Ann. N.Y. Acad. Sci. ISSN 0077-8923

ANNALS OF THE NEW YORK ACADEMY OF SCIENCES
Issue: *Ecological Economics Reviews*

Agent-based modeling in ecological economics

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Interconnected social and environmental systems are the domain of ecological economics, and models can be used to explore feedbacks and adaptations inherent in these systems. Agent-based modeling (ABM) represents autonomous entities, each with dynamic behavior and heterogeneous characteristics. Agents interact with each other and their environment, resulting in emergent outcomes at the macroscale that can be used to quantitatively analyze complex systems. ABM is contributing to research questions in ecological economics in the areas of natural resource management and land-use change, urban systems modeling, market dynamics, changes in consumer attitudes, innovation, and diffusion of technology and management practices, commons dilemmas and self-governance, and psychological aspects to human decision making and behavior change. Frontiers for ABM research in ecological economics involve advancing the empirical calibration and validation of models through mixed methods, including surveys, interviews, participatory modeling, and, notably, experimental economics to test specific decision-making hypotheses. Linking ABM with other modeling techniques at the level of emergent properties will further advance efforts to understand dynamics of social-environmental systems.

Keywords: multi-agent systems; validation; decision making; emergent properties

Modeling complex systems in ecological economics with agents

Both ecology and economics are concerned with interactions between organisms and their environment, from individuals within resource-scarce conditions through to populations. Interactions occur along social and kinship networks and within communities, along supply chains, and within markets, economies, and ecosystems. As both disciplines are concerned with interactions among individuals, both have much to gain from computer modeling tools for complex systems, including agent-based modeling (ABM).

Agent-based models have been widely used in ecology where they tend to be termed individual-based models.[1] They have contributed significantly to ecological theory, including population dynamics, group behavior and speciation, forestry and fisheries management, conservation planning, and species re-introductions.[2] ABMs have also been widely used in economics, although perhaps to a lesser extent than in ecology. The field of agent-based computational economics (ACE) has explored features of economies as complex systems by representing economic agents in computer models as autonomous and interacting decision makers.[3] This opens the possibility to explore assumptions about decision making beyond that of self-interested rational and fully informed actors.

Ecological economics is interested in the interactions between human behavior and the environment as a social-ecological system. The environment and the economy are interconnected at many levels, and neither can be understood without considering the social context.[4] Socio-ecological systems can be thought of as complex systems comprising feedbacks, sensitivity to initial conditions, stochastic and nonlinear processes, and expressing self-organizing behavior across scales. The aim of modeling such systems is to capture the principal laws behind the exciting variety of new phenomena that become apparent when the many units of a complex system interact.[5]

The ability of ABM to explicitly represent adaptive decision making and interactions provides an opportunity to explore issues in ecological economics which are defined by heterogeneity, feedbacks through interactions, and adaptation. Topics that could benefit are market dynamics,[6] changes in consumer attitudes,[7] consumption and sustainable behavior,[8] and psychological aspects, such as subjective well-being,[9] natural resource management and land-use change,[10,11] common pool resource use,[12,13] and dynamics of urban systems.[14–16]

This paper identifies current contributions of ABM to ecological economics, and briefly summarizes the history of ABM to the present. We identify two frontiers for further research: (1) advancing empirical calibration and validation of models, notably through integrating with experimental economics; and (2) designing ABMs that can interact through macrolevel emergent properties with other modeling techniques.

Agent-based modeling defined

ABM is the computational study of systems of interacting autonomous entities, each with dynamic behavior and heterogeneous characteristics. The "agents" interact with each other and their environment, resulting in emergent outcomes at the macroscale. Interactions can be direct, such as communication and physical interaction, or indirect via multiple-pathway feedbacks and from aggregate outcomes. Dynamic behavior of heterogeneous agents is represented by decision-making functions, using both rule-based and analytical functions as appropriate for the decision-making situation. Various definitions of ABM include perspectives from the particular research context, including individual-based models (IBM), ACE, multiagent systems, and others. ABM in its different guises has two defining features: (1) interactions leading to emergent outcomes; and (2) explicit representation of dynamic behavior of heterogeneous agents.

Interactions and emergent patterns

Interactions matter in complex systems. Depending on the model's purpose, interactions can be social or environmental, and constitute feedbacks between the individual and its external conditions. Space is commonly used as the medium of interaction, particularly where human–environment feedbacks exist, as in many research areas relevant to ecological economics. ABM allows the researcher to explore how macrophenomena emerge from microlevel behavior among a heterogeneous set of interacting agents,[17] with the structure of interaction networks a critical element affecting the dynamics of systems.[18] Interactions can be direct, such as trading, consuming, communicating, or indirect, such as multiple-pathway environmental responses to agent behavior, or via aggregate outcomes, such as prices in markets. In ABM, higher-order variables (e.g., commodity prices, population dynamics) are not specified but are emergent outcomes of the multitude of interactions.[19] We use the definitions of emergence outlined by Epstein and Axtell[20] and Axelrod[21] as stable macroscopic patterns arising from local interaction of agents.

Complex systems are recognized by the presence of patterns at a system level not reducible to characteristics at the individual level. Such patterns are emergent properties of microlevel interactions and behaviors, in the same sense as the chemical properties of a complex molecule are an emergent property of many subparticles interacting. In such cases we can not analytically derive the properties of the macrosystem from those of its component parts, although we can apply novel mathematical techniques to model the behavior of the emergent properties, and come to recognize emergent patterns therein as "stylized facts."

Grimm and Railsback[22] suggest a perspective of pattern-oriented modeling as a strategy to explain observed patterns that are defining features of systems and therefore indicators of essential underlying processes and structures. Patterns contain information on the internal organization of a system. In social systems, relatively uncomplicated decision-making functions can derive similar statistics as observed stylized facts.[13] Such thinking is used to explain agent markets that reproduce patterns like crashes, unpredictable stock price and volatility, high skew, and kurtosis (fat tails) in the distribution of profits among investors.[23,24]

Adaptive behaviors and decision making of heterogeneous agents

Human behavior itself can be complex and adaptive. Assumptions of human decision makers as a homogenous pool of rational, self-interested economic agents termed homo economicus, are challenged by a wealth of evidence, notably from laboratory economic experiments demonstrating that human decision makers routinely depart from rational and fully informed behavior. People are at best boundedly rational,[25] typically using heuristics rather than optimization for making decisions, and also show a series of consistent "behavioral anomalies." For example, people tend to be risk averse and behave differently when faced with losses or gains.[26] Traits, such as risk aversion, are not constant, but vary between individuals, and people vary in their skills and preferences. Gintis[27] finds that people have different discount rates depending on the decision-making context, are not solely self-regarding, and are strong reciprocators who cooperate and also retaliate against free-riders even at personal cost.

In reality, decisions are usually based on incomplete information, and the preferences and behaviors that underlie decision making can change as new information becomes available. Herein lies a strength of ABM in the ability to encapsulate decision making, allowing for dynamic behavior, adaptation, and learning. ABM can explicitly formalize simple to complex representations of human decision making,[28] with examples including goal-oriented decisions, such as the belief-desire-intention (BDI) framework,[29] utility-seeking agents using preference functions calibrated from econometric techniques,[30,31] and fulfillment of aspirational thresholds.[11,32] Modeling of boundedly rational agents is outlined in Ebenhoeh and Pahl-Wostl,[33] and rule-based decision making using heuristics[12] and decision trees are commonly used to work through sequential conditional decisions an individual may face.[34,35] Decision making can also incorporate interactions in networks of agents, such as diffusion of innovation along social networks or spatial media,[36] the effect of leaders with weighted social influence, imitation of others' behaviors,[11] reputation, and the influence of "skilled" agents in information-limited networks.[37–39]

Encapsulating decision making in agents opens the door to loosening assumptions in contravention of the homo economicus paradigm. Behaviors and patterns seen in socio-economic systems are open for explanation. For example, people routinely show social preferences, valuing the welfare of others in addition to their own. In public goods experiments, many people contribute to a public good even though their income would be higher if they did not.[40] Participants in laboratory experiments are motivated by fairness and reciprocity, and are more trusting than homo economicus.[27,41] They are particularly concerned by equity, proving highly averse to inequitable outcomes.[42] It is also clear outside the economics laboratory that most people do not act in a purely self-interested manner. For instance, people donate to charity, give blood, or work to protect the environment. It is necessary to understand heterogeneities in social preferences in order to understand many key questions in economics.[43] While the homo economicus paradigm may provide a reasonable approximation of behavior in impersonal competitive market settings, it is of far less relevance to many of the questions of interest to ecological economists. Imagine the outcomes of homo economicus participating in the classic example of the tragedy of the commons, rather than an empathetic, skilled, and learned agent. Along this line Jager *et al.*[8] present the "consumat" agent as an alternative to homo economicus, which includes features of decision making, such as social comparison, imitation, and repetitive behavior (habits) under agents' limited cognitive resources.

Agents and other complex system science tools

The impetus to construct a model stems from a limited set of basic motivations: to predict, explore, or understand. We may want to simulate how a well-known system might react to a new situation and anticipate the outcome. With the same model we might want to explore the landscape of many possible situations. Alternatively, we may have incomplete knowledge of the system but, armed with some data and understanding of processes, wish to investigate new hypotheses about the structure and function of the system. These types of questions can be found throughout ecological economics. Practical direction on modeling social systems is given in Gilbert and Troitzsch,[44] also providing an overview of techniques and history of applications. In this section we discuss use of complex systems models and outline the characteristics of different complex systems modeling approaches and their strengths and weaknesses. In attempting to *describe* social-ecological and other complex systems, equation-based models, systems dynamics, and statistical techniques have been used to good effect. In the attempt to *explain* complex and particularly complex adaptive systems, these approaches have different strengths and weaknesses.[15]

As described above, ABM involves autonomous decision makers interacting, a level of detail which is not always required. It is instructive to provide some guidance on alternatives—when to use ABM and what features of the target system guide that choice. Among the candidate modeling approaches that are capable of representing decision making, behavior, adaptation, and other complex dynamics are Bayesian networks, evolutionary models, and SD. Each of these can be considered alongside equation-based and statistical approaches. Figure 1 outlines a possible decision tree to determine the type of complex systems modeling approach to use for a given application.

Statistical approaches, such as regression techniques and factor analysis, are powerful ways to characterize complex systems' aggregate attributes and relationships. Microdynamics are implicitly represented, but statistical models are at a disadvantage when the subject of the model is not a homogenous population or when that population has coordinated or coherent interactions.

SD models represent feedbacks and can describe macrolevel processes and complexity. Whereas they may not necessarily seek the equilibrium results expected by equation-based models, they often deal in aggregate variables and parameters. SD is certainly the most used modeling tool for complex systems, and ecological economics has benefitted in the ability to develop modular SD components connecting phenomena that typically are treated in isolation in some disciplines. Interconnected elements of real-world systems, such as the biosphere, hydrosphere, atmosphere, and anthroposphere, can be represented and linked to allow feedbacks, as in Costanza *et al.*[45] Van den Belt[46] outlines techniques to use SD modeling in participation with stakeholders to build models directed toward participant learning, awareness, and coordination across scales.

However, decisions and actions of multiple actors and potentially multiple spatial relationships are generally absent from SD models. While pure SD models survive this test and are capable of representing complex systems, they are fundamentally not adaptive. The equations and feedbacks in SD are structural, and their ability to evolve is limited

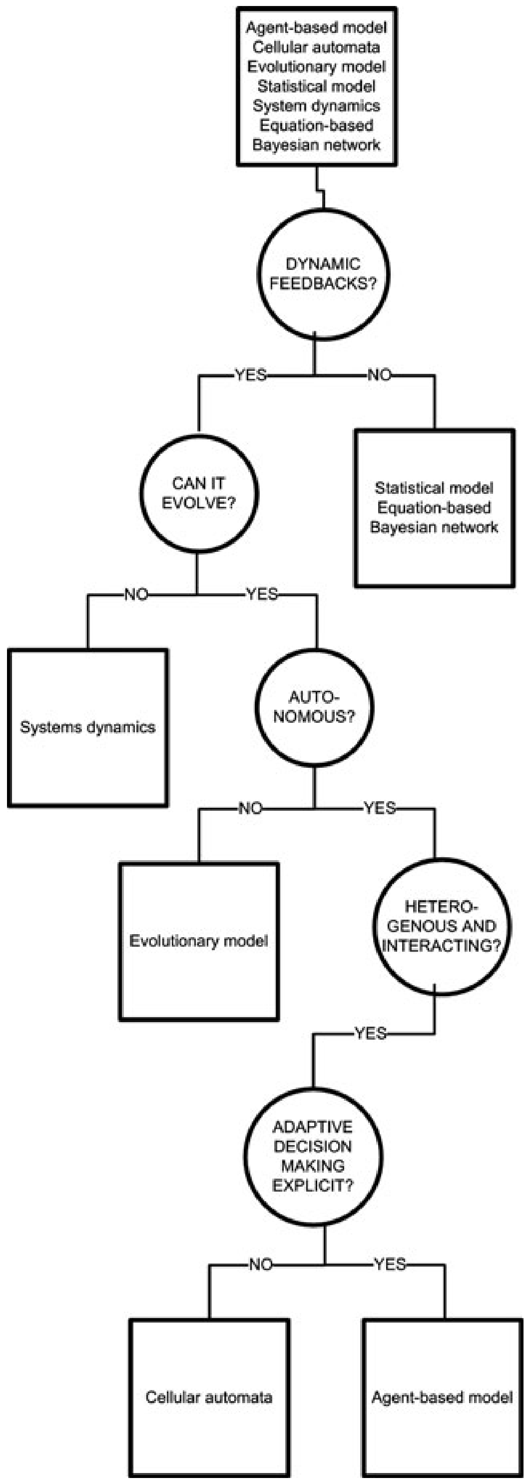

Figure 1. **Possible decision tree for using agent-based modeling and other complex systems modeling tools.**

to variations in parameter values. SD do not lay claim to representing microdynamics and disaggregate features, yet they can be used more directly to explain macrolevel characteristics.

Bayesian networks can incorporate qualitative information and behavior alongside quantitative data and statistical distributions, but they do not easily represent feedbacks. A strong first criterion for choosing ABM is adaptive decision making, which, in turn, may involve interactions with other adaptive decision makers. These dynamic aspects to modeling make outcomes at one point in time influence future events, and the linear mathematics of Bayesian networks, statistical techniques, and equation-based models excludes them from further consideration. Neural networks are included in the category of evolutionary models, and they can exhibit learning and adaptation, but not necessarily heterogeneity or autonomy.

Cellular automata is a special case of ABM whereby the agent locations are fixed, and behaviors amount to a state change based on spatial metrics and neighbor conditions. Cellular automata rules describe observed patterns but do not necessarily explain them. ABM's human decision-making functions require explicit assumptions about this process. Interactions may occur over space using mobile agents or over nonspatial arrangements, such as social networks.

The structure of most ABM platforms is flexible enough to incorporate equations, SD, and statistical techniques whereas the converse is not always the case. The disaggregated form of computation in ABM can always be aggregated up, but statistics and macrolevel variables can not always be disaggregated.

Ways to use agents

ABM's ability to represent interactions and decision making presents opportunities for novel uses. Models can be used as constructive tools to explore how interactions generate emergence, by "growing" patterns that characterize systems. Using ABM as a constructive tool is presented in Epstein,[47] who terms it a "generative" social science technique, which offers a natural environment for the study connectionist phenomena in social science and permits the study of how rules of individual behavior give rise to macroscopic regularities and organizations. This can be used to decouple the events at an individual scale to the macroscopic outcome.

The use of ABMs as a virtual laboratory for testing assumptions and theories of human decision making is valuable for providing a sound empirical and theoretical basis for understanding and predicting behavior, and allows assumptions to be modified.[29,44,47] Controlled experiments test alternative hypotheses of the behavior of interacting agents, revealing how dynamics of systems from molecules to ecosystems and economies emerge from bottom-level processes.[48] From this we can understand how social-ecological systems function and can quantitatively explain many of the deviations that ecological economics takes from mainstream disciplines.

History and current use of ABM

In recent years a number of ABM reviews have been published, attempting to summarize experiences and also categorize general trends. Grimm[1] is among the first collection of learning from the field, coming from the perspective of ecology, which uses the term IBM, addressing systems of interacting individuals but with little dynamic decision making. In social sciences, initial summaries include Levin,[49] Arthur *et al.*,[50] Gilbert and Troitzsch,[44] and Epstein and Axtell.[20] Advances in modeling decision making of human agents led to the uptake of ABM in a variety of social sciences, with Tesfatsion[51] providing a review of models applied to ACE, which continues to have an ongoing community of practice with online resources. Proceedings from several papers are summarized in Berry *et al.*[52] on the topic of economic agents and markets as emergent phenomena, cooperation, and competition as factors in human organization, and platforms and methodologies for ABM. Human agents within an environmental context are reviewed in Parker *et al.,*[10] focusing on land-use and land-cover change, including models with varying degrees of refinement of interactions and human decision making. Notably, land-use and land-cover change models provide the opportunity to simulate a system where economic tradeoffs with spatial environmental systems can occur. Janssen[53] and Bousquet and Le Page[29] review and categorize applications, presenting overviews of issues relating to the growing field that are still relevant today, particularly validation of models. Janssen[54] presents an overview of contributions to ecological economics, outlining applications in diffusion of ideas and technology, representing mental models and learning, land-use change, and participatory approaches. The "handbook" of Tesfatsion and Judd[3] contributes to ABM being an accessible and accepted methodology for computational economics, and more broadly to macrodynamics of social systems including institutions.

Through this history, ABM has increasingly moved from exploratory models with ad hoc representations of underlying processes to face the rigor of empirical validation. Standards in presenting models have begun to emerge through efforts, such as the Object Design Details (ODD) framework,[55] and the Mr. Potatohead framework for comparing models,[56] making publications better communicated and comparable in their fundamental purpose, and models easier to replicate. Rouchier *et al.*[57] presents an overview of efforts to compare and replicate ABM models with contributions from a number of researchers in the field. On the technical front, ABM software has greatly improved in recent years, from initial models that required extensive programming skills, to software packages that are accessible to researchers in multiple fields.[58] ABM has also improved its ability to represent space through integration with geographical information systems (GIS).[31,59–61,62] These reviews of ABM show a progressive story of an improving methodology, with refinement of tools, targeted applications, increasingly concise and comparable communication of models, and evidence of increased experience of the research community.

Expanding on the reviews and journal special editions listed above, we can view a progress of ABM over time into various disciplines. ABM has roots in ecology's IBM, which represent nonhuman entities interacting within ecological systems,[22] whereas ABM generally refer to human decision makers. IBM gained attention with early applications, such as Boids,[63] which demonstrated how realistic flocking behavior of birds can be recreated using simple interaction rules. Ecology recognized the advantages of modeling systems of autonomous agents,[64] with early IBM able to reproduce familiar macroscale outcomes based on simple interaction rules between individuals within a population. Most applications represent animals and plants as autonomous "particles" with simple interaction rules, but there is increasing sophistication of representing individuals and advancing to cross-scale relationships.[48] Some studies suggest that the distinctions between social systems and ecological systems are the information-processing capacity of human actors and the ability to engage in purposeful action and reflexive learning[12]; however, IBM are increasingly taking into account adaptive decision of animals and plants.[65]

In economics, ABM has had wide use in markets given the appropriate individual scale and focus on interactions which is analogous to traders within a market.[6,23,66] ABMs have also been used directly to simulate markets related to natural resources. Filatova *et al.*[67] use rules from standard urban economics to model price formation and market transactions in an urban land market. Polhill *et al.*[11] assess land-use decision strategies and land market exchange decisions, deriving results that show that land market modeling decisions do affect outcomes, which, in turn, improves the relative success of innovators. Heckbert[61] presents an ABM of emissions trading for agricultural pollutants using a market-based instrument for managing water quality on the Great Barrier Reef. The model explores equity issues associated with emissions trading and the functioning of markets while land-use change occurs. Berger *et al.*[68] describe an application of watersheds in Chile and explore the impacts of technical change and rental markets on household income and water-use efficiency. Applications in economics are presented in an accessible fashion in Tesfatsion and Judd[3] with ACE as a constructive methodology to explore economies as emergent systems and to understand how microinteractions lead to persistent observed regularities at the level of society. The ability to represent decision-making functions and interactions in markets opens the door for ecological economics research domains of market dynamics, consumer attitudes, and sustainable behavior and psychological aspects of human well-being.

Also relevant to ecological economics are applications in natural resource management and land-use change. These involve simulated feedbacks between human actions and the environmental system. Bithell and Brasington[69] model cropping and forestry in subsistence farming communities. In this model, rainfall, crop growth, land clearing, and the hydrological cycle involve feedbacks. The forest is represented as an IBM of different tree species, and the community of households is represented as a rudimentary ABM, operating over a catchment-scale spatial hydrological landscape. Schlüter and Pahl-Wostl[12] examine resilience of irrigated cropping and fishing systems under different governance arrangements. Here, a community of agents makes irrigation and cropping decisions that affect the aquatic ecology and fishing, and climate scenarios under different water-use policies are explored. Interactions between different natural resource uses, and cumulative environmental impacts caused through multiple use are studied in Heckbert *et al.*[30] This application models the development of forestry roads and the effect on wildlife through hunting access. Agents are calibrated from a number of stated and revealed preference studies that econometrically estimate preference functions, and additional learning algorithms allow for adaptation. Different configurations of agent preferences result in different spatial patterns in the use of renewable resources. Persistent emergent properties are found to exist under certain parameter combinations—some which "crash" the renewable resource, and some which create a resonating spatial pattern as an example of a self-organizing sustainable renewable resource harvest. A number of applications relate to reforestation and agriculture, such as Evans *et al.*,[70] which models land-use decision making and resulting configuration of forest and agriculture using agents calibrated from surveys, spatial regression, and laboratory experiments. Testing the effect of various agricultural policies has been explored by Happe *et al.*[71,72] for agriculture in the European Union and the effect of farm business structure from agricultural subsidies. An *et al.*[73] apply an ABM of fuelwood harvest and giant panda habitat conservation. Little and McDonald[37] examine the role of social networks and information on resource exploitation. Interestingly, introducing "skilled" agents in an information-limited system has the effect of producing hierarchical performance among agents.

Many ABM applications have explored economic choices within urban environments.[14,15,31,74,75] Cities provide rich territory for research into the complex relationships between decision making and landscapes affected by human activity. In cities there is a concentration of features that match well with the strengths of ABM: heterogeneity (in households, businesses, neighborhoods, land use); autonomous decision making (e.g., by residents, industry, utilities); direct and indirect interactions (e.g., in property markets, planning and policy); and cross-scale effects (from local development choices to urban expansion). In the same arena, ecological economics has questions about the loss of environmental amenity, equitable access to resources (land, education, employment), and the intergenerational effects of development.

On a metropolitan scale, the number of local councils or utility companies or the number of major industries is often at a critical quantity that is neither small enough to be represented easily in equations nor large enough to be usefully represented by statistical techniques. Each of those entities is operating according to rules of limited awareness, limited jurisdiction, and self-interest, and with selected connections to other agents.

Several studies have examined how residential land-use patterns and urban sprawl evolve. Irwin and Bockstael[76] found that negative interactions, (e.g., externalities like congestion) could explain fragmented patterns of urban development in Maryland. Li and Muller[77] simulate settlement patterns in Colorado using ABM to represent household decision making and preferences regarding accessibility, landscape, and visual amenity, and incorporate 2D GIS information (e.g., about transport routes) and 3D information about scenic views. Guzy *et al.*[78] use ABM to incorporate urban containment and nonurban land-use policies in Oregon. Brown and Robinson[16] found that agent heterogeneity augments the potential for landscape change. Zellner *et al.*[79] used a combination of ABM and game theory to represent the characteristics and priorities of neighboring municipalities and demonstrated how zoning policy games can emerge from intermunicipal interactions. Daniell *et al.*[80] and Baynes[81] have explored urban resource consumption and opportunities for urban industrial ecology.

Frontier 1: Empirical calibration and validation of ABM

The paper now turns to the first of two research frontiers for ABM, namely the calibration and validation of models. The field arguably suffers from a lack of success and effort in validating models. Criticism has been aired that model outputs rest on weak theoretical representations of human decision making; empirical data is absent often because data is collected and available only at a coarse resolution, and key model functions may be deeply buried in lengthy code requiring great skill to develop and debug. Model development issues aside, validating models of complex systems with their nefarious feedbacks poses unique challenges, and was identified early and remains an ongoing challenge for ABM.[53]

Although most models have been inspired by observation of real biological and social systems, many of them have not been rigorously tested using empirical data. In fact, most ABM efforts do not go beyond a "proof of concept."[18] Grimm *et al.*[55] report that no general framework for designing, testing, and analyzing bottom-up models has yet been established. Epstein[47] concludes that the field lacks standards for model comparison and replication of results. Janssen and Ostrom[18] call for efforts to develop methods that select from alternatives that fit data and are generalizable. As a result, a concerted effort has been applied to improve the defensibility of ABM through empirical validation of models, with examples in Marks,[82] Brown *et al.*,[62] and notably Robinson *et al.*,[28] Matthews *et al.*,[19] Windrum *et al.*,[83] Fagiolo *et al.*,[84] Moss,[85] and Janssen and Ostrom.[18]

The complex nature of ABM and its emergent properties at a system level make validation of models challenging, and some argue inherently impossible due to the irreducibility of emergent properties. Identification of underlying organization of ABMs has been hampered by the lack of an explicit strategy for coping with complexity and uncertainty. Consequently, model structure is often ad hoc,[48,59] and thus, the strengths of ABMs' flexibility to represents all manner of behaviors comes at a cost. Couclelis[86] asks whether the benefits of that flexibility exceed the considerable costs of the added dimensions of complexity, concluding that it likely does not.

The flexibility of ABM allows the modeler to use any number of parameters and functions, making it difficult to restrict the ranges model parameters based on empirical data.[87] Similarly, Grimm *et al.*[48] conclude that ABMs include too many degrees of freedom, and conceptual models may too much reflect the perspective of the observers without an understanding of specific interests, beliefs, and scales of perception.

The response of many modelers is to not attempt empirical validation. Moss[85] argues that complex systems are "volatile," and "soft" calibration with stakeholder knowledge is perhaps the best strategy. The problem with this approach is that if different stakeholders have different subjective understandings of the system, the model might be an accurate representation of some views but an inaccurate (though precise) representation of others.[85] Subjective understandings of systems are most likely incomplete, if not incorrect. For example, Abel *et al.*[88] show that mental models of graziers, scientists, and government rangeland managers are to some degree inconsistent. Given world views and prejudices change, there is no guarantee that modeling the same system with similar stakeholders at different times should result in consistent modeled outcomes. The difficulty is in collecting empirical data on a system level and identifying its underlying causes. Matthews *et al.*[19] argue that models can be used to organize knowledge from other studies and for developing rules of thumb, rather than be used as decision support systems.

Because of the difficult task of validating complex systems models and the response of many modelers to not sufficiently address validation, the field struggles with reputability, sometimes deserved. Bousquet and Le Page[29] find some model credibility lacking, and there have been problems reported in reimplementation and replication work.[89] Programming errors are not uncommon, especially initially when ABM software platforms were not user-friendly. Without a team of skilled computer scientists, early models were likely to contain unresolved bugs. Polhill *et al.*[90,91] highlight the dramatic changes in model outcomes that can arise from floating point errors. Rouchier[92] reports that a finding of a trading market ABM was not replicable. Gintis[93] goes further, suggesting that authors sometimes suffer from self-delusion, seeing emergent properties and explanations of model findings that can not be supported by operations actually occurring in the code, and listing the presence of significant problems with verification, let alone attempting validation of system-level outcomes.

These critiques can perhaps be viewed as birthing pains of a new methodology, and a certain amount of honest (and other) errors can be perhaps be expected. Nevertheless, a consistent effort has been make over time to improve the empirical validation of models. Janssen and Ostrom[18] report an increasing confidence in ABM as a valid technical methodology that can provide novel insights, particularly because relevant data are more available, and an increasing use of experiments in social sciences. To assist in communication and replicability of models, Grimm *et al.*[55] have proposed a standard protocol dubbed ODD, which has been used for ABMs of land-use change in Polhill *et al.*[65] Parker *et al.*[56] outline a method of communicating models, termed *Mr.Potatohead*, which also attempts to make models comparable and communication consistent.

The challenge is in validating emergent phenomena. A number of studies suggest using patterns or stylized facts of a system. Grimm *et al.*[48] suggest the use of patterns to guide model structure and reduce parameter uncertainty. First, alternate theories of agents' decision are formulated, and patterns at both individual and higher levels are identified. Theories are tested by how well they reproduce the patterns, rejecting those that fail to do so. Finally, additional patterns with more falsifiable power can be used to design experiments and analyze data. Similarly, Windrum *et al.*[83] suggest indirect calibration where stylized facts are identified and empirical experimental evidence about behavior and interactions is gathered. Model outcomes for stylized facts are compared to the evidence, and parameter sets are limited to those which reproduce the stylized facts.

Calibration techniques have been used where full parameter ranges are used for sets of initial conditions, and simulations are completed for all parameter configurations. Parameter sets that do not yield outcomes which match empirical realizations are discarded and the remaining parameter sets can be further filtered using expert witness. Another approach described uses multiple secondary sources to design agent decisions making functions and initial conditions likely to generate observed history. Historical data are compared with model outcomes to support selection of decision-making functions.[83]

Turning now to the calibration of ABM, increasingly, researchers are using multiple methods to calibrate models which are planned at the outset of the modeling exercise, including primary data collections and data drawn from existing secondary sources. These include surveys, semistructured interviews, existing data sources, such as GIS and census data, direct participant observation, role-playing games, and laboratory experiments. From the data gained through these media, statistical functions can be derived, and/or decision-making rules constructed. Heterogeneous types of agents can be described from empirical data and coded into models, as outlined in Valbuena *et al.*[94] Reviews of validation techniques for ABM are presented in Robinson *et al.*,[28] Matthews *et al.*,[19] Windrum *et al.*,[83] Fagiolo *et al.*,[84] Moss,[85] and Janssen and Ostrom.[18]

Surveys can gather information to derive individual or household behavioral models based on microeconomic theory, or to generate statistical descriptions of the attributes of agents. Brown and Robinson[16] and Heckbert *et al.*[30] use econometric estimates from survey data to design agent preference functions. To learn directly why people reveal behavior, semistructured interviews can be used to explore drivers behind dynamic decision making. Participant observation from anthropological techniques can capture in-depth information.[95] Assignment of data to the agent population is outlined in Berger and Schreinemachers,[96] which presents a way to parameterize ABMs using a common sampling frame to randomly select observation units for both biophysical measurements and socio-economic surveys, which are then extrapolated over the landscape based on probability functions. The resulting landscape and agent population are statistically consistent with empirical data.[96]

Participatory modeling using ABM has been conduced with success using the companion modeling technique, where stakeholders participate in model development through role-playing games laden with context. Participants play their roles while information is gathered to be used in developing the associated ABM.[78,97–100] Behavior functions are evaluated by stakeholders and transformed into rule-based agents in the model. This process can offer system-level awareness building and an opportunity to observe agent–agent interactions, but is limited by issues of objective knowledge of stakeholders.

Laboratory experiments and ABM each have much to gain through combined use. Agent behaviors can be calibrated from results of experiments with humans to create a population of agents whose behaviors are consistent with those revealed by the participants. Experiments can also help identify the type of decision-making strategy used in different contexts, whether decision makers optimize, use heuristics, learn, or imitate others. The use of combined ABM and experiments is limited, namely being conducted in Evans *et al.*,[70] Duffy and Unver,[23] Janssen *et al.*,[101] and Heckbert.[102] ABMs have been designed with the results of lab experiments in mind, and experiments can reveal behaviors at an individual level whose inclusion in ABM would have novel applications. Janssen *et al.*[101] uses laboratory experiments to examine endogenous rule changes from open access to private property as a solution to private versus public commons dilemmas, testing the rate at which participants invested in creating private property arrangements. ABMs have much to gain by using experiments to better select agent behaviors, calibrate decision-making functions based on revealed behaviors, and validate outcomes of ABMs against laboratory findings.

Laboratory experiments are generally highly abstract and controlled in order to test specific hypotheses of particular decision-making situations, as opposed to companion modeling which is context rich. Using laboratory experiments alongside other validation techniques can narrow into a specific decision-making function of agents, while leaving the surveys and interviews to gather more general information. Duffy and Unver[66] examine a trading market with a modified zero-intelligence agent,[103] used to explore patterns observed in trading laboratory experiments by generating asset price bubbles and crashes. Heckbert[102] presents an ABM with an integrated experimental economics interface. Human participants log in to a running ABM and take control of an artificial agent, making agricultural production and trading decisions. Participant decisions in response to information and production conditions are recorded to derive decision-making functions for calibrating artificial agents.

An example of using ABM and experiments alongside other calibration techniques is presented in Evans *et al.*,[70] examining transitions between reforestation and agricultural production. Remote sensing data are used to estimate agent preference parameters by regression, and surveys are used to collect social and demographic information. Only a weak relationship between income and reforestation was found using this approach, and other factors, such as learning, information, knowledge, risk aversion, and influence of social networks, were hypothesized to play a role, but were not captured in surveys.[60] Lab experiments were designed to further test hypotheses about decision making and resulting behaviors. The experiment assesses how people make allocation decisions between agricultural use and reforestation, and subjects are allocated areas to one of two land uses, receiving revenue according to an increasing price for one, and a decreasing price for the other. Considerable variance was found in behaviors of allocating land to each of the two uses, and where a "rational" decision maker would have changed land use, the majority of experiment participants took many rounds complete the reallocation, and some persisted in allocating to the disadvantaged option.[60] The overall macro-outcome of such participant decision making was to generate reforestation patterns with more "edge" compared to a model populated with agents who make fully rational choices.

Frontier 2: Linking macro-outcomes

The second research frontier identified is the ability to link emergent properties of ABMs to other modeling tools, including macroscopic patterns of other ABMs. In ABM related to ecological economics, there is a long history of representing interconnected human and environmental systems. In most cases, one—not both—is modeled as an ABM that operates against a "backdrop" of static equation-based models. Most ABMs of social-ecological systems have considered feedbacks on just one side of the system. For example, most land-use ABMs either consider the environment as fixed, or changing according to simple rules.[19]

ABMs have been linking with other techniques such as SD models, and appear as modules in larger system models, such as Miller *et al.*,[104] which model urban transport systems as series of modules, some of which run as ABMs. Scaling up from ABM is common, but few applications integrate with other modeling techniques other than SD and cellular automata. The use of ABM in SD can play the role of a "breakout model," which solves for certain emergent properties, such as demographic trends, stock market bubbles and crashes, and distributional effects of resources and information which can not be meaningfully represented at aggregated levels. This brings to light issues of model coupling, and Matthews *et al.*[19] propose three types: loose, tight, and integrated models. With the advent of flexible software, there is increasing movement toward the latter where once integration with other modeling techniques was prohibitively difficult. The argument can be made that ABM has not made significant inroads into planning, where SD models are the norm. However, a series of hybrid tools, such as Geertman *et al.*,[105] are beginning to use ABMs selectively alongside other techniques in so-called planning support systems. Nevertheless, the conditions under which ABM are required above other modeling tools will continue to be best placed where emergent properties are critical features of the system.

With increasing realization that the social ecological system acts as a coevolving system,[106] there is a need to further refine the one-sided complexity of ABMs. This perspective presents models of interaction between human decision making and nonhuman organisms and examines these systems as coevolving systems. The ABM approach may be useful for modeling coevolution between human activities and biological populations. For example, fish populations respond to selective human harvesting of large individuals by changing their life history to grow more rapidly and mature at smaller sizes.[107] Populations subject to consistent and strong "harvest selection" show particularly rapid and dramatic changes in phenotype compared to wild populations.[108] These changes may in turn impact human harvesting behavior, for instance, encouraging fishers to move to new areas, which may in turn reduce the selection pressure in the initial area. A better understanding of such feedbacks could prove useful for the prove useful for managing fishing, hunting, invasive species and weeds. In the case of hunting, there may be selection for "wariness" as more visible individuals are removed from populations. Some long-lived mammal species may also be able to learn to avoid areas used by hunters based on earlier encounters. The development of insecticide and pesticide resistance is another promising research area. Farmers spraying pest and weed populations create selection pressure for resistance genes, which can spread rapidly through populations. The build-up of resistance will depend in part on when and where in a landscape farmers are spraying. As resistance builds up, farmers may need to increase their rate of spraying in order to get the same result, which further enhances the development of resistance—the so-called pesticide treadmill.

ABM can represent feedbacks between macro-features of economic and ecological systems. Two-way feedbacks are discussed in Wu and Irwin[109] for SD models; however, the niche of ABM are emergent properties from interacting adaptive decision makers. This begs the challenge of having two-way feedbacks between emergent properties, of which examples appear to be rare if not absent. Linking macro-outcomes of agent-based models to other emergent macropatterns and into other modeling techniques offers an opportunity for ABM to contribute to specific questions in ecological economics. Ecological economics deals with the complex nature of social-environmental systems, and will continue to benefit from ABM as a tool to explicitly represent the interacting and adaptive elements of complex systems.

Conflicts of interest

The authors declare no conflicts of interest.

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