AdS7/CFT6, Gauss-Bonnet gravity, and viscosity bound

de Boer, J.; Kulaxizi, M.; Parnachev, A.

DOI
10.1007/JHEP03(2010)087
Publication date
2010
Document Version
Final published version
Published in
The Journal of High Energy Physics

JHEP

Published for SISSA by Springer

Received: January 28, 2010
Accepted: February 27, 2010
Published: March 16, 2010

AdS_7/CFT_6, Gauss-Bonnet gravity, and viscosity bound

Jan de Boer,[a] **Manuela Kulaxizi**[a] **and Andrei Parnachev**[b]

[a] *Department of Physics, University of Amsterdam, Valckenierstraat 65, 1018XE Amsterdam, The Netherlands*

[b] *C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794-3840, U.S.A.*

E-mail: jdeboer@science.uva.nl, M.Kulaxizi@uva.nl, andreipv@max2.physics.sunysb.edu

Abstract: We study the relation between the causality and positivity of energy bounds for Gauss-Bonnet gravity in an AdS_7 background. Requiring the group velocity of metastable states to be bounded by the speed of light places a bound on the value of Gauss-Bonnet coupling. To find the positivity of energy constraints we compute the parameters which determine the angular distribution of the energy flux in terms of three independent coefficients specifying the three-point function of the stress-energy tensor. We then relate the latter to the Weyl anomaly of the six-dimensional CFT and compute the anomaly holographically. The resulting upper bound on the Gauss-Bonnet coupling coincides with that from causality and results in a new bound on viscosity/entropy ratio.

Keywords: AdS-CFT Correspondence, Black Holes in String Theory

ArXiv ePrint: 0910.5347

doi:10.1007/JHEP03(2010)087

JHEP03(2010)087

Contents

JHEP03(2010)087

1 Introduction and summary

Recently an intriguing connection between positivity of energy flux and causality has been addressed in the context of higher derivative gravity [1–5]. Consider Gauss-Bonnet gravity with negative cosmological constant in five dimensions. Since AdS space is a solution of the equations of motion, one can hypothesize the existence of a dual CFT. The theory furthermore possesses exact black hole solutions which are asymptotically AdS. Brigante, Liu, Myers, Shenker and Yaida [2, 3] considered gravitons propagating in these backgrounds and found long lived excitations which correspond to metastable states in the boundary theory. Restricting the group velocity of these states to be bounded by the speed of light (requiring causality of the boundary theory at finite temperature) places non-trivial constraints on the value of Gauss-Bonnet coupling λ [2, 3].

At first sight, a completely unrelated set of constraints was proposed for CFTs by Hofman and Maldacena [1]. By requiring positivity of the energy flux measured in a collision of certain CFT states, they deduced a set of constraints on the quantities t_2 and t_4 which determine the angular distribution of the energy flux. The values of t_2 and t_4 are completely determined by the two- and three-point functions of the stress energy tensor. The constraints can be reformulated as bounds on the ratio $a^{(d=4)}/c^{(d=4)}$ of the coefficients which appear in the Weyl anomaly of a four-dimensional CFT. In the supersymmetric case $t_4 = 0$ and the bounds are

$$\frac{1}{2} \leq \frac{a^{(d=4)}}{c^{(d=4)}} \leq \frac{3}{2} \tag{1.1}$$

These bounds have been verified for a number of interacting superconformal theories [6]. (More stringent bounds exist for $\mathcal{N} = 2$ superconformal four dimensional field theories [7] .) In the non-supersymmetric case $a^{(d=4)}$ and $c^{(d=4)}$ do not completely determine t_2 and t_4,

and therefore the bounds on $a^{(d=4)}/c^{(d=4)}$ are slightly relaxed. In [1] it has been noted that for Gauss-Bonnet gravity the lower bound in (1.1) precisely translates into the upper bound on λ coming from causality.[1] Recently the upper bound in (1.1) has been reproduced by examining a differently polarized graviton (shear channel) [4] (see also [5]).

At first sight this relation is puzzling. Why would the $\mathcal{N} = 1$ superconformal result have anything to do with Gauss-Bonnet gravity? It is worth to note however that the R^3 terms are absent in supersymmetric string theories, so one may fantasize that some supersymmetric string compactification would yield Gauss-Bonnet gravity as the low energy theory. In this paper we investigate the state of the correspondence between causality and positivity of the energy flux in the case of Gauss-Bonnet (GB) gravity in an AdS_7 background possibly dual to some six-dimensional CFT. An important feature of the six-dimensional CFT is that, unlike the four-dimensional case, the knowledge of the Weyl anomaly completely determines the values of t_2 and t_4. In particular, we find that $t_4 = 0$. Note that vanishing t_4 is a necessary feature of any supersymmetric CFT. We compute the upper bound on λ by requiring both causality of the boundary theory at finite temperature and positivity of the energy flux. The two bounds coincide exactly and result in a new bound on η/s. We also compute the lower bound on λ.

The rest of the paper is organized as follows. In the next section we review Gauss-Bonnet gravity and asymptotically AdS black holes solutions. In section 3 we study small fluctuations around these black holes, and find metastable states which can lead to causality violation. By requiring the group velocity of these states to be bounded by the speed of light, we find an upper bound on the value of λ. In section 4 we generalize the results of [1] to six dimensions. We compute the values of t_2 and t_4 in terms of the parameters a, b, c which determine the two- and three-point functions of the stress-energy tensor. In section 5 we compute the coefficients a, b, c (and hence t_2 and t_4) in GB gravity. We do it by relating the values of a, b, c to the coefficients of the B-type terms in the Weyl anomaly and computing the anomaly holographically. We then find the bounds on λ and observe that the upper bound is precisely the same as the one found in section 3. We discuss our results in section 6. Some technical results appear in the appendix.

2 Gauss-Bonnet gravity

Among gravity theories which involve higher derivative terms of the Riemann tensor in their actions, there is a special class which shares many of the properties of Einstein-Hilbert gravity. It is usually referred to as Lovelock gravity [25–27] and is the most general theory of gravitation whose equations of motion contain at most second order derivatives of the metric. Recently, the Palatini and metric formulations of Lovelock gravity have been shown to be equivalent [28].

[1] The upper bound on λ implies a new bound on the viscosity/entropy ratio in higher derivative gravity [2, 3]. The original KSS bound $\eta/s \geq 1/4\pi$ [8, 9] has been shown to be violated in a controlled setting [10]. Other recent work on the higher derivative corrections to η/s includes [11–24].

JHEP03(2010)087

The Lovelock action for a $d+1$-dimensional spacetime can be written as

$$S = \frac{1}{l_P^{d-1}} \int d^{d+1}x\sqrt{-g} \sum_{p=1}^{[\frac{d}{2}]} \lambda_p \mathcal{L}_p \tag{2.1}$$

Here l_P is Planck's length, $[\frac{d}{2}]$ denotes the integral part of $\frac{d}{2}$, λ_p is the p-th order Lovelock coefficient and $\mathcal{L}_p$ is the Euler density of a $2p$-dimensional manifold. In $d+1$ dimensions all $\mathcal{L}_p$ terms with $p \geq [\frac{d}{2}]$ are either total derivatives or vanish identically.

The Gauss-Bonnet action is the simplest example of a Lovelock action, with only the 4-dimensional Euler density included. In the following we set $d = 6$

$$S = \int d^7x\sqrt{-g}\mathcal{L} = \int d^7x\sqrt{-g}\left(R + \frac{30}{L^2} + \frac{\lambda}{12}L^2\mathcal{L}_{(2)}\right) \tag{2.2}$$

Note that in eq. (2.2) we set $l_P = 1$, included a cosmological constant term $\Lambda = -\frac{30}{L^2}$ and rescaled the Lovelock parameter by L^2. The Gauss-Bonnet term $\mathcal{L}_{(2)}$ is

$$\mathcal{L}_{(2)} = R_{MNPQ}R^{MNPQ} - 4R_{MN}R^{MN} + R^2 \tag{2.3}$$

Equations of motion derived from (2.2) can be expressed in the following way

$$-\frac{1}{2}g_{MN}\mathcal{L} + R_{MN} + \frac{1}{6}\lambda\mathcal{H}^{(2)}_{MN} = 0 \tag{2.4}$$

with $\mathcal{H}^{(2)}_{MN}$ defined as

$$\mathcal{H}^{(2)}_{MN} = R_{MLPQ}R_N{}^{LPQ} - 2R_{MP}R_N{}^P - 2R_{MPNQ}R^{PQ} + RR_{MN} \tag{2.5}$$

Eq. (2.4) admits a solution of the form

$$ds^2 = -\tilde{a}^2 f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}d\Sigma^2_{5,k} \tag{2.6}$$

with $d\Sigma^2_{5,k}$ the metric of a 5-dimensional manifold of constant curvature equal to $20k$ [29, 30]. Note that $\tilde{a}$ in eq. (2.6) is an arbitrary constant which allows one to fix the speed of light of the boundary theory to unity. Given that we are interested in black hole solutions with flat horizon, we set $k = 0$ in the following. In this case the solution, known to be thermodynamically stable, reduces to[2]

$$\begin{aligned} ds^2 &= -\tilde{a}^2 f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\sum_{i=1}^{5} dx_i^2 \\ f(r) &= \frac{r^2}{L^2}X(r), \qquad X(r) = \frac{1}{2\lambda}\left[1 - \sqrt{1 - 4\lambda\left(1 - \frac{r_+^6}{r^6}\right)}\right] \end{aligned}$$

[2]Gauss-Bonnet gravity admits another AdS solution with $\tilde{a}^2 = \frac{1}{2}\left[1 - \sqrt{1-4\lambda}\right]$. This is however unstable and contains ghosts [29].

JHEP03(2010)087

$$\tilde{a}^2 = \left[\lim_{r\to\infty}\frac{L^2}{r^2}f(r)\right]^{-1} = \frac{1}{2}\left[1+\sqrt{1-4\lambda}\right] \tag{2.7}$$

The horizon is located at $r = r_+$ whereas the Hawking temperature of the black hole is

$$T = \tilde{a}\frac{3}{2}\frac{r_+}{L^2} \tag{2.8}$$

In the limit $\frac{r_+}{r}\to 0$ one recovers AdS_7 space from (2.7). The curvature scale of the AdS space is related to the cosmological constant via

$$L_{\rm AdS} = \tilde{a}L \tag{2.9}$$

3 Fluctuations

In this section we study fluctuations around the black hole solution (2.7). In particular, we consider small perturbations of the metric h_{MN} in the scalar channel $\phi = h_{12}$. In this case, the form of the perturbed metric is

$$ds^2 = -\tilde{a}^2 f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}\left[\sum_{i=1}^{5} dx_i^2 + 2\phi(t,r,x_5)dx_1dx_2\right] \tag{3.1}$$

Note that ϕ only depends on the (t, r, x_5) directions of spacetime and its Fourier transform can be written as

$$\phi(t,r,x_5) = \int \frac{d\omega dq}{(2\pi)^2}\varphi(r)e^{-i\omega t+iqx_5} \qquad k = (\omega,0,0,0,0,q) \tag{3.2}$$

The equations of motions for φ can be found by substituting the ansatz (3.1) into (2.4) and expanding to linear order in the fluctuating field. The result is

$$T_2\varphi''(r) + T_1\varphi'(r) + T_0\varphi(r) = 0 \tag{3.3}$$

with

$$\begin{aligned}
T_0 &= 3r\tilde{\omega}^2\left[-2r^3+\lambda L^2r^2f'(r)+2r\lambda L^2f(r)\right]+\\
&\quad +\tilde{q}^2\tilde{a}^2L^2f(r)\left[-4\lambda L^2rf'(r)+6r^2-\lambda L^2r^2f''(r)-2\lambda L^2f(r)\right]\\
T_1 &= 3\tilde{a}^2L^4rf(r)\left[r^2f'(r)\left(-2r+\lambda L^2f'(r)\right)+6\lambda L^2f(r)^2\right.\\
&\quad \left.+rf(r)\left(8\lambda L^2f'(r)-10r^2+r\lambda L^2f''(r)\right)\right]\\
T_2 &= 3\tilde{a}^2L^4rf(r)^2\left[-2r^3+\lambda L^2r^2f'(r)+2r\lambda L^2f(r)\right]
\end{aligned} \tag{3.4}$$

where primes indicate differentiation with respect to the variable r and we defined $\tilde{\omega} = \omega L^2$ and $\tilde{q} = qL^2$.

It is convenient to place this equation in Schrödinger form. To do this, we follow two steps: We first define a new function $\Phi(r)$ through

$$\ln\Phi = \ln\varphi + \frac{1}{2}\int\frac{T_1}{T_2} \tag{3.5}$$

JHEP03(2010)087

which brings eq. (3.3) into the standard form

$$\Phi''(r) + W(r)\Phi(r) = 0 \qquad W(r) = \frac{T_0}{T_2} - \frac{1}{2}\left[\frac{T_1' T_2 - T_2' T_1}{T_2^2}\right] - \frac{1}{4}\left(\frac{T_1}{T_2}\right)^2 \tag{3.6}$$

Eq. (3.4) then allows us to express $T_0(r)$ as $T_0(r) = b_\omega(r)\tilde{\omega}^2 + b_q(r)\tilde{q}^2$ where $b_\omega(r), b_q(r)$ are functions of the radial coordinate r alone. This implies that $W(r)$ can be written as

$$W(r) = \frac{b_\omega}{T_2}\tilde{\omega}^2 + \frac{b_q}{T_2}\tilde{q}^2 + h(r) \qquad h(r) = -\frac{1}{2}\left[\frac{T_1' T_2 - T_2' T_1}{T_2^2}\right] - \frac{1}{4}\left(\frac{T_1}{T_2}\right)^2 \tag{3.7}$$

We then substitute $\Phi(r)$ with $\Psi(r)$ defined as $\Psi(r) = \left(\frac{b_\omega}{T_2}\right)^{\frac{1}{4}}\Phi(r)$ and subsequently make a coordinate transformation from r to y according to

$$\partial_r y(r) = \sqrt{\frac{b_\omega}{T_2}} \tag{3.8}$$

Eq. (2.4) is finally expressed as

$$-\partial_y^2\Psi + \left[\tilde{q}^2 c_g^2(y) + V_1(y)\right]\Psi = \tilde{\omega}^2\Psi \tag{3.9}$$

where

$$c_g^2 = -\frac{b_q}{b_\omega} \qquad V(y) = -\frac{T_2}{b_\omega}h(y) - \left(\frac{b_\omega}{T_2}\right)^{-\frac{1}{4}}\partial_y\left[\frac{b_\omega}{T_2}\partial_y\left(\frac{b_\omega}{T_2}\right)^{-\frac{1}{4}}\right] \tag{3.10}$$

We are now ready to study the full graviton wave function (3.9). Note that $y(r)$ is a monotonically increasing function of r with $y \to 0$ at the boundary $r \gg r_+$ and $y \to -\infty$ at the horizon $r = r_+$. $V_1(y)$ blows up as y^{-2} for $y \to 0$.

Following [2, 3] we consider (3.9) in the limit $\tilde{q} \to \infty$. In this case, $\tilde{q}^2 c_g^2(y)$ provides the dominant contribution to the potential except for a small region $y > -\frac{1}{\tilde{q}}$. It is therefore reasonable to approximate the potential with $c_g^2(y)$ for all $y < 0$ and replace it with an infinite wall at $y = 0$. Consider now the behaviour of $c_g^2(y)$ near the boundary $y = 0$. This is easier to analyze in the original variable r. In particular,

$$c_g^2 = 1 + C\frac{r_+^6}{r^6} + \mathcal{O}(\frac{1}{r^7}) \qquad C = -\frac{1 - 8\lambda + \sqrt{1 - 4\lambda}}{2 - 8\lambda} \tag{3.11}$$

Note that when C is positive, $c_g^2(r) > 1$ which implies (through WKB quantization) the existence of the metastable states whose group velocity is greater than one [3]. Hence, for values of λ such that $C > 0$, the boundary theory violates causality. That is, C should remain negative for the dual field theory to be consistent. The values of the Gauss-Bonnet parameter λ for which causality is preserved are determined from the solutions of the inequality $C \leq 0$. To be precise,

$$-\frac{1 - 8\lambda + \sqrt{1 - 4\lambda}}{2 - 8\lambda} \leq 0 \qquad \Rightarrow \qquad \lambda \leq \frac{3}{16} \tag{3.12}$$

JHEP03(2010)087

Note that although we have analyzed only the leading behaviour of c_g^2 close to the boundary our results are exact since c_g^2 given by

$$c_g^2 = a^2(1-4\lambda)X(r)\frac{(1-4\lambda)+7\lambda\frac{r_+^6}{r^6}}{\left[(1-4\lambda)+\lambda\frac{r_+^6}{r^6}\right]\left[(1-4\lambda)+4\lambda\frac{r_+^6}{r^6}\right]} \tag{3.13}$$

is a monotonically increasing function of r for all $\lambda \leq \frac{3}{16}$. $X(r)$ is defined in (2.7).

This completes the discussion of this section. The gravity analysis imposes an upper bound (3.12) on the Gauss-Bonnet parameter λ. As we will see below, the AdS/CFT correspondence relates λ to the coefficients of the stress-energy tensor three point function of the boundary CFT. Therefore, the bound on λ can be translated into constraints on these coefficients. In the next section we will consider how similar constraints arise in field theory.

4 Energy flux one point functions and positivity of energy bounds

Consider the integrated energy flux per unit angle measured through a very large sphere of radius r

$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^{d-2}\int dt\hat{n}^i T_i^0(t, r\hat{n}^i) \tag{4.1}$$

where n^i denotes a unit vector in R^{d-1}, the space where the field theory lives. This unit vector specifies the position on S^{d-2} where energy measurements may take place. Integrating over all angles yields the total energy flux at large distances.

An interesting object to consider [1] is the energy flux one point function [31]. It is defined as the expectation value of the energy flux operator (4.1) on states created by local operators $\mathcal{O}_q$

$$\langle\mathcal{E}(\hat{n})\rangle = \frac{\langle 0|\mathcal{O}_q^\dagger\mathcal{E}(\hat{n})\mathcal{O}_q|0\rangle}{\langle 0|\mathcal{O}_q^+\mathcal{O}_q|0\rangle} \tag{4.2}$$

The simplest case to examine is when the external states $\mathcal{O}_q|0\rangle$ are produced by operators with energy $q_0 \equiv q$ and zero momentum, i.e., $q^\mu = (q, 0, 0, 0)$

$$\mathcal{O}_q \equiv \int d^dx\mathcal{O}(x)e^{iqt} \tag{4.3}$$

A state with generic four momentum q^μ can be obtained by performing a simple boost on $\mathcal{O}_q$ of (4.3).

Here we will be interested in six dimensional conformal field theories. In particular, we will analyze the energy flux one point function on states produced by the stress-energy tensor operator

$$\mathcal{O}_q = \epsilon_{ij}T_{ij}(q) \tag{4.4}$$

where ϵ_{ij} is a symmetric, traceless polarization tensor with indices purely in the spatial directions. In complete analogy with [1] $O(5)$ rotational symmetry allows us to express $\langle\mathcal{E}(\hat{n})\rangle$ as

$$\langle\mathcal{E}(\hat{n})\rangle_{T_{ij}} = \frac{\langle\epsilon_{ik}^*T_{ik}\mathcal{E}(\hat{n})\epsilon_{lj}T_{lj}\rangle}{\langle\epsilon_{ik}^*T_{ik}\epsilon_{lj}T_{lj}\rangle} = \frac{q_0}{\Omega_4}\left[1+t_2\left(\frac{\epsilon_{il}^*\epsilon_{lj}n_in_j}{\epsilon_{ij}^*\epsilon_{ij}}-\frac{1}{5}\right)+t_4\left(\frac{|\epsilon_{ij}n_in_j|^2}{\epsilon_{ij}^*\epsilon_{ij}}-\frac{2}{35}\right)\right] \tag{4.5}$$

JHEP03(2010)087

where Ω_4 is the volume of the unit four-sphere $\Omega_4 = \frac{8\pi^2}{3}$. Hence, the energy flux one point function is fixed by symmetry up to two coefficients, t_2 and t_4.

This result is in agreement with expectations from conformal invariance. As explained in [32], in any d-dimensional CFT the three point function of the stress-energy tensor can be expressed in terms of three independent coefficients. They are denoted by a, b, c in eqs. (3.15) to (3.21) of [32]. On the other hand, the two point function of the stress-energy tensor depends on a unique parameter C_T which is related to a, b, c though

$$C_T = 4\frac{2\pi^{\frac{d}{2}}}{\Gamma\left[\frac{d}{2}\right]}\frac{(d-2)(d+3)a - 2b - (d+1)c}{d(d+2)} \tag{4.6}$$

It is then convenient to "change basis" and express the three point function in terms of C_T and any other two linear combinations of a, b, c. Given that the energy flux one point function is actually the ratio between a three- and a two- point function, it should be completely determined up to *two* independent parameters, i.e., the ratios of the two linear combinations of a, b, c with C_T.

To obtain the numbers $(-\frac{1}{5}, -\frac{2}{35})$ which appear in (4.5), we require that the integral of the energy flux one point function over the four dimensional sphere yields the total energy $q = q_0$. To see this one can use rotational invariance to set all components of the polarization tensor ϵ_{ij} to zero except for $\epsilon_{11} = -\epsilon_{22}$. Then

$$\frac{\epsilon^*_{il}\epsilon_{lj}n_i n_j}{\epsilon^*_{ij}\epsilon_{ij}} = \frac{1}{2}\left(n_1^2 + n_2^2\right) \tag{4.7}$$

integrated over the four sphere and divided by its volume yields $\frac{1}{5}$. The constant $-\frac{2}{35}$ in the last term of (4.5) is obtained in an identical manner.

Much like in [1], positivity of the energy flux implies constraints on the CFT parameters t_2 and t_4

$$\begin{aligned} 1 - \frac{1}{5}t_2 - \frac{2}{35}t_4 &\geq 0 \\ \left(1 - \frac{1}{5}t_2 - \frac{2}{35}t_4\right) + \frac{1}{2}t_2 &\geq 0 \\ \left(1 - \frac{1}{5}t_2 - \frac{2}{35}t_4\right) + \frac{4}{5}(t_2 + t_4) &\geq 0 \end{aligned} \tag{4.8}$$

To obtain these inequalities we use rotational symmetry to set $\hat{n} = \hat{x}_5$. Then ϵ_{ij} can be separated into a tensor, vector and scalar components with respect to rotations in $x^1 \ldots x^4$. The tensor component has $\epsilon_{5i} = \epsilon_{i5} = \epsilon_{55} = 0$ and yields the first line in (4.8). The vector and scalar components give rise to the second and third line respectively. Note that each of the constraints (4.8) is saturated in a free field theory without antisymmetric tensor fields, fermions or scalars respectively. This is similar to the situation in four dimensions.

It is possible to calculate the energy one point function explicitly and thus derive the precise relations between t_2, t_4 and the coefficients which appear in the stress-energy tensor three-point function a, b, c. The computation is outlined in appendix A. Here we present

JHEP03(2010)087

the results

$$t_2 = \frac{14}{3}\frac{220a+54b-39c}{36a-2b-7c} \qquad \Rightarrow \qquad t_2 = -140\frac{2n_a-n_f}{90n_a+20n_f+n_s}$$
$$t_4 = -28\frac{65a+24b-14c}{36a-2b-7c} \qquad \Rightarrow \qquad t_4 = \frac{35}{2}\frac{6n_a-8n_f+n_s}{90n_a+20n_f+n_s} \tag{4.9}$$

The expressions for free theories can be obtained with the help of [33]. First write a, b, c of [32] in terms of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ of [34]

$$a = \frac{\mathcal{A}}{8}, \qquad b = \frac{\mathcal{B}-2\mathcal{A}}{8}, \qquad c = \frac{\mathcal{C}}{2} \tag{4.10}$$

Then use [33] to relate $\mathcal{A}, \mathcal{B}, \mathcal{C}$ with n_a, n_f, n_s, i.e., the number of free antisymmetric two-tensors, Dirac fermions and real scalars.[3]

$$\begin{aligned}
\mathcal{A} &= -\frac{1}{\pi^3}\left[-\frac{6^3}{5^3}n_s + \frac{6^3}{3}6n_a\right] \\
\mathcal{B} &= -\frac{1}{\pi^3}\left[4\frac{6^3}{5^3}n_s + \frac{6^2}{2}8n_f + 4\frac{6^3}{3}6n_a\right] \\
\mathcal{C} &= -\frac{1}{\pi^3}\left[\frac{4^2 6^2}{4\cdot 5^3}n_s + \frac{6^2}{4}8n_f + 2\frac{6^3}{3}6n_a\right]
\end{aligned} \tag{4.11}$$

Note that for the supersymmetric $(2,0)$ multiplet with an anti-selfdual two form, five scalars and one Dirac fermion, t_4 vanishes identically. In general, superconformal Ward identities result in an additional linear constraint on the three point functions of the stress energy tensor which reduces the number of independent parameters to two. (See [35] where the explicit form of the constraint is worked out in four dimensional CFT.) While the precise form of the constraint is not known in six dimensions, it can be easily determined. Recall that supersymmetry in six dimensions implies $6n_a + n_s - 8n_f = 0$ (this relation is satisfied by both scalar multiplet and (2,0) multiplet). Using (4.11) we can express this constraint in terms of $\mathcal{A}, \mathcal{B}, \mathcal{C}$ as $17\mathcal{A} + 24\mathcal{B} - 56\mathcal{C} = 0$. This fixes the form of the constraint in six-dimensional superconformal theories. Note that supersymmetry in six dimensions implies $t_4 = 0$, just like in four dimensions [1].

With $t_4 = 0$ the inequalities in (4.8) reduce to

$$-\frac{5}{3} \le t_2 \le 5 \tag{4.12}$$

thus constraining the domain of t_2 in any supersymmetric CFT. Even when there is no supersymmetry, explicit bounds on t_2, t_4 can be derived from (4.8). In fact, in the space of t_2 and t_4 the solutions of (4.8) lie within a triangle defined by the vertex points $(-\frac{28}{9}, \frac{7}{6})$, $(0, \frac{35}{2})$ and $(7, -7)$.

[3] Note a factor of 2 missing from the last term of the last line in eq. (3.22) of [33] for $d = 6$.

JHEP03(2010)087

5 Weyl anomaly and t_2 and t_4 from Gauss-Bonnet gravity

To compare the bounds (4.8) with the causality constraint (3.12) we need to compute t_2 and t_4. More specifically, we need to determine the coefficients a, b, c in a CFT hypothetically dual to GB gravity. In principle, one can compute the three-point functions of the stress-energy tensor directly by computing the scattering of three gravitons in the bulk of Anti de Sitter space. However this route is technically more challenging than the one we take below. Instead, we use the relation between a, b, c and the coefficients of the Weyl anomaly. In a six-dimensional CFT the latter contains terms of three possible types

$$\mathcal{A}_W = E_6 + \sum_i^3 b_i I_i + \nabla_i J^i \tag{5.1}$$

where E_6 is the Euler density in six dimensions, $I_i, i = 1, \ldots 3$ are three independent conformal invariants composed out of the Weyl tensor and its derivatives, and the last term is a total derivative of a covariant expression. We will use E_6 and I_i in the form quoted in [36].

As explained in [36], the second term in $\mathcal{A}_W$ (B-type anomaly) comes from the effective action which contributes to the three-point functions of the stress-energy tensor. Hence, there is a linear relation between the coefficients b_i in (5.1) and the values of a, b, c. We explain how to obtain this relation below. To determine b_i one needs to compute the Weyl anomaly in GB gravity. The procedure for computing the Weyl anomaly of a CFT in a holographically dual theory has been introduced in [37, 38]. Consider a d-dimensional CFT formulated on a space with Euclidean metric $g^{(0)}_{ij}$. (We will be interested in the specific case $d = 6$.) Under a small Weyl transformation the metric changes as $\delta g^{(0)}_{ij} = 2\delta\sigma g^{(0)}_{ij}$. The CFT action is not invariant, but rather picks up an anomalous term,

$$\delta W[g^{(0)}_{ij}] = \int d^d x \sqrt{\det g^{(0)}}\, \mathcal{A}_W \delta\sigma \tag{5.2}$$

To compute the anomaly in GB gravity consider the following ansatz for the metric [37, 38]

$$ds^2 = L^2_{\text{AdS}} \left(\frac{1}{4\rho^2} d\rho^2 + \frac{g_{ij}}{\rho} dx^i dx^j \right) \tag{5.3}$$

where

$$g_{ij} = g^{(0)}_{ij} + \rho g^{(1)}_{ij} + \rho^2 g^{(2)}_{ij} + \mathcal{O}(\rho^3) \tag{5.4}$$

is an expansion in powers of the radial coordinate ρ. One can now solve the equations of motions of GB gravity order by order in the ρ expansion and determine $g^{(i)}_{ij}, i = 1, \ldots$ in terms of $g^{(0)}_{ij}$. One should think of $g^{(0)}_{ij}$ as the metric which sources the stress-energy tensor of the boundary CFT. From the point of view of the AdS/CFT correspondence, specifying the source completely determines the solution (5.3).

To compute the anomaly, one needs to substitute the resulting expansion (5.4) back into the lagrangian (more precisely, into $\sqrt{\det g}\,\mathcal{L}$), and extract the coefficient of the $1/\rho$ term. This is because when integrated over ρ with the UV cutoff at $\rho = \epsilon$, this term gives rise to a $\log \epsilon$ term in the six-dimensional effective action. This term is not removed by

JHEP03(2010)087

local counterterms and gives rise to the anomaly (5.2) under Weyl transformation. Clearly, the leading term in the expansion of the lagrangian comes from $\sqrt{\det g} \sim 1/\rho^4$. One may therefore be surprised that it is sufficient to find g_{ij} up to next-to-next-to leading order since an $\mathcal{O}(\rho^3)$ term in (5.4) may naively contribute to the $\mathcal{O}(\rho^{-1})$ term in $\sqrt{\det g}\,\mathcal{L}$. However one can check that this term [which is of the type $g^{(0)ij} g^{(3)}_{ij}$] contains a multiplicative factor which vanishes when the solution for the AdS radius (2.9) is substituted. This has been observed previously in the context of gravity with R^2 terms in four dimensions [39].

In principle, the procedure outlined above can be performed analytically to obtain the anomaly $\mathcal{A}_W$ in the form (5.1). However we opted to use the computer. Consider the boundary metric of the form

$$g_{ij}dx^i dx^j = f(x^3, x^4)\left[(dx^1)^2 + (dx^2)^2\right] + \sum_{i=3}^{6}(dx^i)^2 \tag{5.5}$$

One can check that for this metric $E_6 = 0$ and $I_i, i = 1, \ldots 3$ are linearly independent combinations of terms which contain six derivatives of $f(x^3, x^4)$ distributed in various ways. An example of such a term would be

$$[f^{(0,1)}(x^3, x^4)]^3 f^{(2,1)}(x^3, x^4)/[f(x^3, x^4)]^6 \tag{5.6}$$

where $f^{(p,q)} \equiv (\partial/\partial x^3)^p (\partial/\partial x^4)^q f(x^3, x^4)$. One can now use Mathematica to solve the equations of motion order by order in ρ. The leading non-trivial term relates the value of the AdS radius with the cosmological constant. The next to leading term in the equations of motion determines $g^{(1)}$. The non-vanishing components are $g^{(1)}_{11} = g^{(1)}_{22}$, $g^{(1)}_{33}$, $g^{(1)}_{44}$, $g^{(1)}_{34} = g^{(1)}_{43}$ $g^{(1)}_{55} = g^{(1)}_{66}$. The explicit expressions can be obtained by using the following formula

$$g^{(1)}_{ij} = -\frac{1}{4}\left(R_{ij} - \frac{1}{10} R g^{(0)}_{ij}\right) \tag{5.7}$$

This is the result in Einstein-Hilbert gravity [37, 38]. It is not modified by the inclusion of the finite Gauss-Bonnet term. The number of linearly independent algebraic equations at this order is equal to the number of the nontrivial components of $g^{(1)}_{ij}$. At the same order, there are more equations which contain derivatives of $g^{(1)}_{ij}$ with respect to x^3, x^4. However upon substitution of (5.7) these equations are identically satisfied, which provides a good consistency check. This story repeats itself at the next order as well. However now λ enters nontrivially into the solution for $g^{(2)}_{ij}$ which is somewhat cumbersome, so we will not quote it here.

The next step involves substituting (5.3) together with the solution (5.5) into the action (2.2) and extracting the $1/\rho$ term in the integrand. The resulting expression is too long to quote here, but it must be of the form $\int d^6x \sqrt{\det g^{(0)}} \mathcal{A}_W$, where $\mathcal{A}_W$ admits the representation (5.1). Both the anomaly $\mathcal{A}_W$ and the invariants I_i are long expressions involving terms of the type (5.6). Fortunately, the last (total derivative) term in the anomaly can also be represented in a convenient way. In fact, any total derivative term must be of the form [36]

$$\nabla_i J^i = \sum_{i=1}^{7} c_i C_i \tag{5.8}$$

JHEP03(2010)087

where C_i are certain combinations of curvature invariants which can be found in appendix A of [36], and c_i are arbitrary coefficients. Now we compute C_i for our choice of boundary metric (5.5) and demand that the coefficient in front of every term of the type (5.6) in expression

$$\mathcal{A}_W - \sum_{i=1}^{3} b_i I_i - \sum_{i=1}^{7} c_i C_i = 0 \tag{5.9}$$

vanishes. This uniquely fixes b_i and c_i; the result is

$$\begin{aligned} b_1 &= \frac{208}{3}\left(-9(1+\sqrt{1-4\lambda})+2\lambda(31+22\sqrt{1-4\lambda}-52\lambda)\right) \\ b_2 &= \frac{70}{3}\left(-9(1+\sqrt{1-4\lambda})+2\lambda(35+26\sqrt{1-4\lambda}-68\lambda)\right) \\ b_3 &= 70(1+\sqrt{1-4\lambda}-2\lambda)(1-4\lambda) \end{aligned} \tag{5.10}$$

and

$$\begin{aligned} c_1 &= c_2 = 0 \\ c_3 &= 70\left(3(1+\sqrt{1-4\lambda})-2\lambda(11+8\sqrt{1-4\lambda})-20\lambda)\right) \\ c_4 &= 84\left(-3(1+\sqrt{1-4\lambda})+2\lambda(11+8\sqrt{1-4\lambda}-20\lambda)\right) \\ c_5 &= \frac{140}{9}\left(-9(1+\sqrt{1-4\lambda})+\lambda(57+39\sqrt{1-4\lambda}-76\lambda)\right) \\ c_6 &= 14\left(-3(1+\sqrt{1-4\lambda})+2\lambda(11+8\sqrt{1-4\lambda}-20\lambda)\right) \\ c_7 &= \frac{280}{3}\left(9(1+\sqrt{1-4\lambda})-2\lambda(30+21\sqrt{1-4\lambda}-40\lambda)\right) \end{aligned} \tag{5.11}$$

where we neglected an overall factor common to all b_i's and c_i's since only the ratios of c_i will enter the expressions for t_2 and t_4 which we are after. One can check that for $\lambda = 0$ the expressions (5.10) and (5.11) reduce to the numbers which appear in [36]. In this case the values of b_i are consistent with the anomaly of a free (2,0) multiplet in six dimensions. The coefficients in front of the total derivative terms are scheme-dependent and therefore should not be compared.

The coefficients b_i in (5.10) are related linearly to the parameters that determine the two- and three-point functions of the stress-energy tensor, $\mathcal{A}, \mathcal{B}, \mathcal{C}$. To determine this relation we use the free field results for the Weyl anomaly [36]:

$$\mathcal{A}_W = -\left(\frac{28}{3}n_s + \frac{896}{3}n_f + \frac{8008}{3}n_a\right) I_1 + \left(\frac{5}{3}n_s - 32n_f - \frac{2378}{3}n_a\right) I_2 + (2n_s + 40n_f + 180n_a)\, I_3 \tag{5.12}$$

where n_s, n_f, n_a are the same as those in eqs. (4.9) and (4.11). In (5.12) we omitted the A-type anomaly term since its coefficient is related to the four-point function of the stress-energy tensor in $d = 6$. We have also omitted the total derivative term in (5.12). Using (5.12) together with the free field results for $\mathcal{A}, \mathcal{B}, \mathcal{C}$, (4.11), we arrive at

$$\mathcal{A} = \frac{864}{25}\left(3(1+\sqrt{1-4\lambda}) - \lambda(25+19\sqrt{1-4\lambda}-52\lambda)\right)$$

JHEP03(2010)087

$$\mathcal{B} = \frac{72}{25}\left(181(1+\sqrt{1-4\lambda}) - \lambda(1275+913\sqrt{1-4\lambda}-2204\lambda)\right)$$
$$\mathcal{C} = \frac{108}{25}\left(59(1+\sqrt{1-4\lambda}) - \lambda(425+307\sqrt{1-4\lambda}-756\lambda)\right) \tag{5.13}$$

Finally, using (4.10) and (4.9) we determine the values of a, b, c, and t_2 and t_4:

$$t_2 = 5\left(\frac{1}{\sqrt{1-4\lambda}} - 1\right), \qquad t_4 = 0 \tag{5.14}$$

As explained in the previous section, vanishing t_4 is a necessary feature of any superconformal field theory.

We are now in position to substitute the values of t_2 and t_4 in (5.14) into the constraints (4.8) and find out what the constraints are in terms of λ. The result is

$$-\frac{5}{16} \leq \lambda \leq \frac{3}{16} \tag{5.15}$$

Note that the upper bound on λ precisely coincides with the upper bound (3.12) obtained in section 2 by demanding causality of the boundary theory. The lower bound in (5.15) presumably can be obtained from considering excitations with different polarization, just as it has been in the four-dimensional setup [4, 5].

6 Discussion

In this paper we considered Gauss-Bonnet gravity in an AdS_7 background. This theory has exact black hole solutions for values of the Gauss-Bonnet coupling λ smaller than 1/4. We studied small fluctuations around these backgrounds and showed that causality imposes an upper bound (3.12) on the value of λ. We expect a lower bound to follow from studying gravitons of different helicity. To compare the causality bound with the positivity of energy bounds we computed t_2 and t_4 in a six-dimensional CFT in terms of the constants a, b, c which specify the three-point functions of the stress-energy tensor. We then computed the holographic Weyl anomaly, and found that $t_4 = 0$. The resulting bounds on t_2 are translated into bounds on λ (5.15). Note that the upper bound in (5.15) is precisely the same as the causality bound (3.12).

We found that the results in six dimensions are very similar to those in four dimensions. The fact that $t_4 = 0$ is related to the truncation of the gravity action at $\mathcal{O}(R^2)$. It would be interesting to include R^3 terms and see if the nonsupersymmetric bounds can be addressed in this situation. A natural generalization of the Gauss-Bonnet theory to $\mathcal{O}(R^3)$ is third order Lovelock gravity. Its equations of motion again contain only terms with at most second derivatives acting on the metric. There are two independent coefficients which multiply the Gauss-Bonnet and the third order Lovelock term in the gravitational action. Unfortunately, at least naively, the third order Lovelock term does not contribute to the three-point functions of the stress-energy tensor, and hence would not affect the value of t_4. However the situation needs to be analyzed more carefully [40].

One may view our result as an additional argument in favor of the robustness of the correspondence between the positivity of energy and causality conditions. The understanding of this correspondence is at present somewhat incomplete although it was argued in [5]

JHEP03(2010)087

that for Gauss-Bonnet gravity the equation for a graviton propagating in a shock wave background receives contributions only from the energy flux one-point function. In fact, the form of eq. (3.12) suggests that the leading small temperature behaviour of the thermal two-point function might be determined by the three-point function at zero temperature alone; and it is this leading term that determines causality of the boundary theory.

Finally, let us consider the implications of our result on the viscosity to entropy ratio. For a CFT hypothetically dual to $d+1$ dimensional Gauss-Bonnet gravity, this ratio has been computed in [2] with the result[4]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 2\frac{d}{d-2}\lambda\right) \quad \Longrightarrow_{d=6} \quad \frac{\eta}{s} = \frac{1}{4\pi}\left(1 - 3\lambda\right) \tag{6.1}$$

Combining this with the bound on λ leads to

$$\frac{\eta}{s}\Big|_{d=6} \geq \frac{1}{4\pi}\frac{7}{16} \tag{6.2}$$

Note that the viscosity to entropy ratio is bounded from below by a number smaller than that in four dimensions [3]. Hence, the lower bound on viscosity in Gauss-Bonnet gravity depends on the dimensionality of the corresponding field theory. More precisely, the bound decreases as one goes from $d = 4$ to $d = 6$.

This curious fact lead us to examine the restriction causality imposes on η/s for all $d+1$-dimensional Gauss-Bonnet theories with $d \leq 10$. The analysis is similar to the one done in section 3. We find that the viscosity to entropy bound attains the smallest value when $d = 8$:

$$\frac{\eta}{s}\Big|_{d=8} \geq \frac{1}{4\pi}\frac{219}{529} \tag{6.3}$$

It is interesting to note that the correspondence between causality and positivity of energy also leads to an upper bound on viscosity in the Gaus-Bonnet theories. Perhaps further understanding of the bounds of the type (6.3) in generalized theories of gravity may shed some light on the existence of a universal viscosity bound.

Acknowledgments

We thank F. Bastianelli, D. Kutasov, K. Papadodimas, L. Rastelli, K. Skenderis, M. Taylor, A. Tseytlin and E. Verlinde for useful discussions and correspondence. M.K. and A.P. thank the Aspen Center for Physics, the Centro de Ciencias de Benasque and the Simons Center for Geometry and Physics where part of this work has been completed. A.P. thanks the University of Amsterdam and the University of Chicago for hospitality. This work was partly supported by NWO Spinoza grant and the FOM program *String Theory and Quantum Gravity*.

[4]Here $d \geq 4$ otherwise the Gauss-Bonnet term identically vanishes.

JHEP03(2010)087

A Computation of t_2 and t_4

To determine t_2 and t_4 with respect to the coefficients which appear in the three- point function of the stress-energy tensor, an explicit computation of the energy flux is necessary.

Let us start with a careful consideration of eq. (4.5). Without loss of generality we can use rotational symmetry to set the detector along the $\hat{x}_5$ direction. It is then convenient to define new coordinates $x^{\pm} = t \pm x_5$ and express the energy flux measured at large distances as

$$\mathcal{E} = \lim_{x^+ \to \infty} \left(\frac{x^+ - x^-}{2} \right)^4 \int dx^- T_{--}(x^+, x^-) \tag{A.1}$$

Since our main objective is to obtain t_2, t_4 it is sufficient to extract the energy correlation function for two specific choices of the polarization tensor as long as they yield two independent linear combinations of t_2, t_4. The simplest cases to consider are $\epsilon_{ij} = 0$ for all i, j except for $\epsilon_{12} = \epsilon_{21}$ and $\epsilon_{ij} = 0$ for all i, j except for $\epsilon_{15} = \epsilon_{51}$. With these two choices, the following linear combinations of t_2, t_4 can be computed

$$1 - \frac{1}{5} t_2 - \frac{2}{35} t_4, \qquad 1 + \frac{3}{10} t_2 - \frac{2}{35} t_4 \tag{A.2}$$

In what follows we will analyze the former case in detail. The latter can be treated in an almost identical manner.

In practice, we need to separately consider the numerator and denominator of eq. (4.5). That is, we should compute the Fourier transform of the two- point function

$$f_2(q_0) \equiv \int d^6 x e^{i q_0 t} \langle T_{12}(x) T_{12}(0) \rangle \tag{A.3}$$

as well as the three- point function.

$$f_3(q_0) \equiv \int d^6 x e^{i q_0 t} \lim_{x_1^+ \to \infty} \left(\frac{x_1^+ - x_1^-}{2} \right)^4 \int dx_1^- \langle T_{12}(x) T_{--}(x_1) T_{12}(0) \rangle \tag{A.4}$$

As a warm up consider first the case of the two point function. Its form is fixed by conformal invariance and according to [32] can be expressed as

$$\langle T_{12}(x) T_{12}(0) \rangle = \frac{C_T}{(x^2)^6} \left[1 - \frac{2}{x^2} \left(y_1^2 + y_2^2 \right) + \frac{8}{(x^2)^2} y_1^2 y_2^2 \right] \tag{A.5}$$

Here x is a six vector parametrized as $x = (x^+, x^-, y_1, y_2, y_3, y_4)$ and C_T satisfies eq. (4.6) with $d = 6$. To Fourier transform the above expression recall that the operators T_{12} in (A.3) are ordered as written. This implies the $i\epsilon$ prescription $t \to t - i\epsilon$ which in light cone coordinates is replaced by $x^{\pm} \to x^{\pm} - i\epsilon$. Integrating over y_1, y_2 using a spherical parametrization results in

$$f_2(q_0) = \frac{\pi^5}{2 \cdot 12 \cdot 28} (36a - 2b - 7c) \, I_2 \tag{A.6}$$

where we substituted C_T in terms of the coefficients a, b, c which determine the three point function and denoted by I_2 the integral

$$I_2 = -\frac{1}{2} \int \frac{dx^+}{(x^+ - i\epsilon)^4} e^{\frac{i q_0 x^+}{2}} \int \frac{dx_2^-}{(x^- - i\epsilon)^4} e^{\frac{i q_0 x^-}{2}} = -\frac{1}{2} \left(\frac{1}{3!} \right)^2 (2i\pi)^2 \left(\frac{i q_0}{2} \right)^6 \tag{A.7}$$

JHEP03(2010)087

We are now ready to move on to the calculation of (A.4). The starting point is once more the result of [32] where the form of the three- point function of the stress-energy tensor is determined by conformal invariance up to three numbers a, b, c. Adapting eq. (3.15) of [32] to the case of interest and taking the limit $\lim_{x_1^+\to\infty}$ yields

$$\lim_{x_1^+\to\infty}\left(\frac{x_1^+-x_1^-}{2}\right)^4\langle T_{12}(x)T_{--}(x_1)T_{12}(0)\rangle=\frac{h(x)}{64\left(x_1^--x^-+i\epsilon\right)^4\left(x_1^--i\epsilon\right)^4\left(x^2\right)^6} \tag{A.8}$$

with

$$\begin{aligned}h=&\left(x^-\right)^2\Big\{\Big[(48a+8b-7c)\left(y_1^4+y_2^4\right)+c\left(y_3^2+y_4^2\right)+(256a+96b-54c)\,y_1^2y_2^2+\\&+(48a+8b-6c)\left(y_3^2+y_4^2\right)\left(y_1^2+y_2^2\right)+2cy_3^2y_4^2\Big]+\\&+x^-x^+\left[-(48a+8b-6c)\left(y_1^2+y_2^2\right)-2c\left(y_3^2+y_4^2\right)\right]+c\left(x^+\right)^2\left(x^-\right)^2\Big\}\end{aligned} \tag{A.9}$$

Note that the $i\epsilon$ prescription here is such that the operator to the left of another acquires a more negative imaginary part in the time direction. When integrating over x_1^- we have the option of a contour closing on either the upper or the lower x_1^- plane thus including only one of the two poles in (A.8). This results in

$$\int dx_1^-\lim_{x_1^+\to\infty}\left(\frac{x_1^+-x_1^-}{2}\right)^4\langle T_{12}(x_2)T_{--}(x)T_{12}(0)\rangle=\frac{5i\pi}{8}\frac{h(x)}{\left(x^--2i\epsilon\right)^7\left(x^2\right)^6} \tag{A.10}$$

Integrating now over the transverse coordinates y_1, y_2 yields

$$f_3(q_0)=\frac{5i\pi^3}{8}\frac{1}{12}\left(28a+6b-3c\right)I_3 \tag{A.11}$$

where

$$I_3=-\frac{1}{2}\int\frac{dx_2^+}{\left(x_2^+\right)^2}e^{\frac{iq_0x^+}{2}}\int\frac{dx_2^-}{\left(x_2^-\right)^7}e^{\frac{iq_0x^-}{2}}=-\frac{1}{2}\frac{1}{6!}\left(2i\pi\right)^2\left(\frac{iq_0}{2}\right)^7 \tag{A.12}$$

Let us gather the results from eq. (A.6), (A.7), (A.11) and (A.12) to form the following ratio

$$\frac{\frac{8\pi^2}{3}}{q_0}\frac{f_3(q_0)}{f_2(q_0)}=-\frac{7}{3}\frac{28a+6b-3c}{36a-2b-7c} \tag{A.13}$$

As previously explained eq. (4.5) combined with (A.13) leads to

$$1-\frac{1}{5}t_2-\frac{2}{35}t_4=-\frac{7}{3}\frac{28a+6b-3c}{36a-2b-7c} \tag{A.14}$$

In a similar but slightly more complicated manner it is possible to compute the other linear combination of t_2, t_4 in (A.2)

$$1+\frac{3}{10}t_2-\frac{2}{35}t_4=28\frac{16a+4b-3c}{36a-2b-7c} \tag{A.15}$$

Solving then for t_2, t_4 reproduces eq. (4.9).

JHEP03(2010)087

References

[1] D.M. Hofman and J. Maldacena, *Conformal collider physics: energy and charge correlations*, *JHEP* **05** (2008) 012 [arXiv:0803.1467] [SPIRES].

[2] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *Viscosity bound violation in higher derivative gravity*, *Phys. Rev.* **D 77** (2008) 126006 [arXiv:0712.0805] [SPIRES].

[3] M. Brigante, H. Liu, R.C. Myers, S. Shenker and S. Yaida, *The viscosity bound and causality violation*, *Phys. Rev. Lett.* **100** (2008) 191601 [arXiv:0802.3318] [SPIRES].

[4] A. Buchel and R.C. Myers, *Causality of holographic hydrodynamics*, *JHEP* **08** (2009) 016 [arXiv:0906.2922] [SPIRES].

[5] D.M. Hofman, *Higher derivative gravity, causality and positivity of energy in a UV complete QFT*, *Nucl. Phys.* **B 823** (2009) 174 [arXiv:0907.1625] [SPIRES].

[6] A. Parnachev and S.S. Razamat, *Comments on bounds on central charges in $N = 1$ superconformal theories*, *JHEP* **07** (2009) 010 [arXiv:0812.0781] [SPIRES].

[7] A.D. Shapere and Y. Tachikawa, *Central charges of $N = 2$ superconformal field theories in four dimensions*, *JHEP* **09** (2008) 109 [arXiv:0804.1957] [SPIRES].

[8] P. Kovtun, D.T. Son and A.O. Starinets, *Viscosity in strongly interacting quantum field theories from black hole physics*, *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231] [SPIRES].

[9] P. Kovtun, D.T. Son and A.O. Starinets, *Holography and hydrodynamics: diffusion on stretched horizons*, *JHEP* **10** (2003) 064 [hep-th/0309213] [SPIRES].

[10] Y. Kats and P. Petrov, *Effect of curvature squared corrections in AdS on the viscosity of the dual gauge theory*, *JHEP* **01** (2009) 044 [arXiv:0712.0743] [SPIRES].

[11] I.P. Neupane and N. Dadhich, *Higher curvature gravity: entropy bound and causality violation*, arXiv:0808.1919 [SPIRES].

[12] R. Brustein and A.J.M. Medved, *The ratio of shear viscosity to entropy density in generalized theories of gravity*, *Phys. Rev.* **D 79** (2009) 021901 [arXiv:0808.3498] [SPIRES].

[13] A. Adams, A. Maloney, A. Sinha and S.E. Vazquez, *$1/N$ effects in non-relativistic gauge-gravity duality*, *JHEP* **03** (2009) 097 [arXiv:0812.0166] [SPIRES].

[14] A. Buchel, R.C. Myers and A. Sinha, *Beyond $\eta/s = 1/4\pi$*, *JHEP* **03** (2009) 084 [arXiv:0812.2521] [SPIRES].

[15] X.-H. Ge and S.-J. Sin, *Shear viscosity, instability and the upper bound of the Gauss-Bonnet coupling constant*, *JHEP* **05** (2009) 051 [arXiv:0903.2527] [SPIRES].

[16] S. Cremonini, K. Hanaki, J.T. Liu and P. Szepietowski, *Higher derivative effects on η/s at finite chemical potential*, *Phys. Rev.* **D 80** (2009) 025002 [arXiv:0903.3244] [SPIRES].

[17] N. Banerjee and S. Dutta, *Shear viscosity to entropy density ratio in six derivative gravity*, *JHEP* **07** (2009) 024 [arXiv:0903.3925] [SPIRES].

JHEP03(2010)087

[18] X.-H. Ge, S.-J. Sin, S.-F. Wu and G.-H. Yang, *Shear viscosity and instability from third order Lovelock gravity*, *Phys. Rev.* **D 80** (2009) 104019 [arXiv:0905.2675] [SPIRES].

[19] S.S. Pal, *η/s at finite coupling*, *Phys. Rev.* **D 81** (2010) 045005 [arXiv:0910.0101] [SPIRES].

[20] F.-W. Shu, *The quantum viscosity bound in Lovelock gravity*, *Phys. Lett.* **B 685** (2010) 325 [arXiv:0910.0607] [SPIRES].

[21] M.F. Paulos, *Transport coefficients, membrane couplings and universality at extremality*, *JHEP* **02** (2010) 067 [arXiv:0910.4602] [SPIRES].

[22] R.-G. Cai, Y. Liu and Y.-W. Sun, *Transport coefficients from extremal Gauss-Bonnet black holes*, arXiv:0910.4705 [SPIRES].

[23] R.-G. Cai, Z.-Y. Nie, N. Ohta and Y.-W. Sun, *Shear viscosity from Gauss-Bonnet gravity with a dilaton coupling*, *Phys. Rev.* **D 79** (2009) 066004 [arXiv:0901.1421] [SPIRES].

[24] S. Cremonini, J.T. Liu and P. Szepietowski, *Higher derivative corrections to R-charged black holes: boundary counterterms and the mass-charge relation*, arXiv:0910.5159 [SPIRES].

[25] D. Lovelock, *The Einstein tensor and its generalizations*, *J. Math. Phys.* **12** (1971) 498 [SPIRES].

[26] B. Zumino, *Gravity theories in more than four-dimensions*, *Phys. Rept.* **137** (1986) 109 [SPIRES].

[27] R.C. Myers, *Higher derivative gravity, surface terms and string theory*, *Phys. Rev.* **D 36** (1987) 392 [SPIRES].

[28] Q. Exirifard and M.M. Sheikh-Jabbari, *Lovelock gravity at the crossroads of Palatini and metric formulations*, *Phys. Lett.* **B 661** (2008) 158 [arXiv:0705.1879] [SPIRES].

[29] D.G. Boulware and S. Deser, *String generated gravity models*, *Phys. Rev. Lett.* **55** (1985) 2656 [SPIRES].

[30] R.-G. Cai, *Gauss-Bonnet black holes in AdS spaces*, *Phys. Rev.* **D 65** (2002) 084014 [hep-th/0109133] [SPIRES].

[31] N.A. Sveshnikov and F.V. Tkachov, *Jets and quantum field theory*, *Phys. Lett.* **B 382** (1996) 403 [hep-ph/9512370] [SPIRES].

[32] H. Osborn and A.C. Petkou, *Implications of conformal invariance in field theories for general dimensions*, *Ann. Phys.* **231** (1994) 311 [hep-th/9307010] [SPIRES].

[33] F. Bastianelli, S. Frolov and A.A. Tseytlin, *Three-point correlators of stress tensors in maximally-supersymmetric conformal theories in $D = 3$ and $D = 6$*, *Nucl. Phys.* **B 578** (2000) 139 [hep-th/9911135] [SPIRES].

[34] J. Erdmenger and H. Osborn, *Conserved currents and the energy-momentum tensor in conformally invariant theories for general dimensions*, *Nucl. Phys.* **B 483** (1997) 431 [hep-th/9605009] [SPIRES].

[35] H. Osborn, *$N = 1$ superconformal symmetry in four-dimensional quantum field theory*, *Ann. Phys.* **272** (1999) 243 [hep-th/9808041] [SPIRES].

[36] F. Bastianelli, S. Frolov and A.A. Tseytlin, *Conformal anomaly of $(2, 0)$ tensor multiplet in six dimensions and AdS/CFT correspondence*, *JHEP* **02** (2000) 013 [hep-th/0001041] [SPIRES].

JHEP03(2010)087

[37] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *JHEP* **07** (1998) 023 [hep-th/9806087] [SPIRES].

[38] M. Henningson and K. Skenderis, *Holography and the Weyl anomaly*, *Fortsch. Phys.* **48** (2000) 125 [hep-th/9812032] [SPIRES].

[39] S. Nojiri and S.D. Odintsov, *On the conformal anomaly from higher derivative gravity in AdS/CFT correspondence*, *Int. J. Mod. Phys.* **A 15** (2000) 413 [hep-th/9903033] [SPIRES].

[40] J. de Boer, M. Kulaxizi and A. Parnachev, *Holographic Lovelock gravities and black holes*, arXiv:0912.1877 [SPIRES].

JHEP03(2010)087