Invited Paper

Advances in Nonlinear Predictive Control: A Survey on Stability and Optimality

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Abstract: Some recent advances in stability and optimality for the nonlinear receding horizon control (NRHC) or the nonlinear model predictive control (NMPC) are assessed. The NRHCs with terminal conditions are surveyed in terms of a terminal state equality constraint, a terminal cost, and a terminal constraint set. Other NRHCs without terminal conditions are surveyed in terms of a control Lyapunov function (CLF) and cost monotonicity. Additional approaches such as output feedback, fuzzy, and neural network are introduced. This paper excludes the results for linear receding horizon controls and concentrates only on the analytical results of NRHCs, not including applications of NRHCs. Stability and optimality are focused on rather than robustness.

Keywords: Nonlinear predictive control (NPC), nonlinear receding horizon control (NRHC), nonlinear model predictive control (NMPC), terminal state equality, dual-mode control, terminal cost, control lyapunov, cost monotonicity.

1. INTRODUCTION

Receding horizon control (RHC) or model predictive control (MPC) is a scheme that, at each instant of time, implements the first element of an optimal control minimizing a performance criterion. The terminology "RHC" is used throughout this paper. Since the RHC has several advantages, it has emerged as a successful control strategy in both academic and industrial fields and has been employed in diverse applications. Advantages of the RHC can be summarized as follows.

First, it requires simpler computation algorithms than the widely known optimal control on the infinite horizon. Second, when finite future command is available, it presents good tracking performance, which is an important issue in industrial applications. Third, the RHC presents a proper control strategy for time-varying systems. While the optimal control on the infinite horizon requires all future systems parameters, which are unavailable in actual problems, the RHC needs only finite future system parameters. Fourth, the RHC can handle input and state (I/S) constraints, which derive from the physical limitations and safety requirements of real plants.

RHCs for linear systems have similar properties to the conventional optimal controls. However, RHCs for nonlinear systems have different characteristics from the conventional optimal controls as will be explained later. There are several literatures for survey on the linear and nonlinear receding horizon controls. The five papers in the first part of the book [1] provide a review on the existing results. In [2], a very wide ranging list of references is provided. The three survey papers [3-5] were presented at the Chemical Process Control conference in 1996. Recently, the review articles on MPC have appeared in [6, 7]. In [7], the stability and the optimality for the constrained RHC are focused on.

While the RHC for linear systems has been popular since the 1970s, the 90s have witnessed the fact that nonlinear receding horizon control (NRHC) has received much interest in the academic community. Practical interest in the industry is also driven by the fact that the NRHC is more suitable than conventional controls for today's processes possessing nonlinearities and constraints. Additionally, this interest is due to the availability of computers with high performance.

This paper focuses on stability and optimality for NRHC. It excludes the results for linear receding horizon controls and concentrates solely on the analytical results of NRHCs, not including applications of NRHCs. Stability and optimality are focused on rather than robustness.

The major approaches are as follows. The basic and old approach is to stabilize the systems with a terminal state equality constraint. Since the solution for the

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terminal state equality constraint may not be feasible in many cases, a terminal cost approach is used as an alternative to forcing the state to zero. In order to design stabilizing controls easily, a terminal constraint set approach is chosen for two kinds of controls. For the disturbance and the model uncertainties, minimax criteria have been adopted instead of minimization. By using an auxiliary function or a cost function in place of terminal condition, stability can be obtained without optimality. We introduce some other variants besides major approaches such as output feedback control, neural network and genetic algorithm.

This paper is organized as follows. In Section 2, the basic underlying concept and the problem setting for the receding horizon control for nonlinear systems are described. In Section 3, the existing main results on the NRHC with terminal conditions are presented. In Section 4, the NRHC with a control Lyapunov function (CLF) and a cost monotonicity condition (CMC) are introduced. In Section 5, other approaches excluded in the previous section are shown. Finally, we present our conclusion.

2. FORMULATION OF NONLINEAR RECEDING HORIZON CONTROL

We consider the following continuous and discrete time nonlinear systems:

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

$$x_{k+1} = f(x_k, u_k), \tag{2}$$

subject to input and state constraints as

$$u(t) \ (\text{or } u_k) \in U, \qquad t \geq 0, k = 0, 1, \cdots, \tag{3}$$

$$x(t) \ (\text{or } x_k) \in X, \qquad t \geq 0, k = 0, 1, \cdots, \tag{4}$$

where $u(t) \in U \subset \Re^m$ is the vector of inputs and $x(t) \in X \subset \Re^n$ is the vector of states. The NRHC is obtained by repeatedly solving the following online finite horizon open-loop optimal control problem:

$$\begin{aligned} &\min_{\bar{u}(\cdot)} J(x(t), \bar{u}(\cdot); T) \\ &= \min_{\bar{u}(\cdot)} \int_t^{t+T} m(\tau, \bar{u}(\tau), \bar{x}(\tau)) d\tau + F(\bar{x}(t+T)) \end{aligned} \tag{5}$$

or

$$\begin{aligned} &\min_{\bar{u}.} J(x_k, \bar{u}.; N) \\ &= \min_{\bar{u}.} \sum_{i=k}^{k+N-1} m(i, \bar{u}_i, \bar{x}_i) dt + F(\bar{x}_{k+N}) \end{aligned} \tag{6}$$

subject to the predictive system

$$\begin{aligned} &\dot{\bar{x}}(\tau) = f(\bar{x}(\tau), \bar{u}(\tau)), \bar{x}(t) = x(t) \\ &\text{or } \bar{x}_i = f(\bar{x}_i, \bar{u}_i), \bar{x}_k = x_k \end{aligned} \tag{7}$$

$$\bar{u}(\tau) (\text{ or } \bar{u}_k) \in U, \tag{8}$$

$$\bar{x}(\tau) (\text{ or } \bar{x}_k) \in X, \tag{9}$$

$$\bar{x}(t+T) (\text{ or } \bar{x}_{k+N}) \in S, \tag{10}$$

where $\tau \in [t, t+T]$, $k = 0, 1, \cdots, N-1$, and functions $m(\cdot,\cdot,\cdot)$ and $F(\cdot)$ are weighting functions for the control and the state trajectories on the horizon, and the terminal state, respectively. T or N is the horizon size and $S \subset X$ is the terminal constraint set. $\bar{x}(\cdot)$ or $\bar{x}.$ is the solution of (7) driven by the input $\bar{u} : [t, t+T]$ or $k = 0, 1, \cdots, N-1 \mapsto U$ with the initial condition $x(t)$ or x_k.

In order to distinguish the real trajectories of the state and the input from the predicted trajectories of them, we use different notations such as $\bar{x}$ and x. The optimal control is recalculated over a moving finite horizon $[t \ \ t+T]$ or $k = 0, 1, \cdots, N-1$ at every sampling instance. The system model used to predict the future in the calculation of the NRHC is initialized by the actual system state $x(t)$ or x_k. Then, we obtain an optimal solution $\bar{u}^*(\cdot; x(t), T)$: $[t, t+T] \to U$ or $\bar{u}^*(\cdot; x_k, N)$: $k = 0, 1, \cdots, N-1 \to U$ by solving the finite horizon optimal control problem (5) repeatedly at the sampling instances. The NRHC is defined in the form of a state feedback control by the following optimal solution at the sampling instants:

$$u^o(t) \triangleq \bar{u}^*(t; x(t), T), \text{ or } u_k^o \triangleq \bar{u}^*(k; x_k, N). \tag{11}$$

The cost function from the optimal trajectory will be represented in terms of the current state and the horizon size as $J^*(x(t); T)$ or $J^*(x_k; N)$. The optimal cost function plays an important role in the proof of the stability of various nonlinear receding horizon control schemes, as it serves as a Lyapunov function candidate.

How to obtain an optimal control for nonlinear systems is surveyed in [8]. In [9], the relation between the computation burden and the guaranteed stability is discussed.

3. CONTROL DESIGN WITH TERMINAL CONDITION

3.1. Optimal control with terminal equality constraint

As in the RHC for linear systems, the simplest condition guaranteeing the closed-loop stability is based on the terminal state equality constraint [10-12]. The basic idea in this approach is to solve the finite horizon optimal control problem (5) with terminal cost $F(x(t+T))=0$ subject to (7), (8), and (9). In [11], a strong assumption is necessary to guarantee that the optimal value function is continuously differentiable, which is relaxed in [12] only for local Lipschitz continuity of the optimal value. This strategy allows stability to be checked easily and provides a relatively simple conceptual procedure for determining the feedback control of nonlinear systems.

In [13], a comparatively easy implementable formulation is proposed and the existence of the solution is discussed. It is also shown in [13] that the discontinuous MPC with a terminal equality constraint can stabilize a system that cannot be stabilized by continuous feedback controls.

It is assumed in this approach that one can find the finite horizon optimal control $\bar{u}^*(\cdot;x(t),T)$ driving the current state to the origin at time $t+T$, i.e., $x(t+T)=0$ and satisfying the input and state constraints. The main idea behind proving the stability in this approach is to show the monotonicity property of the receding horizon cost function. This is referred to as the cost monotonicity condition (CMC).

One disadvantage of the terminal state equality constraint is that the current state must be brought to the origin in finite time. This leads in general to feasibility problems for short horizon length. In other words, it is possible to have a very small feasible region. The long horizon length can increase the size of the feasible region, but increasing T causes a computational burden. From a computational point of view, a terminal state equality constraint does demand an accurate solution through an infinite number of iterations in the nonlinear programming problem. Even a very tight approximation may lead to the loss of stability.

It was shown in [14] that this approach enjoys robustness properties that are analogous to those of the infinite horizon LQ control. In particular, robustness margins with respect to gain and perturbations are obtained.

3.2. Optimal control with terminal cost

In the terminal cost based approach [15, 16], a terminal penalty term $F(\cdot)$ in (5) is used to guarantee the stability. Here, the stability depends on how to select the terminal penalty term. $F(\cdot)$ is determined off-line such that the following CMC is satisfied:

$$\begin{aligned}&\frac{dF(\bar{x}(t+T))}{dt}\le -m(t+T,\bar{u}(t+T),\bar{x}(t+T)),\\&F(\bar{x}_{k+N+1})-F(\bar{x}_{k+N})\le -m(k+N,\bar{u}_{k+N},\bar{x}_{k+N}).\end{aligned} \tag{12}$$

The function $F(\cdot)$ turns out to be the control Lyapunov function, which will be explained in (4.1).

$F(\cdot)$ satisfying the above inequality gives an upper bound of the infinite horizon cost functional as follows:

$$F(\bar{x}(t+T))\ge\int_{t+T}^{\infty}m(\tau,\bar{u}(\tau),\bar{x}(\tau))d\tau. \tag{13}$$

$$F(\bar{x}_{k+N})\ge\sum_{i=k+N}^{\infty}m(i,\bar{u}_i,\bar{x}_i). \tag{14}$$

Using the CMC, we can show that the optimal cost function decreases as horizon size T increases.

$$\begin{aligned}\frac{\partial J^*(x(t);T)}{\partial T}&\le\frac{dF(\bar{x}(t+T))}{dt}\\&\quad+m(t+T,\bar{u}(t+T),\bar{x}(t+T))\\&\le 0\end{aligned} \tag{15}$$

By using (15), the stability is guaranteed as follows:

$$\begin{aligned}J^*(x(t);T)&\ge\int_t^{t+\theta}m(\tau,\bar{u}(\tau),\bar{x}(\tau))d\tau\\&\quad+J^*(x(t+\theta);T),\end{aligned}$$

$$\int_t^{t+\theta}m(\tau,\bar{u}(\tau),\bar{x}(\tau))d\tau\to 0,$$

from which it is guaranteed that $x(t)$ approaches to zero. Note that the CMC for the discrete time system is derived in a similar way. (13) and (14) would imply that the optimal cost function on the finite horizon is larger than the cost function on the infinite horizon.

3.3. Optimal control with terminal constraint set

As seen in the previous subsection, the terminal equality constraint may be unsatisfactory for both performance and implementation issues. Hence, the idea of replacing the equality constraint with an inequality one, which is much easier to handle computationally, is proposed in [17-19]. The purpose of this approach is to force the state into the terminal constraint set in finite time. Inside the terminal constraint set, a local stabilizing control is employed. This approach is sometimes called a dual-mode MPC.

For this RHC scheme, it is assumed that the pair (A,B) defined as

$$A=\left.\frac{\partial f}{\partial x}\right|_{x=u=0},\qquad B=\left.\frac{\partial f}{\partial u}\right|_{x=u=0} \tag{16}$$

can be stabilized. Let's consider the LQ controller $u(t)=K_{LQ}x(t)$ and the associated feasibility region $X(K_{LQ})$. Let $W_\alpha\subset X(K_{LQ})$ denote a (smaller) feasibility region for the LQ controller where α is a scalar parameter such that $W_{\alpha''}>W_{\alpha'}$ if $\alpha''>\alpha'$, and $\lim_{\alpha\to 0} W_\alpha = 0$. The dual-mode controller is based on the finite-horizon optimal control problem (5) subject to (7), (8), (9) and (10) with $S = W_\alpha$.

In nominal conditions, the dual-mode control scheme works as follows. If $x(t)$ is not in W_α, one solves the finite-horizon problem. Then the optimal control sequence $\bar{u}^*(\cdot;x(t),T)$ on $[t\ t+T]$ is obtained so that $x(t+T) \in W_\alpha$. The control is applied during a sampling time. In the next step, the control is computed in a way that the performance criterion decreases. It is known that there exists a finite time t such that $x(t) \in W_\alpha$. If the state enters W_α, the controller switches to the linear state feedback $u(t) = K_{LQ}x(t)$. [17] also includes receding horizon implementation to cope with model uncertainties and disturbances. In particular, a variable horizon T is used in the receding horizon implementation. Closed-loop stability of the dual-mode controller is straightforward because W_α is a feasibility region for the LQ controller. There is a trade-off between feasibility and performance. To improve feasibility, α should be as large as possible but this implies the suboptimal LQ control law in a wide region. The main disadvantage of this scheme is that the switching from nonlinear control law to linear state feedback control is somewhat artificial and introduces a certain discontinuity. The procedure for deciding the terminal region W_α, which is done off-line, is found in [20].

The terminal constraint set can be combined with the terminal cost explicitly or implicitly. By using the cost function corresponding to the terminal state as a Lyapunov function, the stability is guaranteed if a locally stabilizing linear control law is applied after the time $t+T$. The linear control law ensures local exponential stability of the equilibrium $x = 0$, and it is assumed that the region of attraction of the linear controller is large enough to be reachable from the initial state within the horizon time T. The set W_α satisfying the inequality (12) is computed in some neighborhood of the origin, on which the linear feedback control is designed for the linearized system around the origin. In [21], the nonlinear receding horizon closed-loop system is shown to be infinite-horizon optimal with the new definition of terminal cost, provided that the terminal cost exactly captures the infinite-horizon optimal value in a neighborhood of the origin. If we consider the quadratic cost in the receding horizon cost functional as

$$\int_t^{t+T} [\bar{x}(\tau)^T Q\bar{x}(\tau) + \bar{u}(\tau)^T R\bar{u}(\tau)]d\tau$$
$$+\bar{x}(t+T)^T Q_f \bar{x}(t+T),$$

there exist Q_f and the set S that guarantee close-loop stability of the associated RHC scheme [22, 23]. Q_f is chosen in such a way that $x^T Q_f x$ is a Lyapunov function for the nonlinear systems, subject to the linear control law $u(t) = K_{LQ}x(t)$. More precisely, S is a feasible region for the LQ controller and is such that

$$\begin{aligned} &f(x,L_{LQ}x)^T Q_f f(x,L_{LQ}x) - x^T Q_f x \\ &< -x^T[Q + K_{LQ}^T R K_{LQ}]x, \qquad \forall x \in S. \end{aligned} \tag{17}$$

Moreover, if we take

$$S = \{x \mid x^T Q_f x \le \alpha\}, \tag{18}$$

subject to (17), S acts as an invariant ellipsoid for nonlinear systems. Procedures for finding Q_f and S are developed in [23, 23]. There are two main disadvantages in this approach. Firstly, it might be difficult to locate finite horizon T such that the chosen neighborhood of the origin S is reachable from any possible initial state. Secondly, if the linearization of the system around the origin cannot be stabilized, then this approach cannot be used.

This approach is addressed for nonlinear unconstrained systems in [16, 23-25]. In [25], selection of the terminal cost and the terminal constraint sets were proposed to guarantee some desired properties such as infinite horizon optimality around the origin. In particular, in [23, 24], a terminal constraint set is automatically satisfied by choosing proper parameters such as the horizon T and the final weighting matrix or functional. This approach is proposed in [22, 26-29] for nonlinear constrained systems.

3.4. Optimal controls with minimax criteria

Minimax performance criteria are preferable when uncertainties or disturbances are considered. In [30, 31], a minimax criterion on a finite horizon is adopted for robust stability. In [30], both a terminal cost and a terminal constraint set are used while discontinuous feedback strategies are allowed. In [31], the additional Lyapunov function such as the CLF is included for closed-loop stability. [32] proposed the minimax RHC with a terminal cost and a terminal constraint set for the constrained piecewise affine systems that include a large class of nonlinear systems. [33-36] consider a nonlinear receding horizon H_∞ control. In [36], a terminal cost is chosen so that the receding horizon H_∞ control problem is solvable. In [34, 35], both a terminal cost and a terminal constraint are used to guarantee the H_∞ performance criterion.

4. CONTROL DESIGN WITHOUT TERMINAL CONDITION

4.1. Control design with control Lyapunov function

Recently, a control Lyapunov function (CLF) was used in conjunction with a receding horizon strategy to consider the stability and the performance simultaneously. In [37], the NRHC was proposed by using the stability constraint of the CLF as

$$\frac{\partial V}{\partial x}[f(x,u)] \le -\sigma(x), \tag{19}$$

where V is the CLF and $\sigma(x)$ is positive definite. This approach provides both the stability properties of the CLF and the performance advantages of receding horizon techniques. While in Section 3.2 a terminal cost was used to satisfy a control Lyapunov function (CLF) for stability, in this approach, a terminal cost is not used and only the CLF is utilized.

The stability comes from the CLF, not from the performance criterion. In order to apply this scheme, we have to know the global CLF for the systems *a priori*. The main drawback of this method is that it further increases the amount of online computations required by imposing state inequality constraints (19). Another application of CLF to the NRHC is to use the CLF as the terminal cost [24, 38, 39], which is dealt with similarly in Section 3.2.

In order to ensure the closed-loop stability in this scheme, we can choose the terminal cost as

$$F(x(t+T)) = \int_{t+T}^{\infty} m(\tau, \kappa(x(\tau)), x(\tau))d\tau, \tag{20}$$

where $\kappa(\cdot)$ is the CLF based controller obtained *a priori*. By appropriate choice of scaling parameter μ, it is possible to find the terminal cost satisfying (19) and

$$F(x(t+T)) = \mu V(x(t+T)), \tag{21}$$

where $V(\cdot)$ is a CLF and μ is a positive constant. In [39, 40], a sub-optimal scheme is used to design the NRHC with the help of the CLF.

The main disadvantage of this technique is that we have to know the CLF *a priori*. All the results on the NRHC with the CLF have not considered the constraints on input and state.

4.2. Control design with cost monotonicity

Basically, determining the NRHC includes the non-convex optimization problem. A radical way of avoiding it is to shift the emphasis away from obtaining the optimal solution and focus on keeping some properties such as stability. The approach is different from the one in Section 3.2 in that the optimal control is not considered. The idea for this problem is addressed in [17, 41] where the dual-mode approach can ensure stability by using a feasible solution rather than an optimal solution to the open-loop control problem. In [18], this idea is extended to a fixed horizon receding horizon control problem. A more general analysis for the sub-optimal receding horizon control scheme is presented in [19]. The NRHC law can be chosen not by minimizing (6), but by finding a predicted control providing a sufficient reduction of the cost

$$\begin{aligned} J(x_k) \le J(x_{k-1}) \\ - \mu m(\bar{x}_{k|k-1}, \bar{u}_{k-1|k-1}), \end{aligned} \tag{22}$$

where $0 < \mu \le 1$, while (8) and (9) are satisfied. This sub-optimal scheme gives an asymptotical closed-loop stability result and the region of attraction under mild conditions [19]. It is noted that it removes the need for finding the global minimum solution for a non-convex optimization problem. All we have to do is to determine the control sequence, which gives a sufficient reduction of the cost function at each step. This method is useful for solving the computational difficulties of the dual-mode control scheme due to the discontinuous stage cost. The result regarding the consideration of the hard input constraint for sub-optimal NRHC is presented in [42].

5. OTHER APPROACHES

Aside from major approaches, there are some variant ones for NRHCs. [43] proposed a NRHC that is different from the conventional RHC in that the horizon size is a variable to be minimized and the control on the horizon is implemented. In this work, inequality contraction constraints are required in order to ensure that the state is contracted by a specified factor before optimization starts again. The critical drawback of this approach is to require a strong assumption. It is shown in [44] that the existence of the NRHC corresponding to this approach cannot be guaranteed.

The NRHC is computationally demanding so that there are many results on how to solve the optimization problem [45-49].

In order to obtain a large feasible set, a large terminal constraint set is necessary. Thus, polytopic invariant sets were considered in [50, 51] like linear systems.

In [52, 53], the stabilizing state feedback NRHC is combined with observers to achieve output feedback stabilization. Specifically, [52] uses the state feedback NRHC with a terminal cost and a terminal constraint set by using the high gain observer.

A fuzzy logic, a neural network, and a genetic algorithm are adopted for the NRHC. In [54-56], a fuzzy logic is used for tuning parameters or obtaining a NRHC. In [55], a terminal cost is used. [57, 58] proposed the NRHC based on a neural network model. [59] uses a generic algorithm in order to generate an empirical dynamic model for a process. [60] adopts neural networks in order to reduce the high computation effort involved in NRHC.

6. CONCLUSION

This paper has surveyed some recent advances in stability and optimality for the nonlinear receding horizon control (NRHC) or the nonlinear model predictive control (NMPC). This paper excludes the results for linear receding horizon controls and concentrates only on analytical results of NRHCs, not including applications of NRHCs.

The approaches with terminal conditions are surveyed in terms of a terminal state equality constraint, a terminal cost, and a terminal constraint set. Other approaches without terminal conditions are surveyed in terms of the control Lyapunov function and cost monotonicity. Additional approaches such as output feedback, fuzzy, and neural network are introduced.

Some advantages and disadvantages for each approach were discussed. The terminal constraint condition is a basic approach and is easy to understand. Since it has some drawbacks in the feasibility of the optimal solution, other approaches have been exploited in order to obtain larger feasible regions.

Since the NRHC is especially useful for many nonlinear processes that require a tight performance specification, this survey paper can help the researchers of NRHCs to perceive the current research trend and determine the appropriate direction for future researches.

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