

Advanced Fluid Mechanics

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*I maintained my edge by always being a student.
You will always have ideas, have something new to learn.*
Jackie Joyner-Kersey

Education is not the filling of a pail, but the lighting of the fire.
William Butler Yeats

*I have always believed that 98% of a student's progress is due to his own efforts,
and 2% to his teacher.*
John Philip Sousa

The one thing that matters is the effort.
Antoine de Saint-Exupery

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Preface

This book covers material for second fluid dynamics courses at the senior/graduate level. Students are introduced to three-dimensional fluid mechanics and classical theory, with an introduction to modern computational methods. Problems discussed in the text are accompanied by examples and computer programs illustrating how classical theory can be applied to solve practical problems with techniques that are well within the capabilities of present-day personal computers.

Modern fluid dynamics covers a wide range of subject areas and facets—far too many to include in a single book. Therefore, this book concentrates on incompressible fluid dynamics. Because it is an introduction to basic computational fluid dynamics, it does not go into great depth on the various methods that exist today. Rather, it focuses on how theory and computation can be combined and applied to problems to demonstrate and give insight into how various describing parameters affect the behavior of the flow. Many large and expensive computer programs are used in industry today that serve as major tools in industrial design. In many cases the user does not have any information about the program developers' assumptions. This book shows students how to test various methods and ask the right questions when evaluating such programs.

The references in this book are quite extensive—for three reasons. First, the originator of the work deserves due credit. Many of the originators' names have become associated with their work, so referring to an equation as the Orr-Sommerfeld equation is common shorthand.

A more subversive reason for the number of references is to entice students to explore the history of the subject and how the world has been affected by the growth of science. Isaac Newton (1643–1747) is credited with providing the first solid footings of fluid dynamics. Newton, who applied algebra to geometry and established the fields of analytical geometry and the calculus, combined mathematical proof with physical observation. His treatise *Philosophiae Naturalis Principia Mathematica* not only firmly established the concept of the scientific method, but it led to what is called the Age of Enlightenment, which became the intellectual framework for the American and French Revolutions and led to the birth of the Industrial Revolution.

The Industrial Revolution, which started in Great Britain, produced a revolution in science (in those days called “natural philosophy” in reference to Newton's treatise) of gigantic magnitude. In just a few decades, theories of dynamics, solid mechanics, fluid dynamics, thermodynamics, electricity, magnetism, mathematics, medical science, and many other sciences were born, grew, and thrived with an intellectual verve never before found in the history of mankind. As a result, the world saw the invention of steam engines and locomotives, electric motors and light, automobiles, the telephone, manned flight, and other advances that had only existed in dreams before then. A chronologic and geographic study of the references would show how ideas jumped from country to

country and how the time interval between the advances shortened dramatically in time. Truly, Newton's work was directly responsible for bringing civilization from the dark ages to the founding of democracy and the downfall of tyranny.

This book is the product of material covered in many classes over a period of five decades, mostly at The University of Michigan. I arrived there as a student at the same time as Professor Chia-Shun Yih, who over the years I was fortunate to have as a teacher, colleague, and good friend. His lively presentations lured many of us to the excitement of fluid dynamics. I can only hope that this book has a similar effect on its readers.

I give much credit for this book to my wife, June, who encouraged me greatly during this work—in fact, during all of our 50+ years of marriage! Her proofreading removed some of the most egregious errors. I take full credit for any that remain.