Advantages of calculating shear-wave velocity from surface waves with higher modes

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Summary

The Rayleigh-wave phase velocity of a layered earth model is a function of frequency and four groups of earth parameters: compressional (P)-wave velocity, shear (S)-wave velocity, density, and thickness of layers. For the fundamental mode of Rayleigh waves, analysis of the Jacobian matrix for high frequencies (5-40 Hz) provides a measure of dispersion curve sensitivity to earth model parameters. S-wave velocities are the dominant influence of the four earth model parameters. With the lack of sensitivity of the Rayleigh wave to P-wave velocities and densities, estimations of these parameters can be made for a layered earth model such that dispersive data vary predominantly with S-wave velocities (Xia et al., 1999a). This thesis is valid for higher modes of Rayleigh waves as well. Experimental analysis indicates that energy of higher modes tends to become more dominant as the source distance becomes larger (Park et al., 1999a). In some cases, higher mode data are necessary since shorter wavelength components of fundamental mode Rayleigh waves are obscured by these higher frequency data where higher modes of Rayleigh waves dominate. As well, our modeling results demonstrate at least two quite exciting higher mode properties. First, for fundamental and higher mode Rayleigh wave data with the same wavelength, higher modes can "see" deeper (longer than the wavelength) than fundamental modes (normally shorter than the wavelength). Second, higher mode data can increase the resolution of the inverted S-wave velocities. A much better S-wave velocity picture can be produced from inversion of surface wave data if higher-mode data are included. Real world examples show how resolution can be improved.

Introduction

Elastic properties of near-surface materials and their effects on seismic wave propagation are of fundamental interest in groundwater, engineering, and environmental studies. S-wave velocity is used to determine "stiffness," one of the key earth properties in construction engineering. Shear-wave velocity as a function of depth can be derived from inverting the phase velocity of the surface (Rayleigh and/or Love) wave (Dorman and Ewing, 1962).

Surface waves are guided and dispersive. Rayleigh (1885) waves are surface waves that travel along a "free" surface, such as the earth-air interface and are the result of interfering P and S_v waves. Particle motion of Rayleigh waves moving from left to right is elliptical in a counter-clockwise (retrograde) direction. This motion is constrained to a vertical plane, consistent with the direction of wave propagation (p. 30, Babuska and Cara, 1991). Longer wavelengths penetrate deeper than shorter wavelengths for a given mode and in general exhibit greater phase velocities and are more sensitive to the elastic properties of deeper layers (p. 30, Babuska and Cara, 1991). Shorter wavelengths are sensitive to the physical properties of surficial layers. For this reason, surface waves possess a unique phase velocity for each unique wavelength, which results in dispersion characteristics.

A series of Rayleigh waves of different frequencies can have the same wave velocity. These different frequency Rayleigh waves for a given phase velocity are known as modes and are characterized by their different number of horizontal nodal planes (planes of no particle displacement within the layer) (p. 60, Garland, 1979). In other words, more than one phase velocity can be associated with a given frequency of Rayleigh wave simply because these waves can travel at different velocities for a given frequency. The lowest velocity for any given frequency is called the fundamental-mode velocity (or the first mode). The next higher velocity above the fundamental-mode phase velocity is called the second-mode velocity, and so on.

Ground roll is a particular type of Rayleigh wave that travels along or near the ground surface and is usually characterized by relatively low velocity, low frequency, and high amplitude (p. 143, Sheriff, 1991). Stokoe and Nazarian (1983) presented a surface-wave method, Spectral Analysis of Surface Waves (SASW), that analyzes the dispersion curve of ground roll to produce near-surface S-wave velocity profiles.

A research group of the Kansas Geological Survey (KGS) investigated how to estimate the S-wave velocity from ground roll, focusing mainly on the fundamental mode of Rayleigh waves. The resulting technique consists of: 1) acquisition of wide band ground roll using a multi-channel recording system; 2) creation of efficient and accurate algorithms designed to extract Rayleigh-wave dispersion curves from ground roll using a basic, robust, and pseudo-automated processing sequence; and 3) development of stable and efficient inversion algorithms to obtain S-wave velocity profiles. The main product of this research, called Multi-channel Analysis of Surface Waves (MASW), has been published by Park et al. (1999b), Xia et al. (1998 and 1999a), and Miller et al., (1999).

Based on experience, the fundamental-mode phase velocities when calculated with high accuracy (e.g., Xia et al., 1999b) can generally provide reliable Vs velocities (±15%). However, in cases where estimations of the fundamental-mode phase velocities are associated with high degree of error (e.g., if the fundamental-mode Rayleigh waves are contaminated by body waves and/or higher-mode Rayleigh waves), inversion results will become unstable. It is well known that instability in the inversion of geophysical data generally results in situations where small changes in data result in large fluctuations in a model. The degree of instability could be reduced either by imposing constraints or by including an extra independent data set to the inversion procedure. If no constraints or extra data are available, the inversion can be stabilized by reducing the resolution of the inverted Vs model. This can be demonstrated with a real world example.

Higher modes are independent from the fundament-mode phase velocities. It has been reported that the generation of higher modes has been associated with presence of a velocity reversal (a lower Vs layer between higher Vs layers) (Stokoe et al., 1994) and that higher mode surface waves, when trapped in a layer, are much more sensitive to the fine structure of the S-wave velocity field (Kovach, 1965). Reliable observation of higher modes is possible with multichannel recording. Development of a new technique allows direct construction of a high-resolution image of multimodal dispersion curves from multichannel records with a relatively small number of traces (e.g., 30 traces) covering only a small lateral distance (20 m). Observations of higher modes of Rayleigh waves using the MASW method have been reported (Park et al., 1999a and 1999c).

Analyzing the Jacobian matrix clearly shows the advantages of utilizing higher modes in inversion of surface wave data. Higher mode information from Rayleigh waves not only provides extra information about subsurface geology, but also increase the resolution of the inverted shear wave velocities while improving stability of the inversion procedure.

Modeling Results

First the sensitivity of higher modes of surface waves must be analyzed (Xia et al., 1999a). The same model used previously will be incorporated into this analysis (Table 1). Rayleigh-wave phase velocity of a layered earth model is a function of frequency and four groups of earth parameters: P-wave velocity, S-wave velocity, density, and thickness of layers. Contributions to the higher-mode Rayleigh-wave phase velocity from each parameter are calculated as a 25% change in a particular parameter. For higher-mode Rayleigh-wave data the S-wave velocity is still the dominant influence on the dispersion curve above 10 Hz. Furthermore, a 25% change in P-wave velocity and/or density causes virtually no change in the third-mode Rayleigh wave data (Figure 1b). The effect of layer thickness on Rayleigh-wave data can be minimized by subdividing certain thinner layers within each constant S-wave velocity slice.

The Jacobian matrix of the higher-mode Rayleigh-wave data suggests higher-mode data have deeper investigation depths than do the fundamental mode data (Figure 2). The open circles in Figure 2 are the row vectors of the Jacobian matrix associated with the shortest wavelength data. In Figure 2a a wavelength of 8.7 m reaches zero at a depth of 13 m for the fundamental-mode data. However, zero is reached at a depth of 17 m for the second-mode data with a 10.9 m wavelength (Figure 2b) and for the third-mode data with a 6 m wavelength (Figure 2c). This is also observed in row vectors for other wavelengths. This confirms that high-mode Rayleigh-wave data can "see" deeper when compared to the same wavelength components of the fundamental-mode Rayleigh-wave data.

The most significant result is that higher-mode data stabilizes the inversion process and increases the resolution of inverted S-wave velocities. Figure 3 shows the difference in phase velocities (Figure 3a) calculated from two S-wave velocity models (Figure 3b). Although relative differences between S-wave velocities of model 1 and model 2 at depths of 6 m and 7 m are more than 100%, the standard deviation between the fundamental-mode phase velocities of these two models is only 4.6 m/s. This indicates that the inversion process will choose either one of the models to be a final result at a 4.6 m/s error level. In practice, the "irrational" model 2 will usually be selected if the inversion process is forced to conclude with an error level less than 4.6 m/s. However, because the standard deviations for the higher-mode phase velocities are 33.5 m/s for the second-mode and 27.3 m/s for the third-mode data, an inversion with high-mode data will reject "irrational" model 2 so that a stabilized inversion is achieved.

A Real World Example of Utilizing Higher Mode Surface Wave data

A shallow high frequency surface wave survey was conducted in San Jose, California, in 1998 to determine shear-wave velocities in near-surface materials up to 10 m deep. Thirty-channel P-wave data were acquired by the MASW method. Thirty 4.5 Hz vertical-component geophones were used on a 1 m geophone interval. The source of seismic energy was vertical impacts from a 6.3 kg (14 lb.) hammer on a metal plate. The nearest geophone-source offset was 5 m, evident on P-wave data (Figure 4a) when displaced in the frequency-velocity domain (Figure 4b) (Park et al., 1999a). The high modes were also confirmed by slant-stacking techniques (Figure 4b) (McMechan and Yedlin, 1981).

Three data sets are generated and inverted from the frequency-velocity domain. The first set is only fundamental-mode surface wave data, the second is fundamental-mode data with noise in the frequency range 13 to 19 Hz, simulating a case where the fundamental-mode data are contaminated with higher modes and/or body waves. The standard deviation between these two data sets is only 16 m/s. A third set includes the second data set (noisy data) and the second-mode surface wave data. A fourteen-layer model with each layer 1 m thick was chosen to test these data.

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Figure 4c shows inverted S-wave velocities from all three data sets. All root-mean-square (rms) errors between the measured dispersion curve and calculated dispersion curves are less than 5 m/s, based on inverted S-wave velocities (Figure 4c). Because the fundamental-mode data are accurately calculated, the inverted S-wave velocities (solid squares) are geologically reasonable. They smoothly increase from shallower layers to deeper layers. However, smoothness disappears when data set two is inverted (diamonds with a solid line). The S-wave velocity model changes irrationally in the 3 to 7 m depth range. This instability is caused by forcing the response of the inverted model to fit the error. In the real world, it is non-trivial to determine the error range that will allow the inverted model to be easily forced into an unreasonable space. We have experienced this situation on real data a number of times when processing surface wave data. Better results are obtained if higher-mode surface wave data are included during the inversion (solid triangles). Inverted S-wave velocities that have included higher modes during their calculation are closer to the results obtained from data set one (solid squares). When the second-mode data are used in inversion, the S-wave model with abrupt variation (diamonds with solid lines) is rejected due to higher-mode data by the higher rms error in the higher-mode data.

Discussion and Conclusions

We have shown that by including higher-mode data in the inversion of surface wave data the inversion process becomes much more stable. This stability indeed improves the resolution of inversion results. In the real world, we normally make a choice between error and resolution of a model. The instability that we see in the inverted S-wave velocities of data set two is error in the inverted model, which can be reduced by reducing the resolution of the model. For example, an S-wave velocity model with eight layers, each 2 m thick, with only one-half the resolution of the previous model, will give more stable results. Sacrificing resolution to obtain stable results is a wise strategy in the inversion theory (Backus and Gilbert, 1970). For noisy data, it is critical to define a correct error level and stop the inversion process at or a little above this error level. In most cases, the best fit in data does not necessarily mean the best inverted results.

We showed that higher-mode data have a deeper investigation depth than do the fundamental-mode data and higher-mode data increases the resolution of inverted S-wave velocities. Because the resolution of an inverted model is generally determined by the accuracy of the data, we can determine the resolution of inverted S-wave velocities by forward modeling.

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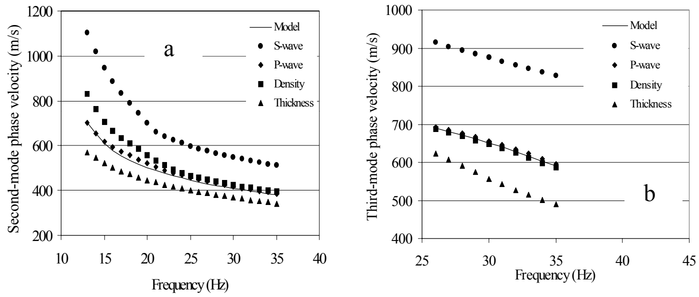

Table 1. An earth model parameters.

Layer number	Vs (m/s)	Vp (m/s)	ρ (g/cm^3)	H (m)
1	194.0	650.0	1.82	2.0
2	270.0	750.0	1.86	2.3
3	367.0	1400.0	1.91	2.5
4	485.0	1800.0	1.96	2.8
5	603.0	2150.0	2.02	3.2
6	740.0	2800.0	2.09	infinite

Fig. 1. Contribution to the second-mode (a) and the third-mode (b) Rayleigh-wave phase velocity by a 25% change in each earth parameter (Table1).

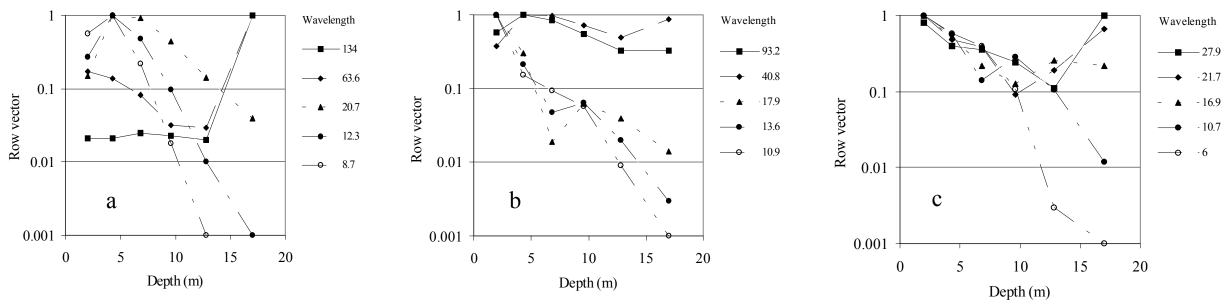

Fig. 2. Row vectors of the Jacobian matrix show data sensitivity varying with depth: (a) the fundamental mode, (b) the second mode, and (c) the third mode.

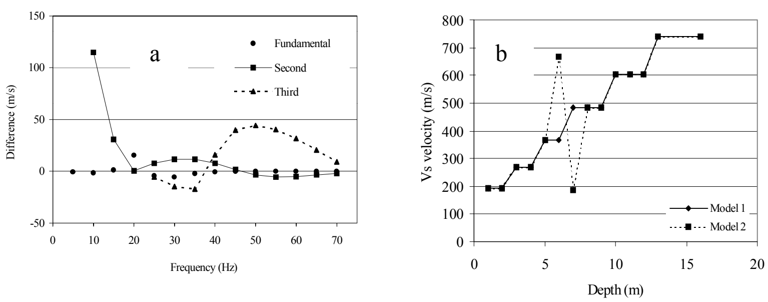

Fig. 3. Differences in phase velocities due to models 1 and 2 (a). More than 100% difference in S-wave velocity models at depths of 6 m and 7 m only result in a standard deviation of 4.6 m/s in the fundamental-mode data (b). However, the changes cause standard deviations of 33.5 m/s and 27.3 m/s for the second-mode data and the third mode data, respectively.

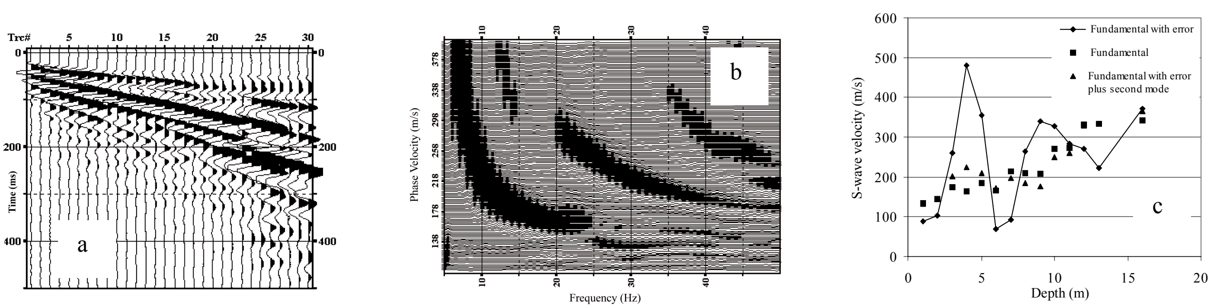

Fig. 4. Surface wave data (a), its image in the frequency-velocity domain (b), and inversion results (c). An “irrational” model (solid line) is found by the inversion process due to errors in the fundamental-mode data. A well behaved S-wave velocity model (solid triangle) is located because higher-mode data are included in the inversion process. This S-wave velocity model is close to an S-wave velocity model (solid square) that is inverted from the fundamental-mode data with high accuracy.