrt{-g_E}\left(\frac{m_p^2}{2}R(g_E) - V + \cdots\right),$$
$$\frac{m_p^2}{2} = \frac{\tilde{V}_6}{(2\pi)^7\alpha'^4}. \tag{D2}$$

We have now included the $\mathcal{N} = 1$ supergravity potential, written in terms of the superpotential and Kähler potential as

$$V = \frac{1}{m_p^2}e^{\mathcal{K}}(\mathcal{K}^{i\bar{j}}D_iWD_{\bar{j}}\bar{W} - 3|W|^2), \tag{D3}$$

where we can write $\Lambda_E = -V/m_p^2$ for the absolute Einstein-frame cosmological constant in an AdS_4 vacuum. Note that this sets our conventions for the cosmological constant, as well. Henceforth, we will take $\tilde{V}_6 = (2\pi\sqrt{\alpha'})^6$, as in the text, though the normalization can be generalized. In this Appendix, we are approximating the warp factor as trivial and the dilaton as constant over X^6.

We take the heterotic superpotential of [14,45,46] (generalizing that of [21]) to be normalized as

$$W = \frac{m_p^3}{\sqrt{4\pi}}\frac{1}{(2\pi\sqrt{\alpha'})^5}\int(H + ibdJ + c\Sigma) \wedge \tilde{\Omega}. \tag{D4}$$

The factors of $2\pi\sqrt{\alpha'}$ remove the dimensionality of the integral, so that only the 4D Planck scale enters the superpotential as a dimensional factor. The relative normalizations b, c of the torsion and condensate terms are addressed in Sec. IV C. As discussed in Sec. IV C, we use the Kähler potential

$$\begin{aligned}\mathcal{K} &= -3\ln(-i(T-\bar{T})) - \ln(-i(S-\bar{S})) \\ &\quad - \ln\left(\frac{i}{(2\pi\sqrt{\alpha'})^6}\int\tilde{\Omega}\wedge\bar{\tilde{\Omega}}\right) + \delta\mathcal{K},\end{aligned} \tag{D5}$$

where we have included a constant $\delta\mathcal{K}$ in order to fix the potential given a normalization of W. In terms of the 10D variables, the 4D moduli are $\mathrm{Im}T = e^{2u}$, $\mathrm{Im}S = e^{6u-2\phi}$ for the heterotic theory.[17] We are ignoring warping and also variation of the dilaton in the compact space.

So to fix $\delta\mathcal{K}$, we consider the tension of a BPS domain wall, which is given by the jump in superpotential over the wall, $T = 2e^{\mathcal{K}/2}|\Delta W|$. If we take a CY_3 compactification, an NS5 brane on a SLAG 3-cycle c is a BPS domain wall. Crossing the domain wall, the flux jumps one unit on the dual cycle, $\int_{\tilde{c}}\Delta H = 4\pi^2\alpha'$ according to the Dirac quan-

[17] The real parts of S and T are axions related to $B_{\mu\nu}$ and B_{mn}, respectively.

tization condition. Since c is calibrated, we find

$$|\Delta W| = \frac{m_p^3}{\sqrt{4\pi}} \frac{\tilde{V}_c}{(2\pi\sqrt{\alpha'})^3}, \tag{D6}$$

where $\tilde{V}_c$ is the volume of c with respect to $\tilde{g}_{mn}$. The domain wall tension is then

$$T = \frac{\tilde{V}_c}{\sqrt{8}} \frac{1}{(2\pi)^5 \alpha'^3} e^{\phi - 6u} e^{\delta\mathcal{K}/2}. \tag{D7}$$

Comparing to the Einstein-frame action of an NS5 brane wrapping c, we find $\delta\mathcal{K} = 3\ln 2$.

[1] S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002).
[2] A. Strominger, Nucl. Phys. **B274**, 253 (1986).
[3] B. de Wit, D. J. Smit, and N. D. Hari Dass, Nucl. Phys. **B283**, 165 (1987).
[4] C. M. Hull, Nucl. Phys. **B267**, 266 (1986).
[5] C. M. Hull, Phys. Lett. B **178**, 357 (1986).
[6] C. M. Hull, in *Proceedings of the Conference on Superunification and Extra Dimensions* (World Scientific, Philadelphia, PA, 1986), pp. 347–375.
[7] M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. **156B**, 55 (1985).
[8] J. P. Derendinger, L. E. Ibanez, and H. P. Nilles, Phys. Lett. **155B**, 65 (1985).
[9] S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003).
[10] S. P. de Alwis, Phys. Rev. D **68**, 126001 (2003).
[11] A. Buchel, Phys. Rev. D **69**, 106004 (2004).
[12] S. P. de Alwis, Phys. Lett. B **603**, 230 (2004).
[13] S. P. de Alwis, Phys. Lett. B **626**, 223 (2005).
[14] G. L. Cardoso, G. Curio, G. Dall'Agata, and D. Lust, Fortschr. Phys. **52**, 483 (2004).
[15] G. Curio, A. Krause, and D. Lust, hep-th/0502168.
[16] V. Balasubramanian, P. Berglund, J. Conlon, and F. Quevedo, J. High Energy Phys. 03 (2005) 007.
[17] F. Denef, M. R. Douglas, B. Florea, A. Grassi, and S. Kachru, hep-th/0503124.
[18] O. DeWolfe, A. Giryavets, S. Kachru, and W. Taylor, J. High Energy Phys. 07 (2005) 066.
[19] J. Polchinski and A. Strominger, Phys. Lett. B **388**, 736 (1996).
[20] T. R. Taylor and C. Vafa, Phys. Lett. B **474**, 130 (2000).
[21] S. Gukov, C. Vafa, and E. Witten, Nucl. Phys. **B584**, 69 (2000); **B608**, 477 (2001).
[22] A. R. Frey, hep-th/0308156.
[23] K. Dasgupta, G. Rajesh, and S. Sethi, J. High Energy Phys. 08 (1999) 023.
[24] B. R. Greene, K. Schalm, and G. Shiu, Nucl. Phys. **B584**, 480 (2000).
[25] K. Becker and K. Dasgupta, J. High Energy Phys. 11 (2002) 006.
[26] G. L. Cardoso *et al.*, Nucl. Phys. **B652**, 5 (2003).
[27] G. Dall'Agata and N. Prezas, Phys. Rev. D **69**, 066004 (2004).
[28] K. Behrndt and M. Cvetic, Phys. Rev. Lett. **95**, 021601 (2005).
[29] D. Lust and D. Tsimpis, J. High Energy Phys. 02 (2005) 027.
[30] T. House and E. Palti, Phys. Rev. D **72**, 026004 (2005).
[31] G. Curio and A. Krause, Nucl. Phys. **B602**, 172 (2001).
[32] J. P. Gauntlett, D. Martelli, S. Pakis, and D. Waldram, Commun. Math. Phys. **247**, 421 (2004).
[33] G. Curio and A. Krause, Nucl. Phys. **B693**, 195 (2004).
[34] K. Behrndt and C. Jeschek, Nucl. Phys. **B694**, 99 (2004).
[35] P. Kaste, R. Minasian, and A. Tomasiello, J. High Energy Phys. 07 (2003) 004.
[36] A. Lukas and P. M. Saffin, Phys. Rev. D **71**, 046005 (2005).
[37] K. Behrndt and C. Jeschek, hep-th/0406138.
[38] A. Franzen, P. Kaura, A. Misra, and R. Ray, hep-th/0506224.
[39] A. Misra, hep-th/0507147.
[40] K. Becker, M. Becker, K. Dasgupta, and P. S. Green, J. High Energy Phys. 04 (2003) 007.
[41] S. Kachru, M. B. Schulz, P. K. Tripathy, and S. P. Trivedi, J. High Energy Phys. 03 (2003) 061.
[42] S. Gurrieri, J. Louis, A. Micu, and D. Waldram, Nucl. Phys. **B654**, 61 (2003).
[43] S. Fidanza, R. Minasian, and A. Tomasiello, Commun. Math. Phys. **254**, 401 (2005).
[44] M. Graña, R. Minasian, M. Petrini, and A. Tomasiello, J. High Energy Phys. 08 (2004) 046.
[45] G. L. Cardoso, G. Curio, G. Dall'Agata, and D. Lust, J. High Energy Phys. 10 (2003) 004.
[46] K. Becker, M. Becker, K. Dasgupta, and S. Prokushkin, Nucl. Phys. **B666**, 144 (2003).
[47] S. Gurrieri, A. Lukas, and A. Micu, Phys. Rev. D **70**, 126009 (2004).
[48] A. Micu, Phys. Rev. D **70**, 126002 (2004).
[49] S. Gurrieri and A. Micu, Classical Quantum Gravity **20**, 2181 (2003).
[50] A. R. Frey and M. Graña, Phys. Rev. D **68**, 106002 (2003).
[51] A. R. Frey, J. High Energy Phys. 06 (2004) 027.
[52] K. Behrndt, M. Cvetic, and P. Gao, Nucl. Phys. **B721**, 287 (2005).
[53] I. Affleck, M. Dine, and N. Seiberg, Nucl. Phys. **B241**, 493 (1984).
[54] K. Becker, M. Becker, M. Haack, and J. Louis, J. High Energy Phys. 06 (2002) 060.
[55] F. Denef, M. R. Douglas, and B. Florea, J. High Energy Phys. 06 (2004) 034.
[56] C. P. Burgess, R. Kallosh, and F. Quevedo, J. High Energy Phys. 10 (2003) 056.
[57] R. Brustein and S. P. de Alwis, Phys. Rev. D **69**, 126006 (2004).

[58] S. Gukov, S. Kachru, X. Liu, and L. McAllister, Phys. Rev. D **69**, 086008 (2004).
[59] E. I. Buchbinder and B. A. Ovrut, Phys. Rev. D **69**, 086010 (2004).
[60] M. Becker, G. Curio, and A. Krause, Nucl. Phys. **B693**, 223 (2004).
[61] E. I. Buchbinder, Phys. Rev. D **70**, 066008 (2004).
[62] K. Becker, M. Becker, P. S. Green, K. Dasgupta, and E. Sharpe, Nucl. Phys. **B678**, 19 (2004).
[63] E. A. Bergshoeff and M. de Roo, Nucl. Phys. **B328**, 439 (1989).
[64] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, UK, 1998), Vol. 2, p. 531.
[65] J. P. Gauntlett, D. Martelli, and D. Waldram, Phys. Rev. D **69**, 086002 (2004).
[66] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).
[67] A. Lichnerowicz, Lett. Math. Phys. **13**, 331 (1987).
[68] A. Gray and L. Hervella, Ann. Math. Pura Appl. **123**, 35 (1980).
[69] S. Chiossi and S. Salamon, in *Procedings of the Conference on Differential Geometry, Valencia, 2001* (World Scientific, River Edge, NJ, 2002), pp. 115–133.
[70] S. G. Chiossi and A. Swann, math.dg/0404554.
[71] R. Rohm and E. Witten, Ann. Phys. (N.Y.) **170**, 454 (1986).
[72] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. **B212**, 413 (1983).
[73] K. Behrndt and M. Cvetic, Nucl. Phys. **B708**, 45 (2005).
[74] A. Gray, J. Diff. Geom. **4**, 283 (1970).
[75] A. Gray, Math. Z. **123**, 290 (1971).
[76] A. Gray, Math. Ann. **223**, 233 (1976).
[77] P.-A. Nagy, math.DG/0110065.
[78] R. L. Bryant, Ann. Math. **126**, 525 (1987).
[79] R. L. Bryant and S. Salamon, Duke Math. J. **58**, 829 (1989).
[80] C. Bar, Commun. Math. Phys. **154**, 509 (1993).
[81] A. Gray, J. Diff. Geom. **7**, 343 (1972).
[82] R. Grunewald, Ann. Global Anal. Geom. **8**, 43 (1990).
[83] M. Cvetic, G. W. Gibbons, H. Lu, and C. N. Pope, Classical Quantum Gravity **20**, 4239 (2003).
[84] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
[85] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000).
[86] I. R. Klebanov and M. J. Strassler, J. High Energy Phys. 08 (2000) 052.
[87] G. Curio and A. Krause, Nucl. Phys. **B643**, 131 (2002).
[88] P. Candelas and D. J. Raine, Nucl. Phys. **B248**, 415 (1984).
[89] M. Nakahara, *Geometry, Topology, and Physics* (Hilger, Bristol, UK, 1990), p. 505.
[90] S. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison-Wesley, San Francisco, 2004), p. 513.
[91] B. de Carlos, S. Gurrieri, A. Lukas, and A. Micu, hep-th/0507173.