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Title: Advances in Artificial Spin Ice

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10 Abstract

11 Artificial spin ices, consisting of nanomagnets arranged on the sites of various periodic and
12 aperiodic lattices, have opened a way to study a variety of fascinating phenomena such as
13 frustration, emergent magnetic monopoles and phase transitions that, in the past, have mainly been
14 the domain of bulk spin crystals and theory. Here we provide a review of recent progress in the
15 field, beginning with emergent magnetic monopoles, which are mobile excitations with magnetic
16 charge that have the potential to be exploited in future spintronic devices and can be constrained
17 to move in particular directions. We then discuss the latest developments concerning thermally
18 active phenomena, in particular the phases and phase transitions that can be observed for a
19 selection of artificial spin ice geometries. Artificial spin ices also show promise as reprogrammable
20 magnonic crystals and, with this in mind, we give an overview of the measurements of fast
21 dynamics in these magnetic metamaterials. In terms of geometries, the focus was originally on
22 elongated nanomagnets placed on the sites of the square and kagome lattices but, more recently,
23 there has been a considerable increase in the diversity of designs. This has included altering the
24 shape and size of the nanomagnets, the decimation or combination of known lattices, and rotation
25 or multiplication of the nanomagnets at the sites of the lattices. In addition, quasicrystal lattices
26 have been implemented and nanofabrication routes to produce artificial spin systems in three
27 dimensions are being explored. Different magnetic materials can be incorporated, for example, to
28 modify anisotropies and blocking temperatures. With this large variety, the way is open to discover
29 startling new phenomena and we complete this review with possible directions for the future.

30 1. Introduction

31 Artificial spin ices are designer metamaterials in the sense that they are created from
32 arrangements of single-domain dipolar-coupled magnets with nanoscale dimensions and exhibit a
33 variety of properties, such as collective dynamics, not inherently present in their building blocks.
34 The nanomagnets are arranged in such a way that the moments are frustrated, which means that
35 not all dipolar interactions between the nanomagnets can be satisfied simultaneously. They were
36 originally conceived ¹ to mimic the behaviour of spins in their crystal counterparts, such as the rare
37 earth titanate pyrochlores ², which have spins positioned on the corners of tetrahedra.
38 Correspondingly, the nanomagnets are placed on the sites of a square or kagome lattice (**Figure 1**).
39 The mesoscopic size of the nanomagnets makes it possible to directly observe the moment
40 configurations with magnetic microscopy techniques. Field-driven phenomena were at first the
41 focus of interest, with observations of magnetization reversal as well as the use of an alternating
42 magnetic field to perform an effective thermal anneal. Later it became possible to create
43 thermodynamic systems where the energy barrier to switching was small enough to allow the
44 moments of the nanomagnets to switch spontaneously, but in a collective manner mediated by the
45 dipolar interaction between the nanomagnets. In the current review, we focus on what has
46 happened in the field since the reviews in 2013 ^{3,4}, which provide a useful foundation for the reader
47 who is unfamiliar with the field.

48 In these frustrated arrangements of nanomagnets, there have been a wide variety of
49 emergent phenomena discovered including emergent magnetic monopoles ^{5,6} (covered in
50 **Section 2**), vertex based frustration ^{7,8}, chiral dynamics ⁹ and phase transitions ¹⁰⁻¹² (see **Section 3**
51 and **5**). The existence of these emergent phenomena, which arise from the collective behaviour of
52 the nanomagnets, underscores why artificial spin ices can be considered to be metamaterials.
53 Artificial spin systems are also very important for high frequency dynamics because of their
54 potential as programmable magnonic crystals ¹³, which is the theme of **Section 4**. Underpinning
55 the research into artificial spin ice is the possibility with electron beam lithography to tailor the
56 geometry at will providing “engineering by design”. This means that any arbitrary design can be
57 realised, including more exotic arrays, not just based on extensions of the square and kagome
58 geometries ^{7,14-16}, but also going beyond periodic systems to artificial quasicrystals ^{17,18}, which we
59 cover in **Section 5**. In addition to systems with separated elongated nanomagnets, there are also
60 noteworthy systems with circular ¹⁹⁻²³ or square magnets ²⁴, or connected systems where domain
61 walls can travel through the network (relevant citations are given in the last paragraph of **Section**
62 **2** and in the subsection concerning magnonics in **Section 4**). We also mention the first efforts to
63 create and characterize three-dimensional artificial lattices.

64

65

66 2. Emergent Magnetic Monopoles

67 The interest in magnetic monopoles in condensed matter systems was inspired by a
68 publication in 2008, where Castelnovo, Moessner and Sondhi discussed the idea of mapping the
69 dipoles in a spin ice system onto charge dumbbells ²⁵. Following this work, it was not long before

70 the signatures of such monopoles were experimentally observed in the rare earth titanate
71 pyrochlores²⁶⁻²⁸ and the term “magnetricity” was coined²⁹ indicating that it might be possible to
72 harness such magnetic charges in devices just as electrons and spins are exploited in electronic and
73 spintronic devices.

74 It was then a natural step to consider how emergent magnetic monopoles would behave in
75 the corresponding artificial spin ices (see description of emergent magnetic monopoles in **Box 1**).
76 In 2009, the first discussion of emergent magnetic monopoles in artificial spin ice appeared³⁰,
77 theoretically considering interactions between charge excitations in artificial square ice, which not
78 only have a Coulombic interaction but also a confining potential, or string tension, binding the
79 charge defect pairs. Shortly afterwards, the first experimental observations of the creation and
80 separation of monopole-antimonopole pairs in an applied magnetic field, leaving behind them
81 strings of reversed magnets referred to as Dirac strings, were reported in disconnected⁶ and
82 connected⁵ artificial kagome spin ice. In the disconnected system, the Dirac strings were found to
83 be one-dimensional while, in the connected artificial kagome spin ice, branching was observed.
84 This highlights the fact that the behaviour during magnetization reversal is sensitive to the local
85 energetics, which in turn are highly dependent on the detailed geometry of the artificial spin ice³¹.
86 Indeed, one of the appeals of artificial spin ice is that the behaviour can be tuned through the lattice
87 type and parameter, as well as nanomagnet shape and size. Furthermore, viewing artificial spin ice
88 in terms of emergent magnetic monopoles also provides a means to visualise the inherent physical
89 phenomena. For example, in an artificial square ice, mapping of the behaviour of magnetic
90 monopoles during magnetization reversal gives important insights into memory effects at the
91 microscopic level³².

92 The first direct observation of the spontaneous creation and separation of charge defects,
93 or Type III vertices (for vertex types, see **Figure 1**), in a thermally-active artificial square ice was
94 reported in 2013³³. On separating, the defects left behind strings of reversed magnets against a
95 Type II vertex background (**Figure 2a-f**). Here, an applied magnetic field was used to set the initial
96 remanent state of the sample with all vertices in a Type II configuration and the authors pointed
97 out that the interaction energy between the charges, attractive or repulsive, is dependent on the
98 background configuration. In a thermally-active artificial kagome spin ice, it was found that the
99 emergent magnetic monopoles remain confined, since increasing the length of the strings beyond
100 one spin flip is energetically unfavourable³⁴. Therefore, important in defining the behaviour of the
101 monopoles are both the background magnetic configuration and the artificial spin ice geometry,
102 which can be modified by introducing defects into the lattice³⁵⁻³⁷. Interestingly, the string tension
103 can be exploited in artificial square ice to obtain spontaneous magnetic currents, first stretching
104 the bound monopoles apart with an applied magnetic field and removing the field in order to
105 release them, and it was suggested that this effect may be of interest for energy storage³⁸.

106 As the charge defect (Type III vertex) pairs in thermally-active artificial square ice continue
107 to be created and spread through the system, the strings eventually coalesce to form domains of
108 ground state vertices separated by Type II boundaries (**Figure 2 g,h**). Charge defects in the form of

109 Type III vertices can propagate along the domain boundaries providing a mechanism for them to
110 decrease in length until they disappear^{33,39}.

111 Even though the string tension between two monopoles decreases with increasing distance
112 between them, an important question to answer is whether it is possible to create an artificial square
113 ice where the string tension is absent to give truly unconfined emergent magnetic monopoles that
114 interact via a Coulomb interaction only. The key to this issue is the fact that the four nanomagnets
115 at a vertex do not have equivalent interactions since the intermagnet separations and angles are not
116 the same. One possible solution is to raise one sublattice with respect to the other⁴⁰⁻⁴⁴ so that, at a
117 critical vertical separation, the so-called Coulomb phase is achieved⁴⁵, with emergent magnetic
118 monopoles freely moving in a highly degenerate, divergence-free background. Another possibility
119 to reduce the string tension is to use two different sublattice parameters^{46,47}. In lattices of
120 connected elements, modifying the size and shape of the vertex, for example by fabricating a hole
121 at the vertex centre or making the elements thinner, can also make the Type I and Type II vertices
122 similar in energy^{48,49}. In this regard, measurements of the magnetic susceptibility can give helpful
123 information on both the monopole motion⁵⁰ and the charge correlation length associated with
124 monopoles⁵¹. Another valuable signature of monopole defects and their Dirac strings appears in
125 the frequency spectrum⁵²⁻⁵⁴, which provides a macroscopic means of identifying monopole defects
126 where the spectral amplitude is proportional to the number of defects. Indeed, artificial spin ices
127 are interesting candidates for reconfigurable or reprogrammable magnonic crystals, which we
128 discuss further in [Section 4](#).

129 Going to geometries beyond the square ice, one would expect a very different behaviour
130 of the effective charges¹⁴. For example, in the shakti lattice⁷, it is possible to see signatures of
131 charge screening involving magnetic monopole polarons analogous to polarons associated with
132 electrons in a crystal⁵⁵. These magnetic charges emerge at the four-magnet vertices in the shakti
133 lattice, which are surrounded by three-magnet vertices that, due to their topology, will always have
134 magnetic charge. By careful design, it is also possible to control the emergent magnetic monopoles,
135 which is important for device applications. For example, charge defects can be constrained to move
136 in particular directions using certain geometries⁵⁶ or a temperature gradient⁵⁷, and they can also
137 be controlled by locally inducing strain with an electric field⁵⁸.

138 This Section would not be complete without mentioning that magnetic domain walls within
139 the nanowires constituting connected artificial spin ice systems can also be considered as charge
140 carriers⁵⁹. These walls are chiral objects where the magnetization within the wall rotates either
141 clockwise or anticlockwise, independent of whether the lattice itself has chiral symmetry.
142 Interestingly, when the walls approach a Y-shaped vertex in an artificial kagome ice on application
143 of a magnetic field ([Figure 2 i,j](#)), this domain wall chirality can determine whether they will move
144 into the left or the right branch^{60,61}. Taking advantage of this, a domain wall passing through
145 tailored Y-junctions, which are the basic unit of connected artificial kagome spin ice, can be used
146 to perform logic operations^{62,63}. However, care needs to be taken since the walls may transform
147 into domain walls of opposite chirality for particular fields and geometries^{64,65}, and the
148 propagation behaviour of domain walls depends on the magnitude and orientation of the applied

149 magnetic field ⁶⁶⁻⁶⁸. To circumvent these effects, the geometry of the Y-junctions can be modified
150 so that the symmetry of the branches is reduced, giving a deterministic domain wall path that is
151 independent of DW chirality ⁶⁹. It is also possible to nucleate domain walls in particular positions
152 in connected artificial spin ice using a magnetic tip, thereby creating specific magnetic
153 configurations at will ⁷⁰. Finally, we note that, just as the chirality of the domain walls provides
154 additional degrees of freedom in connected artificial spin systems, the edge bending of the
155 magnetization at the vertices in disconnected lattices also gives an additional chiral degree of
156 freedom to the vertex monopoles ⁷¹ (**Figure 2k**).

157

158 3. Thermodynamics and kinetics

159 **Thermally active artificial spin systems.** Since the beginning of the field, it has been of
 160 fundamental interest to understand the collective behaviour of the nanomagnets in artificial spin
 161 ices, in particular how they access thermodynamically favourable magnetic phases. **The first**
 162 **experimental observations of a magnetic phase transition in an artificial spin system was on the**
 163 **square geometry, where magnetization measurements indicated a decrease in macrospin order well**
 164 **below the intrinsic ordering temperature of the nanomagnet material**¹⁰. Nevertheless, **for other**
 165 **geometries** it has been a challenge to fabricate arrays of nanomagnets that remain thermally active
 166 down to temperatures where their magnetic moments are predicted to thermodynamically order as
 167 a result of the dipolar interactions. Here we give an overview of the earlier efforts using thermal
 168 annealing and field protocols before reviewing the advances that finally lead to **the flourish of**
 169 **work on** truly thermally active artificial spin ices.

170 The earliest examples of artificial spin ice were athermal at the temperature of the
 171 experiments, which were conducted at room temperature. For these athermal systems, effective
 172 thermodynamics were probed using applied alternating magnetic fields, for which an effective
 173 temperature^{72,73} and effective entropy⁷⁴ were defined. Thermal ordering in these artificial spin
 174 systems could only then be achieved by annealing the samples above the Curie temperature, T_C ⁷⁵⁻
 175 ⁷⁷. Indeed, such thermal protocols proved to be more effective for lowering the residual entropy of
 176 the system than the previously used field protocols for athermal systems^{1,78,79}.

177 In a thermally-active artificial spin system, a nanomagnet is considered to be thermally
 178 active if the time taken for the measurement, t_m , is longer than the time required for the moment
 179 of a nanomagnet to switch, t_s , which is described by the Arrhenius law:

$$180 \quad \frac{1}{t_s} = \frac{1}{t_f} e^{-\frac{KV}{k_B T}} \quad (1)$$

181 where $1/t_f$ is the attempt frequency, V is the magnetic volume of the nanomagnets, k_B is the
 182 Boltzmann constant and K is the anisotropy constant that is related to the shape of the nanomagnets
 183 and is given by $K = \frac{1}{2} \mu_0 M_S^2 \Delta N$ for ellipsoids⁸⁰, where μ_0 is the magnetic permeability of free
 184 space, M_S is the saturation magnetization and ΔN is the difference between the demagnetizing
 185 factors along the in-plane short and long axes of the nanomagnets⁸¹. We can define the ‘blocking
 186 temperature’, T_b , as the lowest temperature where the nanomagnets appear to be thermally active,
 187 or superparamagnetic (or perhaps more correctly ‘paramagnetic superspins’) at the timescale of
 188 the measurement ($t_m < t_s$). Below T_b , the energy required to switch the magnetic moments is higher
 189 than the thermal energy, and the nanomagnets will appear to be frozen with static magnetic
 190 moments at the timescale of observation. For nanomagnets in artificial spin ice, the switching has
 191 been shown to occur through a nonuniform process⁸², leading to a much lower T_b than is expected
 192 for magnets displaying single-domain coherent rotation, for which Equation 1 applies. The
 193 moments of an artificial spin ice comprising weakly coupled nanomagnets have further been
 194 shown to freeze in similar manner to that of a glass on approaching T_b from above, where the

195 system crosses over from the thermally equilibrated state to the frozen state following a Vogel-
196 Fulcher-Tammann law ^{82,83}.

197 To observe thermally active behaviour in artificial spin ice, the blocking temperature T_b
198 needs to be below the temperature of the experiments that are ideally performed close to room
199 temperature. For this, Equation 1 provides some insight into how to lower the blocking temperature
200 T_b by modifying the individual nanomagnets, despite the fact that it does not fully reflect the
201 switching mechanism of the nanomagnets. Firstly, the nanomagnet volume can be decreased by
202 reducing the thickness and/or the lateral dimensions of the magnets ^{33,84,85}. Secondly, the shape
203 anisotropy, K , of the magnets can be reduced by lowering the aspect ratio (nanomagnet length
204 relative to width) or the magnetization of the nanomagnets. Thirdly, the magnetization of the
205 magnetic material can be lowered. Typically, artificial spin ice is manufactured from
206 ferromagnetic elements fabricated from Permalloy, a soft magnetic alloy that has a composition
207 close to 80% Ni / 20% Fe. Therefore, in order to reduce the magnetization of the Permalloy, the
208 Ni content can be increased, thus lowering the Curie temperature T_C of the material ⁷⁵. Another
209 possibility to reduce the magnetization is to alloy the magnetic elements with non-magnetic
210 elements, such as palladium ^{77,86}, or to use a ferrimagnetic material instead of ferromagnetic
211 material ⁸⁷.

212 One important aim is then to observe magnetic ordering in thermally active artificial spin
213 ice. For this, the nanomagnets need to be thermally active below the temperature of the respective
214 phase transition(s). However, all of the routes to lower T_b involving a modification of the
215 individual nanomagnets also simultaneously lower the transition temperature. This is because all
216 such modifications (directly or indirectly) reduce the dipolar coupling between the nanomagnets,
217 which is proportional to m^2/d^3 . Thus, increasing the centre-to-centre distance d between the
218 nanomagnets or decreasing the nanomagnet moment $m = M_s V$ will reduce both the T_b and the
219 phase transition temperature. To counteract this effect on the transition temperature, and ensure it
220 is above T_b , one can minimize the lattice parameter ^{76,85}, which in turn decreases the nanomagnet
221 separation, so giving an increase in the coupling between the nanomagnets. Therefore, great care
222 in both the design of artificial spin systems and the thermal protocol is required to ensure that the
223 system reaches equilibrium, especially at lower temperatures, and a useful discussion on
224 determining whether a system is out-of-equilibrium can be found in a recent colloquium by
225 Rougemaille and Canals ⁸⁸.

226

227 **Phases and phase transitions.** The progress in fabrication, allowing the manufacture of artificial
228 spin systems with spontaneously fluctuating moments, fuelled the research for studying the
229 collective behaviour in thermally-active artificial spin systems. We review here the latest findings
230 regarding the prototypical square and kagome lattices, before we touch on the more exotic and still
231 rather unexplored lattices.

232 The predicted ground state in artificial square ice ⁴² was experimentally verified in 2011 in
233 samples that ordered during the early deposition of the Permalloy film from which they were
234 made ⁸⁹. The ordering was later reproduced using thermal annealing protocols ⁷⁵⁻⁷⁷ and observed

235 in thermally active systems³³. This ordering is a result of the fact that, while all ice-rule obeying
 236 vertex configurations **have low energy**, the non-equal interactions between the nanomagnets at the
 237 vertices lift the degeneracy, favouring the lowest energy Type I vertices¹. As a result the non-
 238 equal interactions at the vertex, the artificial square system does not have an extensive degeneracy,
 239 and has been theoretically suggested to belong to the two-dimensional Ising universality class with
 240 a continuous phase transition^{90,91}. In recent X-ray scattering experiments, the first measurements
 241 of critical behaviour across the phase transition have been obtained, providing evidence of a
 242 continuous two-dimensional Ising transition¹². In addition, it may be possible to alter the nature of
 243 the phase transition by modifying the energies of the vertex configurations, for example using an
 244 exchange bias field⁹².

245 One of several ways to equalize the intermagnet interactions in the square ice geometry is
 246 to lift the nanomagnets on one of the two sublattices out of the plane (**Figure 3a**)⁴⁰⁻⁴², which has
 247 been experimentally realized^{43,44}. While the structure factor deduced from the MFM images of
 248 conventional square ice displayed the expected peaks associated with ordering (**Figure 3b**), ‘pinch
 249 points’ emerged in the structure factor for the fully degenerate square ice (**Figure 3c**). These pinch
 250 points are a signature of a so-called Coulomb phase⁴⁵, which obeys Gauss’ law and supports
 251 magnetic monopole excitations that emerge in highly frustrated systems. **The same effect is**
 252 **predicted to be achieved in a purely two-dimensional system in which the interactions at the**
 253 **vertices are tuned by connecting the nanomagnets having made appropriate choices for their width**
 254 **and thickness**⁴⁸.

255 It is predicted that the artificial kagome spin ice, despite the extensively degenerate
 256 configurations arising from the geometrical frustration, has a rich phase diagram, including a long
 257 range ordered phase, due to the presence of long-range dipolar interactions between the
 258 nanomagnets^{42,93}. The predicted phases are shown schematically in **Figure 3d**. On cooling, there
 259 is first a crossover from a disordered paramagnetic phase to another paramagnetic phase, referred
 260 to as kagome ice I, in which the ice rule is obeyed with magnetic charges of +1 or -1 at each vertex.
 261 **The cooperative paramagnetic behaviour of the kagome ice I phase means that it can be seen in**
 262 **the framework of spin liquid physics**⁸⁸. A spin liquid is a disordered, but correlated magnetic state,
 263 **in which the interactions within the assembly of classical Ising variables lead to pairwise non-zero**
 264 **spin correlations, but decay to zero at large distances. It is, however, important to note that the**
 265 **nanomagnets in artificial spin systems are only Ising-like, firstly because they have a finite size**
 266 **and shape with non-uniform magnetization, and, secondly, because they interact magnetostatically**
 267 **beyond their nearest neighbours. Upon lowering the temperature further, the system goes through**
 268 **two other** phase transitions, first an Ising transition to the kagome ice II phase with ordered charges,
 269 but still with highly fluctuating moments, and then a transition to a long-range ordered phase with
 270 both charge and spin order. This last transition is a second-order three-state Potts transition,
 271 although simulations will give a Kosterlitz-Thouless transition if **charge order-breaking**
 272 **fluctuations are not present**^{93,94}. **Large system sizes are needed to observe the three-state Potts**
 273 **universality because these fluctuations have a very large correlation length**⁹³. Intriguingly, the
 274 emergence of the charge-ordered phase is not easily explained, as no charge degree of freedom is

275 encoded in the underlying dipolar spin Hamiltonian^{88,95}. The emergence of the charge order and
276 the rest of the phase diagram can, however, be understood in terms of fragmentation of the spins
277 into two parts; a divergence full part, which gives a static signature of ordered magnetic charges,
278 and a divergence free part reflecting a disordered magnetic phase with fluctuating moments
279 characteristic of a Coulomb phase⁹⁵⁻⁹⁷. **The kagome ice II phase can thus be regarded as a spin
280 liquid superimposed on a magnetic charge crystal.**

281 In early studies, the difficulty of obtaining ordering in artificial kagome spin ice structures
282 with sizes beyond four rings was demonstrated^{84,98}. Regarding extended kagome lattices,
283 indications of two phase transitions have been observed with muon rotation spectroscopy¹¹ and
284 the first signatures of the relevant phases have been observed with neutron scattering and resistivity
285 measurements performed on connected artificial kagome spin ice⁹⁹⁻¹⁰². However, only small areas
286 of the charge-ordered phase have been directly observed with imaging techniques following
287 annealing^{76,77,103}, and the long-range ordering of the charge (spin ice II phase) or the spins (the
288 ground state) have not yet been unambiguously observed. The difficulty in achieving ordering in
289 the artificial kagome spin ice is due to the high frustration and extensive degeneracy of the system.
290 Achieving the ground-state in such a degenerate energy landscape simply takes a very long time,
291 which may also be hindered by kinetic restrictions. For example, it can be seen that, during
292 ordering of the charges into the kagome ice II phase, domains with an even number of vertices are
293 favoured. This indicates that two vertices with opposite charge prefer to order simultaneously so
294 that charge neutrality is maintained, thus creating restrictions in how the domains grow⁸⁷. Another
295 challenge associated with achieving the ground state in highly frustrated systems is that they are
296 very susceptible to external fields, which can dramatically change the energy landscape¹⁰⁴.

297 As we show in **Section 5**, there are many more geometries to consider beyond the kagome
298 and square geometries. For more exotic lattices, it was found that the low energy states adopted by
299 the extended system could not be predicted by considering the relative energies of the building
300 blocks consisting of only a few nanomagnets^{18,105}. Going beyond Ising-like systems to dipolar XY
301 systems with arrays of coupled discs, complex ordering phenomena have been observed^{19,20,22},
302 and the phase diagrams of such XY systems can be modified by structural disorder to give new
303 phases that favour local flux closure²⁰. Both the lattice and the moment orientations define the
304 ordering behaviour. For example, the kagome Ising system with out-of-plane moments displays
305 behaviour that is quite different from its topologically equivalent kagome lattice with in-plane
306 moments^{88,106,107}. Furthermore, competing ferro- and antiferromagnetic orders have been observed
307 in the quadrupolar ice.¹⁰⁸ Here pairs of elongated nanomagnets representing Ising moments are
308 arranged in a similar manner to the individual nanomagnets in chiral ice and can be considered to
309 be an array of quadrupoles (**Figure 3e-h**) (see **Section 5** for more details about the chiral ice). This
310 system also provides a way to realise the Potts model, which has also been achieved by exploiting
311 magnetocrystalline anisotropy in epitaxial thin films²⁴, and the competing interactions present
312 result in a complex phase diagram dependent on field and temperature.

313

314 4. Fast dynamics

315 **Resonances and spin wave excitations.** Since the nanomagnets in artificial spin
316 ices are typically manufactured from soft magnetic metals such as Permalloy, the magnetic
317 resonances and spin wave excitations typically investigated occur in the 1-100 GHz range. When
318 the artificial spin ices are made up of disconnected magnetic elements, the spin wave excitations
319 can exist within each individual nanomagnet, appearing as modes of precession that can be
320 strongly affected by the element shape, applied magnetic field direction and stray field from
321 neighbouring elements¹⁰⁹. Examples of standing and localised modes within an elongated,
322 rectangular nanomagnet are depicted in [Figure 4a](#), with the corresponding frequencies given in
323 [Figure 4b](#) as functions of an external magnetic field applied along the direction shown¹¹⁰. The
324 resonances here are mostly localised towards the edges of the rectangular nanomagnet. In addition,
325 probing spin wave resonances for a static magnetic field applied at different angles gives valuable
326 information⁵⁴, for example, allowing the identification of resonances from sets of magnetic
327 elements with particular orientations.

328 Collective excitations in artificial spin ice have been predicted and studied for several
329 geometries. In particular, the relationship between the lattice geometry, the magnetic configuration
330 present and the resonant modes that occur within the nanomagnets has been demonstrated with
331 experimental measurements on artificial square ice, which revealed multiple modes in the spectra
332 in addition to the main resonance¹¹¹. With micromagnetic simulations¹¹² and scaling arguments
333⁵¹ it was shown that the modes can be understood in terms of number and position of emergent
334 magnetic monopoles at the vertices, which in turn depends on the strength and direction of the
335 applied field. In particular, the emergent magnetic monopoles created during reversal processes
336 ([Figure 4c](#)) can be detected using ferromagnetic resonance (FMR) that, with its sensitivity to local
337 fields, provides a way to distinguish contributions from different vertex types in the spectra^{113,114}.
338 The frequency spectrum for the artificial square ice in the low field region shown in [Figure 4d](#)
339 reflects the generation of pairs of oppositely charged Type III vertices separated by a string of
340 magnets with reversed moments as shown in [Figure 4c](#)⁵². From these spectra it is possible to
341 identify features related to the string length and the number of defect pairs. The amplitudes of the
342 spectra, for example, are related to the length of strings, while the frequencies are related to the
343 location of the defects relative to the nanomagnet array boundaries. The magnetic configurations
344 therefore have tell-tale signatures associated with the distribution of local fields acting on the
345 associated elements⁵².

346 Signatures in the resonant spectra can also arise from the non-uniformity of magnetization
347 within the individual nanomagnets. In particular, sufficiently large elements will minimize their
348 stray field energies with a canting of spins near the element ends¹¹⁵. This canting modifies the
349 energy of resonances because of the additional possible configurations at the vertices. For example,
350 for a Type I vertex in an artificial square ice (see [Figure 1a](#)), there are two possible canting
351 directions at a vertex for each element, which results in two additional degenerate
352 configurations¹¹⁵.

353 Investigations of spin wave resonances in frustrated artificial spin systems are not just
354 restricted to Ising-like systems. For example, coupled magnetic vortices arranged in a kagome
355 geometry¹¹⁶, display collective oscillation modes of the interacting vortex cores that are sensitive
356 to the application of an external magnetic field. This system is particularly interesting due to
357 frustration that leads to multiple configurations for the vortices reflected by the resonant
358 frequencies.

359
360 **Magnonic crystals.** In addition to using spin waves as a means to study defect formation in
361 frustrated systems, artificial spin ice geometries may offer a platform for programmable spin wave
362 devices³. In this regard, detection of spin waves in artificial spin ice with spin torque spectroscopy
363 may be especially useful¹¹⁷. A connected geometry able to transmit spin wave information through
364 a lattice is particularly suited for modification and control the spin wave manifold of states¹¹⁸. This
365 concept is widely used for control of optical excitations in photonic crystals with patterned
366 structures designed to diffract optical waves at certain wavelengths. The analogous patterned
367 materials for spin waves are referred to as magnonic crystals and metamaterials, and there is a
368 large body of work on the topic of magnonics and magnonic devices, of which composite and
369 three-dimensional materials are possible future directions¹³. The concept of controlling spin wave
370 band gaps and properties by adjusting interactions between active elements are being explored¹¹⁹⁻
371¹²¹. In artificial spin ice, this can be achieved with end canting¹¹⁵, as discussed earlier in **Section**
372 **2**, and engineering interactions using interface induced chiral symmetry breaking. In addition, a
373 periodic artificial spin ice geometry can be used to create a spin wave band structure where the
374 frequencies near the zone centre can be tuned by setting the magnetic configuration¹²². The
375 introduction of Dzyaloshinskii-Moriya chiral symmetry breaking in artificial spin ice is also of
376 interest for magnonic device applications to give nonreciprocity as well as topologically protected
377 spin waves in the long wavelength limit¹²³.

378 Difficulties with using nanometre scale magnetic elements for magnonic applications can
379 arise simply because the material volumes are small. This can result in weak coupling between
380 elements and signal-to-noise issues. Therefore, the material volume can be maximised by creating
381 an array of holes in a magnetic thin film¹²⁴ often referred to as antidot arrays¹²⁵. Here, instead of
382 creating magnetic elements out of continuous films, one instead etches vacancies in the films with
383 the desired shape, **which can be left empty or filled with another material**¹²⁶. The vacancies create
384 patterns in the local magnetic fields generated by the surrounding material, and define the magnetic
385 environment controlling spin wave propagation. For example, in an array of elliptical holes
386 arranged in an artificial square ice geometry that was characterised with Brillouin light scattering
387¹²⁷, a magnonic band structure was observed as well as a type of spin wave channelling. Another
388 key aspect of such connected geometries is the possibility to transmit shorter wavelength
389 exchange-dominated spin waves through the junctions connecting the elements^{13,128}. The example
390 shown in **Figure 4e-g** on connected artificial kagome spin ice⁵⁴ demonstrates how signatures from

391 monopole resonances can be obtained for a two-step magnetization reversal process driven by a
392 magnetic field applied along different lattice directions.

393
394

395 **5. Further geometries and associated phenomena**

396 One of the main attractions of the artificial spin ice approach is that novel phenomena can
397 be observed in nanomagnet arrays where every aspect can be engineered with various
398 nanofabrication methods. In this section, we discuss a broad range of artificial spin systems, many
399 of which go beyond artificial spin ice, as they all share the philosophy of generating new emergent
400 phenomena arising from the collective behaviour of constituent building blocks. We illustrate a
401 selection of artificial spin systems and their relationships in [Figure 5](#) based on geometries that have
402 been implemented to date. Indeed, the only limitation on possible designs is the imagination of the
403 designer and, in this sense, artificial spin ices are examples of designer metamaterials. For
404 example, it is possible to deliberately introduce lattice defects such as an edge dislocation in the
405 artificial square ice, which is a topological defect in the structure ([Figure 6a](#)) and results in the
406 creation of domain boundaries between areas of Type I ground state order that have one end pinned
407 at the defects ³⁷.

408 One of the first alternative lattices to the kagome and square geometries was the
409 ‘brickwork’ lattice ¹²⁹, which keeps the long axes of the nanomagnets orthogonal to each other,
410 just as for square ice, but reduces the number of nanomagnets meeting at each vertex to three,
411 making it topologically equivalent to the artificial kagome spin ice. However, since the lattice is
412 not frustrated, its properties are more similar to the square than kagome spin ice. This demonstrates
413 that the details of the geometry and symmetry are important in artificial frustrated systems, and
414 they cannot be classified on topology alone.

415 As shown in [Figure 1](#), the standard four vertex types of the square ice have a particular
416 hierarchy of energies. This hierarchy can be modified in order to increase the frustration in the
417 lattice. One example of tuning the hierarchy was to stretch the lattice to a rectangular shape so that
418 increased degeneracy was achieved ^{46,47}. In another example, the introduction of magnetic
419 nanodiscs into the gaps at the vertices ²¹, so-called ‘slave’ macrospins, was shown to equalize the
420 energy between Type I and Type II vertices. The macrospin of the ‘slave’ discs is XY-like and
421 therefore its direction is dictated by its magnetostatic interactions with the four elongated
422 nanomagnets that surround it at the vertex. At a critical disc size, there is no longer an energy gap
423 between the Type I and Type II vertex configurations, and pinch points are observed in the
424 magnetic structure factor, indicating that the degeneracy of the ice-rule-obeying vertices has been
425 restored. It is also possible to tune vertex energies by modifying the vertex shape and size in
426 systems of connected elements by inserting a hole at the centre of the vertex or decreasing the
427 width of the elements^{48,49}. These approaches are easier in terms of fabrication compared to the
428 approach of offsetting one of the sublattices out of the plane as discussed in [Section 3](#) and shown
429 in [Figure 3a](#).

430

431 The dipolar trident system is also based on the square ice in which the vertex configuration
432 energies can be tuned. Here, each element in a square ice is reproduced to give a group of three
433 parallel nanomagnets, which then interact among themselves¹³⁰. These interactions modify the
434 energies of the different vertex types that can be adjusted by modifying the distances between the
435 three elements in the group and between the groups.

436 The trident lattice has an inherent chirality arising from the geometry of the trident design.
437 In addition, a dynamic chirality was observed in the pinwheel system^{131,132} referred to as a chiral
438 ice⁹, where the nanomagnets in a square ice are rotated through 45° (**Figure 6b**). Here,
439 minimisation of the stray field energy at the edges of the array results in an unexpected ratchet
440 effect in which thermal relaxation leads to a rotation of the magnetization of the array in a unique
441 direction defined by the edge structure. The chiral ice has also been shown to prefer specific
442 ferromagnetic ordering on tuning both the anisotropy of the system and the topology of its edges
443¹³¹, and this leads to the appearance of flux closure states similar to the those found in mesoscale
444 thin film patterned elements. Further geometries incorporating chiral plaquettes are the square-kite
445 tessellation¹⁰⁵ and the vortex ice, which is based on the artificial kagome spin ice and results in a
446 defined chirality of the vortex states in the hexagonal rings of nanomagnets¹³³. Handedness in the
447 magnetic order has also been observed in an artificial magnetotoroidal crystal, where the toroidal
448 moment can be defined with the magnetic tip of an MFM¹³⁴. In this geometry, the nanomagnets
449 are arranged in squares that are repeated across the array, which is equivalent to replacing each
450 element in artificial square ice with two elements (see also ref.¹²⁴ and supplementary information
451 in ref.¹³⁵). Hence, the lattice geometry is non-chiral, but the magnetic configuration is. Similarly,
452 edge bending occurring at +3/-3 charged monopoles in the kagome lattice, shown in **Figure 2k**,
453 adds magnetic chirality to an otherwise non-chiral lattice geometry.

454 It is possible to make different vertices play different roles by designing them to have
455 different coordination numbers i.e. the number of nanomagnets meeting at a vertex. The
456 pentagonal lattice⁵⁵ is an example of this class of lattices that possess a mixed coordination
457 number, since it has either four or three nanomagnets meeting at each vertex. Mixed coordination
458 number lattices not only house emergent magnetic monopoles, but also associated emergent
459 magnetic polarons. Such screened charges have also been directly visualised as a transient state in
460 dice lattice artificial spin ices, which has vertices with coordination of three or six¹⁶.

461 Allowing mixed coordination numbers permits the construction of many more lattices,
462 including the shakti, tetris, and Santa Fe lattices⁸. The most heavily studied of these is the so-
463 called shakti lattice, constructed by removing elements from the square ice system in such a way
464 that there is a mixture of coordination numbers 3 and 4 (**Figure 6c**). This decimated square ice has
465 been shown theoretically to possess a quasicritical ice phase with critical correlations similar to
466 those found in the Coulomb phase of the pyrochlore spin ices¹³⁶. Artificial spin ices based on the
467 shakti lattice have also been experimentally realised in both static⁷ and thermally active¹³⁷ forms.
468 In this class of vertex models, the frustration does not arise from the properties of an individual
469 vertex, but rather from the inability of neighbouring vertices to take up their lowest energy
470 configuration at the same time. The phenomenon is known as topologically-induced emergent

471 frustration ⁷, which can only be realised with artificial spin systems since they have no known
472 analogues in nature, and remains a fruitful avenue for future work. A modified version of the shakti
473 lattice in which some of the nanomagnets are longer than the others has been shown to order
474 differently due to the difference in switching barriers for the different nanomagnets ¹³⁷. Combining
475 designs with different density and connectivity, for example in the tetris lattice ¹⁵, can also give
476 rise to regions with different energy barriers as well as reduced dimensionality.

477 In addition to Ising-like systems, it is possible to create so-called dipolar-coupled XY
478 systems from circular nanomagnets with stripe ordered low energy states ^{19,22}, and introduction of
479 positional disorder in such artificial spin systems modifies the phase diagram to give a microvortex
480 phase ²⁰. Introducing random interactions and/or lack of crystalline order can be a route to obtain
481 spin glass behaviour¹³⁸, which has been investigated in for example XY systems with positional
482 disorder²⁰ or in systems of randomly placed elongated magnets¹³⁹. In addition to constraining the
483 magnetic moments with shape anisotropy, the in-plane magnetization of the nanomagnets can be
484 defined using single crystal materials, restricting the magnetization to point along crystallographic
485 axes. A tuneable dipolar four-state Potts model system can be created employing single crystal
486 nanomagnets with cubic anisotropy, which can order ferromagnetically, antiferromagnetically or
487 in an ice-like manner depending on the relative orientation between the easy axes and bonds
488 connecting the magnets ²⁴ (See [Section 3](#) and [Figure 3](#)).

489 At the boundary between crystals, with perfect discrete translation symmetry, and glasses,
490 with total absence of any such symmetry, lie the quasicrystals. These possess order – by knowing
491 a small part of the structure, one can follow rules to construct the rest of it and fill all of space –
492 but lack discrete translational symmetry. The bulk quasicrystals contain both rare earth ¹⁴⁰ and
493 transition metal ¹⁴¹ magnetic species and display spin glass-like freezing when the spins are dilute.
494 For a two-dimensional analogue of a quasicrystal, one can consider the Penrose tiling ¹⁴². Indeed,
495 Heisenberg spins on the nodes of a Penrose tiling have been studied theoretically and recreated as
496 a macroscopic model of magnets ¹⁴³, and the resulting magnetic configuration can be shown to
497 correspond to interpenetrating non-collinear sublattices in a higher dimensional structure ¹⁴⁴. This
498 work provided the inspiration to build a microscopic artificial analogue of a magnetic quasicrystal
499 placing nanomagnets with Ising-like macrospins along the edges of a Penrose tiling.

500 Penrose patterns come in two forms, known as kite-and-dart or rhomboid, with the names
501 derived from the shapes of the tiles used to form them. Bhat et al. have constructed Permalloy
502 lattices based on the kite-and-dart form, where the magnetic elements are connected to form a
503 continuous network ¹⁴⁵. These were investigated using the macroscopic probes of SQUID
504 magnetometry and FMR, revealing well-defined switching fields and a rich mode structure.
505 Magnetic imaging of such patterns revealed spatially distinct ordered and frustrated sublattices ¹⁷
506 and reversal by means of two-dimensional avalanches ¹⁴⁶, **with vortices forming metastable**
507 **transient states**¹⁴⁷. An artificial magnetic quasicrystal built by placing discrete nanomagnets along
508 the edges of rhomboid tiles ([Figure 6d](#)) ¹⁸, allows the effects of variable coupling to be studied by
509 varying the spacing between the nanomagnets. In contrast to the kite-and-dart quasicrystal, all the
510 links between vertices of the rhomboid tilings are the same length, and this lattice type is less

511 frustrated than the kite-and-dart lattice¹⁴⁸. With a combination of theoretical considerations and
512 magnetic force microscopy, it was shown that the pattern contains a rigid “skeleton” of moments
513 that spans the patterns and possesses a two-fold degenerate long-range-ordered ground state that
514 surrounds free moments that lead to extensive degeneracy, reminiscent of the heterogeneous
515 ordering in the tetris lattice¹⁵. This behaviour is similar to the decagonal ordering in a Heisenberg
516 system¹⁴³. In addition to the lack of translational symmetry of the lattice, the quasi-one-
517 dimensional nature of the skeleton enhances the frustration compared to other spin ice systems,
518 such as the square ice. This is because 1D Ising models do not order and means that the ground
519 state is very difficult to access, even by thermal annealing, offering a possible microscopic
520 explanation for the glassy freezing in bulk magnetic quasicrystals.

521 As many of the examples above illustrate, topologically equivalent geometries can behave
522 very differently due to different point symmetry at the vertices. Lower symmetry at the vertices
523 leads to non-degenerate interactions between the nanomagnets. For example, both the kagome and
524 the brickwork lattice have vertices where three nanomagnets meet but, while the vertices of the
525 kagome lattice gives perfect degeneracy of the vertex states, the vertices in the brickwork lattice
526 are non-degenerate because they are shaped like a ‘T’. So even though the pairwise interactions
527 are frustrated in both systems, in the artificial kagome spin ice, any of the three interactions at the
528 vertex can be frustrated, while in the brickwork lattice it is favourable to arrange the frustration on
529 parallel moments. This connection between geometry and frustration limits the way we can design
530 new lattices to study frustration. One can, however, bypass this limitation by going beyond systems
531 where the frustration is a result of pairwise interactions, and rather locate the frustration on the
532 vertices themselves. One way to achieve this is in mixed-coordination systems, where the different
533 vertex types have well-defined low-energy configurations, but they are arranged in such a way that
534 not all vertices can take their lowest energy configuration¹⁴⁹. Such ‘unhappy vertices’ thus provide
535 topologically protected excitations above the ground state and, in proper geometries, the
536 degeneracy of the allocation of such vertices grows exponentially with the size of the system
537 leading to a degenerate low-energy manifold^{8,150}.

538

539 6. Conclusions

540 We highlight here the key directions for future research in the field of artificial spin ice
541 (Figure 7). In terms of fundamental science, there are many more geometries and topologies to
542 explore that can lead to the discovery of new emergent phenomena. The exploration of the phase
543 diagrams in artificial spin systems has only recently begun, spurred on by the creation of systems
544 with coupled superparamagnets in thermodynamic equilibrium and the possibilities for
545 characterisation with low energy muon spin spectroscopy,¹¹ as well as neutron and synchrotron x-
546 ray scattering. In particular, the use of indirect measurement techniques such as resonant soft x-
547 ray scattering^{12,97,151,152}, neutron scattering and reflectometry^{100-102,153}, and x-ray photon
548 correlation spectroscopy (XPCS)^{82,154} can contribute to further understanding of the long and
549 short-range spatial correlations in these frustrated systems. The low temperature phases in artificial

550 kagome ice have yet to be observed, and there are many different classes of phase transitions to
551 explore, not only Ising but also Kosterlitz-Thouless^{22,93,155} and Potts transitions⁹⁴ to name a few.
552 For this, not only do the relevant spin-ice geometries need to be created, but characterisation
553 methods need to be developed to measure magnetization dynamics at high spatial and temporal
554 resolution, in order to quantify and classify the transitions unambiguously.

555 The observation of novel emergent phenomena will be enabled by ever improving
556 nanofabrication methods to create new geometries. Indeed, we have only just started to explore
557 3D structures, which bring added degrees of freedom^{156,157}. One route to create 3D structures is
558 to manufacture layered systems^{41,43,158}, applying the same lithography methods used to create the
559 2D systems. Alternatively, 3D structures (see examples in **Figure 5**) can be manufactured with two-
560 photon laser lithography¹⁵⁹⁻¹⁶¹ to create, for example, a buckyball structure¹⁶², which is a 2D
561 connected spin ice structure wrapped around on itself, and also to manufacture more complex
562 structures¹⁶³. Smaller 3D structures can be made with focused electron beam induced deposition
563^{164,165} and self-assembly methods^{153,166,167}. For the creation of novel systems, the only limitation
564 is our imagination and ingenuity to design structures that will generate novel phenomena. In terms
565 of magnetic materials, up to now mainly ferromagnetic thin film materials have been explored, but
566 other materials offer additional functionality to artificial spin ice. For example, in nanomagnets
567 fabricated from ferrimagnetic alloys⁸⁷, one can tune the compensation temperature to give specific
568 temperature dependent behaviour of the magnetization, or one can introduce complex anisotropies
569 with single crystal materials^{24,168}, which can also be adapted by modifying the nanomagnet
570 shape¹⁶⁹. Interesting behaviour can be obtained with hybrid thin film systems combining, for
571 example, different magnetic thin films, magnetic thin films with superconductors^{170,171}, or
572 magnetostrictive and piezoelectric materials in so-called multiferroic composites.

573 In terms of devices, there are many applications where artificial spin ices have the potential
574 to play an important role, including data storage, computation, and encryption, with prospects for
575 their use in the emerging field of unconventional computing, including non-Boolean or
576 neuromorphic computation¹⁷². For these applications, it will be important to find a way to access
577 the large number of moment configurations⁸⁴. Furthermore, it will be important to develop
578 methods to write data electrically in the form of the magnetic state of individual or collections of
579 magnets utilising, for example, spin orbit torques¹⁷³ or spin transfer torques, and to read out the
580 states of individual magnets or an array of magnets with transport measurements^{66,99,174}. One
581 might implement magnetic tunnel junctions to access individual nanomagnets, but care would be
582 needed to ensure that the tunnel junctions do not perturb the magnetic configurations in an
583 untoward manner. In addition to electric currents, other stimuli that can be implemented to control
584 the magnetic configurations include electric and magnetic fields¹³⁵, strain⁵⁸ and heat⁵⁷.

585 The first demonstrations of computation with arrays of magnets exploiting artificial spin ice
586 geometries have been performed¹⁷⁵⁻¹⁷⁸, but there is still quite some way to go in terms of
587 demonstrating functional devices that are CMOS-compatible. Other possibilities for devices
588 include the exploitation of emergent magnetic monopoles, manipulating them in magnetic devices
589 in the same way that the flow of electrons or spins is controlled in electric or spintronic devices.

590 For this, basic control of monopole propagation has already been demonstrated by modifying the
591 shape of individual nanomagnets (see supplementary information in Ref. ⁶). For neuromorphic
592 computing, methods need to be found to mimic the spike-timing-dependent plasticity associated
593 with neurons and eventually to create neural networks. For such bioinspired computation, it will
594 be critical to implement an active or self-regulating control, for example manipulating the
595 magnetic state of individual nanomagnets through strain in a multiferroic composite ⁵⁸. It will also
596 be important to be able to tune the interactions between the nanomagnets, something that is just
597 starting to be implemented^{48,49,173} and is likely to lead to new phenomena as well as new devices.

598 Mobile magnetic monopoles are also of interest for high frequency dynamics, in the spirit of
599 previous studies on pyrochlores, where THz and GHz dynamics were used to investigate
600 possibilities for monopole generated responses ¹⁷⁹. The potential to control avalanche dynamics ⁶
601 provides a first step towards using artificial spin ice systems as platforms for reprogrammable
602 magnonic resonators and crystals. The flexibility in design may allow, for example, the creation
603 of artificial spin geometries that can channel high frequency excitations along a particular pathway
604 as chosen by the trigger of a specific avalanche or some other mechanism of generating a desired
605 magnetic configuration. Such devices would be of interest as elements in reconfigurable
606 microwave circuits.

607 Moreover, concepts such as microwave-assisted reversal, which have been pursued for
608 applications in data storage, can inspire new devices constructed from artificial spin ice systems.
609 For example, GHz frequency magnetic fields could be used to guide a frustrated artificial spin ice
610 system into a specific magnetic configuration, which could be exploited in the logic and
611 neuromorphic computing concepts mentioned above. Another route to manipulate magnetic
612 configurations in artificial spin systems is to employ hybrid elements combining nanomagnets with
613 thermoplasmonic heaters, allowing for fast, spatially-selective, and element-specific optical
614 control of local temperatures ^{180,181}.

615 For devices, both scalability and low power operation are essential. In terms of scalability,
616 it has been shown that artificial spin systems can be fabricated down to the atomic scale with the
617 atoms precisely placed on a surface using a scanning tunnelling microscope tip ¹⁸². While such
618 atomic-scale artificial spin systems can only be realised at low temperatures, the small length
619 scales mean that the interactions can be modified, with the RKKY - like interactions being
620 ferromagnetic or antiferromagnetic depending on the distance between the atoms. These atomic-
621 scale systems may even provide a route to create analogies to the Kitaev model and quantum spin
622 liquids. With a view to low power applications, it would be feasible to create devices working at
623 the Landauer limit ¹⁸³.

624 Finally, it should be mentioned that artificial spin systems are not restricted to nanomagnets
625 but can also be implemented with colloids ¹⁸⁴⁻¹⁸⁷, superconductors ^{188,189}, and buckled polymer
626 structures ¹⁹⁰, and this list is by no means exhaustive. These non-magnetic systems have additional
627 interesting phenomena of their own. For example, in colloidal ice, the ice rule can be destabilised
628 by changing the topology of the lattice by decimation, with effective charges that can rearrange
629 and screen the defects ¹⁹¹. Artificial spin ice designs can also be used to create systems with a

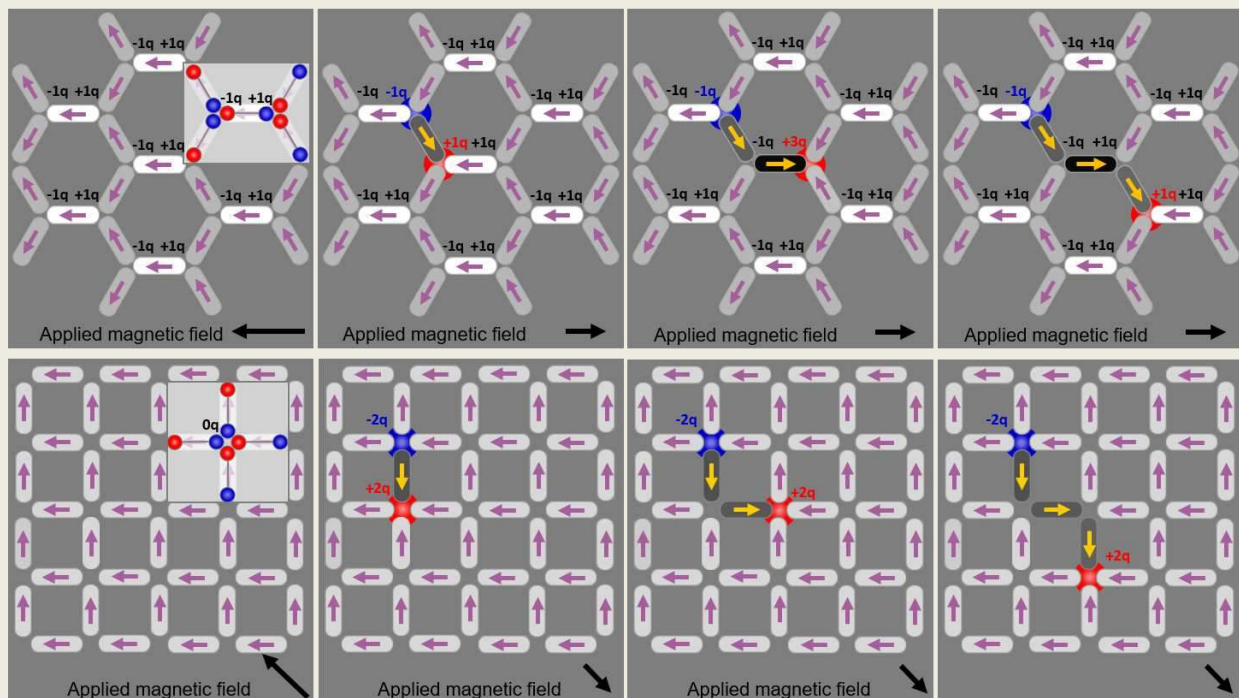
630 strong plasmonic response ¹⁹². In addition, artificial spin ices made of nanomagnet arrays can be
631 used to control other phenomena including particles for biological applications ¹⁹³, the behaviour
632 of vortices in superconducting materials ¹⁷⁰ or skyrmions^{194,195}. The ways in which the field can
633 develop is limited only by the ingenuity of the researchers. Inspiration for the field will continue
634 to increase as the technology evolves to create artificial spin ice geometries at ever decreasing
635 length scales and in three dimensions, to manufacture them from thin film materials with
636 innovative properties and to characterise the spatial correlations at ever faster timescales.
637

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Box 1 | Emergent magnetic monopoles in artificial spin ice

For the identification of emergent magnetic monopoles in a system consisting of discrete mesoscopic spins, we start with artificial kagome spin ice (top panels)^{6,196} and, for simplicity, we create a reference state with all moments pointing to the left by applying a magnetic field to the left (top left panel). We then take the lead from the theoretical description of emergent magnetic monopoles in bulk spin crystals,^{25,197} and map each dipole moment to a charge dumbbell (see inset in top left panel). On doing this, we find that the net charge at the vertices will alternate between $Q_0 = +q$ and $Q_0 = -q$, so forming a charge background that is non-zero at the vertices. A string of magnets can then be reversed by applying a magnetic field in the opposite direction. While the vertex charge within the string is the same as the charge background, at one end of the string it **exceeds** the background vertex charge by $\Delta Q = Q - Q_0 = +2q$ (a monopole) and, at the other end, **the background charge is reduced** by $\Delta Q = Q - Q_0 = -2q$ (an antimonopole). This defines emergent magnetic monopoles, i.e. **those vertices with $\Delta Q = Q - Q_0 = \pm 2q$, as long as there is a dilute population**. However, this definition breaks down as magnetization reversal proceeds. One can therefore consider instead a coarse-grained charge density **that results from convolution with a Gaussian of width larger than the nearest neighbour distance**. The resulting averaged charge density resembles the magnetic flux detected with a magnetic force microscopy (MFM) **that cannot distinguish features smaller than the vertex separation**. The regions of high and low magnetic flux match very well with the position of the $\Delta Q = \pm 2q$ charges, and can now be used to identify the emergent magnetic monopoles at any monopole concentration^{6,34}. It is important to note that, as the string expands magnet-by-magnet, the vertex charge, Q , at the ends of the string changes and the emergent magnetic monopoles are not necessarily associated with ice rule breaking vertices. This is an important point; the vertex charge itself is not **conserved** but the ΔQ charges are topologically protected quasiparticles. Similarly, identifying vertices with $\Delta Q = \pm 2q$ allows us to locate monopoles in the artificial square ice. Here the situation is **simpler** since the charge background is locally zero so that **the change in charge is always equal to the vertex charge ($\Delta Q = Q$)**. Therefore, **not only** the emergent magnetic monopoles, **but also their respective vertex charges** are topologically protected.



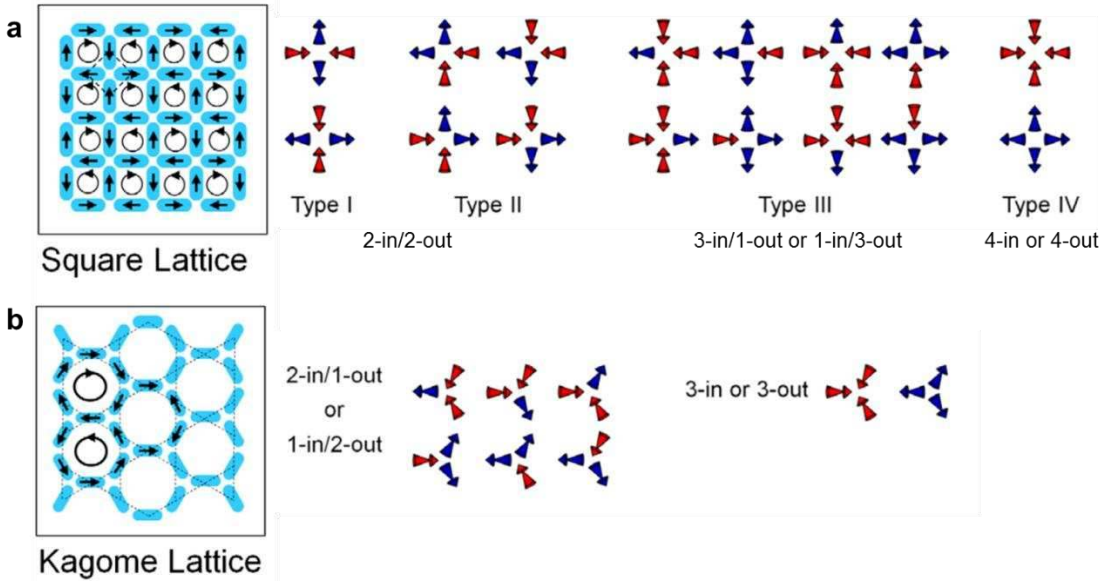


Figure 1 | Classical artificial spin ice systems with elongated single-domain nanomagnets arranged **(a)** on the square lattice and **(b)** on the kagome lattice resulting in a honeycomb structure. To the right, the possible magnetic moment configurations at the vertices are given with the energy of the configurations increasing from left to right.

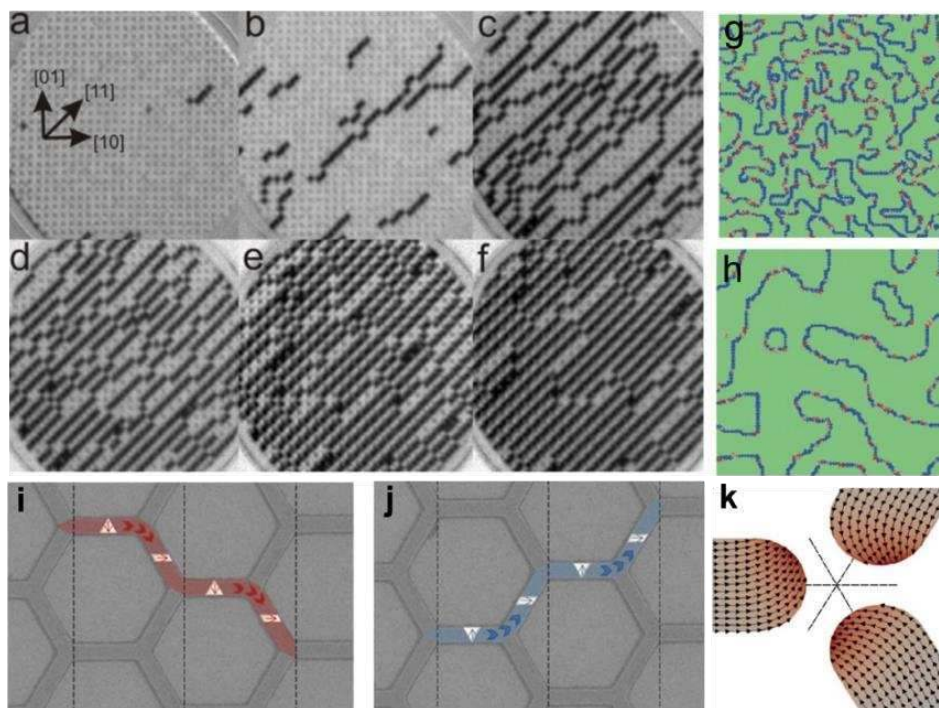


Figure 2 | Propagation and chirality of monopole-like magnetic charges in artificial square ice. a-f | Observation of the creation and separation of charge defects in artificial square ice. Strings of reversed magnets are left behind, which eventually coalesce to form ground state domains separated by domain boundaries [Reprinted from Farhan et al. (supplementary information)³³]. **g,h |** The domain boundaries themselves contain charge defects, whose motion provides a decrease in the domain wall length and eventually their annihilation (reprinted from Budrikis et al. ³⁹). **i,j |** Domain walls within the elements of a connected nanowire network have a chirality, with clockwise or anticlockwise rotation of the magnetization within the wall that influences their path through the network (reprinted from Zeissler et al. ⁶⁰) **k |** Additional degree of freedom of monopoles arising from the chiral nature of edge bending at a vertex (reprinted from Rougemaille et al. ⁷¹).

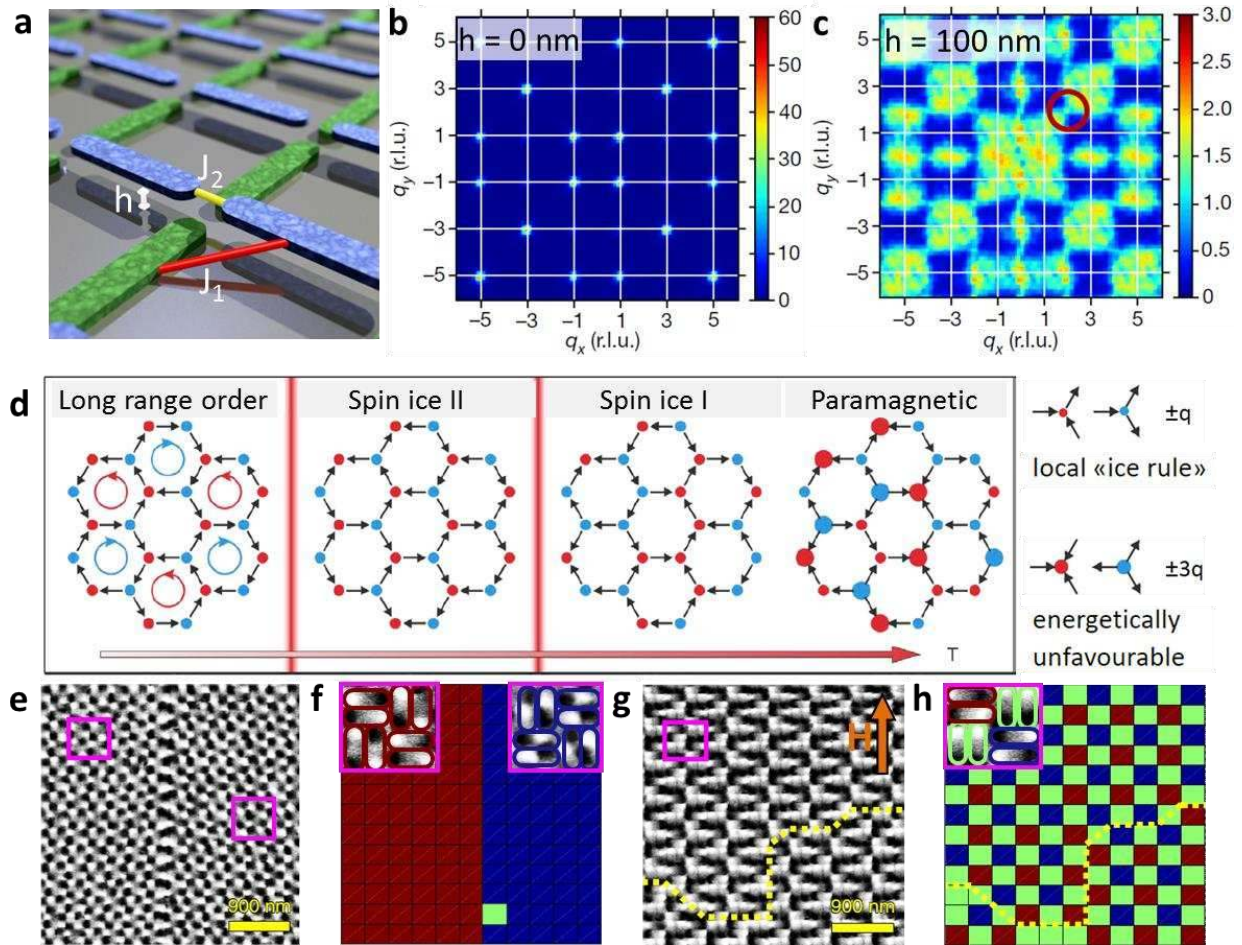


Figure 3 | **a** | Schematic of an artificial square ice in which one sublattice is offset out of the plane by a height, h , in order to modify the J_1 interactions and **b,c** | magnetic structure factors deduced from MFM images for $h=0$ nm (**b**) and $h=100$ nm (**c**). The colour scale refers to the intensity at a given point (q_x, q_y) in reciprocal space. The red circle in **c** encloses a pinch point (Reprinted from Perrin et al. ⁴³). **d** | Schematics of the predicted magnetic phases as a function of temperature in artificial kagome spin ice. Magnetic charges of opposite sign are indicated in red and blue at the vertices with small dots denoting $\pm 1q$ and large dots denoting $\pm 3q$ magnetic charges (Inspired by Möller and Moessner ⁴²). **e-h** | Ordered Potts states in a 'quadrupolar' ice. Magnetic force microscopy (MFM) image (**e**) and corresponding bitmap (**f**) of a zero-field-annealed system favouring a ferro-quadrupolar phase due to strong dipolar interaction between intraplaquette nanomagnet moments. Magnetic force microscopy image (**g**) and corresponding bitmap (**h**) of a lattice annealed in a magnetic field favouring an antiferro-quadrupolar phase due to strong dipolar interaction between interplaquette nanomagnet moments. Pink square frames in the MFM images indicate plaquettes of four moment pairs and the colours in the bitmaps represent the different Potts states for the moment pairs, where red and blue denote the ferro-quadrupolar states with opposite chirality (+1 and -1) and green denotes the antiferro-quadrupolar state (Modified from Sklenar et al. ¹⁰⁸).

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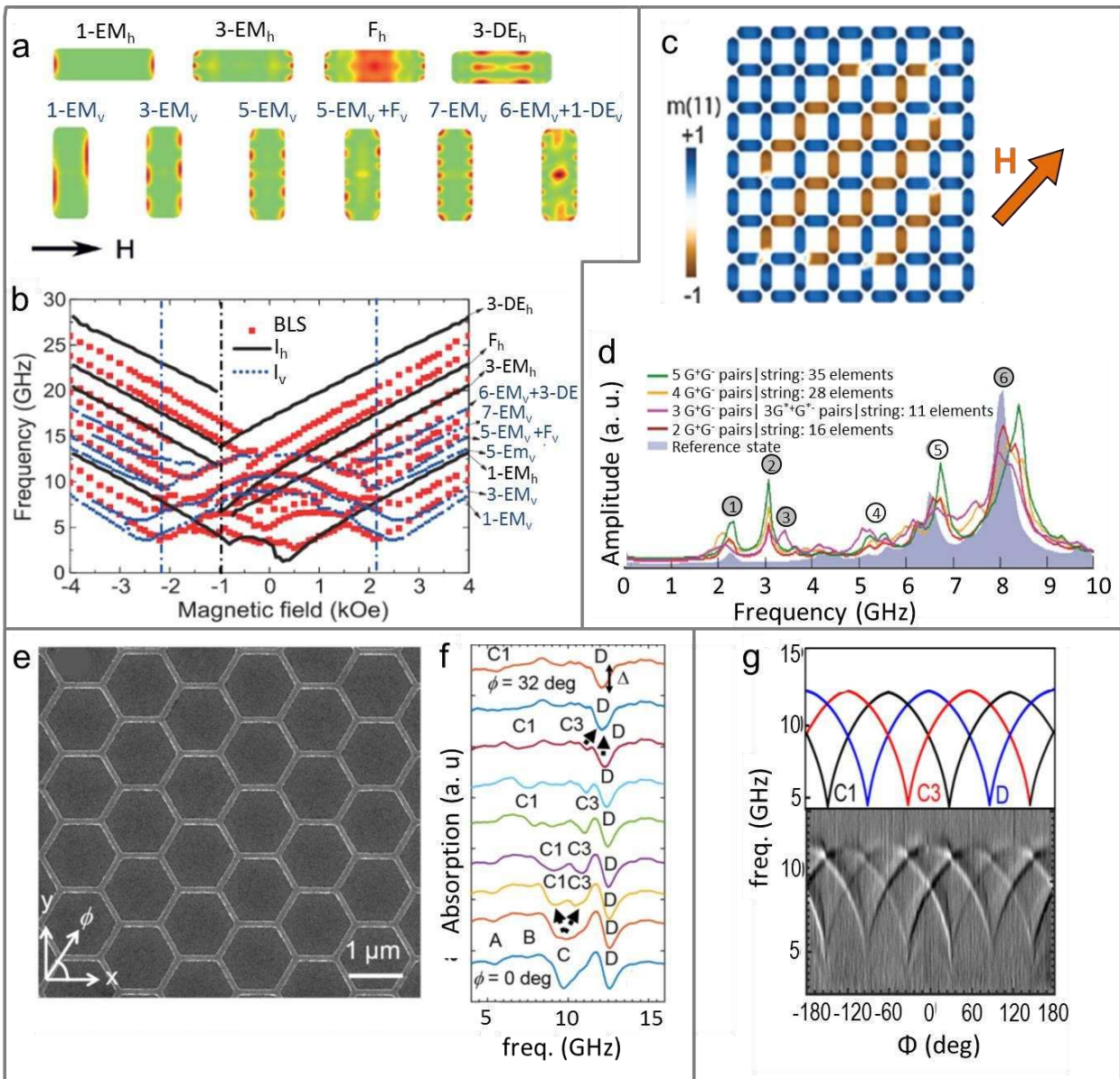


Figure 4 | a,b | Spin wave resonances observed in an artificial square ice using Brillouin light scattering (BLS). Frequencies as a function of magnetic field (**b**) and the corresponding mode profiles (**a**) (reprinted from Li et al. ¹¹⁰). Red squares are BLS data. Black solid and blue dotted lines are simulation results for horizontal (I_h) and vertical (I_v) isolated islands with respect to the field, respectively. The blue and black dashed dotted vertical lines indicate switching fields of the horizontal and vertical islands, respectively. Labels for the modes indicate eigenmodes with specific spatial characteristics such as an edge mode (EM), a fundamental resonance (F) and a Damon-Eshbach mode (DE). The mode labelled F_h is mainly a standing mode centred in the element with some localisation at the ends. The mode labelled 3-DE_h is a higher order standing mode. The mode shown in the bottom furthest right of panel (a) is a hybrid of two closely spaced modes. **c,d** | Artificial square ice composed of 112 magnetic elements used for simulating spin wave spectra for a magnet field applied in the (11) direction (**c**). The lattice contains four monopole-antimonopole (G⁺G⁻ or G⁻G⁺) pairs connected by Dirac strings extending over 28 elements. (**d**) Spin wave spectra calculated for strings of reversed magnetic elements in an artificial square ice for increasing string length and number of monopole-antimonopole pairs compared to the reference state. The grey shaded number labels correspond to the main, distinct signatures of the topological defects. The resonance peaks shift to larger

frequencies when the strings become longer (reprinted from Gliga et al. ⁵²). **e,f,g** | FMR in a connected artificial kagome spin ice. **e** | SEM image of a connected artificial kagome spin ice. **f** | Resonance response for different directions Φ of the applied magnetic field ($H = 1$ kOe). At $\Phi=0^\circ$, four modes can be observed, labelled A, B, C and D. The mode C *splits into two modes (C1 and C3) when Φ increases, and C3 merges with D at $\Phi \approx 30^\circ$* . The three modes C1, C3 and D originate from elements with the three different orientations. **g** | Spin-wave absorption spectra as a function of in-plane angle Φ (*bottom*) combined with a sketch highlighting the extracted branches (top). The branches corresponding to modes C1, C3 and D are represented by black, red and blue lines, respectively, in the sketch. (Reprinted from Bhat et al. ⁵⁴).

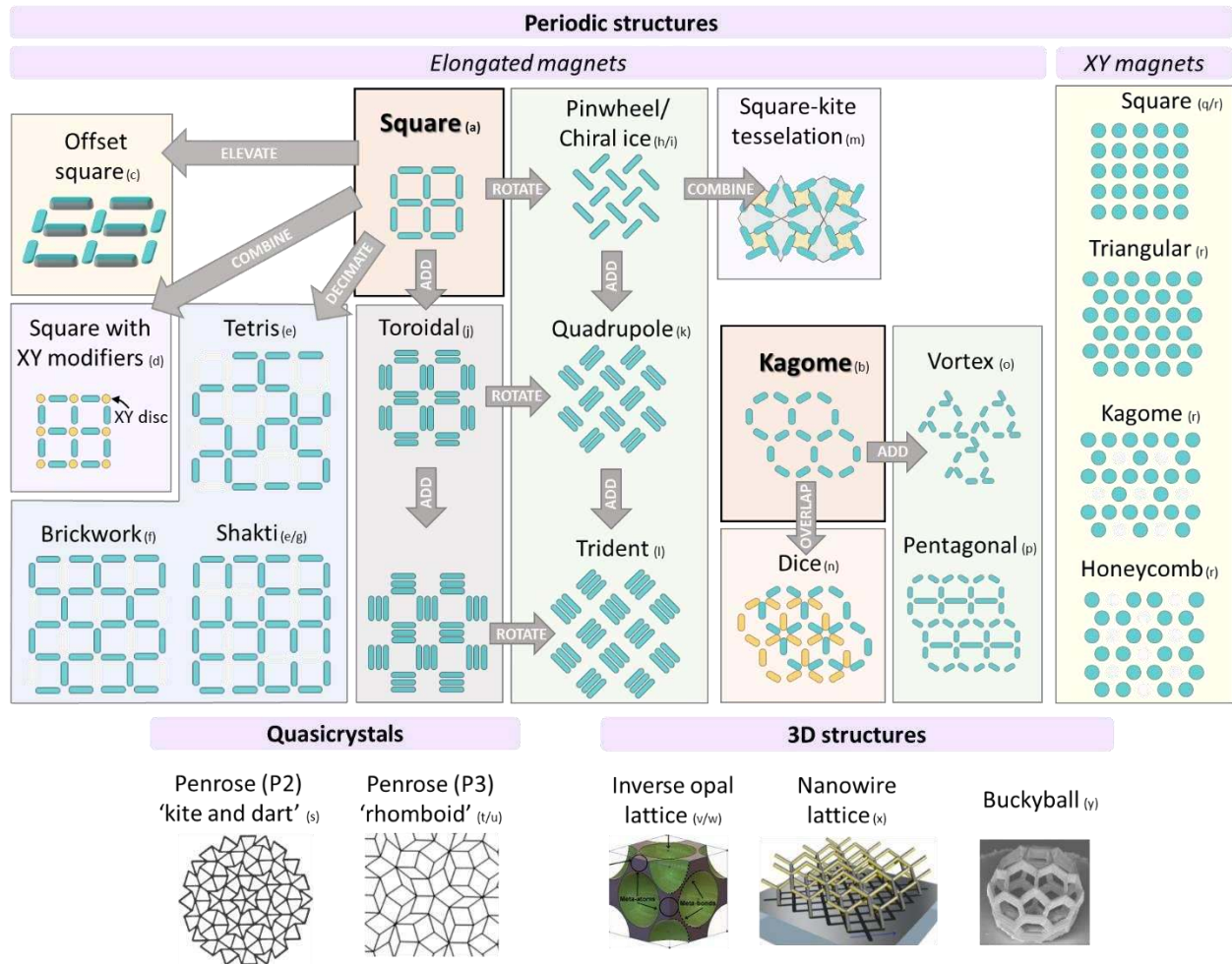


Figure 5 | The family of artificial spin systems going beyond artificial spin ice. Here we highlight some of the connections between the different lattices that have either been realized experimentally or considered theoretically. Other important artificial spin systems omitted from the figure include in-plane magnetization systems with the nanomagnets placed on the Shuriken¹⁹⁸, modified shakti¹³⁷, Santa Fe⁸, Ising stripe¹⁹⁹ and rectangular⁴⁷ lattice. In addition, the in-plane magnetization of the nanomagnets can be further constrained by introducing magnetocrystalline anisotropy, resulting in Potts state magnetic configurations²⁴. Another important class of systems that is not shown here consists of magnetic dots with out-of-plane magnetization, which can be fabricated with the same lattice geometries as the XY systems, although their behaviour is fundamentally different^{106,200,201}. For the quasicrystal lattices, we highlight connected geometries, although quasicrystal systems made up of individual magnets have also been investigated (See Section 5). In addition to the connected quasicrystal lattices, examples of other lattices with connected elements exist in e.g. square¹¹⁷, kagome⁵⁴ and brickwork²⁰² geometries. These are topologically equivalent to the antidot lattices¹²⁵ mentioned in Section 4 on magnonic crystals and are also interesting networks for the observation of domain wall motion (see Section 2). References a-y are found at the end of the document, before the main reference list.

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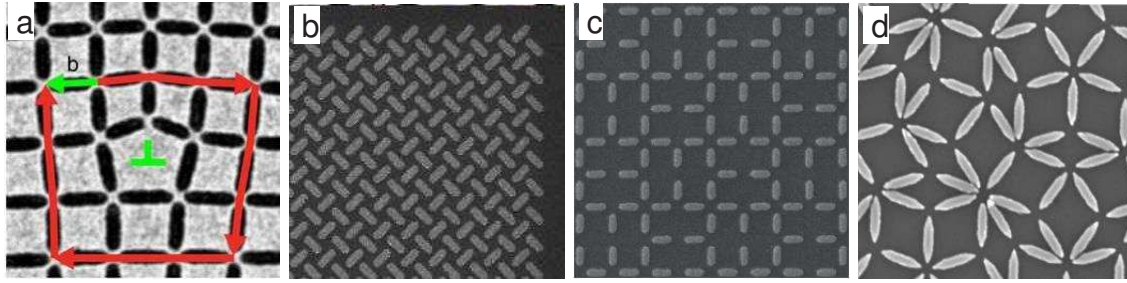


Figure 6 | Experimental realisations of selected artificial spin ice geometries. **a** | In-focus transmission electron micrograph of a dislocation point defect in an artificial square ice. The Burgers circuit and vector **b** are shown in red and green, respectively (reprinted from Drisko et al. ³⁷). **b** | Scanning electron micrograph of part of a chiral ice array, based on rotating every element of a square ice through an angle of 45° . This results in a dynamic chirality with the magnetization rotating in one sense during thermal relaxation (reprinted from Gliga et al. ⁹). **c** | Scanning electron micrograph of part of a shakti lattice (reprinted from Gilbert et al. ⁷). **d** | Scanning electron micrograph of part of a rhomboid quasiperiodic Penrose tiling similar to that studied by Shi et al. ¹⁸.

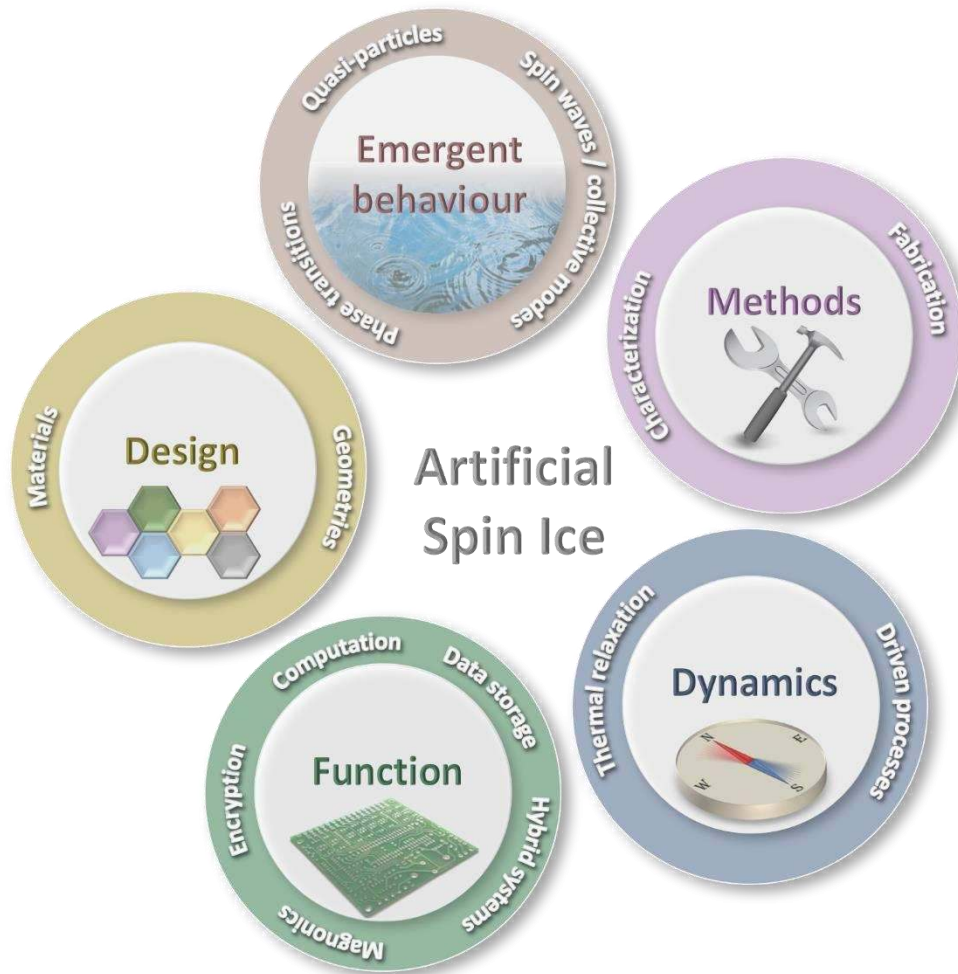


Figure 7 | Key directions for future research in artificial spin ice.

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