Adverse Selection in Competitive Search Equilibrium

Veronica Guerrieri, Robert Shimer, and Randall Wright

Stanford GSB, 2009

Objective

- study adverse selection in environments with search frictions
- competitive search: principals compete to attract agents
- here: uninformed principals compete to attract heterogeneous agents
- principals form rational beliefs about matching probability and *composition of agents* associated to any contract

Objective

- study adverse selection in environments with search frictions
- competitive search: principals compete to attract agents
- here: uninformed principals compete to attract heterogeneous agents
- principals form rational beliefs about matching probability and *composition of agents* associated to any contract
- **ISSUE:** interaction of adverse selection and competitive search
  1. adverse selection affects the search equilibrium
  2. search affects the set of contracts offered in equilibrium

Results

- existence and uniqueness of equilibrium
- equilibrium may be constrained inefficient
- private information may distort terms of trade or market tightness
- three examples:
  - layoff insurance
  - asset market
  - rat race

Literature

- competitive search: Montgomery (1991), Peters (1991), Shimer (1996), Moen (1997), Acemoglu and Shimer (1999), Burdett, Shi, and Wright (2001), Mortensen and Wright (2002)
- competitive search with asymmetric information: Inderst and Müller (1999), Faig and Jerez (2005), Moen and Rosen (2006), Guerrieri (2008)
- adverse selection with different market structures: Rothschild and Stigliz (1976), Miyazaki (1977), Wilson (1977), Riley (1979), Prescott and Townsend (1984), Gale (1992), Dubey and Geanakoplos (2002), Bisin and Gottardi (2006)

Roadmap

- general model
  - environment
  - equilibrium definition
- example:
  - layoff insurance
- general results:
  - existence and uniqueness
- other examples:
  - asset market
  - rat race

Model

Environment

Motivation

Model

- Environment
- Timing
- Contracts
- Incentive Compatibility
- Matching
- Expected Utilities
- Equilibrium

Layoff Insurance

Characterization

Asset Market

Rat Race

Conclusions

- large measure of ex-ante homogeneous principals
- continuum of measure 1 of heterogeneous agents
- $\pi_i > 0$ agents of type $i \in \{1, 2, ..., I\} \equiv \mathbb{I}$
- agent's type is his own private information
- principals and agents have single opportunity to match

Timing

Motivation
Model
❖ Environment
❖ Timing
❖ Contracts
❖ Incentive Compatibility
❖ Matching
❖ Expected Utilities
❖ Equilibrium
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

- each principal can post a contract C at a cost $k > 0$
- agents observe the set of posted contracts $\bar{\mathbb{C}}$
- agents direct search to their preferred one
- principals and agents match in pairs
- matched principals and agents implement the contract
- agents who fail to match get their outside option $= 0$

Contracts

Motivation
Model
❖ Environment
❖ Timing
❖ Contracts
❖ Incentive Compatibility
❖ Matching
❖ Expected Utilities
❖ Equilibrium
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

- $y \in \mathbb{Y}$ is an action
- $\mathbb{Y}$ is a compact metric space (allows for lotteries)
- if a principal and a type-i agent match and undertake y:
  - agent gets $u_i(y)$, continuous
  - principal gets $v_i(y)$, continuous
- WLOG, contracts are revelation mechanisms
- a contract is a vector of actions $C \equiv \{y_1, \ldots, y_I\}$, where y_i is prescribed if matched agent reports type i

Incentive Compatibility

Motivation
Model
❖ Environment
❖ Timing
❖ Contracts
❖ Incentive Compatibility
❖ Matching
❖ Expected Utilities
❖ Equilibrium
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

- a contract $C = \{y_1, \ldots, y_I\}$ is incentive-compatible iff

$$u_i(y_i) \geq u_i(y_j) \text{ for all } i, j$$

- let $\mathbb{C}$ be the set of incentive-compatible contracts ($\bar{\mathbb{C}} \subset \mathbb{C}$)

Matching

Motivation

Model
- Environment
- Timing
- Contracts
- Incentive Compatibility
- Matching
- Expected Utilities
- Equilibrium

Layoff Insurance

Characterization

Asset Market

Rat Race

Conclusions

- constant returns to scale matching function
- $\Theta(C)$: principal/agent ratio associated to contract C
- $\gamma_i(C)$: share of agents of type i seeking C
- $\Gamma(C) = (\gamma_1(C), \ldots, \gamma_I(C))$
- $\mu(\Theta(C))$: probability an agent seeking C matches
- $\eta(\Theta(C))\gamma_i(C)$: probability a principal posting C matches with a type i agent
- μ and η are continuous functions, $\mu(\theta) = \eta(\theta)\theta$

Expected Utilities

Motivation
Model
❖ Environment
❖ Timing
❖ Contracts
❖ Incentive Compatibility
❖ Matching
❖ Expected Utilities
❖ Equilibrium
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

- expected utility of principal offering $C = \{y_1, \ldots, y_I\}$:

$$\eta(\Theta(C)) \sum_{i=1}^{I} \gamma_i(C) v_i(y_i) - k$$

- expected utility of type-i agent seeking $C = \{y_1, \ldots, y_I\}$:

$$\mu(\Theta(C)) u_i(y_i)$$

Competitive Search Equilibrium

Motivation
Model
❖ Environment
❖ Timing
❖ Contracts
❖ Incentive Compatibility
❖ Matching
❖ Expected Utilities
❖ Equilibrium
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

A CSE is a vector $\bar{U} = \{\bar{U}_1, \ldots, \bar{U}_I\}$, a measure λ on $\mathbb{C}$, and two functions $\Theta(C)$ and $\Gamma(C)$ on $\mathbb{C}$ s.t.

(i) **profit maximization and free entry**: $\forall C \in \mathbb{C}$,

$$\eta(\Theta(C)) \sum_{i=1}^{I} \gamma_i(C) v_i(y_i) - k \leq 0, \text{ with equality if } C \in \bar{\mathbb{C}};$$

(ii) **optimal search**: $\forall C \in \mathbb{C}$ and i,

$$\mu(\Theta(C)) u_i(y_i) \leq \bar{U}_i \equiv \max\left\{0, \max_{C'=\{y'_1,\ldots,y'_I\} \in \bar{\mathbb{C}}} \mu(\Theta(C')) u_i(y'_i)\right\},$$

with equality if $\Theta(C) < \infty$ and $\gamma_i(C) > 0$;

(iii) **market clearing**

Layoff Insurance
(Rothschild and Stigliz 1976)

Model

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- principals = homogeneous risk-neutral firms
  - cost k to search for a worker
  - can hire at most one worker
  - productive match produces 1 unit of output
  - unproductive match leads to a layoff
- agents = heterogeneous risk-averse workers
  - $p_i =$ probability of a productive match for type i
  - 2 types with $k < p_1 < p_2$, share π_i
  - p_i is private info, firms only verify ex-post realization
  - utility of workers never employed is normalized to 0

Contracts and Payoffs

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- action: $y = (c^e, c^u)$ with
  - c^e = consumption if productive
  - c^u = consumption if unproductive
- if firm and type-i worker match and undertake (c^e, c^u):
  - worker gets $u_i(c^e, c^u) = p_i U(c^e) + (1 - p_i)U(c^u)$
  - firm gets $v_i(c^e, c^u) = p_i(1 - c^e) - (1 - p_i)c^u$
- contract: $C = \{(c_1^e, c_1^u), (c_2^e, c_2^u)\}$
- matching function: $\mu(\theta) = \min\{\theta, 1\}$

Equilibrium

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- there exists a unique separating equilibrium
- all workers find a job with probability 1
- private info distorts contracts (relative to full info)
  - $c_1^e = c_1^u = p_1 - k \rightarrow$ full insurance
  - $c_2^e > c_1^e$ and $c_2^u < c_1^u \rightarrow$ partial insurance

Rothschild-Stiglitz

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- in RS if there is an equilibrium, least cost separating
- BUT if there are few bad agents, a pooling contract is profitable deviation $\Rightarrow$ **non-existence result**
- if offering pooling contract, optimal offer full insurance
- consider a contract $C^P = \{(c, c), (c, c)\}$ such that

$$U(c) \geq p_2 U\left(c_2^e\right) + (1 - p_2)\, U\left(c_2^u\right) > p_1 U\left(c_1^e\right) + (1 - p_1)\, U\left(c_1^u\right)$$

- if $\pi_1 < 1 - c$ then the deviation is profitable given that

$$(1 - \pi_1) - c > 0$$

Existence

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- why in our model this deviation is not profitable?
- consider the same pooling contract $C^P = \{(c,c),(c,c)\}$
- to attract both types need

$$\begin{aligned} \mu\left(\Theta\left(C^P\right)\right) U(c) &\geq \bar{U}_1 \\ \mu\left(\Theta\left(C^P\right)\right) U(c) &\geq \bar{U}_2 \end{aligned}$$

- as long as one is a strict inequality, $\Theta\left(C^P\right)$ would adjust up to

$$\begin{aligned} \mu\left(\Theta\left(C^P\right)\right) U(c) &= \bar{U}_1 \\ \mu\left(\Theta\left(C^P\right)\right) U(c) &< \bar{U}_2 \end{aligned}$$

- BUT then bad types don't go $\rightarrow$ not profitable

Efficiency

Motivation
Model
Layoff Insurance
❖ Model
❖ Contracts and Payoffs
❖ Equilibrium
❖ Rothschild-Stiglitz
❖ Existence
❖ Efficiency
Characterization
Asset Market
Rat Race
Conclusions

- **equilibrium may be constrained inefficient**
- few low types $\longrightarrow$ cross-subsidization Pareto dominant
- consider a planner that restrict firms to post pooling contracts
- in this case, the associated market tightness will be equal to 1
- an equilibrium is constrained inefficient when it does not exists in RS!

Characterization

Assumptions

Motivation
Model
Layoff Insurance
Characterization
❖ Assumptions
❖ Optimization Problem
❖ Existence
❖ Uniqueness
❖ Positive Utility
Asset Market
Rat Race
Conclusions

- define $\bar{\mathbb{Y}}_i \equiv \{y \in \mathbb{Y} \,|\, \bar{\eta} v_i(y) \geq k$ and $u_i(y) > 0\}$
- **A1. Monotonicity**: for all $y \in \bigcup_i \bar{\mathbb{Y}}_i$

$$v_1(y) \leq v_2(y) \leq \ldots \leq v_I(y)$$

- **A2. Local non-satiation**: for all i, $y \in \bar{\mathbb{Y}}_i$, and $\varepsilon > 0$

$$\exists y' \in B_\varepsilon(y) \text{ s.t. } v_i(y') > v_i(y)$$

- **A3. Sorting**: for all i, $y \in \bar{\mathbb{Y}}_i$, and $\varepsilon > 0$, $\exists y' \in B_\varepsilon(y)$ s.t.

$$u_j(y') > u_j(y) \text{ for all } j \geq i$$
$$u_j(y') < u_j(y) \text{ for all } j < i$$

Optimization Problem

Motivation
Model
Layoff Insurance
Characterization
❖ Assumptions
❖ Optimization Problem
❖ Existence
❖ Uniqueness
❖ Positive Utility
Asset Market
Rat Race
Conclusions

- consider the constrained maximization problem

$$\bar{U}_i = \max_{\theta \in [0,\infty], y \in \mathbb{Y}} \mu(\theta) u_i(y) \qquad \text{(P-}i\text{)}$$

$$\text{s.t. } \eta(\theta) v_i(y) \geq k,$$

$$\mu(\theta) u_j(y) \leq \bar{U}_j \text{ for all } j < i.$$

- call a solution to the collection of (P-i) a solution to (P)
- if for some i the constraint set is empty or the problem has a negative maximum set $\bar{U}_i = 0$
- **Lemma:** (P) has a solution and $\bar{U}$ is unique
  - ▷ recursive structure of (P) $\Rightarrow$ solve (P-1) first...

Motivation
Model
Layoff Insurance
Characterization
❖ Assumptions
❖ Optimization Problem
❖ Existence
❖ Uniqueness
❖ Positive Utility
Asset Market
Rat Race
Conclusions

- **Proposition 1**: assume A1-A3, let $\{\bar{U}_i\}$, $\{\theta_i\}$, and $\{y_i\}$ be a solution to (P)
- there exists a CSE $\{\bar{U}, \lambda, \bar{\mathbb{C}}, \Theta, \Gamma\}$ with

1. $\bar{U} = \{\bar{U}_i\}$
2. $\bar{\mathbb{C}} = \{C_i\}$, where $C_i = (y_i, \ldots, y_i)$
3. $\Theta(C_i) = \theta_i$
4. $\gamma_i(C_i) = 1$

Uniqueness

Motivation
Model
Layoff Insurance
Characterization
❖ Assumptions
❖ Optimization Problem
❖ Existence
❖ Uniqueness
❖ Positive Utility
Asset Market
Rat Race
Conclusions

- **Proposition 2**: assume A1-A3, let $\{\bar{U}, \lambda, \bar{\mathbb{C}}, \Theta, \Gamma\}$ be a CSE
- let $\{\bar{U}_i\} = \bar{U}$
- take any $\{\theta_i, y_i\}$ s.t. $\exists C_i = \{y_1, \ldots, y_i, \ldots, y_I\} \in \bar{\mathbb{C}}$ with $\theta_i = \Theta(C_i) < \infty$, $\gamma_i(C_i) > 0$
- $\{\bar{U}_i\}$, $\{\theta_i\}$, and $\{y_i\}$ solve (P)

Positive Utility

Motivation
Model
Layoff Insurance
Characterization
❖ Assumptions
❖ Optimization Problem
❖ Existence
❖ Uniqueness
❖ Positive Utility
Asset Market
Rat Race
Conclusions

- **Proposition 3**: assume A1-A3, $\{y \in \mathbb{Y} | \eta(0)v_i(y) > k$ and $u_i(y) > 0\} \neq \emptyset$ for all i
- in equilibrium, $\bar{U}_i > 0$ for all i
- NOTE: positive gains of trade for some i do not guarantee $\bar{U}_i > 0$ (next example)

Asset Market
(Akerlof 1970)

Model

Motivation
Model
Layoff Insurance
Characterization
Asset Market
❖ Model
❖ Assumptions
❖ Equilibrium
❖ No Trade
Rat Race
Conclusions

- sellers own heterogeneous apples
- buyers value apples more than sellers
- an action is $y = (\alpha, t)$:
  - α = probability seller gives up apple
  - t = transfer from buyer to seller
- if buyer and type-i seller match and undertake (α, t):
  - seller's payoff: $u_i(\alpha, t) = t - \alpha a_i^S$
  - buyer's payoff: $v_i(\alpha, t) = \alpha a_i^B - t$

Assumptions

Motivation
Model
Layoff Insurance
Characterization
Asset Market
❖ Model
❖ Assumptions
❖ Equilibrium
❖ No Trade
Rat Race
Conclusions

- assume there are only two types $I = 2$
- type 2 sellers have a better apple: $a_2^S > a_1^S \geq 0$
- preferences of buyers and sellers aligned: $a_2^B > a_1^B \geq 0$
- for now assume gains from trade: $a_i^B > a_i^S + k$
- matching $\mu(\theta) = \min\{\theta, 1\}$

Equilibrium

Motivation
Model
Layoff Insurance
Characterization
Asset Market
❖ Model
❖ Assumptions
❖ Equilibrium
❖ No Trade
Rat Race
Conclusions

- there exists a CSE with:

  1. $\alpha_i = 1$ and $t_i = a_i^B - k$ for all i
  2. $\theta_1 = 1$ and $\theta_2 = \dfrac{a_1^B - a_1^S - k}{a_2^B - a_1^S - k} < 1$

- **NOTE: private information affects market tightness**
- rationing through α would be more costly due to k
- Pareto improvement if $\pi_1 < \dfrac{a_2^B - a_2^S - k}{a_2^B - a_1^S - k}$

No Trade

Motivation
Model
Layoff Insurance
Characterization
Asset Market
❖ Model
❖ Assumptions
❖ Equilibrium
❖ No Trade
Rat Race
Conclusions

- now suppose $a_1^B \leq a_1^S + k$
- then, $\bar{U}_1 = \bar{U}_2 = 0$, no contracts are posted
- **bad asset shuts down the market for a good one**

Rat Race
(Akerlof 1976)

Model

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
❖ Model
❖ Assumptions
❖ Benchmark
❖ Equilibrium
Conclusions

- workers heterogeneous in preferences and productivity
- homogeneous firms need to hire a worker to produce
- an action is $y = (c, h)$:
  - c = wage
  - h = hours worked
- if a firm and a type-i worker match and undertake (c, h)
  - worker's payoff: $u_i(c, h) = u_i(c, h)$
  - firm's payoff: $v_i(c, h) = f_i(h) - c$

Assumptions

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
❖ Model
❖ Assumptions
❖ Benchmark
❖ Equilibrium
Conclusions

- assume there are only two types $I = 2$
- wlog type 2 is more productive: $f_2(h) > f_1(h)$ for all h
- single crossing assumption:

$$-\frac{\partial u_2/\partial h}{\partial u_2/\partial c} < -\frac{\partial u_1/\partial h}{\partial u_1/\partial c}$$

- matching $\mu(\theta) = \min\{\theta, 1\}$

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
❖ Model
❖ Assumptions
❖ Benchmark
❖ Equilibrium
Conclusions

- full info equilibrium determined by three equations:
  - optimality for hours

$$-\frac{\partial u_i(c_i, h_i)/\partial h}{\partial u_i(c_i, h_i)/\partial c} = f_i'(h_i)$$

  - optimality for vacancy creation:

$$\mu'(\theta_i)\left(f_i(h_i) - c_i + \frac{u_i(c_i, h_i)}{\partial u_i(c_i, h_i)/\partial c}\right) = k$$

  - free-entry condition

$$\frac{\mu(\theta_i)}{\theta_i}(f_i(h_i) - c_i) = k$$

Equilibrium

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
❖ Model
❖ Assumptions
❖ Benchmark
❖ Equilibrium
Conclusions

- there is a unique separating equilibrium
- private info distorts contracts:
  - low type not distorted
  - high type distorted (overemployment):

$$-\frac{\partial u_2(c_2, h_2)/\partial h}{\partial u_2(c_2, h_2)/\partial c} > f_2'(h_2)$$

- market tightness may be distorted in either direction

Equilibrium

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
❖ Model
❖ Assumptions
❖ Benchmark
❖ Equilibrium
Conclusions

- there is a unique separating equilibrium
- private info distorts contracts:
  - low type not distorted
  - high type distorted (overemployment):

$$-\frac{\partial u_2(c_2, h_2)/\partial h}{\partial u_2(c_2, h_2)/\partial c} > f_2'(h_2)$$

- market tightness may be distorted in either direction

NOTE: equilibrium may be constrained inefficient

- few low types or high cost of screening, cross-subsidization may Pareto dominate

Conclusions

Conclusions

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions

- general framework combining search frictions and adverse selection
- existence and uniqueness
- general algorithm to characterize equilibrium
- private information can affect contracts and/or matching
- equilibrium may be Pareto dominated

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions
Incentive Feasibility
❖ Allocation
❖ Incentive-Feasibility

Incentive Feasibility

Allocation

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions
Incentive Feasibility
❖ Allocation
❖ Incentive-Feasibility

- An allocation is
  - a vector $\bar{U}$ of expected utilities for the agents
  - a measure λ over $\mathbb{C}$ with support $\bar{\mathbb{C}}$
  - a function $\tilde{\Theta} : \bar{\mathbb{C}} \mapsto [0, \infty]$
  - a function $\tilde{\Gamma} : \bar{\mathbb{C}} \mapsto \Delta^I$

Motivation
Model
Layoff Insurance
Characterization
Asset Market
Rat Race
Conclusions
Incentive Feasibility
❖ Allocation
❖ Incentive-Feasibility

- An allocation is incentive feasible if

1.
$$\int \left(\eta(\tilde{\Theta}(C)) \sum_{i=1}^{I} \tilde{\gamma}_i(C) v_i(y_i) - k \right) d\lambda(C) = 0;$$

2. for any $C \in \bar{\mathbb{C}}$ and i s.t. $\tilde{\gamma}_i(C) > 0$ and $\tilde{\Theta}(C) < \infty$,
$$\mu(\tilde{\Theta}(C)) u_i(y_i) = \bar{U}_i = \max_{C' \in \bar{\mathbb{C}}} \mu(\tilde{\Theta}(C')) u_i(y_i')$$

3.
$$\int \frac{\tilde{\gamma}_i(C)}{\tilde{\Theta}(C)} d\lambda(C) \leq \pi_i, \text{ with equality if } \bar{U}_i > 0$$

14.461 Advanced Macroeconomics I
Fall 2009