Advertising versus Brokerage Model for Online Trading Platforms*

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Abstract

The two leading online consumer-to-consumer platforms use very different revenue models: eBay.com in the United States uses a brokerage model in which sellers pay eBay on a transaction basis, whereas Taobao.com in China uses an advertising model in which sellers can use the basic platform service for free and pay Taobao for advertising services to increase their exposure. This paper studies how the chosen revenue model affects a platform's revenue, buyers' payoffs, sellers' payoffs, and social welfare. We find that when little space can be dedicated to advertising under the advertising model, the brokerage model generates more revenue for the platform than the advertising model. When a significant proportion of space is dedicated to advertising under the advertising model, matching probability on a platform plays a critical role in determining which revenue model can generate more revenue: If the matching probability is high, the brokerage model generates more revenue; otherwise, the advertising model generates more revenue. Buyers are always better off under the advertising model because of larger participation by the sellers for the platform's free service. Sellers are better off under the advertising model in most scenarios. The only exception is when the matching probability is low and the platform dedicates a large space to advertising. Under these conditions, the sellers with payoffs similar to the marginal advertiser who is indifferent about advertising can be worse off under the advertising model. Finally, the advertising model generates more social welfare than the brokerage model.

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1 Introduction

eBay.com, the leading online consumer-to-consumer platform in the United States, has been establishing its business for almost two decades. Individual buyers can shop on the eBay platform without any fee, and sellers pay eBay on a transaction basis. Arguably, eBay was one of the biggest innovations and successes in the early e-commerce period. In contrast, Taobao.com, the leading online consumer-to-consumer platform in China, started its business in 2003. While Taobao and eBay share many similar design features, Taobao adopted a radically different revenue model. In addition to providing free service to individual buyers, Taobao offers the basic platform service to sellers for free as well. Meanwhile, Taobao offers an advertising/promotion service to monetize the traffic, and sellers can pay to participate. In other words, in a manner different from eBay but similar to Google search result pages, Taobao provides two lists: One is an "organic" listing, typically on the left of each page, in which sellers are listed for free, and the other is a "paid" listing, typically on the right of each page, in which sellers pay Taobao to increase their exposure to potential buyers. The differences between the revenue models associated with the two largest and most successful online marketplaces raise several questions: Which revenue model is more suitable for an online platform? How should a platform choose and design a revenue model? How does the revenue model affect buyers' and sellers' payoffs? This paper aims to answer these questions.

Founded in 1995, eBay's total transaction volume, or gross merchandise volume, was nearly $62 billion in 2010, according to its annual report. eBay's marketplace charges sellers insertion fees and final value fees. The insertion fee ranges from $0.10 to $2 for auction-style listings at eBay and is $0.50 for fixed-price listings. Depending on the sale format and product category, the final value fees at eBay can range from 7% to 13% of the total buyer cost, including price and shipping costs.[1] Taobao was launched by Alibaba Group in 2003 and has grown remarkably since then. Its sales volume was about $61 billion in 2010.[2] Taobao offers basic market services for free to both buyers and sellers. Its main source of revenue is the advertising paid for by the sellers. Despite the success of Taobao, no formal analysis of its revenue model has been conducted. This study fills this gap and sheds some light on the choice and design of a platform's revenue models.

To do so, we develop a game theoretic model in which a platform faces a group of potential

[1] http://pages.ebay.com/help/sell/fees.html
[2] http://www.techweb.com.cn/internet/2011-01-20/778349.shtml

buyers on the one side and a group of potential sellers on the other. We assume the platform can choose either the brokerage model or the advertising model. Under the brokerage model, the platform charges sellers a transaction fee for each sale. Under the advertising model, the platform offers the basic service for free and meanwhile provides paid advertising service by which sellers can participate to increase their exposure. The platform's choice of the revenue model affects potential sellers' participation decisions. Potential buyers do not have to pay to participate but have different opportunity costs to use the platform. Buyers' participation decisions are affected by the number of participating sellers and the matching probability. The number of participating sellers indicates how likely buyers' trading partners are on the platform: As more sellers participate in the platform, the likelihood that a buyer's trading partner is on the platform increases. The matching probability measures the degree to which a buyer is likely to actually find his or her trading partner (given the partner's being on the platform). Under this framework, we compare the platform's revenues, sellers' payoffs, buyers' payoffs, and social welfare under the two revenue models.

We identify both the space dedicated to advertising and the matching probability as critical factors in comparing the two revenue model. Not surprisingly, when little space can be dedicated to advertising under the advertising model, the brokerage model generates more revenue for the platform than the advertising model. When a significant proportion of space is dedicated to advertising under the advertising model, matching probability on a platform plays a critical role in determining which revenue model can generate more revenue: If the matching probability is high, the brokerage model generates more revenue; otherwise, the advertising model generates more revenue. In the presence of a free basic platform service, when the matching probability is low, the advertising space becomes more valuable, which allows the platform to charge a higher price and potentially to make more revenue under the advertising model.

Buyers are always better off under the advertising model because of greater participation by sellers in the platform's free service. Sellers are better off under the advertising model in most scenarios. The only exception is that, when the matching probability is low and the platform dedicates a large space to advertising, the sellers having payoffs similar to the marginal advertiser (i.e., who is indifferent about advertising) can be worse off under the advertising model. Finally, the advertising model generates more social welfare than the brokerage model because of the increased number of trading pairs.

Our study is mainly related to two streams of research. The first related stream looks at different business models and revenue models. A number of papers focus on one type of revenue model and study the optimal strategies under that model (Anderson and Coate, 2005; Casadesus-Masanell and Zhu, 2010, 2012; Niculescu and Wu, 2012; Cheng and Liu, 2012; Chellappa and Shivendu, 2010). For example, Anderson and Coate (2005) examine equilibrium advertising levels in broadcasting, and Niculescu and Wu (2012) investigate when software firms should commercialize new products via freemium business models. Other papers compare different revenue models. For instance, Casadesus-Masanell and Zhu (2010) analyze the optimal business model choice for a high-quality incumbent facing a low-quality, ad-sponsored competitor in a product market. Among the four business models considered—a subscription-based model, an ad-sponsored model, a mixed model incorporating both subscriptions and advertising, and a dual model in which one product uses the ad-sponsored model and the other uses the mixed model—they find the incumbent prefers the subscription-based or the ad-sponsored model. Lin et al. (2012) consider settings in which online service providers might offer an ad-free service, an ad-supported service, or a combination of these services. They find that in a monopoly case, offering both ad-free and ad-supported services is optimal, and in a duopoly case, exactly one firm offers both services when the ad revenue rate is sufficiently high. Our study differs from theirs in that we compare the advertising model and the brokerage model for online trading platforms. In addition, in our setting, the basic platform service and the advertising service serve the same purpose of presenting sellers to potential buyers and, by their nature, are substitutes; thus, our analysis and insights depart from theirs.

The second related stream involves the studies on two-sided markets (Gallaugher and Wang, 2002; Rochet and Tirole, 2003; Parker and Alstyne, 2005; Bhargava and Choudhary, 2004; Economides and Katsamakas, 2006; Jullien, 2006; Hagiu, 2009). Two-sided markets refer to the situations where "platforms" provide services to facilitate interaction and the operation of exchanges between two types of trading partners (Jullien, 2006). Examples of two-sided markets include credit card systems (cardholders and merchants), health maintenance organizations (HMOs) (patients and doctors), shopping malls (buyers and merchants), travel reservation services (travelers and airlines), video game consoles (gamers and game developers), and online trading platforms (buyers and sellers). In a typical two-sided market, the benefit of users from joining the platform on one side is increasing in the number of users adopting the platform on the other side. For example, an online trading platform that provides services to enable interactions between buyers and sellers is a two-sided market, in which the users on one side (e.g., sellers) are more likely to find their trading partners if more users join the platform on the other side (e.g., buyers). Rochet and Tirole (2003) study platform competition and optimal price allocation between buyers and sellers. They consider a brokerage intermediary that charges prices or registration fees from market participants to be a marketplace. The market has (indirect) network externalities, and the demand on one side of the market depends on the demand from the other side. Bhargava and Choudhary (2004) study the optimal quality and pricing strategies for information intermediaries with aggregation benefits (positive indirect network externalities) and find that intermediaries have strong incentives to provide quality-differentiated versions of services. Gallaugher and Wang (2002) empirically investigate the effects on software price of different factors, including network externalities, in the context of the two-sided market for Web server software. In contrast to these papers, we compare two different revenue models for an online trading platform and examine the effect of the platform's revenue model choice on the players involved.

The rest of the paper is organized as follows. In the next section, we set forth our model. In Section 3 we provide an equilibrium analysis, and in Section 4 we compare the publisher's revenue, buyers' payoffs, sellers' payoffs, and social welfare under the two revenue models. Section 5 concludes the paper.

2 Model

We consider an online platform with multiple sellers on one side and multiple buyers on the other. The platform provides matching as well as other necessary services to facilitate transactions between sellers and buyers. Consistent with popular online platform practices, such as in eBay and in Taobao, buyers can participate without any cost. For sellers, we consider two different business models: a brokerage model and an advertising model. Under the brokerage model, sellers pay a transaction fee t for each sale. Under the advertising model, sellers can participate in the basic platform service for free, and, in addition, they can pay θ to participate in an advertising or promotion service provided by the platform to increase their exposure to potential buyers. The brokerage model resembles eBay's practice, and the advertising model resembles Taobao's practice.

A mass of sellers with measure 1 may sell their products through the platform. Each seller is seen as selling a different product, and sellers have different fixed cost k of providing their products through the platform. A mass of buyers with measure 1 may buy products through the platform. Accessing the platform involves different opportunity cost c for the buyers. We assume that both k and c satisfy uniform distributions with support $[0, 1]$.

Let m be the mass of buyers and n be the mass of sellers participating on the platform. A buyer's probability of finding her trading partner on the platform depends on whether her selling partner is on the platform, and, if so, whether the buyer can find the selling partner on it. In general, as the number of sellers participating in the platform increases, so does the likelihood that a buyer's trading partner is on the platform. We assume that the probability that a buyer's trading partner is on the platform is equal to the mass of participating sellers n. Under the brokerage model, we assume that sellers are listed without differentiation, and each seller receives the same exposure level p, which determines the likelihood that her product is noticed by buyers. In other words, a buyer can find her trading partner with probability p, $p \in (0, 1)$, conditional on the partner's being on the platform. Thus, under the brokerage model, the number of pairs of trading partners meeting on the platform is mnp, given the numbers of participants. For ease of exposition, we simply assume that when a potential buyer finds her trading partner, the trade occurs. Then, the number of transactions is also mnp. Introducing trading probability adds an additional parameter and does not change the results. We call p the *base probability* for a buyer's finding her trading partner, or the *matching probability*. The base probability depends on buyers' online skill and experience, as well as the quality of the search function provided by the platform, among other factors.

Under the advertising model, advertised sellers get more exposure than unadvertised ones and are more likely to be noticed by potential buyers. If we denote p_1 as the exposure that an unadvertised seller receives and p_2 as the exposure that an advertised seller receives, we generally have $p_1 < p_2$. We denote n' as the mass of sellers who participate in the advertising service. For a fair comparison, we assume that the advertising model itself does not increase the number of pairs of trading partners from the brokerage model. Meanwhile, notice that the platform considered in this model is a dedicated trading platform, and buyers come to the platform to find their trading partners. We thus assume that when the advertising is mild and no advertised sellers are "over-exposed" (such that $p_2 \leq 1$), advertising does not decrease the number of pairs of trading partners either. Then we have $m\left[(n-n')p_1+n'p_2\right]=mnp$, or

$$(n-n')p_1+n'p_2=np \tag{1}$$

In a sense, advertising acts as an exposure reallocation device: The total exposure to a buyer is np, and the advertising shifts the exposure more toward the advertised sellers. As a result, the advertised sellers are more likely to be noticed by buyers than unadvertised sellers. Equation (1) can also be interpreted to mean that when the advertising is mild, the (weighted) average matching probability is the same under the two revenue models.

Because $p_1 < p_2$, by Equation (1), we have $p_1 < p$, and, without loss of generality, we can let $p_1=(1-a)p$, where $a \in [0,1]$ reflects the proportion of space dedicated to advertising. For example, when $a=0$, we have $p_1=p_2=p$, which indicates all sellers receive the same exposure and are equally likely to be noticed by potential buyers with base matching probability p, and thus this case is equivalent to the one in which no advertising space is offered.[3] By simple algebra, from Equation (1), we can derive

$$p_2-p_1=\tfrac{apn}{n'} \tag{2}$$

which measures the additional exposure gained from advertising. Evidently, a larger a or a larger space dedicated to advertising increases the additional exposure, given the number of sellers who participate in the advertising service. By substituting in p_1, we have $p_2=(1-a)p+\frac{apn}{n'}$. When considerable space is dedicated to advertising (i.e., a is large) or when a relatively small number of sellers is participating in the advertising service (i.e., n'/n is small), p_2, as previously defined, technically would go above 1. In this case, the advertised sellers are "over-exposed" to buyers—they are noticed by potential buyers with probability 1, and in fact are exposed more than what is necessary to for them to be noticed by potential buyers with probability 1. We call this case *excessive advertising*. In the excessive advertising case, some "attention" from buyers is wasted, and the number of pairs of trading partners is $m\left[p_1(n-n')+n'\right]$. The number and also the (weighted) average matching probability in this case is lower than that under the regular advertising case (with $p_2 \leq 1$).

[3] The other extreme case is $a=1$, which means no free service is provided and sellers have to pay (the advertising fee) to be listed on the platform. In this case, the advertising fee is equivalent to the listing fee.

We denote by s the expected surplus that a buyer derives from finding her trading partner and by π the expected profit that a seller derives from finding her trading partner. We assume $s \leq 1$ and $\pi \leq 1$ to exclude some less interesting cases. For instance, if $\pi > 1$, under advertising model, all sellers participate in the basic platform service (because their fixed costs, being in the range $[0, 1]$, are less than 1) and the mass of participating sellers is simply 1.

Under the brokerage model, the expected payoff that the buyer derives from participating in the platform is

$$nps - c \tag{3}$$

and the expected payoff that a seller derives from participating in the platform is

$$mp(\pi - k - t) \tag{4}$$

where k is the seller's fixed cost and t is the transaction fee paid to the platform.

Under the advertising model, the expected payoff that a buyer derives from participating in the platform is

$$\left[(n - n')p_1 + n' \min\{p_2, 1\}\right] s - c, \tag{5}$$

where $[(n - n')p_1 + n' \min\{p_2, 1\}]$ is the probability that the buyer can find her trading partner—the probability of finding the partner among the unadvertised sellers plus the probability of finding the partner among the advertised sellers. Notice that when $p_2 < 1$, the above equation is simply $(nps - c)$ because of Equation (1).

The expected payoff that a seller derives from participating in the platform is

$$m\left[p_1 + I\left(\min\{p_2, 1\} - p_1\right)\right](\pi - k) - I\theta, \tag{6}$$

where I, $I \in \{0, 1\}$, indicates whether a seller participates in the advertising service. When a seller chooses not to participate in the advertising service (i.e., $I = 0$), the seller's expected payoff from participating in the platform is $mp_1(\pi - k)$. When a seller chooses to participate in the advertising service (i.e., $I = 1$), the seller's expected payoff from participating in the platform is $[m \min\{p_2, 1\}(\pi - k) - \theta]$. The benefit of participating in the advertising service is the additional exposure ($\min\{p_2, 1\} - p_1$), at the cost of θ.

The sequence of events in the game is as follows. The platform owner first announces its business model and its fee structure (i.e., advertising fee θ under the advertising model or transaction fee t under the brokerage model). Then, the potential sellers decide whether to participate in the platform (which determines n), and under the advertising model they also decide whether to participate in the advertising service (which determines n'). Meanwhile, the consumers decide whether to participate in the platform (which determines m). Finally, transactions take place between sellers and buyers.

We next derive and compare the equilibrium outcome under the brokerage model and under the advertising model, considering the complete parameter space of advertising space a and matching probability p.

3 Equilibrium Analysis

In this section, we examine the participation decisions of the potential buyers and sellers in equilibrium, and we derive the equilibrium payoffs of both participating players and the platform under the brokerage model and under the advertising model.

3.1 Equilibrium Under a Brokerage Model

Given the structure of the problem, we can expect monotonicity in both sellers' and buyers' participation decisions because the players with lower costs generally derive higher payoff than their counterparts with higher costs. We summarize this monotonicity in the following lemma.

Lemma 1. *Let $c < c'$ and $k < k'$. Under the brokerage model, if a buyer with cost c' participates in the platform, the buyer with cost c also participates in the platform. If a seller with cost k' participates in the platform, the seller with cost k also participates in the platform.*

Proof. The buyer with c' participates if her payoff, specified in Equation (3), is positive (i.e., if $nps - c' \geq 0$). Because $c < c'$, we have $nps - c \geq 0$, which indicates that the buyer with c also has an incentive to participate. Similar reasoning applies to the participation decision for sellers. □

Based on this monotonicity, we next can characterize the marginal buyer who is indifferent about participating or not. We denote c_B as the cost of the marginal buyer, which satisfies $nps - c_B = 0$, based on the payoff in Equation (3). By Lemma 1, the buyers with costs lower than c_B participate in the platform and those with costs higher than c_B do not participate. Because we assume that the opportunity costs of buyers are uniformly distributed over $[0, 1]$, the mass of participating buyers is $m = c_B$. Similarly, we denote k_B as the cost of the marginal seller who is indifferent about participating or not; that is, $mp(\pi - k_B - t) = 0$, based on the payoff in Equation (4). The sellers who have costs lower than k_B participate in the platform, and thus the mass of participating sellers is $n = k_B$. Together, we can derive

$$k_B = \pi - t \tag{7}$$

$$c_B = k_B ps = (\pi - t)ps \tag{8}$$

Clearly, t should be less than π; otherwise, no sellers participate.

Notice that the total number of transactions $mnp = c_B k_B p$. The platform's owner maximizes its revenue Π_B by optimally choosing its transaction fee:

$$\max_{0<t<\pi} c_B k_B pt = \max_{0<t<\pi} p^2 st(\pi - t)^2$$

By the first-order condition, we conclude the optimal solution as follows.

Proposition 1. *The optimal transaction fee that the platform should charge is* $t^* = \frac{\pi}{3}$, *and the maximum revenue is* $\Pi_B^* = \frac{4p^2 s\pi^3}{27}$.

Proof. All proofs are in the appendix, unless indicated otherwise. □

The above results are derived in a fashion similar to the balance between price and demand. If the transaction fee t (i.e., "price") is high, the participating players and thus the number of transactions is low (i.e., "demand"). The optimal value as derived is the result of the balance. Note that if the number of participating buyers was fixed (such that m was not a function of t), the optimal transaction fee would be $\pi/2$ by maximizing the platform's revenue $mp(\pi - t)t$. In contrast, considering the effect of t on the number of participating buyers via the number of participating sellers, or considering the two-sided market effect, the optimal transaction fee $\pi/3$ is lower than when considering the effect of t on the number of sellers only. The lower transaction fee in the two-sided market occurs because lowering the transaction fee not only increases the number of participating sellers, but also increases the number of participating buyers. This additional benefit induces the platform to lower the transaction fee.

Based on the optimal transaction fee and Equation (7), the mass of the participating sellers in equilibrium is

$$n_B^* = k_B^* = \frac{2}{3}\pi \tag{9}$$

By Equation (8), the mass of the participating buyers in equilibrium is

$$m_B^* = c_B^* = \frac{2}{3}ps\pi \tag{10}$$

Based on Lemma 1, we can conclude the participation of sellers and buyers as follows.

Corollary 1. *Under the brokerage model, the sellers who have costs in* $[0, k_B^*]$ *and the buyers who have opportunity costs in* $[0, c_B^*]$ *participate in the platform in equilibrium.*

By substituting n_B^*, m_B^*, and t^* into Equations (3) and (4), we can formulate the payoffs of the participating buyers and the payoffs of the participating sellers in equilibrium.

3.2 Equilibrium Under an Advertising Model

Similar to the brokerage model, in the advertising model, we have monotonicity in both sellers' and buyers' participation decisions.

Lemma 2. *Let* $c < c'$ *and* $k < k'$*. Under the advertising model, if a buyer with cost* c' *participates in the platform, the buyer with cost* c *also participates in the platform. If a seller with cost* k' *participates in the platform, the seller with cost* k *also participates in the platform. Moreover, if the seller with* k' *participates in the advertising service, the seller with* k *also participates in the advertising service.*

Proof. The proof of buyers' and sellers' decisions of participating in the platform is the same as that of Lemma 1. We next show sellers' decisions about participating in the advertising service. The seller with k' participates in the advertising service if her payoff with advertising is greater than her payoff without advertising; that is, if

$$m\min\{p_2,1\}(\pi-k')-\theta > mp_1(\pi-k')$$

by Equation (6). Because $k < k'$, if the above inequality is true, $[m\min\{p_2,1\}(\pi-k)-\theta] > [mp_1(\pi-k)]$ must be true because $\min\{p_2,1\} > p_1$, which indicates that the seller with c also has incentive to participate in the advertising service. □

Similar to our approach under the brokerage model, we denote c_A as the cost of the marginal buyer who is indifferent about participating or not. The buyers with costs lower than c_A participate in the platform, and the mass of participating buyers thus is $m = c_A$. We denote k_A as the cost of the marginal seller who is indifferent about participating in the platform or not, and k'_A as the cost of the marginal advertiser who is indifferent about participating in the advertising service or not. By Equation (6), if a seller derives a positive payoff from participating in the advertising service (paying advertising cost θ), her payoff from participating in the basic platform service without cost should be positive, which implies $k'_A < k_A$. Therefore, the sellers with costs lower than k_A participate in the platform, and the mass of participating sellers is $n = k_A$. Among these participating sellers, those with costs lower than k'_A participate in the advertising service, and the mass of advertised sellers thus is $n' = k'_A$.

In settings similar to a two-sided market, a "pessimistic" equilibrium exists in which neither side participates. Because such an equilibrium can be easily excluded, we next focus on the equilibrium with positive participation. For internal solutions, we derive the following relationship among these marginal users:

$$\left[(k_A-k'_A)p_1+k'_A\min\{p_2,1\}\right]s-c_A = 0 \tag{11}$$

$$c_Ap_1(\pi-k_A) = 0 \tag{12}$$

$$c_A\min\{p_2,1\}(\pi-k'_A)-\theta = c_Ap_1(\pi-k'_A) \tag{13}$$

Equation (11) is the condition for the marginal buyer who is indifferent about participating in the platform or not, derived by substituting $n = k_A$ and $n' = k'_A$ into Equation (5). Equation (12) is the condition for the marginal seller who is indifferent about participating in the platform or not, derived by letting $I = 0$ and $m = c_A$ in Equation (6). Equation (13) is the condition for the marginal advertiser who is indifferent about participating in the advertising service or not: The left-hand side is her payoff for using advertising (by letting $I = 1$ and $m = c_A$ in Equation (6)), and the right-hand side is her payoff for not using advertising (by letting $I = 0$ and $m = c_A$ in Equation (6)).

From Equation (12), we derive $k_A = \pi$, and therefore, only sellers with fixed costs less than π participate. When the advertising is mild, such that $p_2 < 1$, from Equation (11), we have $c_A = k_A ps = \pi ps$, where the first equality is because of Equation (1) and the second equality is because of $k_A = \pi$. In addition, $\min\{p_2, 1\} - p_1 = \frac{apk_A}{k'_A}$ by Equation (2). From Equation (13), we can then derive $\theta k'_A = \pi^2 p^2 sa(\pi - k'_A)$. First, we notice that k'_A is monotonically decreasing in θ, which makes intuitive sense in that a higher advertising cost leads to participation by fewer sellers in advertising. Second, the equation implies that the platform's advertising revenue $\theta k'_A$ is simply $p^2\pi^2 sa(\pi - k'_A)$. To increase its revenue, the platform has incentive to lower k'_A by charging a higher advertising price. Notice that, by Equation (2), $p_2 = p_1 + \frac{apn}{n'} = p_1 + \frac{ap\pi}{k'_A}$. Lowering k'_A increases the exposure level for advertised sellers and ultimately gives with probability of 1 the notice of advertised sellers by buyers; that is, $\min\{p_2, 1\} = 1$. In other words, $p_2 < 1$ cannot be the platform's optimal choice.

We next analyze the case with $p_2 \geq 1$. We denote

$$\delta \equiv 1 - (1 - a)p, \tag{14}$$

which is the (maximum) increase in the probability of a seller's being noticed by potential buyers through advertising (because $(1 - a)p = p_1$). By substituting in $p_1 = (1 - a)p$ and $\min\{p_2, 1\} = 1$, Equations (11) and (13) change to

$$\begin{aligned} c_A &= \left[(1 - a)p(\pi - k'_A) + k'_A\right] s = \left[(1 - \delta)\pi + \delta k'_A\right] s & (15) \\ \theta &= c_A \left[1 - (1 - a)p\right] (\pi - k'_A) = c_A \delta(\pi - k'_A) & (16) \end{aligned}$$

where the second equality in each equation is achieved by substituting in the definition of δ.

According to Lemma 1, all sellers with cost lower than k'_A participate in the advertising service, and k'_A also measures the mass of sellers who advertise, or the demand for the platform advertising service given the advertising price θ. Equations (15) and (16) define the relationship between the demand k'_A and advertising price θ. By substituting c_A in Equation (15) into Equation (16), we can derive the inverse demand function as $\theta = \delta\pi s(\pi - k'_A) - \delta^2 s(\pi - k'_A)^2$. The platform maximizes its revenue $\Pi_A = \theta k'_A$ by choosing the advertising fee θ (i.e., price) or, equivalently, by choosing the marginal seller who is indifferent about participation in the advertising service (i.e., demand); that is,

$$\max_{0<k'_A<\pi} \delta s \left[\pi(\pi - k'_A) - \delta(\pi - k'_A)^2\right] k'_A \tag{17}$$

$$\text{s.t. } p_2 = (1-a)p + \frac{ap\pi}{k'_A} \geq 1 \tag{18}$$

Solving the above optimization problem, we can conclude the optimal advertising fee and the maximum revenue that the platform can generate as follows.

Proposition 2. *Denote* $\hat{p}(a) \equiv \frac{1+a}{1+2a}$. *Given* a *(*$a \in (0,1)$*), the optimal advertising fee that the platform should charge is*

$$\theta^* = \begin{cases} s\pi^2 \left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right] & \text{if } p > \hat{p}(a) \\ p(1-p)s\pi^2 & \text{if } p \leq \hat{p}(a) \end{cases} \tag{19}$$

The platform's maximum revenue is

$$\Pi_A^* = \begin{cases} \frac{s\pi^3}{27\delta}\left[-2+3\delta+3\delta^2-2\delta^3+2(1-\delta+\delta^2)^{\frac{3}{2}}\right] & \text{if } p > \hat{p}(a) \\ \frac{(1-p)a}{\delta}p^2 s\pi^3 & \text{if } p \leq \hat{p}(a) \end{cases}$$

Similar to the effects of fee increases under the brokerage model, increasing the advertising fee decreases the number of participating advertisers, which in turn affects the number of participating buyers. The optimal advertising fee derived above is the result of the balance between the price and the number of participating players, including both the buyers and the sellers. Depending on the relative value between p and a, we have two scenarios with different results that are segmented by $\hat{p}(a)$. Figure 1 depicts curve $\hat{p}(a)$ in the (a, p) space. When the matching probability p is large (i.e., $p > \hat{p}(a)$), from the proof of the proposition, the constraint in Inequality (18) does not bind, and the equilibrium p_2 thus is above 1, which indicates excessive advertising. When p is small (i.e., $p < \hat{p}(a)$), the constraint binds, and in equilibrium $p_2 = 1$, which indicates non-excessive advertising.

Corollary 2. *When $p > \hat{p}(a)$, the equilibrium advertising is excessive (i.e., $p_2^* > 1$); when $p < \hat{p}(a)$, the equilibrium advertising is non-excessive.*

Notice that a seller's benefit from advertising is $\delta(\pi - k)$, the additional exposure multiplied by the profit margin from each sale. Clearly, the benefit is decreasing in sellers' costs. Given any a, when p is large—which means the exposure from the free listing is large—the additional exposure (by Equation (2)) is small and the benefit from advertising thus is small. As a result, only those with very low fixed costs participate in the advertising service, and the advertising might be excessive. Figure 1 shows the scenarios in which equilibrium advertising is excessive and the scenarios in which advertising is regular. The two scenarios are segmented by the cut-off curve $\hat{p}(a)$, above which the advertising is excessive. Note that the cut-off curve $\hat{p}(a)$ decreases in a. Intuitively, with a larger a, or with more space dedicated to advertising and less space to free organic space, the additional exposure and the benefit from advertising is more significant; in such a case, more sellers are induced to participate in the advertising service, and the advertising is less likely to be excessive.

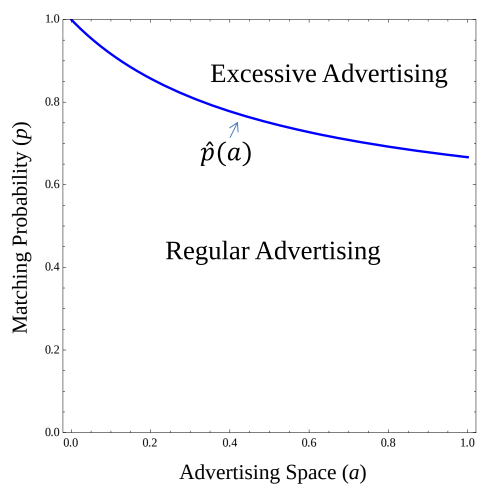


Figure 1: Excessive vs. Regular Advertising

Based on the optimal advertising fee, the mass of the participating seller and the mass of the participating advertisers in equilibrium are

$$n_A^* = k_A^* = \pi \text{ and } n_A'^* = k_A'^* \tag{20}$$

where $k_A'^*$, the cost of the marginal advertiser in equilibrium, is

$$k_A'^* = \begin{cases} \frac{(2\delta-1)+\sqrt{1-\delta+\delta^2}}{3\delta}\pi & \text{if } p > \hat{p}(a) \\ \frac{ap\pi}{\delta} & \text{if } p \leq \hat{p}(a). \end{cases} \tag{21}$$

(The derivation of $k_A'^*$ can be found in the proof of Proposition 2.) By Equation (15), the mass of the participating buyers in equilibrium is

$$m_A^* = c_A^* = \left[(1-\delta)\pi + \delta k_A'^*\right] s \tag{22}$$

Based on Lemma 2, we can conclude the participation of sellers and buyers as follows.

Corollary 3. *Under the advertising model, the sellers with costs in* $[0, k_A^*]$ *and the buyers with opportunity costs in* $[0, c_A^*]$ *participate in the platform in equilibrium. Among the participating sellers, the ones with costs in* $[0, k_A'^*]$ *participate in the advertising service, and the ones with costs in* $[k_A'^*, k_A^*]$ *do not participate in the advertising service.*

By substituting n_A^*, $n_A'^*$, m_A^*, and θ^* into Equations (5) and (6), we can formulate the payoffs of the participating buyers and the payoffs of the participating sellers (with advertising and without advertising) in equilibrium.

4 Equilibrium Comparison

In this section, we compare the platform's revenues under the two revenue models and study the conditions under which the advertising model can generate more revenue than the brokerage model. We also examine the sellers' payoffs, the buyers' payoffs, and the social welfare under the two revenue models.

4.1 The Platform's Revenue

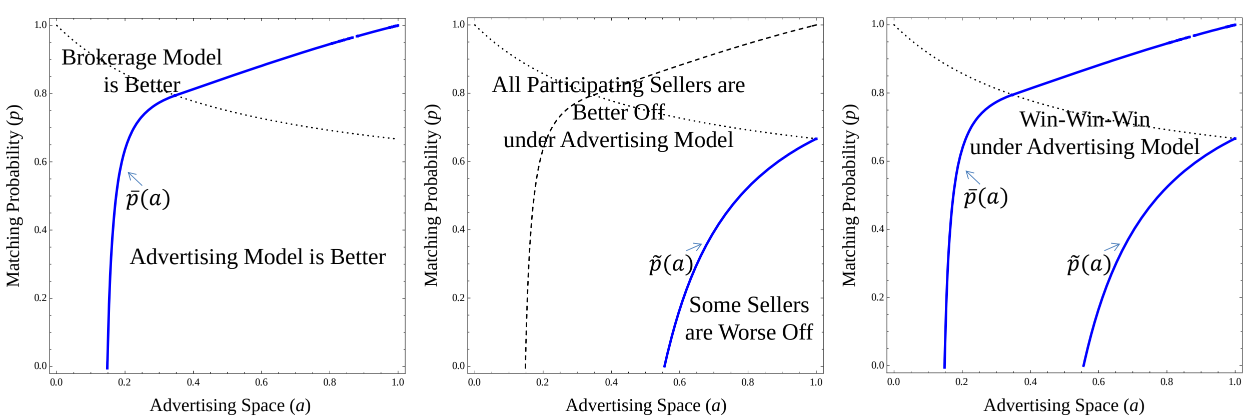
We first consider the platform's revenues under the two revenue models. The following proposition summarizes the results of comparing the equilibrium revenues derived in Propositions 1 and 2.

Proposition 3. *When $p > \bar{p}(a)$, the brokerage model generates more revenue than the advertising model; when $p < \bar{p}(a)$, the advertising model generates more revenue. The cutoff curve $\bar{p}(a)$ is defined as*

$$\bar{p}(a) = \begin{cases} 0 & \text{if } a \in \left[0, \frac{4}{27}\right] \\ \frac{27a-4}{31a-4} & \text{if } a \in \left[\frac{4}{27}, \frac{8}{23}\right] \\ p^*(a) & \text{if } a \in \left[\frac{8}{23}, 1\right] \end{cases}$$

in which $p^*(a)$ is determined by $\left[-2 + 3\delta + 3\delta^2 - 2\delta^3 + 2(1 - \delta + \delta^2)^{\frac{3}{2}}\right] = 4\delta p^2$, and δ is defined in Equation (14).

Figure 2a shows the curve $\bar{p}(a)$ and illustrates the comparison results. When the space dedicated to advertising a is very small, the brokerage model always generates more revenue than the advertising model. The reason is that, under the advertising model, the platform's revenue comes from the advertising space a, and the organic space is offered for free. When the space dedicated to advertising is very small, the revenue generated from advertising is limited. As a result, the advertising model generates less revenue than the brokerage model, in which each transaction is charged by the platform.



(a) Revenue Comparison (b) Sellers' Payoffs (c) Social Welfare

Figure 2: Equilibrium Comparison under the Two Revenue Models

This finding indicates that when a significant proportion of space is dedicated to advertising, the matching probability with which buyers find sellers plays a critical role in determining which revenue model is better. If the matching probability is high, the brokerage model generates more revenue than the advertising model; otherwise, the advertising model generates more revenue. The intuition is as follows. Under the brokerage model, when the matching probability that buyers will find their trading partner is low, buyers' payoffs are low, and a small number of buyers participate. In addition, even if a trading pair is on the platform, a low matching probability indicates that the likelihood of a trade is low. Therefore, the matching probability that trading partners will find one another monotonically affects the platform's revenue: The higher the probability is, the greater the platform's revenue. Under the advertising model, the probability on the buyer side has a similar effect—high matching probability tends to induce more buyers to participate in the platform. However, in sharp contrast to the brokerage model, when the probability of the seller's being noticed by buyers is low, the advertising service is highly valuable to sellers; the platform thus can charge a high price and earn high revenue. This difference—the change in the platform's revenue that occurs with the change in probability that the sellers are noticed by buyers—explains the existence of the cutoff: Once the matching probability falls below a certain threshold, the advertising model generates more revenue than the brokerage model.

This finding might also be used to explain the different business practices established by Taobao in China and by eBay in America. When Taobao started its business in 2003, its matching function, enabled by the underlying search function and categorizations, was in a less advanced stage than eBay's. More importantly, e-commerce was a relatively new phenomenon, and consumers were less experienced and less skillful in shopping online. These factors all contribute to a lower probability that buyers can find trading partners on the platform. Therefore, Taobao's use of the advertising model made—and makes—economic sense.

4.2 Sellers' Payoffs

We next examine sellers' payoffs under the two revenue models.

Under the brokerage model, the sellers with costs in $[0, k_B^*]$ participate in the platform. By

Equation (4), we can formulate the equilibrium payoff of a participating seller with cost k as

$$m_B^* p(\pi - k - t^*) = \tfrac{2}{3}p^2 s\pi(\tfrac{2}{3}\pi - k) \tag{23}$$

where the equality is achieved by substituting in both the optimal transaction fee t^* derived in Proposition 1 and the equilibrium mass of participating buyers outlined in Equation (10).

Under the advertising model, the sellers with costs in $[k_A'^*, k_A^*]$ participate in the platform for the basic service only (without advertising), and the sellers with costs in $[0, k_A'^*]$ participate in the advertising service in addition to the basic service. By letting $I = 0$ in Equation (6), we can formulate the equilibrium payoff of a participating seller with cost k in $[k_A'^*, k_A^*]$ as

$$m_A^*(1-a)p(\pi - k) \tag{24}$$

where the equilibrium mass of participating buyers is outlined in Equation (22). By letting $I = 1$ in Equation (6), we can formulate the equilibrium payoff of a participating advertiser with cost k in $[0, k_A'^*]$ as

$$m_A^*(\pi - k) - \theta^* \tag{25}$$

where the optimal advertising fee θ^* is outlined in Equation (19).

One important feature of the advertising model is that it offers a two-tiered service: the free basic service and the paid advertising service. Because of the free basic service, the advertising model generally attracts more sellers to participate in the platform than the brokerage model. In particular, $k_B^* = \frac{2}{3}\pi < \pi = k_A^*$ by Equations (9) and (20). Meanwhile, we can verify that the number of sellers who opt to advertise under the advertising model is less than the number of participating sellers under the brokerage model; that is, $k_A'^* < k_B^*$.

The sellers with costs in (k_B^*, k_A^*) are all better off under the advertising model. These sellers do not participate in the platform under the brokerage model because their low profit margins (resulting from high costs) cannot compensate for the transaction fee charged by the platform. In contrast, under the advertising model, they have an incentive to participate because the basic service is free and they can reap their profit from sales on the platform. Therefore, they are better off under the advertising model.

For the sellers with costs in $[0, k_B^*]$, we need to compare their payoffs under the two revenue models to determine who is better off under which model.

Proposition 4. *(a) The sellers with costs in (k_B^*, k_A^*) are better off under the advertising model.*

(b) For sellers with costs in $[0, k_B^*]$, if $p > \tilde{p}(a)$, all these sellers are better off under the advertising model; if $p < \tilde{p}(a)$, the sellers with cost $k \in \left(\frac{5\pi p}{3(3-2p)}, \frac{9a-5}{9a-3}\pi\right) \subset [0, k_B^*]$ are worse off under the advertising model, and other sellers are better off, where

$$\tilde{p}(a) = \begin{cases} 0 & \text{if } a \in \left[0, \frac{5}{9}\right] \\ \frac{9a-5}{11a-5} & \text{if } a \in \left[\frac{5}{9}, 1\right] \end{cases}$$

Figure 2b shows the $\tilde{p}(a)$ curve and illustrates the results. The intuition for (b) is as follows. First, the seller with cost k_B^* is better off under the advertising model, because under the brokerage model she is the marginal seller who is indifferent about participating in the market or not and who earns zero payoff, whereas under the advertising model, she earns positive payoff (by participating in the basic platform service for free). Because of the continuity in their payoff functions, the sellers with costs close to k_B^* are also better off under the advertising model. Second, the sellers with costs close to zero are also better off under the advertising model. Compared to the brokerage model, advertised sellers under the advertising model benefit from the increased exposure. Under the brokerage model, all the sellers get the same exposure. Under the advertising model, advertised sellers receive more exposure than they would under the brokerage model because advertising essentially shifts buyers' attention toward the advertised sellers from the unadvertised sellers. The sellers with very low fixed costs who participate in the advertising service (by Lemma 2) benefit more than their high-cost counterpart advertisers because of their different profit margins. As the optimal advertising fee is established on the basis of the average benefit from advertising for different sellers, the advertisers with very low costs benefit more than average and are better off under the advertising model.

Sellers with intermediate costs might be worse off under the advertising model when the space left for organic listing is limited and matching probability is low. First, when the organic space is small (and advertising space is large), the value of free organic listing is limited because of the limited exposure. As a result, the payoff of sellers under the free organic service might not be as good as their payoff under the brokerage model, even though sellers pay transaction fees for each sale. Second, when the matching probability is low, the advertising service is very valuable, and in equilibrium the advertising fee is not proportionally low and could even be high. As a result, when sellers with intermediate costs participate in the advertising service, they might not benefit a lot because of relatively high advertising fees. Thus, their payoff from advertising might not be as good as their payoff under the brokerage model.

4.3 Buyers' Payoffs

We can similarly examine buyers' payoffs under the two revenue models. Under the brokerage model, the buyers with costs in $[0, c_B^*]$ participate in the platform. By Equation (3), we can formulate the equilibrium payoff of a participating buyer with cost c as

$$n_B^* ps - c = \tfrac{2}{3} p\pi s - c \tag{26}$$

where the equality is achieved by substituting in the equilibrium mass of participating sellers outlined in Equation (9). Under the advertising model, the sellers with costs in $[0, c_A^*]$ participate in the platform. By Equation (5), we can formulate the equilibrium payoff of a participating buyer with cost c as

$$\left[(n_A^* - n_A'^*)p_1 + n_A'^*\right] s - c = \begin{cases} \frac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s - c & \text{if } p > \hat{p}(a) \\ p\pi s - c & \text{if } p \leq \hat{p}(a), \end{cases} \tag{27}$$

where the equality is achieved by substituting in $\delta = 1 - p_1$ and the equilibrium mass of participating sellers outlined in Equation (20).

Because consumers have the same opportunity cost under the two revenue models, whether they are better off under one model simply depends on the probability that they can find their trading partners (i.e., the mass of participating sellers times the matching probability). Under regular advertising (when $p < \hat{p}(a)$), buyers derive a higher benefit from the advertising model because more sellers participate in the advertising platform (i.e., $n_A^* = \pi > n_B^* = \frac{2}{3}\pi$), while the average matching probability is the same under the two revenue models.

Under excessive advertising, the average matching probability under the advertising model is lower than the probability under the brokerage model (but the number of participating sellers is larger in the former). With an arbitrarily high advertising fee (such that only few sellers participate in the advertising service), buyers might be worse off because of a significant decrease in the average matching probability resulting from excessive advertising. However, in choosing the optimal advertising fee, the platform considers not only the direct effect of the advertising fee on the number of participating advertisers, but also the indirect effect on buyers' participation, driven by the benefit that buyers derive from the platform. As a result, we can verify that even with excessive advertising in equilibrium, buyers are better off under the advertising model.

Proposition 5. *Buyers are better off under the advertising model.*

Also, because the benefits for buyers under the advertising model are higher than they are under the brokerage model, in equilibrium more buyers participate in the platform under the advertising model.

4.4 Social Welfare

We next examine the social welfare under the two revenue models. In our setting, social welfare is the value created by the platform, which can be measured by the total value realized by the transactions on the platform net the costs associated with both the sellers and buyers.

Under the brokerage model, the number of transactions is $m_B^* n_B^* p$, and the value created from each transaction is $s+\pi$. The average cost on the seller side is $k_B^*/2$, and the average opportunity cost on the buyer side is $c_B^*/2$. Therefore, the social welfare under the brokerage model is

$$W_B = m_B^* n_B^* p \left(s+\pi-\frac{k_B^*}{2}\right) - m_B^* \frac{c_B^*}{2} = \frac{2}{3}ps\pi\frac{2}{3}\pi p\left(s+\pi-\frac{\pi}{3}\right)-\frac{2}{9}(ps\pi)^2 \tag{28}$$

where the equality is achieved by substituting in $(m_B^*,\ k_B^*)$ and $(n_B^*,\ c_B^*)$ from Equations (9) and (10). Similarly, we can formulate social welfare under the advertising model as

$$W_A = m_A^* \left[(n_A^* - n_A'^*)\, p_1 \left(s+\pi-\frac{k_A^*+k_A'^*}{2}\right) + n_A'^* \left(s+\pi-\frac{k_A'^*}{2}\right)\right] - m_A^* \frac{c_A^*}{2} \tag{29}$$

where $(n_A^*,\ k_A^*)$, $(n_A'^*,\ k_A'^*)$, and $(m_A^*,\ c_A^*)$ are specified in Equations (20), (21), and (22), respectively. Comparing the social welfare under the two revenue models as derived leads to the following conclusion.

Proposition 6. *The advertising model generates more social welfare than the brokerage model.*

The intuition is as follows. First, compared with the brokerage model, more buyers participate in the trading platform under the advertising model, as explained in the previous section, because the probability (i.e., the mass of participating sellers times the matching probabilities) that they find their trading partners is higher under the advertising model. As a result of the greater participation by buyers and the higher trading probability, more transactions take place under the advertising model. Second, the lower cost sellers participate in the advertising service and receive more attention from buyers. Therefore, a product from a lower cost seller is more likely to be sold than a product of a high-cost counterpart under the advertising model, and lower cost products are sold more often under the advertising model than under the brokerage model. Both the increased volume of transactions and the increased transactions of low-cost products under the advertising model increase social welfare, compared to the brokerage model.

Note that, compared to the brokerage model, the advertising model may lead to a win-win-win result in equilibrium; that is, the platform, the (participating) sellers, and the (participating) buyers might all be better off under the advertising model at the same time. Figure 2c depicts the win-win-win area in the (a, p) space; that is, $\tilde{p}(a) < p < \bar{p}(a)$. By Proposition 3, when $p < \bar{p}(a)$, the platform is better off under the advertising model, and by Proposition 4, when $p > \tilde{p}(a)$, all participating sellers are better off as well. Meanwhile, by Proposition 5, buyers are always better off under the advertising model. Therefore, when both conditions $p < \bar{p}(a)$ and $p > \tilde{p}(a)$ are satisfied, the win-win-win outcome occurs. The win-win-win result is possible because the advertising model generates more social welfare, so that the total "pie" is bigger under the advertising model.

5 Conclusion

In this paper, we study how the choice of revenue model—between the brokerage model and the advertising model—affects the platform's revenue, sellers' payoffs, buyers' payoffs, and social welfare. We find that both the size of the advertising space and matching probability play critical roles in the comparison. We also find that when a significant proportion of space is dedicated to advertising, if the matching probability is low, the advertising model generates more revenue; otherwise, the brokerage model generates more. Sellers are better off under the advertising model in most scenarios.

The only exception is that when a limited space is left for organic listing and matching probability is low, some sellers with intermediate costs might be worse off under the advertising model. Buyers are always better off, and social welfare is higher under the advertising model.

Our research has several implications. First, we underscore the importance for platform owners to tailor their revenue models according to the platform technologies and user experience with online platform shopping. With the advances of a platform's search and categorization technologies, the platform owner should consider adapting its revenue model from advertising to brokerage. In addition, different from that under an advertising model, platform owners under a brokerage model should always make the platform easy to navigate and provide necessary help for users to locate what they seek, which increases the probability that consumers can find their sellers and could also benefit the platform owners. Our analysis thus illustrates that the choice of revenue model should be assessed in line with technologies development.

Our results also imply that platform owners who switch from one revenue model to the other might encounter resistance from users. In particular, switching to a brokerage model might hurt sellers with either very low or high profit margins because, under the advertising model, the former benefit from the free service and the latter benefit from the additional exposure from advertising. To prevent the sellers from leaving the platform (e.g., by establishing their own direct-sell websites) after the switch, the platform could consider offering some special term for their business on its platform.

Our research also has implications for social planners. As we illustrated in the analysis, the advertising model could generate more social welfare than the brokerage model. Therefore, the choice of the social planner is not necessarily aligned with that of a platform owner. To induce platform owners to choose the right revenue model for social welfare purposes, social planners might need to subsidize platform owners.

As a future research direction, we can consider the competition between the two platforms using different business models. In fact, eBay entered China in 2002, the year before Taobao launched its trading platform. The dominant role that Taobao holds in China can be viewed as the result of the competition. Also, although Google does not offer a trading platform, the competition between eBay and Google bears a similar flavor as the competition between eBay and Taobao because Google, similar to Taobao, offers basic search service for free and sponsored search service for sellers to purchase. Competitive positioning has long been discussed in the literature (e.g., Adner et al., 2012). We believe that a study of the competition between platforms would reveal additional insights beyond the extant literature.

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A Appendix

A.1 Proof of Proposition 1

Proof. Notice that the objective function $p^2 st(\pi - t)^2$ crosses zero at $t = 0$ and $t = \pi$, and it is positive over $[0, \pi]$. Its first-order derivative $p^2 s\left[(\pi - t)^2 - 2t(\pi - t)\right] = p^2 s(\pi - t)(\pi - 3t)$ is positive over $(0, \pi/3)$ and is negative over $(\pi/3, \pi)$. Therefore, the objective function reaches the maximum at $t^* = \pi/3$. Substituting t^* into the objective function results in the maximum revenue. □

A.2 Proof of Proposition 2

Proof. Notice that the objective function $\delta s\left[\pi(\pi - k'_A) - \delta(\pi - k'_A)^2\right] k'_A$ crosses zero three times at $k'_A = \pi - \pi/\delta$, $k'_A = 0$, and $k'_A = \pi$. We can verify that the objective function is positive over $[0, \pi]$.

By letting the first-order derivative of the objective function be zero, we have (after removing the constant term δs)

$$\left[\pi(\pi - k'_A) - \delta(\pi - k'_A)^2\right] + \left[-\pi + 2\delta(\pi - k'_A)\right] k'_A = 0$$

which can be reorganized as

$$-3\delta k'^2_A - 2(1 - 2\delta)\pi k'_A + (1 - \delta)\pi^2 = 0$$

Because $\frac{(1-\delta)\pi^2}{-3\delta} < 0$, one root of the above equation is negative and the other is positive. The positive one is

$$k'^+_A = \frac{2(1-2\delta)\pi - \sqrt{4(1-2\delta)^2\pi^2 + 12\delta(1-\delta)\pi^2}}{-6\delta} = \frac{(2\delta-1) + \sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which can be verified to be less than π because $\sqrt{1 - \delta + \delta^2} < 1 + \delta$. Therefore, its first-order derivative is positive over $\left(0, k'^+_A\right)$ and is negative over $\left(k'^+_A, \pi\right)$, which indicates that the objective function is increasing over $\left(0, k'^+_A\right)$ and decreasing over $\left(k'^+_A, \pi\right)$.

Notice that the constraint in Inequality (18) is equivalent to $k'_A \leq \frac{ap\pi}{\delta}$. Therefore, if $\frac{ap\pi}{\delta} \geq k'^+_A$, the objective function reaches the maximum at $k'^*_A = k'^+_A$; otherwise, it reaches the maximum at

$k'^{*}_A = \frac{ap\pi}{\delta}$ (when the constraint binds). The condition $\frac{ap\pi}{\delta} \geq k'^{+}_A$ can be rewritten as

$$\frac{ap\pi}{\delta} > \frac{(2\delta-1)+\sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which is equivalent to $(\delta - 2 + 3p) > \sqrt{1-\delta+\delta^2}$. By substituting in $\delta = 1-(1-a)p$, the above condition can be simplified to $p > \frac{1+a}{1+2a} = \hat{p}(a)$.

Therefore, if $p < \hat{p}(a)$, substituting $k'^{*}_A = \frac{ap\pi}{\delta}$ into θ^*, we have

$$\theta^* = \delta\pi s(\pi - k'_A) - \delta^2 s(\pi - k'_A)^2 = s\pi^2\left[(\delta - ap) - (\delta - ap)^2\right] = p(1-p)s\pi^2$$

and thus $\Pi^*_A = \theta^* k'^{*}_A = \frac{a}{\delta}p^2(1-p)s\pi^3$.

If $p > \hat{p}(a)$, substituting $k'^{*}_A = k'^{+}_A$ into θ^*, we have

$$\theta^* = s\pi^2\left[(\delta - \frac{-(1-2\delta)+\sqrt{1-\delta+\delta^2}}{3}) - (\delta - \frac{-(1-2\delta)+\sqrt{1-\delta+\delta^2}}{3})^2\right] = s\pi^2\left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right]$$

and thus

$$\Pi^*_A = \theta^* k'^{*}_A = s\pi^2\left[\frac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right]\frac{(2\delta-1)+\sqrt{1-\delta+\delta^2}}{3\delta}\pi$$

which can be simplified to the one in the proposition. □

A.3 Proof of Corollary 2

Proof. From the proof of Proposition 2, if $p > \hat{p}(a)$, the constraint in Inequality (18) does not bind, and therefore $p^*_2 > 1$; otherwise, the constraint binds and $p^*_2 = 1$. □

A.4 Proof of Proposition 3

Proof. We denote $\Delta \equiv \Pi^*_B - \Pi^*_A$. When $p < \hat{p}(a)$ (in which $\hat{p}(a)$ is as defined in Proposition 2),

$$\Delta = \frac{4p^2 s\pi^3}{27} - \frac{(1-p)a}{\delta}p^2 s\pi^3 = p^2 s\pi^3\left(\frac{4}{27} - \frac{(1-p)a}{\delta}\right) = \frac{p^2 s\pi^3}{27\delta}\left[p(31a-4) - (27a-4)\right] \tag{30}$$

For any $a \in [0, \frac{4}{27}]$, $\Delta > 0$ because when $a < \frac{4}{31}$, $4 - 27a > 4 - 31a$, and when $a > \frac{4}{31}$, $31a - 4 > 0$ and $27a - 4 < 0$. For any $a \in [\frac{4}{27}, 1]$, $\Delta = 0$ defines a curve $p(a) = \frac{27a-4}{31a-4}$ on the (a, p) space, which intersects with $\hat{p}(a)$ at $a^* = \frac{8}{23}$. When $a < a^*$, we can verify $\frac{27a-4}{31a-4} < \hat{p}(a)$, and therefore if

$p > \frac{27a-4}{31a-4}$, $\Delta > 0$; otherwise, $\Delta < 0$. When $a > a^*$, $\frac{27a-4}{31a-4} > \hat{p}(a)$, and therefore, within $p < \hat{p}(a)$, $\Delta < 0$.

When $p > \hat{p}(a)$,

$$\Delta = \frac{s\pi^3}{27\delta}\left[4p^2\delta - \left(-2 + 3\delta + 3\delta^2 - 2\delta^3 + 2(1-\delta+\delta^2)^{\frac{3}{2}}\right)\right]$$

Notice that $\Delta = 0$ defines a curve $p^*(a)$ on the (a,p) space, which intersects with $\hat{p}(a)$ at $a^* = \frac{8}{23}$. When $a < a^*$, we can verify $p^*(a) < \hat{p}(a)$, and therefore, within $p > \hat{p}(a)$, $\Delta > 0$. When $a > a^*$, $p^*(a) > \hat{p}(a)$, and therefore if $p > p^*(a)$, $\Delta > 0$; otherwise, $\Delta < 0$.

To summarize, for $a \in [0, \frac{4}{27}]$, we have $\Delta > 0$. For $a \in [\frac{4}{27}, \frac{8}{23}]$, if and only if $p > \frac{27a-4}{31a-4}$, $\Delta > 0$. For $a \in [\frac{8}{23}, 1]$, if and only if $p > p^*(a)$, $\Delta > 0$. Then $\bar{p}(a)$ in the proposition follows. □

A.5 Proof of Proposition 4

Proof. (a) The payoffs of these sellers under the advertising model, by Equation (24), are positive because the marginal seller with k_A^* derives zero payoff, and these sellers have lower costs than the marginal sellers. These sellers do not participate under the brokerage model and derive zero payoff. Therefore, they are all better off under the advertising model.

(b) For sellers in $[k_A'^*, k_B^*]$, when $p > \hat{p}(a)$ (in which $\hat{p}(a)$ is defined as in Proposition 2), sellers are better off under the advertising model if

$$\frac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s(1-a)p(\pi - k) > \frac{2}{3}p^2 s\pi(\frac{2}{3}\pi - k) \tag{31}$$

where the term in the left-hand side is the sellers' payoffs under the advertising model by substituting m_A^* from Equation (22) into Equation (24), and the term in the right-hand side is the sellers' payoff under the brokerage model from Equation (23). We can verify that Inequality (31) is true for all $p > \hat{p}(a)$ and that these sellers are better off under the advertising model.

Similarly, when $p < \hat{p}(a)$, sellers are better off under the advertising model if

$$ps\pi(1-a)p(\pi - k) > \frac{2}{3}p^2 s\pi(\frac{2}{3}\pi - k)$$

The condition can be simplified to $k(a - \frac{1}{3}) > (a - \frac{5}{9})\pi$. For $a \in [0, \frac{1}{3}]$, the condition is satisfied because $k \leq \frac{2\pi}{3} = k_B^*$. For $a \in [\frac{1}{3}, \frac{5}{9}]$, the condition is satisfied because the right-hand side is non-positive. When $a > \frac{5}{9}$, the inequality condition reduces to

$$k > \tfrac{9a-5}{9a-3}\pi \tag{32}$$

We next examine the condition for $\frac{9a-5}{9a-3}\pi > k_A'^* = \frac{ap\pi}{\delta}$. Substituting δ in, by simple algebra, the condition is equivalent to $p < \frac{9a-5}{11a-5}$. Therefore, if $p > \frac{9a-5}{11a-5}$, $\frac{9a-5}{9a-3}\pi < k_A'^*$ and all $k \in [k_A'^*, k_B^*]$ satisfy Equation (32) and sellers are better off; otherwise, sellers with $k > \frac{9a-5}{9a-3}\pi$ are better off under the advertising model and other sellers are worse off.

For sellers in $[0, k_A'^*]$, when $p > \hat{p}(a)$, sellers are better off under the advertising model if

$$\tfrac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s(\pi - k) - s\pi^2 \left[\tfrac{1+2\delta-2\delta^2+(2\delta-1)\sqrt{1-\delta+\delta^2}}{9}\right] > \tfrac{2}{3}p^2 s\pi(\tfrac{2}{3}\pi - k) \tag{33}$$

where the term in the left-hand side is sellers' payoffs under the advertising model by substituting m_A^* from Equation (22) and θ^* from Equation (19) into Equation (25), and the term in the right-hand side is sellers' payoff under the brokerage model from Equation (23). We can verify that Inequality (33) is true for all $p > \hat{p}(a)$ and these sellers are better off under the advertising model.

Similarly, when $p < \hat{p}(a)$, sellers are better off under the advertising model if

$$ps\pi(\pi - k) - p(1-p)s\pi^2 > \tfrac{2}{3}p^2 s\pi(\tfrac{2}{3}\pi - k)$$

By simple algebra, the condition can be reduced to $(1 - \frac{2}{3}p)k < \frac{5}{9}p\pi$. Therefore, any $k < \frac{5p\pi}{3(3-2p)}$ satisfies the condition. We next check the condition for $\frac{5\pi p}{3(3-2p)} < k_A'^* = \frac{ap\pi}{\delta}$. Substituting δ in, by simple algebra, the condition is equivalent to $p < \frac{9a-5}{11a-5}$. Therefore, if $p > \frac{9a-5}{11a-5}$, $\frac{5\pi p}{3(3-2p)} > k_A'^*$ and all $k \in [0, k_A'^*]$ satisfy $k < \frac{5\pi p}{3(3-2p)}$ and sellers are better off; otherwise, sellers with $k < \frac{5p\pi}{3(3-2p)}$ are better off under the advertising model and other sellers are worse off.

All together, we can summarize the results using function $\tilde{p}(a)$ specified in the proposition. $\square$

A.6 Proof of Proposition 5

Proof. When $p < \hat{p}(a)$, the result follows because $p\pi s - c > \frac{2}{3}p\pi s - c$ by Equations (26) and (27). When $p > \hat{p}(a)$, the result can be established if

$$\tfrac{2-\delta+\sqrt{1-\delta+\delta^2}}{3}\pi s - c > \tfrac{2}{3}p\pi s - c$$

or, equivalently, if $2-\delta+\sqrt{1-\delta+\delta^2} > 2p$. Furthermore, the condition is equivalent to $1-\delta+\delta^2 > (2p-2+\delta)^2$, which reduces to $3+a-4ap > 0$ by substituting into δ. Because a and p are in $[0,1]$, the inequality $3+a-4ap > 0$ is true. □

A.7 Proof of Proposition 6

Proof. When $p \leq \hat{p}(a)$, the average matching probability under the advertising model is the same as under the brokerage model; that is, $[(n_A^* - n_A'^*)p_1 + n_A'^*] = pn_A^*$ (which can also be seen from Equation (27)). Notice that $m_B^* < m_A^*$ and $n_B^* < n_A^*$. We check the social welfare under the brokerage model generated by the low-cost buyers with a mass m_B^* (among m_A^*) and the low-cost sellers with a mass n_B^* (among n_A^*). These segments of buyers and sellers under the advertising model generate the same total value $m_B^* n_B^* p(s+\pi)$ as under the brokerage model. The total fixed cost on the seller side is lower than the fixed cost under the brokerage model because, among m_B^*, the lower cost sellers participate in the advertising and sell their products more often. The total opportunity costs on the buyer side are the same under the two models. Therefore, these segments of buyers and sellers under the advertising model generate more social welfare than that under the brokerage model. Other participating buyers and sellers generate additional social welfare because their decisions to participate means that the expected benefits are greater than their costs.

When $p > \hat{p}(a)$, we can also verify that the advertising model generates more social welfare than the brokerage model. □