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Affective Factors Associated with Computational Estimation of Seventh Graders

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Abstract – This study examined affective factors associated with the seventh grade students' computational estimation ability. Data was collected from five high achievers via two interview sessions. During the interviews, students were required to solve numerical computational estimation questions and share their perspectives about computational estimation. Additionally, teacher interviews and observations were conducted for triangulation purposes about students' performance in mathematics and attitude toward mathematics. A theme oriented matrix was used to analyze the data. The results of the study found that students' confidence in ability to do mathematics, perceptions of mathematics, confidence in ability to do the estimation, perceptions of estimation and tolerance for error were identified as affective factors related to students' computational estimation ability
Key words: computational estimation, affective factors, seventh grade

Introduction

Although computational skills are valued in today's society, it would be detrimental to ignore the importance of logical judgment. In many situations, the ability to judge the appropriateness of results of computations by computational estimation, may be greater appreciated than simply being able to find exact answers. For instance, using computational estimation is more practical than exact computation for comparing countries' economic

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growth by using Gross Domestic Product per person or checking the appropriateness of the answers given by calculators.

Global reports by mathematical councils and curriculum reform initiatives, deem computational estimation as an important component of students becoming proficient in mathematics (The Turkish Ministry of National Education-MoNE, 2005; National Council of Teachers of Mathematics-NCTM, 2000; Segovia and Castro, 2009; The England Department for Education and Skills- DfES, 2007; Australian Education Council, 1991). These reports emphasize computational estimation as a needed skill for engaging in problem solving activities, performing mental computations and interpreting and making judgments of mathematical results. Therefore, computational estimation plays an important role in the teaching and learning of mathematics and it is imperative that students be taught both computation and computational estimation in their mathematics courses.

A review of the mathematics education literature highlights that many researchers have investigated computational estimation ability. In many instances, researchers have studied cognitive aspects of computational estimation. For instance, studies examined achievement levels of computational estimation in a variety of age groups (e.g., Baroody & Gatzke, 1991; Berry, 1998; Bestgen, Reys, Rybolt & Wyatt, 1980; Boz, 2004; Case & Sowder, 1990; Goodman, 1991; Reys, Reys, & Penafiel, 1991; Volkova, 2006); while other studies investigated students' achievement levels of computational estimation based on the formats of the questions (word vs numeric) (e.g., Bestgen et al., 1980; Cilingir & Turnuklu, 2009; Rubenstein, 1985; Sowder, 1992; Goodman, 1991). Additionally, researchers explored students' computational estimation performance based on the type of tasks posed (i.e., multiple choices, reference number, open-ended, order of magnitudes) (Blair, 2001; Goodman, 1991; Rubenstein, 1985); and conceptions of numbers (such as whole numbers, fractions, decimals, and percent) (Bobis, 1991; Goodman, 1991; Hanson & Hogan, 2000; Reys, Reys, Nohda, Ishida, & Shimizu, 1991; Reehm, 1992; Rubenstein, 1985; Volkova, 2006).

Furthermore, researchers have revealed that computational estimation is a complex process with a various components (Reys et al., 1982; Sowder & Wheerler, 1989; Volkova, 2006). Aiken (1976) and Ma and Kishor (1997) asserted that cognitive processes are not independent of feelings and beliefs. However, few research studies took into consideration affective factors of computational estimation (Hogan, Wyckoff, Krebs, Jones and Fitzgerald, 2004; Hogan & Parlapiano, 2008; LeFevre, Greenham, & Waheed, 1993; Sowder &Wheeler,

1989; Reys, Rybolt, Bestgen & Wyatt, 1982). Sowder and Wheeler's (1989) framework for computational estimation had four components, one of which was called affective component. Although the Sowder and Wheeler's framework included affective factors, the researchers did not utilize the affective component for data analysis. Similarly, Reys et al. (1982) identified three distinct dimensions of good computational estimators, namely: number skills, cognitive processes and affective attributes.

Although Sowder and Wheeler (1989), LeFevre et al. (1993) and Reys et al. (1982) used both qualitative inquiry and quantitative approach to identify the affective factors, Hogan et al. (2004) and Hogan and Parlapioano (2008) approached only quantitative perspective and searched sole affective factor called as "tolerance for ambiguity [error]", which was determined by Reys et al. (1982) study. In an examination of the correlation between "tolerance for ambiguity [error]" and computational estimation using the personality test, Hogan et al. (2004) found no statistically significant correlation between these variables.

Researchers have indicated that there may be specific affective factors related with the computational estimation, which include confidence in ability to do mathematics, confidence in ability to estimation, recognition of estimation as useful and tolerance for error (Hogan et. al, 2004; Hogan and Parlapiano, 2008; Reys et al., 1982; Sowder & Wheeler, 1989). Notwithstanding the empirical findings thus far, additional research is needed, particularly in terms of how students reveal their affective reactions for computational estimation. In educational and psychological literature affective factors are listed in variety of ways, such as Reyes (1984) suggested that self-concept, anxiety and perceived usefulness were the affective factors and some other researchers considered enjoyment of mathematics, self-perception, motivation and interest as affective factors (Skaalvik & Rankin, 1995; Mitchell, 1993; Watson, 1983). However, McLeod (1992) claimed that on based on Mandler's Theory affective factors could be listed in three titles which were emotions, beliefs and attitudes. Here we prefer to use the term affective as participants' feelings about computational estimation (emotions), aspects of using computational estimation (attitudes), or feelings about themselves as users of computational estimation (beliefs).

The overarching question of the study is "What are the affective factors that are associated with the seventh grade students' computational estimation ability?" Using qualitative inquiry and giving detail explanations from participants' answers, we examined their belief and thoughts about computational estimation, in an effort to illuminate the role of affective components of computational estimation. We believe the findings of this study make a significant contribution to understanding what are the affective factors associated with computational estimation.

Methodology

Participants

The participants in the study were determined by a two-step sampling process. First, the elementary school was chosen randomly from forty-six elementary schools in a city located in the Aegean region of Turkey. As a second step purposive sampling was used to select high achieving seventh graders according to the results of the Computational Estimation Test (CET). The selection of the seventh grades guaranteed that the participants studied estimation in their previous year so called 6^{th} grade in elementary school. In Turkey, the elementary school mathematics curriculum consist of computational estimation concepts in whole numbers, decimal and fraction topics in the 6^{th} grade mathematics objectives of mathematics curriculum. Among the all seventh grade we applied the CET and we chose the most successful students as participants of the study.

According to Merriam (1998) “purposive sampling emphasizes a criterion based selection of information rich cases from which a researcher can discover, understand and gain more insight on issues crucial for the study” (p. 61). This study employed getting high scores from the CET as a criterion for purposive sampling. Hence our study sought only high achieving students on CET because we wanted students who were knowledgeable of estimation and used it effectively during the estimation required questions regarding to their opponents.

To succeed in choosing most successful students, the scores gathered from CET were ranked and the top seven performing students were asked to participate in the study. The reason that we chose only seven students for interview is the big score gap between the seventh (7 points) and eighth (4 points) student in the achievement list. This score difference forced us to pick only seven students for the interview group. Although seven students originally were selected to participate in the study, only five students’ data were used for the analysis due to the incompleteness of responses by two of the selected seven students in the interviews. When five participants’ computational estimation ability performances are examined according to the Computational Estimation Test (CET), all interviewees showed similar levels of achievement to each other (see Table 1). Hence the participants for this study were Deniz, Mert, Sergen, Nevzat and Ayşe (all names are pseudonyms) selected for interviews. Among the participants Ayşe was the only female participant.

Table 1. Interviewees' scores on whole class application of CET

Interviewee	Score (out of 15)
Ayşe	10
Mert	10
Deniz	9
Nevzat	9
Sergen	7

According to students' records all participants have working mothers and fathers; such as Sergen's parents are both civil servants, Ayşe's mother is a teacher and father is a financial consultant, Deniz's mother is a dentist and father is administrator in city health department, Nevzat's both parents are teachers, Mert's mother is a nurse and father is a doctor. The selected participants were enrolled in the same mathematics class, which resulted in them having the same mathematics teacher (Teacher A). The Teacher A refers all five students successful in his mathematics classes. Unlike Sergen and Nevzat who got 4 out of 5, the other participants got 5 from mathematics as previous semester grade.

Because of the participants were required to take the Level Determination Exam (*Seviye Belirleme Sınavı*- abbreviated SBS), which is a nationwide exam for elementary students at the end of each academic year, all of the students were enrolled in after school private classes to get higher score for the national exam. In these classes students are exposed to sample exams on a weekly basis that are similar to the national exam. The national exams contain 80, 90 and 100 questions for 6th, 7th and 8th grades, respectively. The exams contain questions which are based on topics in Mathematics, Turkish, Science and Technology, Social Science and Foreign Language (English, French, German and Italian). The questions for Mathematics, Science and Technology and Social Science are the same amount for each grade level but they increased as the grade level increased. The success in these exams is important for students because according to result of these exams, they can choice well known high schools such as Science High Schools after their graduation from elementary school. The dense population of young age group in Turkey makes it difficult to be enrolled in well-known high schools for students and this creates a big competition among the students.

Data Collection Tools

CET. The CET was administered to 116 seventh grade students in numeric format. CET consists of 15 open-ended questions that combined computational estimation tasks from different research studies (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys, Rybolt, Bestgen & Wyatt, 1982). Reliability (Cronbach's Alpha was .78) and validity of the CET was examined in a pilot study through credibility (internal validity), dependability (reliability), and confirmability (objectivity). The credibility criterion requires demonstrating that the results of qualitative research are credible or believable from the perspective of the participant in the research (Lincoln & Guba, 1985). Therefore, we presented the analysis' results through the defined themes in an understandable and clear way. The inferences and interpretations are supported with the interviewees' quotations. Dependability is analogous to reliability, that is, the consistency of observing the same finding under similar circumstances (Lincoln & Guba, 1985). In the present study, detailed explanations of how the data collected and analyzed were provided in the study. Lodico, Spaulding, and Voegtle (2006) stated that the dependability is often the difference between an experiential report that simply summarizes a researcher's conclusions and research-based qualitative study that includes a systematic explanation of methods. Although in the study no raw data were served, certain explanations about the procedures are given. Confirmability refers to the extent that the research findings can be confirmed or corroborated by others. It is analogous to objectivity, that is, the extent to which a researcher is aware of or accounts for individual subjectivity or bias (Lincoln & Guba, 1985). Some crucial examples of transcripts are embedded in the report to enhance the confirmability of the study. Naturally, if the reader examines these examples of transcripts, then the interpretations and results are maximally confirmable.

In the CET there were equal numbers (five items each of them) of whole numbers, decimal and fraction related questions. Some examples from CET with exact answers and accepted interval are given in Table 2.

Table 2. The example questions of CET with accepted intervals and exact results

		Numerical Format of CET	Accepted Interval	Exact Answers
1.	Whole Numbers	87 419 + 92 765 + 90 045 + 81 974 + 98 102=	400 000-500 000	450 305
2.	Decimals	835.67— 0.526=	829-835	835.67
3.	Fractions	$7^1/6 – 4^1/3$ =	2-3	$^{17}/6$=2.83

Interview protocols. We collected data via two individual interview sessions of each participant. During the first interview, students were asked each questions of the CET, without anytime restrictions. Students were asked to explain their reasoning and process used to generate the responses provided. A week after the first interview, the second interview was conducted. This interview session sought to explicate students' thoughts about using computational estimation in daily life applications and in their classroom mathematical applications, as well as their feelings about estimated solution(s). The second interview consisted of twelve semi-structured interview questions on computational estimation. Examples of some of the affective questions asked are represented in Table 3.

Table 3. Some questions that were posed the second interview

	Questions	**Related themes**
Question7	On a scale of 1 to 10, how would you rate your success in mathematics? Why?	Mathematics related affective factors
Question8	How successful are you at computational estimation?	Estimation related affective factors
Question9	On a scale of 1 to 10, how would you rate your success in computational estimation? Why?	Estimation related affective factors
Question11	How successful are you at mathematics? Why?	Mathematics related affective factors

The first interview results are concern of another research report because of this in the current study we discuss only the second interview's data analysis and results. The first interview data are used only to support the results of the second interview. Moreover, we also conducted a semi-structured interview with Teacher A and observed mathematics class for 4 weeks as a means to triangulate the data collected from students. In addition to the planned interview session, the researchers and Teacher A engaged in conversation that further contributed to the enhancement of the data collected.

Procedure

A pilot study of the CET and interview protocol were conducted during the Fall of 2008. Based on the responses, possible themes for data analysis were identified. Subsequently, the data for this study were collected in Spring 2009 (February – April). We administered the CET over a two week period to 116 seventh grade students. To reduce the likelihood of students seeking to find exact answers rather than use estimation, questions were projected using an overhead projector, and a time limitation for each question was enforced. The questions were projected for 10 seconds on the board with size 36 pt Times New Roman. Due to the time constraints, students were cautioned, “Not to copy the problem but do the work in their heads.”

For the whole class application of CET, students received 1 point for each answer that was within a specified interval. The accepted interval ranged from 25% below the exact answer to 25% above the exact answer. Students’ responses that were not in the specified interval received no points. Students’ scores were totaled out of 15 possible points. Students who scored exceptionally high on the CET were invited to participate in the study.

Both interview sessions were conducted by the first author and they were video-recorded and transcribed verbatim by an assistant. The first author spent four weeks in the mathematics classes where all interviewees were enrolled in and she tried to get rapport with students during the break times. After identify the participants of the study the first author concentrated on them during the classroom observation and took some filed notes about them.

The first interview provided the means of learning about processes participants used to solve different estimation problems. Students were presented each question on a card, in the same order. No time limit was imposed, but students were instructed to estimate their answers rather than use algorithms to compute exact answers. Interviewees were asked to explain aloud their thoughts for finding estimated solutions. The first interview sessions took approximately 30- 45 minutes for each interviewee.

A week after the first interview, the second interview sessions were conducted. Students were asked twelve semi-structured questions in this session. The first author probed students for depth of understanding, and clarity. The second interview sessions lasted approximately 40-45 minutes each of the students.

Data Analysis

The second interview data were coded based on key words and phrases that were identified in the review of the literature, and pilot study. The coding outline consisted of two main themes, namely: *mathematics related affective factors* and *estimation related affective factors.* The themes and related codes were placed into a four-way matrix, which essentially involved crossing the four dimensions to see how they interact with each other (Miles & Huberman, 1994). Therefore, we placed interviewees, themes, sub-themes and codes into the theme-matrix, conducted a theme-oriented analysis, and subsequently expanded a more holistic case-oriented analysis. Each interviewee's transcription was carefully examined through the themes and codes individually to observe uniqueness, and then collectively in an effort to observe similarities and differences among participants.

The first author coded the interview transcriptions, which was validated by a second coder who was an experienced researcher in qualitative researches and coding procedure. A three round coding procedure was applied by two researchers. In the first round they were assigned to independently code one of the interviewee's transcripts. To increase the coding reliability, in the second round, they compared the coded transcripts and negotiated any discrepancies. Subsequently, in the last round common coding language was produced and five interviewee's transcriptions were coded according to constructed theme-matrix which is given in Table 4.

Triangulation was achieved by conducting a semi-structured interview with the students' mathematics teacher and classroom observations. Additionally, the first author engaged in unstructured conversations with interviewees, as well as with the Teacher A who is participants' mathematics teacher, to confirm students' assertions about their behavior in mathematics classes.

Table 4. Theme-Matrix

Participants	Main Themes	Sub-themes	Codes
Deniz Sergen Mert Ayşe Nevzat	Mathematics related affective factors	Confidence in ability to do mathematics	Students' *high/ low* point rating of success in mathematics.
			Mental computation ability coded as *strong and weak* based on students responses during interview sessions.
		Perceptions of mathematics	*Exactness*- Students perceives mathematics as an exact science and rejects estimated values.
	Estimation related affective factors	Confidence in to do estimation	Students provide *high/ low* rating on their achievement in estimation.
		Perceptions of estimation	Students' feelings about estimation-*like/do not like* estimation and *disturbance/lack* thereof by using estimation.
			Recognition of estimation as a *useful tool* for mathematics classes and/or daily life. Furthermore, students explained whether or not estimation is deem *important*, and their preference as to whether or not they should *use* it.
		Tolerance for Error	*Ambiguity* of estimated results.
			Acceptance of *pay off* (used as an amount of distance between exact answer and estimated answer).
			Participants determine answers in a *small/big range* for estimation.

Results

We present our findings around two themes: mathematics related affective factors and estimation related affective factors. The mathematics related affective factors are presented through the lens of students' confidence in their ability to do mathematics, and their perceptions of mathematics. The estimation related affective factors are discussed via three sub themes (students' confidence in their ability to do estimation, perceptions of estimation and tolerance for error).

Mathematics Related Affective Factors

For this sample, there was variation in students that possessed high confidence in their ability to do mathematics and the likelihood that they exhibit high computational estimation skills. Furthermore, we found that students who did not restrict mathematics to only exact solutions, were more willing to accept the practicality of estimation, and subsequently were better estimators than their counterparts.

Confidence in ability to do mathematics.

During the second interview session, interviewees were asked to rate their mathematics achievement on a scale 1 to10; the responses ranged between 5-10. Table 5 depicts the participants' self-ratings. Sergen was the sole student that rated his mathematics success was average, while the other students provided above-average ratings. Surprisingly, although all students were successful in mathematics, when they were asked "how successful they are in mathematics", three of five students provided the same response as "not very successful".

Table 5. Students' self-ratings of achievement in mathematics

Interviewees	Self-rating on a scale of 1 to 10	How successful are you at mathematics?
Ayşe	9,5	Enough
Mert	10	Successful
Nevzat	7	Not very successful
Deniz	7	Not very successful
Sergen	5	Not very successful

According to the follow up questions, Mert had very high achievement expectation for himself and he was not pleased about his actual level of achievement. For instance during his second interview Mert stated:

[Excerpt 1]

Researcher: How successful are you at mathematics?

Mert: Successful…Himm**,** actually I am not that successful at mathematics.

Researcher: But you gave 10 points for your mathematics achievement in the previous question. Why do you think you are not successful enough?

Mert: Because my success is not enough for my aim. I should and could be more successful.

On the other hand, Deniz, Sergen and Nevzat thought that they were not very successful because of their low pre-test scores at the after school private classes (in preparation for the national mathematics exam).

Mert and Ayşe's high scores from CET corresponded to self-rating of themselves (as depicted in Table 5). However, the other 3 students obtained high scores, but their perceptions of their mathematical performance were average (Table 1 and Table 4). Sergen is the only person who thought he was poor at mathematics.

Researcher: Why did you give 5 points to yourself?

Sergen: Because I am not successful any more like last year.

Researcher: How successful were you last year?

Sergen: I could give 8 points last year to myself but not this year.

Researcher: Why do you think your success is decrease like that?

Sergen: Actually I am a successful student but this year our mathematics teacher is asking very difficult questions in the exams. Although I thought I made correct my exams' results are so bad.

Deniz has same thought like Sergen who claimed that his previous year success was better than this year. However Deniz accused himself rather than his mathematics teachers' difficult exam questions. He confessed that because of his studying habits changed and he spent more time with computer games and outdoor activities he could not give enough time to study mathematics.

During the classroom observation, it was noticed that Ayşe and Mert were two exceptional students. They usually give answers to mathematics questions in a very short time like they are in a competition. It was observed that Mert had strong mental computation ability because when a computation question was asked, he used mental computation rather paper-pencil for computing the question. However, Ayşe generally gives the exact answers after Mert's approximate answers or giving a range that is determined by Mert.

Ayşe is very confident in her solutions of each mathematics questions, she does not hesitate to raise her hand to say the answers of each questions. When the researcher asked why she gave 9.5 points to herself for in the interview session, she explained as follow:

Ayşe: I gave that point according to my exams' result.

Researcher: OK. What do you think about yourself? Are you a successful student at mathematics?

Ayşe: Yes, I am.

Researcher: But you did not give 10 points to yourself, why?

Ayşe: Because, some people can compute all questions in their head so quickly, they should be given 10 points. Although I am not that kind of student I am still very successful student. Because of this I gave 9.5 points to myself.

Therefore, although the five interviewees were all successful in mathematics based on their mathematics teachers' assessments and school exams; their confidence in ability to do mathematics varies as well as their desire to provide an estimated response.

Perceptions of mathematics.

Mathematics needs exactness. When participants were asked about their perceptions of mathematics, and using estimation in their mathematics classes, two concepts influenced their perspectives, namely: exactness of mathematics, and their after school private class. Most students (except Mert) viewed mathematics as a collection of rules and formulas that are needed to deduce exact solutions, and did not consider estimation an important element in mathematics classrooms.

Except from Sergen, the participants' numeric CET achievement level is same (Table 1); however, their perceptions and answers varied on the estimation questions during the second interview. For example, Ayşe and Sergen were two of the participants who insisted on exact computation rather than using estimation. Ayşe was eager to conduct exact computation because she wanted to show her strong mathematical computational skills. For example, when she was asked to provide an estimated response for the task $7\ {}^{1}/_{6} - 4\ {}^{1}/_{3}$, Ayşe persisted in using an exact computation, and only provided an estimated response after multiple probing.

[Excerpt 2]

Ayşe: May I compute the problem?

Researcher: Please, give an estimated answer.

Ayşe: I can do this computation in my head without writing if you want.

During the interviews, Ayşe and Sergen admitted that mathematics needs exactness. The following excerpts from interviews with Ayşe and Sergen highlights their adherence to such viewpoints.

[Excerpt 4]

Researcher: What do you think about to use of estimation in mathematics lesson?

Ayşe: In mathematics class, I don't prefer to use it.

Researcher: Why don't you prefer?

Ayşe: Because mathematics needs exactness. Estimation contains ambiguity. In math lesson, you should find exact answers. Therefore, I don't prefer to use it in math classes.

………………………

[Excerpt 5]

Researcher: What do you think about to use of estimation in mathematics lesson?

Sergen: Estimation gives approximate solutions. It doesn't give exact result. I don't want to use it in math classes. But if I solve test questions…then estimation may help me in tests…may be…

Researcher: How can estimation help you in the tests?

Sergen: When you find estimated solution in the test you can choose the exact answer according to estimated answer that you found. But in the written exam you should find the exact answer…Estimated one is not helpful.

As can be seen from these dialogues, Sergen and Ayşe believe that mathematics does not contain approximate answers and estimated solutions. Ayşe acknowledged that even if she finds an estimated response, she would further attempt to find the exact solution (Excerpt 6).

[Excerpt 6]

Researcher: Don't you ever comment on a mathematics problem without exact result?

Ayşe: Sometimes I do. Then I find the exact solution. I feel disregardful to the math question if I do not find the exact result.

Ayşe believed that she could show her respect to mathematics by computing an exact answer for the question, rather than providing an estimated result. This way of thinking may be due to the influence of the directions provided by teachers to students. For instance, Mert claimed that he wanted to use estimation in math classes but his mathematics teacher made him use exact computation rather than estimation.

[Excerpt 7]

Researcher: What do you think about finding approximate solutions in math class?

Mert: I use it sometimes for gauging the answers. But most of the time my mathematics teacher wants me to give exact answers. Therefore, I generally find exact solution in math class. Sometimes, this makes me angry.

Researcher: What makes you angry?

Mert: When I give the approximate result to teacher, he thinks it is a wrong answer. But I only say approximate one, not exact one. When I explain that is not an exact answer, he says "find the exact one". Therefore, I don't want to use estimation in math classes.

Although Ayşe and Mert's had high self-confidence in their ability to do mathematics their perceptions of estimation were quite different. Unlike Ayşe, Mert was comfortable providing estimated results; he gave approximate results for each question quite naturally during the interview, without expressing any objections or difficulties. Mert was not reluctant to use estimation, and was content with using approximate computation for the mathematics questions. His initial answer was always an estimated response, rather than an exact solution. During the classroom observation sessions, the first author noticed that he was the first person who provided approximate answers for the mathematics questions. He was cognizant of the place value of numbers, and was skilled in providing estimated response not simply for whole number but also for decimal places as well. The following excerpt shows his response for estimating the difference between two numbers (835.67 - 0.526):

[Excerpt 3]

Mert: The first number's decimal part could be ignored. Then the number becomes 835 and the second number is rounded as 1. Therefore, the result of subtraction is 834.

Researcher: OK. Nevertheless, this answer is far away from the exact answer. Would you find a result closer to exact answer?

Mert: OK. I can round the first number as 835.650 and the second as 0.500. The operation became 835.650 minus 0.500, the result is around 835.150, and this may be rounded to 835.

During the interviews, students claimed that test anxiety increased their desire to provide exact answers for mathematical problems. All interviewees are attending after school classes for preparing the national exams (SBS). Ayşe perceived that estimated responses were distractors for the test, and hence selecting estimated responses might result in wrong answers.

[Excerpt 8]

Researcher: You do not want to use estimation in math classes. That`s OK but what about the after school private classes? During tests?

Ayşe: I don't think so… this may mislead me.

Researcher: Why do you think so?

Ayşe: If I estimate an answer for a question in test, I will find approximate result. This probably is more or less than the exact one. What if there are options close to these answers. No, no… Estimation should not be used in math classes in school and at tests in the private classes.

Conversely, Mert confessed that he used estimation on his after school private class tests. He emphasized that although his mathematics teacher did not encourage estimation at school, his after school program mathematics teacher encouraged him to use estimation to eliminate distractors.

[Excerpt 9]

Mert: At the after school private classes my mathematics teacher allowed me to use estimation at the test.

Researcher: What did he say?

Mert: He said that I can use estimation during the tests for eliminating the distractors.

Overall, the interviewees considered high achievement on multiple-choice tests of mathematics very important because of the national exam. However, multiple choice testing encouraged some students to believe that mathematics always requires a single correct answer. Additionally, teachers may vary in their emphasis of exact computation and estimation.

Estimation Related Affective Factors

We observed the participants presented inverse relationship between high self-confidence in ability to do estimation and low computational estimation ability. Some interviewees do not like to use estimation in mathematics questions had lower performance on computational estimation than who prefer to use it in mathematics questions. Furthermore, we found that the interviewees had better performance on computational estimation when they had high tolerance for error while solving the estimation questions.

Confidence in ability to do estimation.

When the interviewees were asked to rate their computational estimation level on a scale of 1 to10 the responses ranged between 6 and10 (Table 6). Of the participants' responses, Deniz and Mert responses were unexpected, because although Deniz was not as good estimator as Mert (based on his first interview responseswhich are presented in first author's dissertation), he thought that he was a "good" estimator and gave himself the highest score for self-confidence question. Furthermore, Mert who was considered to be a good estimator (according to his first interview responses) gave himself a lower score than expected.

Table 6. The self-rating points given by interviewees

Interviewees	Self-rating on a scale of 1 to 10	How successful are you at estimation?
Deniz	10	Very successful
Ayşe	9	Moderately successful
Mert	8	Not enough successful
Sergen	6	Moderately successful
Nevzat	6	Not successful

Deniz believed that estimation and rounding numbers are the same concepts, and that if he is able to round the numbers then he is a good estimator. On the other hand Mert, believed that being a good estimator was a factor of time. Mert suggested that if he is to be a very good estimator, he should be able to provide an estimated result within a few seconds. Excerpts 10 and 11 provide insight into participants' self-confidence in their computational estimation ability.

[Excerpt 10]

Researcher: Why did you give 10 points on your estimation ability?

Deniz: Because I can round the numbers easily.

Researcher: You mean that you are a good estimator because you are able to round the given numbers?

Deniz: Yes, exactly.

……………….

[Excerpt 11]

Researcher: Why did you give 8 points on your estimation ability, Mert?

Mert: I am not good enough at computational estimation. I can not find the answer in a few seconds, it takes time.

Moreover, when students were asked about the success of their computational estimation, only Deniz considered himself to be very successful. The other responses varied along the pendulum from not successful to moderately successful.

Deniz, Ayşe and Nevzat responses were consistent for the two self-confidence questions. Deniz, and Ayşe believed that they had high computational estimation ability, however, Nevzat rated himself very low (see Table 6). On the other hand, Mert and Sergen responses were inconsistent with their self-rating questions. For instance, Mert rated himself highly in the first question but he added that he was "not enough successful" in computational estimation for the last question. Similarly, Sergen varied in his responses, he initially rated himself low and asserted that he was moderately successful in computational estimation for the remaining question.

Nevzat was the only student who was consistently negative about his computational ability. However, his negative disposition about his computational ability were not based on his computational estimation performance, but rather because he was not enrolled in after school private classes in the previous years.

[Excerpt 12]

Researcher: How successful are you at computational estimation?

Nevzat: I am not very successful to do estimation but I am trying to my best.

Researcher: Why do you think so?

Nevzat: I did not go to after school private classes in my early years of the school so my mathematical background is not good like others.

Researcher: Do you think estimation was taught in those classes?

Nevzat: It should be because my classmates could do that kind of things better than ne. I know they went to that after school private classes in previous years, they should practice it there.

Amongst the students', their responses varied for their self-confidence in computational estimation. The results suggest that the more confident students have lower computational estimation ability, and the student who possessed higher computational estimation ability had less confidence in their ability to do estimation.

Perceptions of estimation.

Feelings on estimation. Three of the five students had positive feelings about estimation (Mert, Nevzat, and Deniz). Mert acknowledged that he felt comfortable using estimation during the mathematics to solve mathematical problem because he can use the skill to gauge the accuracy of his answer.

[Excerpt 13]

Researcher: How do you feel when you're doing questions that request computational estimation?

Mert: I feel good because I can use the estimation for gauging the answers. This makes me confident about my exact result.

For the same question, Ayşe gave a different answer. She claimed that estimation makes her uncomfortable and she did not prefer to use it.

[Excerpt 14]

Researcher: How do you feel when you are solving computational estimation requested questions?

Ayşe: Actually, I do not prefer to use computational estimation. I can compute mentally the given question, or I use pencil to compute exactly. Therefore, I don't need to use estimation.

Researcher: OK. What if you have to do estimation? What do you feel?

Ayşe: It makes me uncomfortable. The estimated answer is not an exact one. If I could find the exact result then I should find it.

Researcher: Why do you feel uncomfortable?

Ayşe: Because, the ambiguity makes me uncomfortable. By doing estimation, I can only find approximate solutions. I don't feel good with estimation.

During the first interview, Sergen was uneasy when asked to estimate for a mathematical problem. He said he felt uncomfortable because he preferred mental calculations.

[Excerpt 15]

Sergen: I have to use rounding.

[He made a noise to show estimation made him uncomfortable]

Researcher: Do you feel discontent, as you use rounding?

Sergen: Yes. A little.

Researcher: Why do you feel discontent?

Sergen: Since, I usually compute math questions mentally.

Researcher: Why don't you prefer to use estimation?

Sergen: I do not prefer, since computing questions mentally is usually very easy for me and I mostly use mental computation. "To estimate" is different for me. I rarely conduct estimation for a math question. I usually prefer directly to solve it.

Thoughts on using estimation. Three of the students (Mert, Deniz and Nevzat) said they used estimation in everyday life, while the other two students (Ayşe and Sergen) saw no use for it. Among the three students who saw the value in estimation, shopping was frequently presented as an example of a practical use of computational estimation. Mert and Nevzat added that they used computational estimation in mathematics classes to gauge the accuracy of answers; however, Deniz deemed the practicality of estimation was not for mathematics classes and was only appropriate in social settings outside of school.

Ayşe and Sergen disagreed with using computational estimation in mathematics classes. They asserted that using computational estimation during shopping is meaningless because cashiers perform exact calculations with the cash register; hence there is no need to estimate.

[Excerpt 16]

Researcher: Where do you use computational estimation in your daily life?

Ayşe: I think, I use estimation, for example while computing how far away the school from home is.

Researcher: It is not a computational estimation procedure. It is measurement estimation. Do you use computational estimation in your daily life?

Ayşe: Hımmm. I think, no.

Researcher: During shopping at the supermarket?

Ayşe: No. There is a cashier at markets. Therefore, I do not have to compute. I only use estimation when I have to measure something like length or weight.

Tolerance for error. Tolerance for error is defined as feeling comfortable with: results that are not exact answers, and with the pay off of the computations (Reys et al., 1982). Although tolerance for error is not assessed using exact questioning, evidence of this phenomenon is visible in students' interview responses. For example, in question three of the first interview (7465—572), Ayşe explained her feelings about adhering to school taught rounding procedures to find a narrow range for an estimated answer.

[Excerpt 17]

Ayşe: I could round the first one seven thousand five hundred. No, it is going to be too much.

Researcher: What is going to be too much?

Ayşe: If I rounded the first one to seven thousand five hundred, there will be more difference between my answer and the exact one. Therefore, I should round them as seven thousand four hundred seventy and five hundred seventy.

Ayşe repeatedly reaffirmed that the approximate computations made her uncomfortable, and estimation is ambiguous.

[Excerpt 18]

Ayşe: When I compute approximate solutions, this make me uncomfortable.

Researcher: Why do you feel that way?

Ayşe: It is not exact solution. I mean it is not clear, it is uncertain.

Similar to Ayşe, Sergen thought that mathematics requires exact results and had a low tolerance for error. For example, in question fifteen of the first interview (87 419 + 92 765 + 90 045 + 81 974 + 98 102) he preferred to use the compatible numbers for reformulating the numbers, to find an exceptionally close approximation to the exact answer.

[Excerpt 19]

Sergen: The first one could be rounded to 87 000 and the second one to 93 000.

Researcher: Why would you round the numbers like this?

Sergen: I don't want to exaggerate the result.

Researcher: Exaggerate?

Sergen: I mean that I want to find an answer closer to the exact one. I don't want to go far away from the exact answer. This is not true.

Researcher: Why do you think so?

Sergen: Actually, I am more comfortable with exact computation. But you want me to estimate solution then I perform the rounding to find the addition as specific as I can. This kind of rounding is reasonable for that.

Contrary to Ayşe and Sergen, Mert has a high tolerance for error. Mert was at ease when rounding numbers, and frequently provided approximate results using a broad range. An example of Mert's error of tolerance is illustrated in Excerpt 20 when he provided an estimated response for 835.67 - 0.526 .

[Excerpt 20]

Mert: I round 835.67 to 836. Then the second one can be 1. Therefore, the result is 834.

Researcher: Would you give a closer answer?

Mert: The closer answer should be eight hundred thirty five point a hundred fifty but the other one is also OK for me.

Similarly in question four for the first interview ($7^{1}/_{6} – 4\ ^{1}/_{3}$), Mert provided estimated answers using a broad range and stated that he had no objections with this kind of pay off. In the following excerpt, he explained his perspective.

[Excerpt 21]

Mert: The result is three.

Researcher: That's it?

Mert: Actually, a bit less than three but does not matter. Because it is too small.

Furthermore, Mert illustrated tolerance for error for pay off for a CET question (31 x 68 x 296) by utilizing multiple rounding procedures. Although Mert initially considered multiplying 2100 x 300, he conducted a second rounding, and multiplied 2000 x 300, because he perceived it to be easier. This shows that Mert's ignorance of pay off, which was cut from numbers while operating the multiplication. Moreover it shows he is flexible and has multiple estimation tools to employ.

Discussion

Computational estimation is a complex process with cognitive and affective components. In this study we focused our attention on the affective components and subsequently identified five affective factors associated with computational estimation, including confidence in ability to do mathematics, perceptions of mathematics, confidence in ability to do estimation, perceptions of estimation and tolerance for error. We further found possible influences that might potentially bias students' disposition towards computational estimation (inclusive of the perceptions: that estimation and rounding are the same, mathematics needs exactness, estimation may not be useful in mathematics classes neither at school nor at after school classes, and that there exist uncertainty of the benefits of computational pay off). Our results have implication on the affective nature of computational estimation, and in some instances did not align with previous research.

For example, our results not fully supported to Gliner's (1991) finding that students' self-confidence in their ability to do mathematics was positively correlated with their estimation score. More specifically, Ayşe had high self-confidence; however she was not a good estimator, while Mert possessed both high self-confidence in his ability to do mathematics and strong computational estimation ability. Hence we can conclude that self-confidence in ability to do mathematics is a component of good estimator but not a distinctive factor.

Among the interviewees Ayşe and Sergen insisted on using exact computation rather than computational estimation during their first interview session. These students believed that mathematics needs exactness and approximate answers are not enough for mathematics. This belief might have prevented students from using computational estimation. The interviewees who believed that mathematics needs exactness confessed that they felt uncomfortable while giving estimated results for mathematics questions, so that they prefer to use exact calculation for questions that asked students to estimate. Similarly, Baroody (1987) concluded that in most cases the belief that mathematical calculations must produce a single correct answer might contribute to a preference for exact solutions.

Moreover the influence of testing can bias students' perceptions in the practicality of computational estimation. Ayşe believed that estimated results were often placed as distractors on multiple-choice exams (resulting in a wrong answer) in mathematics assessment during school and after-school private classes; hence estimation should not be encouraged. On the other hand, Mert believed that estimation could help to identify the range of the exact answer. Admittedly, the students' mathematics teacher emphasis on exact results during mathematics lessons and assessments might have influenced their perceptions about estimation as well. For example, Mert admitted that although he desired to use estimation in mathematics classes his mathematics teacher did not encourage it. Hence the teachers' beliefs and disposition toward estimation might have influenced students' perceptions about estimation. Likewise, Alajmi and Reys (2007) found that middle school mathematics teachers perceived computational estimation as an important skill for daily life outside of the school environment, but not appropriate for school mathematics.

Furthermore, we found that there exist an apparent inverse relationship between students' self-rating scores on computational estimation ability and achievement on estimation. For example, Mert rated himself low for his computational estimation success, however he was a high achiever in finding estimated results; while Deniz who exhibited poor computational estimation skills (when compared to his counterparts in the interview group) rated himself highly for his achievement level on computational estimation. This finding is consistent with the result of Reehm's (1992) study that found that students in low ability range tended to overestimate their ability to estimate, conversely, students in the high ability range tended to underestimate their ability. Follow-up questions revealed that Deniz believed rounding and estimation were the same, so because he could round the numbers he assumed that he was a good estimator. Reys (1993) acknowledged that most students believed that estimation and rounding concepts are synonymous.

LeFevre et al. (1993) reported that students' high self-reported estimation skill correlated with high math marks, and also with the belief that estimation is useful in everyday situation. The results of our study suggest that the positive relationship between high self rating and high marks on mathematics, as well as the relationship between high self rating and belief that estimation was useful in everyday situation may not be generalizable, considering that Deniz was the only student whose results mirrored the positive relationships identified by LeFevre et al (1993). Sergen and Ayşe clearly contradicted LeFevre et al (1993) results because although they both had high mathematics scores, they rated themselves moderately on computation estimation and did not believe estimation was useful in everyday life. Although Nevzat and Mert did not rate themselves highly on estimation they believed that computational estimation should be used in social contexts. Therefore, this could suggest that recognizing estimation as useful in everyday life is independent of one self-rating of computational estimation. Considering that having positive feelings about usefulness of computational estimation did not infer high computational estimation ability, neither did negative feelings about the usefulness of computational estimation, reflect weak computational estimation ability.

Furthermore, our results suggest that although a student may be a high achiever and possess a strong self-confidence in their ability to do mathematics, it does not guarantee that the student will be a good estimator. For example, Ayşe was a high achiever in mathematics, possessed strong mental computational ability, and exhibited very high self-confidence in her ability to do mathematics, however she was not a good estimator. In fact, Ayşe did not believe that computational estimation is useful for any purpose. Our findings confirm Reys et al. (1982) results that stated a person can be competent at mental computation but very poor at computational estimation simultaneously. Students not being successful in estimation maybe related with a low tolerance for error. Contrary to Hogan et al. (2004) finding, which suggested that the correlation between tolerance for error and computational estimation ability was statistically insignificant, we found that good estimator had high tolerance for error. For example, Mert had high tolerance for error as well as being a successful estimator. Hence our results more readily aligned with Reys et al. (1982) findings that high tolerance for error contributes to having good estimation ability. Researchers (Sowder, 1992; Reys et al., 1982) claimed that tolerance for error in estimation could diffuse good estimators' thoughts, thus making the individuals more comfortable with pay-offs. A person who seeks only to obtain exact answers is unlikely to value estimation because it involves the acceptance of the possibility and usefulness of inexact answers.

These findings highlight the importance of affective factors that are associated with the computational estimation ability. Although our sample consisted of high mathematical achievers that performed exceptionally well on CET exam, our results revealed that variations existed among the students according to their beliefs, feelings and thoughts about computational estimation. These affective factors make distinction between good estimators and poor estimators. It should be recognized that not only cognitive factors but also affective factors are influencing students' computational estimation achievement.

Recommendation

The findings of this study showed that students prefer to use exact computation rather than estimation for a mathematical question, even when they were asked to estimate. Upon further inspection, it was revealed that the students' mathematics teacher encouraged exact results as a socio-mathematical norm of the classroom, which reinforced students placing great importance on exact calculations. This result highlights the need for mathematics teachers to promote an appreciation for estimation in classes, emphasize and use real-world examples of estimation, facilitate students supplying multiple acceptable answers of computational estimation within an appropriate range, and be flexible in acceptance of exact answers as well as estimated answers. Such changes in a mathematics teachers' instructional practice, relative to estimation, can potentially foster students developing positive dispositions towards the usability and practicality of estimated answers.

Since multiple-choice exams (which were used in after school private classes and were similar to standardized exams) were given more importance by the community than open-ended exams (in which they were required to fill in the blank or provide worked out solutions), students should be taught how to use estimation in eliminating distractors, and identifying the correct answers on multiple-choice exams. Estimation can be used to gauge the accuracy of responses provided, and pending the context, it may be more efficient than using an algorithm to find an exact answer. Therefore, teachers should encourage students to use computational estimation in mathematics classes and for making decisions about solutions.

In closing, we have identified affective factors that are associated with computational estimation and do believe further research is needed regarding the impact of affective factors on estimation. Future research ought to find means to improve students' disposition about estimation and to evaluate the roles teachers play in influencing their students' disposition and learning of estimation. Additionally, researchers should assess whether older students and adults have similar affective factors associated with computational estimation.

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Yedinci Sınıfların Hesaplamalı Tahmin Becerilerine İlişkin Duyuşsal Faktörler

Geniş Özet

Giriş

Yaklaşık sonuçlar elde edebilmek ve bu sonuçlar hakkında yorum yapabilmek ilköğretim matematik öğretimi programında akıl yürütme ve tahmin becerisi altında vurgulanmaktadır. Yapılan çalışmalar bu becerinin birçok bileşenlerinin olduğunu ortaya koymuştur. Bu bileşenlerinden biri duyuşsal faktörlerdir. McLeod (1992) duyuşsal faktörü inanç, davranış ve duygu olmak üzere üç başlık altında incelemiştir. Bu çalışmada katılımcıların hesaplamalı tahmin üzerine duyguları, bu beceriyi kullanmaya dair yaklaşımları ve onu davranışa dönüştürmeyi ve tahmin becerisini kullanmaya dair inançları incelenmek istenmiştir.

Metot

Çalışmanın katılımcıları iki aşamalı örneklem seçim sürecinden geçmişlerdir. Seçilen 116 yedinci sınıf öğrencisine Tahmin Beceri Testi (TBT) uygulanmıştır. Uygulanan TBT sonucunda öğrenciler başarı sıralamasına göre listenmiş ve ilk yedi öğrenci nitel çalışmada yer almak üzere belirlenmiştir, ancak bu araştırma raporunda sadece beş öğrencinin verileri değerlendirilebilmiştir.

TBT, literatürde bulunan ölçeklerden derlenen sorulardan oluşturulmuştur (Berry, 1998; Goodman, 1991; Heinrich, 1998; Reys, Rybolt, Bestgen & Wyatt, 1982). Yapılan pilot çalışmada ölçeğin güvenirliliği ve geçerliliği test edilmiş olup Croanbach Alpha katsayısı 0,78 olarak belirlenmiştir. Ölçek, doğal sayılar, ondalıklı kesirler ve kesirlere dair 15 sorudan oluşmaktadır.

Görüşme seanslarında öğrencilere TBT soruları zaman sınırlaması olmaksızın sorulmuş ve her cevap izleme soruları ile irdelenmiştir. İkinci tur görüşmelerde ise 12 açık uçlu soru sorularak katılımcıların duyuşsal boyutta tahmin becerisine bakış açıları, duyguları ve inançları anlaşılmaya çalışılmıştır. Bu araştırma raporu ikinci tur görüşmelerden elde edilen sonuçlar doğrultusunda oluşturulmuştur. Görüşme sonuçlarını desteklemek adına matematik öğretmeni ile planlı ve planlı olmayan görüşmeler düzenlenmiş ve öğretmenin dersleri 4 hafta boyunca gözlemlenmiştir.

Testin puanlanması için araştırmacılar tarafından her bir soru için doğru yanıtlar aralığı belirlenerek cevapların bu aralıkta yer alması durumunda 1 puan, aralıkta yer almaması durumunda 0 puan olarak puanlanmıştır. Buna göre testten en yüksek 15, en düşük 0 puan alınabilmektedir. Doğru cevap aralığı net cevabın %25 i üstünü ve altını kapsayacak şeklinde temel alınmıştır.

Görüşme sürecinde TBT soruları kartlara yazılarak öğrencilerden yüksek sesle okumaları ve cevaplamaları istenmiş, ardından öğrencilere izleme soruları sorulmuştur. Bir hafta sonra başlatılan ikinci görüşmelerde her öğrenciye 12 açık uçlu sorular sorulmuştur. Ardından tüm görüşmeler yazıya dökülmüş ve kodlayıcı güvenirliliğini sağlamak için matematik eğitimi ve nitel çalışmada uzman bir akademisyene başvurulmuştur.

Bulgular

İkinci tur görüşmelerin sonucunda pilot çalışma ve literatür temel alınarak iki ana tema oluşturulmuştur. Bunlar; *matematiğe dair duyuşsal faktörler* ve *tahmin etmeye dair duyuşsal faktörler*dir. Yapılan incelemelerde *matematiğe dair duyuşsal faktörler* temasının içeriği "matematik yapmaya dair güven ve matematiğe dair algı" olarak ikiye, *tahmin etmeye dair duyuşsal faktörler* teması ise "tahmin etmeye dair güven, tahmin etmeye dair algı ve hataya duyarlılık" olarak üç ana temaya ayrılmıştır.

Matematiğe dair algıları incelendiğinde öğrencilerin çoğunluğunun matematiğin kesinliklerden ibaret olduğu inancı baskın çıkmaktadır. Sorulan tahmin sorularına net cevapları bulmaya çalışmaları bunun en açık göstergesidir. Özellikle Ayşe ve Mert arasındaki yorum farklılığı ilginçtir. Ayşe, tahmini sonuçlardan öte net cevaplara bağımlı olmakla beraber kendisinin matematik yapma konusunda yeteri kadar başarılı olduğunu düşünmektedir. Mert ise tahmini sonuçlara daha yatkın bir duruş sergileyerek matematiğin sadece net cevaplar değil, yaklaşık sonuçlarla da yapılabileceğine inanmakta ve matematik yapma konusunda kendisini başarılı, ancak daha da başarılı olabilecek bir kişi olarak görmektedir. Ne var ki, tahmin etmeyi derslerde kullanamadığını çünkü öğretmeninin onu net ve tek cevaplara teşvik ettiğini söylemektedir.

Dikkat çeken bir başka bulgu, öğrencilerin SBS sınavlarına dair hissettikleri kaygı nedeniyle testlerde istenen net cevaplara dair farklı düşünceleridir. Bir yandan testte sorulan matematik sorularına tahmin becerisini kullanarak daha kolay çözüme gidebileceğini savunan öğrencilere karşın, diğer yandan tahmin yardımıyla elde edilen sonuçların çeldiriciler arasında yer alabileceğini ve bu yolu kullanmanın yanlış sonuçlara götürebildiğini savunan öğrenciler bulunmaktadır. Çoktan seçmeli testte başarılı olmak, katılımcılar için önemli olmakla beraber bazı öğrenciler bu başarıyı tahmin yardımıyla elde edebileceğini, bazıları ise matematiğin tek ve net doğru cevabı istediğine inanarak tahmin kullanmanın yararlı olmayacağını düşünmektedir.

Öğrencilere ne kadar başarılı bir tahminci oldukları sorulduğunda, Deniz ve Mert'in cevapları dikkat çekmektedir. Deniz, diğer öğrencilerden farklı olarak kendisini oldukça başarılı bir tahminci olarak nitelendirmiştir. Buna karşın içlerinde tahmin etmeye daha olumlu bakan ve bunu sıklıkla kullandığını söyleyen Mert, kendisini yeteri kadar başarılı görmemektedir. Bu cevaplar izleme soruları ile detaylarına inilerek incelendiğinde, Deniz'in tahmin etmekten anladığının sadece sayıları yuvarlamak olduğu, Mert'in ise daha hızlı tahmin etmesi gerektiğine dair bir beklentisi olduğu tespit edilmiştir.

Verilere göre bazı katılımcıların tahmini cevapların hata paylarına dair olumsuz yaklaşımları olduğu tespit edilmiştir. Örneğin Ayşe'nin izleme sorusuna verdiği cevap şöyledir: "Eğer net cevabı bulmazsam matematiğe saygısızlık yapmış hissediyorum." Bunun yanında yaptığı işlemlerde kendiliğinden bazı ondalıklı kesirlerin kesir kısmını görmezden gelerek "buna (*bu sayıya*) gerek yok, çünkü cevabı çok etkilemez." şeklinde yorum yapan Mert'in hataya karşı Ayşe kadar duyarlı olmadığı tespit edilmiştir.

Tartışma

Gliner (1991) matematik yapmaya dair güven ile tahmin etmede başarı arasında pozitif bir ilişki olduğunu ortaya koymuştur. Ancak, yapmış olduğumuz çalışmanın sonuçları Gliner'in (1991) sonuçlarını tam olarak desteklememektedir. Katılımcılardan Mert, kendine güven konusunda Ayşe kadar olumlu cevaplar vermese de tahmin etmeye dair Ayşe'ye göre çok daha başarılı olduğu gözlemlenmiştir. Ayşe'nin matematik yapma konusundaki özgüveni oldukça fazla olmasına karşı tahmin etmede Mert kadar istekli ve başarılı olmadığı gözlenmiştir. Matematiğin net cevaplardan oluşan kurallar yığını olduğuna dair inançları olan ve tahmin etmeye olumlu bakmayan Ayşe ve Sergen; bu becerinin ne derslerde ne de günlük yaşamda kullanımının gereksiz olduğunu düşünmektedir. Öğrencilerin tahmin etmeye dair yaklaşımları okuldaki matematik öğretmenleri ve ulusal sınav kaygıları ile pekiştirmektedir. Okul öğretmenlerinin çocukların duygu ve düşüncelerini etkiledikleri kaçınılmaz bir gerçektir. Reys ve Alajmi'nin (2007) çalışması ilköğretim matematik öğretmenlerinin okul matematiğinde tahmin etmeye yer olmadığını savunduklarını göstermiştir. Yaptığımız çalışmada, katılımcıların kendi tahmin becerilerine dair görüşleri ile bu konudaki gerçek başarıları arasında Rheem'in (1992) araştırmasında da olduğu gibi ters bir ilişki tespit edilmiştir.

Bütün bu sonuçlar ışığında tahmin etmeye dair duyuşsal faktörlerin önemi ortaya çıkmaktadır. Bu çalışmada, katılımcılarının benzer şekilde matematik başarılarının yüksek olmasına ve TBT den yüksek puan almalarına karşın tahmin etmeye dair inançlarının, duygu ve düşüncelerinin ne kadar farklı olduğu tespit edilmiştir. Katılımcıların farklı duygu, düşünce ve yaklaşımları onların tahmin etme konusundaki performanslarının daha iyi/kötü olmasına etki etmektedir. Tahmin etme konusunda başarılı olmanın sadece bilişsel süreçlere bağlı olmadığı duyuşsal faktörlerin de etkili olduğu ortadadır.