

Aggregate implications of corporate debt choices*

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Abstract

The composition of corporate borrowing between bank loans and market debt varies substantially, both across countries and over the business cycle. This paper develops a new model of firm dynamics, where firms choose both the scale and composition of their borrowing, in order to understand the aggregate implications of corporate debt choices. In choosing debt composition, firms trade-off the flexibility that banks provide in case of financial distress, with the lower intermediation costs offered by markets in normal times. The steady-state of the model captures key cross-sectional facts about debt composition: firms sort between a mixed-debt and a market-debt regime, and shift from the former to the latter as their credit risk declines. Aggregate shocks to banks' intermediation costs generate a large recession, accompanied by a sharp drop in banks' share of total intermediation. The decline in borrowing and investment reflects not only lower bank loan issuance by mixed-debt firms (the traditional "credit channel"), but also a reduction in borrowing by firms that switch from the mixed-debt to the market-debt regime (a new "substitution channel"). By switching to the market-debt regime, firms indeed lose the flexibility provided by banks in bad times. They offset this increase in financial fragility by reducing borrowing and investment. I show that this mechanism can also manifest itself in the response of firms to policies that encourage credit disintermediation.

Keywords : banks, bonds, financial structure, financial frictions, firm dynamics, output, investment, productivity risk.

JEL Classification Numbers: E22, E23, G21, G33.

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1 Introduction

How do financial frictions affect aggregate activity? Since the onset of the Great Recession, this question has attracted renewed attention among macroeconomists. A central hypothesis is that financial market imperfections limit firms' ability to issue debt in order to finance investment. To formalize this mechanism, much of the literature has focused on total debt issued by firms, and narrowed down the analysis of financial frictions to a situation where firms face a single borrowing constraint.

An important limitation of this approach is that it ignores the fact that, in reality, firms use a broad array of debt contracts to finance investment. This heterogeneity is reflected in the balance sheet of the corporate sector: in the US, for example, bank loans only accounted for 27.5% of the stock of outstanding debt of non-financial corporations on the eve of the Great Recession.¹ Moreover, there is substantial evidence that the use of these different debt instruments varies with aggregate conditions, and in turn affects firms' investment possibilities. [Becker and Ivashina \(2014\)](#) find evidence of substitution between loans and bonds when lending standards are tight, and show that this substitution is a strong predictor of reductions in investment by small firms. Analogously, [Adrian, Colla, and Shin \(2012\)](#) study debt issued for investment purposes, and show that during the 2007-2009 recession, bank-dependent firms experienced a larger decline in debt issuance than firms that with access to markets. This evidence echoes previous empirical work, such as [Kashyap, Lamont, and Stein \(1994\)](#), who show that bank-dependent firms experience larger declines in inventory investment during recessions.

Motivated by this evidence, this paper studies the relationship between aggregate outcomes and debt heterogeneity, defined as the coexistence and availability of different corporate debt instruments. I address three specific questions. First, what drives long-run differences in the aggregate composition of corporate debt across countries, and how do these differences in financial structure relate to long-run differences in output and investment?² Second, how does debt heterogeneity affect the transmission of aggregate shocks? Third, can policy affect debt composition, and if so, what are the effects on aggregate investment and output?

In order to answer these questions, I develop a macroeconomic model with heterogeneous firms in which both the scale and composition of borrowing are endogenous. [Section 2](#) describes this model. It is the first model of firm dynamics with financial frictions to endogenize jointly the borrowing composition, investment choices, and firm growth. In this economy, firms can finance investment either internally (through the accumulation of retained earnings), or by issuing two types of debt: bank loans and market debt. Credit is constrained by the fact that firms have limited liability, and default entails deadweight losses of output.

¹The remaining 72.5% are mostly accounted for by corporate bonds (63%). Data is from the Flow of Funds for 2007Q3; see [appendix A](#) for details.

²By contrast to the 27.5% bank share in the US, in Italy bank loans accounted for 65.7% of outstanding corporate debt in 2007Q3; see [appendix A](#) for details on the data.

The central assumption of the model is that banks and market lenders differ in their ability to deal with financial distress. Specifically, I assume that bank loans can be restructured when firms' revenues are low, whereas market liabilities cannot be reorganized. In this sense, banks offer more flexibility than market lenders when a firm is in financial distress. On the other hand, outside of financial distress, I assume that bank lending is more restrictive than market lending. In the model, this difference is captured through differences in intermediation costs between bank and market lenders. The higher intermediation costs of banks are reflected in the equilibrium terms of lending contracts, which firms must honor outside of financial distress. In choosing the scale and composition of borrowing, firms therefore trade-off the higher flexibility of bank debt in financial distress, with the lower costs associated with market financing in normal times.³

Section 3 analyzes firms' financial policies in steady-state. The model endogenously generates a distribution of firms across levels of internal finance. Two main results characterize variation in the composition and scale of borrowing across this distribution. First, some firms choose to borrow simultaneously from bank and market lenders. This is a key empirical finding of Rauh and Sufi (2010), who nevertheless stress that few models of debt structure have this feature. Intuitively, simultaneous borrowing occurs in the model because issuing liabilities with market lenders may sometimes relax the bank borrowing constraint. Crucially, this complementarity arises because investment and debt structure are jointly determined: borrowing more from markets helps firms increase their scale of operation; this in turn raises the liquidation value of the firm, and therefore relaxes its bank borrowing constraint. Second, a firm's stock of internal finance is negatively related to its bank share, defined as the ratio of bank loans to total debt. As a firm grows, it will therefore reduce its reliance on bank debt. In the model, this arises because of the interaction between decreasing returns, leverage, and bank flexibility. Because of decreasing returns, firms have an ex-ante optimal investment scale, irrespective of internal resources. For firms that have large internal resources, reaching that scale requires less leverage. Those firms will therefore be unlikely to face financial distress, and for them the flexibility gains associated with bank debt are of little value. The model thus predicts a tight link between the likelihood of financial distress and debt composition, consistently with another key finding of Rauh and Sufi (2010), who show that, as credit quality declines, firms tend to "spread out" debt structure more across different types of debt instruments, and in particular increase their reliance on bank loans.⁴

The remainder of the paper uses this model to study the aggregate implications of debt heterogeneity. Section 4 focuses on long-run differences in aggregate debt composition. I show that this composition only depends on structural parameters that affect the trade-off between flexibility in distress and costs outside

³In section 2, I discuss at length the empirical support for the view that banks are more flexible creditors than markets in financial distress. I also provide different interpretations of the assumption that banks provide a more constraining form of debt in normal times than markets.

⁴A negative correlation between credit quality and bank loans' share of total credit is also documented by Denis and Mihov (2003). Additionally, appendix A reports direct evidence on the link between the bank share of total credit, and firms' net worth, using firm-level data for a sample of advanced and developing economies.

of distress. In particular, a lower wedge between the intermediation costs of banks and markets increases the aggregate bank share, by making bank borrowing relatively less constraining outside of bankruptcy. Likewise, higher idiosyncratic productivity risk increases the aggregate bank share, because it exposes firms to financial distress more frequently. By contrast, average productivity and average lending costs have no effect on aggregate debt composition. I calibrate the model using US and Italian data on idiosyncratic productivity risk and intermediation costs, and show that these two sources of exogenous variation alone can account for between one half and two thirds of the gap between the US bank share (27.5%) and the Italian bank share (65.7%). In this sense, the flexibility/cost trade-off gives a simple, yet powerful account of cross-country differences in aggregate debt structure. At the same time, the model suggests that these differences are manifestations of costly inefficiency in the intermediation of credit. For example, in the model, lowering Italian market lenders' costs to a level comparable to the US would generate, *ceteris paribus*, increases in steady-state output and investment in the order of 5%.

Section 5 focuses on the response of the model to financial shocks. I study the perfect foresight response of the model to a shock that permanently increases the intermediation costs of banks relative to markets. The magnitude of this shock is chosen so as to match the large and persistent decline in the composition of corporate non-financial debt in the US during the Great Recession, which fell from 28.7% in 2008Q3 to 19.1% in 2011Q3. The long-run increase in the intermediation wedge required to match that drop is 0.45%, from a initial steady-state level of 1.255% – an increase of roughly a third.⁵ In addition to its effects on debt composition, the shock also generates a substantial drop in output and investment, with output dropping by 4.1% over the first three years. The shock affects investment and output in two ways. On one hand, firms react to the permanently higher bank intermediation costs by reducing their borrowing from banks. This traditional “credit channel” effect accounts for about two thirds of the response of output, mostly through its impact on borrowing and investment policies of mixed-finance firms, who were heavily relying on bank loans before the shock⁶ The remaining third of the response of output is accounted for by a novel propagation mechanism. Firms that, prior to the shock, were close to (but smaller than) the threshold for between the mixed and the market finance regimes, respond by switching to the mixed finance regime. In effect, these firms are substituting market debt for bank loans. However, the model predicts that these firms substitute market debt for bank loans less than one for one, and as a result, reduce total borrowing and investment. The impact of this “substitution channel” on borrowing and investment can be understood as a precautionary response of firms to their qualitative change in debt structure: as firms move from mixed to market finance,

⁵I also show that, aside from its effects on *aggregate* debt composition, the shock generates patterns of debt substitution *in the cross-section* that are consistent with existing empirical evidence: while bank borrowing declined economy-wide, large firms also increased their issuance of non-bank debt, while small and medium-size firms did not.

⁶Market-financed firms, who are forward-looking and anticipate the possibility of shifting back to a mixed-financed debt structure, were they to sustain a sequence of negative shocks, also respond by lowering borrowing and investment somewhat, although they are not directly impacted by the shock.

they lose the flexibility that bank debt traditionally gave them. As a result, for a comparable leverage, they are now more prone to liquidation risk. They respond to this increase in liquidation risk by deleveraging, relative to their pre-shock borrowing policies. The model thus suggests that transitions between financial regimes can have substantial real implications, as they can alter the degree of financial fragility which a particular borrowing and investment policy would entail for a firm.

Section 6 studies the real effects of “disintermediation” policies, that is, policies aimed at encouraging the use of market credit by a broader set of firms. I analyze two specific examples: German efforts to develop a bond market specifically targeted for small and medium-sized firms, and an Italian fiscal reform extending tax deductibility of interest payments to bond issues by private firms. As for the propagation of financial shocks, the effects of these policies are best understood in terms of a “credit channel” and a “substitution channel” effect. On the one hand, they boost aggregate investment by lowering the cost of market debt issuance (the “credit channel” effect). On the other, they induce medium-sized firms that were previously partially bank-financed to switch entirely to market finance. As a result of their increased fragility, these firms borrow less, and their output and investment falls (the “substitution channel” effect). The net effect of the policy on aggregate investment and output in steady-state is in general positive, but this comes at the expense of a precautionary reduction in leverage and activity of medium-sized firms.

Related literature This paper builds on the extensive literature on corporate debt structure, following the seminal contributions of [Diamond \(1991\)](#), [Rajan \(1992\)](#), [Besanko and Kanatas \(1993\)](#) and [Bolton and Scharfstein \(1996\)](#). While I do not model it explicitly, the assumption that bank and market lending differ in their degree of flexibility in times of financial distress builds on the insight of [Bolton and Scharfstein \(1996\)](#) that the dispersion of market creditors reduces individual incentives to renegotiate debt payments. The closest model to the one I develop in this paper is [Hackbarth, Hennessy, and Leland \(2007\)](#). This paper, along with most of theoretical literature on the topic, focuses on the structure of financing, given a fixed scale of investment; by contrast, in the model I propose, the scale of investment is also determined endogenously, which allows me to draw cross-sectional and aggregate implications of debt structure for output and investment.

This paper is also related to the literature on firm growth and financial frictions. My model’s key friction is limited liability, as in [Cooley and Quadrini \(2001\)](#), [Clementi and Hopenhayn \(2006\)](#) or [Hennessy and Whited \(2007\)](#). In particular, the connection between the firms’ optimal financial policies and their steady-state growth dynamics follows closely [Cooley and Quadrini \(2001\)](#). I contribute to this literature by introducing an endogenous debt structure choice, and illustrating its implications for firm growth and the distribution of firms across levels of internal finance in steady-state.

The macroeconomic implications of debt heterogeneity have been addressed by relatively few papers.

Bolton and Freixas (2006), in the context of a static model, show that, by affecting the spread of bank loans over corporate bonds, monetary policy can lower banks' equity-capital base, in turn leading to a contraction in corporate credit. This channel is separate from the traditional "bank lending channel", which operates through reductions in bank reserves. My model does not allow me to distinguish between causes of contractions bank lending; its focus is purely and squarely on their consequences for firm-level and aggregate investment. De Fiore and Uhlig (2011) and De Fiore and Uhlig (2014) study an asymmetric information model of bond and bank borrowing, and show that the model accounts well for long-run differences between the Euro Area and the US to the extent that information availability on corporate risk is lower in Europe. They also provide a model-based assessment of the changes in corporate debt composition in the US during the Great Recession, relying on a combination of different shocks, among which an increase in firm-level uncertainty and in the intermediation costs of banks. Aside from the differences in the role that banks play in our respective setups, the cross-sectional predictions of the models differ: firms in their model indeed specialize in either bank or market borrowing; but these two forms of debt are relatively good substitutes for marginal firms. This, together with the fact that in the setup of my paper, leverage and debt composition are simultaneously chosen by each firm, implies that the effects of financial shocks on borrowing and investment are magnified, in my setup relative to theirs.

Finally, this paper draws from the results of Crouzet (2013). In particular, the characterization of firms' feasible set of debt structures, as a function of their internal resources, is similar in the static setup of that paper, and the dynamic model considered here.

2 A dynamic model of debt composition

This section describes a frictional model of firm investment, in which heterogeneous firms with limited internal finance must raise outside funding from financial intermediaries. The novelty of the model is that outside funding can take two forms: bank debt (b_t) or market debt (m_t). A firm's internal funds are denoted by e_t . Time is discrete; while firms are infinitely-lived, financial contracts are settled each period.

In this economy, firm investment is constrained for two reasons. First, firms have limited liability. A firm can choose to default on its debt obligations, and this default may entail the liquidation of the firm and the transfer of its assets to creditors. Liquidation is inefficient: it involves deadweight losses. Second, firms cannot issue equity. Instead, they accumulate internal finance e_t over time, through retained earnings. Absent either friction, the firm would be able to finance investment to its optimal scale. With frictions to debt and equity issuance, the firm will attempt to accumulate retained earnings in order to fund investment internally, and limit its dependence on debt.⁷

⁷The assumption that firms cannot issue equity *at all* is not necessary for the main results of the paper to hold. It is

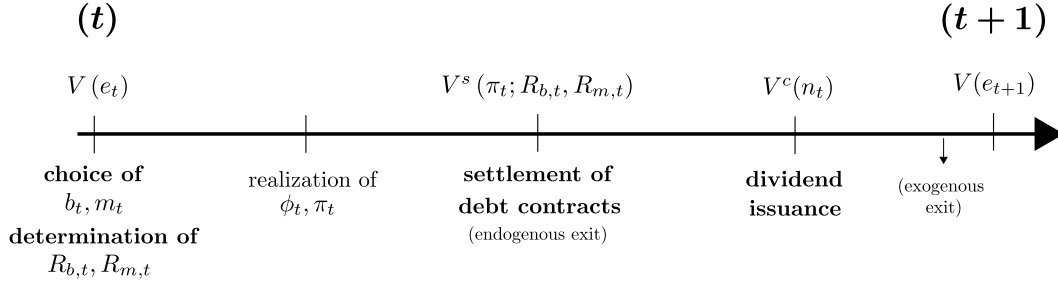


Figure 1: Overview and timing of firms' problem.

In order to clarify the exposition, paragraphs 2.1-2.8 focus on the description of the model, while paragraph 2.9 is devoted to the discussion of key assumptions and their relationship to evidence on bank and market lending.

2.1 Overview of firms' problem

Figure 1 summarizes the timing of each individual firms' problem. There are three stages within each period: first, the choice of debt structure (b_t, m_t) ; second, the settlement of debt contracts; third, the issuance of dividends.

At the beginning of the period, each firm is characterized by its internal finance e_t . The present discounted value of future dividends for a firm with internal finance e_t is given by $V(e_t)$; managers are risk-neutral and all discount dividends using the same discount factor β . The firm operates a decreasing returns to scale technology that takes capital k_t as sole input. The firm's total resources after production are given by:

$$\pi_t = \pi(\phi_t, k_t) = \phi_t k_t^\zeta + (1 - \delta)k_t,$$

where $0 < \zeta < 1$ denotes the degree of returns to scale, $0 < \delta < 1$ denotes the rate of depreciation of capital, and $\phi_t \geq 0$ denotes the productivity level of the firm. ϕ_t is drawn from a distribution $F(\cdot)$ with mean $\mathbb{E}(\phi)$ and standard deviation $\sigma(\phi)$.⁸ Total investment in capital is given by $k_t = e_t + b_t + m_t$. Both the scale of borrowing $(b_t + m_t)$ and its composition $\left(\frac{b_t}{b_t + m_t}, \frac{m_t}{b_t + m_t}\right)$ are chosen by the firm *before* the realization of productivity. Thus, the terms of the debt contracts, denoted by $R_{b,t}$ (for bank debt) and $R_{m,t}$ (for market debt), cannot be indexed by ϕ_t .⁹ The terms of the debt contracts represent the gross promised repayments to the intermediaries (principal plus interest), due once productivity has been realized.

Debt contracts are settled in the interim stage. The value of a firm entering this stage with resources π_t

however necessary to assume that equity issuance is costly. This can be done by introducing a fixed marginal issuance cost of equity, which needs to be strictly larger than the marginal cost of lending of banks. However, the assumption of infinite equity issuance costs, effectively maintained here, allows the paper to focus on the relationship between debt composition and investment. Results on a version of the model with equity issuance are available from the author upon request.

⁸In this section, productivity is i.i.d. across firms and over time. Section 4 deals with time variation in $\mathbb{E}(\phi)$ and $\sigma(\phi)$. Additionally, to ensure unicity of lending contracts, $F(\cdot)$ must have a strictly increasing hazard rate; see [Crouzet \(2013\)](#) for more details.

⁹They also depend on the amounts b_t and m_t borrowed by the firm, as well as its internal funds e_t and potentially its value function $V(\cdot)$. In order to alleviate notation, I omit this in the exposition of the model.

and liabilities $(R_{b,t}, R_{m,t})$ is given by $V^s(\pi_t; R_{b,t}, R_{m,t})$. Given the realization of its current productivity ϕ_t , the firm can choose to either fulfill its promise to repay $(R_{b,t}, R_{m,t})$ to its creditors, or attempt to renegotiate down these liabilities. If the realization of productivity is sufficiently bad, the firm may also be forced into liquidation. The details of the settlement process are spelled out below.

Firms that are not liquidated during debt settlement then allocate remaining funds n_t between dividends and retained earnings, which constitute next period's internal funds e_{t+1} . After this dividend issuance choice, only a fraction $(1 - \eta)$ of firms survive; the rest are destroyed, along with their internal finance stock e_{t+1} .¹⁰ The value of a firm with remaining funds n_t after settlement is denoted by $V^c(n_t)$.

A recursive formulation of the firm's problem can be obtained by solving the model backwards within each period, starting with the dividend issuance stage.

2.2 Dividend issuance

Given the value function $V(\cdot)$, the dividend issuance problem of a firm after debt settlement is given by:

$$\begin{aligned} V^c(n_t) &= \max_{div_t, e_{t+1}} div_t + (1 - \eta)\beta V(e_{t+1}) \\ \text{s.t.} \quad &div_t + e_{t+1} \leq n_t \quad , \quad div_t \geq 0 \end{aligned} \tag{1}$$

Here, div_t denotes dividends issued, and η denotes the probability of exogenous exit. The dividend policy of the firm is given by the following lemma, which is analogous to the results of [Cooley and Quadrini \(2001\)](#).

Lemma 1 (Dividend policy) *If V is continuous on \mathbb{R}_+ and satisfies $V(0) \geq 0$, then V^c is continuous and strictly increasing on \mathbb{R}_+ , and satisfies $V^c(0) \geq 0$. If, moreover, (1) V is strictly increasing and differentiable except at a finite number of points, (2) $\frac{dV}{de}$ is strictly decreasing, and (3) there exists $\bar{e} > 0$ such that:*

$$(1 - \eta)\beta \frac{dV}{de}(\bar{e}) = 1, \tag{2}$$

then, the firm's optimal dividend policy is given by:

$$\hat{div}(n_t) = \begin{cases} 0 & \text{if } 0 \leq n_t < \bar{e} \\ n_t - \bar{e} & \text{if } n_t \geq \bar{e} \end{cases} .$$

Intuitively, a firm with earnings n_t below \bar{e} will find that the marginal value of keeping these earnings inside the firm exceeds the marginal value of dividends, which is equal to 1.¹¹ The firm will therefore choose the corner solution $e_{t+1} = n_t$, and issue no dividends. On the other hand, a firm with cash above \bar{e} will be

¹⁰The assumption of exogenous exit is not necessary to guarantee the existence of a solution to each firm's individual problem, but it guarantees the existence of a stationary distribution of firms across levels of e_t . See the proof of proposition 3 for details.

¹¹The properties of the value function discussed in this proposition will hold as a result of the existence of decreasing returns at the firm level. This will be formally established as part of the proof of proposition 3.

able to choose the interior solution $e_{t+1} = \bar{e}$, and will use its remaining earnings, $n_t - \bar{e}$, to issue dividends. The relevant state-space of the firm's problem is therefore $[0, \bar{e}]$.¹²

2.3 Debt settlement

At the settlement stage, the firm has three options: liquidation; debt restructuring; or full payment of its liabilities. Let V_t^L , V_t^R and V_t^P denote the respective values of the firm under each option. The firm faces the discrete choice problem:

$$V^s(\pi_t, R_{b,t}, R_{m,t}) = \max_{L,R,P} (V_t^L, V_t^R, V_t^P) \quad (3)$$

I next describe in more detail each of these three options. I delay the discussion of key assumptions embodied in this description to the next paragraph.

Liquidation (V_t^L) In liquidation, the firm is shut down and its resources π_t are passed onto creditors. I make the following assumption about liquidation:

Assumption 1 (Liquidation losses) *The liquidation value of the firm given by:*

$$\tilde{\pi}_t = \chi \pi_t \quad , \quad 0 \leq \chi \leq 1.$$

When $\chi < 1$, the transfer of the firm's resources to creditors involves a deadweight loss. In that case, lenders charge the firm a liquidation premium. Absent this loss (when $\chi = 1$), if all participants are risk-neutral, lenders charge the risk-free rate, and firms can reach their ex-ante optimal size.

With multiple creditors, one must take a stance on how liquidation resources are allocated among stakeholders. I assume that the split follows a rule similar to the Absolute Priority Rule (APR) which governs chapter 7 corporate bankruptcies in the US: a claim by a stakeholder to liquidation resources can be activated only if all stakeholders placed higher in the priority structure have been made whole. In this model, there are three stakeholders: bank lenders; market lenders; and the firm itself. The firm is the last residual claimant. I furthermore assume that bank lenders are senior to market lenders in the priority structure. Payoffs to

¹²The key constraint in this problem is that firms are not allowed to issue equity: $div_t \geq 0$. This is a sufficient assumption to guarantee a non-degenerate solution to the problem. In general, following Cooley and Quadrini (2001), it is necessary that equity issuance be costly, that is, that the marginal cost of equity issuance be strictly larger than 1. Indeed, imagine that the firm were allowed to issue negative dividends at a marginal cost of 1. In that case, the firm can always achieve the interior optimum, by using the dividend policy $div_t = n_t - \bar{e}$, even when $\bar{e} > n_t$. All firms would then operate under identical debt structures and scales, and would only differ in the interim period, after idiosyncratic productivity has been realized.

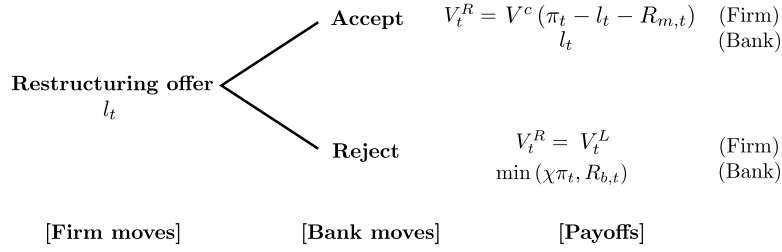


Figure 2: Two-stage game for debt restructuring

stakeholders are then given by:

$$\begin{aligned}
 \tilde{R}_{b,t} &= \min(R_{b,t}, \chi\pi_t) && \text{(bank lenders)} \\
 \tilde{R}_{m,t} &= \min(\max(0, \chi\pi_t - R_{b,t}), R_{m,t}) && \text{(market lenders)} \\
 V_t^L &= \max(0, \chi\pi_t - R_{b,t} - R_{m,t}) && \text{(firm)}
 \end{aligned} \tag{4}$$

Restructuring (V_t^R) Restructuring is the process through which firms renegotiate their liabilities, conditional on the realization of their productivity. The crucial distinction between banks and market lenders lies with their ability to participate in the renegotiation process.

Assumption 2 (Debt flexibility) *Only bank debt can be restructured; market debt is not flexible.*

I model the restructuring process as a two-stage game between firm and bank. This game is summarized in figure 2. The firm moves first, and offers to repay the bank an amount l_t , instead of the promised amount $R_{b,t}$. The bank can choose to accept or reject the offer. In case the offer is rejected, liquidation ensues, and all parties receive the liquidation payoffs described by (4). In this process, market lenders only have an indirect role, and their liabilities remain untouched in any successful restructuring agreement.

The optimal action of the bank is to accept the offer, if and only if it exceeds the banks' reservation value, that is, if and only if $l_t \geq \min(R_{b,t}, \chi\pi_t)$. The value of an offer l_t to the firm is therefore:

$$\tilde{V}^R(l_t; \pi_t, R_{b,t}, R_{m,t}) = \begin{cases} V^c(\pi_t - l_t - R_{m,t}) & \text{if } l_t \geq \min(R_{b,t}, \chi\pi_t) \\ V_t^L & \text{if } l_t < \min(R_{b,t}, \chi\pi_t) \end{cases}$$

The firm chooses its restructuring offer, l_t , in order to maximize this value, subject to the constraint that its net resources, after the restructuring offer, must be positive:

$$\begin{aligned}
 V_t^R &= \max_{l_t} \tilde{V}^R(l_t; \pi_t, R_{b,t}, R_{m,t}) \\
 \text{s.t. } &\pi_t - l_t - R_{m,t} \geq 0
 \end{aligned} \tag{5}$$

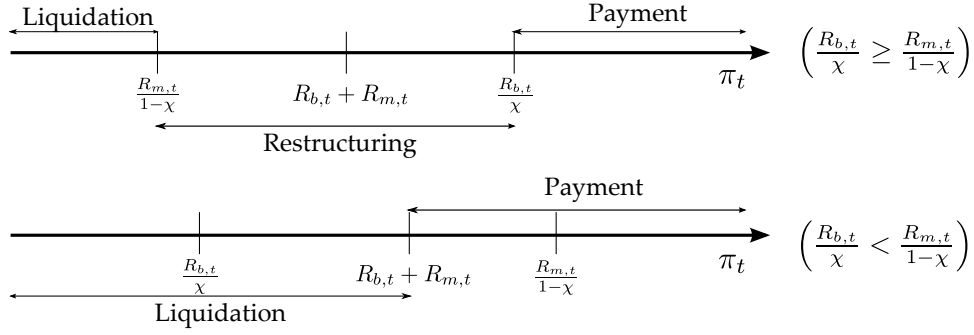


Figure 3: Debt settlement outcomes

Payment (V_t^P) A firm can only fully pay its creditors if its resources exceed its total liabilities; otherwise, it is liquidated. Therefore,

$$V_t^P = \begin{cases} V^c(\pi_t - R_{m,t} - R_{b,t}) & \text{if } \pi_t \geq R_{b,t} + R_{m,t} \\ V_t^L & \text{if } \pi_t < R_{b,t} + R_{m,t} \end{cases} \quad (6)$$

Debt settlement outcomes Given values of π_t , $R_{m,t}$ and $R_{b,t}$, I refer to a solution to problem (3), subject to (4), (5) and (6) as a *debt settlement outcome*. The following proposition describes the optimal choice of the firm between liquidation, restructuring and repayment.

Proposition 1 (Debt settlement outcomes) *There are two types of debt settlement outcomes:*

- **When** $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t}}{1-\chi}$, *the firm chooses to repay its creditors in full, if and only, $\pi_t \geq \frac{R_{b,t}}{\chi}$. It successfully restructures its debt, if and only if, $\frac{R_{m,t}}{1-\chi} \leq \pi_t < \frac{R_{b,t}}{\chi}$, and it is liquidated when $\pi_t < \frac{R_{m,t}}{1-\chi}$.*
- **When** $\frac{R_{b,t}}{\chi} < \frac{R_{m,t}}{1-\chi}$, *the firm repays its creditors in full if and only if $\pi_t \geq R_{b,t} + R_{m,t}$, and it is liquidated otherwise.*

Moreover, in all debt settlement outcomes resulting in restructuring, the bank obtains its reservation value $\chi\pi_t$, and in all debt settlement outcomes resulting in liquidations, $V_t^L = 0$.

Figure 3 offers a graphical representation of the two situations. Two points are worth noting. First, the firm does not necessarily prefer restructuring to repayment. The surplus that the firm can extract from the bank in restructuring is the difference between its bank liabilities and the banks' reservation value, $R_{b,t} - \chi\pi_t$. The firm will only attempt a restructuring if this surplus is positive. Although the firm has bargaining power, since it moves first, the bank's ability to force liquidation in restructuring renegotiations effectively limits the firm's incentives to "abuse" restructuring.

Second, restructuring will not always save the firm from liquidation. This is because of the indirect role of market lenders in the restructuring process. When market liabilities are relatively large $\left(\frac{R_{b,t}}{\chi} < \frac{R_{m,t}}{1-\chi}\right)$, even though the firm can extract a positive surplus from the bank in restructuring, that surplus is never sufficient to avoid liquidation. On the other hand, when market liabilities are relatively small $\left(\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t}}{1-\chi}\right)$,

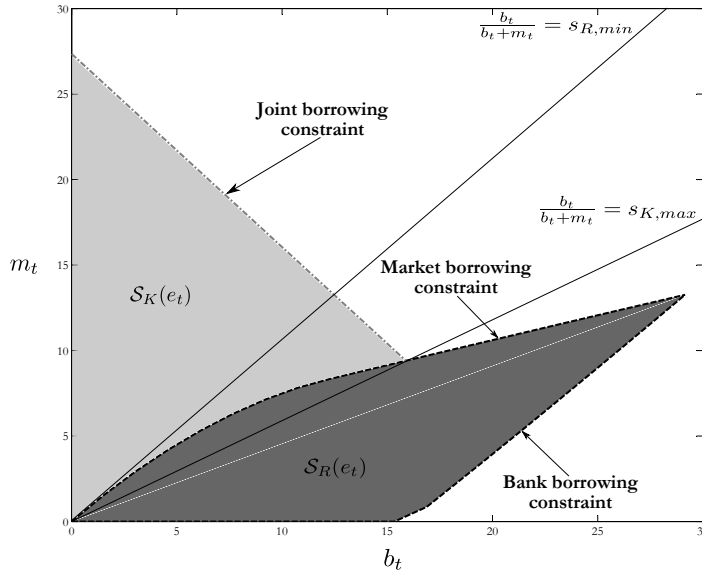


Figure 4: The lending menu $\mathcal{S}(e_t)$.

restructuring can be the best option for the firm. In some cases $\left(\frac{R_{m,t}}{1-\chi} \leq \pi_t \leq R_{b,t} + R_{m,t}\right)$, it is because restructuring allows the firm to avoid liquidation. In others $\left(R_{b,t} + R_{m,t} \leq \pi_t \leq \frac{R_{b,t}}{\chi}\right)$, the firm restructures for opportunistic reasons: it could pay in full both creditors, but instead decides to exert its bargaining power over the bank and reduce its bank liabilities.

Additionally, proposition 1 indicates that liquidation never involves a strictly positive payment to the firm ($V_t^L = 0$). This result is intuitive. Imagine indeed that $V_t^L > 0$. This is possible only if both bank and market lenders have been repayed, so that necessarily $\chi\pi_t - R_{b,t} - R_{m,t} > 0$. But it is then also the case that $\pi_t - R_{b,t} - R_{m,t} \geq \chi\pi_t - R_{b,t} - R_{m,t} > 0$. In that case, the firm would be better off by simply paying its creditors.¹³

2.4 Debt pricing and feasible debt structures

Banks and market lenders are perfectly competitive financial intermediaries, and have constant marginal lending costs r_b (for banks) and r_m (for markets); I come back below to the equilibrium determination of these lending costs. Perfect competition implies that lenders will make zero expected profits, in equilibrium, on each loan.¹⁴ Therefore, equilibrium lending terms $R_{b,t}$ and $R_{m,t}$ must satisfy:

$$\begin{aligned} \int_{\phi_t \geq 0} \tilde{R}_b(\phi_t, e_t + b_t + m_t, R_{b,t}, R_{m,t}) dF(\phi_t) &= (1 + r_b)b_t \\ \int_{\phi_t \geq 0} \tilde{R}_m(\phi_t, e_t + b_t + m_t, R_{b,t}, R_{m,t}) dF(\phi_t) &= (1 + r_m)m_t \end{aligned} \quad (7)$$

Here, $\tilde{R}_i(\phi_t, k_t, R_{b,t}, R_{m,t})$, $i = b, m$, denote the gross return on lending for banks and market lenders.

¹³Note that the decision to liquidate, repay or restructure does not depend on the continuation value function $V^c(\cdot)$. Since, by lemma 1, this continuation value is increasing, the decision to restructure or repay takes the form of a simple threshold rule for π_t , or equivalently, for ϕ_t . Analogously, because $V^c(0) \geq 0$, the liquidation decision also takes the form of a simple threshold rule.

¹⁴In particular, perfect competition precludes financial intermediaries from imposing tougher lending terms on certain firms in order to subsidize lending to others.

This return is conditional on the realization of the idiosyncratic productivity shock as well as the firm's debt structure, since both determine the debt settlement outcomes, and therefore the payments to financial intermediaries.¹⁵

The lending menu $\mathcal{S}(e_t)$ is defined as the set of all debt structures $(b_t, m_t) \in \mathbb{R}_+^2$ for which there exists a unique solution to (7). It is the set of feasible debt structures for a firm with internal finance e_t .

Proposition 2 *The lending menu $\mathcal{S}(e_t)$ is non-empty and compact for all $e_t \geq 0$. Moreover, $\mathcal{S}(e_t)$ can be partitioned into two non-empty and compact subsets $\mathcal{S}_K(e_t)$ and $\mathcal{S}_R(e_t)$, such that:*

- *The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t}}{1-\chi}$, if and only if, $(b_t, m_t) \in \mathcal{S}_R(e_t)$;*
- *The lending terms $(R_{b,t}, R_{m,t})$ satisfy $\frac{R_{b,t}}{\chi} < \frac{R_{m,t}}{1-\chi}$, if and only if, $(b_t, m_t) \in \mathcal{S}_K(e_t)$.*

Proposition 2 provides a partition of the feasible set of debt structures according to the nature of the debt settlement outcomes that they lead to. In the first subset, $\mathcal{S}_R(e_t)$, debt structures (b_t, m_t) imply liabilities $(R_{b,t}, R_{m,t})$ such that restructuring is sometimes successful during debt settlement. On the other hand, in the second subset, $\mathcal{S}_K(e_t)$, debt structures are such that restructuring never occurs during debt settlement. The subsets $\mathcal{S}_K(e_t)$ and $\mathcal{S}_R(e_t)$ are depicted in figure 4. Their relative location conveys the same intuition that underpins proposition 1: ex-post, debt structures tend to lead to successful restructuring when they are tilted towards bank loans (i.e., towards the lower left part of the positive orthant in figure 4); when they are tilted towards market debt (i.e., towards the upper part of the positive orthant in figure 4), restructuring does not occur during debt settlement outcomes.¹⁶

2.5 The firm's dynamic debt structure problem

The firm's problem can now be written recursively as:

$$V(e_t) = \max_{(b_t, m_t) \in \mathcal{S}(e_t)} \int_{\max(n_t^P, n_t^R) \geq 0} V^s(\pi(\phi_t, k_t), R_{b,t}, R_{m,t}) dF(\phi_t) \quad (8)$$

¹⁵For example, when $\frac{R_{m,t}}{\chi} \leq \frac{R_{b,t}}{1-\chi}$, the gross lending return function for market lenders is given by:

$$\tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}) = \begin{cases} 0 & \text{if } \pi(\phi_t, k_t) \leq \frac{R_{m,t}}{\chi} \\ R_{m,t} & \text{if } \pi(\phi_t, k_t) > \frac{R_{m,t}}{\chi} \end{cases}$$

The first case corresponds to realization of the productivity shock sufficiently low that the firm is forced into liquidation. The second case corresponds to realizations of the productivity shock such that the firm will either choose to pay its creditors in full, or will be able to successfully restructure debt payments with the bank.

¹⁶Specifically, debt structures in $\mathcal{S}_R(e_t)$ have a ratio of bank to total debt strictly above a threshold $s^{R,min}$, where $\frac{1-s^{R,min}}{s^{R,min}}$ is the slope of the upper solid line in figure 4. On the other hand, debt structures in $\mathcal{S}_K(e_t)$ have a ratio of bank to total debt strictly below a threshold $s^{K,max}$, where $\frac{1-s^{K,max}}{s^{K,max}}$ is the slope of the lower solid line in figure 4.

$$\begin{aligned}
\text{s.t.} \quad k_t &= e_t + b_t + m_t && \text{(Capital structure)} \\
\pi(\phi_t, k_t) &= \phi_t k_t^\zeta + (1 - \delta)k_t && \text{(Production)} \\
(1 + r_b)b_t &= \int_{\phi_t} \tilde{R}_b(\phi_t, k_t, R_{b,t}, R_{m,t}) dF(\phi_t) && \text{(Debt pricing, bank)} \\
(1 + r_m)m_t &= \int_{\phi_t} \tilde{R}_m(\phi_t, k_t, R_{b,t}, R_{m,t}) dF(\phi_t) && \text{(Debt pricing, market)} \\
V^s(\pi(\phi, k_t), R_{b,t}, R_{m,t}) &= V^c(\max(n_t^P, n_t^R)) && \text{(Debt settlement)} \\
n_t^P &= \pi(\phi_t, k_t) - R_{b,t} - R_{m,t} && \text{(Repayment)} \\
n_t^R &= \pi(\phi_t, k_t) - \chi\pi(\phi_t, k_t) - R_{m,t} && \text{(Restructuring)} \\
V^c(n_t) &= \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}) && \text{(Dividend issuance)}
\end{aligned}$$

This formulation incorporates results from the three stages of the firm's problem discussed in the previous paragraphs. First, the debt structure chosen by the firm at the beginning of the period must be feasible: $(b_t, m_t) \in \mathcal{S}(e_t)$. Second, the expression for the value of the firm at the debt settlement stage, $V^s(\pi(\phi_t, k_t), R_{m,t}, R_{b,t})$, uses the results of proposition 1.¹⁷ Finally, the value of the firm at the dividend issuance is the sum of the current value of dividends issued and the discounted value of future dividends. A solution to this problem is characterized by a threshold for internal finance \bar{e} as well as a value function V and policy functions \hat{b} and \hat{m} , all defined on $[0, \bar{e}]$.

2.6 Entry and exit

There are two sources of firm exit in this economy. First, some firms are endogenously liquidated at the debt settlement stage. Given that the realization of ϕ_t is independent of e_t , the fraction of existing firms with internal finance e_t that are liquidated is given by $F(\underline{\phi}(e_t, \hat{b}(e_t), \hat{m}(e_t)))$. Here, $\underline{\phi}(e_t, b_t, m_t)$ denotes the threshold such that firms with a productivity realization $\phi_t \leq \underline{\phi}(e_t, b_t, m_t)$ are liquidated.¹⁸ Second, a fraction η of firms is exogenously destroyed after the dividend issuance stage.

Let μ_t denote the distribution of firms over $[0, \bar{e}]$ at the beginning of period t . The total mass of firms exiting during period t is given by:

$$\delta^e(\mu_t) = \int_{e \in [0, \bar{e}]} \left(\overbrace{F(\underline{\phi}(e_t, \hat{b}(e_t), \hat{m}(e_t)))}^{\text{liquidations}} + \underbrace{((1 - F(\underline{\phi}(e_t, \hat{b}(e_t), \hat{m}(e_t)))) \eta)}_{\text{exogenous exits}} \right) d\mu_t(e_t).$$

The fraction $\delta^e(\mu_t)$ of exiting firms is replaced by an identical number of entering firms at the beginning of the following period. Entry involves two costs: the internal funds e_t^e of entering firms; and a fixed entry cost κ . The surplus associated to entering with internal finance e_t^e is given by $\beta V(e_t^e) - (\kappa + e_t^e)$. There is

¹⁷In particular, the firm is liquidated if and only if its resources under both restructuring and payment are strictly negative, so that the integrand in the objective function of the firm is truncated at 0.

¹⁸A complete characterization of the restructuring and liquidation thresholds is given in appendix B.

free entry, so that the surplus associated with entering is 0 and e_t^e must solve:

$$\beta V(e_t^e) = \kappa + e_t^e. \quad (9)$$

Given the entry scale, the mass of exiting firms, and the firm's optimal policy functions along with their dividend issuance policies, the law of motion for the distribution of firms across levels of internal finance can be expressed as: $\mu_{t+1} = M(\mu_t)$ where $M : \mathcal{M}(\bar{e}) \rightarrow \mathcal{M}(\bar{e})$ is a transition mapping over firm measures, and $\mathcal{M}(\bar{e})$ denotes the set of measures on $[0, \bar{e}]$ that are absolutely continuous with respect to the Lebesgue measure.¹⁹

2.7 Financial intermediation

Intermediaries use their funds to extend credit to firms. I assume that they face an identical opportunity cost of funds, but different intermediation costs.

Assumption 3 (Financial intermediation costs) *Banks and market lenders have a common opportunity cost of funds, given by $r = \frac{1}{\beta} - 1$. Their intermediation costs per unit of credit extended are given γ_b and γ_m . The wedge between bank and market-specific intermediation costs is strictly positive: $\theta = \gamma_b - \gamma_m > 0$.*

Equilibrium financial intermediation costs for banks and markets are thus given by:

$$r_m = r + \gamma_m, \quad r_b = r + \gamma_b, \quad r = \frac{1}{\beta} - 1. \quad (10)$$

The restriction that $r = \frac{1}{\beta} - 1$ can be thought of as a general equilibrium outcome. Indeed, it would hold in a model in which intermediaries raise deposits from a representative, risk-neutral household. In such a model, perfect competition in the market for deposits would impose that $\beta(1+r) = 1$.²⁰ Alternatively, the restriction $\beta(1+r) = 1$ would hold if both financial intermediaries and firms had access to a risk-free technology offering a rate of a rate of return r . I come back to the assumptions about intermediation costs below.

2.8 Equilibrium

Definition 1 (Recursive competitive equilibrium) *A recursive competitive equilibrium of this economy is given by value functions V , V^s and V^c , an upper bound on internal finance \bar{e} , policy functions \hat{d}^v , \hat{b} , \hat{m} ,*

¹⁹The detailed expression of the mapping M is reported in appendix B.

²⁰The online appendix to the paper spells out a general equilibrium version of this model. This model is notationally more burdensome, but leads to identical equilibrium outcomes.

equilibrium lending costs r_b and r_m , equilibrium lending terms R_b and R_m , an entry size e^e , a distribution of firm size μ and a transition mapping M , such that:

- given \bar{e} , the value functions solve problem (8), and $\hat{d}iv$, \hat{b} , \hat{m} , are the associated policies;
- given the value function V , the upper bound \bar{e} satisfies condition (2);
- equilibrium lending costs satisfy (10);
- equilibrium lending terms satisfy the zero profit conditions of intermediaries (7);
- the entry scale e^e satisfies the free-entry condition (9);
- the transition mapping M is consistent with firms' policies and with the entry scale of firms e^e ;
- the distribution μ is a fixed point of M .

Proposition 3 (Existence of a recursive competitive equilibrium) *There exists a recursive competitive equilibrium of this economy.*

The proof of proposition 3 is given in appendix B. It uses two key insights mentioned in the exposition of the model: (1) the structure of the feasible set of debt contracts does not depend on the value function V , but only on internal funds e_t ; (2) the feasible set can be split into two subsets, associated with different types of debt settlement outcomes. Both follow from proposition 2.

The first step of the proof is to establish the existence of a unique solution to problem (8). In general, the firm's problem involves solving a triple fixed point problem, where the value function $V(\cdot)$, the upper bound \bar{e} , and the constraint correspondence $\mathcal{S}(\cdot) : [0, \bar{e}] \rightarrow \mathbb{R}_+^2$ must be simultaneously determined. However, given the first insight, the problem reduces to a double fixed point problem in \bar{e} and $V(\cdot)$, analogous to Cooley and Quadrini (2001). However, unlike that paper, standard approaches do not directly apply, because of the non-convexities in the interim value function of the firm. The second insight suggests using the fact that the problem is equivalent to $V(e_t) = \max_{K,R} (V_K(e_t), V_R(e_t))$, where $V_K(e_t)$ denotes the continuation value of a firm restricted to use debt structures that are in $\mathcal{S}_K(e_t)$, and $V_R(e_t)$ is analogously defined. These two sub-problems can be analyzed using standard methods.

The second step of the proof of proposition 3 is to derive the expression of the transition mapping M , and show that it has a fixed point. This part of the proof also uses the decomposition of the feasible set of debt structures, because transition probabilities between levels of internal finance depend on the type of debt settlement outcomes the firm will face (given the debt structure it chooses). Given the expression for M provided in appendix B, standard approaches, such as those described in Stokey, Lucas, and Prescott (1989), can be used to prove that M has a fixed point.²¹

²¹The sufficient conditions of monotonicity of the transition kernel developed by Stokey, Lucas, and Prescott (1989), or the weaker conditions of Hopenhayn and Prescott (1987), cannot be established for every point in the state-space of the firms, because of discrete nature of the choice between types of debt settlement outcomes. However, the issue of unicity never arises numerically.

2.9 Discussion

I now come back to the discussing key assumptions about the settlement of debt contract (in particular the flexibility of banks relative to markets); and about the intermediation costs of banks, relative to markets.

Liquidation The first key assumption about the liquidation process (assumption 1) is that it involves losses of output: $\chi < 1$. This assumption is common to many models in which the underlying financial friction is limited liability. It embodies the notion that bankruptcy and liquidation are costly processes, and is supported by evidence on changes in asset values of firms that go through bankruptcy proceedings (see, for example, [Bris, Welch, and Zhu \(2006\)](#)).

The second key assumption about liquidation is the seniority of bank lenders. This is motivated by two considerations. First, empirically, bank loans tend to be either senior, or secured by liens on assets, as documented by [Rauh and Sufi \(2010\)](#). Second, in this model, putting bank debt ahead in the priority structure improves the firm's ability to issue bank debt, because it enhances the bank's claim in liquidation. Moreover, the ability to issue bank debt cheaply is valuable, in particular to small firms, because it allows them to expand faster. Thus, given the choice, firms would likely prefer to put bank loans ahead of the priority structure.²²

Bank flexibility The assumption that banks are more flexible in distress than markets (assumption 2) receives considerable support in the data. [Gilson, Kose, and Lang \(1990\)](#) show that, in a sample of 169 financially distressed firms, the single best predictor of restructuring success is the existence of bank loans in the firm's debt structure. [Denis and Mihov \(2003\)](#), in a sample of 1560 new debt financings by 1480 public companies, show that bank debt issuances have more flexibility in the timing of borrowing and payment, and that firms with higher revenue volatility tend to issue more bank debt. [Bolton and Scharfstein \(1996\)](#) provide a theoretical rationale for bank flexibility, by noting that ownership of market debt tends to be more dispersed than ownership of bank debt. This creates a free-rider problem, as market creditors have little individual incentive to participate in debt renegotiations. The assumption of bank debt flexibility maintained in this model thus captures the consequences of differences in creditor concentration between classes of debt, on firms' ability to successfully restructure debt contracts.

Intermediation costs The motivations underlying assumption 3 are the following. First, the fact that financial intermediation is costly ($\gamma_b, \gamma_m > 0$) is not controversial: [Philippon \(2012\)](#) provides recent and

²²In a closely related setup, [Hackbarth, Hennessy, and Leland \(2007\)](#) establish that bank debt seniority is the optimal priority structure. The optimality of bank seniority is also a feature of other models of debt structure, in which banks' role is to provide ex-ante monitoring of projects, such as for example [Besanko and Kanatas \(1993\)](#), [Park \(2000\)](#), or [DeMarzo and Fishman \(2007\)](#). The rationale for bank seniority, in these models, is that increases banks' return on monitoring, by allowing them to seize more output in liquidation. This is distinct from, but related to the model I consider here, where seniority allows firms to operate at larger scales early on.

comprehensive evidence that overall intermediation costs in the US financial sector have averaged approximately 2% between 1870 and 2012. The assumption specific to this model is that these intermediation costs are larger for banks than for markets. This assumption captures three key differences between the costs associated with bank and market lending:

1. Bank lenders place more stringent requirements on lenders outside of financial distress than markets, in particular, tighter loan covenants, as documented by [Demiroglu and James \(2010\)](#) and [Rauh and Sufi \(2010\)](#). The positive lending wedge is a reduced-form way of capturing tighter bank lending requirements: indeed, in the model, the wedge will be reflected in higher equilibrium lending terms for banks loans ($R_{b,t}$) outside of financial distress.
2. Banks specialize in costly activities related to lending and which markets typically shun. In particular, banks engage in screening and monitoring of borrowers, as documented in, for example, [Berger and Udell \(1995\)](#), [Houston and James \(1996\)](#) or [Mester, Nakamura, and Renault \(2007\)](#). The positive lending wedge then captures costs associated to these bank-specific activities.
3. Banks face specific regulatory environments that have an impact on their lending costs. In particular, capital requirements require firms to issue additional equity in order to expand their deposit and lending base. However, banks typically finds it costly to adjust their equity base (see [Adrian and Shin \(2011\)](#) for evidence on this topic). This mechanism contributes to making marginal loan issuance more costly for banks.

3 Financial policies in steady-state

I now turn to a description of the equilibrium financial policies of firms. The model has no closed form solution. Instead, this section focuses on a baseline calibration of the model, summarized in table 1.²³

3.1 A baseline calibration of the model

The frequency of the model is annual. Standard model parameters (the discount rate, the rate of depreciation of capital and the degree of returns to scale) are calibrated using existing estimates from the literature.

Idiosyncratic productivity shocks follow a Weibull distribution.²⁴ I pick the location and scale parameters to match two targets: (1) $\bar{k} = \left(\frac{\mathbb{E}(\phi)\zeta}{\delta+r+\gamma_m} \right)^{\frac{1}{1-\zeta}} = 100$ and (2) $\sigma(\log(\phi)) = 0.62$. The former is a normalization

²³The numerical solution procedure is standard and is described in the online appendix to the paper.

²⁴The Weibull distribution has a strictly increasing hazard rate, which is sufficient to ensure the unicity of the lending terms that solve equation 7. I discuss sufficient conditions for the unicity of lending terms in more detail in [Crouzet \(2013\)](#).

that ensures that firms' total size is smaller than or equal to \bar{k} . The latter value corresponds to estimates of the cross-sectional standard deviation of output-based measures of firm-level (log) productivity reported by [Bartelsman, Haltiwanger, and Scarpetta \(2009\)](#) for the US manufacturing sector.²⁵ This value is line with other estimates of output-based productivity measures for the US, such as for example [Hsieh and Klenow \(2009\)](#). The choice of the fixed entry cost κ imply that the size of entrants is 5% of the maximum size of firms in the economy.

In order to calibrate the magnitude of deadweight losses in liquidation, I use evidence reported by [Bris, Welch, and Zhu \(2006\)](#). They analyze a sample of 61 chapter 7 liquidations in Arizona and New York between 1995 and 2001. Their median estimate of the change in asset values pre- to post- chapter 7 liquidation is 38%.²⁶ I therefore use $\chi = 0.38$.

As a proxy for market-specific lending costs γ_m , I use existing estimates of underwriting fees for corporate bond issuances. [Fang \(2005\)](#) studies a sample of bond issuances in the US, and finds an average underwriting fee of 0.95%, while [Altinkilic and Hansen \(2000\)](#), in a sample including lower-quality issuances, find a an average underwriting fee of 1.09%.²⁷ Given this evidence, I set market-specific intermediation costs to $\gamma_m = 0.0100$.

Measuring analogously intermediation costs of banks, for example from operating expenses reported in income statements of commercial banks, has two potential drawbacks. First, operating expenses of banks can be associated with a number of non-lending activities. Second, operating expenses may miss some costs associated with credit intermediation by banks, such as for example, equity issuance costs associated with capital or liquidity requirements. Therefore, instead of trying to construct a direct measure of γ_b , I match the aggregate bank share of US non-financial corporations reported in the Flow of Funds in 2007Q3. Loans and advances from banks and bank-like institutions accounted for 27.5% of credit market liabilities of US non-financial corporations at that date. Given other parameters of the model, matching this aggregate share requires bank-specific intermediation costs of $\gamma_b = 0.02255$.²⁸

3.2 Optimal debt structure

The key properties of the firm's optimal debt structure are reported in figures [5\(a\)](#) and [5\(b\)](#), and summarized in the following result. This result has no analytical proof, but holds for all numerical calibrations

²⁵See their online data appendix at http://econweb.umd.edu/~haltiwan/BHS_jobflows_productivity/.

²⁶See their table III, p. 1265. This number adjusts for the value of collateralized assets that creditors may have seized outside of the formal bankruptcy proceedings.

²⁷In both cases, the authors report the "underwriting spread", that is, the ratio of proceeds to underwriters to the size of the issuance, which matches the definition of γ_m in the model.

²⁸Note that combining a risk-free rate of $r = 4.00\%$ with the value of bank intermediation costs γ_b , one arrives at a risk-free bank lending rates of 6.25%. This aligns relatively well with the average of the monthly Bank Prime Loan Rate during 2007Q3 (8.19%), once inflation is taken into account. In the model, firms borrowing from banks are never entirely risk-free, so that this number cannot be used to directly match the lending wedge.

Parameter	Description	Value	Parameter	Description	Value
ζ	degree of returns to scale	0.900	$\mathbb{E}(\phi)$	average productivity	0.264
δ	depreciation rate	0.100	$\sigma(\phi)$	s.d. of productivity	0.181
β	discount rate	1/1.040	χ	liquidation efficiency	0.380
η	exogenous exit rate	0.0095	γ_m	market intermediation costs	0.010
κ	entry cost	29.151	γ_b	bank intermediation costs	0.023

Table 1: Baseline calibration of the model.

under which I have solved the model, provided that $\theta = \gamma_b - \gamma_m > 0$.

Numerical Result 1 (The firm’s optimal debt structure) *Let $\hat{b}(e_t)$ and $\hat{m}(e_t)$ denote the policy functions associated to the solution to problem (8). There exists a unique value of internal finance $e^* \in [0, \bar{e}]$ such that:*

- For $e_t \in [0, e^*]$, $\hat{b}(e_t) > 0$, $\hat{m}(e_t) \geq 0$, and $(\hat{b}(e_t), \hat{m}(e_t)) \in \mathcal{S}_R(e_t)$;
- For $e_t \in]e^*, \bar{e}]$, $\hat{b}(e_t) = 0$, $\hat{m}(e_t) \geq 0$, and $(\hat{b}(e_t), \hat{m}(e_t)) \in \mathcal{S}_K(e_t)$.

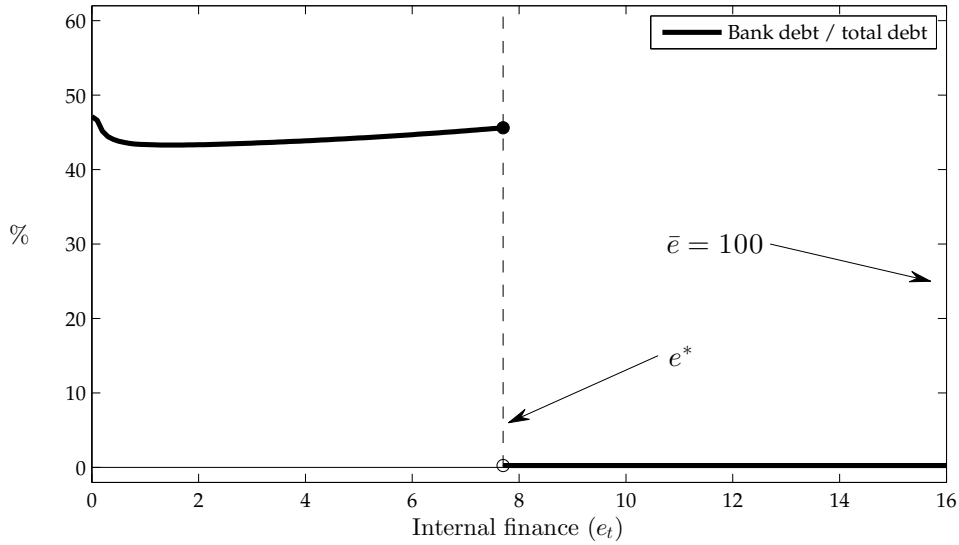
This result, illustrated in figure 5(a), indicates that firms’ debt structures fall under two broad categories. Firms with internal funds below e^* will choose a “mixed” debt structure, involving a combination of bank and market debt.²⁹ On the other hand, large firms – those firms with internal funds strictly above e^* – choose a “market-only” debt structure. As firms grow by accumulating internal funds from retained earnings, they will therefore switch from a mixed debt structure to a market-only debt structure.

The intuition for this result is as follows. Because of decreasing returns, firms with small e_t have a larger financing gap, and tend to be more leveraged than firms with large e_t . This also implies that they have a higher probability of financial distress. Since, all other things equal, borrowing more from banks reduces the expected losses associated with financial distress, firms with small e_t , seeking high leverage, have a strong incentive to use bank debt. One should therefore expect the composition of their debt to be more tilted towards bank loans. More generally, the trade-off between bank flexibility in times of financial distress, and the costs associated with using banks loans in normal times, changes with the level of the firm’s internal resources e_t , and ultimately affects the firms’ choice of debt structure.

This logic does not account for the fact that firms switch in a discrete manner to market finance when $e_t \geq e^*$. To understand this, it is useful to think back to the results of proposition 1. This proposition indicates that, if a firm’s market liabilities are sufficiently large, it never restructures bank debt in bad times. In that case, the flexibility associated with bank debt is irrelevant to the firm since, in equilibrium, the firm never uses that flexibility. But borrowing from banks results in large liabilities $R_{b,t}$ outside of financial distress, since the lending wedge $\theta = \gamma_b - \gamma_m$ is strictly positive. The net benefit of substituting a unit

²⁹The result indicates that $(\hat{b}(e_t), \hat{m}(e_t)) \in \mathcal{S}_R(e_t)$, so the share of bank debt in that mix is bounded from below by $s_{R,min}$ (see figure 4). These firms therefore always borrow in strictly positive amounts from banks.

(a) Optimal composition of debt.



(b) Optimal level of bank and market debt, and total assets.

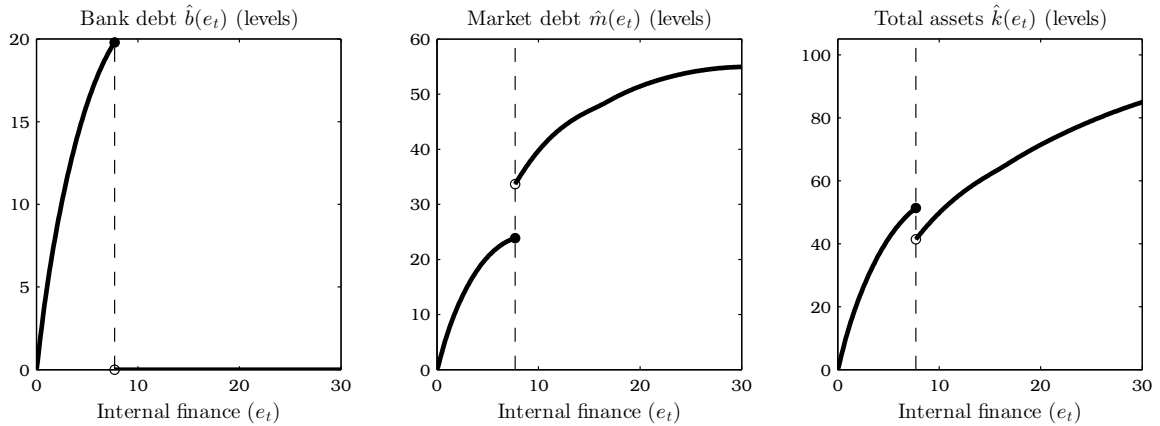


Figure 5: Optimal debt structure in steady-state.

of bank debt for a unit of market debt, for these firms who do not restructure ex-post, is therefore always strictly positive. As a result, these firms choose the corner solution $\hat{b}(e_t) = 0$, $\hat{m}(e_t) > 0$.

Apart from the decline of the bank share with internal finance, the second key aspect of firms' optimal debt choices is that they imply non-monotonicities in the borrowing and investment policies of firms. This is illustrated in figure 5(b), which reports bank borrowing $\hat{b}(e_t)$ (left panel), market borrowing $\hat{m}(e_t)$ (middle panel), and total assets $\hat{k}(e_t) = e_t + \hat{b}(e_t) + \hat{m}(e_t)$ (right panel), as a function of internal funds e_t . In each of the two regions $e_t \leq e^*$ and $e_t > e^*$, the amounts borrowed from banks and markets are increasing (or equal to 0 for bank borrowing when $e_t > e^*$). However, when a firm crosses the threshold e^* , total assets and total borrowing fall: bank debt is not replaced one for one by market debt. This effect can be interpreted as a precautionary response of firms that migrate from a mixed to a market debt structure. At the point

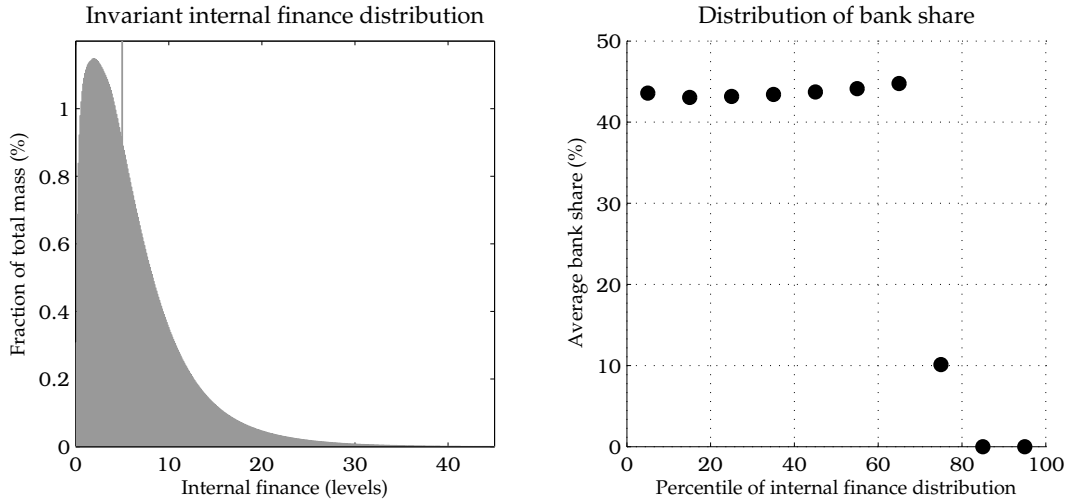


Figure 6: Bank share and internal finance in the cross-section of the model.

$e_t = e^*$, firms are exactly indifferent between mixed and market debt structures. Imagine that firms were to choose the same overall leverage under the market debt structure than under the mixed debt structure. Since, under the market debt structure, firms have no option to restructure debt in bad times, their ex-ante likelihood of liquidation would be much higher than under the mixed debt structure. Firms offset this effect by operating under a lower leverage, as they switch between financial regimes. This feature of the optimal debt structure is one example of the real implications of imperfect substitutability between types of debt.

3.3 Debt structure in the cross-section

Are the model's cross-sectional predictions about variation in debt structure consistent with the data? Figure 6 reports, on the left panel, the steady-state distribution of firms across levels of internal finance. The key property of this distribution is that it is strongly skewed to the left. This occurs both because the entry size of firms is relatively small, and because small firms are much more frequently liquidated than large firms. Because of this left-skewness, the share of bank loans as a fraction of total borrowing is high for most firms, and only declines for the largest firms. This is reported in the right panel of figure 6. Each point in this graph corresponds to the median bank share of firms within a particular decile of the distribution of internal finance. The bottom 72.5% of firms use both bank and market debt, while the top 27.5% use only market debt.

Rauh and Sufi (2010) provide evidence on the composition of debt in a sample of 305 publicly traded and rated US firms between 1996 and 2006, combining data from their 10-K filings from data on issue origination. They analyze debt structure both from the standpoint of type and priority. They have two key findings. First, a majority of firms simultaneously have outstanding issues of different types. 68.3% of firm-year observations use at least two types of debt; among firms using bank debt (52.6% of the total),

70.5% also use straight or convertible bond debt.³⁰ Second, the degree to which the debt structure of firms is “spread out” across types and priorities is strongly related to firms’ credit ratings. Investment-grade firms (with ratings of BBB and above) mostly use senior unsecured debt (bond and program debt), while firms speculative grade firms (with ratings of BB and below) use a combination of secured debt, senior unsecured debt, and subordinated bonds.³¹ Moreover, their results indicate that “the increase in secured debt as credit quality deteriorates is driven almost exclusively by an increase in secured bank debt, and the increase in subordinated debt is driven almost exclusively by an increase in subordinated bonds and convertibles”.

The cross-sectional variation of debt structure in the model captures these two facts well. First, a majority of firms use a “mixed” debt structure, combining bank with market debt.³² The intuition for this results is that by increasing market debt issuance, firms increase the scale of total investment, and therefore liquidation value. This in turn loosens firms’ borrowing constraint with respect to banks, who are senior in the priority structure, and allows firms to increase bank debt issuance. In this manner, the endogenous investment scale decision creates a complementarity between types of debt, which accounts for the existence of an interior solution.³³ Second, in the model, as firms rely increasingly on internal finance, they move from a mixed debt structure to a market-only debt structure. The key driver of this shift is that, as firms’ internal finance increases and their leverage falls, the probability that they will face financial distress ex-post declines, making the use of bank debt less attractive. Generically, firms with “market-only” debt structures, in the model, have lower ex-ante liquidation probabilities. To the extent that credit ratings proxy for a firm’s credit risk, the model thus captures well the correlation between credit ratings and the concentration of debt structure.

Colla, Ippolito, and Li (2013) provide additional evidence on the debt structure of all publicly traded US firms, rated and unrated, using information on debt structure provided by Capital IQ and gathered from 10-K filings. Their findings partially support the evidence of Rauh and Sufi (2010) for rated firms, but suggest that unrated firms have a higher degree of concentration in their debt structures. Specifically, they find that 36.6% of firm-year observations of rated firms concentrate 90% or more of their debt within a single type (mostly senior bonds and notes); the remainder are diversified across debt types. On the other hand,

³⁰Table 2, p. 11.

³¹Program debt encompasses debt issuances which do not require SEC filing, such as commercial paper, medium-term notes and shelf-registered debt.

³²Note that, while the model does explicitly target the ratio of “mixed” firms to “market-only” firms, the baseline calibration indicates that 72.5% of firms use a mixed debt structure, close to the 68.3% of observations which simultaneously have issues of different types.

³³Many existing models of the choice between bank and non-bank debt, in which investment scale is not endogenous, instead have the prediction that firms issue a single type of debt; this is the case, for example, in Diamond (1991), Chemmanur and Fulghieri (1994), Cantillo and Wright (2000) or Bolton and Freixas (2000). Among models addressing simultaneously the choice of priority and type of debt in an optimal contracting framework, Besanko and Kanatas (1993) and Park (2000) both find interior solutions. However, in these models, the generic intuition for the result is different from the model studied here: combining a senior, monitoring creditor (a bank) and junior creditors reduces the bank’s stake in the firm, making any liquidation threats by the bank more credible. By allowing other creditors in the debt structure, banks are therefore better able to deter sub-optimal effort or risk-shifting behavior by the firm.

by the same measure, 51.3% of unrated firms specialize in a single type of debt, 27.2% of them in credit lines or term loans. Thus, while the model provides a good account of the debt structure choices of rated firms, its predictions underestimate the degree of debt specialization among unrated firms. This shortcoming of the model is mitigated by the fact that rated firms account for the majority of debt issuance and assets of public firms. [Faulkender and Petersen \(2006\)](#) find that, in their sample of publicly traded firms between 1986 and 2000, traded firms account for 78% of all debt issues, while [Colla, Ippolito, and Li \(2013\)](#) report that rated firms account for 91% of total assets in their sample.³⁴

There is limited existing evidence, to my knowledge, focusing specifically on the relationship between debt structure and internal finance, the state variable of firms in this model.³⁵ [Figure 14](#), in appendix, looks at this relationship in the data, for publicly traded firms in various (OECD and non-OECD) countries. These graphs document a pattern qualitatively similar to the predictions of the model: the average share of bank debt rapidly declines as firms move up in the distribution of internal finance, in most OECD countries, as well as in some non-OECD countries in which the use of public debt is more wide-spread.³⁶

3.4 Other aspect of firms' policies

The previous discussion indicates that the model's predictions about firm-level debt structure align well with the data. Does this come at the expense other predictions? [Figure 7](#) addresses this question. With respect to financial policies, the model has the feature that (1) firms with small internal resources are generally more leveraged (top row, left panel), as previously mentioned; and that (2) firms with small internal resources have a higher rate of profit, but distribute more dividends (top row, middle panel). These features of firms' financial policies are broadly similar to the results of [Cooley and Quadriani \(2001\)](#), and consistent with empirical facts on financial behavior of firms documented in, e.g., [Fazzari, Hubbard, and Petersen \(1988\)](#) and [Gilchrist and Himmelberg \(1998\)](#). With respect to firm dynamics, the key predictions of the model can be summarized as follows: (1) small firms experience a higher rate of growth, either measured in terms of internal funds, output, or total assets (top row, right graph; bottom row, left and middle graphs); (2) small firms have a higher volatility of growth (bottom row, right graph).³⁷ These facts also align well with empirical evidence on the relationship between size and growth dynamics.³⁸ Note that, as was the case for financial policies, the switch between financial regimes is associated with changes in the growth dynamics of

³⁴See p.2118.

³⁵An exception is [Colla, Ippolito, and Li \(2013\)](#), who report correlations between the degree of debt specialization and various firm characteristics. Their measure of debt specialization is most positively correlated with cash holdings, consistent with the model's predictions; see their table VII, p.2131.

³⁶The definition of internal finance adopted in this graph follows closely that of the model: namely, it is measured as the difference between the book value of firms' assets and liabilities. Details on the definition of variables are given in [appendix A](#).

³⁷The online appendix to the paper gives the exact definition of these variables in terms of the optimal policy functions of firms.

³⁸See for example [Evans \(1987\)](#), or more recently [Davis, Haltiwanger, Jarmin, and Miranda \(2006\)](#).

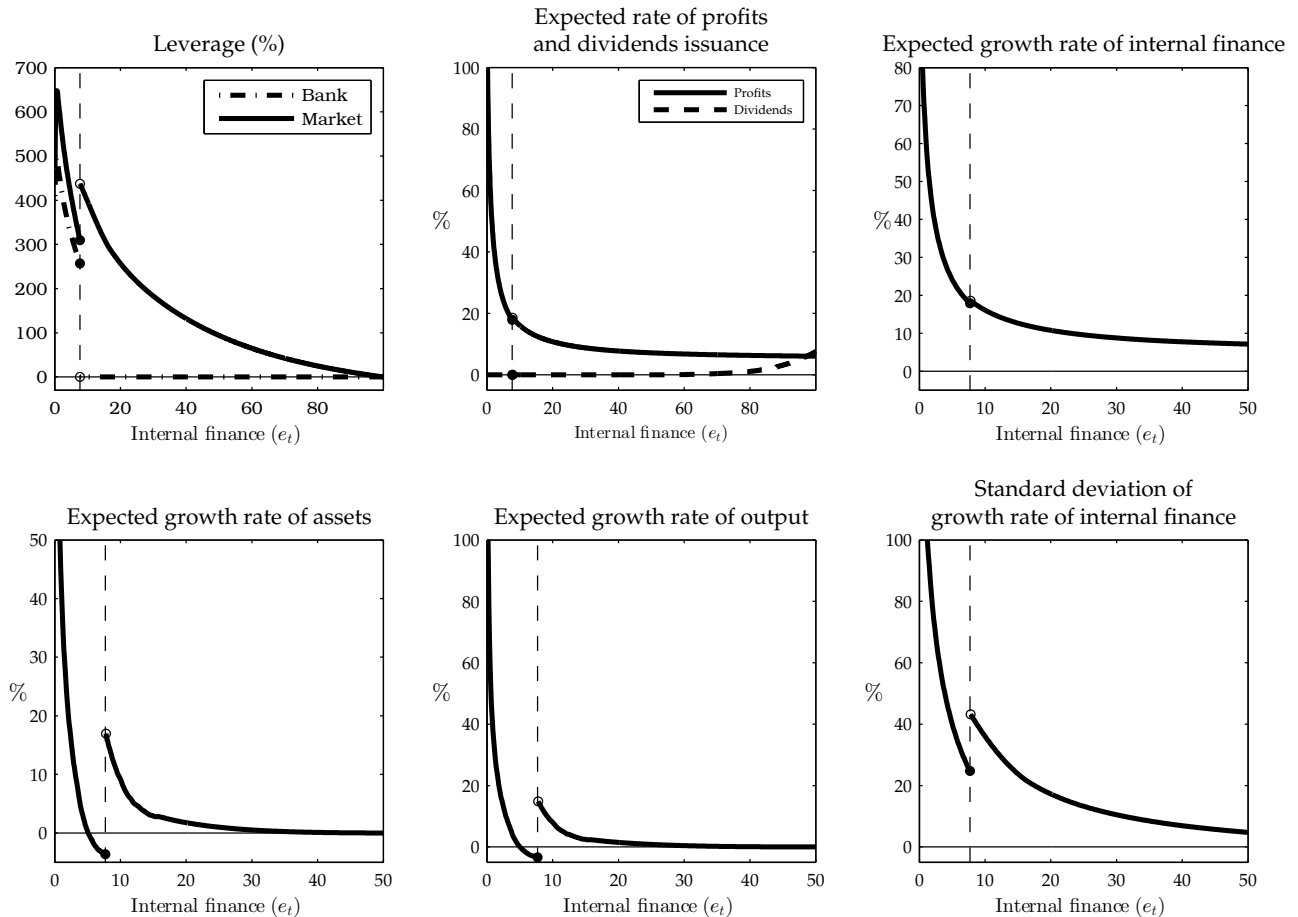


Figure 7: Other aspects of firms' financial policies and implied growth dynamics.

firms. Namely, firms immediately to the left of the switching threshold experience negative expected growth rates, as they anticipate the fact that reaching the switching threshold will imply a reduction in the scale of their operations. The volatility of growth of firms that switch to market-only debt also increases, in line with the intuition that the debt structure adopted by these firms increases their exposure to liquidation risk.

This section has established that the model's firm-level and cross-sectional predictions are consistent with documented patterns, in particular regarding debt structure. I next turn to studying its aggregate implications.

4 Long-run differences in credit intermediation

Figure 8 illustrates long-run differences in the structure of credit intermediation across countries, by plotting the ratio of bank loans to total debt of non-financial firms, in sample of developed and developing countries.³⁹ While the aggregate bank share is broadly negatively related to output per capita, there is considerable variation, even within developed countries. This section investigates the extent to which the

³⁹This graph uses balance sheet data on public firms to construct the ratio, so that the US bank share is lower than that obtained using Flow of Funds data, which include all firms in the economy. See appendix A for more details on the data.

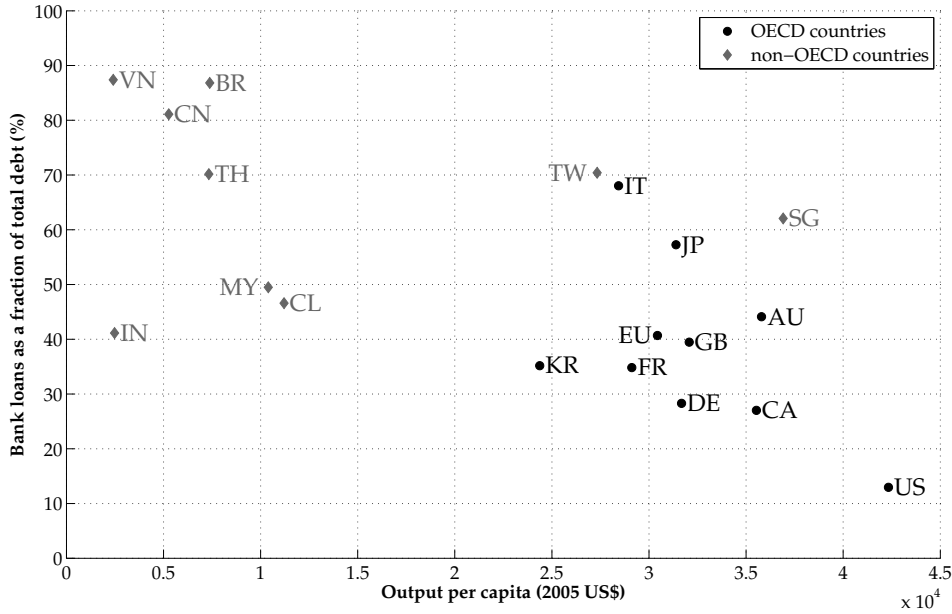


Figure 8: Average ratio of bank loans to total debt in the corporate sector across countries, 2000-2007.

model of section 2 can account for this variation.

4.1 Comparative statics of the aggregate debt structure

The aggregate bank share, in the model, is given by:

$$S = \frac{\int_{e_t \in [0, \bar{e}]} \hat{b}(e_t) d\mu(e_t)}{\int_{e_t \in [0, \bar{e}]} (\hat{b}(e_t) + \hat{m}(e_t)) d\mu(e_t)}.$$

Variation in any structural parameter of this economy always affects the composition of debt at the firm level, that is, the schedules $\hat{b}(e_t)$ and $\hat{m}(e_t)$. However, at the aggregate level, these changes may wash out. The comparative statics of the model indeed reveal that the aggregate bank share S only depends on a subset of structural parameters.

What leaves the composition of credit unchanged? Figure 9 (1) looks at economies where average productivity $\mathbb{E}(\phi)$ is lower than in the baseline economy of table 1. Naturally, in these lower-productivity economies, total output is lower, as reported in the right panel of figure 9 (1).⁴⁰ The bank share, however, is very close to that of the baseline calibration. Lower aggregate productivity is associated with lower total borrowing by all firms, and also reduces the maximum operation size of firms \bar{e} , but it does not alter the firm-level composition of borrowing.⁴¹ Differences in average productivity thus have level effects, but do no composition effects, on borrowing. Analogously, figure 9 (2) indicates that differences in the risk-free rate r

⁴⁰ Aggregate output, in the steady-state of the model, is defined as $Y = \int_{\phi_t \geq 0, e_t \in [0, \bar{e}]} \phi_t (e_t + \hat{b}(e_t) + \hat{m}(e_t))^\zeta d\mu(e_t) dF(\phi_t) = \mathbb{E}(\phi) \int_{e_t \in [0, \bar{e}]} (e_t + \hat{b}(e_t) + \hat{m}(e_t))^\zeta d\mu(e_t)$.

⁴¹ Firm-level borrowing policies for the baseline case and alternative calibrations, for all comparative statics considered in this section, are reported in the online appendix.

(or equivalently, the discount factor β) do not have aggregate composition effects.

What affects the composition of credit? First, differences in firm-level characteristics can be associated with large differences in the composition of debt. The left panel of figure 9 (3) shows that higher productivity dispersion is associated with a higher aggregate bank share. Two factors contribute to this: first, among mixed debt structure firms, the amount of bank borrowing is higher when productivity dispersion is higher (as it makes bank debt flexibility more attractive); second, more firms use a mixed debt structure when productivity is higher (as liquidation is more frequent, the steady-state distribution of is more skewed to the left, and fewer firms reach the maximum size \bar{e}). Note that aggregate output is also lower in high-dispersion economies. This reflects more constrained borrowing by small and medium-sized firms; borrowing policies of large firms, relative to the baseline, are almost unchanged.

Second, structural parameters capturing the intensity of financial frictions (intermediation costs γ_b and γ_m , and liquidation efficiency χ) also affect the composition of credit. Figure 9 (4) shows that economies in which market intermediation costs are high relative to bank intermediation costs (for example, because of costly underwriting, or because of the costs associated with obtaining a credit rating) have a higher aggregate bank share.⁴² This is mostly due to fewer firms operating under a market debt structure, rather than mixed debt structure firms relying more on bank loans. Figure 9 (5) shows that, somewhat counter-intuitively, a higher liquidation efficiency leads to a higher aggregate bank share. On one hand, more medium-sized firms use a market-only debt structure when liquidation efficiency is higher; on the other, mixed debt structure firms have higher liquidation values, and therefore higher bank debt capacities. Quantitatively, the latter effect dominates.

The right panels of figures 9 (4) and 9 (5) indicate that variation in these parameters also has a large impact on output. However, while variation in the intermediation wedge $\theta = \gamma_b - \gamma_m$ generates a negative relationship between bank share and output, variation in liquidation efficiency generates a positive relationship.

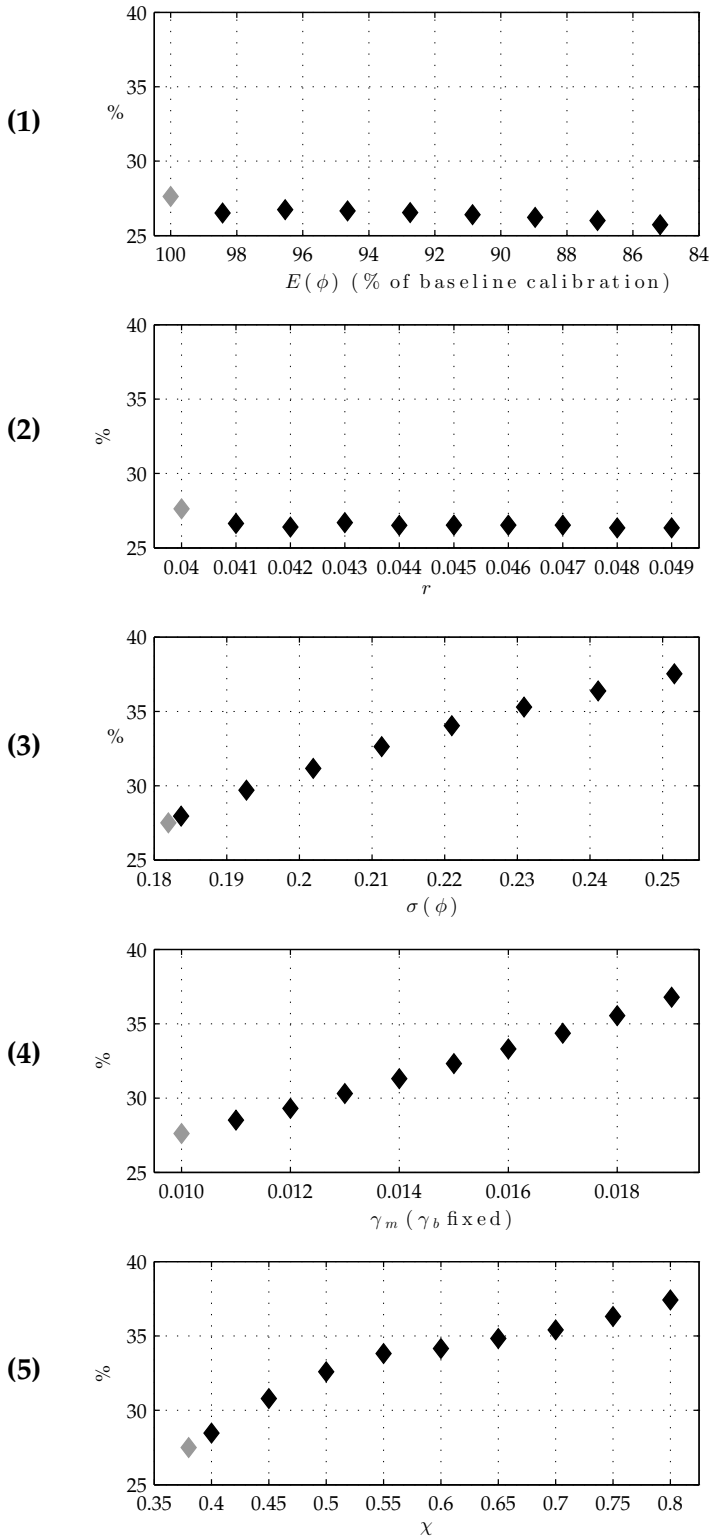
4.2 A quantitative example: the US and Italy

Given the previous comparative statics, can the model account quantitatively for observed differences in the aggregate bank share? If so, do firm characteristics (as captured by $\sigma(\phi)$) or differences in the intensity of financial frictions (as captured by γ_b , γ_m and χ) account for the bulk of these differences? I study this question in the case of two specific countries: Italy and the US. Italy is of particular interest because, among OECD economies, it is one of the most bank-oriented economies. In 2007Q3, bank loans accounted for 65.7%

⁴²The aggregate bank share does not depend on the absolute levels of intermediation costs, for reasons analogous to the effects of average productivity.

Aggregate bank share

(Percent; grey marker indicates baseline calibration)



Aggregate output

(Percentage of steady-state output in baseline calibration; grey marker indicates baseline calibration)

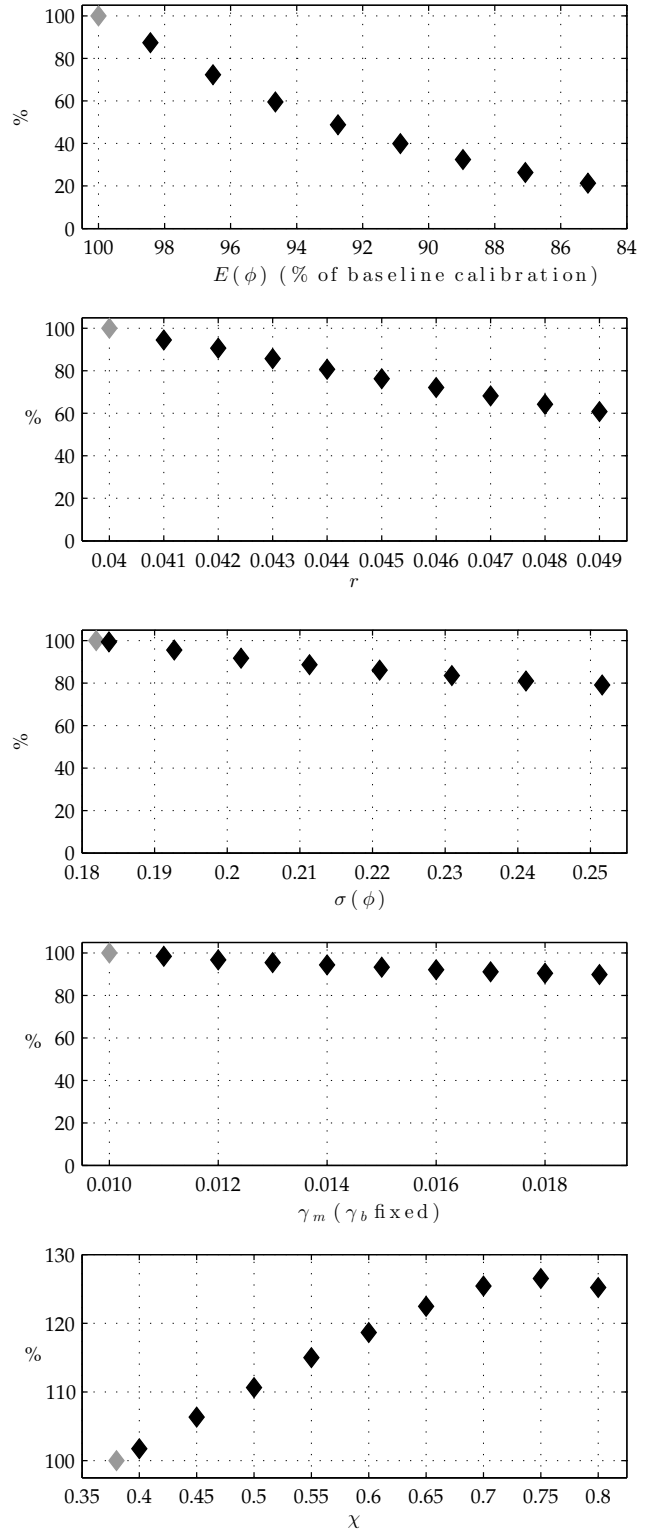


Figure 9: Comparative statics for aggregate bank share and aggregate output. The left column reports the ratio S . The right column reports aggregate output Y in different calibrations, expressed as a percentage of the baseline calibration reported in table 1. The grey point in each graph corresponds to that baseline calibration. Each line corresponds to a particular comparative static exercise; see text for details.

	Bank share	Debt/Assets	Output per capita
Data (US)	27.5%	71.9%	100%
Data (Italy)	65.7%	56.3%	67.1%
Baseline calibration (US)	27.3%	83.7%	100%
Alternative calibrations (Italy)			
High market intermediation costs			
$\gamma_{m,IT} = 0.016, \gamma_{b,IT} = \gamma_{b,US}$	31.1%	80.0%	91.8%
$\gamma_{m,IT} = \gamma_{b,US}, \gamma_{b,IT} = \gamma_{b,US}$	36.8%	82.1%	85.8%
High productivity dispersion			
$\sigma_{IT} = 1.14\sigma_{US}$	32.6%	82.4%	89.0%
$\sigma_{IT} = 1.30\sigma_{US}$	37.0%	80.3%	80.1%
High productivity dispersion and high market costs			
$\gamma_{m,IT} = 0.016, \sigma_{IT} = 1.14\sigma_{US}$	47.6%	78.1%	86.9%
$\gamma_{m,IT} = \gamma_{b,IT}, \sigma_{IT} = 1.30\sigma_{US}$	54.8%	76.2%	72.1%
High liquidation efficiency			
$\chi_{IT} = 0.75$	35.6%	86.6%	127.4%

Table 2: Sources of differences in aggregate debt structure between the US and Italy. Output per capita is expressed relative to the US. For data sources, see appendix A.

of financial liabilities of Italian non-financial corporations, a high ratio even among European countries.⁴³

Table 2 reports the aggregate bank share, aggregate output (relative to the baseline US calibration) and the ratio of debt to assets, in a versions of the model calibrated to match evidence Italian evidence on productivity dispersion and intermediation costs. The first lines of the panel focus on calibrations in which bank intermediation costs are identical to the US ($\gamma_{b,IT} = \gamma_{b,US}$), but market intermediation costs are higher. This is motivated by the following evidence. First, OECD data on income statements of Italian and US banks indicates that, in 2007, the ratio of their total income to total expenses were close (12.3% for Italy and 13.8% for the US). This suggests that bank-specific intermediation costs in the two countries could be similar.⁴⁴ For market lending costs, no direct evidence is available on Italian underwriting costs. However, Santos and Tsatsaronis (2003) estimate that the average difference in underwriting fees between the Euro-area and the US, between 1996 and 2001, stood at 56bps.⁴⁵ This motivates choosing higher values for $\gamma_{b,IT}$. I first set $\gamma_{m,IT} = 0.006 + \gamma_{m,US} = 0.016$, resulting in a lending wedge of $\theta_{IT} = 0.010$. In that alternative calibration, the bank share rises to 32.4%. I also consider the limit case in which market intermediation costs are as large as banks': $\gamma_{m,IT} = \gamma_{b,IT}$. In that case, the bank share rises to 36.8%. In those calibrations, consistently with the data, both the Italian debt to assets ratio and output are smaller than in the baseline US calibration.

The following lines of table 2 look at the potential contribution of productivity dispersion. The US

⁴³See appendix A for data sources.

⁴⁴As emphasized before, operating expenses do not only reflect monitoring and screening costs, as they include expenses associated with other, e.g. fee-based, activities of banks. However, for the underlying level of γ_b to be substantially different from the US's, given similar operating expenses, the allocation of banks' expenses would need to be substantially different.

⁴⁵See their table 3.

calibration uses direct evidence on the dispersion of quantity-based total factor productivity to calibrate the standard deviation of log productivity, σ_{US} , which are not available for Italy. Michelacci and Schivardi (2013) estimate proxies for idiosyncratic business risk across a panel of countries, using data on firm-level stock price volatility; Italian idiosyncratic business risk is, according to this measure, 14% larger than the US's.⁴⁶ Accordingly, I first look at a calibration in which productivity dispersion in Italy is 14% larger than in the US; this increases the bank share to 32.8%. With a productivity dispersion 30% larger than in the US, the bank share increases to 37.5%. Taken separately, these calibrations suggest that the quantitative contribution of productivity dispersion and intermediation costs to differences credit composition are broadly similar.

The last lines of table 2 looks at their combined effects. When $\sigma_{IT} = 1.14\sigma_{US}$ and $\gamma_{m,IT} = 0.016$, the model's aggregate bank share is 47.6%. This calibration thus accounts for 20.1% out of the 38.0%, or roughly half, of the difference between the bank shares of the US and Italy. I also look at a calibration in $\gamma_{m,IT} = 0.016$ and $\sigma_{IT} = 1.30\sigma_{US}$. This calibration can be viewed as an upper bound on the joint effects of productivity dispersion and the lending wedge on the aggregate bank share. It leads to an aggregate bank share of 52.1%, accounting for two thirds of the gap between the US and Italy. Both calibrations are also consistent with a lower debt to assets ratio and lower output per capita in Italy than in the US.⁴⁷

Aside from providing insight into the potential drivers of cross-country differences in debt composition, this exercise indicates that there are potentially large “static” output gains associated with changes in intermediation costs: for example, lowering market-specific intermediation costs, in Italy, to levels comparable to those of the US, would lead to long-run output gains of 9.0% ($= \frac{78.6}{72.1} - 1$). Section 6 analyzes this point in more detail; I next turn to analyzing the business-cycle implications of the model.

5 Aggregate shocks and the corporate debt structure

This section focuses on the business cycle implications of debt heterogeneity. The discussion centers mostly on shocks to the lending wedge $\gamma_b - \gamma_m$. From the standpoint of the model, they offer the best account of observed changes in debt structure in the US during the Great Recession.

The central finding of this section is that taking into account debt heterogeneity introduces a new channel of propagation of financial shocks, which operates through substitution between bank and market finance, and affects both the scale and composition of aggregate borrowing. Furthermore, the model suggests that the contribution of this channel to the decline in aggregate borrowing and investment is quantitatively large.

⁴⁶See table 2 of their online appendix.

⁴⁷A higher liquidation efficiency in Italy, on the other hand, would lead to higher output and a lower debt to asset ratio, inconsistent with the data, as indicated by the last line of the table.

5.1 Debt structure during the Great Recession

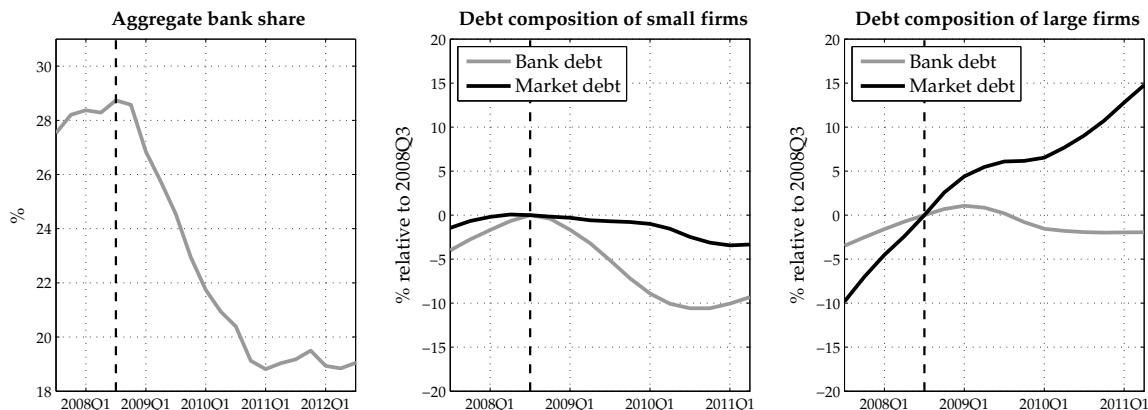


Figure 10: Changes in debt structure in the US during the Great Recession. Data from the Flow of Funds and the Quarterly Financial Report of Manufacturing firms. Small firms are those with assets below \$1bn; large firms are those with assets above 1bn\$. See appendix A for details on the construction of the series.

In the US, the 2007-2009 recession was accompanied by large and persistent changes in debt composition. Figure 10 illustrates these changes. First, in aggregate, the bank share fell substantially, from 27.5% in 2007Q3 to 19.5% in 2009Q2 (left panel). Second, at the firm level, the debt structure of small firms evolved very differently from that of large firms. The middle and right panels of figure 10 show the changes, relative to 2008Q3, in bank and non-bank liabilities of small and large manufacturing firms (respectively, those with less and more than 1 bn\$ in assets).⁴⁸ Both bank and non-bank liabilities of small firms declined, with bank debt showing the largest drop relative to peak. Large firms also experienced a decline in bank liabilities, but this decline was accompanied by an increase in non-bank liabilities. Thus, the data suggests that large firms resorted to “debt substitution” to a larger extent than small firms.⁴⁹

5.2 The effects of a shock to the lending wedge

In the model, these patterns emerge naturally in response to a shock to the lending wedge θ . To illustrate this, I compute the perfect foresight response of the model to an exogenous increase in the lending wedge, driven by an increase in bank lending costs costs $\gamma_{b,t}$. The economy starts from the steady-state described in section 2. I choose the path of $\gamma_{b,t}$ to match, quantitatively, the fall in the aggregate bank share documented in figure 10. The path of the aggregate bank share, and the implied path of the lending wedge, are reported on the left column of figure 11. The model requires a long-run increase in γ_b of 45bps in order to match the observed decline in the aggregate bank share.

⁴⁸This graph uses data from the Quarterly Financial Report of manufacturing firms (QFR). Appendix A discusses this data and the construction of the time series of figure 10 in more detail.

⁴⁹These patterns echo other evidence on the 2007-2009 recession, most notably, Adrian, Colla, and Shin (2012). They are also consistent with evidence from other periods or other countries on the effects of shocks to the supply of bank credit on debt composition, such as Becker and Ivashina (2014) and Baumann, Hoggarth, and Pain (2003).

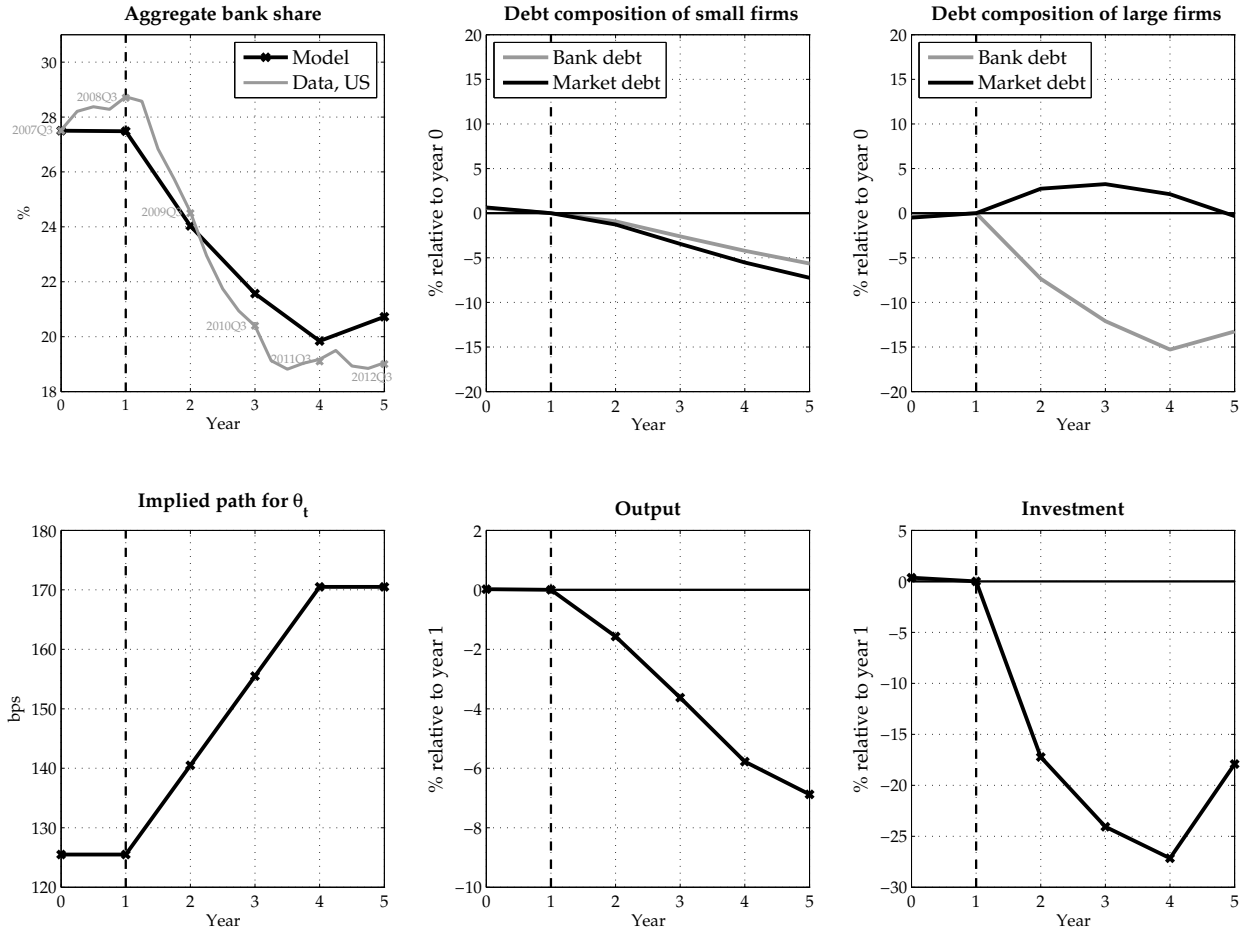


Figure 11: Response of the model to an increase in the lending wedge.

Figure 11 indicates that the increase in γ_b is associated with a 3.9% fall in output and a 24.7% fall in investment over the first three years, in line with the peak to trough drop in output and private investment observed during the 2007-2009 recession. The response of output displays endogenous persistence, since output continues declining even after θ has reached its new long-run value. At that point, investment instead starts recovering, having overshoot relative to its long-run value.⁵⁰ In the model, an unexpected increase in the lending wedge thus has large and persistent aggregate effects.

Moreover, the effect of the shock on borrowing in the cross-section lines up well with the evidence on small and large firms discussed above. In order to maintain comparability with that figure, I compute the change in bank and market liabilities of small and large firms in the model, using a threshold for internal finance to define size categories.⁵¹ Figure 11 reports the changes in borrowing of these two firm groups, relative

⁵⁰Aggregate investment is defined as: $I_t = K_t - (1 - \delta)K_{t-1}$, with: $K_t \equiv \int_{e_t \in [0, \bar{e}_t]} (e_t + \hat{b}(e_t) + \hat{m}(e_t)) d\mu(e_t)$.

⁵¹Specifically, I construct the “small” and “large” firms of the model as follows. I first determine a cutoff for internal finance, $e_{S/L}$, such that, in year 0, firms with $e_0 > e_{S/L}$ account for a fraction $s_{S/L}$ of the total stock of internal finance in the economy. For $t \geq 1$, I define borrowing by small and large firms, in bank and market debt, as:

$$BS_t = \int_{0 \leq e_t \leq e_{S/L}} \hat{b}_t(e_t) d\mu_t(e_t) \quad , \quad MS_t = \int_{0 \leq e_t \leq e_{S/L}} \hat{m}_t(e_t) d\mu_t(e_t) \quad ;$$

to year 1. For small firms, bank and market borrowing both decline, while for large firms, an increase in market borrowing accompanies the decline in bank liabilities. “Debt substitution” in the model is thus also a phenomenon concentrated among larger firms.

5.3 Propagation mechanisms

In order to understand the results, it is useful to first analyze the short-run response of firms’ policy functions for total assets. Figure 12 illustrates the two key mechanisms through which the shock affects these policies in the short-run.

The first mechanism is illustrated in the top panel of figure 12: the increase in the lending wedge results in a higher cost of bank borrowing outside of financial distress, and makes bank lending less attractive for mixed-finance firms. Those firms issue less bank debt and reduce their scale of operation. This is the traditional “bank credit” channel of financial shocks in models with a single borrowing constraint. The model however makes two additional predictions about this channel. First, for mixed-debt firms, the shock also leads to a fall in market borrowing. This is a manifestation of the complementarity between bank and market debt for mixed-debt firms, which intuitively comes from the fact that issuing market liabilities can partially relax a firm’s bank borrowing constraint by increasing its scale of operation and therefore its value in liquidation. The second additional prediction of the model is that, although market-financed firms are not directly affected by the increase in the lending wedge, they still somewhat reduce their borrowing from markets. These firms anticipate the fact that, given a sufficiently bad sequence of shocks ϕ_t , they will have to revert to mixed debt structures and face the now tighter lending terms of banks. They seek to avoid this by reducing their leverage.

The second mechanism by which the shock affects output and investment is that it changes the threshold at which firms switch from a mixed debt structure to a market-financed debt structure. This is illustrated in the bottom panel of figure 12. This threshold shifts to the left, and all firms between the pre- and post-shock threshold switch to a market-financed debt structure. A central prediction of the model is that this switch has real effects: it affects the scale at which switching firms choose to operate. Firms that switch lose the flexibility associated with bank finance, and become more exposed to liquidation risk. Put differently, these firms would face excessively high liquidation premia, under a market-financed structure, in order to continue operating at the same scale. As a result, their total borrowing, and therefore their output and investment,

$$BL_t = \int_{e_{S/L} \leq e_t \leq \bar{e}_t} \hat{b}_t(e_t) d\mu_t(e_t) \quad , \quad ML_t = \int_{e_{S/L} \leq e_t \leq \bar{e}_t} \hat{m}_t(e_t) d\mu_t(e_t) \quad ,$$

where, along the perfect foresight path, $\hat{b}_t(\cdot)$, $\hat{m}_t(\cdot)$ and \bar{e}_t characterize firms’ policies, and μ_t is the distribution of firms. This definition is analogous to that of the data because it uses fixed cut-offs on successive cross-sections to characterize the small and large firm groups. Qualitative results on the borrowing of these aggregate “small” and “large” firms are similar if one uses a cutoff for assets. I choose $s_{S/L} = 55.7\%$. This is the ratio of internal finance of manufacturing firms with more than 1 bn\$ in assets, to total internal finance of manufacturing firms in the QFR in 2007Q3, with internal finance computed as the difference between non-financial assets and total debt, as in the model.

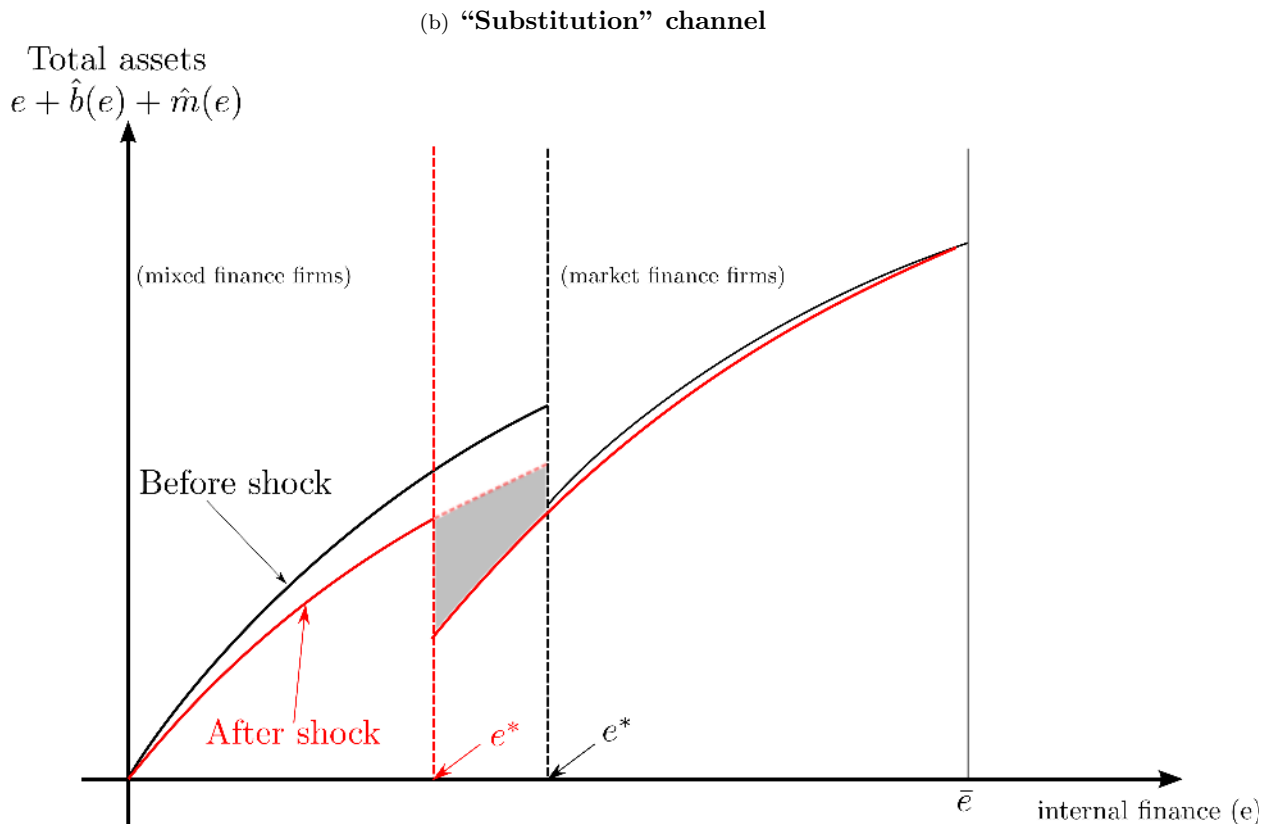
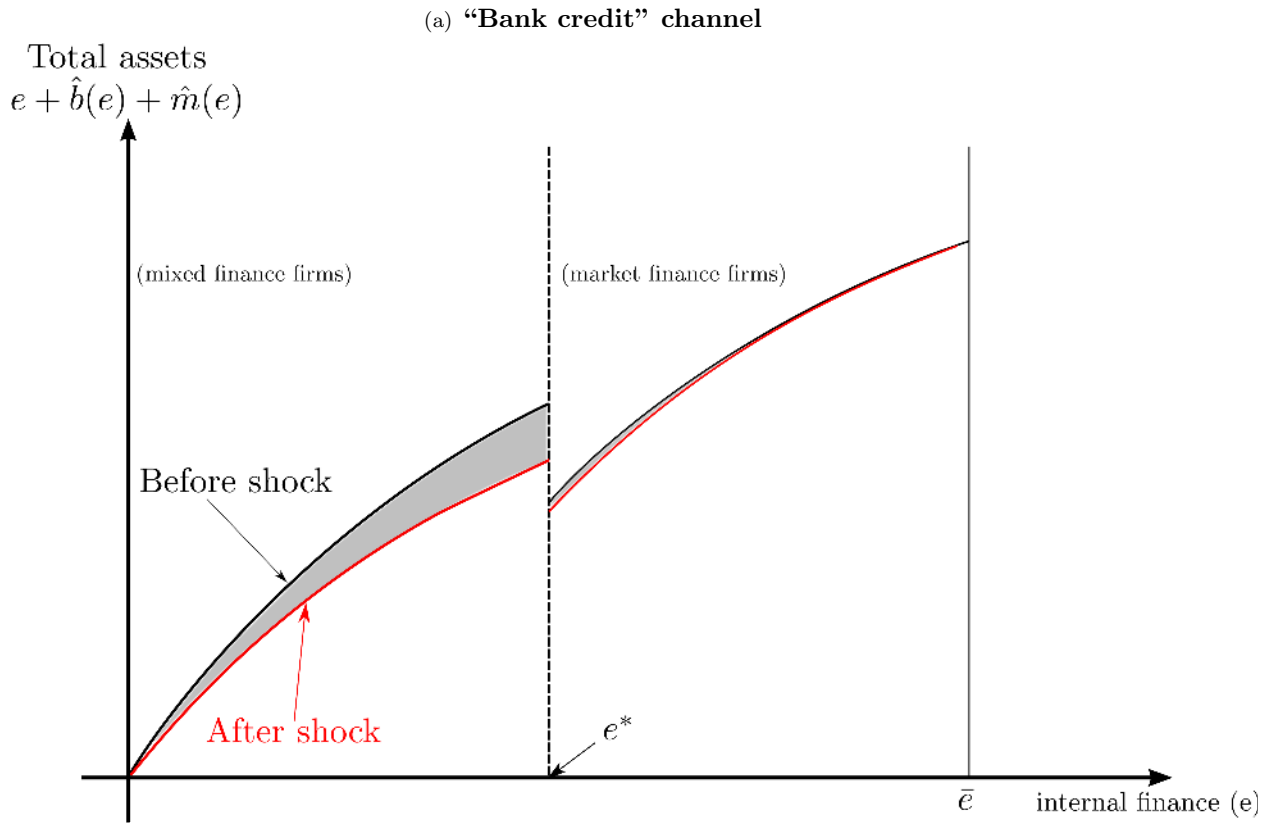


Figure 12: “Credit” and “substitution” channels in the response of firms to the lending wedge shock.

drop below what they would be under a mixed debt structure. The additional fall in borrowing due to this “substitution” channel corresponds to the shaded area in the bottom panel of figure 12.

Quantitatively, the contribution of the “substitution” channel to the total decline in borrowing can be large. One way to gauge this contribution is to compute what total investment would have been in period 2 (once $\gamma_{b,t}$), had switching firms been constrained to keep using a mixed debt structure. In that counterfactual case, the drop in aggregate investment between year 1 and year 2 would have been 12.2%, instead of 17.2%. The “substitution” channel thus accounts for about a third of the decline in aggregate investment in the model.

The previous discussion helps understand the short-run response of the economy to the shock. In the long-run, the shock has protracted effects on firms’ ability to accumulate internal finance, both because lending terms of banks worsen, and because the shock induces firms to leverage less. As a result, the steady-state distribution of firms across levels of internal finance shifts to the left: in the long run, the median stock of internal finance of firms in the economy falls. The slow adjustment of firms’ internal funds in the new environment with tougher lending bank lending standards is what drives the endogenous persistence in the response output and investment, since, after the lending wedge has stabilized to a new, higher level, the threshold e^* does not change.

5.4 Can other aggregate shocks account for the behavior of the bank share?

The previous discussion focused on the effects of an increase in the lending wedge $\gamma_{b,t} - \gamma_{m,t}$: as discussed, such a shock generates a recession accompanied by a fall in the aggregate bank share. Shocks to $\mathbb{E}(\phi)$, and shocks that increase γ_b and γ_m jointly (and thus do not affect the lending wedge), do generate large recessions, but have limited effects on debt composition at the aggregate and firm level.⁵² A recession driven by an increase in idiosyncratic volatility, on the other hand, is accompanied by an *increase* in the aggregate bank share. This reflects the greater demand for debt flexibility when firm-level uncertainty is high. The response of small and large firms features debt substitution, but in the *opposite* direction, as medium and large firms substitute away from market debt and into bank debt. Thus, in this model, dispersion-driven recessions are associated with changes in corporate debt structure that are at odds with the patterns documented in figure 10.

Summarizing, this section has shown that lending wedge shocks can generate large recessions accompanied by substitution toward market borrowing, both at the firm and aggregate levels. However, debt substitution also forces some firms to reduce leverage, and therefore investment. Deleveraging is optimal from the standpoint of switching firms: relying only on market debt deprives them from the flexibility offered by banks

⁵²Detailed results for the perfect foresight response of the economy to shocks to aggregate productivity, productivity dispersion are available from the author upon request.

in bad times. But it amplifies the response of investment and aggregate activity to the shock. The following section investigates how this mechanism may play out in the context of programs aimed at encouraging firms to use market-based intermediation.

6 Corporate finance policies and their real effects

The comparative statics of section 4 suggest that a more market-oriented intermediation sector can lead to higher long-run output and investment, independent of the underlying productivity distribution of firms. At first blush, this suggests that improving firms’ access to market credit should be beneficial to aggregate output and investment.

In Europe, some countries indeed have embraced market-based intermediation as a potential remedy to the slowdown in bank lending that has taken place since 2008.⁵³ This section discusses two examples: a new exchange for bond issuances by small and medium size firms in Germany, and an Italian tax reform introducing tax deductibility of interest payments on bonds issued by private firms. Given the firm-level heterogeneity which it generates, the environment analyzed in this paper is a particularly convenient laboratory to study the effects of programs such as these, which target specific groups of firms.

6.1 The Bondm market

The Bondm exchange was launched by Boerse Stuttgart in 2010 with the goal of providing a more favorable environment for bond issuance for SMEs than existing markets. Bondm provides firms with a primary market for new issuances, and also operates a secondary exchange in which private and retail investors trade existing issues. Participation is restricted to firms with less than €250m in assets, and to issuances between €50m and €150m. Crucially, it aims at making bond issuances attractive to SMEs by reducing intermediation costs. Bondm issuances do not need to be individually rated, and do not need to be underwritten by a bank. Underwriting requirements are particularly costly for SMEs, as few European investment banks specialize in underwriting small issuers.⁵⁴

In the context of the model described in section 2, a simple way to capture the advantages offered by Bondm to SME bond issuance is to let market intermediation costs depend on internal funds e_t :

$$\gamma_m(e_t) = \begin{cases} \tilde{\gamma}_m & \text{if } e_t \leq e_{sm} \\ \gamma_m & \text{if } e_t > e_{sm} \end{cases}, \quad \tilde{\gamma}_m < \gamma_m.$$

⁵³The Euro Area Bank Lending Survey indicates that between 2008 and 2009, half of Euro area banks tightened their margins on “normal” loans; two thirds tightened their margins on “risky” loans. An additional tightening of bank lending standards also took place in 2010, and coincided with a pick-up in the issuance of new corporate bonds.

⁵⁴For more details on the German corporate bond market during the recession, and the Bondm exchange, see the case study of the German mid-cap Dürr in Hillion et al. (2012).

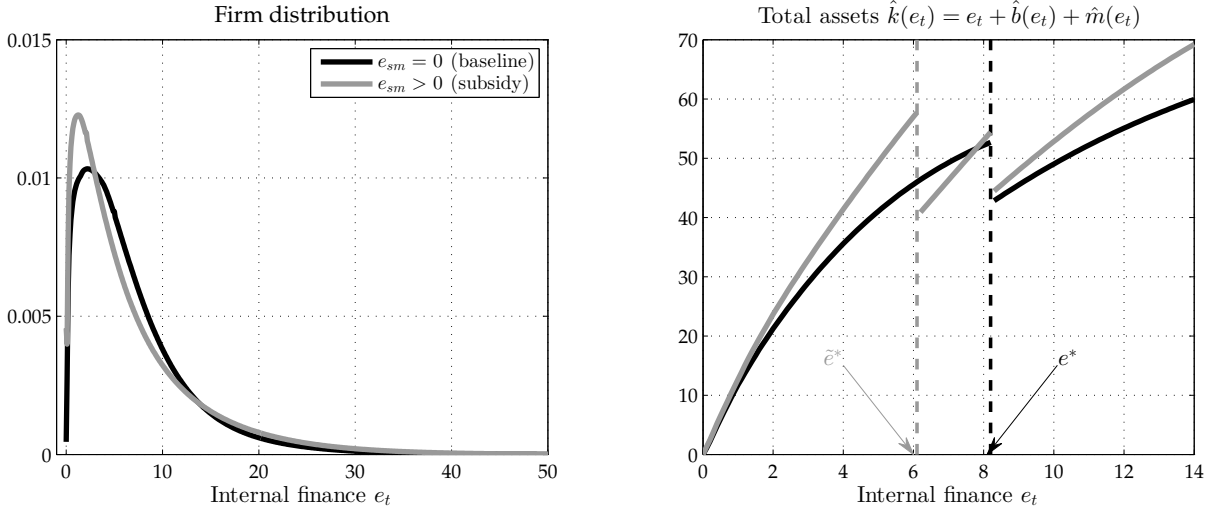


Figure 13: The effect of lower intermediation costs on firms' choice of scale.

The introduction of the Bondm exchange corresponds to an economy in which $e_{sm} > 0$, as opposed to the baseline case where $e_{sm} = 0$. $\tilde{\gamma}_m \leq \gamma_m$ denotes intermediation costs for small firms ($e_t \leq e_{sm}$) that use Bondm for their issuances.

Figure 13 compares the baseline economy (black line), to the economy with low market intermediation costs for SMEs (grey line). In the baseline economy, market intermediation costs are given by $\gamma_m = 0.016$, consistent with the evidence provided by Santos and Tsatsaronis (2003) on the differences in issuance costs between the US and the Euro area. In the alternative economy, the intermediation costs for Bondm-type issuances is set to $\tilde{\gamma}_m = 0.010$.⁵⁵

The left panel of figure 13 indicates that the policy has limited effects on the size distribution. Its effects on total output and investment are best understood by looking at changes in firms' total assets $\hat{k}(e_t) = e_t + \hat{b}(e_t) + \hat{m}(e_t)$, which are reported in the right panel of figure 13.

This response depends on internal funds e_t . There are three cases. First, the lower intermediation costs create a new threshold \tilde{e}^* . Firms with internal funds $e_t \leq \tilde{e}^*$ keep relying on bank loans even when there are lower market intermediation costs. These firms however increase total borrowing: lower intermediation costs indeed encourage them increase market debt issuance, and additionally relax their bank borrowing constraint, because of the complementarity between forms of borrowing in the mixed-finance regime.

The second case is that of firms with internal funds $e_t > e^*$, where e^* is the threshold above which firms cannot take direct advantage from low market intermediation costs (i.e., cannot issue on the Bondm market because they are too large). These firms nevertheless increase bond issuance in response to the lower intermediation costs. This is because they anticipate that, now that market borrowing is cheaper at small scales, they will be able to operate more profitably if a string of bad shocks pushes them back to a mixed

⁵⁵The threshold e_{sm} is set to $e_{sm} = e^*$, where e^* is the switching threshold in the baseline economy. The qualitative effects of the policy would be identical if $e_{sm} < e^*$, but this choice makes the graphical discussion that follows clearer.

debt structure.

The third case relates to intermediate firms, those with $\tilde{e}^* < e_t \leq e^*$. These firms are large enough that the policy will induce them to switch to an entirely market-financed debt structure, but small enough that they will still benefit from the Bondm intermediation costs for their bond issuances. The switch fragilizes the debt structure of these firms, as they loose the ability to restructure debt in bad times. As a result, some of these firms deleverage: their total borrowing is *smaller* than in the world with higher intermediation costs, and they operate at a *smaller* scale.⁵⁶ Surprisingly, the lower intermediation costs therefore result in a precautionary reduction in total debt issuance and total investment by these firms.

How do these two effects – the “credit channel” effect that stimulates borrowing and investment by firms with $e_t \leq \tilde{e}^*$ and $e_t > e^*$, and the “substitution channel” effect that depresses borrowing and investment by firms with $\tilde{e}^* < e_t \leq e^*$ – measure up against each other? Overall, the effect of the policy $e_{sm} > 0$ on total output and investment is positive: they increase, respectively, by 6.3% and 7.5% under the policy of lower intermediation cost for SMEs. However, the negative effect on investment by medium-sized firms is sizeable, because the mass of firms that are in this region is large (see the left panel of figure 13). Specifically, these firms account for a -3.8% decline in aggregate investment, and a -2.3% decline in aggregate output, relative to the initial steady-state.

6.2 An Italian tax reform

To the extent that they discriminate between bank and market liabilities, taxes are an alternative tool that can be used as an instrument to influence firms’ choice of debt structure. A 2012 Italian tax reform meant to improve access to bond markets for private companies includes the tax treatment of interest payment to corporate bonds as a part of a larger array of policy tools.⁵⁷ Specifically, the reform allows private firms to deduct interest paid on bonds in the same way as interest paid on other debt, in line with the tax rules imposed on large firms.

The model of section 2 does not explicitly incorporate tax deductibility of debt. However, in appendix C, I show that a simple extension can accomodate this. Let cash on hand of the firm in repayment be given by:

$$n_t^P = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t},$$

where τ denotes the marginal coporate tax rate. Then, $\tau_b \leq \tau$ and $\tau_m \leq \tau$ reflect preferential tax treatments of different types of debt instruments.

⁵⁶Visually, this is indicated by the fact that the light grey line is below the black line on the right panel of figure 13.

⁵⁷The reform also relaxes the requirements to find a sponsor to guarantee the issuance of the bond, and eliminates existing limits on indebtedness as a fraction of net worth for private firms. See <http://www.paulhastings.com/Resources/Upload/Publications/2351.pdf> for more details on the contents of the reform.

The two tax regimes that existed in Italy prior to the reform were, respectively, $\tau_b = \tau$ and $\tau_m = 0$ (only bank debt is interest-deductible); and which $\tau_b = \tau_m = \tau$ (both types of debt are treated indentically for tax purposes). The policy experiment then consists in comparing an economy where only sufficiently large firms enjoy preferential tax treatment for market debt ($\tau_b = \tau_m = \tau$ for $e_t > e^*$; $\tau_b = \tau$ and $\tau_m = 0$ for $e_t \leq e^*$), to an economy in which all firms enjoy the same tax shields for market and bank debt ($\tau_m = \tau_b = \tau$ for all e_t).

The impact of this change in tax policy on borrowing and investment are qualitatively similar to the previous experiment.⁵⁸ Intuitively, a differential treatment of bank and market debt ($\tau_m < \tau_b$) directly affects the relative cost of market and bank debt outside of liquidation. Introducing a tax shield on market debt issuance for SMEs will therefore have similar effects on borrowing as lowering market intermediation costs. Namely, the tax shield boosts total borrowing by the smallest and the largest firms, because of the “credit channel” effect discussed above. However, it reduces total borrowing by the mass of intermediate firms that switch entirely to market-financed debt structure in response to the introduction of the tax shield.

While the comparative statics of the model provided rationales for encouraging firms to rely more on markets as a source of funds, the results of both policy exercises in this section show that attempts to do so may also result in inadvertent effects on firm-level investment. Encouraging market-based intermediation may indeed create an incentive for firms to adopt more fragile debt structures. There are real implications to this increased financial fragility, as switching firms reduce borrowing and investment.

7 Conclusion

The composition of corporate borrowing between bank loans and market debt exhibits substantial variation, across firms, across countries, and over time. This paper starts from the view that banks specialize in providing a flexible form of borrowing to firms, albeit at the cost of tougher lending standards in normal times. I showed that embedding this trade-off between flexibility and cost into a simple model of firm dynamics leads to cross-sectional predictions that align well with cross-sectional data on the composition of firm debt. The model is useful to parse some simple explanations of cross-country variation in debt structure; I argued that productivity dispersion and differences in intermediation costs between countries can account for a large share of US-Italy differences in debt structure. Additionally, in response to an increase in bank-specific intermediation costs, the model generates a drop in output and investment, accompanied by changes in debt composition which, both at the aggregate and the firm level, resemble those experienced by the US during the 2007-2009 recession.

⁵⁸Appendix C reports these results.

One of the central findings of the paper is that, aside from the traditional “credit channel” effects of financial shocks or policies, debt heterogeneity introduces a new “substitution channel”, according to which firms that switch away from bank loans and into corporate bonds may also choose to deleverage, for precautionary reasons. This result speaks to the idea that bond markets could provide an alternative to traditional, bank-based intermediation in times when the latter impaired. Providing support to corporate bond markets has indeed been one the explicitly stated goals of the recent quantitative easing policies in, for example, the UK and the US.⁵⁹ While understanding the transmission of these policies to corporate investment would require a more fully-fleshed model of the banking sector, the policy experiments of section 6 suggest that these interventions could inadvertently have adverse effects on the investment of firms of intermediate size and credit risk, by inducing them to adopt a fragile debt structure. I leave this topic to future research.

⁵⁹For a description of the quantitative easing policies of the Bank of England and their effects on corporate bond issuance, see [Joyce, Tong, and Woods \(2011\)](#). For the effects of Fed’s Large Scale Asset Purchase programs on corporate bond yields, see [Gilchrist and Zakrajšek \(2013\)](#).

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A Data appendix

Aggregate balance sheet data For the US, the data on the aggregate bank share is obtained from table L.102 of the Flow of Funds, the balance sheet of the the nonfinancial corporate sector. The series in the left panel of figure 10 is the ratio of the sum of depository institution loans (line 27) and other loans and advances (line 28) to total credit market instruments outstanding (line 23). The ratio of debt to assets is measured as the ratio of total credit market instruments outstanding (line 23), to miscellaneous assets (line 16), a measure of assets excluding credit market instruments and deposits or money market fund shares. I exclude these financial assets from this ratio because the model’s firms do not hold cash and do not lend to other firms.

For Italy, the data on the aggregate bank share is obtained from [Bank of Italy \(2008\)](#), table 5 (TDHET000). The aggregate bank share is computed as the ratio of total loans (short and long-term) to total debt. Total debt is measured as total liabilities minus shares and other equities issued by residents.⁶⁰ Similarly to the US, I construct the ratio of debt to assets as the ratio of total debt to a measure of total assets which excludes cash and cash-like securities, as well as credit market securities. I obtain this measure by subtracting deposits, short-terms securities, bonds, derivatives, short-term loans and mutual fund shares from total liabilities of firms.

Data on business-cycle changes in debt composition in the US I use the data from F.102 to construct changes in the aggregate bank share during the recession, reported on the left panel of figure 10. The middle and right panels of 10 report changes in outstanding bank and non-bank debt for small and large manufacturing firms, in the US, from 2008 onwards. The data is from the Quarterly Financial Report of manufacturing firms. This dataset contains information on firms’ balance sheets and income statements, and is reported in semi-aggregated form (by asset size categories). Because it is a quarterly dataset, the QFR is well suited for illustrating business-cycle changes in debt composition. The “small firm” category is defined as firms with less than \$1bn in assets, and the “large firm” category as the remainder. For both categories, total debt is defined as total liabilities excluding non-financial liabilities (such as trade credit) and stockholders’ equity. It includes both short and long-term debt. Bank debt b_t is reported as a specific item in the QFR; I define market credit m_t as the remaining financial liabilities. The series called “bank debt” in figure 10 is given by: $\gamma_{b,small,t} = \frac{b_{small,t_0}}{b_{small,t_0} + m_{small,t_0}} \left(\frac{b_{small,t}}{b_{small,t_0}} - 1 \right)$. The series called “market debt” is defined similarly. The series reported in figure 10 are smoothed by a 2 by 4 MA smoother to remove seasonal variation.

Data on firm-level debt composition I additionally use firm-level data to illustrate cross-sectional variation in debt structure, and to construct bank shares for countries for which aggregate balance sheets are not readily available. For all countries except the US, I use the OSIRIS database, maintained by Bureau Van Dijk. This dataset contains balance sheet information for publicly traded firms in emerging and advanced economies. I focus on the subsample of non-financial firms that are active between 2000 and 2010, and keep only firms that report consolidated financial

⁶⁰The Bank of Italy distinguishes between loans from MFI’s (monetary financial institutions, comprising the Central Bank, banks, money-market funds, electronic money institutions and the Cassa Depositi e Prestiti), and loans from other financial institutions. Excluding other financial institutions, the aggregate bank share for 2007Q3 is 50.3 %, closer to the aggregate bank share which can be matched by the model.

statements. Total debt $d_{j,t}^c$ is defined as total long-term interest bearing debt (data item number 14016), and bank debt $b_{j,t}^c$ as bank loans (data item number 21070). I focus on long-term debt (inclusive of the currently due portions of it) because bank loans are a subset of this category in the OSIRIS database; currently due debt only features a "loans" category which include potentially credit instruments other than bank loans. Additionally, I define internal funds $e_{j,t}^c$ as shareholders funds (data item 14041). I keep only observations for which $e_{j,t,c} \geq 0$ and $s_{j,t,c} \in [0, 1]$ (and for which both are non-missing). The resulting sample has 51921 firm-year observations, corresponding to 12931 distinct company names.

For the US, firm-level data is drawn from the dataset created by [Rauh and Sufi \(2010\)](#), arguably the highest-quality dataset on debt structure for publicly traded firms.⁶¹ This dataset draws from Compustat (for balance-sheet data) and Dealscan (for bond issuances), and has direct measures of total debt $d_{j,t,US}$. Internal finance $e_{j,t,US}$ is defined as the difference between the "debt plus equity" and the "debt" variables. Finally, bank debt $b_{j,t,US}$ is defined as outstanding bank loans. All other variable definitions are identical.

Figure 8 uses this firm-level balance sheet data to construct aggregate bank shares for a larger sample of countries. For each country-year, the aggregate bank share is computed as $S_{c,t} = \frac{\sum_{j \in F_{c,t}} b_{j,t}}{\sum_{j \in F_{c,t}} d_{j,t}}$, where $F_{c,t}$ denotes the set of active firms in country c and year t . The aggregate bank shares reported in the graph are the average of the $S_{c,t}$ for $t = 2000, \dots, 2007$.

The relationship between internal finance and the bank share Finally, figure 14 uses the firm-level data to illustrate the relationship between internal finance and debt composition in a sample of advanced and developing countries. For each firm-year observation, the firm-level bank share is defined as: $s_{j,t,c} = \frac{b_{j,t}^c}{d_{j,t}^c}$. Observations are pooled by country-year (t, c) . For each country-year cell, let $\tilde{e}_{k,t,c}$ denote the k -th quantile of the empirical distribution of firms across levels of internal funds $e_{j,t,c}$. I use $k \in \{5, 15, 25, \dots, 95\}$. Define the average bank share within each quantile group as: $\tilde{s}_{k_i,t,c} = \frac{1}{N_{k_i,t,c}} \sum_{\tilde{e}_{k_i,t,c} \leq e_{j,t,c} < \tilde{e}_{k_{i+1},t,c}} s_{j,t,c}$, where $N_{k_i,t,c}$ is the number of firms in (t, c) with $\tilde{e}_{k_{i-1},t,c} \leq e_{j,t,c} < \tilde{e}_{k_i,t,c}$. I then average out these shares over time: $\tilde{s}_{k_i,c} = \frac{1}{T} \sum_t \tilde{s}_{k_i,t,c}$. Figure 14 reports the pairs $(k_i, \tilde{s}_{k_i,c})$, for the subset of 8 countries that have the largest number of observations among advanced and emerging economies, respectively.

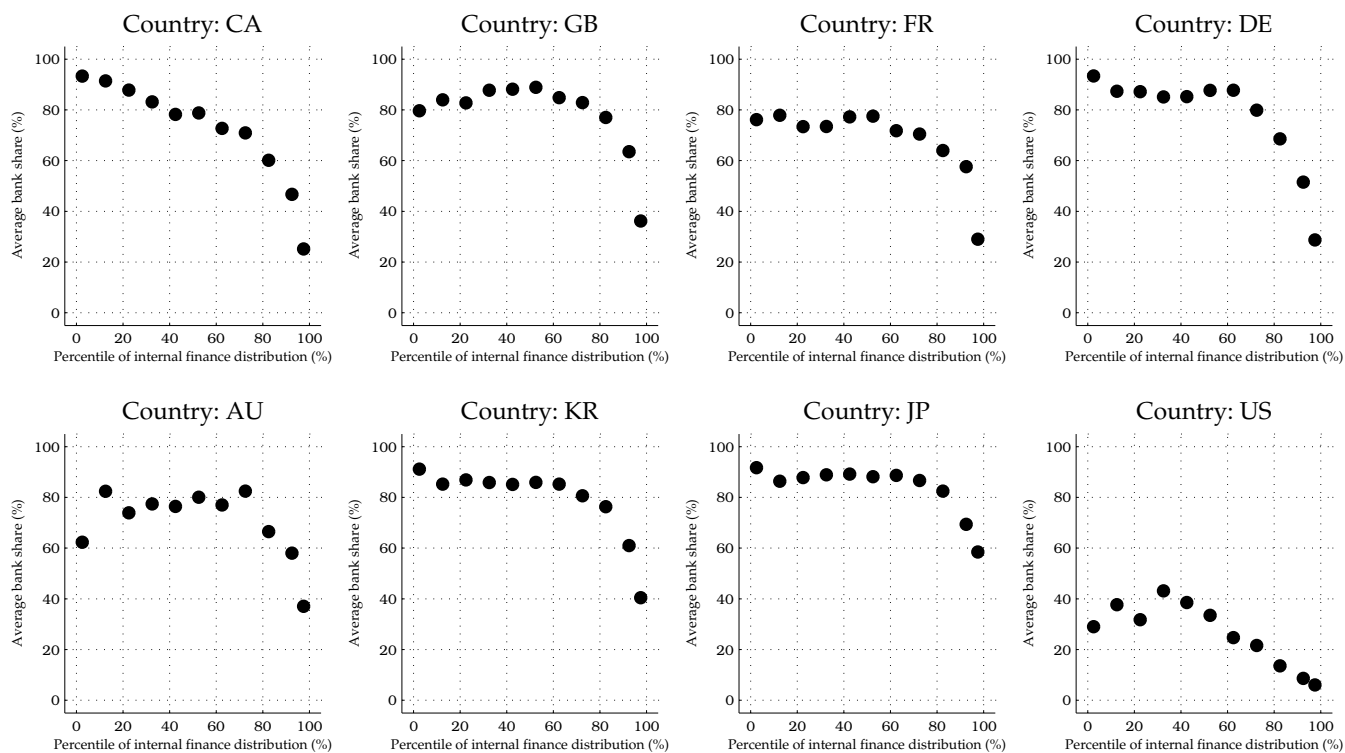
B Proofs

Proof of lemma 1.

When V is continuous, the objective function in problem (1) is continuous. The constraint correspondence in problem (1) is compact-valued and continuous. The theorem of the maximum then implies that V^c is continuous. Let $(n_t^1, n_t^2) \in \mathbb{R}_+^2$ such that $n_t^1 > n_t^2$, and let e_{t+1}^2 be a value for next period net worth that solves problem (1), when $n_t = n_t^2$. We have $e_{t+1}^2 \leq n_t^2 < n_t^1$, so e_{t+1}^2 is also feasible when $n_t = n_t^1$. Therefore, $V^c(n_t^1) \geq n_t^1 - e_{t+1}^2 + (1 - \eta)\beta V(e_{t+1}^2) > n_t^2 - e_{t+1}^2 + (1 - \eta)\beta V(e_{t+1}^2) = V^c(n_t^2)$. This proves that V^c is strictly increasing.

⁶¹The dataset is available at: http://faculty.chicagobooth.edu/amir.sufi/data-and-appendices/RauhSufi_RFSdebtstructure_20100216.zip.

(a) Internal finance and bank share in OECD-countries



(b) Internal finance and bank share in non-OECD-countries

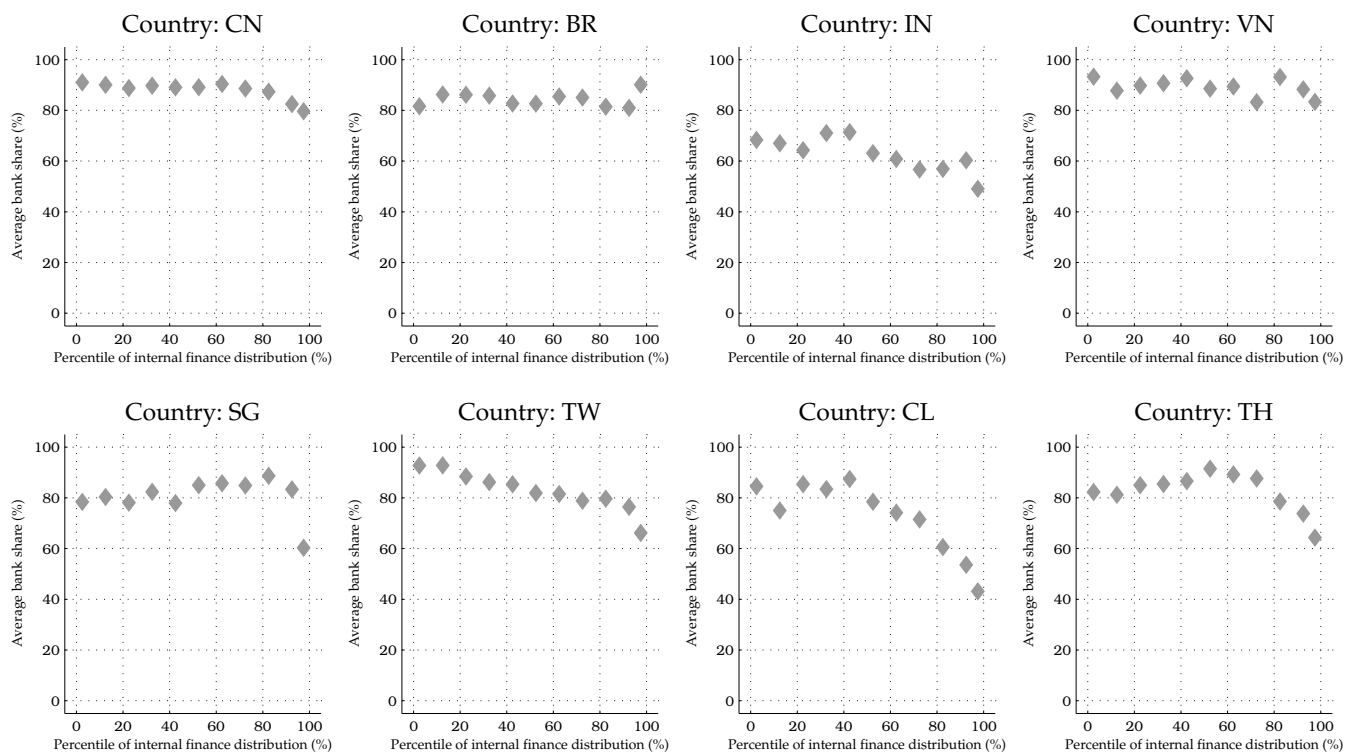


Figure 14: Bank share and internal finance in the cross-section. Each graph reports, for a particular country, the median ratio of bank loans to total firm liabilities, in each decile of the distribution of internal finance. For the US, data from taken from [Rauh and Sufi \(2010\)](#); for other countries, data from Bureau Van Djik. See online appendix for details on the definitions of variables.

Finally, when $n_t = 0$ the feasible set contains only $div_t = 0, e_{t+1} = 0$. So $V^c(0) = (1 - \eta)\beta V(0)$. Therefore when $V(0) \geq 0, V^c(0) \geq 0$. The rest of the lemma is identical to [Cooley and Quadrini \(2001\)](#). ■

Proof of proposition 1. A useful result for the proof is that $V^c(n_t) \geq n_t$ when $V(0) \geq 0$. This is established by noting that the dividend policy $e_{t+1} = 0$ is always feasible at the dividend issuance stage, and that the value of this policy is $n_t + (1 - \eta)\beta V(0) \geq n_t$.

Assume, first, that $\frac{R_{m,t}}{1-\chi} \leq \frac{R_{b,t}}{\chi}$. Then, $\frac{R_{m,t}}{1-\chi} \leq R_{b,t} + R_{m,t} \leq \frac{R_{b,t}}{\chi}$. The proof proceeds by comparing V_t^L, V_t^R and V_t^P , the values of the firm under liquidation, restructuring or repayment, for each realization of π_t . There are five possible cases:

- when $\pi_t \geq \frac{R_{b,t} + R_{m,t}}{\chi}$, we have $V_t^L = \chi\pi_t - R_{b,t} - R_{m,t} < \pi_t - R_{b,t} - R_{m,t} \leq V^c(\pi_t - R_{b,t} - R_{m,t}) = V_t^P$. Moreover, since $\pi_t \geq \frac{R_{b,t} + R_{m,t}}{\chi} \geq \frac{R_{b,t}}{\chi}$, the reservation value of the bank is $R_{b,t}$, so the best restructuring offer for the firm is $l_t = R_{b,t}$. Therefore $V_t^P = V_t^R$; I will assume the firm chooses repayment.
- when $\frac{R_{b,t} + R_{m,t}}{\chi} > \pi_t \geq \frac{R_{b,t}}{\chi}$, we have $V_t^L = 0 \leq V^c(\pi_t - R_{b,t} - R_{m,t}) = V_t^P$, since $\pi_t \geq \frac{R_{b,t}}{\chi} \geq R_{b,t} + R_{m,t}$, $V^c(0) \geq 0$ and V^c is strictly increasing. ($V_t^L < V_t^R$ so long as $\pi_t > R_{b,t} + R_{m,t}$). Moreover, $V_t^R = V_t^P$ for the same reason as above. Again, the firm chooses repayment.
- when $\frac{R_{b,t}}{\chi} > \pi_t \geq R_{b,t} + R_{m,t}$, the reservation value of the bank is $\chi\pi_t$. The restructuring offer at which the participation constraint of the bank binds, $l_t = \chi\pi_t$, is feasible because $\pi_t - l_t - R_{m,t} = (1 - \chi)\pi_t - R_{m,t} \geq 0$. So $V_t^R \geq V^c((1 - \chi)\pi_t - R_{m,t})$. This implies $V_t^R > V^c(\pi_t - R_{b,t} - R_{m,t}) = V_t^P$, since V^c is strictly increasing and $(1 - \chi)\pi_t - R_{m,t} > \pi_t - R_{b,t} - R_{m,t}$. For the same reasons as above, $V_t^R \geq V_t^L$. So the firm chooses to restructure. Because V^c is increasing, the optimal restructuring offer makes the participation constraint of the bank bind: $\hat{l}_t = \chi\pi_t$.
- when $R_{b,t} + R_{m,t} \geq \pi_t \geq \frac{R_{m,t}}{1-\chi}$, we have $V_t^L = 0 < V^c((1 - \chi)\pi_t - R_{m,t}) = V_t^R$, where again the properties of V^c were used. Moreover, $V_t^P = V_t^L$, since the firm does not have enough funds to repay both its creditors. So the firm chooses to restructure, again with $\hat{l}_t = \chi\pi_t$.
- when $\pi_t < \frac{R_{m,t}}{1-\chi}$, the firm is liquidated because any restructuring offer consistent with the participation constraint of the bank will leave the firm unable to repay market creditors. Since in that case, $0 > \chi\pi_t - R_{b,t} - R_{m,t}$, the liquidation value for the firm is $V_t^L = 0$.

This shows that when $\frac{R_{m,t}}{1-\chi} \leq \frac{R_{b,t}}{\chi}$, the firm repays when $\pi_t \geq \frac{R_{b,t}}{\chi}$, restructures when $\frac{R_{b,t}}{\chi} \geq \pi_t \geq \frac{R_{m,t}}{1-\chi}$, and is liquidated otherwise. Moreover, this also establishes the two additional claims of the proposition, in the case $\frac{R_{m,t}}{1-\chi} \leq \frac{R_{b,t}}{\chi}$: $V_t^L = 0$ whenever liquidation is chosen, and the restructuring offer always makes the participation constraint of the bank bind: $\hat{l}_t = \chi\pi_t$. The claims of the proposition when $\frac{R_{m,t}}{1-\chi} > \frac{R_{b,t}}{\chi}$ can similarly be established, by focusing on the three sub-cases $\pi_t \geq \frac{R_{b,t} + R_{m,t}}{\chi}$, $\frac{R_{b,t} + R_{m,t}}{\chi} > \pi_t \geq R_{b,t} + R_{m,t}$ and $R_{b,t} + R_{m,t} > \pi_t$. ■

Proof of proposition 2. Given the results of proposition 1, the debt settlement outcomes yield the same conditional return functions for banks and market lenders, $\tilde{R}_{b,t}(\pi_t, R_{b,t}, R_{m,t})$ and $\tilde{R}_{m,t}(\pi_t, R_{b,t}, R_{m,t})$, as those reported in the appendix to [Crouzet \(2013\)](#). Proposition 2 is a subset of the results of proposition 2 in that paper. ■

Proof of proposition 3.

The proof of the existence of a recursive competitive equilibrium of the economy of section 2 proceeds by first establishing the existence and unicity of a solution to the individual firm problem, then by establishing the existence of a steady-state distribution.

Firm problem Throughout, the firm's problem is restated in terms of the variables $d_t = b_t + m_t$ and $s_t = \frac{b_t}{b_t + m_t}$. d_t denotes total borrowing by a firm, and s_t denotes the share of borrowing that is bank debt. Note that $(d_t, s_t) \in \mathbb{R}_+ \times [0, 1]$. With some abuse of notation, the set of feasible debt structures (d_t, s_t) is still denoted by $\mathcal{S}(e_t)$, and its partition established in proposition 2 as $(\mathcal{S}_K(e_t), \mathcal{S}_R(e_t))$. Additionally, the functions $G : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $I(\cdot; e_t + d_t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $M(\cdot; e_t + d_t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ by:

$$\begin{aligned} G(x) &= x(1 - F(x)) + \int_0^x \phi dF(\phi) \\ I(x; e_t + d_t) &= x(1 - F(x)) - F(x)(1 - \delta)(e_t + d_t)^{1-\zeta} \\ M(x; e_t + d_t) &= (1 - \chi)I(x; e_t + d_t) + \chi G(x) \end{aligned}$$

Following lemmas 2 to 4 in the appendix to Crouzet (2013), G is strictly increasing on \mathbb{R}^+ , while I and M have unique maxima. I denote these inverse mappings by $G^{-1}(\cdot)$, $I^{-1}(\cdot; e_t + d_t)$ and $M^{-1}(\cdot; e_t + d_t)$. They are defined, respectively, on $[0, \mathbb{E}(\phi)]$, $[0, \hat{I}(e_t + d_t)]$ and $[0, \hat{M}(e_t + d_t)]$, where $\hat{I}(e_t + d_t)$ is the global maximum of I and similarly $\hat{M}(e_t + d_t)$ is the global maximum of M .

These functions are useful because their inverse mappings determine the terms of debt contracts $(R_{b,t}, R_{m,t})$ for given (e_t, d_t, s_t) .⁶² Following the results of proposition 1, the thresholds for restructuring and liquidation, in terms of the productivity shock ϕ_t , are given by:

$$\begin{aligned} \underline{\phi}_R(e_t, d_t, s_t) &= \frac{R_m(d_t, s_t, e_t) - (1 - \chi)(1 - \delta)(e_t + d_t)}{(1 - \chi)(e_t + d_t)^\zeta} && \text{(liquidation threshold when } (d_t, s_t) \in \mathcal{S}_R(e_t)) \\ \bar{\phi}_R(e_t, d_t, s_t) &= \frac{R_b(d_t, s_t, e_t) - \chi(1 - \delta)(e_t + d_t)}{\chi(e_t + d_t)^\zeta} && \text{(restructuring threshold when } (d_t, s_t) \in \mathcal{S}_R(e_t)) \\ \underline{\phi}_K(e_t, d_t, s_t) &= \frac{R_b(d_t, s_t, e_t) + R_m(d_t, s_t, e_t) - (1 - \delta)(e_t + d_t)}{(e_t + d_t)^\zeta} && \text{(liquidation threshold when } (d_t, s_t) \in \mathcal{S}_K(e_t)) \end{aligned}$$

The proof combines these expressions with the inverse mappings defined above to express liquidation thresholds directly as functions of e_t, d_t and s_t . There are three steps:

Step 1: Reformulate the problem as the combination of a discrete choice and continuous choice problem.

Step 2: Show that the functional mapping T associated with this new formulation maps the space $C(E)$ of real-valued,

⁶²For example, the terms of bank contracts when $(d_t, s_t) \in \mathcal{S}_R(e_t)$ are given by:

$$R_{b,t} = R_b(d_t, s_t, e_t) = \begin{cases} (1 + r_b)d_t s_t & \text{if } 0 \leq \frac{(1 + r_b)d_t s_t}{\chi(e_t + d_t)^\zeta} < (1 - \delta)(e_t + d_t)^{1-\zeta} \\ \chi(1 - \delta)(e_t + d_t) & \text{if } (1 - \delta)(e_t + d_t)^{1-\zeta} \leq \frac{(1 + r_b)d_t s_t}{\chi(e_t + d_t)^\zeta} \\ + \chi(e_t + d_t)^\zeta G^{-1}\left(\frac{(1 + r_b)d_t s_t - \chi(1 - \delta)(e_t + d_t)}{\chi(e_t + d_t)^\zeta}\right) & \leq \mathbb{E}(\phi) + (1 - \delta)(e_t + d_t)^{1-\zeta} \end{cases}$$

Expressions for $R_b(d_t, s_t, e_t)$ and $R_m(d_t, s_t, e_t)$ that use these inverse mappings, for other cases, are reported in the appendix 1 of Crouzet (2013).

continuous functions on $[0, E]$, with the sup norm $\|\cdot\|_s$, onto itself, where $E > 0$ is an arbitrarily large upper bound for equity. Additionally, show that $T(C_0(E)) \subseteq C_0(E)$, where $C_0(E) = \{V \in C(E) \text{ s.t. } V(0) \geq 0\}$. Since $(C(E), \|\cdot\|_s)$ is a complete metric space and $C_0(E)$ is a closed subset of $C(E)$ under $\|\cdot\|_s$ which is additionally stable through T , if T is a contraction mapping, then its fixed point must be in $C_0(E)$.

Step 3: Check that T satisfies Blackwell's sufficiency conditions, so that it is indeed a contraction mapping.

Note that step 2 is crucial because lemma 1 requires the continuity of V and the fact that $V(0) \geq 0$ for V^c to be continuous, strictly increasing and satisfy $V^c(0) \geq 0$. In turn, these three conditions are necessary for characterizing of the set of feasible debt structures, that is, for proposition 2 to hold.

Step 1: Define the mapping T on $C(E)$ as:

$$\forall e_t \in [0, E], \quad TV(e_t) = \max_{R,K} \left(T_R V(e_t), T_K V(e_t) \right) \quad (\text{A1})$$

where the mappings T_R and T_K , also defined on $C(E)$, are given by:

$$\begin{aligned} \forall e_t \in [0, E], \quad T_R V(e_t) &= \max_{(d_t, s_t) \in \mathcal{S}_R(e_t)} \int_{\phi_t \geq \underline{\phi}_R(e_t, d_t, s_t)} V^c(n_R(\phi_t; e_t, d_t, s_t)) dF(\phi_t) & (\text{A1-R}) \\ \text{s.t.} \quad V^c(n_t) &= \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}) \\ n_R(\phi_t, e_t, d_t, s_t) &= \begin{cases} \left(\phi_t - \chi \bar{\phi}_R(e_t, d_t, s_t) - (1 - \chi) \underline{\phi}_R(e_t, d_t, s_t) \right) (e_t + d_t)^\zeta & \text{if } \bar{\phi}_R(e_t, d_t, s_t) \leq \phi_t \\ (1 - \chi) \left(\phi_t - \underline{\phi}_R(e_t, d_t, s_t) \right) (e_t + d_t)^\zeta & \text{if } \underline{\phi}_R(e_t, d_t, s_t) \leq \phi_t \leq \bar{\phi}_R(e_t, d_t, s_t) \end{cases} \\ \underline{\phi}_R(e_t, d_t, s_t) &= \begin{cases} 0 & \text{if } 0 \leq (1 + r_m)d_t(1 - s_t) < (1 - \chi)(1 - \delta)(e_t + d_t) \\ I^{-1} \left(\frac{(1 + r_m)d_t(1 - s_t) - (1 - \chi)(1 - \delta)(e_t + d_t)}{(1 - \chi)(e_t + d_t)^\zeta}; e_t + d_t \right) & \text{if } (1 - \chi)(1 - \delta)(e_t + d_t) \leq (1 + r_m)d_t(1 - s_t) \\ & \leq (1 - \chi)(1 - \delta)(e_t + d_t) + (1 - \chi)(e_t + d_t)^\zeta \hat{I}(e_t + d_t) \end{cases} \\ \bar{\phi}_R(e_t, d_t, s_t) &= \begin{cases} 0 & \text{if } 0 \leq (1 + r_b)d_t s_t < \chi(1 - \delta)(e_t + d_t) \\ G^{-1} \left(\frac{(1 + r_b)d_t s_t - \chi(1 - \delta)(e_t + d_t)}{\chi(e_t + d_t)^\zeta}; e_t + d_t \right) & \text{if } \chi(1 - \delta)(e_t + d_t) \leq (1 + r_b)d_t s_t \\ & \leq \chi(1 - \delta)(e_t + d_t) + \chi(e_t + d_t)^\zeta \mathbb{E}(e_t + d_t) \end{cases} \end{aligned}$$

$$\text{and:} \quad \forall e_t \in [0, E], \quad T_K V(e_t) = \max_{(d_t, s_t) \in \mathcal{S}_K(e_t)} \int_{\phi_t \geq \underline{\phi}_K(e_t, d_t, s_t)} V^c(n_K(\phi_t; e_t, d_t, s_t)) dF(\phi_t) \quad (\text{A1-K})$$

$$\begin{aligned} \text{s.t.} \quad V^c(n_t) &= \max_{0 \leq e_{t+1} \leq n_t} n_t - e_{t+1} + (1 - \eta)\beta V(e_{t+1}) \\ n_K(\phi_t, e_t, d_t, s_t) &= \left(\phi_t - \underline{\phi}_K(e_t, d_t, s_t) \right) (e_t + d_t)^\zeta & \text{if } \phi_t \geq \underline{\phi}_K(e_t, d_t, s_t) \\ \underline{\phi}_K(e_t, d_t, s_t) &= \begin{cases} 0 & \text{if } 0 \leq (1 + r_m(1 - s_t) + r_b s_t)d_t < (1 - \delta)(e_t + d_t) \\ M^{-1} \left(\frac{(1 + r_m(1 - s_t) + r_b s_t)d_t - (1 - \delta)(e_t + d_t)}{(e_t + d_t)^\zeta}; e_t + d_t \right) & \text{if } (1 - \delta)(e_t + d_t) \leq (1 + r_m(1 - s_t) + r_b s_t)d_t \\ & \leq (1 - \delta)(e_t + d_t) + (e_t + d_t)^\zeta \hat{M}(e_t + d_t) \end{cases} \end{aligned}$$

Consider a solution V to problem (8) and a particular value of $e_t \in [0, E]$. Since $(\mathcal{S}_K(e_t), \mathcal{S}_R(e_t))$ is a partition of $\mathcal{S}(e_t)$, the optimal policies (\hat{d}_t, \hat{s}_t) (there may be several) must be in either $\mathcal{S}_R(e_t)$ or $\mathcal{S}_K(e_t)$. Assume that they are in $\mathcal{S}_R(e_t)$. Then the contracts $R_{b,t}, R_{m,t}$ associated with the optimal policies satisfy $\frac{R_{b,t}}{\chi} \geq \frac{R_{m,t}}{1 - \chi}$. Given the results of proposition 2, the constraints and objectives in problem (8) can be rewritten as in (A1-R). Since V solves (8), this implies that $V(e_t) = T_R V(e_t)$. Moreover, in that case $T_R V(e_t) = V(e_t) \geq T_K V(e_t)$, by optimality of (\hat{d}_t, \hat{s}_t) . Thus, $TV(e_t) = V(e_t)$. The same equality obtains if $(\hat{d}_t, \hat{s}_t) \in \mathcal{S}_K(e_t)$. Any solution to V to problem (8) must thus satisfy

$TV = V$. The rest of the proof therefore focuses on the properties of the operators T , T_K and T_R .

Step 2: Let $V \in C(E)$. By lemma 1, the associated continuation value V^c is continuous on \mathbb{R}_+ . Moreover, since I^{-1} and G^{-1} are continuous functions of e_t, d_t and s_t , the functions n_R , $\underline{\phi}_R$ and $\bar{\phi}_R$ are continuous in their (e_t, d_t, s_t) arguments. Define the mapping $O_R : [0, E] \times [0, \bar{d}(E)] \times [0, 1] \rightarrow \mathbb{R}_+$ by $O_R(e_t, d_t, s_t) = \int_{\phi_t \geq \underline{\phi}_R(e_t, d_t, s_t)} V^c(n_t^R(\phi_t; e_t, d_t, s_t)) dF(\phi_t)$. Here $\bar{d}(E)$ denotes the upper bound on borrowing for the maximum level of equity E .⁶³ By continuity of $V^c, n_t^R, \underline{\phi}_R$ and $\bar{\phi}_R$, the integrand in O_R is continuous on the compact set $[0, E] \times [0, \bar{d}(E)] \times [0, 1]$, and therefore uniformly continuous. Hence, O_R is continuous on $[0, E] \times [0, \bar{d}(E)] \times [0, 1]$. The constraint correspondence $\Gamma_R : e_t \rightarrow \mathcal{S}_R(e_t)$ maps $[0, E]$ into $[0, \bar{d}(E)] \times [0, 1]$. The characterization of the set $\mathcal{S}_R(e_t)$ in proposition 4 of [Crouzet \(2013\)](#) moreover shows that the graph of the correspondence Γ_R is closed and convex. Theorems 3.4 and 3.5 in [Stokey, Lucas, and Prescott \(1989\)](#) then indicate that Γ_R is continuous. Given that O_R is continuous and Γ_R compact-valued and continuous, the theorem of the maximum applies, and guarantees that $T_R V \in C(E)$. In analogous steps, one can prove that $T_K V \in C(E)$. Therefore, $TV = \max(T_R V, T_K V) \in C(E)$. Moreover, let $V \in C_0(E)$. Then $V^c(0) \geq 0$ and V^c is increasing, by lemma 1. Moreover $\mathcal{S}_R(0) \neq \emptyset$, so one can evaluate O_R at some $(d_{t,0}, s_{t,0}) \in \mathcal{S}_R(0)$. Since $n_R \geq 0$, $V^c(0) \geq 0$ and V^c is increasing, $O_R(0, d_{t,0}, s_{t,0}) \geq 0$. Therefore $T_R V(0) \geq 0$, so $TV(0) \geq 0$ and $TV \in C_0(E)$.

Step 3: Finally, I establish that the operator T has the monotonicity and discounting properties. First, let $(V, W) \in C(E)$ such that $\forall e_t \in C(E)$, $V(e_t) \geq W(e_t)$. Pick a particular $e_t \in [0, E]$. By an argument similar to the proof of lemma 1, $\forall n_t \geq 0$, $V^c(n_t) \geq W^c(n_t)$, where W^c denotes the solution to the dividend issuance problem when the continuation value is W (and analogously for V). Since the functions $\underline{\phi}_R, \bar{\phi}_R$ and n_R are independent of V , this inequality implies $O_R^V(e_t, d_t, s_t) \geq O_R^W(e_t, d_t, s_t)$ for any $(d_t, s_t) \in \mathcal{S}_R(e_t)$, where the notation O_R^W designates the objective function in problem (A1-R) when the continuation value function is W (and analogously for V). Thus $T_R V(e_t) \geq T_R W(e_t)$. Similarly, one can show that $T_K V(e_t) \geq T_K W(e_t)$. Therefore, $TV(e_t) \geq TW(e_t)$, and T has the monotonicity property. To establish the discounting property, it is sufficient to note that $(V+a)^c(n_t) = V^c(n_t) + \beta a$, so that for any $e_t \in [0, E]$ and $(d_t, s_t) \in \mathcal{S}_R(e_t)$, $O_R^{V+a}(e_t, d_t, s_t) = O_R^V(e_t, d_t, s_t) + \left(1 - F\left(\underline{\phi}_R(e_t, d_t, s_t)\right)\right) \beta a \leq O_R^V(e_t, d_t, s_t) + \beta a$. This shows that $T_R(V+a)(e_t) \leq T_R V(e_t) + \beta a$. A similar claim can be made for T_K . Therefore, the operator T has the discounting property. The Blackwell sufficiency conditions hold, so that T is a contraction mapping. As a contracting mapping on a complete metric space, it has a unique fixed point.

Properties of the solution to problem (8) Let V denote the unique solution to problem (8).

Monotonicity: First, it can be shown that $\forall (e_t^1, e_t^2) \in [0, E]$ s.t. $e_t^1 < e_t^2$, $T_R V(e_t^1) < T_R V(e_t^2)$. To show this, let $(d_t, s_t) \in \mathcal{S}_R(e_t^1)$. I proceed in three steps:

1 : First, since $e_t^1 < e_t^2$, $\mathcal{S}_R(e_t^1) \subset \mathcal{S}_R(e_t^2)$ (a proof for this can be obtained using proposition 4 of [Crouzet \(2013\)](#); intuitively, this result indicates that increasing internal finance relaxes borrowing constraints). Therefore, $(d_t, s_t) \in \mathcal{S}_R(e_t^2)$.

2 : Next, I show that $\bar{\phi}_R(e_t^1, d_t, s_t) > \bar{\phi}_R(e_t^2, d_t, s_t)$. (The functions at this point are well-defined because $(d_t, s_t) \in$

⁶³See the results of proposition 4 of [Crouzet \(2013\)](#) for a proof that such an upper bound always exist.

$S_R(e_t^2)$). Given the expression for $\bar{\phi}_R$ in (A1-R), since G^{-1} is strictly increasing on \mathbb{R}_+ , it is sufficient to show that $e_t \rightarrow \frac{(1+r_b)d_t s_t - \chi(1-\delta)(e_t + d_t)}{\chi(e_t + d_t)^\zeta}$ is strictly decreasing in e_t . This is true because $\zeta < 1$. Next I prove that $\underline{\phi}_R(e_t^1, d_t, s_t) > \underline{\phi}_R(e_t^2, d_t, s_t)$. To see this, note that:

$$\frac{\partial \underline{\phi}_R(e_t, d_t, s_t)}{\partial e_t} = \frac{\frac{\partial y_I}{\partial e_t} - I_2(I^{-1}(y_I(e_t); e_t + d_t); e_t + d_t)}{I_1(I^{-1}(y_I(e_t); e_t + d_t); e_t + d_t)},$$

where $y_I(e_t) \equiv \frac{(1+r_m)d_t(1-s_t) - (1-\chi)(1-\delta)(e_t + d_t)}{(1-\chi)(e_t + d_t)^\zeta}$. Note that, letting $x = I^{-1}(y_I(e_t); e_t + d_t)$:

$$\frac{\partial y_I}{\partial e_t} - I_2(x; e_t + d_t) = -\zeta \frac{(1+r_m)d_t(1-s_t)}{(1-\chi)(e_t + d_t)^{\zeta+1}} - (1-\zeta)(1-\delta)(e_t + d_t)^{-\zeta} (1-F(x)) < 0$$

Since $I_1 > 0$, this implies that $\underline{\phi}_R$ is strictly decreasing in e_t .

3 : Given the fact that $\underline{\phi}_R$ and $\bar{\phi}_R$ are strictly decreasing in e_t , the expression for $n_R(\phi_t; e_t, d_t, s_t)$ in (A1-R) then implies that $\forall \phi_t \geq \underline{\phi}_R(e_t^1, d_t, s_t)$, $n_R(\phi_t; e_t^1, d_t, s_t) < n_R(\phi_t; e_t^2, d_t, s_t)$. Thus, since V^c is increasing,

$$\begin{aligned} O_R(e_t^1, d_t, s_t) &= \int_{\phi_t \geq \underline{\phi}_R(e_t^1, d_t, s_t)} V^c(n_R(\phi_t; e_t^1, d_t, s_t)) dF(\phi_t) \\ &< \int_{\phi_t \geq \underline{\phi}_R(e_t^1, d_t, s_t)} V^c(n_R(\phi_t; e_t^2, d_t, s_t)) dF(\phi_t) \\ &\leq \int_{\phi_t \geq \underline{\phi}_R(e_t^2, d_t, s_t)} V^c(n_R(\phi_t; e_t^2, d_t, s_t)) dF(\phi_t) \\ &= O_R(e_t^2, d_t, s_t). \end{aligned}$$

where the second line exploits the fact that $\underline{\phi}_R(e_t^1, d_t, s_t) > \underline{\phi}_R(e_t^2, d_t, s_t)$ and $V^c \geq 0$.

Since last inequality has been established for any $(d_t, s_t) \in S_R(e_t^1) \subset S_R(e_t^2)$, it shows that the objective function is uniformly increasing (strictly) in e_t , so that $T_R V(e_t^1) < T_R V(e_t^2)$. A similar but simpler proof using the expression for $\underline{\phi}_K$ in (A1-K) shows that $T_K V(e_t^1) < T_K V(e_t^2)$. Therefore, $TV(e_t^1) < TV(e_t^2)$, so that $V(e_t^1) < V(e_t^2)$. Therefore, the solution to problem 8 is strictly increasing in e_t .

Existence and unicity of an invariant measure I next prove that, given a solution to problem (8), an invariant measure of firms across levels of e_t exists. I start by introducing some preliminary notation.

Preliminary notation \bar{e} denotes the level of net worth above which firms start issuing dividends. $\bar{E} = [0, \bar{e}]$ denotes the state-space of the firm problem (8). $(\bar{E}, \bar{\mathcal{E}})$ is the measurable space composed of \bar{E} and the family of Borel subsets of \bar{E} . For any value $e_t \in \bar{E}$, $\hat{d}(e_t)$ and $\hat{s}(e_t)$ denote the policy functions of the firms. The fact that these policy functions are such that $(\hat{d}(e_t), \hat{s}(e_t)) \in \mathcal{S}_R(e_t)$ will be denoted by $e_t \in \bar{E}_R$, and $e_t \in \bar{E}_K$ for the other case. $\hat{\phi}_R(e_t)$ and $\hat{\phi}_K(e_t)$ denote the liquidation threshold implied by the firm's policy functions when $e_t \in \bar{E}_R$, while $\hat{\phi}_K(e_t)$ denotes the liquidation threshold when $e_t \in \bar{E}_K$. I use the notation: $\hat{r}(e_t) = r_m(1 - \hat{s}(e_t)) + r_b \hat{s}(e_t)$. Finally, recall that $F(\cdot)$ denotes the CDF of ϕ_t , the idiosyncratic productivity shock, and η denotes the exogenous exit probability.

Transition function Let $N(e_t, e_{t+1})$ denote the probability that a firm will a level of internal finance of at

least e_{t+1} , given that its current internal finance level is e_t . Three elements contribute to this probability: the exogenous exit probability; conditional on survival, the firm's borrowing and dividend policy; and the replacement of exiting firms by new ones, entering at scale e^e . The exact expression of transition probabilities depend both on whether $e_t \in \bar{E}_K$, or $e_t \in \bar{E}_R$, and on the amount borrowed by firms in each case; it is straightforward but tedious to construct, and is reported in detail in the online appendix. This expression indicates that $N(e_t, \cdot)$ is weakly increasing, has limits 0 and 1 at $-\infty$ and $+\infty$ and is everywhere continuous from above. Following theorem 12.7 of [Stokey, Lucas, and Prescott \(1989\)](#), there is therefore a unique probability measure $\hat{Q}(e_t, \cdot)$ on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ such that $\hat{Q}(e_t,]-\infty, e]) = N(e_t, e) \forall e \in \mathbb{R}$. This measure is 0 for $e < 0$ and 1 for $e > \bar{e}$, and so the restriction of \hat{Q} to $\bar{\mathcal{E}}$ is also a probability measure on $(\bar{E}, \bar{\mathcal{E}})$, which can be denoted $Q(e_t, \cdot)$. Moreover, fixing $e \in \bar{E}$, the function $Q(\cdot,]-\infty, e]) : \bar{E} \rightarrow [0, 1]$ is measurable with respect to $\bar{\mathcal{E}}$. Indeed, given $z \in [0, 1]$, the definition of $N(e_t, e)$ indicates that the set $H(z, e) = \{e_t \in \bar{E} \text{ s.t. } Q(e_t, [0, e]) \leq z\} = \left\{e_t \in \bar{E} \text{ s.t. } \hat{Q}(e_t,]-\infty, e]) \leq z\right\} = \{e_t \in \bar{E} \text{ s.t. } N(e_t, e) \leq z\}$ is an element of $\bar{\mathcal{E}}$.⁶⁴ This in turn implies that the function $Q(\cdot, \mathbf{E})$ is $\bar{\mathcal{E}}$ -measurable for any $\mathbf{E} \in \bar{\mathcal{E}}$. The function $Q : \bar{E} \times \bar{\mathcal{E}} \rightarrow [0, 1]$ is such that $Q(e_t, \cdot)$ is therefore a probability measure for any $e_t \in \bar{E}$, and $Q(\cdot, \mathbf{E})$ is $\bar{\mathcal{E}}$ -measurable $\forall \mathbf{E} \in \bar{\mathcal{E}}$; hence, Q is a transition function.

Feller property To establish that the transition function Q has the Feller property, one must show that $\forall e_t \in \bar{E}$ and $\forall (e_{n,t})_n \in \bar{E}^{\mathbb{N}}$ such that $e_{n,t} \rightarrow e_t$, $Q(e_{n,t}, \cdot) \Rightarrow Q(e_t, \cdot)$, where \Rightarrow denotes weak convergence. To establish this, given the definition of Q it is sufficient to show that $N(e_{n,t}, e_{t+1}) \rightarrow N(e_t, e_{t+1})$ pointwise, at all values of e_{t+1} and e_t where $N(\cdot, e_{t+1})$ and $N(e_t, \cdot)$ are continuous. This excludes, in particular, the cases $e_{t+1} = \bar{e}$ or $e_t = \bar{e}$. I give the proof for the case $e_t \in \bar{E}_K$; the proof for the other case ($e_t \in \bar{E}_R$) is similar.

First consider a simple case, when e_t satisfies:

$$(1 + \hat{r}(e_t))\hat{d}(e_t) > (1 - \delta)(e_t + \hat{d}(e_t)) \quad (11)$$

Because $e_t \mapsto \eta + (1 - \eta)F\left(\frac{\hat{\phi}_K(e_t)}{\hat{\phi}_K(e_t) + \hat{d}(e_t)}\right)$ is continuous on \bar{E}_K , $e \mapsto N(e, e_{t+1})$ is continuous in a neighborhood of e_t . Additionally, since \hat{r} and \hat{d} is continuous, the inequality (11) holds for $e_{n,t}$, when n is sufficiently large. Combining these two observations implies that $N(e_{n,t}, e_{t+1}) \rightarrow N(e_t, e_{t+1})$. The case when the inequality above holds in the reverse direction is handled similarly.

Now consider a knife-edge case:

$$(1 + \hat{r}(e_t))\hat{d}(e_t) = (1 - \delta)(e_t + \hat{d}(e_t)) \quad (12)$$

⁶⁴For example, if $z \leq \eta$ and $0 < e < \bar{e}$, the intersection of the set $H(z, e)$ with \bar{E}_K is given by:

$$\begin{aligned} H(z, e) \cap \bar{E}_K &= \left\{e_t \in \bar{E}_K \text{ s.t. } (1 - \delta)(e_t + \hat{d}(e_t)) \geq (1 + \hat{r}(e_t))\hat{d}(e_t) + \bar{e}\right\} \cup \\ &\quad \left\{e_t \text{ s.t. } (1 + \hat{r}(e_t))\hat{d}(e_t) + \bar{e} > (1 - \delta)(e_t + \hat{d}(e_t)) \geq (1 + \hat{r}(e_t))\hat{d}(e_t) \text{ and } \underline{e}(e_t) > e\right\} \\ &= \left\{e_t \in \bar{E}_K \text{ s.t. } (1 - \delta)(e_t + \hat{d}(e_t)) > (1 + \hat{r}(e_t))\hat{d}(e_t) + e\right\} \end{aligned}$$

This set is the inverse image of $]0, +\infty[$ by the function $g : \bar{E}_K \rightarrow \mathbb{R}, x \rightarrow (1 - \delta)(x + \hat{d}(x)) - (1 + \hat{r}(x))\hat{d}(x) - e$. Since the policy functions are continuous, g is continuous. Since the inverse image of an open set by a continuous function is an open set, $H(z, e) \cap \bar{E}_K$ is an open set, and hence a Borel set. The intersection $H(x, e) \cap \bar{E}_R$ has a more complicated expression, but also boils down to a finite union of sets that are open because of the continuity of policy functions. Given that \bar{E}_K and \bar{E}_R are intervals that form a partition of \bar{E} , this implies that $H(z, e) = (H(z, e) \cap \bar{E}_R) \cup (H(z, e) \cap \bar{E}_K)$ is a finite union of Borel sets, and therefore a Borel set. This line of reasoning applies for all $0 \leq z \leq 1$ and $0 \leq e \leq \bar{e}$.

The problem is that the sequence $(e_{n,t})_n$ can have elements that satisfy either $(1 + \hat{r}(e_{n,t}))\hat{d}(e_{n,t}) \geq (1 - \delta)(e_{n,t} + \hat{d}(e_{n,t}))$, which correspond to different expressions for $N(e_{n,t}, e_{t+1})$; one must check therefore check that $N(\cdot, e_{t+1})$ is continuous at e_t such that (12) holds. At such a point, $\underline{e}(e_t) = 0$, so that:

$$\lim_{e \uparrow e_t} N(e_t, e_{t+1}) = \lim_{e \uparrow e_t} \eta + (1 - \eta)F\left(\frac{e_{t+1} - \bar{e}(e_t)}{(e_t + \hat{d}(e_t))^\zeta}\right) = \eta + (1 - \eta)F\left(\frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\zeta}\right).$$

Moreover, $\hat{\phi}_K(e_t) = 0$, so that:

$$\lim_{e \downarrow e_t} N(e_t, e_{t+1}) = \lim_{e \downarrow e_t} \eta + (1 - \eta)F\left(\hat{\phi}_K(e_t) + \frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\zeta}\right) = \eta + (1 - \eta)F\left(\frac{e_{t+1}}{(e_t + \hat{d}(e_t))^\zeta}\right).$$

This establishes the continuity of $N(\cdot, e_{t+1})$ at points e_t such that equation (12) holds; thus, $\forall e_{t+1} \in \bar{E}$, $\bar{e}_{t+1} \neq \bar{e}$, $N(\cdot, e_{t+1})$ is continuous on \bar{E}_K .

Finally, given that the transition function Q has the Feller property, theorem 12.10 of [Stokey, Lucas, and Prescott \(1989\)](#) indicates that there exists a probability measure on \bar{E} that is invariant under Q . ■

C The italian tax reform

In this appendix, I first briefly describe the introduction of tax shields into the model. I then illustrate the effects of the Italian reform described in section 6.2 on the borrowing and investment choices of firms.

Introducing tax shields As mentioned in section 6.2, when gross income and debt payments are subject to differential taxation, the cash on hand of firm that repays its creditors can generally be written as:

$$n_t^R = (1 - \tau)\pi_t - (1 - \tau_b)R_{b,t} - (1 - \tau_m)R_{m,t}.$$

One must specify how the firm and its creditor's income are taxed under payment, restructuring and liquidation. I make two key assumptions in this regard:

Assumption 4 (Tax treatment of restructuring and liquidation)

- *Income tax liabilities are senior to bank and market debt payments in liquidation;*
- *There are no tax shields for debt payments that have been restructured.*

The first assumption is innocuous, and simply guarantees that firms will not find it beneficial to default in order to avoid the payment of tax liabilities. The second assumption guarantees that, when tax shields are identical ($\tau_b = \tau_m$), the restructuring choices of the firm are similar to the baseline model; it therefore helps to focus the discussion on the effects of *asymmetric* tax treatment of debt.

With these assumptions, the payoffs to stakeholders in liquidation are given by:

$$\begin{aligned}
\tilde{R}_{b,t} &= \min(R_{b,t}, (1-\tau)\chi\pi_t) && \text{(bank lenders)} \\
\tilde{R}_{m,t} &= \min(\max(0, (1-\tau)\chi\pi_t - R_{b,t}), R_{m,t}) && \text{(market lenders)} \\
n_t^L &= \max(0, (1-\tau)\chi\pi_t - R_{b,t} - R_{m,t}) && \text{(firm)}
\end{aligned} \tag{13}$$

Moreover, in restructuring the firm will drive the bank down to its reservation value, $(1-\tau)\chi\pi_t$. In that case, given the second assumption, the cash on hand of the firm after restructuring will be given by:

$$n_t^R = (1-\tau)\pi_t - (1-\tau)\chi\pi_t - (1-\tau_m)R_{m,t}.$$

Given this, the following lemma is straightforward to establish.

Lemma 2 (Debt settlement outcomes) *Assume that $V^c(\cdot)$ is increasing, and $V^c(0) \geq 0$. Then, there are two types of debt settlement outcomes:*

- **When** $\frac{(1-\tau_b)R_{b,t}}{\chi} \geq \frac{(1-\tau_m)R_{m,t}}{1-\chi}$, *the firm chooses to repay its creditors in full, if and only if, $\pi_t \geq \frac{(1-\tau_m)R_{b,t}}{\chi}$. It successfully restructures its debt, if and only if, $\frac{(1-\tau_m)R_{m,t}}{1-\chi} \leq \pi_t < \frac{(1-\tau_b)R_{b,t}}{\chi}$, and it is liquidated when $\pi_t < \frac{(1-\tau_m)R_{m,t}}{1-\chi}$.*
- **When** $\frac{(1-\tau_b)R_{b,t}}{\chi} < \frac{(1-\tau_m)R_{m,t}}{1-\chi}$, *the firm repays its creditors in full if and only if $\pi_t \geq \frac{(1-\tau_b)R_{b,t} + (1-\tau_m)R_{m,t}}{(1-\tau)}$, and it is liquidated otherwise.*

Moreover, in any successful restructuring offer, the bank obtains its reservation value $(1-\tau)\chi\pi_t$, and in all debt settlement outcomes resulting in liquidations, $n_t^L = 0$.

Thus, under the two assumptions above, the structure of the debt settlement outcomes is similar to the baseline model: when the firm's bank liabilities are large enough, it will sometimes restructure debt contracts conditional on its productivity realizations; otherwise, it never uses the restructuring option. The difference is that, when $\tau_b \neq \tau_m$, the firm's decision to restructure debt contracts also depends on the relative values of the tax shield on bank and market debt, and not simply on liquidation losses (associated with the parameter χ).

Effects of the reform Lemma 2 be used to fully characterize the set of feasible debt contracts, and therefore the general formulation of firms' optimal debt structure problem when there are differential tax treatments of debt. These derivations are available upon request.

I next turn to the effects of the policy experiment described in section 6.2. Namely, I compare firm-level borrowing and investment in an economy without and with the tax reform. In the baseline economy, all firms enjoy tax shields for bank debt, but only large firms enjoy a tax shield for market debt issuance:

$$\begin{aligned}
\tau_b(e_t) &= \tau \quad \forall e_t, \\
\tau_m(e_t) &= \begin{cases} 0 & \text{if } e_t \leq e_{sm} \\ \tau & \text{if } e_t > e_{sm} \end{cases}
\end{aligned}$$

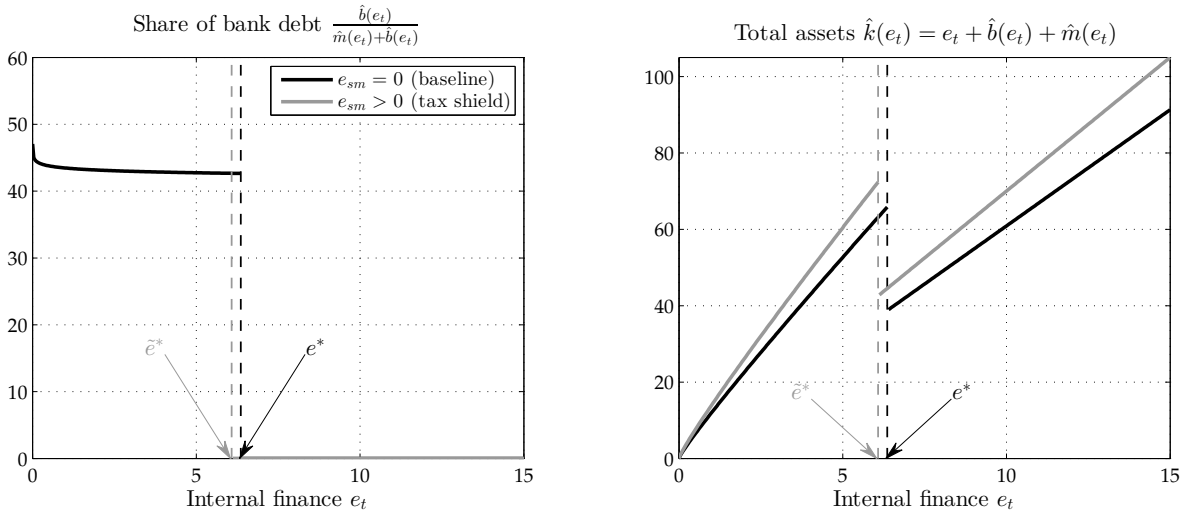


Figure 15: The effect of the tax reform on borrowing and investment.

In the reformed economy, however, the tax shield applies to all debt issuances of all firms:

$$\tau_b(e_t) = \tau_m(e_t) = \tau \quad \forall e_t.$$

As in the case of the experiment of section 6.1, so as to clarify the exposition of the effects of the subsidy, I set the threshold for the reform at $e_{sm} = e^*$. Moreover, I focus on the borrowing policies of firms in the static version of the model. Indeed, the results of section 6.1 suggest that the bulk of the effects of this type of policy is mediated by firms' borrowing decisions, rather than by long-run changes in the firm size distribution.

Figure 15 reports the results of the experiment, when the tax rate is $\tau = 2.5\%$. The reform has little incidence on the debt composition of firms, but induces a measure of firms to switch to pure market finance (left panel). In doing so, these firms operate at a lower scale than they otherwise would have (right panel). The investment of all other firms, however, gains from the introduction of the tax shield. These results are qualitatively analogous to those obtained in the experiment of section 6.1.