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AdS_2 type-IIA solutions and scale separation

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Abstract: In this note we examine certain classes of solutions of IIA theory without sources, of the form $\mathrm{AdS}_2 \times \mathcal{M}^{(1)} \times \cdots \times \mathcal{M}^{(n)}$, where $\mathcal{M}^{(i)}$ are Riemannian spaces. We show that large hierarchies of curvatures can be obtained between the different factors, however the absolute value of the scalar curvature of AdS_2 must be of the same order or larger than the absolute values of the scalar curvatures of all the other factors.

Keywords: Black Holes in String Theory, D-branes, Flux compactifications, Superstring Vacua

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Contents

1 Introduction **1**

2 p-branes in $D = 10$ **3**

3 The brane configuration **4**
- 3.1 Near-horizon limit 5
- 3.2 Validity 6
- 3.3 Scale separation 7
- 3.4 Comparison with [13] 7

4 Generalizations **8**
- 4.1 M_6 nearly-Kähler with $m \neq 0$ 8
- 4.2 M_6 nearly-Kähler with $m = 0$ 10
- 4.3 M_6 Kähler Einstein 10
- 4.4 M_6 Ricci-flat 12
- 4.5 $M_{10} = M_2^{(1)} \times \cdots \times M_2^{(5)}$ 12
  - 4.5.1 Flux quantization 14

5 Discussion **16**

A IIA supergravity **17**

1 Introduction

The question of scale separation of AdS vacua in string theory [1–3] is the subject to a lot of recent discussion. In many of the previous works, backgrounds of the form $\mathrm{AdS}_d \times \mathcal{M}^{(p)}$ were considered, and special emphasis was given to the case with $d = 4$ and $p = 6$. Scale separation is possible whenever the radius L_d of AdS_d is parametrically larger than the inverse Kaluza-Klein (KK) mass scale $L_p \simeq \frac{1}{m_{\mathrm{KK}}}$ of $\mathcal{M}^{(p)}$, i.e. $L_d \gg L_p$. In the case where scale separation is possible, the theory possesses a limit in which the solution can be regarded as d-dimensional AdS_d space. On the other hand, if scale separation is not possible, i.e. if $L_d \simeq L_p$, the solutions are not really d-dimensional, and the gravitational background has to be considered as $(d + p)$-dimensional. Whether scale separation in supergravity backgrounds of the above form is possible or not has also profound consequences for the holographically dual CFT in $(d - 1)$ dimensions.

We do not want to review all arguments which were given in favor or against scale separation for $\mathrm{AdS}_d \times \mathcal{M}^{(p)}$ background spaces. Some general arguments against scale separation were given in [2]. On the other hand, one of the early papers addressing this issue is the work of DGKT [4], where it was argued that in the presence of orientifold planes scale separation is possible. This discussion was recently refined and extended in [5–10]. The question of scale separation was also recently addressed in the general context of the quantum gravity swampland discussion [11], namely as the AdS Distance Conjecture (ADC) [12]. This conjecture states that the limit of small AdS cosmological constant, $\Lambda \simeq \frac{1}{L_d^2} \to 0$, is at infinite distance in the space of AdS metrics, and that it is related to an infinite tower of states with typical masses that scale as,

$$\text{ADC}: \quad m \sim \Lambda^{\alpha}\,, \tag{1.1}$$

with $\alpha = \mathcal{O}(1)$. The strong version of the ADC proposes that for supersymmetric backgrounds $\alpha = \frac{1}{2}$, and that in this case scale separation is not possible, since $L_d \sim \frac{1}{m}$.

In this paper we will consider several AdS$_2$ solutions in string theory, where the total space is of the form AdS$_2 \times \mathcal{M}^{(1)} \times \cdots \times \mathcal{M}^{(n)}$. Assuming that the scale, or radius, is related to the scalar curvature, or cosmological constant, via $\Lambda \simeq \frac{1}{L^2}$, we will see that scale separation for these backgrounds will never be possible in the sense that for all the considered cases the radius L_2 of AdS$_2$ can never be much larger than at least one of the radii of the other factors. E.g. if $\mathcal{M}^{(1)}$ is a two-sphere S^2 of radius L_2', then $L_2 \leq L_2'$. However there are cases where the rest of the radii, e.g. the radius of a Ricci-flat space $M_{(6)}$, can be much smaller than L_2, L_2'. In particular, $L_2 = L_2' \gg L_6$ is possible. This means that there can be scale separation between AdS$_2 \times S^2$ and $M_{(6)}$, even within a regime of weak coupling and curvature where the supergravity approximation is valid. The reason this is possible is that in the Ricci-flat case the radius is no longer related to the inverse of the scalar curvature (which vanishes). Instead the radius becomes a free parameter of the solution, only constrained by flux quantization.

The case of AdS$_2 \times S^2$ is of special interest, since it corresponds to the near horizon geometry of four-dimensional extremal, supersymmetric black holes. The radii $L_2 = L_2'$ are directly related to the entropy $\mathcal{S}$ of the corresponding black hole solutions:

$$\mathcal{S} \sim L_2^2\,. \tag{1.2}$$

As it was recently discussed in [13], the limit of large entropy, $\mathcal{S} \to \infty$, is at infinite distance in the space of 4D black hole metrics. Therefore, similarly to the ADC, a black hole entropy conjecture (BHEC) was put forward in [13], stating that the large entropy limit of black holes is also accompanied by a tower of light modes. However these modes cannot be given in terms of the internal KK modes of $M_{(6)}$. This was already seen in [13] from the so-called attractor equations, since, as a function of the electric and magnetic black hole charges, $\mathcal{S}$ can be made large, while keeping the internal scale L_6 small. Here we will confirm this result by investigating the supergravity solutions of the corresponding intersecting D-branes, and reading off from the supergravity solutions the corresponding length scales. However, as we will discuss, there are other classes of AdS$_2$ supergravity solutions, where scale separation is only possible in the other direction: namely there are solutions where the AdS$_2$ space and some of the internal factors are more highly curved than the rest of the internal factors. We call this the "*wrong*" kind of scale separation.

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The paper is organized as follows. In the next section we will briefly review the background spaces of supergravity p-branes in ten spacetime dimensions. In section 3 we will then discuss the construction of supergravity solutions of intersecting D-branes, which lead to supersymmetric 4D black holes with $\mathrm{AdS}_2 \times S^2$ near-horizon geometry. We will see that scale separation $L_2 = L_2' \gg L_6$ is possible within the validity regime of the supergravity approximation. We also compare these results with those of [13]. In section 4, we discuss various generalizations of spaces M_6 and show that scale separation works in a different way than before. The case of M_6 Ricci-flat, discussed in section 4.4, includes the brane set up of section 3.1 as a special case. In section 4.5 the case of backgrounds of the form $M_{10} = M_2^{(1)} \times \cdots \times M_2^{(5)}$ is analyzed. In section 4.5.1 we discuss solutions of the form $\mathrm{AdS}_2{\times}\mathrm{S}^2{\times}\mathrm{S}^2{\times}\mathrm{S}^2{\times}\mathrm{T}^2$, $\mathrm{AdS}_2{\times}\mathrm{S}^2{\times}\mathrm{S}^2{\times}\mathrm{T}^4$, or $\mathrm{AdS}_2{\times}\mathrm{S}^2{\times}\mathrm{S}^2{\times}\mathrm{K3}$, obeying flux quantization within the validity regime of the supergravity approximation. We conclude with a discussion in section 5.

2 p-branes in $D = 10$

For a review of brane solutions see e.g. [14–16]. In ten dimensions, in the string frame, p-branes are solutions of the action $S = S_{\text{bulk}} + S_{\text{sources}}$ where,

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{|g_{10}|} \left(e^{-2\phi}(\mathcal{R} + 4|\mathrm{d}\phi|^2) - \frac{1}{2}|F_{p+2}|^2 \right) , \tag{2.1}$$

g_{10} is the determinant of the metric g_{MN}, $M, N = 0 \ldots 9$, $F_{p+2} = \mathrm{d}C_{p+1}$ is the abelian $(p+2)$-form field strength, and ϕ is the dilaton. The square of a q-form A_q is defined by $|A_q|^2 = A_{q\,M_1 \ldots M_q}\, g^{M_1 N_1} \ldots g^{M_q N_q} A_{q\,N_1 \ldots N_q}/q!$. Moreover,

$$S_{\text{sources}} = -T_p \int_{\Sigma_{p+1}} \mathrm{d}^{p+1}\xi \; e^{-\phi} \sqrt{|\imath^*[g_D]|} + \mu_p \int_{\Sigma_{p+1}} \imath^*[C_{p+1}] \,, \tag{2.2}$$

where Σ_{p+1} is the world-volume of the p-brane with coordinates ξ^i, $i = 0 \ldots p$, and $\imath^*[\cdot]$ the pull-back to Σ_{p+1}. The gravitational constant and the tension are given by,

$$2\kappa_{10}^2 = (2\pi)^7 (\alpha')^4 \,, \quad T_p^2 = \frac{\pi}{\kappa_{10}^2} (4\pi^2 \alpha')^{3-p} \,, \tag{2.3}$$

where $\alpha' = l_s^2$, with l_s the string length. For BPS sources as here, one has $\mu_p = T_p$. The p-brane solutions in the string frame are then given by,

$$\begin{aligned} \mathrm{d}s^2 &= H^{-\frac{1}{2}} \eta_{ij} \mathrm{d}x^i \mathrm{d}x^j + H^{\frac{1}{2}} \delta_{mn} \mathrm{d}y^m \mathrm{d}y^n \\ e^{\phi} &= e^{\phi_0}\, H^{-\frac{(p-3)}{4}} \;; \quad C_{p+1} = (H^{-1} - 1)\, e^{-\phi_0}\, \mathrm{vol}_{p+1} \,, \end{aligned} \tag{2.4}$$

with $x^{i=0\ldots p}$ the coordinates along the brane, $\mathrm{vol}_{p+1} = \mathrm{d}x^0 \wedge \ldots \wedge \mathrm{d}x^p$, and $y^{m=p+1\ldots 9}$ the coordinates of the space transverse to the brane. $H(\vec{y})$ is a harmonic function with localized source in the (unwarped) $\mathbb{R}^{9-p}$ space transverse to the p-brane,

$$\delta^{mn} \partial_m \partial_n H(\vec{y}) = Q\, \delta(\vec{y} - \vec{y}_0) \,, \tag{2.5}$$

where the brane is located at $\vec{y}_0$ in the transverse space. For D_p-branes we have,

$$Q_{D_p} = -2\kappa_{10}^2 T_p g_s = -(2\pi l_s)^{7-p} g_s \ . \tag{2.6}$$

The constant $e^{\phi_0} = g_s$ can be used to define the string coupling as the value of the dilaton at asymptotic infinity, where the harmonic functions tend to unity. However, once the near-horizon limit is taken (see section 3.1), the asymptotic region is no longer accessible. More generally one should think of g_s as a free parameter of the solution, related to the string coupling via (2.4).

3 The brane configuration

One can form superpositions of brane solutions according to the *harmonic superposition rule* [17, 18]. Consider the following system of intersecting D4/D0-branes:

	t	x^1	x^2	x^3	y^1	y^2	y^3	y^4	y^5	y^6
$D4_1$	$\otimes$				$\otimes$	$\otimes$	$\otimes$	$\otimes$		
$D4_2$	$\otimes$						$\otimes$	$\otimes$	$\otimes$	$\otimes$
$D4_3$	$\otimes$				$\otimes$	$\otimes$			$\otimes$	$\otimes$
D0	$\otimes$									

where $y^{m=1,\ldots,6}$ are assumed to parameterize a T^6, and $x^{i=1,2,3}$ are coordinates of $\mathbb{R}^3$. We use the notation $D4_\alpha$, $\alpha = 1, 2, 3$ to distinguish the three different types of D4-branes in the configuration of the table above. We shall assume there are n_0 D0-branes located at $\vec{x}_0 \in \mathbb{R}^3$, and n_α $D4_\alpha$-branes located at $\vec{x}_\alpha$, $\alpha = 1, 2, 3$.

The explicit form of the metric reads,

$$\begin{aligned} ds_{10}^2 = &-\left(\prod_{\alpha=0}^{3} H_\alpha\right)^{-\frac{1}{2}} dt^2 + \left(\prod_{\alpha=0}^{3} H_\alpha\right)^{\frac{1}{2}} \sum_{i=1}^{3}(dx^i)^2 + \sqrt{\frac{H_0 H_2}{H_1 H_3}}\left((dy^1)^2 + (dy^2)^2\right) \\ &+ \sqrt{\frac{H_0 H_3}{H_1 H_2}}\left((dy^3)^2 + (dy^4)^2\right) + \sqrt{\frac{H_0 H_1}{H_2 H_3}}\left((dy^5)^2 + (dy^6)^2\right) , \end{aligned} \tag{3.1}$$

where H_α, $\alpha = 1, 2, 3$, are the harmonic functions of the $D4_\alpha$-branes, and H_0 is the harmonic function of the D0-branes. We have,

$$H_\alpha = 1 + \frac{c_\alpha}{|\vec{x} - \vec{x}_\alpha|} \ ; \quad c_\alpha = \frac{N_\alpha g_s}{4\pi}(2\pi l_s)^{7-p} , \tag{3.2}$$

for $\alpha = 0, \ldots, 3$, and we took into account that (2.5) implies $c = -\frac{Q}{4\pi}$, for the case where the transverse space is $\mathbb{R}^3$. The N_α are proportional to the number of D-branes n_α. The precise relation will be derived below using flux quantization.[1] More explicitly,

$$c_0 = \frac{N_0 g_s}{4\pi}(2\pi l_s)^7 \ ; \quad c_\alpha = \frac{N_\alpha g_s}{4\pi}(2\pi l_s)^3 , \ \alpha = 1, 2, 3 \ . \tag{3.3}$$

[1]In the case of a single set of parallel D-branes, we would simply have $N_\alpha = n_\alpha$. However the brane solution (3.1) was obtained using harmonic superposition, which results in smearing the D-branes along the directions of the T^6. As a consequence, N_α, n_α are not equal to each other.

3.1 Near-horizon limit

We shall now assume that all branes are located at the origin: $\vec{x}_\alpha = 0$, $\alpha = 0, \dots, 3$. Let us define $r := \sqrt{\vec{x}^2}$. In the *near-horizon* limit $r \to 0$, (3.1) reads,

$$\begin{aligned}\frac{1}{C}\,\mathrm{d}s^2_{10} = &- r^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}\Omega^2 \\ &+ \frac{1}{c_1c_3}\left((\mathrm{d}y^1)^2 + (\mathrm{d}y^2)^2\right) + \frac{1}{c_1c_2}\left((\mathrm{d}y^3)^2 + (\mathrm{d}y^4)^2\right) + \frac{1}{c_2c_3}\left((\mathrm{d}y^5)^2 + (\mathrm{d}y^6)^2\right),\end{aligned} \tag{3.4}$$

where $\mathrm{d}\Omega^2$ is the line element of the unit two-sphere. Moreover we defined $C := \left(\prod_{\alpha=0}^3 c_\alpha\right)^{\frac{1}{2}}$, and rescaled the time coordinate: $t \to t/C$.

The fluxes read,

$$\begin{aligned}g_sF_2 &= \frac{C}{c_0}\mathrm{d}r\wedge\mathrm{d}t\\ g_sF_6 &= C\mathrm{d}r\wedge\mathrm{d}t\wedge\Big(\frac{1}{c_1}\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^3\wedge\mathrm{d}y^4\\ &\qquad + \frac{1}{c_2}\mathrm{d}y^3\wedge\mathrm{d}y^4\wedge\mathrm{d}y^5\wedge\mathrm{d}y^6 + \frac{1}{c_3}\mathrm{d}y^1\wedge\mathrm{d}y^2\wedge\mathrm{d}y^5\wedge\mathrm{d}y^6\Big)\\ g_s\star F_2 &= c_0\mathrm{d}\Omega_2\wedge\mathrm{d}y^1\wedge\cdots\wedge\mathrm{d}y^6\\ g_s\star F_6 &= \mathrm{d}\Omega_2\wedge\left(c_1\mathrm{d}y^5\wedge\mathrm{d}y^6 + c_2\mathrm{d}y^1\wedge\mathrm{d}y^2 + c_3\mathrm{d}y^3\wedge\mathrm{d}y^4\right),\end{aligned} \tag{3.5}$$

where $\mathrm{d}\Omega_2$ is the volume form of the unit 2-sphere, and we have taken into account that the time coordinate has been rescaled as indicated below (3.4). All fluxes, as well as their Hodge-duals, can readily be seen to be everywhere well-defined and closed, $\mathrm{d}F = \mathrm{d}\star F = 0$, indicating the absence of sources. In other words, the near-horizon limit is a pure gravity background, all branes having dissolved into fluxes in the limit.

We shall assume that the areas, in units of string length, of the three 2-tori Σ_i spanned by the internal coordinates are given by three moduli v_i,

$$v_1 = \frac{1}{l_s^2}\int_{\Sigma_1}\mathrm{d}y^1\mathrm{d}y^2\ ;\quad v_2 = \frac{1}{l_s^2}\int_{\Sigma_2}\mathrm{d}y^3\mathrm{d}y^4\ ;\quad v_3 = \frac{1}{l_s^2}\int_{\Sigma_3}\mathrm{d}y^5\mathrm{d}y^6\ . \tag{3.6}$$

The flux quantization conditions,

$$\begin{aligned}n_0 &= \frac{1}{(2\pi l_s)^7}\int_{S^2\times T^6}\star F_2\ ;\\ n_1 &= \frac{1}{(2\pi l_s)^3}\int_{S^2\times\Sigma_3}\star F_6\ ;\quad n_2 = \frac{1}{(2\pi l_s)^3}\int_{S^2\times\Sigma_1}\star F_6\ ;\quad n_3 = \frac{1}{(2\pi l_s)^3}\int_{S^2\times\Sigma_2}\star F_6\,,\end{aligned} \tag{3.7}$$

then relate N_α to the number of D-branes $n_\alpha \in \mathbb{N}$, which have dissolved into flux quanta in the near-horizon geometry,

$$N_0 = \frac{n_0}{l_s^6 v_1v_2v_3}\ ;\quad N_1 = \frac{n_1}{l_s^2v_3}\ ;\quad N_2 = \frac{n_2}{l_s^2v_1}\ ;\quad N_3 = \frac{n_3}{l_s^2v_2}\,, \tag{3.8}$$

where we have substituted (3.5) into (3.7), taking (3.3), (3.6) into account.

The geometry of (3.4) is $AdS_2 \times S^2 \times T^6$. The radii L_2, L_2' of AdS_2, S^2 respectively, can be seen to be equal to each other. The 4D part of the geometry, $AdS_2 \times S^2$, is thus characterized by a radius $L_4 := L_2 = L_2'$. The latter and the radius L_6 of T^6 can be read off of (3.4),

$$\begin{aligned} L_4 &= C^{\frac{1}{2}} = 4\pi^3 l_s g_s (n_0 n_1 n_2 n_3)^{\frac{1}{4}} (v_1 v_2 v_3)^{-\frac{1}{2}} \ ; \\ L_6 &= \frac{C^{\frac{1}{2}}}{(c_1 c_2 c_3)^{\frac{1}{3}}} l_s (v_1 v_2 v_3)^{\frac{1}{6}} = 2\pi l_s \left(\frac{n_0^3}{n_1 n_2 n_3} \right)^{\frac{1}{12}} \ . \end{aligned} \tag{3.9}$$

Unlike the individual values of the radii L_4, L_6, their ratio is frame-independent,

$$\frac{L_4}{L_6} = 2\pi^2 \, g_s (n_1 n_2 n_3)^{\frac{1}{3}} (v_1 v_2 v_3)^{-\frac{1}{2}} \ . \tag{3.10}$$

The dilaton is constant,

$$e^{\phi} = g_s (2\pi)^3 \left(\frac{n_0^3}{n_1 n_2 n_3} \right)^{\frac{1}{4}} (v_1 v_2 v_3)^{-\frac{1}{2}} \ . \tag{3.11}$$

3.2 Validity

The metric, fluxes and dilaton (3.4), (3.5), (3.11) give an exact bulk supergravity solution without sources. The solution is parameterized by the parameters g_s, n_a, v_i, which can be tuned independently.

Let us denote by T_i the effective areas (in string frame and in string units) of the three 2-tori in the near-horizon limit,

$$T_2 := \frac{C}{c_1 c_3} v_1 \ ; \quad T_3 := \frac{C}{c_1 c_2} v_2 \ ; \quad T_1 := \frac{C}{c_2 c_3} v_3 \ . \tag{3.12}$$

Taking (3.3), (3.8) into account, this is equivalent to,

$$T_i = 4\pi^2 n_i \sqrt{\frac{n_0}{n_1 n_2 n_3}} \ . \tag{3.13}$$

For the validity of the supergravity regime we must require,

$$T_i \gg 1 \ . \tag{3.14}$$

In addition we must require that the radius of curvature of the background is much larger than the string scale,

$$L_4, L_6 \gg l_s \, , \tag{3.15}$$

and that the string coupling obeys

$$e^{\phi} < 1 \ ; \quad e^{\phi} |F_p| < 1 \, , \tag{3.16}$$

in order for the higher-order flux corrections to be under control. In the second inequality above, the norm of the p-form flux is given by $|F_p|^2 := \frac{1}{p!} |F_{M_1 \cdots M_p} F_{N_1 \cdots N_p} g^{M_1 N_1} \ldots g^{M_p N_p}|$.

Conditions (3.14), (3.15), (3.16) are necessary and sufficient for the supergravity solution given in (3.4), (3.5), (3.11) to be within its regime of validity.

3.3 Scale separation

Let us first note that any rescaling of the v_i's can be cancelled by a corresponding rescaling in g_s. So in the following we can keep v_i fixed without loss of generality.

Suppose there is a solution parameterized by $\{g_s, n_a, v_i\}$. Let us moreover rescale,

$$n_0 \to p\, n_0 \ ; \quad n_{1,2,3} \to q\, n_{1,2,3} \ ; \quad g_s \to t \cdot q^{-1} g_s \,, \tag{3.17}$$

for some non-vanishing $p, q \in \mathbb{N}^*$, $t \in \mathbb{R}_+$. Under this rescaling we have,

$$\begin{aligned}
&L_4 \to t \left(\frac{p}{q}\right)^{\frac{1}{4}} L_4 \ ; && L_6 \to \left(\frac{p}{q}\right)^{\frac{1}{4}} L_6 \,, \\
&\frac{L_4}{L_6} \to t\, \frac{L_4}{L_6} \ ; && T_i \to \left(\frac{p}{q}\right)^{\frac{1}{2}} T_i \\
&e^{\phi} \to t\, q^{-1} \left(\frac{p}{q}\right)^{\frac{3}{4}} e^{\phi} \ ; && e^{\phi}|F_2| \to t^{-1} \left(\frac{p}{q}\right)^{-\frac{1}{4}} e^{\phi}|F_2| \,, \qquad e^{\phi}|F_6| \to t^{-1} \left(\frac{p}{q}\right)^{-\frac{1}{4}} e^{\phi}|F_6| \,,
\end{aligned} \tag{3.18}$$

where $i = 1, 2, 3$. Scale separation ($L_4 \gg L_6$) is thus equivalent to taking $t \gg 1$.

If in addition we want to respect conditions (3.14), (3.15), we also must take $p \gg q$. Then the second of the two inequalities in (3.16) is automatically satisfied. To satisfy the first inequality in (3.16), it suffices to take $p = q^2 \gg 1$, and $t = q^r$, with $0 < r < \frac{1}{4}$. We are then guaranteed to be within the validity regime of the supergravity approximation.

3.4 Comparison with [13]

In order to compare with [13] let us first redefine the constant $g_s \to g_s (v_1 v_2 v_3)^{-\frac{1}{2}}$, and also set $l_s = 1$, so that the formulae of section 3.1 become,

$$\frac{L_4}{L_6} = 2\pi^2\, g_s (n_1 n_2 n_3)^{\frac{1}{3}} \ ; \quad e^{\phi} = g_s (2\pi)^3 \left(\frac{n_0^3}{n_1 n_2 n_3}\right)^{\frac{1}{4}} , \tag{3.19}$$

and the 4d dilaton reads,

$$e^{\phi_4} = \frac{e^{\phi}}{\sqrt{V}} = g_s \,, \tag{3.20}$$

where $V := L_6^6$.

We see that the dependence on the 2-tori areas v_i disappears, having been absorbed in the independent constant g_s. This is consistent with the attractor mechanism according to which the near-horizon geometry is fixed by the charges, and in particular is independent of the values of the Kähler moduli at asymptotic infinity. The latter correspond to the areas of the 2-tori, v_i. On the other hand, the values of the Kähler moduli at the horizon are given in (3.13), and correspond to the effective values of the areas of the 2-tori at the horizon. Indeed the T_i's here are essentially the same as defined in eq. (73) of [13].

The ratio L_4/L_6 here, cf. (3.19), corresponds to $m_{\mathrm{KK}}(S_{\mathcal{N}=2})^{1/2}$ of [13], up to a numerical factor of order one. We find agreement with [13], cf. eq. (79) therein, provided we include the string coupling constant g_s there.

4 Generalizations

We will look for solutions of (massive) IIA supergravity the form $M_2 \times M_2' \times M_6$, where M_2 is a two-dimensional maximally-symmetric Lorentzian manifold (i.e. $\mathbb{R}^{1,1}$, dS_2, AdS_2), M_2' is a two-dimensional maximally-symmetric Riemannian manifold (i.e. $\mathbb{R}^2$, S^2, H^2) or discrete quotients thereof, and M_6 is a six-dimensional nearly-Kähler (NK), Calabi-Yau (CY), or Einstein-Kähler manifold. In section 4.5 we will also consider the case where M_6 is a product of two-dimensional Riemannian manifolds.

4.1 M_6 nearly-Kähler with $m \neq 0$

The ansatz of the present section can be obtained from the consistent truncation of [19] §5 therein, by setting the scalars to constants and taking the one-forms therein to obey $\gamma = \chi\alpha$. We are following the conventions of that reference. The ten-dimensional metric reads,

$$\mathrm{d}s^2_{(10)} = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + g_{ij}\mathrm{d}x^i \mathrm{d}x^j + g_{mn}\mathrm{d}y^m \mathrm{d}y^n \,, \tag{4.1}$$

where $\{x^\mu,\ \mu = 0,1\}$ are coordinates on M_2, $\{x^i,\ i = 2,3\}$ are coordinates on M_2' and $\{y^m,\ m = 1,\ldots,6\}$ are coordinates on M_6. The respective Ricci tensors are given by,

$$R_{\mu\nu} = \Lambda_1 g_{\mu\nu} \ ; \quad R_{ij} = \Lambda_2 g_{ij} \ ; \quad R_{mn} = \Lambda g_{mn} \,, \tag{4.2}$$

where the signs of $\Lambda_{1,2}$ are unconstrained at this point. The NK manifold M_6 possesses a real two form J and a complex three-form ω obeying,

$$\begin{aligned} \mathrm{d}J &= -6\omega \mathrm{Re}\Omega \\ \mathrm{dIm}\Omega &= 4\omega J \wedge J \ . \end{aligned} \tag{4.3}$$

With these conventions, the constant $\omega \in \mathbb{R}$ above is related to Λ in (4.2) via,

$$\Lambda = 20\omega^2 \ . \tag{4.4}$$

We will assume that the dilaton is constant. Moreover, our ansatz for the forms reads,

$$\begin{aligned} F &= \beta + m\chi J \ ; \quad H = -6\omega\chi \mathrm{Re}\Omega \\ G &= \varphi \mathrm{vol}_2 \wedge \mathrm{vol}_2' + \frac{1}{2}\xi J \wedge J + \chi J \wedge \beta \,, \end{aligned} \tag{4.5}$$

where χ, φ, ξ are real constants, and vol_2, vol_2' are the volume forms of M_2, M_2' respectively.[2] The two-form β is given by,

$$\beta = -\left(f \mathrm{vol}_2 + f' \mathrm{vol}_2'\right) \,, \tag{4.6}$$

where f, f' are real constants. It can then be seen that (4.5) automatically obeys the Bianchi identities (A.6).

[2] Our conventions for the volume form in D dimensions are: $\mathrm{vol}_D = \frac{1}{D!}\varepsilon_{M_1 \ldots M_D}\mathrm{d}x^{M_1} \ldots \mathrm{d}x^{M_D}$. In the case of Lorentzian signature we assume $\varepsilon_{0\ldots D-1} = +1$.

Plugging the above ansatz into the ten-dimensional equations of motion we obtain the following: the internal (m,n)-components of the Einstein equations read,

$$\Lambda = \frac{1}{16}(1+5\chi^2)\,m^2 + \frac{1}{16}(1+\chi^2)(f^2-f'^2) + 18\omega^2\chi^2 + \frac{3}{16}\varphi^2 + \frac{7}{16}\xi^2\ . \tag{4.7}$$

The (μ,ν)-components read,

$$\Lambda_1 = -\frac{1}{2}(1+3\chi^2)\,f^2 + \frac{1}{16}\Big[(1+9\chi^2)(f^2-f'^2) + (1-3\chi^2)\,m^2 - 5\varphi^2 - 288\omega^2\chi^2 - 9\xi^2\Big]\ . \tag{4.8}$$

The (i,j)-components read,

$$\Lambda_2 = \Lambda_1 + \frac{1}{2}(1+3\chi^2)(f^2+f'^2)\,, \tag{4.9}$$

where we have taken (4.8) into account. All the mixed (μ,m), (i,m), (μ,i) components are automatically satisfied.

The dilaton equation reads,

$$0 = 3(1+\chi^2)(f^2-f'^2) - (5+9\chi^2)\,m^2 + 288\omega^2\chi^2 + \varphi^2 - 3\xi^2\ . \tag{4.10}$$

The F-form equation of motion is automatically satisfied. The H-form equation reduces to the following three equations,

$$0 = \xi\varphi - 48\omega^2\chi - 2m\xi\chi - m^2\chi + 2ff'\chi^2 + (f^2-f'^2)\,\chi\,, \tag{4.11}$$

and,

$$\begin{aligned} 0 &= -3f\xi\chi - f\varphi + mf' + 3m\chi^2 f' \\ 0 &= 3f'\xi\chi + f'\varphi + mf + 3m\chi^2 f\ . \end{aligned} \tag{4.12}$$

The G-form equation of motion reduces to,

$$\omega(\xi - \chi\varphi) = 0\ . \tag{4.13}$$

For ω, $m \neq 0$ this system of equations implies $\xi = \chi\varphi$ and $f = f' = 0$, from which we see in particular that $\Lambda_1 = \Lambda_2$, so that no scale separation is possible. There are three classes of solutions, as given in [20] §11.4 therein.[3] Explicitly we have:

- *First class*:

$$\Lambda_1 = \Lambda_2 = -\frac{3}{2}m^2\ ; \quad \Lambda = m^2\ ; \quad \varphi^2 = 5m^2\ ; \quad \chi = 0\ . \tag{4.14}$$

[3]The solutions of [20] are of the form $\mathrm{AdS}_4 \times M_6$. However, this gives rise to the exact same equations of motion as in the present case: a space of the form $\mathrm{AdS}_2\times\mathrm{H}^2$ subject to $\Lambda_1 = \Lambda_2$. Moreover we may replace H^2 by a discrete quotient thereof, i.e. a Riemann surface Σ_g of genus $g > 1$. For a given curvature, the minimum volume is attained for $g = 2$, cf. (4.43) below.

- *Second class*:

$$\Lambda_1 = \Lambda_2 = -2m^2 \ ; \quad \Lambda = \frac{5}{3}m^2 \ ; \quad \varphi^2 = 3m^2 \ ; \quad \chi^2 = \frac{1}{3} \ . \tag{4.15}$$

- *Third class*:

$$\Lambda_1 = \Lambda_2 = -\frac{48}{25}m^2 \ ; \quad \Lambda = \frac{4}{3}m^2 \ ; \quad \varphi^2 = \frac{27}{5}m^2 \ ; \quad \chi^2 = \frac{1}{15} \ . \tag{4.16}$$

We only expect the third class, given in (4.16) above, to be supersymmetric: it can be obtained from the solutions of [21], which are special cases of [22], by replacing AdS_4 by an $\mathrm{AdS}_2{\times}\mathrm{H}^2$ (or $\mathrm{AdS}_2 \times \Sigma_g$) space subject to $\Lambda_1 = \Lambda_2$, cf. footnote 3. A similar substitution of AdS_4 by an $\mathrm{AdS}_2{\times}\mathrm{H}^2$ (or $\mathrm{AdS}_2 \times \Sigma_g$) space can also be performed for all known AdS_4 solutions. The converse is not true, however, as the $\mathrm{AdS}_2{\times}\mathrm{H}^2$ space allows for more general fluxes, which would otherwise break the symmetries of AdS_4. As already mentioned, scale separation is not possible in any of the three classes of solutions above, since all curvatures are of the same order.

4.2 M_6 nearly-Kähler with $m = 0$

The ansatz for the forms reads,

$$\begin{aligned} &F = \beta \ ; \quad H = -6\omega\chi \mathrm{Re}\Omega \\ &G = \varphi \mathrm{vol}_2{\wedge}\mathrm{vol}'_2 + \frac{1}{2}\xi J{\wedge}J + \chi J{\wedge}\beta \, , \end{aligned} \tag{4.17}$$

with χ, φ, ξ, β as in section 4.1, and satisfies the Bianchi identities (A.6) for $m = 0$. The equations of motion are obtained from (4.7)–(4.13) by setting $m = 0$ therein.

In this case the equations of motion can be solved to give a one-parameter solution of the form $\mathrm{AdS}_2{\times}\mathrm{S}^2$, without scale separation,

$$\begin{aligned} &-\Lambda_1 = \Lambda_2 = \frac{3}{2}(f^2 + f'^2) \ ; \quad \Lambda = \frac{1}{3}(f'^2 - f^2) \\ &\varphi = \xi = 0 \ ; \quad \chi = \pm\sqrt{\frac{5}{3}} \ ; \quad f = \pm\frac{1}{27}(4\sqrt{69} - 5\sqrt{15})f' \, , \end{aligned} \tag{4.18}$$

where the sign of χ is correlated with the sign of f/f'. Note that $|f'/f| < 1$ as it should, since $\Lambda > 0$ for a nearly-Kähler manifold, cf. (4.4). Scale separation is again not possible.

4.3 M_6 Kähler Einstein

The manifold M_6 is now assumed to be Kähler-Einstein with Kähler form J, $\mathrm{d}J = 0$. The form ansatz reads,

$$\begin{aligned} &F = \beta + \chi J \ ; \quad H = 0 \\ &G = \varphi \mathrm{vol}_2{\wedge}\mathrm{vol}'_2 + \frac{1}{2}\xi J{\wedge}J + J{\wedge}\gamma \, , \end{aligned} \tag{4.19}$$

with χ, φ, ξ, β as before, while the two-form γ is given by,

$$\gamma = -\left(g \mathrm{vol}_2 + g' \mathrm{vol}'_2\right) \ , \tag{4.20}$$

where g, g' are real constants. It can then be seen that (4.19) automatically obeys the Bianchi identities (A.6).

Plugging the above ansatz into the ten-dimensional equations of motion we obtain the following: the internal (m,n)-components of the Einstein equations read,

$$\Lambda = \frac{1}{16}m^2 + \frac{5}{16}\chi^2 + \frac{1}{16}(f^2 - f'^2) + \frac{1}{16}(g^2 - g'^2) + \frac{3}{16}\varphi^2 + \frac{7}{16}\xi^2 \ . \tag{4.21}$$

The (μ,ν)-components read,

$$\Lambda_1 = -\frac{7}{16}f^2 - \frac{3}{2}g^2 + \frac{1}{16}\Big[-f'^2 + 9(g^2 - g'^2) + m^2 - 3\chi^2 - 5\varphi^2 - 9\xi^2 \Big] \ . \tag{4.22}$$

The (i,j)-components read,

$$\Lambda_2 = \Lambda_1 + \frac{1}{2}(f^2 + f'^2) + \frac{3}{2}(g^2 + g'^2) \, , \tag{4.23}$$

where we have taken (4.22) into account. All the mixed (μ,m), (i,m), (μ,i) components are automatically satisfied.

The dilaton equation reads,

$$0 = 3(f^2 - f'^2) + 3(g^2 - g'^2) - 5m^2 - 9\chi^2 + \varphi^2 - 3\xi^2 \ . \tag{4.24}$$

Both the F-form and G-form equation of motion is automatically satisfied. The H-form equation reduces to the following three equations,

$$0 = \xi\varphi - 2\xi\chi - m\chi + 2gg' + (fg - f'g') \, , \tag{4.25}$$

and,

$$\begin{aligned} 0 &= mf + f'\varphi + 3\chi g + 3g'\xi \\ 0 &= mf' - f\varphi + 3\chi g' - 3g\xi \ . \end{aligned} \tag{4.26}$$

One way to solve this system of equations would be to view (4.25), (4.26) as a linear system of three equations for three unknowns f, f', χ.[4] The solution can then be substituted into (4.24) to obtain one constraint on the remaining unknowns: g, g', m, φ, ξ. Equations (4.21)–(4.23) then simply determine the curvatures Λ_1, Λ_2, Λ.

Let us now examine whether we can obtain a hierarchy between the curvature scales. From (4.21)–(4.24) we obtain,

$$\begin{aligned} \Lambda &= -\frac{1}{3}(\Lambda_1 + \Lambda_2) \\ &= \frac{1}{6}(\varphi^2 + m^2) + \frac{1}{2}(\chi^2 + \xi^2) \ . \end{aligned} \tag{4.27}$$

Moreover from (4.23), (4.27) it follows that $\Lambda_1 \pm \Lambda_2 \leq 0$, so that,

$$\Lambda_1 \leq 0 \ ; \quad |\Lambda_1| \geq |\Lambda_2| \ . \tag{4.28}$$

If Λ_1, $\Lambda_2 \neq 0$, this then implies that the external space M_2 is at least as highly curved as the internal space M'_2.

[4] The cases for which the system (4.25), (4.26) does not admit solutions for f, f', χ are special and must be considered separately.

JHEP07(2020)060

- If $\Lambda_1 = 0$ then, as we can see immediately from (4.23), (4.27), also Λ_2, Λ vanish as well as all flux, and the solution reduces to empty $\mathbb{R}^{1,3} \times M_6$ space, with M_6 Ricci-flat.

- If $\Lambda_2 = 0$ then from (4.27) it follows that $|\Lambda_1| = 3\Lambda$. Hence the curvatures of the external space M_2 and the internal space M_6 are of the same order.

- The equations of motion can easily be solved (e.g. numerically) for $|\Lambda_1| \approx 3\Lambda \gg |\Lambda_2|$, so that the spaces M_2, M_6 are much more highly curved than M_2'. Of course this is the "wrong" kind of scale separation.

4.4 M_6 Ricci-flat

As a special case of section 4.3 we can impose that M_6 is Ricci-flat, $\Lambda = 0$, so that (4.27) implies,

$$m = \varphi = \chi = \xi = 0 \ . \tag{4.29}$$

The remaining equations of motion can then be solved to give a two-parameter solution without scale separation,

$$-\Lambda_1 = \Lambda_2 = \frac{1}{4}(f^2 + f'^2) + \frac{3}{4}(g^2 + g'^2) \, , \tag{4.30}$$

where the constants f, f', g, g' are constrained to obey,

$$f^2 - f'^2 + g^2 - g'^2 = 0 \ ; \quad fg - f'g' + 2gg' = 0 \ . \tag{4.31}$$

The solution corresponding to the brane configuration of section 3 is a special (supersymmetric) case of the above solution: as we can see by comparing (3.5) with (4.19), taking (4.6), (4.20), (4.29) into account, it corresponds to setting $f' = g = 0$. Indeed this is a solution of (4.31) above, provided $f = \pm g'$.

In this case, to make contact with section 3, we can associate to the curvatures Λ_1, Λ_2, radii L_2, L_2' via: $|\Lambda_1| = \frac{1}{L_2^2}$, $\Lambda_2 = \frac{1}{L_2'^2}$, and $L_4 := L_2 = L_2'$. Moreover, in the case where M_6 is Ricci-flat, the radius L_6 is a free parameter (only subject to flux quantization) and does not enter the equations of motion (4.29)–(4.31).

4.5 $M_{10} = M_2^{(1)} \times \cdots \times M_2^{(5)}$

Let us now consider ten-dimensional spacetimes of the form $M_2^{(1)} \times \cdots \times M_2^{(5)}$, where $M_2^{(1)}$ is a two-dimensional maximally-symmetric space of Lorentzian signature whereas $M_2^{(i)}$, for $i = 2, \ldots, 5$, are two-dimensional maximally-symmetric spaces of Euclidean signature, or discrete quotients thereof. The solutions we present here generalize those in [23, 24], in which the choice of flux is not the most general.

Let us set,

$$R_{\mu\nu} = \Lambda_1 g_{\mu\nu} \ ; \quad R_{mn(i)} = \Lambda_i g_{mn(i)} \, , \tag{4.32}$$

where we have denoted by $R_{mn(i)}$, $g_{mn(i)}$ the Ricci tensor, resp. the metric components along $M_2^{(i)}$, $i = 2, \ldots, 5$. The form ansatz will be taken to be,

$$\begin{aligned} F &= -\sum_{i=1}^{5} f_{(i)} \text{vol}^{(i)} \ ; \quad H = 0 \\ G &= \frac{1}{2} \sum_{i,j=1}^{5} g_{(ij)} \text{vol}^{(i)} \wedge \text{vol}^{(j)} \ , \end{aligned} \tag{4.33}$$

where we have denoted by $\text{vol}^{(i)}$ the volume element of $M_2^{(i)}$, and $f_{(i)}$, $g_{(ij)}$ are constants obeying: $g_{(ij)} = g_{(ji)}$, $g_{(ii)} = 0$. This gives,

$$\begin{aligned} & F^2_{mn(i)} = f^2_{(i)} g_{mn(i)} \ ; \quad F^2_{\mu\nu} = -f^2_{(1)} g_{\mu\nu} \ ; \quad F^2 = 2\Big(- f^2_{(1)} + \sum_{i \neq 1} f^2_{(i)} \Big) \\ & G^2_{mn(i)} = 6\Big(- g^2_{(i1)} + \sum_{j \neq 1} g^2_{(ij)} \Big) g_{mn(i)} \ ; \quad G^2_{\mu\nu} = -6 g_{\mu\nu} \sum_i g^2_{(i1)} \\ & \quad G^2 = 24\Big(- \sum_i g^2_{(1i)} + \sum_{1 \neq i < j} g^2_{(ij)} \Big) \ , \end{aligned} \tag{4.34}$$

It is also useful to list the Hodge duals,

$$\begin{aligned} \star F &= f_{(1)} \widehat{\text{vol}}^{(1)} - \sum_{i \neq 1} f_{(i)} \widehat{\text{vol}}^{(i)} \\ \star G &= - \sum_i g_{(1i)} \widehat{\text{vol}}^{(1i)} + \sum_{1 \neq i < j} g_{(ij)} \widehat{\text{vol}}^{(ij)} \ , \end{aligned} \tag{4.35}$$

where we have denoted $\widehat{\text{vol}}^{(i)} := \frac{\text{vol}_{10}}{\text{vol}^{(i)}}$, $\widehat{\text{vol}}^{(ij)} := \frac{\text{vol}_{10}}{\text{vol}^{(i)} \wedge \text{vol}^{(j)}}$, $\text{vol}_{10} := \text{vol}^{(1)} \wedge \ldots \wedge \text{vol}^{(5)}$.

The equations of motion are as follows: the Einstein equations reduce to,

$$\begin{aligned} \Lambda_1 &= \frac{1}{16} m^2 - \frac{7}{16} f^2_{(1)} - \frac{1}{16} \sum_{i \neq 1} f^2_{(i)} - \frac{1}{2} \sum_i g^2_{(1i)} - \frac{3}{16} \Big(\sum_{1 \neq i < j} g^2_{(ij)} - \sum_i g^2_{(1i)} \Big) \\ \Lambda_i &= \Lambda_{(1)} + \frac{1}{2} \big(f^2_{(1)} + f^2_{(i)} + \sum_{j \neq 1} g^2_{(ij)} + \sum_{j \neq i} g^2_{(1j)} \big) \ ; \quad i = 2, \ldots, 5 \ . \end{aligned} \tag{4.36}$$

The dilaton equation reads,

$$0 = 3\Big(- f^2_{(1)} + \sum_{i \neq 1} f^2_{(i)} \Big) - \sum_i g^2_{(1i)} + \sum_{1 \neq i < j} g^2_{(ij)} + 5m^2 \ . \tag{4.37}$$

Equivalently, the modified dilaton equation reads,

$$\sum_i \Lambda_i = 0 \ . \tag{4.38}$$

The equations of motion for the RR forms F, G are automatically satisfied. The H-form equation of motion reads,

$$
\begin{aligned}
0 &= mf_{(1)} + \sum_{i\neq 1} g_{(1i)}f_{(i)} - \big(g_{(23)}g_{(45)} + g_{(24)}g_{(35)} + g_{(25)}g_{(34)}\big)\\
0 &= mf_{(i)} + \sum_{p\neq 1} g_{(ip)}f_{(p)} - g_{(1i)}f_{(1)} + \big(g_{(1j)}g_{(kl)} + g_{(1k)}g_{(lj)} + g_{(1l)}g_{(jk)}\big) \; ; \quad i = 2,\dots,5\,,
\end{aligned}
\tag{4.39}
$$

where in the second equation above it is assumed that $j < k < l$ and $j, k, l \neq 1, i$.

This system of equations can be solved in a similar fashion as that of section 4.3:[5] in general we can solve the linear system of five equations (4.39) for the five unknowns $f_{(i)}$. The solution can then be substituted into (4.37) to obtain one constraint on the remaining unknowns $g_{(ij)}$, m. Equations (4.36) then simply determine the curvatures $\Lambda_{(i)}$, while (4.38) is automatically satisfied.

It can easily be seen that the system of equations (4.36)–(4.39) admits solutions such that the curvatures Λ_i are not necessarily equal. However, similarly to the case of section 4.3, it is impossible to achieve $|\Lambda_1| < |\Lambda_i|$, for $i = 2,\dots,5$. This can be seen as follows: equations (4.36) can be solved for Λ_1, Λ_i, in terms of the fluxes. Then using the dilaton equation (4.37) we find that $-\Lambda_1$ and $-\Lambda_1 \pm \Lambda_i$, $i = 2,\dots,5$, can all be expressed as sums of squares, so that,

$$
\Lambda_1 \leq 0 \; ; \quad |\Lambda_1| \geq |\Lambda_i|\,,
\tag{4.40}
$$

for all $i = 2,\dots,5$. Therefore, assuming the Λ_i are not all vanishing, we conclude that the AdS_2 radius is bounded above by at least one of the radii of the internal factors. If we set $\Lambda_i = 0$ for $i = 2,\dots,5$, so that the internal space is flat and the radii of the internal factors become free parameters, then (4.38) would also imply $\Lambda_1 = 0$, so that all the fluxes vanish and we obtain a solution with flat ten-dimensional spacetime.

4.5.1 Flux quantization

For the supergravity solutions to be promoted to full-fledged solutions of the quantum theory, flux quantization must be imposed. For simplicity, let us set the $2\pi l_s = 1$ in the following. For $i, j = 2,\dots,5$, flux quantization constrains the constants $f_{(i)}$, $g_{(ij)}$ in (4.33) to obey,

$$
f_{(i)} = \frac{n_i}{V_i} \; ; \quad g_{(ij)} = \frac{n_{ij}}{V_iV_j}\,,
\tag{4.41}
$$

where $n_i, n_{ij} \in \mathbb{Z}$ and $V_i := \int_{M_2^{(i)}} \mathrm{vol}^{(i)}$ is the volume of $M_2^{(i)}$. The constants $f_{(1)}$, $g_{(1i)}$ are constrained to obey,

$$
f_{(1)} = \frac{n_1}{V_2\dots V_5} \; ; \quad g_{(1i)} = \frac{n_{1i}V_i}{V_2\dots V_5}\,,
\tag{4.42}
$$

where $n_1, n_{1i} \in \mathbb{Z}$. Moreover, the Romans mass is constrained to be an integer, $m \in \mathbb{Z}$.

[5] Note that the equations of motion of section 4.3 can be recovered from (4.36)–(4.39) by setting,

$$
\begin{gathered}
\Lambda_3 = \Lambda_4 = \Lambda_5 = \Lambda \; ; \quad f_{(1)} = -f \; ; \quad f_{(2)} = -f' \; ; \quad f_{(3)} = f_{(4)} = f_{(5)} = -\chi \; ; \quad g_{(12)} = \varphi\\
g_{(13)} = g_{(14)} = g_{(15)} = g \; ; \quad g_{(23)} = g_{(24)} = g_{(25)} = g' \; ; \quad g_{(34)} = g_{(35)} = g_{(45)} = \xi\,.
\end{gathered}
$$

Let us also note that for the two-sphere S^2, or a discrete quotient of the hyperbolic space H^2, the volume is related to the scalar curvature by the Gauss-Bonnet theorem. Indeed, a (compact) Riemann surface Σ_g of genus $g > 1$ can be obtained as a discrete quotient of the two-dimensional (non-compact) hyperbolic space H^2, $\Sigma_g = H^2/\Gamma$, where Γ is a discrete subgroup of $SO(1,2)$. Let the Ricci tensor of Σ_g be given by $R_{mn} = \Lambda g_{mn}$. The Gauss-Bonnet theorem then implies,

$$|\Lambda| = 4\pi(g-1)V^{-1}\,, \tag{4.43}$$

where $V = \int_{\Sigma_g}\mathrm{vol}$ is the volume of Σ_g. For the two-sphere S^2 the corresponding relation reads,

$$\Lambda = 4\pi V^{-1}\ . \tag{4.44}$$

We have not been able to find a solution with general flux to the system of equations of motion subject to (4.41), (4.42). However, as we now show, special solutions are possible for $m = 0$, $g_{(ij)} = 0$, i.e. for vanishing Romans mass and four-form flux. In this case the dilaton equation, (4.37) reads,

$$f_{(1)}^2 = \sum_{i\neq 1} f_{(i)}^2\ . \tag{4.45}$$

Taking this into account, the Einstein equations (4.36) read,

$$\Lambda_1 = -\frac{1}{2}f_{(1)}^2\ ;\quad \Lambda_i = \frac{1}{2}f_{(i)}^2\,, \tag{4.46}$$

for $i = 2,\ldots,5$, while the remaining equations of motion are automatically satisfied.

We have already discussed solutions, obeying flux quantization, of the form $AdS_2\times S^2\times T^6$. Let us instead suppose that the curvatures of the internal manifolds are all strictly positive, $\Lambda_i > 0$, for $i = 2,\ldots,5$. Flux quantization, (4.41), (4.42), taking (4.44) into account, implies,

$$f_{(1)} = \frac{n_1}{(4\pi)^4}\Lambda_2\ldots\Lambda_5\ ;\quad f_{(i)} = \frac{n_i}{4\pi}\Lambda_i\,, \tag{4.47}$$

for $i = 2,\ldots,5$. Then (4.46) solves for the curvatures in terms of the quanta,

$$\Lambda_1 = -\frac{(8\pi)^8 n_1^2}{2n_2^4\ldots n_5^4}\ ;\quad \Lambda_i = \frac{2(4\pi)^2}{n_i^2}\,, \tag{4.48}$$

while the dilaton equation imposes,

$$(8\pi)^6 n_1^2 = n_2^4\ldots n_5^4\sum_{i=2}^{5}\frac{1}{n_i^2}\ . \tag{4.49}$$

This equation clearly does not admit any solutions for integer n_i. Therefore solutions of the form $AdS_2\times S^2\cdots\times S^2$, while admissible in supergravity, are excluded in the quantum theory.

Let us now consider the case where the curvatures of the internal manifolds are strictly positive, $\Lambda_i > 0$, for $i = 2,\ldots,4$, while $\Lambda_5 = 0$. Unlike the previous case, now the volume

V_5 of $M_2^{(5)}$=T^2 is not related to its curvature, and thus does not enter the equations of motion. Flux quantization now implies,

$$f_{(1)} = \frac{n_1}{(4\pi)^3}\frac{\Lambda_2\Lambda_3\Lambda_4}{V_5} \ ; \quad f_{(i)} = \frac{n_i}{4\pi}\Lambda_i \ ; \quad f_{(5)} = \frac{n_5}{V_5} \,, \tag{4.50}$$

where $i = 2, 3, 4$. Then (4.46) solves for the curvatures in terms of the quanta,

$$\Lambda_1 = -\frac{(8\pi)^6 n_1^2}{2V_5^2 n_2^4 n_3^4 n_4^4} \ ; \quad \Lambda_i = \frac{2(4\pi)^2}{n_i^2} \ ; \quad \Lambda_5 = n_5 = 0 \,, \tag{4.51}$$

for $i = 2, 3, 4$, leaving V_5 a free parameter. The dilaton equation imposes,

$$(8\pi)^4 n_1^2 = V_5^2 n_2^4 n_3^4 n_4^4 \sum_{i=2}^{4} \frac{1}{n_i^2} \ . \tag{4.52}$$

This equation simply determines V_5 in terms of the flux quanta, and always admits a solution. Therefore solutions of the form $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{S}^2\times\mathrm{S}^2\times\mathrm{T}^2$, are possible in the quantum theory. Note also that by taking large enough quanta we can make sure we are in the regime of small curvature and large volume (in string units). Moreover the dilaton is a free parameter of the solution, and can be tuned to weak coupling so as to ensure we remain within the validity regime of the supergravity approximation.

Similarly one can show that solutions of the form $\mathrm{AdS}_2\times\mathrm{S}^2\times\mathrm{S}^2\times\mathrm{T}^4$, obey flux quantization. In the latter case we may also replace T^4 by a K3 surface.

5 Discussion

We have investigated superstring and supergravity backgrounds of the form $\mathcal{M}^{(1)} \times \cdots \times \mathcal{M}^{(n)}$ with special emphasis on the question of whether or not scale separation between the different factors is possible. We have seen that in all our solutions the scalar curvature of AdS_2 (in absolute value) must be of the same order or larger than the curvatures of all the other factors. Moreover, the other factors cannot all be (Ricci-)flat: in the solutions presented here this would also force the curvature of AdS_2 and all the flux to vanish. One might therefore invoke the relation between the radius, L, and the curvature: $\Lambda \simeq \frac{1}{L^2}$, to conclude that the radius of AdS_2 will be of the same order or smaller than the radius of at least one of the other factors.

Aside from the fact that the relation between scalar curvature and radius is more involved than what the previous paragraph suggests (many different definitions of the "radius" of a space are possible), there is a caveat to the argument of the previous paragraph: taking (possibly singular) discrete quotients of the internal spaces considered here, would leave invariant their local properties such as their curvature, while changing their global properties such as the radius. I.e. curvature hierarchies only concern the local properties of the spaces and do not immediately translate to corresponding hierarchies of radii. One way to directly address the question of the radius of the internal space is to study the spectrum of the (scalar) Laplacian on that space, whose first non-vanishing eigenvalue in particular can serve to read off the radius. Indeed this was the approach used in [1, 25] to establish the absence of scale separation in the vacua of [26].

For the supergravity solutions of section 4 to be promoted to full-fledged superstring solutions, flux quantization must be imposed. As we saw in section section 4.5.1, this is indeed possible to carry out in special cases, notably when the internal space includes a T^2, K3 or CY factor. However the general problem seems rather involved and we have been unable to find a solution obeying flux quantization in the case of the most general flux ansatz. It would be interesting to examine whether this can be addressed algorithmically with the help of a computer.

A possible issue with the supergravity solutions of section 4 is their potential instabilities, given the fact that we expect them to be non-supersymmetric in general. For example the solutions of section 4.1, for which a supersymmetry analysis has been performed, come in three distinct classes only one of which is supersymmetric. On general grounds we would expect the non-supersymmetric solutions to be unstable [27, 28]. It would also be interesting to establish the supersymmetry (or absence thereof) of the remaining solutions of section 4.

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A IIA supergravity

Setting the fermions to zero, the IIA action reads,

$$S = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{g}\Big(-R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2\cdot 2!}e^{3\phi/2}F^2 \\ + \frac{1}{2\cdot 3!}e^{-\phi}H^2 + \frac{1}{2\cdot 4!}e^{\phi/2}G^2 + \frac{1}{2}m^2 e^{5\phi/2}\Big) + S^{\mathrm{CS}}\,, \tag{A.1}$$

and S^{CS} is the Chern-Simons term. The equations of motion (EOM) following from the action (A.1) read:

Einstein EOM,

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{16}m^2 e^{5\phi/2}g_{MN} + \frac{1}{4}e^{3\phi/2}\Big(2F^2_{MN} - \frac{1}{8}g_{MN}F^2\Big) \\ + \frac{1}{12}e^{-\phi}\Big(3H^2_{MN} - \frac{1}{4}g_{MN}H^2\Big) + \frac{1}{48}e^{\phi/2}\Big(4G^2_{MN} - \frac{3}{8}g_{MN}G^2\Big)\,, \tag{A.2}$$

where we have set: $\Phi^2_{MN} := \Phi_{MM_2\dots M_p}\Phi_N{}^{M_2\dots M_p}$, for any p-form Φ.

Dilaton EOM,

$$0 = -\nabla^2\phi + \frac{3}{8}e^{3\phi/2}F^2 - \frac{1}{12}e^{-\phi}H^2 + \frac{1}{96}e^{\phi/2}G^2 + \frac{5}{4}m^2 e^{5\phi/2}\,. \tag{A.3}$$

Combining the trace of (A.2) with (A.3), we obtain the modified dilaton equation,

$$0 = 2R - \nabla^2\phi - (\partial\phi)^2 - \frac{1}{6}e^{-\phi}H^2 \ . \tag{A.4}$$

Form EOM's,

$$\begin{aligned} 0 &= \mathrm{d}\star\big(e^{3\phi/2}F\big) + e^{\phi/2}H{\wedge}{\star}G \\ 0 &= \mathrm{d}\star\big(e^{-\phi}H\big) + e^{\phi}F{\wedge}{\star}G - \frac{1}{2}G{\wedge}G + e^{3\phi/2}m{\star}F \\ 0 &= \mathrm{d}\star\big(e^{\phi/2}G\big) - H{\wedge}G \ . \end{aligned} \tag{A.5}$$

The forms obey in addition the Bianchi identities,

$$\mathrm{d}F = mH \ ; \quad \mathrm{d}H = 0 \ ; \quad \mathrm{d}G = H \wedge F \ . \tag{A.6}$$

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