

# AFFINE LIE ALGEBRAS AND QUANTUM GROUPS

AN INTRODUCTION, WITH APPLICATIONS  
IN CONFORMAL FIELD THEORY

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