

Agents for Traffic Simulation

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Vehicular traffic is a classical example of a multi-agent system in which autonomous drivers operate in a shared environment. The article provides an overview of the state-of-the-art in microscopic traffic modeling and the implications for simulation techniques. We focus on the short-time dynamics of car-following models which describe continuous feedback control tasks (acceleration and braking) and models for discrete-choice tasks as a response to the surrounding traffic. The driving style of an agent is characterized by model parameters such as reaction time, desired speed, desired time gap, anticipation etc. In addition, internal state variables corresponding to the agent's "mind" are used to incorporate the driving experiences. We introduce a time-dependency of some parameters to describe the frustration of drivers being in a traffic jam for a while. Furthermore, the driver's behavior is externally influenced by the neighboring vehicles and also by environmental input such as limited motorization and braking power, visibility conditions and road traffic regulations. A general approach for dealing with discrete decision problems in the context of vehicular traffic is introduced and applied to mandatory and discretionary lane changes. Furthermore, we consider the decision process whether to brake or not when approaching a traffic light turning from green to amber. Another aspect of vehicular traffic is related to the heterogeneity of drivers. We discuss a hybrid system of coupled vehicle and information flow which can be used for developing and testing applications of upcoming inter-vehicle communication techniques.

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1 Introduction

Efficient transportation systems are essential to the functioning and prosperity of modern, industrialized societies. Mobility is also an integral part of our quality of life, sense of self-fulfillment and personal freedom. Our traffic demands of today are predominantly served by individual motor vehicle travel which is the primary means of transportation. However, the limited road capacity and ensuing traffic congestion has become a severe problem in many countries. Nowadays, we additionally have to balance the human desire for personal mobility with the societal concerns about its environmental impact and energy consumption. On the one hand, traffic demand can only be affected indirectly by means of policy measures. On the other hand, an extension of transport infrastructure is no longer an appropriate or desirable option in densely populated areas. Moreover, construction requires high investments and maintenance is costly in the long run. Therefore, engineers are now seeking solutions to the questions of how the capacity of the road network could be used more efficiently and how operations can be improved by way of intelligent transportation systems (ITS).

In the presence of increasing computing power, realistic microscopic traffic simulations are becoming a more and more important tool for diverse purposes ranging from generating surrounding traffic in a virtual reality driving simulator to large-scale network simulations for a model-based prediction of travel times and traffic conditions [Min07]. The primary application for traffic simulations is the evaluation of hypothetical scenarios for their impact on traffic. Computer simulations can be valuable in making these analyses in a cost-effective way. For example, simulations can be used to estimate the impact of future driver assistance systems and wireless communication technologies on traffic dynamics. Another example is the prediction of congestion levels in the future, based on demographic forecasts.

Before going into detail about possible traffic flow models, it is worth mentioning differences between modeling the short-term traffic dynamics on a single road section and the approach used for transportation planning describing behavioral pattern in a network on a larger time scale. Figure 1 shows typical time scales ranging over nine orders of magnitude including vehicle dynamics, traffic dynamics and transportation planning. While dynamic flow models explicitly describe the *physical propagation of traffic flows* of a given traffic volume in a road network, transportation planning tools deal with the calculation of the traffic demand by considering the *decisions of travelers* to participate in economical, social and cultural activities. The need for transportation arises because these activities are spatially separated. The classical approach in trip-based transportation models is based on a four-step methodology of *trip generation, trip distribution, mode split* and *traffic assignment* [OW01, SL97, Dag97, MM05, HN04]. In the fourth step, the origin-destination matrix of trips with a typical minimum disaggregation of one hour (comprising a typical peak-hour analysis) is assigned to routes in the actual (or prospective) transportation network while taking into account the limited capacity of the road infrastructure by means of simplified effective models. Recently, even large-scale multi-agent transportation simulations have been performed in which each traveler is represented individually [NER00, RCV⁺03, CN05]. For the purposes of demand-modeling, mobility-simulation and infrastructure re-planning the open-source software MATSIM provides a toolbox to implement large-scale agent-based transport simulations [MAT08].

Time Scale	Subject	Models	Aspects	
0.1 s	Vehicle Dynamics	Sub-microscopic	Drive-train, brake, ESP	
1 s			Reaction time, Time gap	
10 s	Traffic Dynamics	Car-following models	Accelerating and braking	
1 min			Fluid-dynamic models	Traffic light period
10 min				Period of stop-and-go wave
1 h	Transport. Planning	Traffic assignment models	Peak hour	
1 day			Traffic demand model	Day-to-day human behavior
1 year			Building measures	
5 years		Statistics	Changes in spatial structure	
50 years		Prognosis	Changes in Demography	

Table 1: Subjects in transportation systems sorted by typical time scales involved.

1.1 Aim and Overview

The chapter will review the state-of-the-art in microscopic traffic flow modeling and the implications for simulation techniques. In Sec. 2, we will introduce the concept of a *driver-vehicle agent* within in the context of common traffic modeling approaches.

In order to perform traffic simulations, we will take a “bottom-up” approach and present concrete models for describing the behavior of an agent. In Sec. 3.1, the Intelligent Driver Model [THH00] serves as a basic example of a car-following model representing the operational level of driving. As a first contribution, we will give special attention to the heterogeneity in traffic. Different drivers behave differently in the same situation (so called “inter-driver variability”) but can also change their behavior over the course of time (“intra-driver variability”). While the first aspect can be addressed by individual parameter sets for the agents (Sec. 3.2), the latter can be modeled by introducing a time-dependency of some parameters (e.g. to model the frustration of drivers after being in a traffic jam for a period, Sec. 3.3).

Realistic simulations of multi-lane freeway traffic and traffic in city networks also require discrete decisions by the agents. For example, lane-changing decisions allow faster cars to pass slower trucks. Another decision is related to the decision process of whether to brake or not to brake when approaching a traffic light turning from green to amber. In Sec. 4.1, we will introduce a general framework for dealing with these discrete decision processes. The presented “meta-model” MOBIL [KTH07] is an example of how complexity can in fact be reduced by falling back on the agent’s model for calculating longitudinal accelerations.

After having modeled the agent’s acceleration and lane-changing behavior, we will consider multi-agent simulations. Section 5 addresses the design of microscopic traffic simulators. In order to be specific, we will discuss the explicit numerical integration scheme, input and output quantities and visualization possibilities.

In Sec. 6, we will demonstrate the expressive power of the agent-based approach for handling current research questions. Traffic simulations will illustrate the emergence of collective dynamics from local interaction between agents. By way of example, we will show how the desired individual behavior of agents to move forward fast can lead to contrary effects such as the breakdown of traffic and self-organized stop-and-go waves. Another simulation will evaluate the effect of traffic flow homogenization by means of a speed limit (Sec. 6.2). Last but not least, we will discuss an application of inter-vehicle communication for propagating traffic-related

information in a decentralized way. Inter-vehicle communication has recently received much attention in the academic and engineering world as it is expected to be a challenging issue for the next generation of vehicle-based Intelligent Transportation Systems (ITS). Finally, in Sec. 7, we will discuss such trends in traffic modeling and simulation.

2 Agents for Traffic Simulation

Vehicular traffic is a typical example of a *multi-agent system*: Autonomous agents (the drivers) operate in a shared environment provided by the road infrastructure and react to the neighboring vehicles. Therefore, the activities include both *human interaction* (with the dominant influence originating from the directly leading vehicle) and *man-machine-interactions* (driver interaction with the vehicle and the physical road environment). The *microscopic modeling* or *agent-based* approach describing the motion of each individual vehicle has grown in popularity over the last decade. The following Sec. 2.1 will provide an overview of common mathematical approaches for describing traffic dynamics. In Sec. 2.2, we will introduce the concept of a “driver-vehicle agent” within the context of microscopic traffic modeling.

2.1 Macroscopic vs. Microscopic Approaches

The mathematical description of the dynamics of traffic flow has a long history already. The scientific activity had its beginnings in the 1930s with the pioneering studies on the fundamental relations of traffic flow, velocity and density conducted by Greenshields [Gre59]. By the 1950s, scientists had started to describing the physical propagation of traffic flows by means of dynamic macroscopic and microscopic models. During the 1990s, the number of scientists engaged in traffic modeling grew rapidly because of the availability of better traffic data and higher computational power for numerical analysis.

Traffic models have been successful in reproducing the observed *collective, self-organized traffic dynamics* including phenomena such as breakdowns of traffic flow, the propagation of stop-and-go waves (with a characteristic propagation velocity), the capacity drop, and different spatiotemporal patterns of congested traffic due to instabilities and nonlinear interactions [Hel01, Ker04, KR96a, CB99, DCB99, SH07]. For an overview of experimental studies and the development of miscellaneous traffic models, we refer to the recently published extensive review literature [Hel01, CSS00, Nag02, MD05, HB01, Leu88].

As mentioned, there are two major approaches to describe the spatiotemporal propagation of traffic flows. *Macroscopic traffic flow models* make use of the picture of traffic flow as a physical flow of a fluid. They describe the traffic dynamics in terms of aggregated macroscopic quantities like the traffic density, traffic flow or the average velocity as a function of space and time corresponding to partial differential equations (cf. Fig. 1). The underlying assumption of all macroscopic models is the conservation of vehicles (expressed by the continuity equation) which was initially considered by Lighthill, Whitham and Richards [LW55, Ric56]. More advanced, so-called “second-order” models additionally treat the macroscopic velocity as a dynamic variable in order to also consider the finite acceleration capability of vehicles [KK94, THH99].

By way of contrast, *microscopic traffic models* describe the motion of each individual vehicle. They model the action such as accelerations, decelerations and lane changes of each driver as a response to the surrounding traffic. Microscopic traffic models are especially suited to the

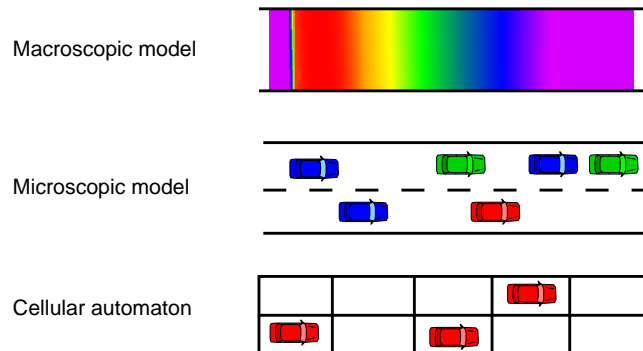


Figure 1: Illustration of different traffic modeling approaches: A snapshot of a road section at time t_0 is either characterized by *macroscopic traffic quantities* like traffic density $\rho(x, t_0)$, flow $Q(x, t_0)$ or average velocity $V(x, t_0)$, or, *microscopically*, by the positions $x_\alpha(t_0)$ of single driver-vehicle agent α . For cellular automata, the road is divided into cells which can be either occupied by a vehicle or empty.

study of heterogeneous traffic streams consisting of different and individual types of *driver-vehicle units* or *agents*. The result is individual trajectories of all vehicles and, consequently, any macroscopic information by appropriate aggregation. Specifically, one can distinguish the following major subclasses of microscopic traffic models (cf. Fig. 1):

- *Time-continuous models* are formulated as ordinary or delay-differential equations and, consequently, space and time are treated as continuous variables. *Car-following models* are the most prominent examples of this approach [BHN⁺95, THH00, JWZ01, TH98]. In general, these models are deterministic but stochasticity can be added in a natural way [TKH06b]. For example, a modified version of the Wiedemann model [Wie74] is used in the commercial traffic simulation software PTV-VISSIMTM.
- *Cellular automata* (CA) use integer variables to describe the dynamic state of the system. The time is discretized and the road is divided into cells which can be either occupied by a vehicle or empty. Besides rules for accelerating and braking, most CA models require additional stochasticity. The first CA for describing traffic was proposed by Nagel and Schreckenberg [NS92]. Although CA lack the accuracy of time-continuous models, they are able to reproduce some traffic phenomena [LBSK04, HS99, KSS01]. Due to their simplicity, they can be implemented very efficiently and are suited to simulating large road networks [Min07].
- *Iterated coupled maps* are between CA and time-continuous models. In this class of model, the update time is considered as an explicit model parameter rather than an auxiliary parameter needed for numerical integration [KT08b]. Consequently, the time is discretized while the spatial coordinate remains continuous. Popular examples are the Gipps model [Gip81] and the Newell model [New61]. However, these models are typically associated with car-following models as well.

At first glance, it may be surprising that simple (and deterministic) mathematical models aimed at describing the complexity of and variations in the human behavior, individual skills

and driving attitudes would lead to reasonable results. However, a traffic flow can (in a good approximation) be considered as a one-dimensional system (with reduced degrees of freedom). Furthermore, traffic models typically assume rational and safe driving behavior as a reaction to the surrounding traffic while at the same time taking into account the fundamental laws of kinematics.

Another aspect concerns the important issue of traffic safety. The traditional models for describing traffic dynamics assume rational drivers that are programmed to avoid collisions.¹ Therefore, traffic safety simulation belongs to the field of human centered simulation where the perception-reaction system of drivers with all its weak points has to be described. Up to now, a general modeling approach is still lacking.

2.2 Driver-Vehicle Agents

Let us now adopt the concept of an agent to implicate the complex human driving behavior into a general modeling framework. We therefore introduce the term *driver-vehicle agent* which refers to the idea that an atomic entity includes internal characteristics of human drivers as well as external properties of a vehicle. Figure 2 provides an overview of relevant influences and features affecting human driving. In this context, the relevant time scales are a first characteristic feature: The *short-term operations* are constituted by control tasks such as acceleration and braking, and typically take place in the range of a second. Specific *behavioral attributes* vary between individual drivers and affect the resulting driving characteristics on an intermediate time scale of seconds up to minutes. Finally, a strategic level of driving includes time periods of hours, e.g. the decision to start a trip or to find a route in a network.

The driving task can be considered as a cognitive and therefore *internal* process of an agent: The driver's perception is limited to the observable *external* objects in the neighborhood while his or her reaction is delayed due to a non-negligible reaction time as a consequence of the physiological aspects of sensing, perceiving, deciding, and performing an action. On the intermediate time scale, the agent's actions are affected by his or her individual driving behavior which may be characterized in terms of, e.g. preferred time gaps when following a vehicle and smooth driving with a desired acceleration and a comfortable deceleration. Moreover, the individual driving style may be influenced by the experience and history of driving. For example, it is observed that people change their behavior after being stuck in traffic congestion for a period [BP96, FSSe03]. Such features can be incorporated by internal state variables corresponding to the agent's "mind" or "memory".

However, short-time driving operations are mainly direct responses to the stimulus of the surrounding traffic. The driver's behavior is externally influenced by environmental input such as limited motorization and braking power of the vehicle, visibility conditions, road characteristics such as horizontal curves, lane narrowings, ramps, gradients and road traffic regulations. In the following sections, we will address a number of these defining characteristics of a driver-vehicle agent.

¹Of course, collisions happen in numerical simulations due to instable models and for kinematic reasons. However, these collisions do not have explanatory or predictive power.

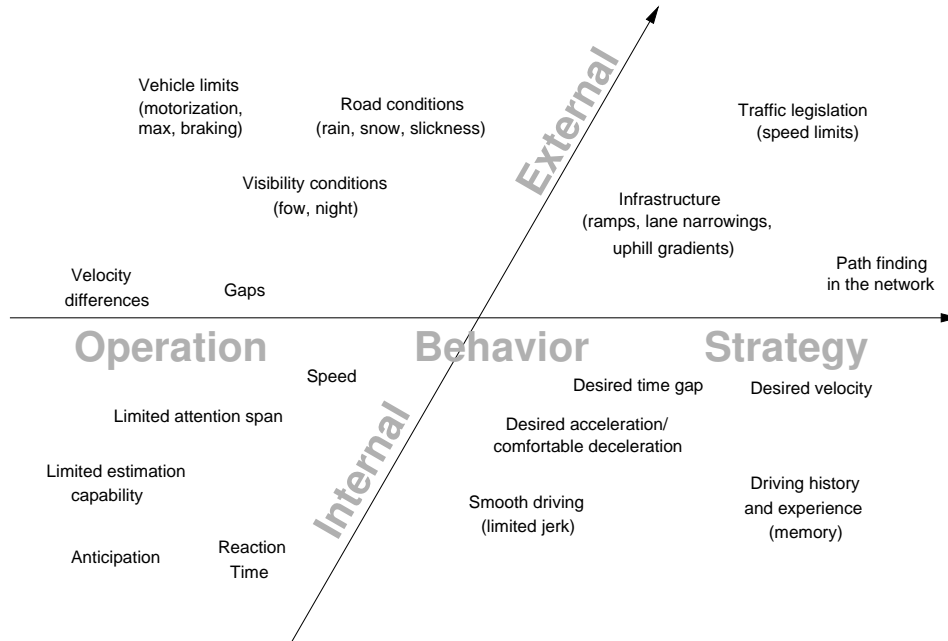


Figure 2: Characteristics of a driver-vehicle agent. The operation of driving can be classified according the involved time scales ranging from short-term actions in terms of acceleration and braking via intermediate time scales describing behavioral characteristics to long-term strategic decisions. In addition, the agent’s behavior is influenced by the physical properties of the vehicle, by interactions with other agents and by the environment.

3 Models for the Driving Task

Microscopic traffic models describe the motion in longitudinal direction of each individual vehicle. They model the action of a driver such as accelerations and decelerations as a response to the surrounding traffic by means of an *acceleration strategy* towards a desired speed in the free-flow regime, a *braking strategy* for approaching other vehicles or obstacles, and a *car-driving strategy for maintaining a safe distance* when driving behind another vehicle. Microscopic traffic models typically assume that human drivers react to the stimulus from neighboring vehicles with the dominant influence originating from the directly leading vehicle known as “follow-the-leader” or “car-following” approximation.

By way of example, we will consider the Intelligent Driver Model (IDM) [THH00] in Sec. 3.1. The IDM belongs to the class of deterministic follow-the-leader models. Like other car-following models, the IDM is formulated as an ordinary differential equation and, consequently, space and time are treated as continuous variables. This model class is characterized by an acceleration function $\dot{v} := \frac{dv}{dt}$ that depends on the actual speed $v(t)$, the gap $s(t)$ and the velocity difference $\Delta v(t)$ to the leading vehicle (see Fig. 3). Note that the dot is the usual shorthand notation for the time derivative of a function. The acceleration is therefore defined as the time derivative of the velocity $\dot{v} := dv/dt$.

In Sec. 3.2, we will model inter-driver variability by defining different classes of drivers which is an inherent feature of microscopic agent approaches. A model for intra-driver variability

(changing behavior over the course of time) will then be discussed in Sec. 3.3.

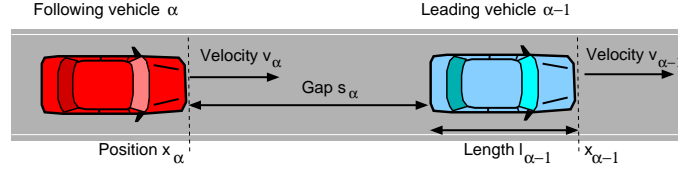


Figure 3: Illustration of the input quantities of a car-following model: The bumper-to-bumper distance s for a vehicle α with respect to the vehicle $(\alpha - 1)$ in front is given by $s_\alpha = x_{\alpha-1} - x_\alpha - l_{\alpha-1}$, where l_α is the vehicle length and x the position on the considered road stretch. The approaching rate (relative speed) is defined by $\Delta v_\alpha := v_\alpha - v_{\alpha-1}$. Notice that the vehicle indices α are ordered such that $(\alpha - 1)$ denotes the preceding vehicle.

3.1 The Intelligent Driver Model

The IDM acceleration is a continuous function incorporating different driving modes for all velocities in freeway traffic as well as city traffic. Besides the distance to the leading vehicle s and the actual speed v , the IDM also takes into account velocity differences Δv , which play an essential stabilizing role in real traffic, especially when approaching traffic jams and avoiding rear-end collisions (see Fig. 3). The IDM acceleration function is given by

$$\frac{dv_\alpha}{dt} = f(s_\alpha, v_\alpha, \Delta v_\alpha) = a \left[1 - \left(\frac{v_\alpha}{v_0} \right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \right)^2 \right]. \quad (1)$$

This expression combines the acceleration strategy $\dot{v}_{\text{free}}(v) = a[1 - (v/v_0)^\delta]$ towards a *desired speed* v_0 on a free road with the parameter a for the *maximum acceleration* with a braking strategy $\dot{v}_{\text{brake}}(s, v, \Delta v) = -a(s^*/s)^2$ serving as repulsive interaction when vehicle α comes too close to the vehicle ahead. If the distance to the leading vehicle, s_α , is large, the interaction term \dot{v}_{brake} is negligible and the IDM equation is reduced to the free-road acceleration $\dot{v}_{\text{free}}(v)$, which is a decreasing function of the velocity with the maximum value $\dot{v}(0) = a$ and the minimum value $\dot{v}(v_0) = 0$ at the desired speed v_0 . For denser traffic, the deceleration term becomes relevant. It depends on the ratio between the effective “desired minimum gap”

$$s^*(v, \Delta v) = s_0 + vT + \frac{v\Delta v}{2\sqrt{ab}}, \quad (2)$$

and the actual gap s_α . The *minimum distance* s_0 in congested traffic is significant for low velocities only. The main contribution in stationary traffic is the term vT which corresponds to following the leading vehicle with a constant *desired time gap* T . The last term is only active in non-stationary traffic corresponding to situations in which $\Delta v \neq 0$ and implements an “intelligent” driving behavior including a braking strategy that, in nearly all situations, limits braking decelerations to the *comfortable deceleration* b . Note, however, that the IDM brakes stronger than b if the gap becomes too small. This braking strategy makes the IDM collision-free [THH00]. All IDM parameters v_0 , T , s_0 , a and b are defined by positive values. These parameters have a reasonable interpretation, are known to be relevant, are empirically

measurable and have realistic values [KT08a]. We will discuss parameter values in detail in Sec. 3.2 and will use their clear meaning to characterize different driving styles, that is, inter-driver variability.

For a simulation scenario with a speed limit (which we will study in Sec. 6.2), we consider a refinement of the IDM for the case when the actual speed is higher than the desired speed, $v > v_0$. For example, an excess of $v = 2v_0$ would lead to an unrealistic braking of $-15a$ for $\delta = 4$. This situation may occur when simulating, e.g. a speed limit on a road segment that reduces the desired speed locally. Therefore, we replace the free acceleration for the case $v > v_0$ by

$$\dot{v}_{\text{free}}(v) = -b \left[1 - \left(\frac{v_0}{v} \right)^\delta \right]. \quad (3)$$

That is, the IDM vehicle brakes with the comfortable deceleration b in the limit $v \gg v_0$. Further extensions of the IDM can be found in Refs. [TKH06a, KT08b, TKH06b].

The *dynamic properties* of the IDM are controlled by the maximum acceleration a , the acceleration exponent δ and the parameter for the comfortable braking deceleration b . Let us now consider the following scenario: If the distance s is large (corresponding to the situation of a nearly empty road), the interaction \dot{v}_{brake} is negligible and the IDM equation (1) is reduced to the free-road acceleration $\dot{v}_{\text{free}}(v)$. The driver accelerates to his or her desired speed v_0 with the maximum acceleration $\dot{v}(0) = a$. The acceleration exponent δ specifies how the acceleration decreases when approaching the desired speed. The limiting case $\delta \rightarrow \infty$ corresponds to approaching v_0 with a constant acceleration a while $\delta = 1$ corresponds to an exponential relaxation to the desired speed with the relaxation time $\tau = v_0/a$. In the latter case, the free-traffic acceleration is equivalent to that of the Optimal Velocity Model [BHN⁺95]. However, the most realistic behavior is expected between the two limiting cases of exponential acceleration (for $\delta = 1$) and constant acceleration (for $\delta \rightarrow \infty$). Therefore, we set the acceleration exponent constant to $\delta = 4$.

In Fig. 4, acceleration periods from a standstill to the desired speed $v_0 = 120$ km/h are simulated for two different settings of the maximum acceleration (the other model parameters are listed in the caption): For $a = 1.4$ m/s², the acceleration phase takes approximately 40 s while an increased maximum acceleration of $a = 3$ m/s² reduces the acceleration period to ~ 15 s. Notice that the acceleration parameter a of 1.4 m/s² (3 m/s²) corresponds to a free-road acceleration from $v = 0$ to $v = 100$ km/h within 23 s (10.5 s).

The *equilibrium properties* of the IDM are influenced by the parameters for the desired time gap T the desired speed v_0 and the minimum distance between vehicles at a standstill s_0 . Equilibrium traffic is defined by vanishing speed differences and accelerations of the driver-vehicle agents α :

$$\Delta v_\alpha = 0, \quad (4)$$

$$\frac{dv_\alpha}{dt} = 0, \quad (5)$$

$$\text{and } \frac{dv_{\alpha-1}}{dt} = 0. \quad (6)$$

Under these stationary traffic conditions, drivers tend to keep a velocity-dependent *equilibrium gap* $s_e(v_\alpha)$ to the leading vehicle. In the following, we consider a homogeneous ensemble of

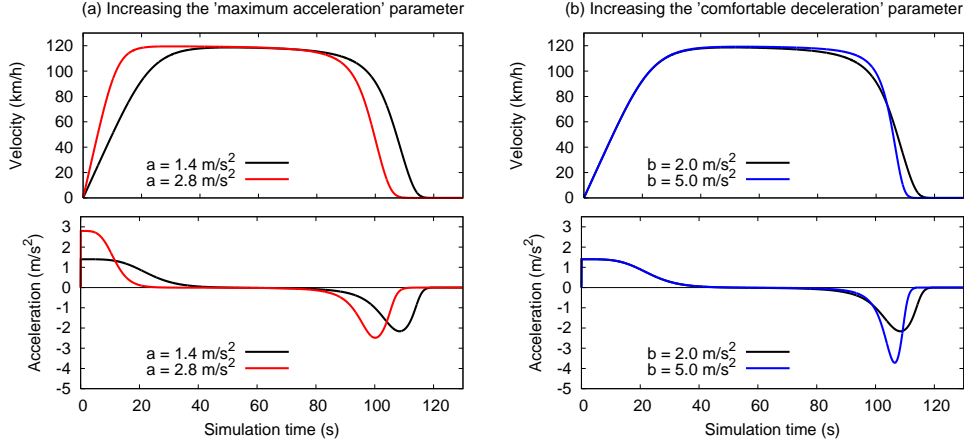


Figure 4: Simulation of a single driver-vehicle agent modeled by the IDM: The diagrams show the acceleration to the desired speed $v_0 = 120$ km/h followed by braking as a reaction to a standing obstacle located 3000 m ahead for several combinations of the IDM acceleration parameters a [in diagram (a)] and b [in (b)]. The remaining parameters are $a = 1.4$ m/s², $b = 2.0$ m/s², $T = 1.5$ s, $s_0 = 2$ m.

identical driver-vehicle agents corresponding to identical parameter settings. Then, the IDM acceleration equation (1) with the constant setting $\delta = 4$ simplifies to

$$s_e(v) = \frac{s_0 + vT}{\sqrt{1 - \left(\frac{v}{v_0}\right)^4}}. \quad (7)$$

The equilibrium distance depends only on the minimum jam distance s_0 , the safety time gap T and the desired speed v_0 . The diagrams (a) and (b) in Fig. 5 show the equilibrium distance as a function of the velocity, $s_e(v)$, for different v_0 and T parameter settings while keeping the minimum distance constant at $s_0 = 2$ m. In particular, the equilibrium gap of homogeneous *congested* traffic (with $v \ll v_0$) is essentially equal to the desired gap, $s_e(v) \approx s^*(v, 0) = s_0 + vT$. It is therefore composed of the minimum bumper-to-bumper distance s_0 kept in stationary traffic at $v = 0$ and an additional velocity-dependent contribution vT corresponding to a constant safety time gap T as shown in the diagrams by straight lines. For $v \rightarrow 0$, the equilibrium distance approaches the minimum distance s_0 . If the velocity is close to the desired speed, $v \approx v_0$, the equilibrium distance s_e is clearly larger than the distance vT according to the safety time gap parameter. For $v \rightarrow v_0$, the equilibrium distance diverges due to the vanishing denominator in Eq. (7). That is, the free speed is reached *exactly* only on a free road.

In the literature, the equilibrium state of homogeneous and stationary traffic is often formulated in macroscopic quantities such as traffic flow Q , (local) average velocity V and traffic density ρ . The translation from the microscopic net distance s into the density is given by the *micro-macro relation*

$$s = \frac{1}{\rho} - l, \quad (8)$$

where l is the vehicle length. In equilibrium traffic, ρ is therefore given by s_e , the mean velocity

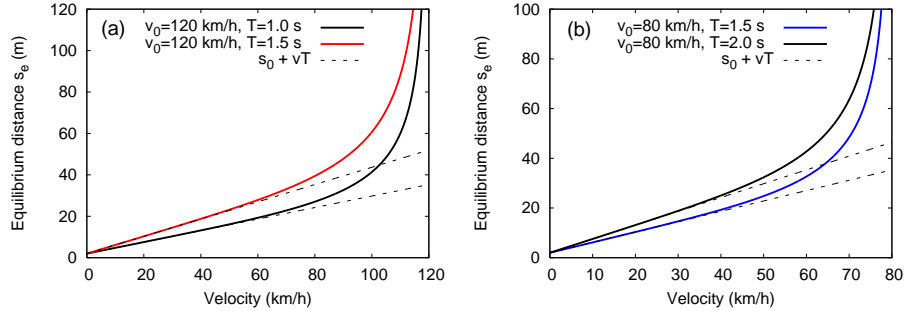


Figure 5: Equilibrium distance $s_e(v)$ according to Eq. (7) as functions of the speed for different settings of the desired speed v_0 and the safety time gap T . The deviations from the dotted lines are discussed in the main text. The other parameters are those listed in the caption of Fig. 4.

is simply $V = v_e$ and the traffic flow follows from the hydrodynamic relation

$$Q = \rho V. \quad (9)$$

So, the equilibrium velocity v_e is needed as a function of the distance s_e . An analytical expression for the inverse of Eq. (7), that is the *equilibrium velocity* as a function of the gap, $v_e(s)$, is only available for the acceleration exponents $\delta = 1, 2$ or $\delta \rightarrow \infty$ [THH00]. For $\delta = 4$, we only have a parametric representation $\rho(v)$ with $v \in [0, v_0]$ resulting from Eqs. (8) and (7). Figures 6(a) and (b) show the equilibrium velocity-density relation $V_e(\rho)$ for the same parameter settings as in Fig. 5. The assumed vehicle length $l = 5$ m together with the minimum jam distance $s_0 = 2$ m results in a maximum density $\rho_{\max} = 1/(s_0 + l) \approx 143$ vehicles/km. Using the relation (9), we obtain the so-called *fundamental diagram* between the traffic flow and the vehicle density, $Q(\rho) = V\rho(v)$ which is displayed in Fig. 6(c) and (d). Notice that Q is typically given in units of vehicles per hour and the density ρ in units of vehicles per km.

According to Eqs. (7) and (8), the *fundamental relations* of homogeneous traffic depend on the desired speed v_0 (low density), the safety time gap T (high density) and the jam distance s_0 (jammed traffic). In the low-density limit $\rho \ll 1/(v_0 T)$, the equilibrium flow can be approximated by $Q \approx v_0 \rho$. In the high density regime, one has a linear decrease of the flow,

$$Q(\rho) \approx \frac{1 - \rho(l + s_0)}{T}, \quad (10)$$

which can be used to determine the effective length $l + s_0$ and T . Notice that the vehicle length is not a model parameter but only a scaling quantity that determines the (static) maximum density ρ_{\max} together with the IDM parameter s_0 .

3.2 Inter-Driver Variability

An important aspect of vehicular traffic is the heterogeneity of agents, a term which includes characteristics of the drivers as well as features of the vehicle. The multi-agent simulation approach is appropriate for representing this heterogeneity on a microscopic level. In order to address *inter-driver variability* (different drivers behave differently in identical traffic situations) and vehicle properties (such as length, width, weight and motorization) we propose to

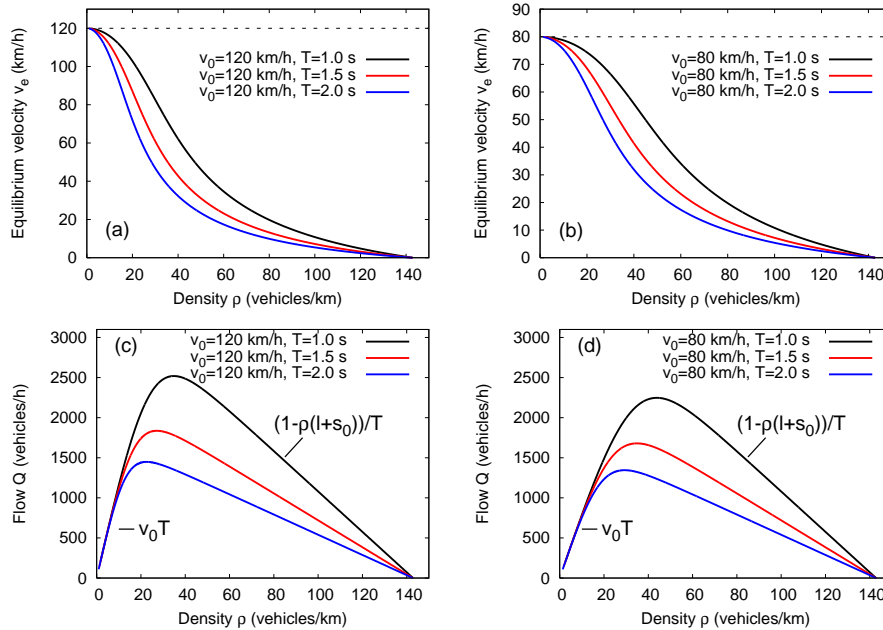


Figure 6: Equilibrium velocity-density relations of the IDM (top) and corresponding flow-density relations, so-called *fundamental diagrams* (bottom). The equilibrium properties depend on the minimum distance s_0 (here set to 2 m), the desired speed v_0 (here displayed for 120 and 80 km/h) and the time gap T (here 1.0, 1.5 and 2.0 s). The safety time gap is the most important parameter determining the maximum flow (stationary freeway capacity).

group driver-vehicle agents into classes defining their specific driving styles and vehicle properties. For this purpose, it is advantageous that the parameters of the Intelligent Driver Model do have an intuitive meaning and are directly related to driving behavior. In the following, we discuss the parameter settings for three classes of passenger car drivers representing “normal”, “timid” and “aggressive” driving styles. In addition, we model a typical truck driver. The corresponding parameter values are listed in Table 2.

- The *desired speed* v_0 is the maximum speed a driver-vehicle agent aims to reach under unobstructed driving conditions. A natural value and upper limit for this parameter would be the typical (highest) speed on the considered road element. The normal driver chooses for instance 120 km/h on a freeway while a timid driver prefers a lower value and a more aggressive driver likes to go faster. The desired speed could be limited by legislation. In city traffic, the speed is typically limited to 50 km/h (cf. the simulation scenario in Sec. 4.2). In this case, a timid driver likes to drive a bit below this limit while an aggressive driver can easily be modeled by an individual “disobedience factor”. Notice that strict speed limits apply to trucks on the whole road network in most countries.
- The *desired time gap* T refers to the preferred distance vT while driving at speed v , cf. Eq. (2), and mainly determines the maximum capacity (cf. Fig. 6). A typical value in dense traffic is about 1.4 s while German road authorities recommend 1.8 s. A common observation on European freeways is that very small time gaps are kept [TKH06b, KSS02].

IDM Parameter	Normal	Timid	Aggressive	Truck
Desired speed v_0 in km/h	120	100	140	85
Desired time gap T in s	1.5	1.8	1.0	2.0
Jam distance s_0 in m	2.0	4.0	1.0	4.0
Maximum acceleration a in m/s^2	1.4	1.0	2.0	0.7
Desired deceleration b in m/s^2	2.0	1.0	3.0	2.0

Table 2: Model parameters of the Intelligent Driver Model for three classes of passenger car drivers and a typical truck driver.

- The parameter s_0 describes the *minimum bumper-to-bumper distance* at a standstill, cf. Eq. (2). Typical gaps in a queue of vehicles standing at traffic lights are in the range between 1 m and 5 m. While a normal driver typically keeps a minimum gap of 2 m, a cautious driver prefers larger gaps and an aggressive driver likes tailgating. It is natural to assume that truck drivers prefer slightly larger gap than the normal car driver due to larger vehicle dimensions. Notice that the vehicle length is not a model parameter. However, it determines the maximum density together with the minimum distance s_0 according to Eq. (8). Typical vehicle lengths are for instance 5 m for cars and 12 m for trucks.
- The *desired acceleration* a describes the acceleration behavior of the driver. Notice that the acceleration depends on the actual vehicle speed as shown, for example, in Fig. 4. Since the acceleration *behavior* is based on a physical movement, the value of a has to respect the limits of motorization. Consequently, a truck has to be modeled by a lower desired acceleration a than a passenger car. An aggressive driver prefers to accelerate fast (e.g. 3 m/s^2) while a timid driver prefers a lower value (e.g. 1 m/s^2). The acceleration exponent $\delta = 4$ is kept constant for all driver classes, cf. Eq. (1).
- The *comfortable braking deceleration* b determines the approaching process toward slower leaders or stationary objects such as traffic lights (see Sec. 4.2). As the IDM tries to limit the braking deceleration to b , a low value ($b = 1 \text{ m/s}^2$) represents a driver who breaks accurately in an anticipative way corresponding to a smooth driving style. By way of contrast, a higher value ($b = 3 \text{ m/s}^2$) describes an aggressive driver who prefers to approach the leader with a large velocity difference.

Taking these average parameters for each driver class as a starting point, it is straightforward to distribute individual agent parameters randomly within given limits, e.g. according to a uniform distribution with a variation of 20%.

3.3 Intra-Driver Variability

Besides reacting to the immediate traffic environment, human drivers adapt their driving style on longer time scales to the traffic situation. Thus, the actual driving style depends on the traffic conditions of the last few minutes which we call *memory effect* [TH03]. For example, it is observed that most drivers increase their preferred temporal headway after being stuck in congested traffic for some time [BP96, FSSe03]. Furthermore, when larger gaps appear or

when reaching the downstream front of the congested zone, human drivers accelerate less and possibly decrease their desired speed as compared to a free-traffic situation.

In contrast to inter-driver variability considered in Sec. 3.2, the memory effect is an example of *intra-driver variability* meaning that a driver behaves differently in similar traffic situations depending on his or her individual driving history and experience. Again, the multi-agent approach can easily cope with this extension of driving behavior as soon as one has a specific model to implement. By way of example, we present a model that introduces a time-dependency of some parameters of the Intelligent Driver Model to describe the *frustration* of drivers being in a traffic jam for a period [TH03].

We assume that adaptations of the driving style are controlled by a single internal dynamical variable $\lambda(t)$ that represents the “subjective level of service” ranging from 0 (in a standstill) to 1 (on a free road). The subjective level of service $\lambda(t)$ relaxes to the instantaneous level of service $\lambda_0(v)$ depending on the agent’s speed $v(t)$ with a relaxation time τ according to

$$\frac{d\lambda}{dt} = \frac{\lambda_0(v) - \lambda}{\tau}. \quad (11)$$

This means that for each driver, the subjective level of service is given by the exponential moving average of the instantaneous level of service experienced in the past:

$$\lambda(t) = \int_0^t \lambda_0(v(t')) e^{-(t-t')/\tau} dt'. \quad (12)$$

We have assumed the instantaneous level of service $\lambda_0(v)$ to be a function of the actual velocity $v(t)$. Obviously, $\lambda_0(v)$ should be a monotonically increasing function with $\lambda_0(0) = 0$ and $\lambda_0(v_0) = 1$ when driving with the desired speed v_0 . The most simple “level-of-service function” satisfying these conditions is the linear relation

$$\lambda_0(v) = \frac{v}{v_0}. \quad (13)$$

Notice that this equation reflects the level of service or efficiency of movement from the agent’s point of view, with $\lambda_0 = 1$ meaning zero hindrance and $\lambda_0 = 0$ meaning maximum hindrance. If one models inter-driver variability (Sec. 3.2) where different drivers have different desired velocities, there is no objective level of service, but rather only an individual and an average one.

Having defined how the traffic environment influences the degree of adaptation $\lambda(t)$ of each agent, we now specify how this internal variable influences driving behavior. A behavioral variable that is both measurable and strongly influences the traffic dynamics is the desired time gap T of the IDM. It is observed that, in congested traffic, the whole distribution of time gaps is shifted to the right when compared with the data of free traffic [TH03, TKH06b]. We model this increase by varying the corresponding IDM parameter in the range between T_0 (free traffic) and $T_{\text{jam}} = \beta_T T_0$ (traffic jam) according to

$$T(\lambda) = \lambda T_0 + (1 - \lambda) T_{\text{jam}} = T_0 [\beta_T + \lambda(1 - \beta_T)]. \quad (14)$$

Herein, the adaptation factor β_T is a model parameter. A value for the frustration effect is $\beta_T = T_{\text{jam}}/T_0 = 1.8$ which is consistent with empirical observations. A typical relaxation

time for the driver's adaptation is $\tau = 5$ min. Notice that other parameters of the driving style are probably influenced as well, such as the acceleration a , the comfortable deceleration b or the desired velocity v_0 . This could be implemented by analogous equations for these parameters. Furthermore, other adaption processes as well as the presented frustration effect are also relevant [TKH06b].

4 Modeling Discrete Decisions

On the road network, drivers encounter many situations where a decision between two or more alternatives is required. This relates not only to lane-changing decisions but also to considerations as to whether or not it is safe to enter the priority road at an unsignalized junction, to cross such a junction or to start an overtaking maneuver on a rural road. Another question concerns whether or not to stop at an amber-phase traffic light. All of the above problems belong to the class of *discrete-choice problems* that, since the pioneering work of McFadden [HM84], has been extensively investigated in an economic context as well as in the context of transportation planning. In spite of the relevance to everyday driving situations, there are fewer investigations attempting to incorporate the aforementioned discrete-choice tasks into microscopic models of traffic flow, and most of them are restricted to modeling lane changes [Gip86]. Only very recently acceleration and discrete-choice tasks have been treated more systematically [TKBA07, KTH07].

The modeling of lane changes is typically considered as a multi-step process. On a *strategic* level, the driver knows about his or her route on the network which influences the lane choice, e.g. with regard to lane blockages, on-ramps, off-ramps or other mandatory merges [TCBA05]. In the *tactical* stage, an intended lane change is prepared and initiated by advance accelerations or decelerations of the driver, and possibly by cooperation of drivers in the target lane [Hid05]. Finally, in the *operational* stage, one determines if an immediate lane change is both safe and desired [Gip86]. While mandatory changes are performed for strategic reasons, the driver's motivation for discretionary lane changes is a perceived improvement of the driving conditions in the target lane compared with the current situation.

In the following, we will present a recently formulated general framework for modeling traffic-related discrete-choice situations in terms of the acceleration function of a longitudinal model [KTH07]. For the purpose of illustration, we will apply the concept to model mandatory and discretionary lane changes (Sec. 4.1). Furthermore, we will consider the decision process whether or not to brake when approaching a traffic light turning from green to amber (Sec. 4.2).

4.1 Modeling Lane Changes

Complementary to the longitudinal movement, lane-changing is a required ingredient for simulations of multi-lane traffic. The realistic description of multi-agent systems is only possible within a multi-lane modeling framework allowing faster driver-vehicle agents to improve their driving conditions by passing slower vehicles.

When considering a lane change, a driver typically makes a trade-off between the expected own advantage and the disadvantage imposed on other drivers. For a driver considering a lane change, the subjective utility of a change increases with the gap to the new leader in the target lane. However, if the speed of this leader is lower, it may be favorable to stay in the

present lane despite the smaller gap. A criterion for the utility including *both* situations is the difference between the accelerations after and before the lane change. This is the core idea of the lane-changing algorithm MOBIL [KTH07] that is based on the expected (dis)advantage in the new lane in terms of the difference in the acceleration which is calculated with an underlying microscopic longitudinal traffic model, e.g. the Intelligent Driver Model (Sec. 3.1).

For the lane-changing decision, we first consider a *safety constraint*. In order to avoid accidents by the follower in the prospective target lane, the safety criterion

$$\dot{v}_{\text{follow}} \geq -b_{\text{safe}} \quad (15)$$

guarantees that the deceleration of the successor \dot{v}_{follow} in the target lane does not exceed a safe limit $b_{\text{safe}} \simeq 4 \text{ m/s}^2$ after the lane change. In other words, the safety criterion essentially restricts the deceleration of the lag vehicle on the target lane to values below b_{safe} . Although formulated as a simple inequality, this condition contains all the information provided by the longitudinal model via the acceleration \dot{v}_{follow} . In particular, if the longitudinal model has a built-in sensitivity with respect to *velocity differences* (such as the IDM) this dependence is transferred to the lane-changing decisions. In this way, larger gaps between the following vehicle in the target lane and the own position are required to satisfy the safety constraint if the speed of the following vehicle is higher than the own speed. In contrast, lower values for the gap are allowed if the back vehicle is slower. Moreover, by formulating the criterion in terms of safe braking decelerations of the longitudinal model, crashes due to lane changes are *automatically* excluded as long as the longitudinal model itself guarantees crash-free dynamics.

For discretionary lane changes, an additional *incentive criterion* favors lane changes whenever the acceleration in one of the target lanes is higher. The incentive criterion for a lane change is also formulated in terms of accelerations. A lane change is executed if the sum of the own acceleration and those of the affected neighboring vehicle-driver agent is higher in the prospective situation than in the current local traffic state (and if the safety criterion (15) is satisfied of course). The innovation of the MOBIL framework [KTH07] is that the immediately affected neighbors are considered by the “politeness factor” p . For an egoistic driver corresponding to $p = 0$, this incentive criterion simplifies to $\dot{v}_{\text{new}} > \dot{v}_{\text{old}}$. However, for $p = 1$, lane changes are only carried out if this increases the combined accelerations of the lane-changing driver and all affected neighbors. This strategy can be paraphrased by the acronym “*Minimizing Overall Braking Induced by Lane Changes*” (MOBIL). We observed realistic lane-changing behavior for politeness parameters in the range $0.2 < p < 1$ [KTH07]. Additional restrictions can easily be included. For example, the “keep-right” directive of most European countries is implemented by adding a bias to the incentive criterion. A “keep-lane” behavior is modeled by an additional constant threshold when considering a lane change.

4.2 Approaching a Traffic Light

When approaching a traffic light that switches from green to amber, a decision has to be made whether to stop just at the traffic light or to pass the amber-phase light with unchanged speed. For an empirical study on the stopping/running decision at the onset of an amber phase we refer to Ref. [RESS07]. If the first option is selected, the traffic light will be modeled by a standing “virtual” vehicle at the position of the light. Otherwise, the traffic light will be ignored. The criterion is satisfied for the “stop at the light” option if the own braking deceleration at the

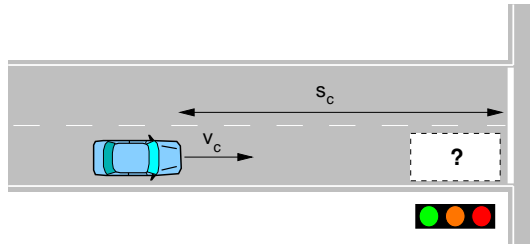


Figure 7: Approaching a traffic switching from green to amber. The two options of the decision situation are to stop in front of the light or to pass the amber-phase traffic light with unchanged speed.

time of the decision does not exceed the safe deceleration b_{safe} . The situation is illustrated in Fig. 7. Denoting the distance to the traffic light by s_c and the velocity at decision time by v_c and assuming a longitudinal model of the form (1), the safety criterion (15) can be written as

$$\dot{v}(s_c, v_c, v_c) \geq -b_{\text{safe}}. \quad (16)$$

Notice that the approaching rate and the velocity are equal ($\Delta v_c = v_c$) in this case. The incentive criterion is governed by the bias towards the stopping decision because legislation requires that one stop at an amber-phase traffic light if it is safe to do so. As a consequence, the incentive criterion is always fulfilled, and Eq. (16) is the only decision criterion in this situation.

Similarly to the lane-changing rules, the “stopping criterion” (16) will inherit all the sophistication of the underlying car-following model. In particular, when using realistic longitudinal models, one obtains a realistic stopping criterion with only one additional parameter b_{safe} . Conversely, unrealistic microscopic models such as the Optimal Velocity Model [BHN⁺95] or the Nagel-Schreckenberg cellular automaton [NS92] will lead to unrealistic stopping-decisions. In the case of the Optimal Velocity Model, it is not even guaranteed that drivers deciding to stop will be able to stop at the lights.

For the purpose of illustration, we apply the concept to the following situation in city traffic: A car is driving at speed $v_c = 50$ km/h towards an amber traffic light located at a distance $s_c = 50$ m. Applying the IDM parameters of a “normal” driver listed in Table 2 in combination with an adapted desired speed of $v_0 = 50$ km/h, the acceleration function (1) results in an initial braking of $\dot{v}(0) \approx 3.6$ m/s² at $t = 0$ s. For a safe deceleration equal to the desired deceleration of the IDM, that is $b_{\text{safe}} = b = 2.0$ m/s², the MOBIL decision says “drive on”. If, however, a safe braking deceleration of $b_{\text{safe}} = 4$ m/s² is assumed, the driver agent would decide to brake resulting in the approaching maneuver shown in Fig. 8. The initial braking stronger than -2 m/s² makes the situation manageable for the agent. After 2 s, the situation is “under control” and the vehicle brakes approximately with the comfortable deceleration $b = 2$ m/s². In order to reach a standstill in a smooth way, the deceleration is reduced to limit the *jerk* which defines the change in the acceleration. In addition, Fig. 8 shows the behavior of the second vehicle following the leader. The acceleration time series shows the important feature of the IDM in limiting braking decelerations to the comfortable limit b as long as safety is warranted. From these results it is obvious that the setting $b_{\text{safe}} = b$ is a natural assumption to model the decision process realistically. Notice, however, that a human reaction time of about 1 s [Gre00] has to be taken into account as well.

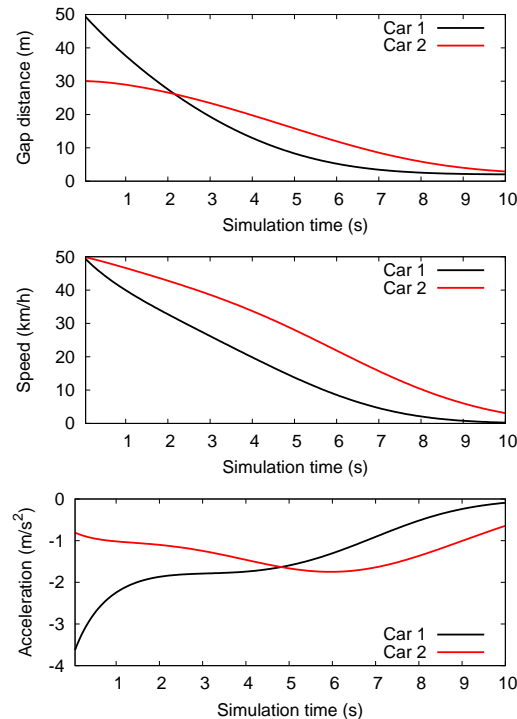


Figure 8: Maneuver of approaching a traffic light initially 50 m with a speed of 50 km/h according to the Intelligent Driver Model. The braking deceleration is limited to the comfortable braking deceleration (IDM parameter b) whenever possible. The stronger braking of the first car is needed to keep the situation under control. The parameters for the simulation are listed in Table 2.

5 Microscopic Traffic Simulation Software

So far, we have discussed models describing the longitudinal movement and discrete decisions of individual driver-vehicle agents. Let us now address the issue of a simulation framework that integrates these components into a microscopic multi-lane traffic simulator. Typical relations among functions in a microscopic traffic simulator are shown in Fig. 9. On the level of input data, simulation settings can be provided by input files, e.g. encoded in XML, by command line or via a graphical user interface (GUI). The main simulation loop is organized by a *Simulation Controller* which keeps track of the program operations and user actions. This central control unit calls the update methods of the road-section objects. We will elaborate on these components in Sec. 5.1. Since the calculation of the vehicle accelerations is the very core of a traffic simulation, we will pinpoint the issue in Sec. 5.2. Simulation results can be written to data files and, in addition, visualized by 2D and 3D computer graphics on the screen (see Sec. 5.3). Furthermore, we will extend the simulator in order to simulate inter-vehicle communication (see Sec. 6.3 below).

There are a number of interactive simulators available publicly. The website [Tre07] deploys the Intelligent Driver Model [THH00] introduced in Sec. 3.1 for cars and trucks together with the lane-changing algorithm MOBIL [KTH07]. This demonstrator simulates typical bottleneck scenarios such as on-ramps, lane-closings, uphill grades and or traffic lights. Another open

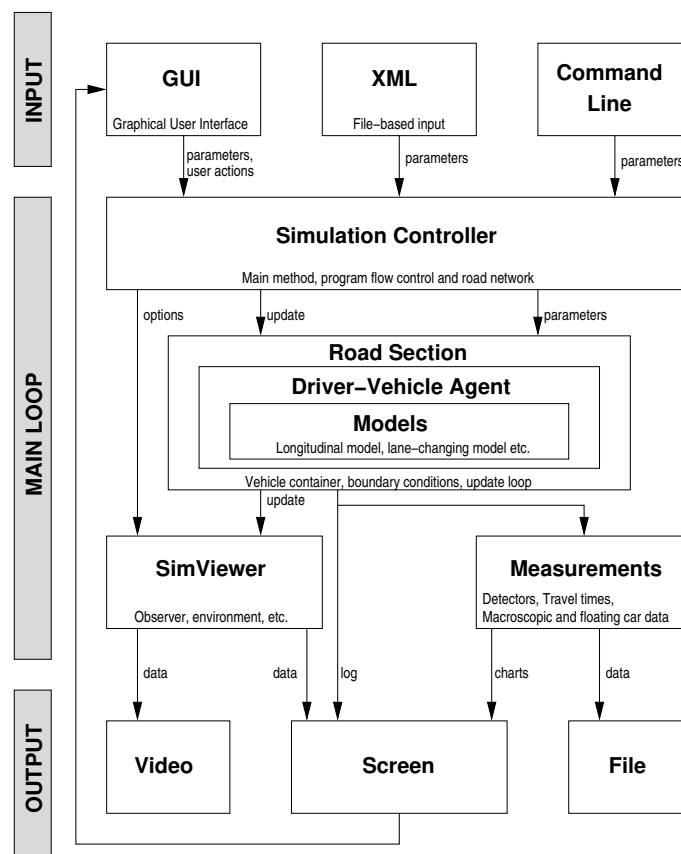


Figure 9: Illustration of possible relations among functions in a traffic simulation framework. The input data defining a simulation setting can be provided by data files, the command line or a graphical user interface (GUI). The main simulation loop is organized by a “Simulation Controller” which controls the update of the road network, the graphical representation (“SimViewer”) and the output functions corresponding to measurements of several microscopic and macroscopic quantities.

source simulator for whole traffic networks is SUMO [SoUM]. The software uses the Krauss model [KWG97]. Recently, FREESIM has been made available to the public [Mil07]. Furthermore, commercial traffic simulation software tools (for instance VISSIMTM, AIMSUNTM or PARAMICSTM) offer a variety of additional modules such as emission or pedestrian models and interfaces, e.g. for controlling simulation runs by remote and for implementing additional features. These commercial products incorporate sophisticated virtual environment 3D engines. Note, however, that the underlying models are generally not well documented.

5.1 Simulator Design

Next to the functional view shown in Fig. 9, a hierarchical view can be used to represent the dependencies and inherited properties which makes use of the object-oriented programming paradigm by representing and abstracting functional units as classes. The best example is the representation of a driver-vehicle agent as an abstract class with several possible designs for

human drivers, vehicles equipped with adaptive cruise control [KTSH07, KTSH08] or even driverless ones as recently demonstrated in reality [(DA07)]. However, each agent has a number of defining properties such as length, width, weight, form and color. Furthermore, each agent requires a model for the lengthwise movement which is in turn an abstract class with the presented IDM as a specific implementation. Further components are required in order to model other aspects of driver behavior such as lane changes, memory, etc. Since each agent is represented by an individual object, it is straightforward to assign individual parameter values to account for driver diversity (Sec. 3.2).

The road network can be represented by connected road sections such as main roads, on-ramps and off-ramps. A road section is defined by its properties like length, number of lanes, etc. In addition, an element may contain attributes representing the concrete infrastructure relevant to the driver-vehicle agents such as lane closures, lane narrowings, speed limits, uphill gradients and/or traffic lights. Notice that the set of attributes which is relevant for the behavior and decision-making has to be available to the agent.

The most detailed view on the innermost update loop of a road section is given in terms of the following pseudo code:

```
updateRoadSection(){
  updateNeighborhood(); // organizing set of vehicles in multiple lanes
  updateInfrastructure(); // active road attributes (e.g. traffic lights)
  updateAgentsRoadConditions(); // attributes affect agents
  calculateAccelerations(); //evaluate longitudinal models of agents
  laneChanges(); // decision making and performing lane changes
  updatePositionsAndSpeeds(); // integration within discrete update
  updateBoundaries(); // inflow and outflow
  updateOutput(); // log observable quantities and update detectors
}
```

5.2 Numerical Integration

The explicit integration in the `updatePositionsAndSpeeds` function of all driver-vehicle agents α is the very core of a traffic simulator. In general, the longitudinal movement of the vehicles is described by car-following models which take into account the direct leader and result in expressions for the acceleration function of the form

$$\frac{dv_\alpha}{dt} = f(s_\alpha, v_\alpha, \Delta v_\alpha), \quad (17)$$

that is the acceleration depends only on the own speed v_α , the gap s_α , and the velocity difference (approaching rate) $\Delta v_\alpha = v_\alpha - v_{\alpha-1}$ to the leading vehicle ($\alpha - 1$). Note that we discussed the Intelligent Driver Model (IDM) as an example for a car-following model in Sec. 3.1. Together with the gap $s_\alpha(t) = x_{\alpha-1}(t) - x_\alpha(t) - l_{\alpha-1}$ and the general equation of motion,

$$\frac{dx_\alpha}{dt} = v_\alpha, \quad (18)$$

Eq. (17) represents a (locally) coupled system of *ordinary differential equations* (ODEs) for the positions x_α and velocities v_α of all vehicles α .

As the considered acceleration functions f are in general nonlinear, we have to solve the set of ODEs by means of numerical integration. In the context of car-following models, it is natural to use an explicit scheme assuming constant *accelerations* within each update time interval Δt . This leads to the explicit numerical update rules

$$\begin{aligned} v_\alpha(t + \Delta t) &= v_\alpha(t) + \dot{v}_\alpha(t)\Delta t, \\ x_\alpha(t + \Delta t) &= x_\alpha(t) + v_\alpha(t)\Delta t + \frac{1}{2}\dot{v}_\alpha(t)(\Delta t)^2, \end{aligned} \tag{19}$$

where $\dot{v}_\alpha(t)$ is an abbreviation for the acceleration function $f(s_\alpha(t), v_\alpha(t), \Delta v_\alpha(t))$. For $\Delta t \rightarrow 0$ s, this scheme locally converges to the exact solution of (17) with consistency order 1 for the velocities (“Euler update”, cf. [PTVF92]) and consistency order 2 for the positions (“modified Euler update”) with respect to the L^2 -norm.² Because of the intuitive meaning of this update procedure in the context of traffic, the update rule (19) or similar rules are sometimes considered to be part of the model itself rather than as a numerical approximation [KT08b]. A typical update time interval Δt for the IDM is between 0.1s and 0.2s. Nevertheless, the IDM is approximately numerically stable up to an update interval of $\Delta t \approx T/2$, that is half of the desired time gap parameter T .

5.3 Visualization

Besides the implementation of the simulation controller with the focus on quantitative models, the visualization of vehicle movements is also an important aspect of simulation software. In the case of vehicular traffic it is straightforward to envision the vehicle trajectories over the course of time whether in 2D or 3D. The latter representation is of course more demanding. Figure 10 illustrated an example of a “bird’s eye view” of a two-lane freeway with an on-ramp, while Fig. 11 illustrates a “cockpit perspective” of a driving vehicle on the road. Note that the 3D engine was programmed from scratch as an exercise by the authors. However, higher level tools and open source 3D engines for OpenGL are available. For more details on this subject we refer to the chapter “Crowd Behavior Modeling: From Cellular Automata to Multi-Agent Systems” by Bandini, Manzoni and Vizzari.

The animated visualization demonstrates both the individual interactions and the resulting collective dynamics. In particular, the graphical visualization turns out to be an important tool when developing and testing lane-changing models and other decisions based on complex interactions with neighboring vehicles for their plausibility. In fact, the driving experiences of programmers offer the best measure of realism and also provide stimulus for further model improvements.

Last but not least, scientists and experts have to keep in mind that computer animations have become an important tool for a fast and intuitive knowledge transfer of traffic phenomena to students, decision-makers and the public. In particular, visualization in *real time* allows for

²A time-continuous traffic model is mathematically consistent if a unique local solution exists and if a numerical update scheme exists whose solution locally converges to this solution when the update time interval goes to zero. It has the consistency order q if $\|\epsilon\| = O(\Delta t^q)$ for $\Delta t \rightarrow 0$ s where ϵ denotes the deviation of the numerical solution for x_α or v_α with respect to the exact solution, and $\|\cdot\|$ is some functional norm such as the L^2 -Norm.

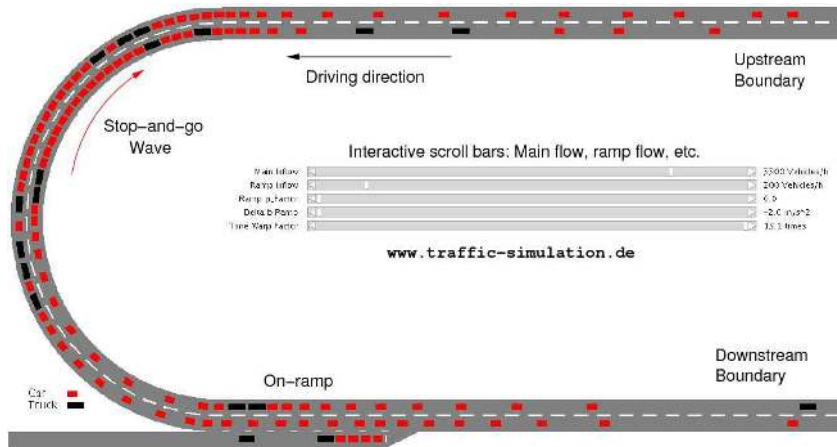


Figure 10: Example of 2D visualization of a two-lane freeway with an on-ramp from the website [Tre07]. The screenshot shows the breakdown of traffic flow at the on-ramp serving as a bottleneck. A stop-and-go wave (cluster of vehicles) is propagating against the driving direction. The source code of the simulator is publicly available as an open source.

direct user interaction influencing the simulation run, e.g. by changing the simulation conditions (in terms of inflows and driver population) and parameter settings of the underlying models. In this way, the complexity of simulation techniques (which are based on assumptions, mathematical models, many parameters and implementational details) can become more accessible.

Finally, we remark that animation is a playground for the programmers. For instance, the full cup of coffee in Fig. 11 represents not just a comforting habit of the agent but is also a vivid way to illustrate the simulated longitudinal acceleration as well as transverse accelerations due to lane-changing. Moreover, the coffee level is a measure of the riding comfort because it is also sensitive to the derivative of the acceleration which is perceived as a jerk by the driver. The hydrodynamic equations of the coffee surface in the cup with a diameter $2r$ are astonishingly realistically approximated by a harmonious pendulum with two degrees of freedom ϕ_x and ϕ_y denoting the angles of the surface normal:

$$\begin{aligned}\ddot{\phi}_x + \frac{2\pi}{\tau}\dot{\phi}_x + \omega_0^2\phi_x + \frac{\ddot{x}}{r} &= 0, \\ \ddot{\phi}_y + \frac{2\pi}{\tau}\dot{\phi}_y + \omega_0^2\phi_y + \frac{\ddot{y}}{r} &= 0.\end{aligned}\tag{20}$$

The second time derivatives \ddot{x} and \ddot{y} denote the vehicle accelerations in longitudinal and transversal direction. The angular frequency is $\omega_0 = \sqrt{g/r}$ where g is the gravitational constant. During a coffee break, the damping time τ of Java coffee was empirically determined as $\tau = 12$ s by the authors.

6 From Individual to Collective Properties

After having constructed the driver-vehicle agents, let us now adopt them in a multi-agent simulation in which they interact with each other. The process of simulating agents in parallel is



Figure 11: Example of a 3D animation from the driver perspective. Notice that the “Coffeemeter” visualizes the acceleration and the jerk (the changes of all acceleration with time) which are difficult to visualize by other means. It therefore serves as a measure of the driving comfort.

one of emergence from the microscopic level of pairwise interactions to the higher, macroscopic level in order to reproduce and predict real phenomena.

In this section, we will present three simulation applications. In Sec. 6.1, we will demonstrate the *emergence of a collective pattern from individual interactions* between driver-vehicle agents by simulating the breakdown of traffic flow and the development of a stop-and-go wave. The simulation will show the expressive power of the Intelligent Driver Model in reproducing the characteristic backwards propagation speed which is a well-known constant of traffic world wide. In Sec. 6.2, we will apply the traffic simulation framework to analyze the impact of a speed limit as an example of a traffic control task. By way of this example, we will demonstrate the predictive power of microscopic traffic flow simulations.

Last but not least, we apply the simulation framework to study a coupled system consisting of communicating driver-vehicle agents using short-range wireless networking technology in Sec. 6.3. Since the multi-agent approach is a flexible general-purpose tool, one can additionally equip an agent with short-range communication devices that can self-organize with other devices in range into ad-hoc networks. Such inter-vehicle communication has recently gained much attention in the academic and engineering world. It is expected to provide great enhancement to the safety and efficiency of modern individual transportation systems. By means of simulation, we will demonstrate the dissemination of information about the local traffic situation over long distances even for small equipment rates in the vehicle fleet.

6.1 Emergence of Stop-and-Go Waves

Let us first study the emergence of a collective traffic phenomenon in a simple ring road scenario as depicted in Fig. 12(a). Note that this scenario can be used interactively on the website [Tre07]. Such a closed system is defined by an initial value problem. The control parameter is the homogeneous traffic density which essentially determines the long-term behavior of the system. In the simulation, the initial traffic density is too high to be able to retain free

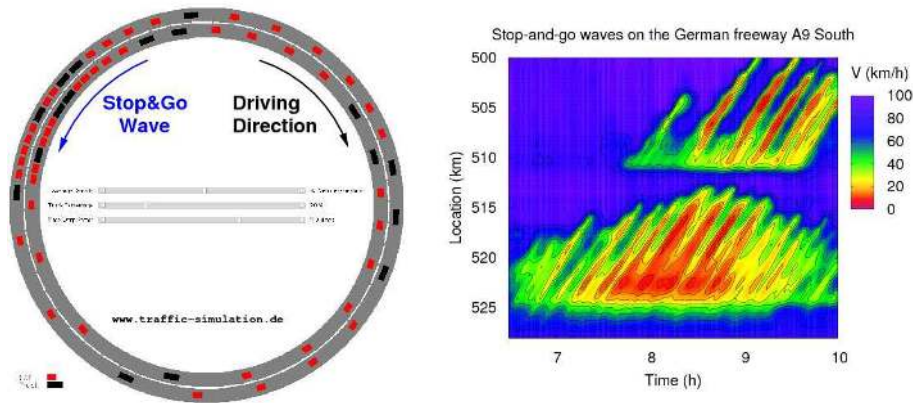


Figure 12: (a) Emergence of a stop-and-go wave in a simulation and (b) stop-and-go traffic on the German freeway A9 South in the north Munich region. Notice that the spatiotemporal traffic dynamics in diagram (b) have been reconstructed from loop detector data using an interpolation method [TH02]. The characteristic propagation speed of stop-and-go waves of about 15 km/h against the driving direction is a self-organized “constant” of traffic flow which is reproduced by the Intelligent Driver Model used in the simulation (a).

flow conditions.

In the course of time, a vehicle eventually changes lanes resulting in a smaller gap for the following vehicle, which, in turn, has to brake in order to re-establish a safe distance to the new leader. After the initial braking, the next follower again needs some time to respond to this new situation by decelerating. The perturbation therefore increases while propagating in upstream direction, that is against the driving direction of the vehicle flow (see Fig. 12(a)). This response mechanism acts like a “vicious circle”: Each following driver has to reduce his or her speed a bit more to regain the necessary safety distance. Eventually, vehicles further upstream in the platoon brake to a standstill. Moreover, the time to re-accelerate to the restored speed of the leading vehicle takes even more time due to limited acceleration capabilities. Finally, we observe the emergence of a *stop-and-go wave*.

Stop-and-go waves are also observed in real traffic as shown in Fig. 12(b) for the German freeway A9 South in the north Munich region. Single stop-and-go waves propagate over more than 10 km leading to their description as “phantom traffic jams”. Their propagation speed is arguably constant. From the time-space diagram in Fig. 12(b), the propagation speed of the downstream front of the stop-and-go wave can be determined as approximately 15.5 km/h. In each country, typical values for this “traffic constant” are in the range 15 ± 5 km/h, depending on the accepted safe time clearance and average vehicle length [KR96b]. Consequently, realistic traffic models should reproduce this self-organized property of traffic flow.

6.2 Impact of a Speed Limit

Microscopic traffic models are especially suited to the study of heterogeneous traffic streams consisting of different and individual types of driver-vehicle agents.

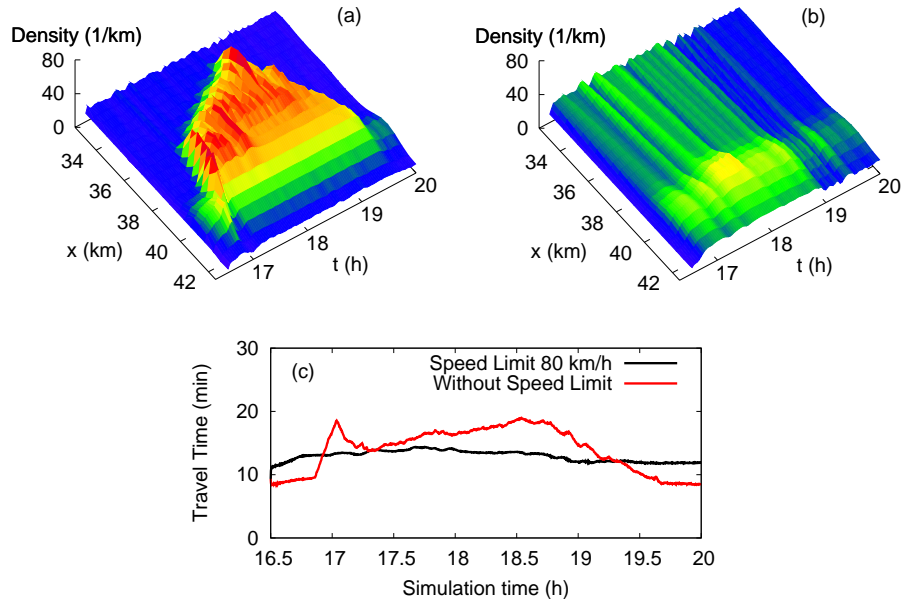


Figure 13: Realistic simulation of an empirical traffic breakdown caused by an uphill gradient located around $x = 40$ km by using velocity and flow data from loop detectors as upstream boundary conditions (a) without speed limit and (b) with a speed limit of 80 km/h. Diagram (c) shows the travel times corresponding to the scenarios (a) and (b). Notice that the microscopic modeling approach allows for an estimation of the current travel time by simply summing up the travel times derived from the speeds of each driver-vehicle agent simultaneously. In accordance with Treiber and Helbing [TH01].

In the following scenario, we will study the effect of a speed limit for a section of the German freeway A8 East containing an uphill section around $x = 40$ km [TH01]. We considered the situation during the evening rush hour on November 2, 1998. In the evening rush hour at about 17 h, traffic broke down at the uphill section. In the simulation shown in Fig. 13(a), we used lane-averaged one-minute data of velocity and flow measured by loop detectors as upstream boundary conditions reproducing the empirical traffic breakdown. In contrast to the ring road scenario in Sec. 6.1, the inflow at the upstream boundary is the natural control parameter for the open system.

We have assumed two vehicle classes: 50% of the drivers had a desired speed of $v_0 = 120$ km/h, while the other half had $v_0 = 160$ km/h outside of the uphill region. A speed limit reduces the desired velocities to 80 km/h. Within the uphill region, both driver-vehicle classes are forced to drive at a maximum of 60 km/h (for example, due to overtaking trucks that are not considered explicitly here).

Figure 13 shows spatiotemporal plots of the locally averaged traffic density for scenarios with and without the speed limit. The simulations show the following:

- During the rush hour ($17 \text{ h} \leq t \leq 19 \text{ h}$), the overall effect of the speed limit is positive. The increased travel times in regions without congestion are overcompensated by the

saved time due to the avoided breakdown.

- For lighter traffic ($t < 17$ h or $t > 19 : 30$ h), however, the effect of the speed limit is clearly negative. Note that this problem can be circumvented by traffic-dependent, variable speed limits.

Although the speed limit reduces the velocity, it can improve the quality of traffic flow while uphill regions obviously results in a deterioration. To understand this counter-intuitive result, we point out that the desired speed v_0 corresponds to the lowest value of (i) the maximum velocity allowed by the motorization, (ii) the imposed speed limits (possibly with a “disobedience factor”), and (iii) the velocity actually “desired” by the driver. Therefore, speed limits act selectively on the *faster* vehicles, while uphill gradients reduce the speed especially of the *slower* vehicles. As a consequence, speed limits reduce velocity differences, thereby stabilizing traffic, while uphill gradients increase them. For traffic consisting of *identical* driver-vehicle combinations (one driver-vehicle class), these differences are neglected and both speed limits and uphill gradients have in fact the same (negative) effect. Since global speed limits always raise the travel time in off-peak hours when free traffic is unconditionally stable (cf. Fig. 13(c)), traffic-dependent speed limits are an optimal solution. Note that the impact of a speed limit on the homogenization of traffic flow can be studied interactively on the website [Tre07] for a lane-closing scenario instead of an uphill bottleneck.

6.3 Store-and-Forward Strategy for Inter-Vehicle Communication

Recently, there has been growing interest in wireless communication between vehicles and potential applications. In particular, inter-vehicle communication (IVC) is widely regarded as a promising concept for the dissemination of information on the local traffic situation and short-term travel time estimates for advanced traveler information systems [JR06, YR05, SKTH06, STKH07, WER05]. In contrast to conventional communication channels which operate with a centralized broadcasting concept via radio or mobile phone services, IVC is designed as a *local service* based on the Dedicated Short Range Communication standard enabling data transmission at a frequency of 5.8 GHz. These devices broadcast messages which are received by all other equipped vehicles within a *limited broadcasting range*. As IVC message dissemination is not controlled by a central station, no further communication infrastructure is needed. For example, wireless local-area networks (IEEE 802.11 a/b/g) have already shown their suitability for IVC with typical broadcasting ranges of 200-500 m [SBSC02, OK04].

In the context of freeway traffic, information on the local traffic situation has to be propagated in an upstream direction. In general, there are two transport strategies: Either a message “hops” from an equipped car to a subsequent equipped car within the same driving direction (“longitudinal hopping”) or the message is transmitted to an IVC-equipped vehicle in the other driving direction which transports the message upstream and delivers it back by broadcasting it to cars in the original driving direction (“transversal hopping”, “cross-transference” or *store-and-forward*). The latter strategy is illustrated in Fig. 14. Although the longitudinal hopping process allows for a quasi-instantaneous information propagation, the connectivity due to the limited broadcasting range is too weak in the presence of low equipment rates [SKTH06]. A concept using IVC for traffic-state detection must therefore tackle the problem that both the required transport distances into upstream direction and the distances between two equipped

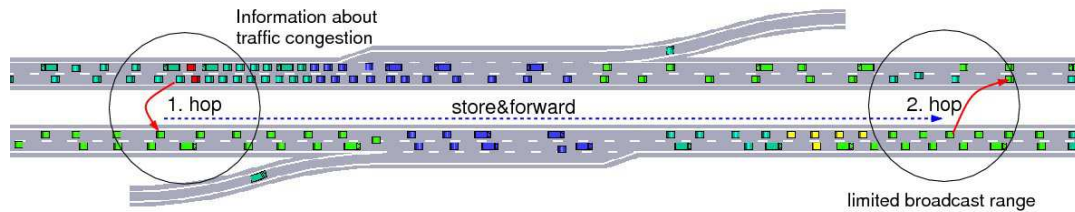


Figure 14: Illustration of the store-and-forward strategy using the opposite driving direction for propagating messages via short-range inter-vehicle communication in upstream direction. First, a message is generated on the occasion of a local change in speed. The broadcasted message will be picked up by an equipped vehicle in the opposite driving direction (first hop). After a certain traveling distance, the vehicle starts broadcasting the message which can be received by vehicles in the original driving direction (second hop).

vehicles are typically larger than the broadcasting range. The transversal hopping mechanism overcomes this problem by using vehicles in the opposite driving direction as relay stations. Despite the time delay in receiving messages, the messages propagate faster than typical shock waves (which are limited to a speed of -15 km/h, cf. Sec. 6.1).

The microscopic simulation approach is well suited to coupling traffic and information flows: The movement of vehicles represents a dynamic network of nodes which determines the spread of information on the network. For the purpose of demonstration, let us now simulate the chain of message propagation by means of IVC in an integrated simulation:

1. The generation of traffic-related messages by individual vehicles,
2. the transmission of up-to-date information in upstream direction using store-and-forward strategy via the opposite driving direction and
3. the receipt of the messages for predicting the future traffic situation further downstream.

The object-oriented design of the traffic simulation software (cf. Sec. 5) can be extended in a straightforward way: First, the simulation of the store-and-forward strategy requires two independent freeways in opposite directions. Second, each equipped driver-vehicle agent autonomously detects jam fronts (by means of velocity gradients) and generates traffic-related messages based on locally available time series data. To this end, the design of a vehicle has been extended by a **detection** unit which generates traffic-relevant messages and a **communication** unit for broadcasting and receiving messages. Finally, the exchange of messages has been realized by a **message pool** which organizes the book-keeping of message broadcast and reception between equipped cars within a limited broadcasting range (corresponding to the outdated ether concept). As the routing in this system is obviously given by the two traffic streams in opposite directions, no further rules are necessary for modeling the message exchange process.

We consider a scenario with an assumed fraction of only 3% communicating vehicles. The resulting trajectories of equipped vehicles in both driving directions together with the generation of messages and their reception by a considered vehicle are illustrated in Fig. 15. In this scenario, a temporary road blockage has triggered a stop-and-go wave reflected by horizontal trajectory curves in one driving direction while the traffic flow in the opposite driving

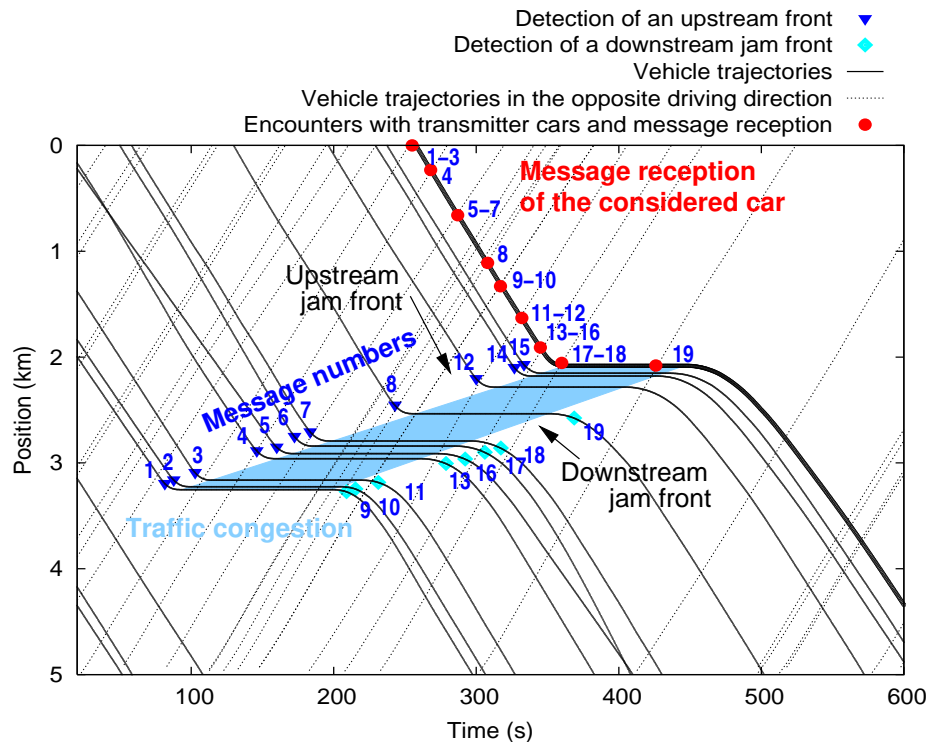


Figure 15: Space-time diagram of the simulated traffic scenario. The trajectories of the IVC-equipped vehicles (3%) are displayed by solid or dotted lines depending on the driving direction. The vehicles in the opposite driving direction serve as transmitter cars for the store-and-forward strategy. For the purpose of illustration, we have set the maximum broadcasting range to 10 m. When cars pass the upstream or downstream jam front of the moving jam, they broadcast messages (marked by numbers) containing the detected position and time. They are later received by the considered vehicle further upstream (thick solid line). Note that the crossing trajectories of equipped vehicles (e.g. in the upper-left corner of the diagram) reflect passing maneuvers due to different desired velocities.

direction was free. When cars encountered the propagating stop-and-go wave, they started to broadcast messages about the detected position and time of the upstream jam front and the following downstream jam front. The event-driven messages were received and carried forward by vehicles in the other driving direction via the store-and-forward mechanism.

As shown in Fig. 15, the considered vehicle already received the first message about the upcoming traffic congestion 2 km before reaching the traffic jam. Further received messages from other equipped vehicles could be used to confirm and update the upcoming traffic situation further downstream. Thus, based on a suitable prediction algorithm, each equipped vehicle could autonomously forecast the moving jam fronts by extrapolating the spatiotemporal information of the messages. In the considered simulation scenario, the upstream jam fronts were already accurately predicted with errors of ± 50 m 1 km ahead of the jam, while the errors for the predicted downstream jam amounted to ± 100 m. Obviously, the quality of the jam-front anticipation improves with the number and the timeliness of the incoming messages. More details about the used prediction algorithm can be found in Ref. [STKH07].

7 Conclusions and Future Work

Agent-based traffic simulations provide a flexible and customizable framework for tackling a variety of current research topics. Simulation of control systems as a part of traffic operations is an important topic in transport telematics. Due to the interrelation of the control systems with traffic, both the control systems and the driver reactions must be described in a combined simulation framework. Examples are variable message signs and speed limits, on-ramp metering, lane-changing legislation and dynamic route guidance systems. An interesting research challenge is adaptive self-organized traffic control in urban road networks [LDH07].

Furthermore, traffic simulations are used to assess the impacts of upcoming driver assistance systems such as *adaptive cruise control* systems on traffic dynamics. The microscopic modeling approach is most appropriate because it allows for a natural representation of heterogeneous driver-vehicle agents and for a detailed specification of the considered models, parameters and vehicle proportions [VSK⁺01, VSMK02, KTSH07, HM02, Min99, Dav04, TvA01]. The challenging question is whether it is possible to design vehicle-based control strategies aimed at improving the capacity and stability of traffic flow [KTSH08].

With rapid advances in wireless communication technologies, the transmission of information within the transportation network is a challenging issue for the next generation of Intelligent Transportation Systems (ITS). Agent-based systems form the basis for a simulation of hybrid systems coupling vehicle and information flow. The decentralized propagation of information about the upcoming traffic situation has been discussed as an application for inter-vehicle communication. Many other applications are conceivable based on the integration of vehicles and infrastructures implying vehicle-to-infrastructure communication technologies. However, realistic and predictive simulations are essential for developing and testing applications of upcoming communication technologies and applications.

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