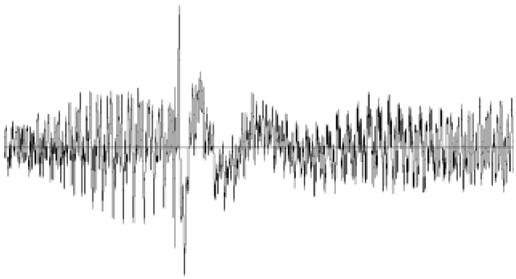
Advanced Digital Signal Processing and Noise Reduction, Second Edition.
Saeed V. Vaseghi
Copyright © 2000 John Wiley & Sons Ltd
ISBNs: 0-471-62692-9 (Hardback): 0-470-84162-1 (Electronic)

13



TRANSIENT NOISE PULSES

Transient noise pulses differ from the short-duration impulsive noise studied in the previous chapter, in that they have a longer duration and a relatively higher proportion of low-frequency energy content, and usually occur less frequently than impulsive noise. The sources of transient noise pulses are varied, and may be electromagnetic, acoustic or due to physical defects in the recording medium. Examples of transient noise pulses include switching noise in telephony, noise pulses due to adverse radio transmission environments, noise pulses due to on/off switching of nearby electric devices, scratches and defects on damaged records, click sounds from a computer keyboard, etc. The noise pulse removal methods considered in this chapter are based on the observation that transient noise pulses can be regarded as the response of the communication channel, or the playback system, to an impulse. In this chapter, we study the characteristics of transient noise pulses and consider a template-based method, a linear predictive model and a hidden Markov model for the modelling and removal of transient noise pulses. The subject of this chapter closely follows that of Chapter 12 on impulsive noise.

13.1 Transient Noise Waveforms

Transient noise pulses often consist of a relatively short sharp initial pulse followed by decaying low-frequency oscillations as shown in Figure 13.1. The initial pulse is usually due to some external or internal impulsive interference, whereas the oscillations are often due to the resonance of the communication channel excited by the initial pulse, and may be considered as the response of the channel to the initial pulse. In a telecommunication system, a noise pulse originates at some point in time and space, and then propagates through the channel to the receiver. The noise pulse is shaped by the channel characteristics, and may be considered as the channel pulse response. Thus we expect to be able to characterize the transient noise pulses with a similar degree of consistency to that of characterizing the channels through which the pulses propagate.

As an illustration of the distribution of a transient noise pulse in time and frequency, consider the scratch pulses from a damaged gramophone record shown in Figures 13.1 and 13.2. Scratch noise pulses are acoustic manifestations of the response of the stylus and the associated electro-mechanical playback system to a sharp physical discontinuity on the recording medium. Since scratches are essentially the impulse response of the playback mechanism, it is expected that for a given system, various scratch pulses exhibit a similar characteristics. As shown in Figure 13.1, a typical scratch waveform often exhibits two distinct regions:

(a) the initial high-amplitude pulse response of the playback system to the physical discontinuity on the record medium; this is followed by
(b) decaying oscillations that cause additive distortion.

The initial pulse is relatively short and has a duration on the order of 1–5 ms, whereas the oscillatory tail has a longer duration and may last up to 50 ms. Note in Figure 13.1 that the frequency of the decaying oscillations decreases with time. This behaviour may be attributed to the nonlinear modes of response of the electro-mechanical playback system excited by the physical scratch discontinuity. Observations of many scratch waveforms from damaged gramophone records reveal that they have a well-defined profile, and can be characterised by a relatively small number of typical templates.

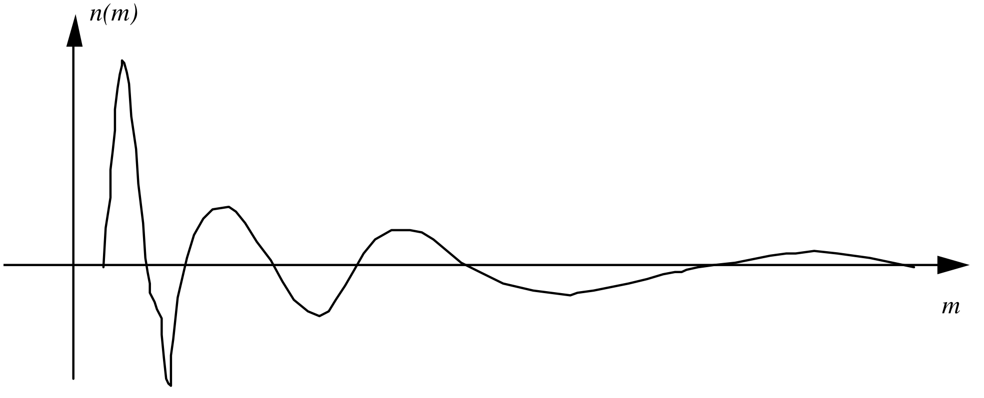
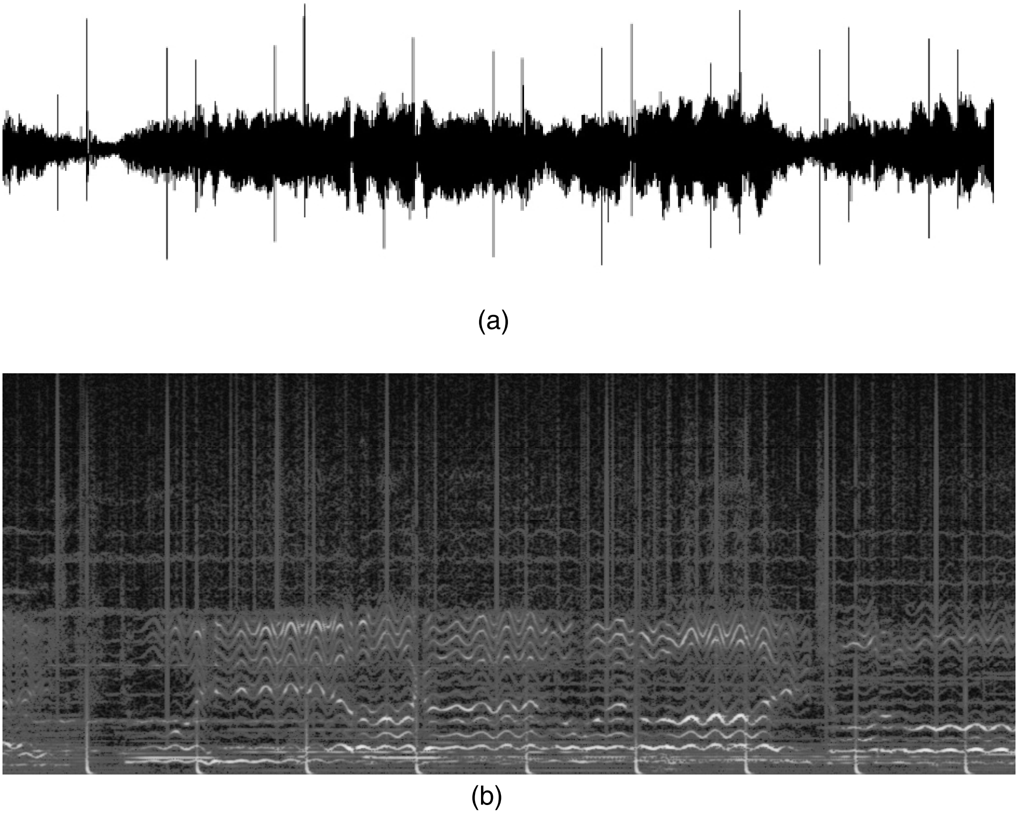


Figure 13.1 The profile of a transient noise pulse from a scratched gramophone record.



(a)

(b)

Figure 13.2 An example of (a) the time-domain waveform and (b) the spectrogram of transient noise scratch pulses in a damaged gramophone record.

A similar argument can be used to describe the transient noise pulses in other systems as the response of the system to an impulsive noise. Figure 13.2(a) (b) show the time-domain waveform and the spectrogram of a section of music and song with scratch-type noise. Note that as the scratch defect on the record was radial, the scratch pulses occure periodically with a period of 78 pulses per scratch per minute. As can be seen, there were in fact two scratches on the record.

The observation that transient noise pulses exhibit certain distinct, definable and consistent characteristics can be used for the modelling detection and removal of transient noise pulses.

13.2 Transient Noise Pulse Models

To a first approximation, a transient noise pulse $n(m)$ can be modelled as the impulse response of a linear time-invariant filter model of the channel as

$$n(m)=\sum_{k} h_k A\delta(m-k)=Ah_m \tag{13.1}$$

where A is the amplitude of the driving impulse and h_k is the channel impulse response. A burst of overlapping, or closely spaced, noise pulses can be modelled as the response of a channel to a sequence of impulses as

$$n(m)=\sum_{k} h_k \sum_{j} A_j \delta\big((m-T_j)-k\big)=\sum_{j} A_j h_{m-T_j} \tag{13.2}$$

where it is assumed that the j^{th} transient pulse is due to an impulse of amplitude A_j at time T_j. In practice, a noise model should be able to deal with the statistical variations of a variety of noise and channel types. In this section, we consider three methods for modelling the temporal, spectral and durational characteristics of a transient noise pulse process:

(a) a template-based model;
(b) a linear-predictive model;
(c) a hidden Markov model.

13.2.1 Noise Pulse Templates

A widely used method for modelling the space of a random process is to model the process as a collection of signal clusters, and to design a code book of templates containing the "centroids" of the clusters. The centroids represent various typical forms of the process. To obtain the centroids, the signal space is partitioned into a number of regions or clusters, and the "centre" of the space within each cluster is taken as a centroid of the signal process.

Similarly, a code book of transient noise pulses can be designed by collecting a large number of training examples of the noise, and then using a clustering technique to group, or partition, the noise database into a number of clusters of noise pulses. The centre of each cluster is taken as a centroid of the noise space. Clustering techniques can be used to obtain a number of prototype templates for the characterisation of a set of transient noise pulses. The clustering of a noise process is based on a set of noise features that best characterise the noise. Features derived from the magnitude spectrum are commonly used for the characterisation of many random processes. For transient noise pulses, the most important features are the pulse shape, the temporal–spectral characteristics of the pulse, the pulse duration and the pulse energy profile. Figure 13.3 shows a number of typical noise pulses. The design of a code book of signal templates is described in Chapter 4.

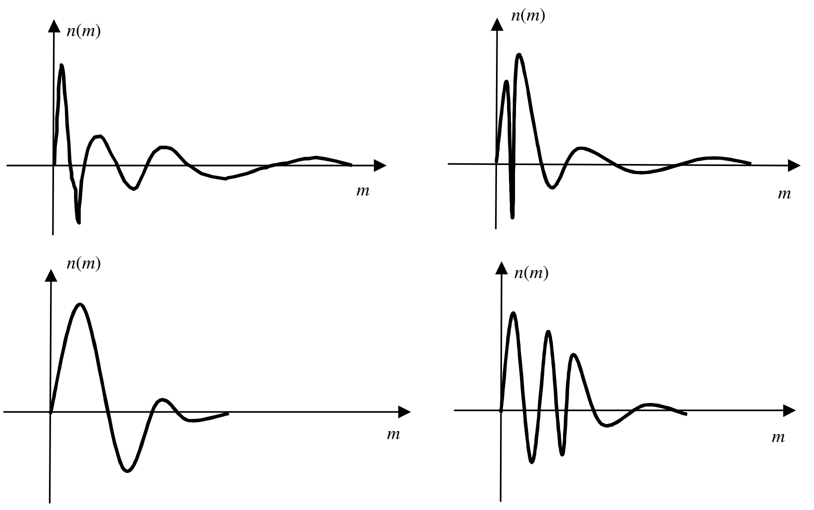


Figure 13.3 A number of prototype transient pulses.

13.2.2 Autoregressive Model of Transient Noise Pulses

Model-based methods have the advantage over template-based methods that overlapped noise pulses can be modelled as the response of the model to a number of closely spaced impulsive inputs. In this section, we consider an autoregressive (AR) model of transient noise pulses. The AR model for a single noise pulse $n(m)$ can be described as

$$n(m)=\sum_{k=1}^{P} c_k n(m-k)+A\delta(m) \tag{13.3}$$

where c_k are the AR model coefficients, and the excitation is an impulse function $\delta(m)$ of amplitude A. A number of closely spaced and overlapping transient noise pulses can be modelled as the response of the AR model to a sequence of impulses:

$$n(m)=\sum_{k=1}^{P} c_k n(m-k)+\sum_{j}^{M} A_j\delta(m-T_j) \tag{13.4}$$

where it is assumed that T_j is the start of the j^{th} pulse in a burst of M excitation pulses.

An improved AR model for transient noise, proposed by Godsill, is driven by a two-state excitation: in the state S_0, the excitation is a zero-mean Gaussian process of small variance σ_0^2, and in the state S_1, the excitation is a zero-mean Gaussian process of relatively larger variance $\sigma_1^2 >> \sigma_0^2$. In the state S_1 a short-duration, and relatively large-amplitude, excitation generates a linear model of the transient noise pulse. In the state S_0 the model generates a low-amplitude excitation that partially models the inaccuracies of approximating a transient noise pulse by a linear predictive model. The binary-state excitation signal can be expressed as

$$e_n(m)=\left[\sigma_1 b(m)+\sigma_0 \bar{b}(m)\right]u(m) \tag{13.5}$$

where $u(m)$ is an uncorrelated zero-mean unit-variance Gaussian process, and $b(m)$ indicates the state of the excitation signal: $b(m)=1$ indicates that the excitation has a variance of σ_1^2, and $b(m)=0$ (or its binary complement

$\bar{b}(m)=1$) indicates the excitation has a smaller variance of σ_0^2. The time-varying variance of $e_n(m)$ can be expressed as

$$\sigma_{e_n}^2(m)=\sigma_1^2 b(m)+\sigma_0^2 \bar{b}(m) \tag{13.6}$$

Assuming that the excitation pattern $b(m)$ is given, and that the excitation amplitude is Gaussian, the pdf of an N-sample long noise pulse $\boldsymbol{n}$ is given by

$$f_N(\boldsymbol{n})=\frac{1}{(2\pi)^{N/2}\left|\boldsymbol{\Lambda}_{e_n e_n}\right|^{1/2}}\exp\left(-\frac{1}{2}\boldsymbol{n}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda}_{e_n e_n}^{-1}\boldsymbol{C}\boldsymbol{n}\right) \tag{13.7}$$

where $\boldsymbol{C}$ is a matrix of coefficients of the AR model of the noise (as described in Section 8.4), and $\boldsymbol{\Lambda}_{e_n e_n}$ is the diagonal covariance matrix of the input to the noise model. The diagonal elements of $\boldsymbol{\Lambda}_{e_n e_n}$ are given by Equation (13.6).

13.2.3 Hidden Markov Model of a Noise Pulse Process

A hidden Markov model (HMM), described in Chapter 5, is a finite state statistical model for non-stationary random processes such as speech or transient noise pulses. In general, we may identify three distinct states for a transient noise pulse process:

(a) the periods during which there are no noise pulses;
(b) the initial, and often short and sharp, pulse of a transient noise;
(c) the decaying oscillatory tail of a transient pulse.

Figure 13.4 illustrates a three-state HMM of transient noise pulses. The state S_0 models the periods when the noise pulses are absent. In this state, the noise process may be zero-valued. This state can also be used to model a different noise process such as a white noise process. The state S_1 models the relatively sharp pulse that forms the initial part of many transient noise pulses. The state S_2 models the decaying oscillatory part of a noise pulse that usually follows the initial pulse of a transient noise. A code book of waveforms in states S_1 and S_2 can model a variety of different noise pulses. Note that in the HMM model of Figure 13.4, the self-loop transition

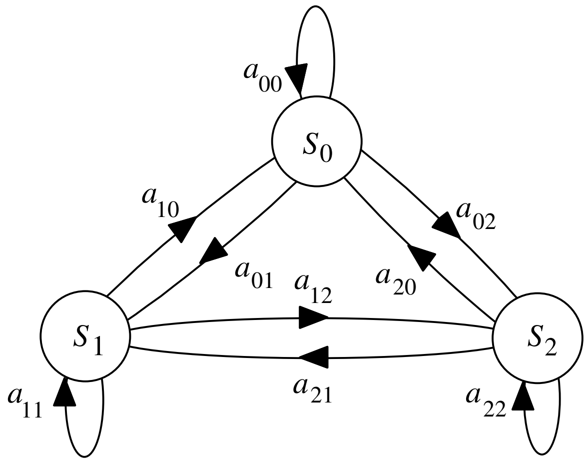


Figure 13.4 A three-state model of a transient noise pulse process.

provides a mechanism for the modelling of the variations in the duration of each noise pulse segment. The skip-state transitions provide a mechanism for the modelling of those noise pulses that do not exhibit either the initial non-linear pulse or the decaying oscillatory part.

A hidden Markov model of noise can be employed for both the detection and the removal of transient noise pulses. As described in Section 13.3.3, the maximum-likelihood state-sequence of the noise HMM provides an estimate of the state of the noise at each time instant. The estimates of the states of the signal and the noise can be used for the implementation of an optimal state-dependent signal restoration algorithm.

13.3 Detection of Noise Pulses

For the detection of a pulse process $n(m)$ observed in an additive signal $x(m)$, the signal and the pulse can be modelled as

$$y(m)=b(m)n(m)+x(m) \tag{13.8}$$

where $b(m)$ is a binary "indicator" process that signals the presence or absence of a noise pulse. Using the model of Equation (13.8), the detection of a noise pulse process can be considered as the estimation of the underlying binary-state noise-indicator process $b(m)$. In this section, we consider three different methods for detection of transient noise pulses, using the noise template model within a matched filter, the linear predictive model of noise, and the hidden Markov model described in Section 13.2.

13.3.1 Matched Filter for Noise Pulse Detection

The inner product of two signal vectors provides a measure of the similarity of the signals. Since filtering is basically an inner product operation, it follows that the output of a filter should provide a measure of similarity of the filter input and the filter impulse response. The classical method for detection of a signal is to use a filter whose impulse response is *matched* to the shape of the signal to be detected. The derivation of a matched filter for the detection of a pulse $n(m)$ is based on maximisation of the amplitude of the filter output when the input contains the pulse $n(m)$. The matched filter for the detection of a pulse $n(m)$ observed in a "background" signal $x(m)$ is defined as

$$H(f)=K\frac{N^*(f)}{P_{XX}(f)} \tag{13.9}$$

where $P_{XX}(f)$ is the power spectrum of $x(m)$ and $N^*(f)$ is the complex conjugate of the spectrum of the noise pulse. When the "background" signal process $x(m)$ is a zero mean uncorrelated signal with variance σ_x^2, the matched filter for detection of the transient noise pulse $n(m)$ becomes

$$H(f)=\frac{K}{\sigma_x^2}N^*(f) \tag{13.10}$$

The impulse response of the matched filter corresponding to Equation (13.10) is given by

$$h(m)=Cn(-m) \tag{13.11}$$

where the scaling factor C is given by $C=K/\sigma_x^2$. Let $z(m)$ denote the output of the matched filter. In response to an input noise pulse, the filter output is given by the convolution relation

$$z(m)=Cn(-m)*n(m) \tag{13.12}$$

where the asterisk * denotes convolution. In the frequency domain Equation (13.12) becomes

$$Z(f) = N(f)H(f) = C|N(f)|^2 \tag{13.13}$$

The matched filter output $z(m)$ is passed through a non-linearity and a decision is made on the presence or the absence of a noise pulse as

$$\hat{b}(m) = \begin{cases} 1 & \text{if } |z(m)| \geq \text{threshold} \\ 0 & \text{otherwise} \end{cases} \tag{13.14}$$

In Equation (13.14), when the matched filter output exceeds a threshold, the detector flags the presence of the signal at the input. Figure 13.5 shows a noise pulse detector composed of a bank of *M* different matched filters. The detector signals the presence or the absence of a noise pulse. If a pulse is present then additional information provide the type of the pulse, the maximum cross-correlation of the input and the noise pulse template, and a time delay that can be used to align the input noise and the noise template. This information can be used for subtraction of the noise pulse from the noisy signal as described in Section 13.4.1.

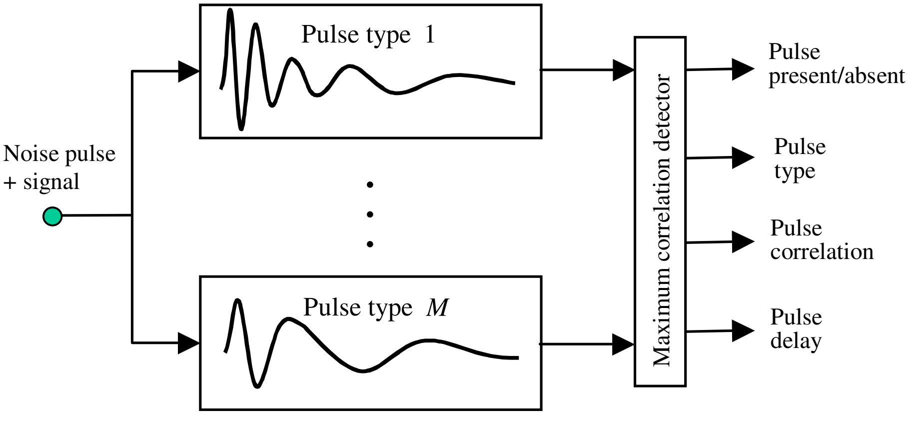


Figure 13.5 A bank of matched filters for detection of transient noise pulses.

13.3.2 Noise Detection Based on Inverse Filtering

The initial part of a transient noise pulse is often a relatively short and sharp impulsive-type event, which can be used as a distinctive feature for the detection of the noise pulses. The detectibility of a sharp noise pulse $n(m)$, observed in a correlated "background" signal $y(m)$, can often be improved by using a differencing operation, which has the effect of enhancing the relative amplitude of the impulsive-type noise. The differencing operation can be accomplished by an inverse linear predictor model of the background signal $y(m)$. An alternative interpretation is that the inverse filtering is equivalent to a spectral whitening operation: it affects the energy of the signal spectrum whereas the theoretically flat spectrum of the impulsive noise is largely unaffected. The use of an inverse linear predictor for the detection of an impulsive-type event was considered in detail in Section 12.4. Note that the inverse filtering operation reduces the detection problem to that of detecting a pulse in additive white noise.

13.3.3 Noise Detection Based on HMM

In the three-state hidden Markov model of a transient noise pulse process, described in Section 13.2.3, the states S_0, S_1 and S_2 correspond to the noise-absent state, the initial noise pulse state, and the decaying oscillatory noise state respectively. As described in Chapter 5, an HMM, denoted by $\mathcal{M}$, is defined by a set of Markovian state transition probabilities and Gaussian state observation pdfs. The statistical parameters of the HMM of a noise pulse process can be obtained from a sufficiently large number of training examples of the process.

Given an observation vector $\boldsymbol{y}=[y(0), y(1), ..., y(N-1)]$, the maximum likelihood state sequence $\boldsymbol{s}=[s(0), s(1), ..., s(N-1)]$, of the HMM $\mathcal{M}$ is obtained as

$$\boldsymbol{s}_{ML}=\arg\max_{\boldsymbol{s}} f_{\boldsymbol{Y}|\boldsymbol{S}}(\boldsymbol{y}\,|\,\boldsymbol{s},\mathcal{M}) \tag{13.15}$$

where, for a hidden Markov model, the likelihood of an observation sequence $f_{\boldsymbol{Y}|\boldsymbol{S}}(\boldsymbol{y}|\boldsymbol{s},\lambda)$ can be expressed as

$$f_{Y|S}\big(y(0),y(1),\ldots,y(N-1)\,\big|\,s(0),s(1),\ldots,s(N-1)\big)$$
$$=\pi_{s(0)}f_{s(0)}\big(y(0)\big)a_{s(0),s(1)}f_{s(1)}\big(y(1)\big)\cdots a_{s(N-2),s(N-1)}f_{s(N-1)}\big(y(N-1)\big)$$
(13. 16)

where $\pi_{s(i)}$ is the initial state probability, $a_{s(i),s(j)}$ is the probability of a transition from state $s(i)$ to state $s(j)$, and $f_{s(i)}(y(i))$ is the state observation pdf for the state $s(i)$. The maximum-likelihood state sequence $\boldsymbol{s}_{\mathrm{ML}}$, derived using the Viterbi algorithm, is an estimate of the underlying states of the noise pulse process, and can be used as a detector of the presence or absence of a noise pulse.

13.4 Removal of Noise Pulse Distortions

In this section, we consider two methods for the removal of transient noise pulses: (a) an adaptive noise subtraction method and (b) an autoregressive (AR) model-based restoration method. The noise removal methods assume that a detector signals the presence or the absence of a noise pulse, and provides additional information on the timing and the underlying the states of the noise pulse

13.4.1 Adaptive Subtraction of Noise Pulses

The transient noise removal system shown in Figure 13.6 is composed of a matched filter for detection of noise pulses, a linear adaptive noise subtractor for cancellation of the linear transitory part of a noise pulse, and an interpolator for the replacement of samples irrevocably distorted by the initial part of each pulse. Let $x(m)$, $n(m)$ and $y(m)$ denote the signal, the noise pulse and the noisy signal respectively; the noisy signal model is

$$y(m)=x(m)+b(m)\,n(m) \tag{13.17}$$

where the binary indicator sequence $b(m)$ indicates the presence or the absence of a noise pulse. Assume that each noise pulse $n(m)$ can be modelled as the amplitude-scaled and time-shifted version of the noise pulse template $\overline{n}(m)$ so that

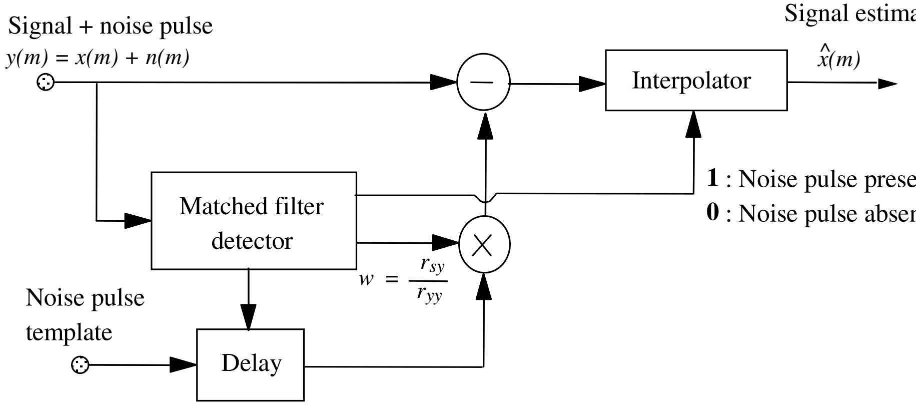


Figure 13.6 Transient noise pulse removal system.

$$n(m) \approx w\bar{n}(m-D) \tag{13.18}$$

where w is an amplitude scalar and the integer D denotes the relative delay (time shift) between the noise pulse template and the detected noise. From Equations (13.17) and (13.18) the noisy signal can be modelled:

$$y(m) \approx x(m) + w\bar{n}(m-D) \tag{13.19}$$

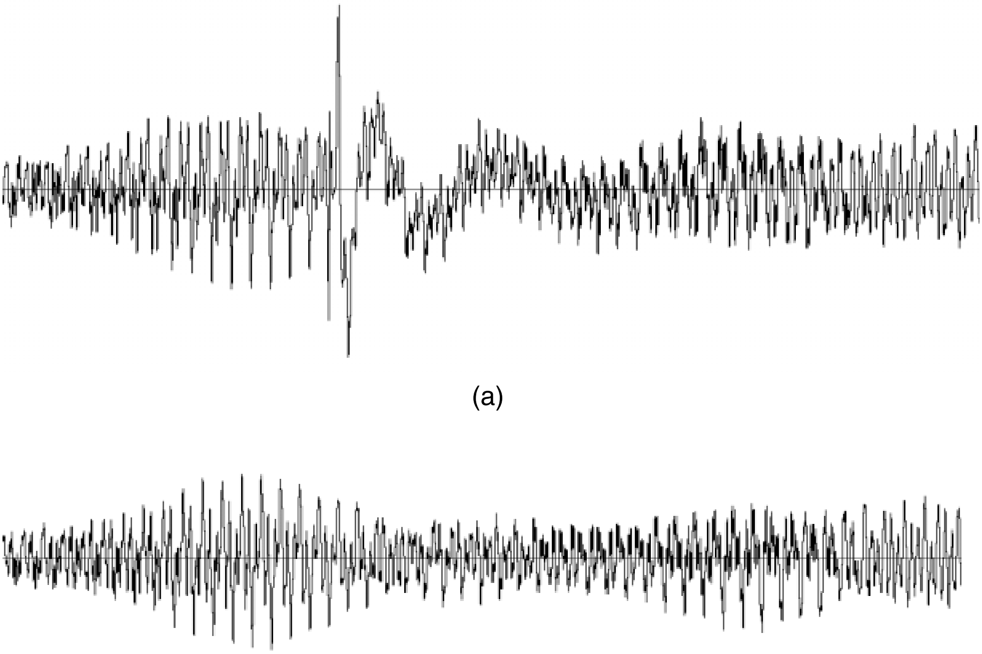
From Equation (13.19) an estimate of the signal $x(m)$ can be obtained by subtracting an estimate of the noise pulse from that of the noisy signal:

$$\hat{x}(m) = y(m) - w\bar{n}(m-D) \tag{13.20}$$

where the time delay D required for time-alignment of the noisy signal $y(m)$ and the noise template $\bar{n}(m)$ is obtained from the cross-correlation function CCF as

$$D = \arg\max_{k} \left[CCF\left(y(m), \bar{n}(m-k) \right) \right] \tag{13.21}$$

When a noise pulse is detected, the time lag corresponding to the maximum of the cross-correlation function is used to delay and time-align the noise pulse template with the noise pulse. The template energy is adaptively matched to that of the noise pulse by an adaptive scaling coefficient w. The scaled and time-aligned noise template is subtracted



(a)

(b)

Figure 13.7 (a) A signal from an old gramophone record with a scratch noise pulse. (b) The restored signal.

from the noisy signal to remove linear additive distortions. The adaptive scaling coefficient w is estimated as follows. The correlation of the noisy signal $y(m)$ with the delayed noise pulse template $\bar{n}(m-D)$ gives

$$\begin{aligned}\sum_{m=0}^{N-1} y(m)\bar{n}(m-D) &= \sum_{m=0}^{N-1}[x(m)+w\bar{n}(m-D)]\bar{n}(m-D) \\ &= \sum_{m=0}^{N-1} x(m)\bar{n}(m-D)+w\sum_{m=0}^{N-1}\bar{n}(m-D)\bar{n}(m-D)\end{aligned} \tag{13.22}$$

where N is the pulse template length. Since the signal $x(m)$ and the noise $n(m)$ are uncorrelated, the term $\Sigma\, x(m)\bar{n}(m-D)$ on the right hand side of Equation (13.22) is small, and we have

$$w \approx \frac{\sum_m x(m)\bar{n}(m-D)}{\sum_m \bar{n}^2(m-D)} \tag{13.23}$$

Note when a false detection of a noise pulse occurs, the cross-correlation term and hence the adaptation coefficient w could be small. This will keep the signal distortion resulting from false detections to a minimum.

Samples that are irrevocably distorted by the initial scratch pulse are discarded and replaced by one of the signal interpolators introduced in Chapter 10. When there is no noise pulse, the coefficient w is zero, the interpolator is bypassed and the input signal is passed through unmodified. Figure 13.7(b) shows the result of processing the noisy signal of Figure 13.7(a). The linear oscillatory noise is completely removed by the adaptive subtraction method. For this signal 80 samples irrevocably distorted by the initial scratch pulse were discarded and interpolated.

13.4.2 AR-based Restoration of Signals Distorted by Noise Pulses

A model-based approach to noise detection/removal provides a more compact method for characterisation of transient noise pulses, and has the advantage that closely spaced pulses can be modelled as the response of the model to a number of closely spaced input impulses. The signal $x(m)$ is modelled as the output of an AR model of order P_1 as

$$x(m)=\sum_{k=1}^{P_1} a_k x(m-k)+e(m) \tag{13.24}$$

Assuming that $e(m)$ is a zero-mean uncorrelated Gaussian process with variance σ_e^2, the pdf of a vector $\boldsymbol{x}$ of N successive signal samples of an autoregressive process with parameter vector $\boldsymbol{a}$ is given by

$$f_X(\boldsymbol{x})=\frac{1}{(2\pi\sigma_e^2)^{N/2}}\exp\left(-\frac{1}{2\sigma_e^2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{x}\right) \tag{13.25}$$

where the elements of the matrix $\boldsymbol{A}$ are composed of the coefficients a_k of the linear predictor model as described in Section 8.4. In Equation (13.25), it is assumed that the P_1 initial samples are known. The AR model for a single noise pulse waveform $n(m)$ can be written as

$$n(m)=\sum_{k=1}^{P_2} c_k n(m-k)+A\delta(m) \tag{13.26}$$

where c_k are the model coefficients, P_2 is the model order, and the excitation is a assumed to be an impulse of amplitude A. A number of closely spaced and overlapping noise pulses can be modelled as

$$n(m)=\sum_{k=1}^{P_2} a_k n(m-k)+\sum_{j}^{M} A_j\delta(m-T_j) \tag{13.27}$$

where it is assumed that T_k is the start of the k^{th} excitation pulse in a burst of M pulses. A linear predictor model proposed by Godsill is driven by a binary-state excitation. The excitation waveform has two states: in state "0", the excitation is a zero-mean Gaussian process of variance σ_0^2, and in state "1", the excitation is a zero-mean Gaussian process of variance $\sigma_1^2 >> \sigma_0^2$. In state "1", the model generates a short-duration large amplitude excitation that largely models the transient pulse. In state "0", the model generates a low excitation that partially models the inaccuracies of approximating a nonlinear system by an AR model. The composite excitation signal can be written as

$$e_n(m)=\left[b(m)\sigma_1+\bar{b}(m)\sigma_0\right]u(m) \tag{13.28}$$

where $u(m)$ is an uncorrelated zero-mean Gaussian process of unit variance, $b(m)$ is a binary sequence that indicates the state of the excitation, and $\bar{b}(m)$ is the binary complement of $b(m)$. When $b(m)$=1 the excitation variance is σ_1^2 and when $b(m)$=0, the excitation variance is σ_0^2. The binary-state variance of $e_n(m)$ can be expressed as

$$\sigma_{e_n}^2(m)=b(m)\sigma_1^2+\bar{b}(m)\sigma_0^2 \tag{13.29}$$

Assuming that the excitation pattern $\boldsymbol{b}=[b(m)]$ is given, the pdf of an N sample noise pulse $\boldsymbol{x}$ is

$$f_{\boldsymbol{N}}(\boldsymbol{n} \mid \boldsymbol{b})=\frac{1}{(2\pi)^{N/2}\left|\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}\right|^{1/2}}\exp\left(-\frac{1}{2}\boldsymbol{n}^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}^{-1}\boldsymbol{C}\boldsymbol{n}\right) \tag{13.30}$$

where the elements of the matrix $\boldsymbol{C}$ are composed of the coefficients c_k of the linear predictor model as described in Section 8.4. The posterior pdf of the signal $\boldsymbol{x}$ given the noisy observation $\boldsymbol{y}$, $f_{\boldsymbol{X}|\boldsymbol{Y}}(\boldsymbol{x}|\boldsymbol{y})$,can be expressed, using Bayes' rule, as

$$\begin{aligned} f_{\boldsymbol{X} \mid \boldsymbol{Y}}(\boldsymbol{x} \mid \boldsymbol{y}) &= \frac{1}{f_{\boldsymbol{Y}}(\boldsymbol{y})} f_{\boldsymbol{Y} \mid \boldsymbol{X}}(\boldsymbol{y} \mid \boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) \\ &= \frac{1}{f_{\boldsymbol{Y}}(\boldsymbol{y})} f_{\boldsymbol{N}}(\boldsymbol{y}-\boldsymbol{x}) f_{\boldsymbol{X}}(\boldsymbol{x}) \end{aligned} \tag{13.31}$$

For a given observation $f_{\boldsymbol{Y}}(\boldsymbol{y})$ is a constant. Substitution of Equations (13.30) and (13.25) in Equation (13.31) yields

$$\begin{aligned} f_{\boldsymbol{X}|\boldsymbol{Y}}(\boldsymbol{x}|\boldsymbol{y}) =& \frac{1}{f_{\boldsymbol{Y}}(\boldsymbol{y})} \frac{1}{(2\pi\sigma_e)^N \left|\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}\right|^{1/2}} \\ &\times\exp\left(-\frac{1}{2}(\boldsymbol{y}-\boldsymbol{x})^{\mathrm{T}}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}^{-1}\boldsymbol{C}(\boldsymbol{y}-\boldsymbol{x}) - \frac{1}{2\sigma_e^2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{x}\right) \end{aligned} \tag{13.32}$$

The MAP solution obtained by maximisation of the log posterior function with respect to the undistorted signal $\boldsymbol{x}$ is given by

$$\hat{\boldsymbol{x}}^{MAP} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A}/\sigma_e^2 + \boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}^{-1}\boldsymbol{C}\right)^{-1}\boldsymbol{C}^{\mathrm{T}}\boldsymbol{\Lambda}_{\boldsymbol{e}_n \boldsymbol{e}_n}^{-1}\boldsymbol{C}\,\boldsymbol{y} \tag{13.33}$$

13.5 Summary

In this chapter, we considered the modelling, detection and removal of transient noise pulses. Transient noise pulses are non-stationary events similar to impulsive noise, but usually occur less frequently and have a longer duration than impulsive noise. An important observation in the modelling of transient noise is that the noise can be regarded as the impulse response of a communication channel, and hence may be modelled by one of a number of statistical methods used in the of modelling communication channels. In Section 13.2, we considered several transient noise pulse models including a template-based method, an AR model-based method and a hidden Markov model. In Sections 13.2 and 13.3, these models were applied to the detection and removal of noise pulses.

Bibliography

GODSILL S.J. (1993), The Restoration of Degraded Audio Signals. PhD Thesis, Cambridge University.

VASEGHI S.V. (1987), Algorithm for Restoration of Archived Gramophone Recordings. Ph.D. Thesis, Cambridge University.