

Advanced Interferometry for Gravitational Wave Detection

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B. Sc. (Hons), Australian National University , 1997.

A thesis submitted for the degree of
Doctor of Philosophy
at the Australian National University

Submitted
3rd November, 2000

To

Jan, Ann and Gran.

Declaration

This thesis is an account of research undertaken between March 1997 and September 2000 at the *Department of Physics, Faculty of Science, Australian National University, Canberra, Australia*.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted in whole or part for a degree in any university.

Daniel Anthony Shaddock
November, 2000

Acknowledgements

I sincerely thank my supervisors Associate Professor David McClelland, Professor Hans Bachor and Dr Malcolm Gray for their support of the work presented here.

David has managed to strike the perfect balance between providing the group with strong leadership and direction, whilst allowing researchers the intellectual freedom to pursue new ideas. I have thoroughly enjoyed my time as a PhD student and much of the credit for this must go to David.

I also thank Hans Bachor, who has always remained approachable, despite his heavy work load as the Head of the Physics Department during a turbulent time. I do not envy him this thankless job. Hopefully, after submission of this thesis I will finally become eligible to play on his team during the annual staff vs. students cricket match.

The most thanks for the work in this thesis is owed to Malcolm Gray. Mal has contributed to many aspects of this work both theoretically and experimentally. I will forever be in awe of his breadth of knowledge, and his amazing personal library.

Throughout my time at ANU I have been privileged to work with many exceptional researchers. I thank Bram Slagmolen, Ben Buchler, Dr Ping Koy Lam, Conor Mow-Lowry, Karl Baigent, Warwick Bowen, Cameron Fletcher, Jessica Lye, Dr John Close and Dr Charles Harb for their help and friendship over the years. I am also grateful to researchers from the international gravitational wave community for their help and hospitality. I thank Jim Mason, Peter Beyersdorf, Mat Lawrence, Thomas Delker, Guido Mueller, David Ottaway, Chris Hollitt, Markus Bode, Sascha Brozek, Michael Peterseim, Volker Queschke, Andreas Friese and Ken Strain for their generosity in sharing their knowledge, their offices, and their homes. The running and maintenance of a difficult experiment is often too much for an individual to handle alone. The word “we” used commonly throughout this thesis is more than mere convention. Specifically, I acknowledge the experimental aid of Conor Mow-Lowry in chapter 7, Bram Slagmolen and Malcolm Gray in section 10.1, Ping Koy Lam, Ben Buchler and Warwick Bowen in section 10.2, and David Ottaway, Malcolm Gray and Chris Hollitt in section 10.3.

As with any experimental work there are many people whose contributions often go unnoticed by the outside world. I take this opportunity to acknowledge Brett Brown, Chris Woodland, Paul MacNamara and Russel Koehne for work consistently of the highest quality, as well as Jenny Willcoxson and Zeta Hall for their friendly assistance with administration. I also thank Mal, David, Conor, Ping Koy, Jan, Mum and Dad for proof reading this thesis. Of course, any remaining errors are my responsibility :)

One of the most difficult things about the last year has been the amount of time that I have missed spending with my friends and family. I thank my parents, grandparents, and brothers for all they have done for Jan and I during an eventful four years. I also thank the entire Taylor family who have been wonderfully supportive from the beginning. I promise to come and visit more often now.

Finally, for her loving company, and enduring faith in me, I give the greatest thanks to my wife, Jan.

Abstract

In this thesis we investigate advanced techniques for the readout and control of various interferometers. In particular, we present experimental investigations of interferometer configurations and control techniques to be used in second generation interferometric gravitational wave detectors. We also present a new technique, tilt locking, for the readout and control of optical interferometers.

We report the first experimental demonstration of a Sagnac interferometer with resonant sideband extraction (RSE). We measure the frequency response to modulation of the length of the arms and demonstrate an increase in signal bandwidth of by a factor of 6.5 compared to the Sagnac with arm cavities only. We compare Sagnac interferometers based on optical cavities with cavity-based Michelson interferometers and find that the Sagnac configuration has little overall advantage in a cavity-based system.

A system for the control and signal extraction of a power recycled Michelson interferometer with RSE is presented. This control system employs a frontal modulation scheme requiring a phase modulated carrier field and a phase modulated subcarrier field. The system is capable of locking all 5 length degrees of freedom and allows the signal cavity to be detuned over the entire range of possibilities, in principle, whilst maintaining lock. We analytically investigate the modulation/demodulation techniques used to obtain these error signals, presenting an introductory explanation of single sideband modulation/demodulation and double demodulation.

This control system is implemented on a benchtop prototype interferometer. We discuss technical problems associated with production of the input beam modulation components and present several solutions. Operation of the interferometer is demonstrated for a wide range of detunings. The frequency response of the interferometer is measured for various detuned points and we observe good agreement with theoretical predictions. The ability of the control system to maintain lock as the interferometer is detuned is experimentally demonstrated.

Tilt locking, a new technique to obtain an error signal to lock a laser to an optical cavity, is presented. This technique produces an error signal by efficient measurement of the interference between the TEM_{00} and TEM_{10} modes. We perform experimental and theoretical comparisons with the widely used Pound-Drever-Hall (PDH) technique. We derive the quantum noise limit to the sensitivity of a measurement of the beam position, and using this result calculate the shot noise limited sensitivity of tilt locking. We show that tilt locking has a quantum efficiency of 80%, compared to 82% for the PDH technique. We present experimental demonstrations of tilt locking in several applications including frequency stabilisation, continuous-wave second harmonic generation, and injection locking of a Nd:YAG slab laser. In each of these cases, we demonstrate that the performance of tilt locking is not the limiting factor of the lock stability, and show that it achieves similar performance to the PDH based system.

Finally, we discuss how tilt locking can be effectively applied to two beam interferometers. We show experimentally how a two beam interferometer typically gives excellent isolation against errors arising from changes in the photodetector position, and exper-

imentally demonstrate the use of tilt locking as a signal readout system for a Sagnac interferometer.

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