Age of Information of Two-way Data Exchanging Systems With Power-Splitting

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Abstract: **We consider a two-way data exchanging system consisting of an access point and a smart device. The access point has a constant power supply and the smart device does not. The access point simultaneously transmits information and energy to the smart device over block fading channels, with fixed powers $\bar{\rho}P_{\mathrm{t}}$ and ρP_{t}, respectively. The smart device receives data and energy at the same time, and stores the harvested energy in an energy buffer. Upon collecting enough energy, the smart device performs one block of transmission immediately. We investigate the timeliness and the efficiency of the system in terms of age of information (AoI) and data rate, respectively. We also investigate the trade-off between downlink communications and uplink communications by optimizing the weighted-sum average AoI and weighted-sum data rate. Moreover, we present the optimal power-splitting ratio ρ and the optimal weighting coefficient w explicitly or via efficient algorithm.**

Index Terms: **Age of information, status updating system, two-way data exchange, wireless power transfer.**

I. INTRODUCTION

IN modern real-time monitoring/controlling applications such as status-sensing of environment [1], position/speed/acceleration monitoring of vehicles [2], [3], providing timely information updates is an important and critical objective of the system design [4]. To be specific, only those timely received updates can reflect the current status of the system, and the outdated updates are generally useless. In this kind of real-time applications, however, neither traditional delay nor throughput is an adequate timeliness measure [5]. Note that when updates arrive very infrequently, the latest received update might be generated a long time ago and hence is not fresh, even if its delay is small; when the throughput is high, the received updates often suffer from large queueing delays and would also be not fresh. In [6], therefore, *age of information* (AoI) was proposed as a new measure characterizing the freshness of received updates. In particular, AoI is defined as the difference between the current epoch and the generation time of the latest received update.

The AoI measure has been used to evaluate the timeliness of many communication systems. In multi-source systems [7], for example, the authors formulated an AoI optimization problem and derived several general results that are applicable to a wide variety of multiple-source service systems. In multi-link systems [8], the authors considered a set of links to deliver all messages and addressed the link transmission scheduling problem. In [10], a multicast network was considered in which real-time status updates are replicated and sent to multiple interested receiving nodes through independent links. Since the AoI of an updating system is jointly determined by its arrival process and its service process, most related studies were developed based on queuing theory. For instance, the average AoI of $M/M/1$, $M/D/1$, $D/M/1$ queueing systems under the first-come-first-served (FCFS) policy was presented in [6]. Subsequently, the average AoI in other queuing models such as $D/G/1$ and $M/G/1/1$ queues was given in [11] and [12], respectively. Moreover, the average AoI of updating systems under other service policies such as the last-come-first-served (LCFS) policy [13], the zero-wait policy [14] was also well studied. In particular, it was shown in [15] that the LCFS policy with preemption can minimize the average AoI of updating systems. Furthermore, there were also works investigating the peak AoI of updating systems, such as [16]–[18].

In many practical remote monitoring systems, the batteries of sensors are often limited and cannot be recharged due to their unaccessible deployments. To ensure the continuous and reliable functioning of these systems, energy harvesting and wireless power transfer (WPT) have been widely used as two promising solutions [19]–[40]. For example, the average AoI of energy harvesting powered one-hop systems with causal energy constrains was investigated in [20]–[22]. In [23], the authors further considered a two-hop network, where both the energy and the data suffer from the causality constrains both for the source and for the relay. The results in [24], [25] proved that the threshold based update scheduling can minimize the long-term average AoI in many scenarios. In addition, a series of works focused on the timeliness of update transmission over error/erasure-prone channels, e.g., [26]–[30]. Different from previous articles, [31] considered a scenario where the timing index of update transmissions is also encoded to convey some messages, and studied the tradeoff between the achievable message rate and the achievable average AoI. In [32]–[36], the authors studied how the size of energy buffers affects the average AoI and proposed corresponding optimal offline/online update managing policies. For an updating system with a unit-capacity battery, it was shown that the optimal policy has a threshold based structure, which was further generalized to updating systems with arbitrary-sized

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batteries [37]. For a massive MIMO system with hybrid time switching and simultaneous wireless information and power transfer system, the sum-rate maximizing power allocation was studied in [38]. Moreover, the average AoI of a two-way data exchange system was presented in [40], [41], where a master node possesses the unique power supply of the system and transfers energy and data to the slave node alternatively. To be specific, when the master node has no data packet to transmit, the master node transfers energy to the slave node, i.e., using the *time-splitting* scheme. When the slave node has collected enough energy to perform a block of transmission, the slave node starts its transmission immediately to the master node. Although the paper has presented a full characterization of the data exchanging capability of the two-way data exchange system, the master node and the slave node cannot transmit data at the same time.

In this paper, we consider a unilaterally powered two-way data exchanging-system with *power-splitting*, where an access point and a smart device exchange their own data over block Rayleigh fading channels. Morever, we split the transmit power of the access point into two parts: the power used for information transmission with proportion $\bar{\rho}$ and the power for energy transfer with proportion ρ. In doing so, information transmission and energy transfer to the smart device can be performed simultaneously. Accordingly, the smart device receives the transmitted packets and collects the transferred energy from the access point at the same time. Once the smart device has collected enough energy to perform one block of transmission, it starts to transmit immediately. At both the access point and the smart device, we assume that new data packets are generated immediately after the transmission completion of the previous packet, i.e., following the zero-wait policy. For both downlink and uplink transmissions, we then derive closed form average AoI and achievable data rate. We also consider how the power-splitting ratio ρ affects the weighted-sum average AoI and weighted-sum data rate of the system. In particular, we obtained the optimal power-splitting ratio ρ^* of this power-splitting system.

The rest of the paper is organized as follows. In Section II, we describe the specific model of the system, including the channel model and the data exchange model, as well as the definition of AoI. In section III, we obtain the explicit form of average AoI and date rate for both downlink and uplink communications. The weighted-sum average AoI and the weighted-sum data rate are optimized over the power-splitting ratio and the weighting coefficients in Section IV. Finally, we present our simulation results in Section V and conclude the paper in Section VI.

II. SYSTEM MODEL

As shown in Fig. 1, we consider a two-way data exchange system based on power-splitting, where an access point and a smart device exchange their own data in packets via block fading channels. The access point has a constant power supply with power P_{t} while the smart device does not. At the access point, its transmit power is split into two parts with a *power-splitting ratio* ρ, where the first part at power $P_{\mathrm{d}} = \bar{\rho}P_{\mathrm{t}}$ is used for the *downlink* information transmission and the other part at power $P_{\mathrm{e}} = \rho P_{\mathrm{t}}$ is used for the power transfer to the smart device ($\bar{\rho}+\rho = 1$). At the smart device, it stores the energy in an large-

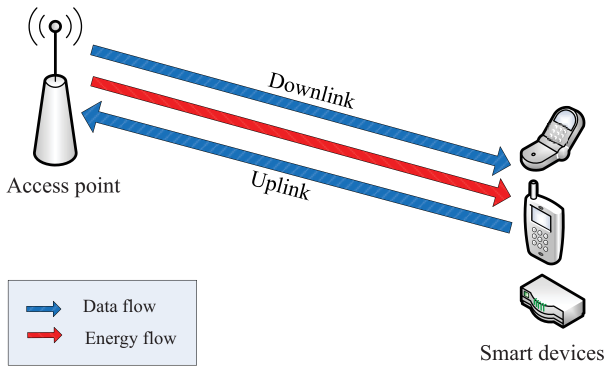


Fig. 1. Two-way data exchanging system with wireless power transfer.

sized energy buffer and performs a block of transmission to the access point over the *uplink channel* when enough energy has been collected. Moreover, data packets are generated according to the zero-wait policy at both the access point and the smart device, which means that a new data packet will be generated when the transmission of previous packet is completed.

A. Channel Model

We have the following assumptions for downlink and uplink channels.

A1 *Block Rayleigh-fading:* Time is discrete and we denote block n as the period between epoch n and epoch $n+1$. In addition, the power gain γ_n follows exponential distribution

$$f_{\gamma}(x) = \lambda e^{-\lambda x}. \tag{1}$$

A2 *AWGN noise:* The received signal is distorted by additive white Gaussian noise (AWGN).

A3 *Low SNR regime:* The total transmit power is small and the received signal-to-noise ratios (SNRs) are much smaller than unity at both the access point and the smart device.

A4 *Frequency division duplex:* The downlink and uplink transmissions are carried out at different frequencies bands.

Let T_{B} be the block length, d be the distance between the access point and the smart device, α be the path-loss exponent, W be the limited system bandwidth, and N_0 be the noise spectrum density. We set the uplink transmit power the same as the average received power from downlink power transfer, i.e., $P_{\mathrm{u}} = \rho P_{\mathrm{t}}/\lambda d^{\alpha}$. That is, the downlink energy transfer exactly meets the demand on the energy for uplink update transmissions. Thus, the downlink and uplink data rates (in nats) that can be transmitted in a block are, respectively, given by

$$b_n^{\mathrm{d}} = T_{\mathrm{B}} W \ln\left(1 + \frac{\bar{\rho}P_{\mathrm{t}}\gamma_n}{d^{\alpha}WN_0}\right) \approx \frac{\bar{\rho}P_{\mathrm{t}}T_{\mathrm{B}}\gamma_n}{d^{\alpha}N_0}, \tag{2}$$

$$b_i^{\mathrm{u}} = T_{\mathrm{B}} W \ln\left(1 + \frac{\rho P_{\mathrm{t}}\gamma_i}{\lambda d^{2\alpha}WN_0}\right) \approx \frac{\rho P_{\mathrm{t}}T_{\mathrm{B}}\gamma_i}{\lambda d^{2\alpha}N_0}, \tag{3}$$

where the approximations follow the *Low SNR* assumption. Hence, both b_n^{d} and b_i^{u} follow the exponential distribution.

B. Data Exchange Model

In order to make full use of the available energy, both the access point and the smart device will generate a new packet

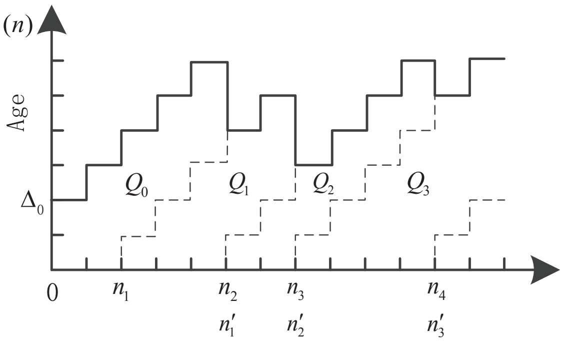


Fig. 2. Sample path of downlink AoI $\Delta_D(n)$ (the upper envelop in bold).

immediately after the previous packet being completely transmitted. However, the uplink and downlink transmissions are slightly different. To be specific, since the access point has a continuous energy supply, the access point would be always busy transmitting information and energy to the smart device over the downlink channel. On the contrary, the smart device will not start transmitting data to the access point until it has collected sufficient energy for a block of transmission. We define the energy transfer efficiency η as the ratio between the power P_r received in smart device and the power ρP_t transmitted by the access point when the smart device is one meter away from the access point, i.e., $\eta = P_\mathrm{r}/\rho P_\mathrm{t}$. Then the energy collected by the smart device in block n can be expressed as

$$E_n = \eta \rho P_\mathrm{t} T_\mathrm{B} d^{-\alpha} \gamma_n. \tag{4}$$

C. Age of Information

In this discrete time model, we assume that AoI does not change within each block. In particular, the downlink AoI is the difference between current epoch n and the generation epoch $U_\mathrm{D}(n)$ of the latest packet received by the smart device.

$$\Delta_\mathrm{D}(n) = n - U_\mathrm{D}(n). \tag{5}$$

Similarly, the uplink AoI is defined as $\Delta_\mathrm{U}(n) = n - U_\mathrm{U}(n)$.

Fig. 2 depicts a sample path of downlink AoI $\Delta_\mathrm{D}(n)$. The access point generates a packet at epoch n_1, and its transmission is completed at epoch n_1'. When the access point detects the completion of the first packet at n_1', a new packet would be generated immediately. It is clear that n_1' and n_2 coincide with each other. Thus, all the packets do not need to wait for its transmission and the waiting time is zero. Therefore, the time that a packet spends in the system is equal to its service time S_k.

In a period of time with N blocks, we suppose that the smart device receives K packets. The data rate of the downlink transmission can then be expressed as $p = K/N$. Furthermore, the average downlink AoI during this period is given by $\bar{\Delta}_\mathrm{D} = (1/N)\sum_{n=1}^{N} \Delta_\mathrm{D}(n)$.

As N approaches infinity, the average downlink AoI and the downlink rate of the system are, respectively, given by

$$\bar{\Delta}_\mathrm{D} = \lim_{N\to\infty} \frac{1}{N}\left(Q_0 + \sum_{k=1}^{K-1} Q_k + \frac{1}{2} S_K(S_K+1)\right), \tag{6}$$

$$p = \lim_{N\to\infty} \frac{K}{N}. \tag{7}$$

Likewise, we assume that M packets are delivered to the access point during N blocks. Then the average uplink AoI and uplink data rate are given by

$$\bar{\Delta}_\mathrm{U} = \lim_{N\to\infty} \frac{1}{N}\left(Q_0 + \sum_{m=1}^{M-1} Q_m + \frac{1}{2} S_M(S_M+1)\right), \tag{8}$$

$$q = \lim_{N\to\infty} \frac{M}{N}. \tag{9}$$

III. DOWNLINK AND UPLINK AOI

In this section, we first investigate the statistic property of downlink service time. After that, we shall derive the average downlink AoI and downlink rate in closed form. Finally, we study the energy harvesting process and derive the average uplink AoI and uplink rate in closed form.

A. Downlink Service Time

The *downlink service time* S_D is the number of blocks required to complete the transmission of a packet. In particular, the probability $p_j^\mathrm{S} = \Pr\{S_\mathrm{D} = j\}$ of S_D is given by the following proposition.

Proposition 1: The probability distribution of downlink service time S_D is given by

$$p_j^\mathrm{S} = \frac{(\frac{\theta}{\bar{\rho}})^{j-1}}{(j-1)!} e^{-\frac{\theta}{\bar{\rho}}}, \text{ for } j = 1, 2, \cdots, \tag{10}$$

where

$$\theta = \frac{\lambda l N_0 d^\alpha}{P_\mathrm{t} T_\mathrm{B}}. \tag{11}$$

Proof: See Appendix A.

The probability generating function and the first two order moments of S_D are, respectively, given by

$$G_\mathrm{S}(z) = \mathbb{E}(z^S) = z e^{\frac{\theta}{\bar{\rho}}(z-1)}, \tag{12}$$

$$\mathbb{E}(S_\mathrm{D}) = \lim_{z\to 1^-} G_\mathrm{S}'(z) = 1 + \frac{\theta}{\bar{\rho}}, \tag{13}$$

$$\mathbb{E}(S_\mathrm{D}^2) = \lim_{z\to 1^-} G_\mathrm{S}''(z) + G_\mathrm{S}'(z) = \left(\frac{\theta}{\bar{\rho}}\right)^2 + \frac{3\theta}{\bar{\rho}} + 1. \tag{14}$$

Note that $\mathbb{E}(S) = 1 + \theta/\bar{\rho}$ is the average downlink service time of packets and $\theta/\bar{\rho}$ is the ratio between packet length l and the average amount of information that can be transmitted in a block.

B. Average Downlink AoI

Based on the analysis in the previous subsection, the average downlink AoI can be readily obtained, as shown in the following theorem.

Theorem 1: If $0 < \rho < 1$, the average downlink AoI and data rate of a two-way data exchanging system with zero-wait policy are, respectively, given by

$$\bar{\Delta}_{\rm D} = 1 + \frac{\theta}{\bar{\rho}} + \frac{(\frac{\theta}{\bar{\rho}})^2 + 4\frac{\theta}{\bar{\rho}} + 2}{2(1+\frac{\theta}{\bar{\rho}})}, \quad (15)$$

$$p(\bar{\rho}) = \frac{1}{1+\frac{\theta}{\bar{\rho}}}. \quad (16)$$

where $\theta = \lambda l N_0 d^\alpha / P_{\rm t} T_{\rm B}$. Otherwise, it would be infinitely large.

Proof: See Appendix B.

Next, we shall discuss two extreme cases about the average downlink AoI and data rate.

Corollary 1: If $\bar{\rho} = o$ where o is an infinitesimal, the average downlink AoI and the downlink rate are, respectively, given by

$$\bar{\Delta}_D = \frac{5}{2} + \frac{3\theta}{2} \cdot \frac{1}{o}, \quad (17)$$

$$p = \frac{1}{\theta} \cdot o. \quad (18)$$

In this case, since almost all the power of the system is used for energy transfer to the smart device, the power allocated to data transmission would close to zero. Thus, the average downlink AoI $\bar{\Delta}_D$ goes to infinity, and the downlink rate p goes to zero.

Corollary 2: If $\bar{\rho} = 1 - o$, where o is an infinitesimal, the average downlink AoI and the downlink rate are, respectively, given by

$$\bar{\Delta}_D = \frac{5}{2} + \frac{3\theta}{2} - \frac{1}{2(1+\theta)}, \quad (19)$$

$$p = \frac{1}{1+\theta}. \quad (20)$$

In this case, almost all the power of the system is used to transmit data to the smart device and the power allocated to energy transfer is close to zero. Therefore, the average downlink AoI $\bar{\Delta}_{\rm D}$ would approach its minimum and the downlink rate p would approach its maximum $p_{\max}$.

C. Energy Harvesting Process

We denote the energy collected in the i-th block and a period consisting of j blocks as E_i and e_j, respectively. We then have

$$e_j = \sum_{i=1}^{j} E_i = \frac{\eta\rho P_{\rm t} T_{\rm B}}{d^\alpha} \sum_{i=1}^{j} \gamma_i. \quad (21)$$

According to [41], we have

$$f_{e_j}(x) = \frac{\mu x^{j-1}}{(j-1)!} e^{-\mu x}. \quad (22)$$

In this data-exchanging system with power splitting and wireless power transfer, the efficiency of energy transmission would be much smaller than unity, since the path loss exponent is large and the energy transfer efficiency is small. The energy collected by the smart device in a block, therefore, is often not enough to perform a block of transmission. In most cases, the smart device has to wait for several blocks to accumulate sufficient energy. Let $\tau_{\rm H}$ be the number of blocks for the smart device to accumulate enough energy to perform a block of transmission. Since $\tau_{\rm H}$ may occasionally be smaller than unity, we further denote $s = \max\{1, \tau_{\rm H}\}$. According to [41], we have

$$\Pr\{\tau_{\rm H} = j\} = \frac{(\frac{1}{\eta})^j}{j!} e^{-\frac{1}{\eta}}, \quad (23)$$

$$\mathbb{E}(s) = \frac{1}{\eta} + e^{-\frac{1}{\eta}}. \quad (24)$$

We denote the number of transmissions to complete an uplink packet as S. Since the uplink service time $S_{\rm U}$ is the sum of S and the time s_i to accumulate energy for each block-of-transmission, we have

$$S_{\rm U} = \sum_{i=1}^{S} s_i. \quad (25)$$

The probability generating function and the first two order moments of $S_{\rm U}$ can be readily obtained as

$$p_j^{\rm S} = \frac{1}{(j-1)!}\Big(\frac{\lambda\theta d^\alpha}{\rho}\Big)^{j-1} e^{-\frac{\lambda\theta d^\alpha}{\rho}}, \quad (26)$$

$$G_{\rm S}(z) = z e^{\frac{\lambda\theta d^\alpha}{\rho}(z-1)}, \quad (27)$$

$$\mathbb{E}(S) = 1 + \frac{\lambda\theta d^\alpha}{\rho}, \quad (28)$$

$$\mathbb{E}(S^2) = \Big(\frac{\lambda\theta d^\alpha}{\rho}\Big)^2 + \frac{3\lambda\theta d^\alpha}{\rho} + 1. \quad (29)$$

The moments of s_i and $S_{\rm U}$ can then be given by the following proposition.

Proposition 2: The first-two order moments of uplink service time $S_{\rm U}$ and s_i are, respectively, given by

$$\mathbb{E}(s_i) = \frac{1}{\eta} + e^{-\frac{1}{\eta}}, \quad (30)$$

$$\mathbb{E}(s_i^2) = \frac{1}{\eta^2} + \frac{1}{\eta} + e^{-\frac{1}{\eta}}, \quad (31)$$

$$\mathbb{E}(S_{\rm U}) = \Big(1 + \frac{\lambda\theta d^\alpha}{\rho}\Big)\Big(\frac{1}{\eta} + e^{-\frac{1}{\eta}}\Big), \quad (32)$$

$$\mathbb{E}(S_{\rm U}^2) = \Big(1 + \frac{\lambda\theta d^\alpha}{\rho}\Big)\Big(\frac{1}{\eta^2} + \frac{1}{\eta} + e^{-\frac{1}{\eta}}\Big) + \bigg(\Big(\frac{\lambda\theta d^\alpha}{\rho}\Big)^2 + \frac{2\lambda\theta d^\alpha}{\rho}\bigg)\Big(\frac{1}{\eta} + e^{-\frac{1}{\eta}}\Big)^2. \quad (33)$$

Proof: See Appendix C.

In the equations, the power-splitting ratio ρ take values between zero and unity, d^α is the path loss, and $1/\lambda$ is the expected channel power gain. Also note that the power transmission efficiency η is much smaller than unity.

D. Average Uplink AoI

In this subsection, we first study the average uplink AoI and uplink rate, and then discuss two extreme cases of them.

Theorem 2: Given that $0 < \rho < 1$, the average uplink AoI of a two-way data exchanging system with zero-wait policy is given by

$$\bar{\Delta}_{\rm U} = \frac{3}{2}\Big(1+\frac{\lambda\theta d^{\alpha}}{\rho}\Big)\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)+\frac{1}{2}+\frac{1}{2}\frac{1}{\eta+\eta^2 e^{-\frac{1}{\eta}}} - \frac{1}{2}\frac{\rho}{\rho+\lambda\theta d^{\alpha}}\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big), \quad (34)$$

and the achievable uplink data rate is

$$q(\rho) = \frac{\rho\cdot\eta}{\big(\rho+\lambda\theta d^{\alpha}\big)\big(1+\eta e^{-\frac{1}{\eta}}\big)}, \quad (35)$$

where $\theta = \lambda l N_0 d^{\alpha}/P_{\rm t}T_{\rm B}$.

Proof: See Appendix D.

We then discuss the following two extreme cases of the system.

Corollary 3: When ρ is an infinitesimal, i.e, $\rho = o$, the average uplink AoI and uplink rate are given, respectively, by

$$\bar{\Delta}_{\rm U} = \frac{3\lambda\theta d^{\alpha}}{2}\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)\cdot\frac{1}{o}, \quad (36)$$

$$q = \frac{\eta}{\lambda\theta d^{\alpha}\big(1+\eta e^{-\frac{1}{\eta}}\big)}\cdot o. \quad (37)$$

In this case, all the power of the system is allocated to the downlink data transmission process. There is almost no energy transferred to the smart device, and thus the packets of the smart device cannot be transmitted to the access point. Therefore, it is easy to understand that the average uplink AoI $\bar{\Delta}_{\rm U}$ goes to infinity and the rate q goes to zero.

Corollary 4: When $\rho = 1 - o$, where o is an infinitesimal, the average uplink AoI and uplink rate are given, respectively, by

$$\bar{\Delta}_{\rm U} = \frac{3}{2}\Big(1+\lambda\theta d^{\alpha}\Big)\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)+\frac{1}{2}+\frac{1}{2}\frac{1}{\eta+\eta^2 e^{-\frac{1}{\eta}}} - \frac{1}{2}\frac{1}{1+\lambda\theta d^{\alpha}}\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big), \quad (38)$$

$$q = \frac{1}{\big(1+\lambda\theta d^{\alpha}\big)\big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\big)}. \quad (39)$$

In this case, all the power of the system is distributed to the downlink energy transfer process. The amount of energy collected by the smart device would then achieve its maximum. Therefore, the average uplink AoI $\bar{\Delta}_{\rm U}$ is close to its minimum and the uplink rate q reaches its maximum.

IV. DOWNLINK AND UPLINK TRADE-OFF

In this section, we shall evaluate the optimal trade-off between downlink communications and uplink communications in terms of weighted-sum average AoI and weighted-sum data rate. In particular, we shall minimize the weighted-sum average AoI and maximize the weighted-sum rate through optimizing the power splitting ratio ρ and the weighting coefficient w.

A. Problem Formulation

Let $0 \le w \le 1$ and $\bar{w} = 1 - w$ be the weighting coefficients for the uplink transmission process and the downlink transmission process, respectively. We are interested in the following optimization problems.

Problem 1: Minimizing the weighted-sum average AoI.

$$\min_{\rho} \quad \bar{\Delta} = \bar{w}\bar{\Delta}_{\rm D} + w\bar{\Delta}_{\rm U}, \quad (40)$$
$$\text{s.t.} \quad 0 \le \rho \le 1, \quad 0 \le w \le 1.$$

Problem 2: Maximizing the weighted-sum data rate.

$$\max_{\rho} \quad R = \bar{w}p + wq, \quad (41)$$
$$\text{s.t.} \quad 0 \le \rho \le 1, \quad 0 \le w \le 1.$$

We shall also investigate how weighting coefficients w and $\bar{w}$ affects the weighted-sum average AoI and weighted-sum data rate.

B. Optimizing AoI

In this section, we shall investigated the relationship between the minimal weighted-sum of average AoI $\bar{\Delta}^*$ and the power-splitting ratio ρ. Next, the relation between the minimal weighted-sum of average AoI $\bar{\Delta}^*$ and weight w is studied for each given ρ.

B.1 Age-Optimal Power-Splitting Ratio

For each given w, by taking the derivative of the objective function in *Problem* 1 with respect to ρ, we have,

$$\frac{\partial\bar{\Delta}}{\partial\rho} = (1-w)\left(\frac{3\theta}{2(1-\rho)^2}+\frac{\theta}{2(1+\theta-\rho)^2}\right) + w\left(-\frac{3a}{2}\frac{\lambda\theta d^{\alpha}}{\rho^2}-\frac{a}{2}\frac{\lambda\theta d^{\alpha}}{(\rho+\lambda\theta d^{\alpha})^2}\right), \quad (42)$$

where $a = 1/\eta + e^{-1/\eta}$.

We note that for each $\rho \in [0, 1]$, (42) is a continuous and derivable function. Also, it can be readily shown that

$$\frac{\partial\bar{\Delta}}{\partial\rho}\Big|_{\rho=0} < 0, \frac{\partial\bar{\Delta}}{\partial\rho}\Big|_{\rho=1} > 0, \frac{\partial^2\bar{\Delta}}{\partial\rho^2}\Big|_{0<\rho<1} > 0. \quad (43)$$

Therefore, there would be exactly one $\rho^{\diamond} \in (0, 1)$ satisfying

$$\frac{\partial\bar{\Delta}}{\partial\rho}\Big|_{\rho=\rho^{\diamond}} = 0, \quad (44)$$

which minimizes $\bar{\Delta}$. Since $\rho^{\diamond}$ would be different if the weighting coefficient w is changed, we shall rewrite the solution $\rho^{\diamond}$ as a function of w, i.e., $\rho^{\diamond}(w)$.

However, explicit $\rho^{\diamond}(w)$ is not available in general. To solve *Problem* 1, therefore, we have proposed an iterative algorithm based on Newton's method, as shown in Algorithm 1. Then we have

$$\bar{\Delta}^* = \bar{\Delta}\,|_{\rho=\rho^{\diamond}(w)}, \quad 0 \le w \le 1. \quad (45)$$

Algorithm 1 Iterative solution to $\rho^*(w)$.

Initialization:

1: Set the initial power-splitting ratio ρ_0,

2: Set the maximum error $\varepsilon_{\max}$, the maximum number of iterations $n_{\max}$;

Iteration:

3: **while** $n \le n_{\max}$, $|\rho(n+1) - \rho(n)| \le \varepsilon_{\max}$ **do**

4: update $\bar{\Delta}$ using (19), (34) and (40);

5: $\rho(n+1) = \rho(n) - d\bar{\Delta}/d^2\bar{\Delta}$;

6: $n = n + 1$;

7: **end while**

B.2 Age-Optimal Weighting Coefficient

We then study how the weighted-sum average AoI $\bar{\Delta}$ varies with coefficient w for each given power-splitting ratio ρ. It is clear that $\bar{\Delta}$ is a linear function of w and the corresponding derivative is given by

$$\frac{\partial \bar{\Delta}}{\partial w} = -\left(\frac{5}{2} + \frac{3\theta}{2(1-\rho)} - \frac{1-\rho}{2(1+\theta-\rho)}\right) + \left(\frac{3a}{2}\left(1 + \frac{\lambda\theta d^\alpha}{\rho}\right) + \frac{1+a\eta^2}{2a\eta^2} - \frac{a}{2}\frac{\rho}{\rho+\lambda\theta d^\alpha}\right), \tag{46}$$

where $a = 1/\eta + e^{-1/\eta}$. Since $\bar{\Delta}$ is a linear function of w, it clear that $\frac{\partial \bar{\Delta}}{\partial w}$ is a constant for each given ρ. Thus, $\frac{\partial \bar{\Delta}}{\partial w}$ can be considered as a continuous and derivable of $\rho \in [0, 1]$ and we can verify that

$$\frac{\partial \bar{\Delta}}{\partial w}\Big|_{\rho=0} > 0, \frac{\partial \bar{\Delta}}{\partial w}\Big|_{\rho=1} < 0, \frac{\partial^2 \bar{\Delta}}{\partial w \partial \rho} < 0. \tag{47}$$

That is, $\frac{\partial \bar{\Delta}}{\partial w}$ is a decreasing function of ρ and there would be only one $\rho^\circ \in (0, 1)$ satisfying

$$\frac{\partial \bar{\Delta}}{\partial w}\Big|_{\rho=\rho^\circ} = 0. \tag{48}$$

When ρ° is used, the average downlink AoI is actually equal to the average uplink AoI. In particular, we have the following proposition.

Proposition 3: If $\rho \in (0, \rho^\circ)$, the weighted-sum average AoI $\bar{\Delta}$ is minimized at $w = 0$. Otherwise (i.e., $\rho \in (\rho^\circ, 1)$), $\bar{\Delta}$ is minimized at $w = 1$. That is,

$$\bar{\Delta}^* = \begin{cases} \bar{\Delta}\,|_{w=0}, & 0 \le \rho \le \rho^\circ, \\ \bar{\Delta}\,|_{w=1}, & \rho^\circ < \rho \le 1. \end{cases} \tag{49}$$

Proof: For $\rho \in (0, \rho^\circ)$, it is clear that $\frac{\partial \bar{\Delta}}{\partial w} > 0$ and $\bar{\Delta}$ would be monotonically increasing with w. Thus, $\bar{\Delta}$ is minimized at $w = 0$. For $\rho \in (\rho^\circ, 1)$, we have $\frac{\partial \bar{\Delta}}{\partial w} > 0$. Then we know that $\bar{\Delta}$ is monotonically decreasing with w and is minimized at $w = 1$. □

In fact, when the splitting ratio is equal to ρ°, the average downlink AoI and the average uplink AoI are equal. Thus, the weighted-sum average AoI $\bar{\Delta}^*|_{\rho^\circ}$ would be a constant independent of weighting coefficient w. If $\rho < \rho^\circ$ holds true, the average uplink AoI would be larger than the average downlink average AoI. It is clear that the weighted-sum average AoI $\bar{\Delta}^*$ becomes smaller when w is reduced, and converges to its minimum as w goes to zero. By a similar induction, we see that in the case $\rho > \rho^\circ$, $\bar{\Delta}^*$ would be smaller when w is increased, and converges to its minimum as w goes to one. In practical two-way data exchanging systems, it is suggested to use a power splitting ratio around ρ°, which ensures the best AoI performance to be achieved when the priority of both downlink and uplink can be guaranteed, other than being dominated by a certain direction.

C. Optimizing Data Rate

C.1 Rate-Optimal Power-Splitting Ratio

First, we study how the weighted-sum data rate R changes with power-splitting ratio ρ for each given weighting coefficient w. By taking the derivative of R with respect to ρ, we have

$$\frac{\partial R}{\partial \rho} = (1-w)\frac{-\theta}{(1-\rho+\theta)^2} + \frac{w}{a}\frac{\lambda\theta d^\alpha}{(\rho+\lambda\theta d^\alpha)^2}, \tag{50}$$

where $a = 1/\eta + e^{-1/\eta}$.

It can be readily verified that (50) is continuous and derivable for $\rho \in [0, 1]$. In particular, we have

$$\frac{\partial^2 R}{\partial \rho^2} < 0, \tag{51}$$

which means that $\frac{\partial R}{\partial \rho}$ is monotonically decreasing with ρ.

When different weighting coefficients w are used, both $\frac{\partial R}{\partial \rho}|_{\rho=0}$ and $\frac{\partial R}{\partial \rho}|_{\rho=1}$ may take positive or negative values. Solving w from equations $\frac{\partial R}{\partial \rho}|_{\rho=0} = 0$ and $\frac{\partial R}{\partial \rho}|_{\rho=1} = 0$, we have the following three cases.

$$\begin{cases} \frac{\partial R}{\partial \rho}\Big|_{\rho=0} < 0, & \frac{\partial R}{\partial \rho}\Big|_{\rho=1} < 0, \quad 0 \le w < w_0, \\ \frac{\partial R}{\partial \rho}\Big|_{\rho=0} > 0, & \frac{\partial R}{\partial \rho}\Big|_{\rho=1} < 0, \quad w_0 < w < w_1, \\ \frac{\partial R}{\partial \rho}\Big|_{\rho=0} > 0, & \frac{\partial R}{\partial \rho}\Big|_{\rho=1} > 0, \quad w_1 < w \le 1, \end{cases} \tag{52}$$

where

$$w_0 = \frac{a\lambda\theta^2 d^\alpha}{a\lambda\theta^2 d^\alpha + (1+\theta)^2}, \text{ and } w_1 = \frac{a(1+\lambda\theta d^\alpha)^2}{a(1+\lambda\theta d^\alpha)^2 + \lambda\theta^2 d^\alpha}.$$

are the corresponding solutions. Then we have the following immediate proposition.

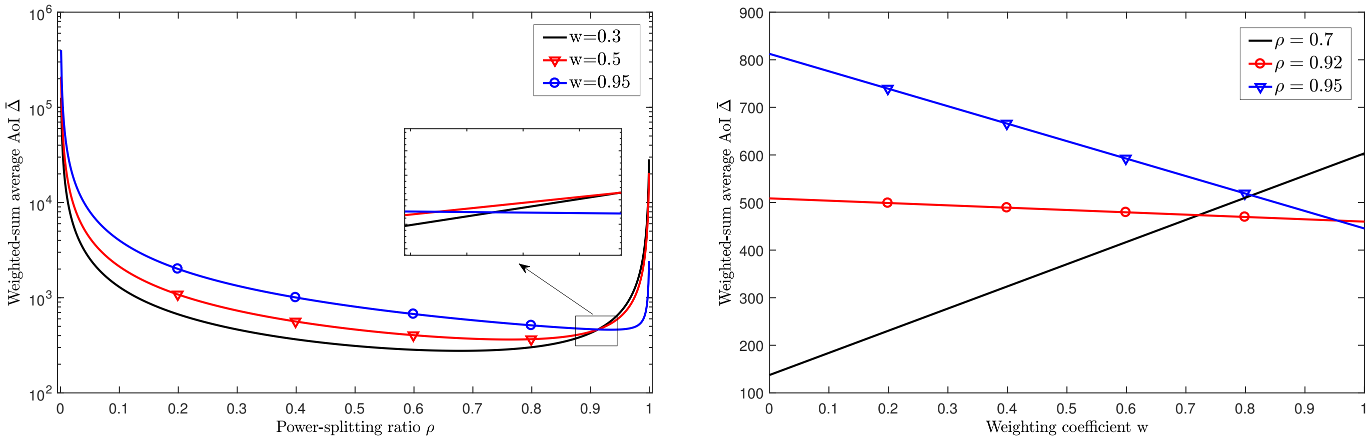
Proposition 4: For each $w \in [0, w_0]$, the maximum weighted-sum rate is achieved when $\rho = 0$; For each $w \in [w_0, w_1]$, the maximum weighted-sum rate is achieved when $\rho = \rho^\triangleright(w)$, which is the solution to $\frac{\partial R}{\partial \rho} = 0$; For each $w \in [w_1, 1]$, the maximum weighted-sum rate is achieved when $\rho = 1$. That is,

$$\bar{\Delta}^* = \begin{cases} \bar{\Delta}\,|_{\rho=0}, & 0 \le w \le w_0, \\ \bar{\Delta}\,|_{\rho=\rho^\triangleright(w)}, & w_0 \le w \le w_1, \\ \bar{\Delta}\,|_{\rho=1}, & w_1 \le w \le 1. \end{cases} \tag{53}$$

C.2 Rate-Optimal Weighting Coefficient

For each given power-splitting ratio ρ, the derivative of R with respect to w would be a constant given by

$$\frac{\partial R}{\partial w} = -\frac{1-\rho}{1-\rho+\theta} + \frac{\rho}{a(\rho+\lambda\theta d^\alpha)}, \tag{54}$$



(a) (b)

Fig. 3. Average AoI of the power-splitting system: (a) Average AoI versus power-splitting ratio ρ and (b) average AoI versus weight w.

where $a = 1/\eta + e^{-1/\eta}$. We then have

$$\frac{\partial R}{\partial w}\Big|_{\rho=0} < 0, \frac{\partial R}{\partial w}\Big|_{\rho=1} > 0, \frac{\partial^2 R}{\partial w \partial \rho}\Big|_{0<\rho<1} > 0, \tag{55}$$

and thus there would be a unique $\rho^{\triangleleft} \in (0,1)$ satisfying

$$\frac{\partial R}{\partial w}\Big|_{\rho=\rho^{\triangleleft}} = 0. \tag{56}$$

We then have the following proposition.

Proposition 5: The weighted-sum data rate is given by

$$R^* = \begin{cases} R^*|_{w=0}, & 0 \le \rho \le \rho^{\triangleleft}, \\ R^*|_{w=1}, & \rho^{\triangleleft} < \rho \le 1. \end{cases} \tag{57}$$

Proof: If $\rho \in (0, \rho^{\triangleleft})$, $\frac{\partial R}{\partial w}$ would be negative and the weighted-sum data rate would be monotonically decreasing with w. Thus, the maximum weighted-sum data rate is obtained at $w = 0$; If $\rho \in (\rho^{\triangleleft}, 1)$, $\frac{\partial R}{\partial w}$ would be positive and the weighted-sum data rate would be monotonically increasing with w. Thus, the maximum weighted-sum data rate is obtained at $w = 1$. □

In particular, it is noted that when $\rho^{\triangleleft}$ is used, the average downlink data rate is equal to the average uplink data rate. That is, $\rho^{\triangleleft}$ is the critical condition for the downlink data rate of the uplink data rate to dominate the efficiency of the system.

V. NUMERICAL RESULT

In this section, we present the obtained results through numerical results. We assume that the system bandwidth W is 1 MHz, the noise spectrum density is $N_0 = 4 \times 10^{-7}$, the distance d between the access point and the smart device is 1.5 m, and the path-loss exponent α is 2. The total power P_t at the access point is 0.01 W, including the power $\bar{\rho}P_t$ for information transmission and the power ρP_t for energy transfer. The transmit power of the smart device is set as $P_u = \rho P_t / \lambda d^{\alpha}$. The length of each packet is $l = 100$ nats and the block length is $T_B = 10^{-3}$ s. The Rayleigh channel parameter is $\lambda = 3$. The energy transfer efficiency of the system is $\eta = 0.5$.

A. Weighted-Sum Average AoI

We present the weighted-sum average AoI $\bar{\Delta} = \bar{w}\bar{\Delta}_D + w\bar{\Delta}_U$ in Fig. 3. As shown in Fig. 3(a), $\bar{\Delta}$ goes to infinity either when the power-splitting ratio ρ approaches to zero or unity and achieves its minimum when ρ is neither too large nor too small. In fact, as ρ approaches zero, the power allocated of downlink information transmission would be very small, leading to large downlink service times and large downlink average AoI. On the other hand, when ρ approaches unity, the power allocated to downlink energy transfer (then for the uplink transmissions) would be very small, leading to large period of energy collecting and large average uplink AoI. Moreover, we see in the left part of Fig. 3(a) that, the larger weighting coefficient w is, the more the average uplink AoI (which is large) influences $\bar{\Delta}$, so we have a larger $\bar{\Delta}$. On right part of Fig. 3(a), the average uplink AoI would be small since ρ is large (i.e., more energy can be harvested). Thus, $\bar{\Delta}$ would be small when w is large. Furthermore, the three curves do not intersect with each other at the same point.

Fig. 3(b) describes how $\bar{\Delta}$ changes with w. As is shown, $\bar{\Delta}$ is decreasing with w when ρ is large, is increasing with w when ρ is small, and do not change when $\rho = \rho^{\circ}$, which validates Proposition 3. To be specific, when ρ is small, the power for downlink information transmission is large. Thus, as the downlink weight $(1 - w)$ decreases, $\bar{\Delta}$ would be increased. On the contrary, when ρ is large, more power is available for uplink transmission. If the uplink weight w is increased, $\bar{\Delta}$ would be decreased.

B. Minimum Weighted-Sum Average AoI

Fig. 4 presents how the minimum weighted-sum average AoI changes with ρ and w. In Fig. 4(a), $\bar{\Delta}$ is minimized over w (see Proposition 3) and we see that the curve is non-differentiable. In particular, at the non-differentiable point $\rho = 0.913$, the average uplink AoI and the average downlink AoI are equal. In the right part of the curve, the optimal weighting coefficient is $w^* = 1$, since ρ is large (more energy available at the smart device) and the average uplink AoI is smaller than that of the downlink trans-

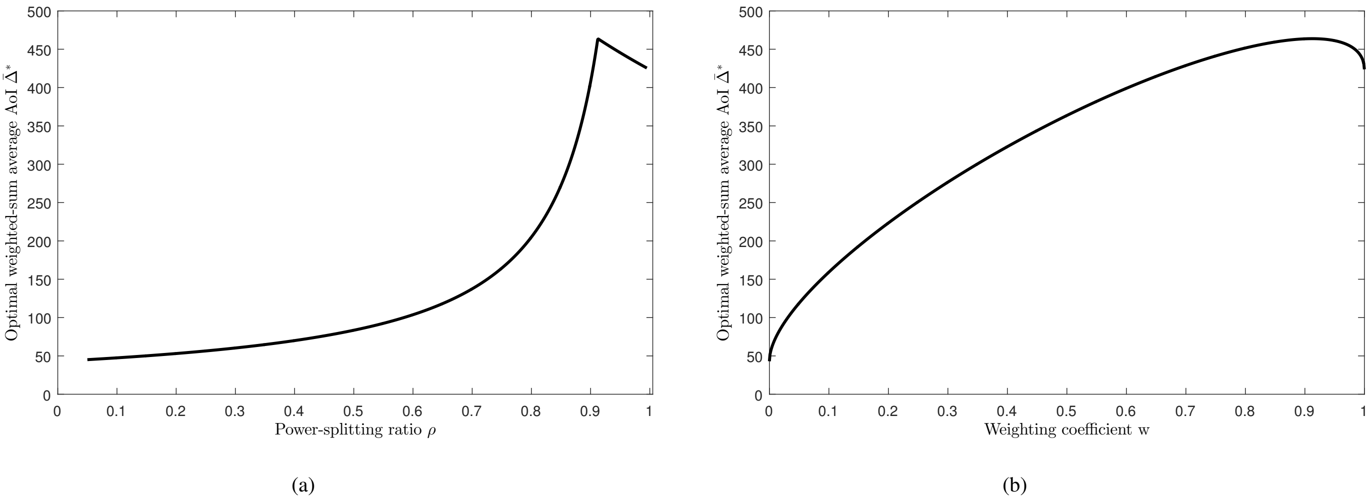


Fig. 4. Optimal average AoI of the power-splitting system: (a) Optimal average AoI versus power-splitting ratio ρ and (b) optimal average AoI versus weight w.

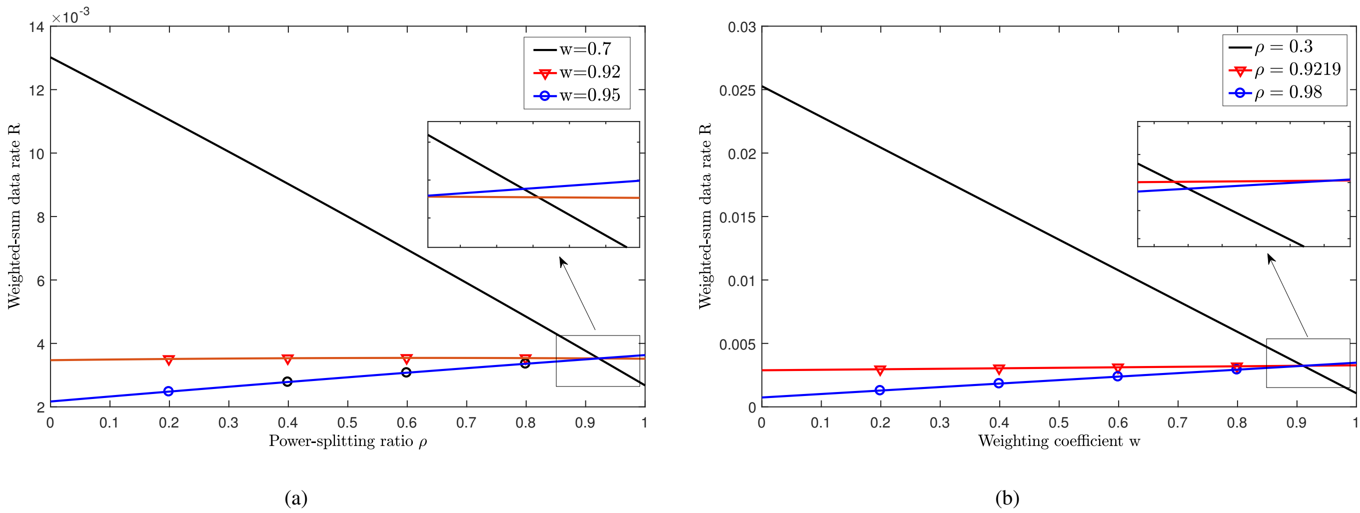


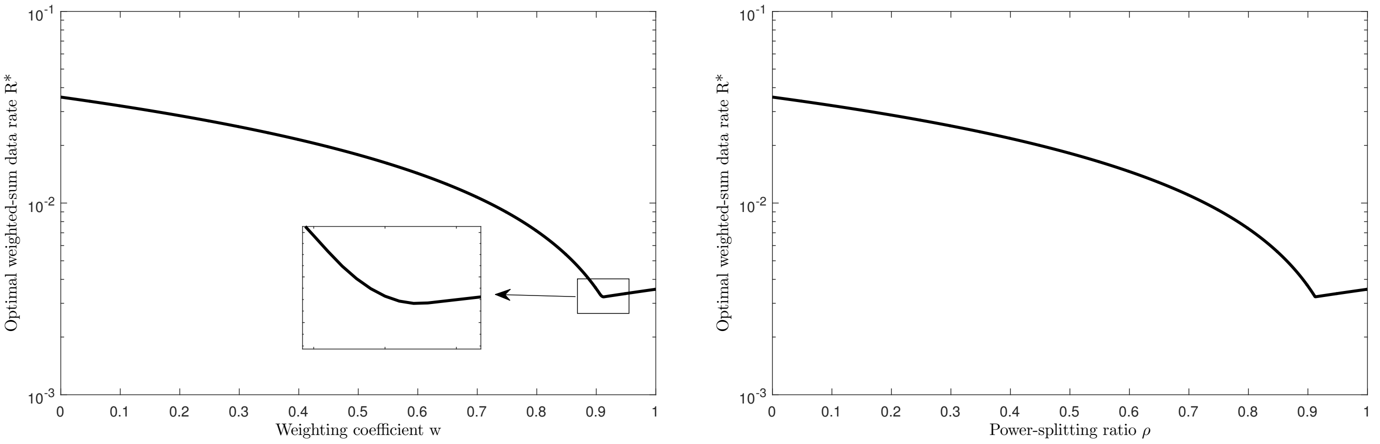
Fig. 5. Average rate of the power-splitting system: (a) Average rate versus power-splitting ratio ρ and (b) average rate versus weight w.

mission. On the contrary, we have $w^* = 0$ for the left part of the curve because the average uplink AoI is larger.

In Fig. 4(b), $\bar{\Delta}$ is minimized over ρ for each given w (see (45)). It is seen that $\bar{\Delta}^*$ has the same value domain in both Fig. 4(a) and Fig. 4(b), where $\bar{\Delta}^*$ is the smallest when $(w, \rho) = (0, 0)$ and is the largest when $(w, \rho) = (0.913, 0.913)$. Since non-zero and non-unity w is not optimal for the system (see Proposition 3 and Fig. 4(a)), the $\bar{\Delta}^*$ presented in Fig. 4(b) is generally larger than that of Fig. 4(a). However, optimizing ρ for some given priority for downlink and uplink communications is more useful for practical communication systems.

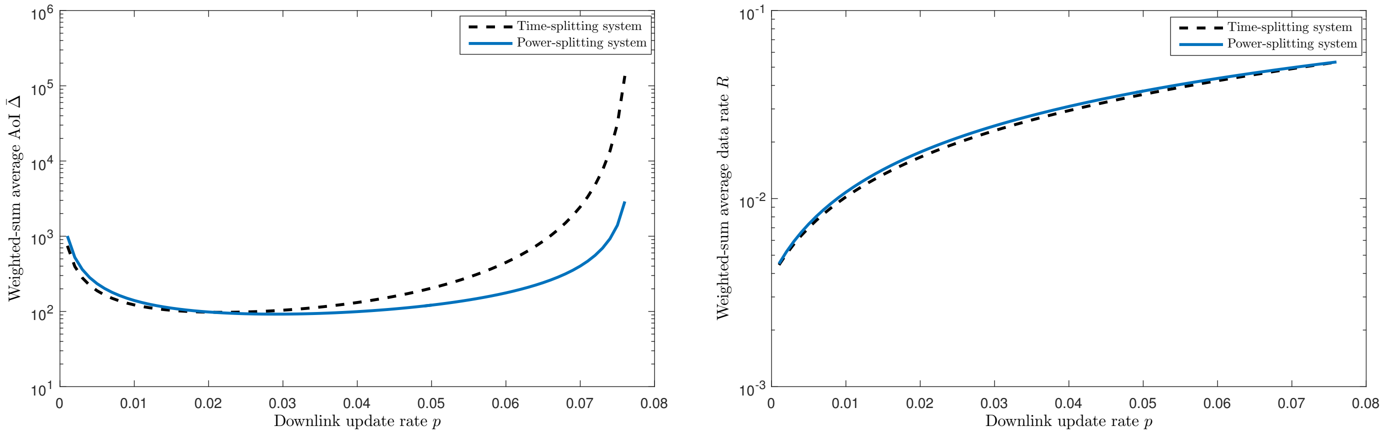
C. Weighted-Sum Data Rate

In Fig. 5, we investigate the weighted-sum data rate $R = \bar{w}p + wq$ of the system. As shown by the black non-marked curve in Fig. 5(a), we observe that when w is relatively small, R is decreasing with ρ. This is because R is mainly depended by the dowlink rate in this case, which is decreasing with ρ. When w is increased to the ratio between downlink energy use efficiency and total energy use efficiency, the downlink rate and the uplink rate paly comparable roles in R. Thus, R is increasing and then decreasing when ρ is increased, as shown by the red triangle marked curve in Fig. 5(a). In particular, as ρ approaches zero or unity, either the uplink rate or the downlink rate term is approaches zero, which decreases R. When ρ lies between zero and unity, the weighted sum of the uplink data rate and the downlink data rate increases. When w is relatively large, it is seen that R is increasing with ρ, as shown by the blue circled curve in Fig. 5(a). Note that in this case, R is mainly determined by the uplink rate, which is increasing with ρ since more power would be allocated to downlink energy transfer and more power would be available for uplink transmissions. Fig. 5(b) further presents how R changes with w for a given ρ. Since both downlink data rate (decreasing with ρ) and uplink data rate (increasing with ρ) would be determined once ρ is given, R is actually a determined linear function of w. For example, we observe that when ρ is small (the black curve), R would be decreased if w is increased. This is because the downlink rate is much larger than



(a) (b)

Fig. 6. Optimal average rate of the power-splitting system: (a) Optimal average rate versus weight w and (b) optimal average rate versus power-splitting ratio ρ



(a) (b)

Fig. 7. Performance comparison between time-splitting systems and the power-splitting systems: (a) Weighted-sum average AoI $\bar{\Delta}$ versus p and (b) weighted-sum average data rate R versus p.

the uplink data rate in this case.

D. Optimal Average Rate

As shown in Proposition 4, the maximum achievable R^* and corresponding ρ^* are depended with w. In particular, when w is small ($w < w_0$), R is determined by downlink rate and the optimal ρ is zero, i.e., all the power is allocated for downlink information transmission and we have $R = \bar{w}p$. Thus, we see from Fig. 6(a) (logarithmic y-axis is used) that the optimal weighted-sum data rate R^* is decreasing with w linearly. On the other hand, if w is close to unity ($w > w_1$), all power would be allocated to downlink energy transfer and thus we have $R = wq$. Accordingly, we see from Fig. 6(a) that R^* is increasing with w. If $w_0 < w < w_1$, the optimal power-splitting ratio $\rho^{\triangleright}(w)$ can be solved from $\frac{\partial R}{\partial \rho} = 0$ as a function of w. The corresponding optimal rate is shown by the smooth curve connecting the two segments.

In Fig. 6(b), we presents the maximum weighted-sum data rate R^* for each given ρ. We have shown in Proposition 5 that the optimal coefficient w^* is either zero or unity. In fact, we have $w^* = 0$ for the the left part of the curve and $w^* = 1$ for the other part of the curve in Fig. 6(b).

E. Connection with Time-Splitting Two-Way Data Exchange

It has been shown in [40], [41] that the two-way data exchanging can also be performed through time-splitting. To be specific, the access point generates a new packet with probability p in each block. Whenever it has data in its queue, the access point transmits its data to the smart device using all of its power. During periods when its queue is empty, the access point transfers energy to the smart device at its full power. Upon receiving enough energy, the smart device can then perform information transmission to the access point. Let S_i be a period transmitting information to the smart device and I_i be a period transferring energy to the smart device, the ratio between the power for energy transfer and the total power can be expressed as

$$\rho_{\text{TS}} = \lim_{N\to\infty} \frac{\sum_{i=1}^{K_{\text{I}}} I_i}{\sum_{i=1}^{K_{\text{I}}} I_i + \sum_{i=1}^{K_{\text{S}}} S_i}, \tag{58}$$

where $K_{\rm I}$ is the number of periods transferring energy and $K_{\rm S}$ is the number of periods transmitting information to the smart device.

Since $\sum_{i=1}^{K_{\rm I}} I_i + \sum_{i=1}^{K_{\rm S}} S_i = N$ and

$$\begin{aligned}\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{K_{\rm S}} S_i &= \lim_{N\to\infty}\frac{K_{\rm S}}{N}\frac{1}{K_{\rm S}}\sum_{i=1}^{K_{\rm S}} S_i\\ &= p\mathbb{E}[S_i]\\ &= p(1+\theta),\end{aligned} \tag{59}$$

where $\theta = \frac{\lambda l N_0 d^\alpha}{P_{\rm t}T_{\rm B}}$, we have

$$\rho_{\rm TS} = 1 - p(1+\theta). \tag{60}$$

In Fig. 7, we present the weighted-sum average AoI and weighted-sum data rate when the same portion of power is transferred to the smart device, i.e., $\rho = \rho_{\rm TS}$. From Fig. 7(b), we observe that the weighted-sum data rates are almost the same for the time-splitting system and the power-splitting system. That is, with the same allocation of the available power between information transmission and energy transfer, we achieve the same data rate in these two systems. In Fig. 7(a), we see that the weighted-sum AoI of the time-splitting system is close to that of a power-splitting system when p is small and is larger than that of a power-splitting system when p is large. Note that when p is large, the weighted-sum AoI is dominated by the average uplink AoI. In the time-splitting system, since the uplink service time of a packet consists of the downlink busy periods (no energy transferred) and the period of energy harvesting, the corresponding variance would be larger than that of a power splitting system, which leads to a larger uplink AoI.

VI. CONCLUSION

In this paper, we considered the efficiency and timeliness of the two-way data exchanging system with an access point and a powerless smart device. Using power splitting scheme and wireless power transfer, the access point transfers a part of its energy to the smart device so that the device can transmit its own data back. We derived closed form expressions for the average downlink AoI, the average uplink AoI, the downlink data rate, and the uplink data rate. We also investigated the weighted-sum average AoI minimizing power-splitting ratios and weighting coefficients of the system. Our results presents a full characterization for the efficiency and timeliness of the two-way data exchanging system and shed lights on the system design for the two-way data exchange with different priorities in the two directions.

APPENDIX A
PROOF OF PROPOSITION 1

Proof: Denote the probability distribution function of the amount of information $b_n^{\rm d}$ transmitted in a block as $F_1(x)$, we have

$$\begin{aligned}F_1(x) &= \Pr\left\{b_n^{\rm d} \le x\right\}\\ &= \Pr\left\{\gamma_n \le \frac{d^\alpha N_0 x}{\bar\rho P_{\rm t}T_{\rm B}}\right\}\\ &= \int_0^{\frac{d^\alpha N_0 x}{\bar\rho P_{\rm t}T_{\rm B}}} \lambda e^{-\lambda x}{\rm d}x.\end{aligned} \tag{61}$$

Thus, the corresponding probability density function (*p.d.f.*) $f_1(x)$ is given by

$$\begin{aligned}f_1(x) &= \frac{{\rm dF_1(x)}}{{\rm dx}}\\ &= \frac{\lambda N_0 d^\alpha}{\bar\rho P_{\rm t}T_{\rm B}} e^{-\frac{\lambda N_0 d^\alpha}{\bar\rho P_{\rm t}T_{\rm B}}x}\\ &= \frac{1}{v}e^{-\frac{x}{v}},\end{aligned} \tag{62}$$

where $v = \frac{\bar\rho P_{\rm t}T_{\rm B}}{\lambda N_0 d^\alpha}$. We further denote the *p.d.f.* of the amount of information transmitted in k ($k = 1,2,3,\cdots$) blocks as $f_k(x)$ and then have

$$f_k(x) = \frac{1}{\Gamma(k)v^k}x^{k-1}e^{-\frac{x}{v}}. \tag{63}$$

Thus, the probability that the downlink service time S equal to j which is the number of blocks is given by

$$\begin{aligned}p_j^{\rm S} &= \Pr\left\{\sum_{i=1}^{j-1} b_i < l, \sum_{i=1}^{j} b_i > l\right\}\\ &= \int_0^l f_{j-1}(y){\rm d}y\int_{\rm l-y}^{\infty}{\rm f_1(x)dx}\\ &= \int_0^l \frac{1}{(j-2)!v^{(j-1)}}y^{(j-2)}e^{-\frac{y}{v}}{\rm d}y\int_{\rm l-y}^{\infty}\frac{1}{\rm v}{\rm e}^{-\frac{\rm x}{\rm v}}{\rm dx}\\ &= \frac{(\frac{\lambda l N_0 d^\alpha}{\bar\rho P_{\rm t}T_{\rm B}})^{j-1}}{(j-1)!}e^{-\frac{\lambda l N_0 d^\alpha}{\bar\rho P_{\rm t}T_{\rm B}}}\\ &= \frac{(\frac{\theta}{\bar\rho})^{j-1}}{(j-1)!}e^{-\frac{\theta}{\bar\rho}},\end{aligned} \tag{64}$$

where $\theta = \lambda l N_0 d^\alpha / P_{\rm t}T_{\rm B}$. This completes the proof of Proposition 1. □

APPENDIX B
PROOF OF THEOREM 1

Proof: According to (6), (7) and Fig. 2, we can get the following conclusions. First, the first terms Q_0 and the third terms $(1/2)S_{\rm DK}(S_{\rm DK}+1)$ is finite in (6). Thus, (6) can be rewritten as

$$\begin{aligned}\bar\Delta_D &= \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{K-1} Q_k\\ &= \lim_{N\to\infty}\frac{K-1}{N}\frac{1}{K-1}\sum_{k=1}^{K-1} Q_k\\ &= p\mathbb{E}(Q_k),\end{aligned} \tag{65}$$

where p is the downlink average data rate.

In addition, the waiting time is $W_k = 0$ because the access point generates new data packet following the zero-wait policy, which means that the downlink system time equals to the service time $T_{\mathrm{D}k} = S_k$. The average area of each Q_k becomes

$$\begin{aligned}\mathbb{E}(Q_k) =&\mathbb{E}\left[\frac{1}{2}(S_{k-1}+S_k)(S_{k-1}+S_k+1)-\frac{1}{2}S_k(S_k+1)\right]\\ =&\mathbb{E}^2(S_k)+\frac{1}{2}\mathbb{E}(S_k^2)+\frac{1}{2}\mathbb{E}(S_k).\end{aligned} \tag{66}$$

Combining the above results, we then have

$$\begin{aligned}\bar{\Delta}_D &= p\mathbb{E}(Q_k)\\ &= \mathbb{E}(S_k)+\frac{1}{2}+\frac{1}{2}\frac{\mathbb{E}(S_k^2)}{\mathbb{E}(S_k)}\\ &= 1+\frac{\theta}{\bar{\rho}}+\frac{(\frac{\theta}{\bar{\rho}})^2+4\frac{\theta}{\bar{\rho}}+2}{2(1+\frac{\theta}{\bar{\rho}})},\end{aligned} \tag{67}$$

$$p(\bar{\rho}) = \frac{1}{\mathbb{E}(S)} = \frac{1}{1+\frac{\theta}{\bar{\rho}}}, \tag{68}$$

which proves Theorem 1. □

APPENDIX C
PROOF OF PROPOSITION 2

Proof: In the uplink data transmission, we set the power as $\rho \mathrm{P}_t/\lambda d^\alpha$. Then the first-two order moments of the number of blocks for the smart device to perform a block of transmission is

$$\begin{aligned}\mathbb{E}(s_i) &= \sum_{j=1}^{\infty}\Pr\{s=j\}j\\ &= \frac{1}{\eta}+e^{-\frac{1}{\eta}},\end{aligned} \tag{69}$$

$$\begin{aligned}\mathbb{E}(s_i^2) &= \sum_{j=1}^{\infty}\Pr\{s=j\}j^2\\ &= \frac{1}{\eta^2}+\frac{1}{\eta}+e^{-\frac{1}{\eta}}.\end{aligned} \tag{70}$$

According to (30), (25), (28), and (29), the first-two order moments of uplink service time S_{U} is

$$\mathbb{E}(S_{\mathrm{U}}) =\mathbb{E}(S)\mathbb{E}(s_i) = \Big(1+\frac{\lambda\theta d^\alpha}{\rho}\Big)\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big), \tag{71}$$

$$\begin{aligned}\mathbb{E}(S_{\mathrm{U}}^2) =&\mathbb{E}(S)\mathbb{E}(s_i^2)+\mathbb{E}(S^2-S)\mathbb{E}^2(s_i)\\ =&\Big(1+\frac{\lambda\theta d^\alpha}{\rho}\Big)\Big(\frac{1}{\eta^2}+\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)\\ &+\bigg(\Big(\frac{\lambda\theta d^\alpha}{\rho}\Big)^2+\frac{2\lambda\theta d^\alpha}{\rho}\bigg)\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)^2.\end{aligned} \tag{72}$$

This completes the proof of Proposition 2. □

APPENDIX D
PROOF OF THEOREM 2

Proof: We use a similar method studying the downlink AoI as that in studying the uplink AoI. In addition, by virtue of (28) and (29), we can get the following results

$$\begin{aligned}\bar{\Delta}_{\mathrm{U}} =&q\mathbb{E}(Q_k)\\ =&\frac{1}{\mathbb{E}(S_{\mathrm{U}})}\Big(\mathbb{E}^2(S_{\mathrm{U}})+\frac{1}{2}\mathbb{E}(S_{\mathrm{U}}^2)+\frac{1}{2}\mathbb{E}(S_{\mathrm{U}})\Big)\\ =&\mathbb{E}(S_{\mathrm{U}})+\frac{1}{2}+\frac{1}{2}\frac{\mathbb{E}^2(S_{\mathrm{U}})}{\mathbb{E}(S_{\mathrm{U}})}\\ =&\frac{3}{2}\Big(1+\frac{\lambda\theta d^\alpha}{\rho}\Big)\Big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\Big)+\frac{1}{2}+\frac{1}{2}\frac{1}{\eta+\eta^2e^{-\frac{1}{\eta}}}\\ &-\frac{1}{2}\frac{\rho}{\rho+\lambda\theta d^\alpha}\big(\frac{1}{\eta}+e^{-\frac{1}{\eta}}\big),\end{aligned} \tag{73}$$

$$\begin{aligned}q(\rho) =&\frac{1}{\mathbb{E}(S_{\mathrm{U}})}\\ =&\frac{\rho\cdot\eta}{(\rho+\lambda\theta d^\alpha)(1+\eta e^{-\frac{1}{\eta}})},\end{aligned} \tag{74}$$

which proves Theorem 2. □

REFERENCES

[1] J. Gong, X. Chen, and X. Ma,"Energy-age tradeoff in status update communication systems with retransmission," in *Proc. IEEE GLOBECOM* Dec. 2018.

[2] S. Kaul, R. Yates, and M. Gruteser,"On piggybacking in vehicular networks," in *Proc. IEEE GLOBECOM*, Dec. 2011.

[3] S. Kaul, M. Gruteser, V. Rai, and J. Kenney, "Minimizing age of information in vehicular networks," in *Proc. IEEE SECON*, June 2011.

[4] Y. Dong, P. Fan, "A queueing characterization of information transmission over block fading Rayleigh channels in the low SNR regime," *IEEE Trans. Veh. Technol.*, vol. 63, no. 8, pp. 3726–3740, Aug. 2014.

[5] C. Kam, S. Kompella, and A. Ephremides, "Experimental evaluation of the age of information via emulation," in *Proc. IEEE MILCOM*, Oct. 2015.

[6] S. K. Kaul, R. D. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Mar. 2012.

[7] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *IEEE Trans. Inf. Theory*, vol. 65, no. 3, pp. 1807–1827, Mar. 2019.

[8] Q. He, D. Yuan, and A. Ephremides, "Optimal link scheduling for age minimization in wireless systems," *IEEE Trans. Inf. Theory*, vol. 63, no. 99, pp. 1–1, Aug. 2017.

[9] A. M. Bedewy, Y. Sun, and N. B. Shroff, "Age-optimal information updates in multihop networks," in *Proc. IEEE ISIT*, June 2017.

[10] J. Zhong, R. D. Yates, and E. Soljanin, "Multicast with prioritized delivery: How fresh is your data?," in *Proc. IEEE SPAWC*, June 2018.

[11] J. P. Champati, H. Al-Zubaidy, and J. Gross, "Statistical guarantee optimization for age of information for the D/G/1 queue," in *Proc. IEEE INFOCOM WKSHPS*, Apr. 2018.

[12] E. Najm and E. Telatar, "Status updates in a multi-stream M/G/1/1 preemptive queue," in *Proc. IEEE INFOCOM WKSHPS*, Apr. 2018.

[13] A. M. Bedewy, Y. Sun, and N. B. Shroff, "Optimizing data freshness, throughput, and delay in multi-server informationupdate systems," in *Proc. IEEE ISIT*, July 2016.

[14] Y. Sun, E. U. Biyikoglu, R. D. Yates, C. E. Koksal, and N. B. Shroff, "Update or wait: How to keep your data fresh," *IEEE Trans. Inf. Theory*, vol. 63, no. 11, pp. 7492–7508, Nov. 2017.

[15] S. Farazi, A. G. Klein, and D. R. Brown, "Age of information in energy harvesting status update systems: When to preempt in service?" in *Proc. IEEE ISIT*, June 2018.

[16] C. Kam, S. Kompella, and A. Ephremides, "Age of information under random updates," in *Proc. IEEE ISIT*, July 2013.

[17] M. Costa, M. Codreanu, and A. Ephremides, "On the age of information in status update systems with packet management," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1897–1910, Apr. 2016.

[18] Q. He, D. Yuan and A. Ephremides, "On optimal link scheduling with min-max peak age of information in wireless systems," in *Proc. IEEE ICC*, May 2016.

[19] S. Shin *et al*., "Wireless power transfer system for high power application and a method of segmentation," in *Proc. IEEE WPT*, May 2013.

[20] B. T. Bacinoglu, E. T. Ceran, and E. Uysal-Biyikoglu. "Age of information under energy replenishment constraints," in *Proc. ITA*, Feb. 2015.

[21] A. Arafa and S. Ulukus. "Age minimization in energy harvesting communications: Energy-controlled delays," in *Proc. 2017 Asilomar Conf. Signals Syst. Comput.*, Oct. 2017.

[22] S. Farazi, A. G. Klein, and D. R. Brown III. "Average age of information for status update systems with an energy harvesting server," in *Proc. IEEE INFOCOM WKSHPS*, Apr. 2018.

[23] A. Arafa and S. Ulukus. "Age-minimal transmission in energy harvesting two-hop networks," in *Proc. 2017 IEEE GLOBECOM*, Dec. 2017.

[24] B. T. Bacinoglu and E. Uysal-Biyikoglu. "Scheduling status updates to minimize age of information with an energy harvesting sensor," in *Proc. 2017 IEEE ISIT*, June 2017.

[25] X. Wu, J. Yang, and J. Wu. "Optimal status update for age of information minimization with an energy harvesting source," *IEEE Trans. Green Commun. Netw.*, vol. 2, no. 1, pp. 193–204, Mar. 2018.

[26] S. Feng and J. Yang. "Optimal status updating for an energy harvesting sensor with a noisy channel," in *Proc. IEEE INFOCOM WKSHPS*, Apr. 2018.

[27] S. Feng and J. Yang. "Minimizing age of information for an energy harvesting source with updating failures," in *Proc. IEEE ISIT*, June 2018.

[28] S. Feng and J. Yang. "Age of information minimization for an energy harvesting source with updating erasures: Without and with feedback," Available Online: arXiv1808.05141.

[29] A. Baknina and S. Ulukus. "Coded status updates in an energy harvesting erasure channel," in *Proc. CISS*, Mar. 2018.

[30] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor. "Online timely status updates with erasures for energy harvesting sensors," in *Proc. Allerton*, Oct. 2018.

[31] A. Baknina, O. Ozel, J. Yang, S. Ulukus, and A. Yener. "Sending information through status updates," in *Proc. IEEE ISIT*, June 2018.

[32] R. D. Yates, "Lazy is timely status updates by an energy harvesting source," in *Proc. IEEE ISIT*, June 2015.

[33] X. Wu, J. Yang, and J. Wu, "Optimal status update for age of information minimization with an energy harvesting source," in *Proc. IEEE ICC*, May 2017.

[34] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor. "Age-minimal online policies for energy harvesting sensors with incremental battery recharges," in *Proc. ITA*, Feb. 2018.

[35] A. Arafa, J. Yang, and S. Ulukus. "Age-minimal online policies for energy harvesting sensors with random battery recharges," in *Proc. IEEE ICC*, May 2018.

[36] A. Arafa, J. Yang, S. Ulukus, and H. V. Poor. "Age-minimal transmission for energy harvesting sensors with finite batteries: Online policies," Available Online: arXiv:1806.07271.

[37] B. T. Bacinoglu, Y. Sun, E. Uysal-Biyikoglu, and V. Mutlu. "Achieving the age-energy tradeoff with a finite-battery energy harvesting source," in *Proc. IEEE ISIT*, June 2018.

[38] K. Xu, Z. Shen, Y. Wang, X. Xia and D. Zhang, "Hybrid Time-Switching and Power Splitting SWIPT for Full-Duplex Massive MIMO Systems: A Beam-Domain Approach," *IEEE Trans. Veh. Technol.*, vol. 67, no. 8, pp. 7257–7274, Aug. 2018.

[39] E. N. Ciftcioglu, Y. E. Sagduyu, R. A. Berry and A. Yener, "Cost-Delay Tradeoffs for Two-Way Relay Networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4100–4109, Dec. 2011.

[40] Y. Dong, Z. Chen and P. Fan, "Uplink age of information of unilaterally powered two-way data exchanging systems," in *Proc. IEEE INFOCOM WKSHPS*, Apr. 2018.

[41] Y. Dong, Z. Chen, and P. Fan, "Timely two-way data exchanging in unilaterally powered fog computing systems," *IEEE Access*, vol. 7, pp. 21103–21117, Feb. 2019.

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