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Aerodynamics of solid bodies in the solar nebula

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Summary. In a centrally condensed solar nebula, the pressure gradient in the gas causes the nebula to rotate more slowly than the free orbital velocity. Drag forces cause the orbits of solid bodies to decay. Their motions have been investigated analytically and numerically for all applicable drag laws. The maximum radial velocity developed is independent of the drag law, and insensitive to the nebular mass. Results are presented for a variety of model nebulae. Radial velocities depend strongly on particle size, reaching values on the order of 10^4 cm/s for metre-sized objects. Possible consequences include: mixing of solid matter within the solar nebula on short timescales, collisions leading to rapid accumulation of planetesimals, fractionation of bodies by size or density, and production of regions of anomalous composition in the solar nebula.

1 Introduction

In theories of cosmogony which involve a disk-shaped solar nebula of gas and dust, it is reasonable to assume some degree of central condensation, i.e. the pressure, density, and temperature in the midplane decrease away from the axis. Such a nebula is supported against the central gravity primarily by its rotation. However, the pressure gradient provides some support to the gaseous component. The condition of hydrostatic equilibrium requires the rotation rate of the gas at any point to be somewhat less than the free orbital velocity. Solid bodies present in the nebula are not supported by the pressure gradient. In the absence of gas, they would pursue Keplerian orbits, but this motion is perturbed by drag. In a centrally condensed nebula, the orbits decay, and the solid bodies spiral inward.

Kiang (1962) analysed in detail the effects of a resisting medium on elliptical orbits. However, he considered only the cases in which the medium was stationary, in uniform rotation, and rotating freely at the Kepler velocity. None of these cases are physically realistic. Whipple (1964) recognized qualitatively that the solar nebula would deviate from Keplerian rotation, and cause the decay of orbits. In later papers (Whipple 1972, 1973), he examined quantitatively the limiting cases of small and large bodies, using the Stokes and Epstein drag laws (defined below). Cameron (1973a) applied Whipple's results to the solar nebula models of Cameron & Pine (1973).

The present paper extends and generalizes Whipple's work. I shall derive expressions which describe the motions of particles of all sizes, from dust grains to protoplanets. All appropriate drag laws will be used to produce quantitative results. These results will not be restricted to a single nebular model, but will show in a general way the effects of different assumptions as to its mass and structure. Finally, some possible consequences of the particle motions will be described, and their cosmogonical implications discussed.

2 Basic relations

Let P be the gas pressure, ρ its density, and r the distance from the nebular axis. The central gravity, g, and the circular Kepler orbital velocity, V_k, are related by

$$g = \frac{GM_\odot}{r^2} = \frac{V_k{}^2}{r}, \tag{1}$$

where G is the gravitational constant, and $M_\odot$ the solar mass. In a reference frame rotating with the gas, the residual gravity is

$$\Delta g = \frac{1}{\rho}\frac{dP}{dr}. \tag{2}$$

Equation (2) is simply the condition for hydrostatic equilibrium. We shall consider explicitly only the centrally condensed case, for which dP/dr and Δg are negative, but the derivations will be valid for either sense of the pressure gradient. If V_g is the rotational velocity of the gas,

$$\frac{V_g{}^2}{r} = \frac{V_k{}^2}{r} + \Delta g, \tag{3}$$

and, if $\Delta g/g \ll 1$,

$$\Delta V = V_k - V_g \simeq -\left(\frac{\Delta g}{2g}\right) V_k \tag{4}$$

is the deviation of the gas velocity from the circular orbital velocity.

Over some range of r, the pressure can always be approximated by a power law,

$$P = P_0(r/r_0)^{-n} \tag{5}$$

For an ideal gas, of molecular weight μ

$$\Delta g = \frac{1}{\rho}\frac{dP}{dr} = \frac{-nRT}{\mu r}, \tag{6}$$

where R is the gas constant, and T the temperature. Note that Δg depends on the exponent n and T, but not on P or ρ. Hence, Δg is independent of the nebular mass, as is ΔV, if the nebula's self-gravitation can be neglected.

3 Drag laws

Assume that a solid particle is spherical, of radius s and density ρ_s. If it moves through a gas at velocity v, and the mean free path of the gas molecules, λ, is small compared with s, then the drag force is

$$F_D = C_D \pi s^2 \rho \frac{v^2}{2} \tag{7}$$

with C_D a dimensionless drag coefficient which depends on the Reynolds number $Re = 2s\rho v/\eta$, where η in the gas viscosity. In an ideal gas, η is independent of the density.

The drag coefficient of a sphere is (Whipple 1972)

$$C_D \simeq 24 Re^{-1} \text{ for } Re < 1 \tag{8a}$$

$$C_D \simeq 24 Re^{-0.6} \text{ for } 1 < Re < 800 \tag{8b}$$

$$C_D \simeq 0.44 \text{ for } Re > 800, \tag{8c}$$

where (8a) is the Stokes drag law. The Epstein drag law,

$$F_D \simeq \frac{4\pi}{3} \rho s^2 v \bar{v}, \tag{9}$$

where $\bar{v}$ is the mean thermal velocity of the gas, is applicable if $\lambda > s$ and $v \ll \bar{v}$; the latter condition holds for all cases of interest. The Stokes and Epstein drag forces are equal when $\lambda/s = 4/9$; this is taken to be the transition point between the two laws.

Using Whipple's notation, the 'stopping time', t_e is

$$t_e = \frac{mv}{F_D}, \tag{10}$$

where $m = 4\pi s^3 \rho_s/3$ is the particle mass; t_e is the time in which v would be reduced by a factor e by a constant drag force. For the drag laws of equations (8),

$$t_e = \frac{2\rho_s s^2}{9\eta} \text{ for } Re < 1 \tag{11a}$$

$$t_e = \frac{2^{0.6} \rho_s s^{1.6}}{9\eta^{0.6} \rho^{0.4} v^{0.4}} \text{ for } 1 < Re < 800 \tag{11b}$$

$$t_e = \frac{6\rho_s s}{\rho v} \text{ for } Re > 800, \tag{11c}$$

and $$t_e = \frac{\rho_s s}{\rho \bar{v}} \tag{12}$$

in the Epstein drag regime.

4 Equations of motion

The type of motion of a particle under the combined forces of gravity and drag depends on the ratio of t_e to the orbital period, t_p. The limiting cases of $t_e/t_p \ll 1$ (small bodies) and $t_e/t_p \gg 1$ (large bodies) were derived by Whipple (1972, 1973), and are briefly restated here.

4.1 $t_e/t_p \ll 1$: RADIAL DRIFT

The particles are carried with the gas, at its angular velocity. In the reference frame rotating with the gas, the residual gravity, Δg, causes them to fall inward. The transverse velocity relative to the gas is negligible. The terminal velocity is given by

$$dr/dt = v = t_e \Delta g. \tag{13}$$

4.2 $t_e/t_p \gg 1$: PERTURBED KEPLER ORBIT

The particle motion is not strongly influenced by the gas. It pursues a Kepler orbit, but experiences a transverse 'headwind' of magnitude ΔV. The drag force causes the orbit to decay at a rate

$$\frac{dr}{dt} = \frac{r}{t_e}\frac{\Delta g}{g}. \tag{14}$$

4.3 STRONGLY PERTURBED CASE

From equations (13) and (14) it can be seen that $dr/dt \to 0$ in the limits of very small or large bodies. The radial velocity reaches a maximum for some particle size; this occurs when $t_e/t_p \simeq 1/2\pi$. If the gas has no radial motion of its own, a particle with radial velocity $-u$ feels a wind of velocity u blowing outward. Let the particle's transverse velocity with respect to the gas be w; its total transverse velocity is $V_g + w$. The total wind velocity felt by the particle is $v = (u^2+w^2)^{1/2}$.

The radial component of the drag force is $F_D\, u/v$. A radial force balance gives

$$\frac{F_D}{m}\frac{u}{v} + \frac{(V_g+w)^2}{r} - g = 0. \tag{15}$$

Using equations (1) and (4), and neglecting terms of order $(\Delta g/g)^2$, this can be written as

$$\frac{F_D}{m}\frac{u}{v} + \Delta g + \frac{2w}{r} V_k = 0. \tag{16}$$

The transverse component of the drag force is $F_D\, w/v$. The particle's specific angular momentum, h, is equal to $r(V_g + w)$. The rate of change of h is equal to the torque:

$$\frac{dh}{dt} = \frac{-rF_D}{m}\frac{w}{v}. \tag{17}$$

However,

$$\frac{dh}{dt} = \frac{d}{dt}(r(V_g+w))$$
$$= \frac{dr}{dt}(V_g+w) + r\frac{\partial}{\partial r}(V_g+w)\frac{dr}{dt}. \tag{18}$$

Since $dr/dt = -u$, and

$$\frac{\partial}{\partial r}(V_g+w) \simeq -\left(1+\frac{\Delta g}{2g}\right)\frac{V_k}{2r} + \frac{\partial w}{\partial r}, \tag{19}$$

equations (17–19) combine to give

$$\frac{F_\mathrm{D}}{m}\frac{w}{v}+\frac{uV_\mathrm{k}}{2r}=0, \tag{20}$$

where we neglect all terms of order $(\Delta g/g)^2$. The term in F_D can be eliminated by multiplying (20) by (u/w) and subtracting from (16)

$$\frac{2V_\mathrm{k}}{r}w^2+\Delta gw-\frac{u^2V_\mathrm{k}}{2r}=0. \tag{21}$$

For any value of u, equation (21) is quadratic in w, with the solutions

$$w(u) = [-\Delta g \pm (\Delta g^2-4gu^2/r)^{1/2}]\,V_\mathrm{k}/4g. \tag{22}$$

These solutions are real whenever $(\Delta g^2-4gu^2/r) \geqslant 0$. Since $g = V_\mathrm{k}{}^2/r$, this condition implies

$$|u| \leqslant \left|\frac{\Delta g}{2g}\right| V_\mathrm{k}. \tag{23}$$

The magnitude of the limiting radial velocity is equal to $|\Delta V|$, where the sign of dr/dt is the same as that of ΔV.

In the derivation of equation (22), we eliminated all information about the drag law and particle properties. The relation between u and w, and the limit of (23), are therefore properties of the nebula itself, and valid in all cases. The only necessary condition is that the drag force be parallel to the wind direction, i.e. that the particle does not develop aerodynamic lift. Whipple (1972, 1973) reported without derivation that the limiting radial velocity was 0.5 ΔV for the Epstein and Stokes drag laws, but we see here that the limit is twice as large, and valid for all drag laws. Equation (22) has two solutions, depending on the sign in the right-hand expression. For the plus sign, $w \to u^2/4\Delta V$ as $u \to 0$; this is the case of small bodies. The minus sign applies to the case of large bodies, for which $w \to \Delta V$ as $u \to 0$. Note that two bodies of different size may have the same radial velocity, but will

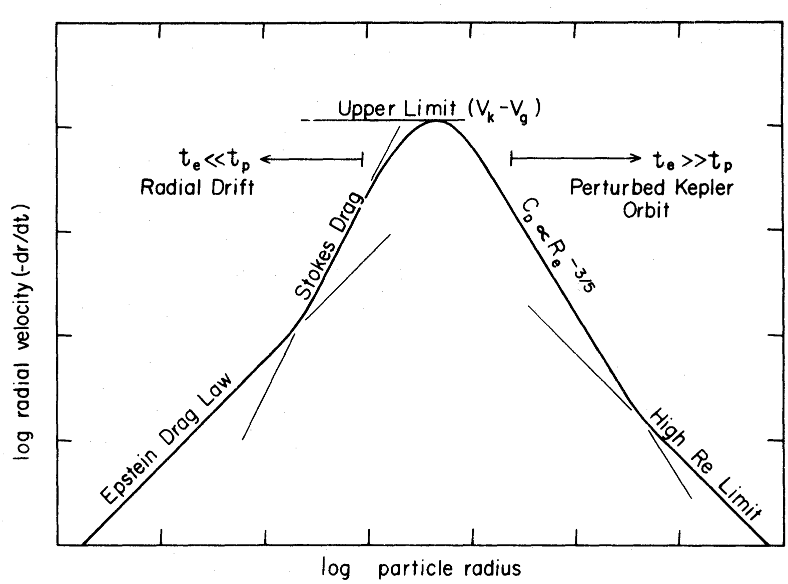

Figure 1. Radial velocity versus particle size (schematic). The shape of the curve is determined by the drag laws, but the peak value depends only on the nebular structure.

always have different transverse velocities. By differentiation of $v = (u^2+w^2)^{1/2}$, we find the maximum total wind velocity is $\simeq 1.15\ \Delta V$.

While equation (22) is generally true, the values of u and w for any given particle depend on its size and density, and on the drag law. The shape of the curve of u versus s is determined by the drag law, as shown schematically in Fig. 1. In principle, equation (22) may be used to rewrite (16) or (20) as a function of u only, and the resulting expression solved numerically. However, the appropriate drag law depends on the wind velocity. In practice, one must assume a drag law and solve for the velocity components. If the computed Reynolds number is not consistent with the assumed drag law, the procedure is repeated until a self-consistent solution is found. A computer program has been written which performs these operations.

5 The model nebula

The results obtained depend to some extent upon the assumed nebular structure. Our procedure will be to define a 'standard' model nebula, and to investigate in detail the behaviour of particles as functions of size and position in the nebula. Then we shall see how this behaviour varies as the nebular structure is changed. It is convenient to assume power laws for the pressure and temperature structures:

$$P(r) = P_0\,(r/r_0)^{-n} \tag{5}$$

$$T(r) = T_0\,(r/r_0)^{-m} \tag{24}$$

For the standard model, we take $n = 3$, $m = 1$. These choices need to be justified and compared with existing models.

5.1 PRESSURE STRUCTURE

Schatzman's (1967) model has $n = 4$. In the massive nebular models of Cameron & Pine (1973), $2 < n < 4$. Safronov (1969) does not explicitly state a pressure structure. However, he shows (chapter 3) that if the nebula's self-gravitation is small and the vertical structure isothermal, then the pressure in the central plane is

$$P = \left(\frac{GM_\odot}{r^3}\right)^{1/2} \frac{\sigma\bar{v}}{4}, \tag{25}$$

where σ is the surface density of gas. It is a straightforward calculation to show that if the vertical structure is adiabatic, the expression of (25) is changed only by a constant factor of order unity. 'Reconstruction' of the solar nebula by adding H and He to restore the planets to solar composition (Kusaka, Nakano & Hayashi 1970) suggests a variation of σ in the range $r^{-1}-r^{-2}$, depending on the assumptions used. Since $\bar{v} \propto T^{1/2}$, the assumption that $m = 1$ would imply $3 \leqslant n \leqslant 4$.

5.2 TEMPERATURE STRUCTURE

The smallest probable value of m is 0.5, corresponding to radiative equilibrium in an optically thin nebula. Schatzman (1967) assumes $m \simeq 0.5$; Safronov's (1969, chapter 4) analysis suggests $0.5 < m < 1.0$. The massive, optically thick nebular models of Cameron & Pine (1973) have $m \simeq 1$ in most regions. Lewis (1974) has argued strongly for a temperature gradient with $m \simeq 1$, on the basis of chemical evidence. The adopted standard model, with $n = 3$, $m = 1$, is equivalent to an adiabatic nebula with γ, the ratio of specific heats, equal to 1.5.

The mass of the model nebula must also be specified. I adopt the conditions, $T = 600$ K, $\rho = 10^{-9}\,\mathrm{g\,cm^{-3}}$, at $r = 1$ AU. The standard model then lies between the 'high' and 'low' adiabats considered by Lewis (1974). It can be shown that $\rho(r) \propto r^{-2}$, $\sigma(r) \propto r^{-1}$, and the total mass within $r = 40$ AU is about $0.05\,M_\odot$. This is a 'minimal' nebula, near the lower limit of mass sufficient to form the planets. The adopted values give $\Delta g/g = -7.5 \times 10^{-3}$ throughout the nebula, and $\Delta V = 3.75 \times 10^{-3}\,V_\mathrm{K}$.

I emphasize that the descriptions of the published nebular models are oversimplified. They differ greatly from each other, and from the standard model used here. The latter is designed for ease of computation, but has Δg and ΔV generally of the same order of magnitude as the more detailed models, and can provide insights to their behaviour. Those who favour particular models are encouraged to examine them; copies of the computer program will be provided upon request.

6 Results

The components of wind velocity at $r = 1$ AU in the standard model nebula are shown in Fig. 2. ΔV is approximately 10^4 cm/s. For the assumed particle density $\rho_s = 3\,\mathrm{g\,cm^{-3}}$, the size corresponding to the maximum radial velocity, s^*, is about 50 cm. The effect of different values of ρ_s is shown in Fig. 3. For 'small' bodies ($s \ll s^*$), $dr/dt \propto \rho_s$ at any given value of s; for 'large' bodies ($s \gg s^*$), $dr/dt \propto \rho_s^{-1}$.

We have seen that Δg and ΔV are not sensitive to the nebular mass. Fig. 4 shows the effect of different values of ρ at $r = 1$ AU, with the temperature held constant, and $P \propto r^{-3}$. Densities less than $10^{-9}\,\mathrm{g\,cm^{-3}}$ in the central plane would imply a nebular mass too small to make the planets, but the curves shown may be taken to illustrate conditions away from the central plane, or at some stage of the nebula's dissipation. For $\rho > 10^{-8}\,\mathrm{g\,cm^{-3}}$, the nebular

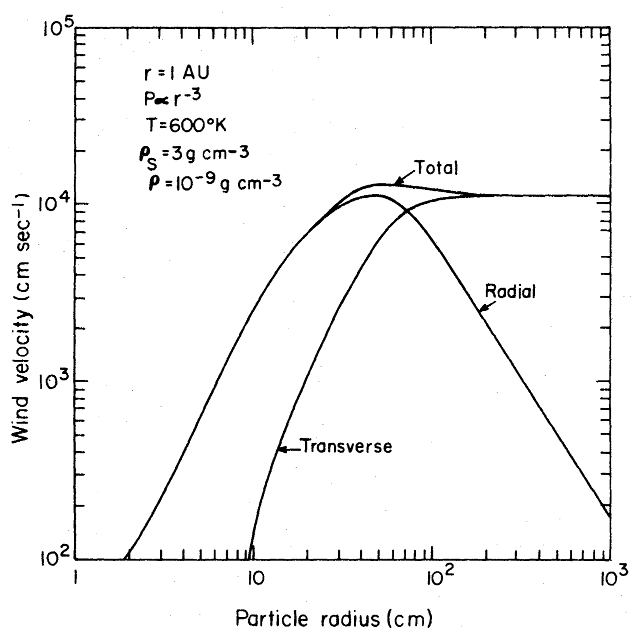

Figure 2. Wind velocity components on a particle of density 3 g cm^{-3} at 1 AU in the model nebula.

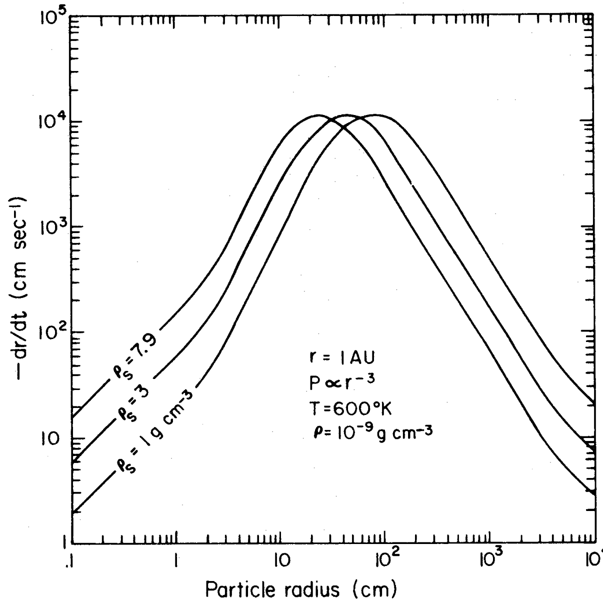

Figure 3. Effect of particle density on radial velocity.

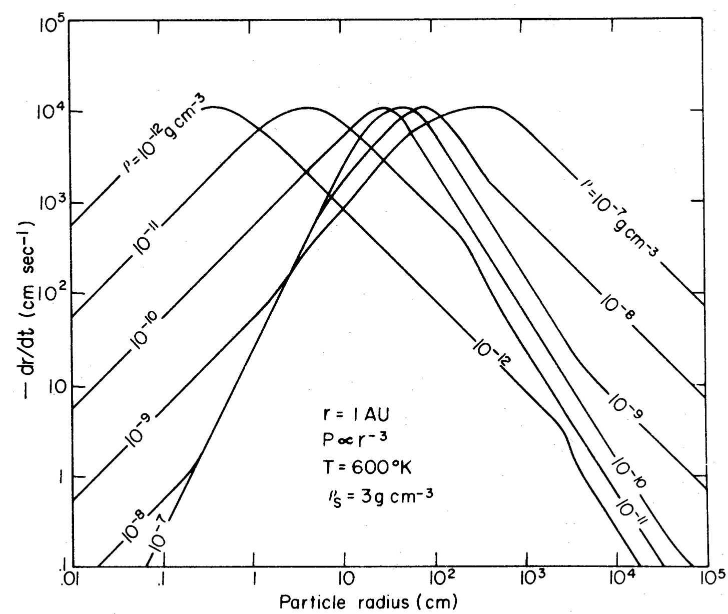

Figure 4. Effect of gas density on particle radial velocity.

mass would be large enough for its self-gravity to increase ΔV, but this effect has been ignored. For sufficiently small and large bodies, $dr/dt \propto \rho^{-1}$ and ρ, respectively. In the Stokes drag regime, dr/dt is independent of ρ. The changes in slope of the curves of dr/dt versus s correspond to the transitions between drag regimes. For very large and small values of ρ, there are no transitions near $s = s^*$, and therefore $s^* \propto \rho$. However, in the range $10^{-10} < \rho < 10^{-8}\,\mathrm{g\,cm^{-3}}$, the drag transitions reduce the effect of ρ on s^*. A variation in ρ of two orders of magnitude causes s^* to change by about a factor of three.

In different parts of the nebula, a body of a particular size encounters different drag regimes and values of t_e/t_p; this situation is shown in Fig. 5. In the model nebula, s^* remains in the range 10–100 cm. There is a general trend for radial velocities of small bodies to increase with r, while the reverse is true for large bodies. This is primarily due to the

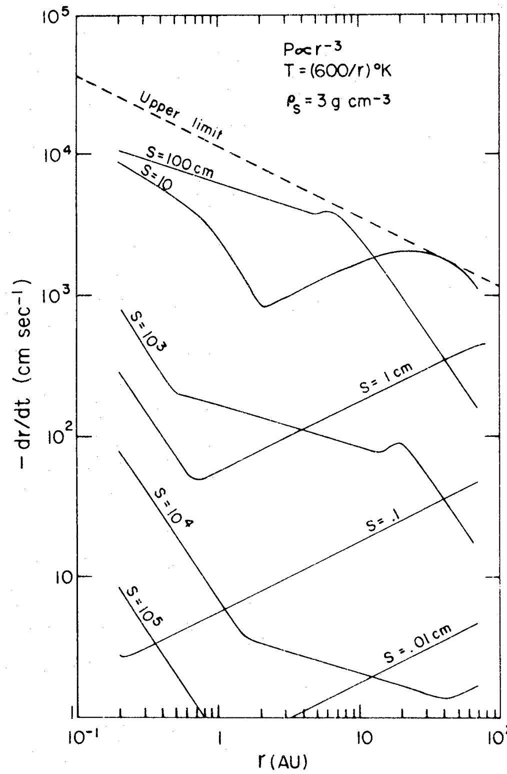

Figure 5. Radial velocity as function of position in the model nebula.

variation of ρ. Transistions between drag regimes cause local extrema in radial velocities. Since the assumed drag laws and particle shapes are idealized, and a range of sizes and densities would be present in a real nebula, these features are probably without physical significance.

A particle would spiral into the sun in a characteristic lifetime of $\tau_c = r/|dr/dt|$. Fig. 6 shows τ_c for particles in the model nebula. The lifetime of the actual solar nebula is generally assumed to be of the order of 10^6 yr, though Cameron & Pine (1973) estimate lifetimes of about 10^4 yr for their models. Even this smaller figure exceeds τ_c for $1 < s < 10^3$ cm in much of the standard model. For $s \simeq 100$ cm, $\tau_c < 100$ yr at $r = 1$ AU. It is unlikely that a particle of this size would maintain its identity while spiralling inward. Collisions with other bodies would cause it to grow by accretion, be disrupted, or be incorporated into a larger body. However, it is apparent that some transport of solids over significant distances can occur on a reasonable timescale, even in a low-mass nebula.

The choice of $T \propto r^{-1}$ in the standard model nebula was based on Lewis' (1974) arguments concerning the bulk chemical compositions of planets and satellites. Even if correct, this evidence refers only to the time at which chemical equilibrium existed between the solid condensates and the nebular gas. The subsequent accumulation of large bodies could result in an optically thin nebula, while preserving the 'frozen' chemistry of the earlier state. For this reason, the calculations were performed for a model nebula with $T \propto r^{-1/2}$, with other parameters identical to those of the standard model. The results, plotted in the manner of Fig. 5, showed no important differences.

7 Limitations

The general behaviour of solid bodies in the solar nebula is not sensitive to explicit assumptions of the nebula's mass or structure. However, the implicit assumptions and

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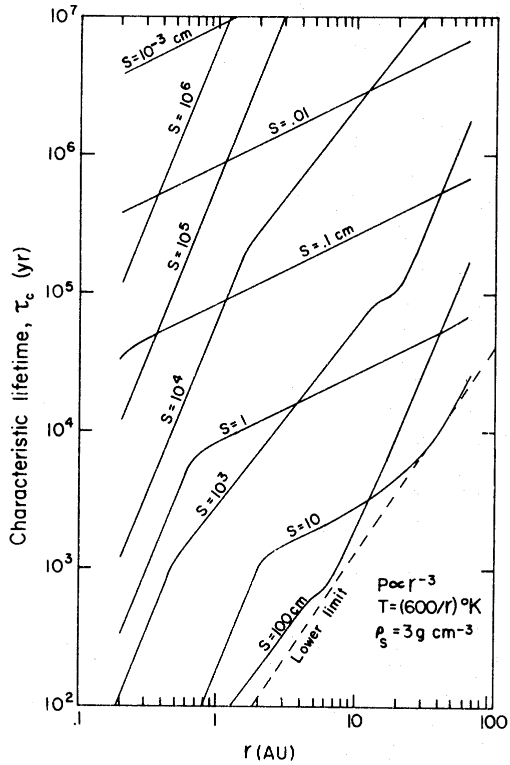

Figure 6. Characteristic lifetime against loss into the sun, $\tau_c = r/|dr/dt|$.

limitations of the preceding treatment should be considered before drawing general conclusions.

7.1 TURBULENCE AND GAS MOTIONS

We have assumed that the only motion of the gas was its rotation. Turbulence is an important feature of Schatzman's (1967) nebular model, while those of Cameron & Pine include out-of-plane convection, and radial circulation currents. Safronov (1969, chapter 4) concludes that rotation tends to stabilize the nebula, making extensive turbulence unlikely, and strongly inhibiting convection in the radial direction. If turbulence was present, small particles would be carried with the gas, but with a net radial motion superimposed. Their mean lifetimes would be larger than the computed values. For large bodies, the effects of turbulence would be averaged over their orbits, and their lifetimes would not be affected significantly.

From equation (6), it is seen that ΔV is independent of z, the distance from the midplane, if T does not vary with z. If the surfaces of the nebula are illuminated by the protosun, dT/dz is very small (Safronov 1969, chapter 4). In the models of Cameron & Pine (1973), T decreases significantly away from the midplane. In that case, ΔV also decreases, and the gas must rotate more rapidly away from the midplane. Drag would cause tangential acceleration of the gas near the midplane, and deceleration of the gas above and below. This process would be especially effective in the presence of out-of-plane convection. The result would be a circulation of gas flowing outward in the central plane, and inward above and below it; this current is opposite in sense to that assumed by Cameron & Pine. If the outward gas velocity exceeded the radial drift velocity of a small particle, it would be carried outward. However, the orbital decay of large bodies is due to tangential drag, and would be virtually unaffected by radial motion of the gas, unless its radial velocity approached ΔV.

7.2 PARTICLE INTERACTIONS

We have assumed that each particle is affected only by the gas, and not by other particles. Goldreich & Ward (1973) considered the case in which dust particles settle to a thin disk in the central plane of the nebula. They showed that in a low-mass nebula, similar to the standard model, gas molecules could not pass freely through the dust disk if the particles were smaller than about 1 cm. The gas in the disk would then be carried with the dust. According to Goldreich & Ward, the disk would behave as a unit, with drag exerted on its surface by the gas above and below it. The characteristic lifetime of the disk was estimated to be of the order of 10^4 yr at 1 AU, independent of particle size. However, in the next section I shall argue that the particles should quickly grow to large sizes, at which they may be treated separately.

V. S. Safronov (private communication) has pointed out that if a large body moves through a swarm of smaller ones, the small objects are, in effect, part of the resisting medium. The amount of resistance depends to some extent on whether the small bodies stick to the larger upon collision, or bounce off. The effect in either case is similar to that of an increase in gas density. If the solids settle into a disk near the central plane of the nebula, their space density may exceed the gas density by more than an order of magnitude. However, the large bodies will encounter increased resistance only while a significant fraction of the mass of solids remains in small bodies.

8 Discussion

The preceding results are applicable, at least in order of magnitude, to a wide range of nebular parameters. We may assume that the pressure gradient in the actual solar nebula gave rise to effects comparable to those indicated for the standard model. A number of cosmogonically significant effects were possible; some of them may have left observable consequences.

8.1 TRANSPORT AND MIXING OF SOLIDS

The pressure gradient provides a plausible mechanism for moving solid objects through large radial distances on a short timescale. Most of this motion was inward, but some outward transport was possible, by circulation currents or by local reversals in the pressure gradient (Whipple 1973). Many meteorites contain inclusions of anomalous composition or mineralogy, the most notable example being chondrules. More or less plausible scenarios can often be suggested to produce the components of a given meteorite at a single heliocentric distance. However, individual inclusions in some C chondrites have been found to contain different proportions of oxygen isotopes. The observed anomalies are incompatible with fractionation processes, and indicate the addition of a component rich in ^{16}O (Clayton, Grossman & Mayeda 1973). More precise analyses (Clayton *et al.* 1976) show that the Earth and ordinary chondrites contain different proportions of the ^{16}O-rich components. One possible explanation is that the solar nebula was isotopically inhomogeneous. In any case, the isotopically distinct components of a single meteorite must have condensed separately, and been brought together as solid bodies. Radial transport by gas drag provides a reasonable mechanism which operates on a much shorter timescale than orbital evolution. Even if a meteorite was assembled after dissipation of the solar nebula, its components may have been brought to the same heliocentric distance in this manner.

8.2 ACCRETION AND GROWTH OF PARTICLES

The amount of mixing which occurred in the solar nebula was limited. Anomalous inclusions comprise only a small fraction of any meteorite's mass, and the planets retain distinct differences in chemical composition (Lewis 1974). Particles with $s \simeq s^*$ have characteristic lifetimes smaller than the probable duration of the nebula. If mixing was not extensive, we must conclude either that most of the mass of solids remained as fine dust until after dissipation of the nebula, or that most particles grew to sizes much larger than s^* in a time shorter than τ_c. The latter event seems more probable.

It is often assumed that in the presence of non-turbulent gas, the relative velocities of solid particles will be damped to some negligible amount. However, their radial and tangential 'wind' velocities depend strongly on size. In any real situation, some range of sizes should exist. Therefore, even in a perfectly quiescent nebula, particles would retain significant relative velocities. This situation is particularly favourable to accretion. Collisional velocities would be on the order of dr/dt, and no greater than ΔV. Most collisions would be between particles of very unequal size. If their structures were somewhat porous, the smaller particle would tend to become embedded in the larger. Bodies of nearly equal size would automatically have low relative velocities, effectively preventing disruptive collisions.

Consider a particle at $r = 1$ AU in the standard nebula. If the particle is small ($t_e/t_p \ll 1$), but larger than the surrounding particles, its velocity relative to them, v, is of the order of its radial velocity. In the Epstein drag regime, $v \simeq 10s$/s. The growth rate of the particle is

$$\frac{ds}{dt} = \frac{\beta \delta v}{4\rho_s}, \tag{26}$$

where β is the effective sticking probability (including possible erosion), and δ the space density of solid matter. In a low-mass nebula, $\delta \simeq 10/d$ g cm^{-3}, where d is the thickness of the dust layer in cm. Then

$$\frac{ds}{dt} \sim \frac{100\beta s}{4\rho_s d} \sim \frac{10\beta s}{d} \text{ cm/s},$$

$$\text{or } s \sim s_0 \exp(10\beta t/d). \tag{27}$$

The particle grows exponentially on a timescale $\tau_g \sim d/10\beta$ s. For gravitational instability to occur in the dust layer, d must decrease to $\sim 10^7$ cm (Safronov 1969; Goldreich & Ward 1973). In that case, if $\beta = 0.1$, the particle would increase in size by a factor of 10^6 in about 5 yr. The exponential growth causes a particle to outgrow its competitors, and justifies the assumption that $v \simeq |dr/dt|$. Actually, growth would begin earlier, at larger values of d, and proceed more slowly, but this does not affect the main conclusion. By the time the solids form a layer near the central plane, most of the mass is not in the form of dust, but bodies of considerable size, even if β is small.

The exponential growth breaks down when $s \to s^*$. Then $v \simeq \Delta V$, and the growth rate is

$$\frac{ds}{dt} \sim \frac{\beta\delta\Delta V}{4\rho_s} \sim \frac{10^4\beta}{d} \text{ cm/s}. \tag{28}$$

For $d \sim 10^7$ cm, and $\beta = 0.1$, $ds/dt \sim 10$ cm/day. Equation (28) is valid only if the accreted mass is in bodies much smaller than s^*. When the accreted matter is also in large bodies, the relative velocities and accretion rate diminish. The accretion rate of the largest body is then determined by the sizes of the smaller bodies. More detailed analysis is needed, but it appears that this process can quickly form kilometre-sized planetesimals, but not objects of planetary size.

Gravitational instability in a dust layer is not necessary to form planetesimals; it may also be impossible. A sufficiently thin layer implies out-of-plane velocities on the order of 10 cm/s (Safronov 1969; Goldreich & Ward 1973). The pressure gradient produces in-plane velocities two or three orders of magnitude larger; collisions would produce similar out-of-plane velocities, particularly if β is small. It seems unlikely that a sufficiently thin layer could form at all. A more detailed investigation of accretion will be reported in a later paper.

8.3 FRACTIONATION

Since the radial velocity of a particle depends on its size and density, there may be fractionation according to these properties. For small particles, radial velocities increase with s and ρ_s; for large bodies, the reverse is true. Separation by density, e.g. Fe/silicate fractionation, can occur in either sense, depending on the size distribution. In a continuous solar nebula, matter lost from a zone by inward motion is replaced by matter coming from farther out; only the net gain or loss is significant. When the nebular surface density decreases with increasing r, the radial velocity of large bodies does also. In this case, it is easily seen that there is a net loss of solid matter at any value of r. Fractionation in this case is a process of selective removal. For sufficiently small bodies, the radial velocity can increase with r, and selective addition to a zone is possible. However, most of the mass was probably in large bodies, as argued above.

The planet Mercury is dense and rich in iron. Lewis (1972, 1974) has proposed that its component matter formed in a P–T regime in which Fe condensed fully, but silicates only partially. This scenario requires condensation to occur within a very narrow temperature range; a difference of a few tens of degrees would have drastically altered the planet's Fe/silicate ratio. It is difficult to explain Mercury's composition as due to density fractionation by the Poynting–Robertson effect (Herczeg 1969); the matter must remain in the form of fine dust for some 10^7yr. However, aerodynamic processes can perform such fractionation on large bodies, on much shorter timescales. It is only required that the matter in Mercury's zone was in bodies larger than about 1 m, and that silicate-rich bodies were not systematically larger than Fe-rich bodies. The low mass of Mercury suggests that such fractionation was not very efficient. The strong dependence of τ_c on r in Fig. 6 suggests that the effect in Mercury's zone could be large, without significant alteration of the composition of Venus. A quantitative examination of this process will be reported in a later paper.

It has been suggested that the regular satellite systems of the giant planets formed from gaseous disks surrounding them, analogous to the solar nebula (Cameron 1973b). The densities of Jupiter's Galilean satellites decrease with increasing orbital radius. This may be entirely explainable by a temperature gradient in the disk (Pollack & Reynolds 1974), but is also consistent with aerodynamic fractionation (both effects could have been significant). If the density of the innermost satellite (JV) is found to indicate an Fe-rich composition, this would strongly suggest fractionation.

8.4 CHEMICAL EFFECTS

The transport of solids could alter the composition of the gas in some regions of the nebula. In its outer part, planetesimals of mainly icy composition would form. These bodies were similar to, or perhaps identical with, comet nuclei. They would spiral inward, some of them entering the warmer region where the icy component was unstable. They would evaporate there, giving up volatiles to the gas. For bodies larger than a few tens of metres, $dr/dt \propto s^{-1}$. Assuming a constant evaporation rate, they would all traverse the same range of *r* during this process, regardless of size. If the gas is not mixed by turbulence, a substantial increase in the content of volatiles could occur in this zone. Such a zone could be heated or cooled after formation, due to fluctuations of solar luminosity and nebular opacity.

The most probable component for local enhancement is H_2O; the presence of NH_3 and CH_4 as hydrates could add these compounds, as well. Comets and carbonaceous meteorites contain complex organic compounds. CO or CO_2 has been suggested as a significant component of some comets (Delsemme & Combi 1976). These compounds are not produced in quantity by chemical equilibrium processes in a solar-composition medium. Some such compounds could be produced under equilibrium conditions in a region of deviant composition. In cases where non-equilibrium reactions must be assumed, the required conditions may be less stringent in such a region.

It should be emphasized that these phenomena are not limited to a particular model of the solar nebula. Significant pressure gradients, and the resulting radial motions of particles, can be expected in any nebular model. Indeed, they can be avoided only by the artificial and unrealistic assumption of an isobaric nebula. It is probable that this effect played an important role in the formation of the solar system.

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