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## AGGREGATE LAND RENTS, EXPENDITURE ON PUBLIC GOODS, AND OPTIMAL CITY SIZE\*

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I. An example, 473.—II. The generality of the Henry George Theorem, 477.—III. On using land rents as a measure of the benefits from public goods, 490.—IV. Competitive attainability of a Pareto optimal distribution of economic activity, 496.—V. Concluding comments, 498.

This paper explores the relationship between aggregate land rents and public expenditure in a residential urban economy. That there are important relationships between aggregate land rents and public goods expenditure has already been recognized in two different contexts. First, the practice of inferring individual valuations of public goods from land values is now widespread (see the literature on the capitalization of fiscal residuals [Oates, 1969; Edel and Sclar, 1974; and Meadows, 1976], for instance).<sup>1</sup> This literature, however, does not directly address the relationship between aggregate land rents and public goods expenditure. Second, Flatters, Henderson, and Mieszkowski [1974], and Stiglitz [1977] have shown that in a simple

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1. The literature on the capitalization of fiscal residuals attempts to explain the values of individual properties. Among the explanatory variables used are jurisdiction-specific tax rates and public expenditures. There is another rather different strand of the capitalization literature that includes the change in aggregate land values induced by a transport improvement in the benefits from the improvement. Notable studies that employ this procedure include Fishlow [1965] and Fogel [1964] in their studies measuring the benefits of the American railroad. The invalidity of this procedure for all but small transport improvements in an open economy is argued in Arnott and Stiglitz [1978].

spatial economy, where the spatial concentration of economic activity is due to a pure local public good and where population size is optimal,<sup>2</sup> aggregate land rents equal expenditure on the pure public good.<sup>3</sup> This result has been dubbed the Henry George Theorem (HGT), since a confiscatory tax on land rents is not only efficient, it is also the "single tax" necessary to finance the pure public good.

This paper is directed at providing insights into the following issues:

1. How general is the Henry George Theorem? We show that it is far more robust than has previously been suspected; in particular, it holds in all large economies in which differential land rents are well defined, and in which the distribution of economic activity over space is Pareto optimal. However, it is still far from completely general.

2. Does unfettered migration in a competitive economy result in a Pareto optimal distribution of population over cities? Our analysis shows that the answer depends on the definition of competitive behavior in a spatial economy.

3. Is there a simple relationship between the local public goods offered by different communities, and aggregate land rents in those communities? We investigate some circumstances in which there is a simple relationship and others in which there is not, and relate the results to the capitalization literature.

In Section I we present an especially simple model that provides an intuitive basis for understanding the more general results derived in Section II. Section II examines the circumstances in which the HGT does and does not hold. In Section III the relationship between aggregate land rents and the benefits from public goods is analyzed. And

2. The basic notion behind the optimal population of a city is a simple one. If communities were collections of individuals enjoying the same pure public good, then since there is no congestion in its consumption, optimal community size would be infinite. But there is a cost to increasing population. In our model this comes from the additional transport costs and crowding of land that the added individual causes. The optimal population is that where the marginal benefits arising from the increasing returns resulting from the public good just offset the increased transport and crowding costs due to the added individual.

3. The theorem is somewhat surprising, since regardless of whether additional population causes more crowding, higher transport costs or both, the optimal city size can be characterized in terms of aggregate land rents and expenditure on public goods, without reference to aggregate transport costs.

This theorem is closely related to a theorem independently proved by Mirrlees [1972], Starrett [1974], and Vickrey [1977]. They consider only completely planned economies; i.e., economies in which a social welfare function is maximized, with no restrictions on the imposition of lump-sum taxation. They show that when the reason for spatial concentration is economies of scale in production, rather than a pure local public good, optimal size is characterized by equality between the degree of increasing returns to scale (defined as the elasticity of output with respect to the input, minus one) times the value of production and differential land rents.

Section IV considers the Pareto optimality of competitive equilibria with free migration.

## I. AN EXAMPLE

### 1.1. *An Algebraic Analysis*

In this section we employ the standard residential location model, but as will be shown later, our central results extend to more general spatial economies. In this model there is a single city center, a point in space, at which all nonresidential urban activity takes place. Land is used only for the housing of identical city residents who live at different distances from the city center. If land is not scarce (i.e., the opportunity rent on land at the boundary of the city is zero), if transport costs are simply a function of the crow-line distance from the city center, and if land is homogeneous, then the city will be circular. However, if land is so scarce that the whole plain is occupied by cities, then under the above conditions, every city will be hexagonal.<sup>4</sup>

The identical individuals derive utility from lot size, a pure (no congestion) public good, and private goods, and have no preference for location per se. The government owns the land and auctions it off competitively, provides a pure local public good, and divides residual resources equally among residents, who use this income to purchase private goods and transport services, one unit of each of which costs one unit of resource, and to obtain a lot in the competitive land market. These assumptions together imply that residents' utilities are equal in equilibrium. Trip frequency is fixed, and tastes are such that everyone lives on a lot of unit size. In competitive equilibrium the benefits to an individual from moving a small distance farther from the city center must equal the costs. Let  $t$  be distance from the city center,  $f(t)$  be the transport costs associated with location  $t$ , and  $R(t)$  be land rent per unit area at  $t$ . The benefit from moving  $dt$  farther from the city center is the decrease in lot rent  $-R'(t)dt$  (where a prime denotes  $d/dt$ ), while the cost is the increase in transport costs  $f'(t)dt$ . Thus,

$$(1.1) \quad R'(t) = -f'(t).$$

For a circular city with boundary  $t^*$  from the center, aggregate land rents ( $ALR$ ) equal

4. This result can be obtained for large economies with identical individuals and homogeneous land by application of theorems presented in Bollobas and Stern [1972].

$$(1.2) \quad ALR \equiv \int_0^{t^*} R(t)2\pi t dt.$$

$ALR$  is calculated as the rent per unit area of land at a distance  $t$  from the center times the number of units of land between  $t$  and  $t + dt(2\pi t dt)$ , integrated over all  $t$ . Similarly, aggregate transport costs ( $ATC$ ) are

$$(1.3) \quad ATC \equiv \int_0^{t^*} f(t)2\pi t dt.$$

Integrating (1.2) by parts, and substituting (1.1), we obtain

$$(1.4) \quad \int_0^{t^*} -R'\pi t^2 dt + R(t^*)\pi t^{*2} = \int_0^{t^*} f'\pi t^2 dt + R(t^*)\pi t^{*2}.$$

The second term on the right-hand side is just the area of the city times the rent on marginal land; hence, the first term is differential rents. Denoting differential land rents by  $DLR$ , we observe, by comparing (1.3) and (1.4), that with linear transport costs,<sup>5</sup>

$$(1.5) \quad DLR = 1/2 ATC,$$

since

$$f't = f \quad \text{for all } t.$$

We now consider the problem of the optimal population for this city. The objective is to maximize per capita utility. With transport costs if individuals consume only private goods and land and have no preference for location per se, and if there are constant or diminishing returns to population, optimal population size is zero. The presence of location-specific pure public goods introduces an inherent non-convexity. If individuals consume only public goods, the optimal population occurs at the point where the marginal productivity of an individual is zero; in the case of the Cobb-Douglas production function, optimal population would be infinite. With individuals con-

5. Mohring [1961] analyzed this case of a circular city with linear transport costs and uniform lot size. He provided an ingenious geometric explanation of this result. From (1.1) each person's expenditure on land rent plus transport costs is the same. Since each person resides on a lot of unit size, land rent plus transport costs are constant per unit area  $h''$  over the settled area of the city. Consider plotting land rent plus transport costs per unit area on the  $z$ -axis, where the  $x$ - $y$  plane is the homogeneous plain on which the city is located. The graphed figure is a cylinder with radius  $t^*$  and height  $h''$ . The volume of the cylinder is aggregate transport costs plus aggregate land rents. Aggregate land rents are given by the volume of a cone with the same base and height. Since the volume of a cone is one-third that of a cylinder with the same base and height, aggregate land rents are one-half aggregate transport costs. Unfortunately, this neat geometric interpretation does not extend to situations in which lot size varies with location.

suming public goods, private goods, and land, there may be a finite, positive optimal population size.

A circular city of radius  $t^*$  has a population of

$$(1.6) \quad N(t^*) = \pi t^{*2}.$$

If transport costs per unit distance are  $e$ , then using (1.3), we obtain

$$(1.7) \quad ATC = e \int_0^{t^*} t(2\pi t)dt = \frac{2e}{3} \pi t^{*3}.$$

Hence, from (1.6),

$$(1.8) \quad ATC = kN^{3/2},$$

where

$$k = \frac{2}{3} e \pi^{-1/2}.$$

Resources available  $Z$  are assumed to be proportional to population; i.e.,  $Z = IN$ . If the resource cost of the public good provided is  $P$ , then per capita consumption of private goods  $C$  is

$$(1.9) \quad C = I - ATC/N - P/N.$$

Substitution of (1.8) into (1.9) gives

$$(1.10) \quad C = I - kN^{1/2} - P/N.$$

The maximization of  $C$  in (1.10) with respect to  $N$ ,  $P$  fixed, is equivalent to the maximization of per capita utility. Thus, independent of the functional form of the utility function and of the level of  $P$ ,  $U$  is maximized with  $P = \frac{1}{2} kN^{3/2}$ , which, using (1.8) and (1.5), gives

$$(1.11) \quad P = \frac{1}{2} ATC = DLR.$$

For any level of the public good, when the city is of optimum population size, public goods expenditures equal one-half aggregate transport costs, which in turn equal differential land rents. The Henry George Theorem holds in this economy, in which the source of economies of scale is a pure local public good, and of diseconomies of scale, transport costs. On a plain dense with hexagonal cities,  $P = \frac{1}{2} ATC$  still obtains, since population goes up as the square of the radius of

the hexagon, while aggregate transport costs go up as the cube.<sup>6</sup>

A limitation of this example is that it is unclear from the analysis which is the more fundamental relationship, that between expenditure on public goods and transport costs, or that between expenditure on public goods and differential land rents.<sup>7</sup> The next subsection resolves this.

### 1.2. A Geometric Analysis

We now consider the dual to the problem considered in subsection 1.1, in which the objective is to minimize the per capita resource cost of providing all city residents with a given level of utility. As before,  $P$  is the cost of public goods, and lots are of unit size.  $C$  units of the private good must be given to each resident to achieve the prespecified utility level. The total resource costs of providing  $N$  residents with this utility are  $AE + P + ATC$ , where  $AE$  is aggregate expenditure on the private good (which equals  $NC$ ). Average resource costs  $\overline{RC}$  equal<sup>8</sup>

$$(1.12) \quad \overline{RC} = C + P/N + ATC/N.$$

Marginal resource costs  $MRC$ , the costs of adding another resident to the city, are the cost of the private good plus the cost of transporting the resident to the boundary lot. Thus,

$$(1.13) \quad MRC = C + f(t^*).$$

With equal-size lots and no land scarcity, (1.1) implies that each resident's expenditure on land rent plus transport costs is the same and equals  $f(t^*)$ . Consequently,

6. The area of a hexagon of (outer) radius  $t^*$  may be calculated as twelve times the area of a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle, where the  $30^\circ$  vertex is the city center, and  $t^*$  is the length of the longest side. Taking the  $30^\circ$  vertex as the origin, and the side of length  $\sqrt{3}/2 t^*$  as base, and using polar coordinates, we obtain that the area of each triangle is

$$A = \int_0^{\pi/6} \int_0^{(t^*\sqrt{3/2})\sec \theta} t \, dt \, d\theta = \frac{\sqrt{3}}{8} (t^*)^2,$$

so that the area of the hexagon is  $(3\sqrt{3}/2) (t^*)^2$ . Since population is proportional to area, population rises as the square of the radius of the hexagon.

Since  $t$  is distance from the city center, then aggregate transport costs are given by

$$ATC = 12 \int_0^{\pi/6} \int_0^{(t^*\sqrt{3/2})\sec \theta} (et) \, t \, dt \, d\theta = \frac{3^{3/2}}{2} (t^*)^3 e \int_0^{\pi/6} \sec^3 \theta \, d\theta,$$

which rises as the cube of the radius of the hexagon.

7. In the way we have constructed the example, the addition of an individual to the community does not reduce existing residents' consumption of land. Thus, in this case the tradeoff determining optimal city size involves public goods and transport costs.

8. We have used  $A$  to denote aggregate ( $ATC$ ,  $ALR$ , e.g.) and use a bar over a variable to denote the average (the aggregate divided by population).

$$(1.14) \quad f(t^*) = ALR/N + ATC/N.$$

Substitution of (1.14) into (1.13) gives

$$(1.15) \quad MRC = C + ALR/N + ATC/N.$$

Finally, average resource costs are minimized when average resource costs equal marginal resource costs. Comparison of (1.15) and (1.12) gives the Henry George Theorem.

We now present this formulation geometrically. Figure I portrays the situation where city population size is optimal; Figure II where it is suboptimal; and Figure III where it is superoptimal. From (1.13) the area under  $MRC$  equals  $AE + ATC$ ; from (1.15),  $MRC(N) \cdot N$  equals  $AE + ALR + ATC$ ; from (1.12),  $\overline{RC}(N) \cdot N$  equals  $AE + P + ATC$ ; and finally, from (1.13),  $MRC(0) \cdot N$  equals  $AE$ . Thus, in Figure II the area 1234 equals expenditure on the public good. And in Figure III the area 134 equals aggregate land rents, the area 146 equals aggregate transport costs, and area 1256 minus area 146 equals expenditure on the public good.<sup>9</sup>

It is easy to see from the figures that in a city with suboptimal population, expenditure on the public good exceeds aggregate land rents, while in a city of superoptimal size the opposite is true; that is if  $N^*$  is the optimal population,

$$(1.16) \quad P \begin{matrix} \geq \\ \leq \end{matrix} ALR \quad \text{as} \quad N \begin{matrix} \leq \\ \geq \end{matrix} N^*.$$

We show later that this result generalizes. Since we made no assumption in this subsection concerning the functional form of  $f(t)$ , it is apparent that *the basic relationship characterizing optimal population size is that between aggregate land rents and expenditure on public goods.*

## II. THE GENERALITY OF THE HENRY GEORGE THEOREM

In this section we investigate the generality of the Henry George Theorem. We first provide a straightforward analysis of the case where all individuals are identical. Unfortunately, the intuition behind the HGT does not emerge clearly from this approach. Accordingly, in subsection 2.3 we consider a more abstract formulation which shows that the HGT holds with remarkable generality. Subsections 2.4 and 2.5 develop generalizations of the Theorem. In subsection 2.6 we

9. If the city is of constant width and if transport costs per unit distance are constant, then  $MRC$  is linear in  $N$ . From Figures I, II, and III this can be seen to imply that  $ALR = ATC$ , which is the analog to (1.5) for a long, narrow city.

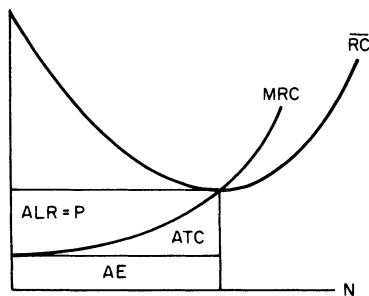


FIGURE I  
Optimal City Size

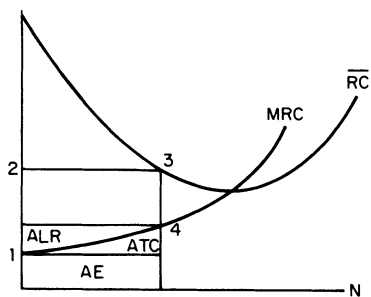


FIGURE II  
Suboptimal City Size

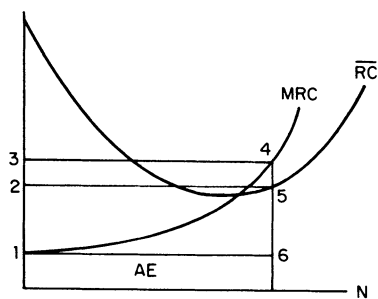


FIGURE III  
Superoptimal City Size



discuss a set of circumstances under which the Theorem holds in a competitive economy. Subsection 2.7 summarizes by indicating circumstances in which the HGT does *not* hold.

### 2.1. No Land Scarcity, Identical Individuals

We wish to maximize social welfare per capita,<sup>10</sup> subject to the relevant resource constraints. All individuals are identical and have a utility function,

$$U(C, T, P),$$

where  $C$  is consumption of the private good,  $T$  is consumption of land, and  $P$  is the supply of the pure public good. Locations are distinguished only by their distance from the city center. We consider the first-best allocation in which each individual is assigned a location and a certain amount of land and the private good, and in which the optimal amount of the public good is supplied. At any given location individuals are treated the same. For simplicity, we denote the level of utility attained by an individual at a distance  $t$  from the center by  $U(t)$ , i.e.,  $U(t) \equiv U(C(t), T(t), P)$ . Then we wish to maximize

$$(2.1) \quad \int_0^{t^*} \frac{W(U(t))}{NT(t)} \phi(t) dt,$$

where  $N$  is the (variable) number of people in the community,  $W$  is the social welfare function, and  $\phi(t) dt$  gives the area of residential land between  $t$  and  $t + dt$  from the city center (we call  $\phi(t)$  the shape of the city; e.g.,  $\phi(t) = 2\pi t$  for a circular city, and  $\phi(t) = w$  for a linear city of width  $w$ ). In the case where there is no land scarcity so that the opportunity rent on land is zero, (2.1) is maximized subject to two constraints:

(a) All individuals in the community must be located somewhere, and all land in the community must be used,

10. We call an economy that is solved for by maximizing social welfare per capita, without reference to constraints imposed by competition, a planned economy. Because of the nonconvexities introduced by space, an economy may have qualitatively different characteristics depending on whether it is planned or organized competitively. Most notably, identical individuals will usually receive different utilities in a planned spatial economy. This result was discovered by Mirrlees [1972] and has been discussed in Arnott and Riley [1977]. Other cases where, due to nonconvexities, social welfare maximization may be associated with the unequal treatment of equals are provided in Stiglitz [1975, 1976, 1977].

We maximize social welfare per capita rather than utility per capita, since we might want our maximand to express the social welfare maximizer's degree of inequality aversion. If the social welfare maximizer has the same cardinalization of utilities as city residents, then the Benthamite social welfare function is appropriate.

$$(2.2) \quad \int_0^{t^*} \frac{\phi(t)}{T(t)} dt = N.$$

(b) Expenditure must equal income,

$$(2.3) \quad \int_0^{t^*} \frac{f(t) + C(t)}{T(t)} \phi(t) dt + P = NI,$$

where  $I$  is per capita resources.<sup>11</sup> The assumption of linear production possibilities is inconsequential.

This can be formulated as a standard Pontryagin problem, the Lagrangean of which is

$$(2.4) \quad \mathcal{L} = \int_0^{t^*} \frac{W(U(t))}{NT(t)} \phi(t) dt + \lambda \left( \int_0^{t^*} \frac{\phi(t)}{T(t)} dt - N \right) - \Omega \left( \int_0^{t^*} \frac{f(t) + C(t)}{T(t)} \phi(t) dt + P - NI \right).$$

The associated Hamiltonian (where the dependence of the variables on  $t$  is suppressed to simplify notation) is

$$(2.5) \quad H = \left[ \frac{W(U(C, T, P))}{N} + \lambda - \Omega(f + C) \right] \frac{\phi}{T}.$$

We obtain as first-order conditions:

(i) The marginal social utility of private goods must be the same for all individuals,

$$(2.6) \quad \frac{\partial H}{\partial C} = \left( \frac{W'U_C}{N} - \Omega \right) \frac{\phi}{T} = 0.$$

(ii) A spatial optimality condition which specifies that the marginal social utility of allocating more land to individuals at a given distance from the center of the city must equal its marginal social cost,<sup>12</sup>

$$(2.7) \quad \frac{\partial H}{\partial T} = - \frac{\phi}{T^2} \left[ \frac{W}{N} + \lambda - \Omega(f + C) \right] + \frac{W'U_T}{NT} \phi = 0.$$

11. There is an implicit assumption in (2.3) that land is effectively owned internal to the community. We may interpret (2.3) as implying constant returns to scale in the production of each of  $P$ ,  $f$ , and  $C$  separately. Alternatively, city residents could be producing an export good with constant returns to the single factor, labor, and be purchasing  $P$ ,  $f$ , and  $C$  at world prices.

12. From this condition we can derive a spatial efficiency condition that states that the difference in the shadow rent on land between any two locations should reflect only the difference in the transport costs to those two locations, i.e.,

$$-T \frac{d(U_T/U_C)}{dt} = f'.$$

(iii) The condition for the optimal supply of public goods, that the sum of the marginal rates of substitution between the private and public goods equal the marginal rate of transformation (unity),

$$(2.8) \quad \frac{\partial \mathcal{L}}{\partial P} = \int_0^{t^*} \frac{W' U_P}{NT} \phi dt - \Omega = 0,$$

which implies, using (2.6), that

$$(2.9) \quad \int_0^{t^*} \frac{U_P}{U_C} \frac{\phi}{T} dt = 1.$$

(iv) Optimal city size condition: the marginal social benefit of increasing population (from the increased resources available) be equal to the marginal social costs (the costs of the private good and transportation given the marginal individual, plus the crowding costs imposed by the marginal individual),

$$(2.10) \quad \frac{\partial \mathcal{L}}{\partial N} = - \int_0^{t^*} \frac{W(U)}{N^2 T} \phi dt - \lambda + \Omega I = 0.$$

If we integrate (2.7), subtract (2.10), and substitute (2.6), we obtain

$$(2.11) \quad P = \int_0^{t^*} \frac{U_T}{U_C} \phi dt.$$

In the city of optimal size, expenditures on public goods just equal (imputed) land rents. Note that this result was obtained without (2.8). Hence, (2.11) holds whether or not the level of the pure public good is optimal. Another interesting feature of the solution is that the ex ante identical individuals may receive different utilities at the social welfare optimum.

## 2.2 Land Scarcity

If the average population density in the city must equal  $1/\eta$ , we have an additional constraint,

$$(2.12) \quad \int_0^{t^*} \phi dt = \eta N,$$

associated with which is a Lagrange multiplier  $\mu$ . The analysis is affected by this density constraint only in that the condition for the optimal boundary,

$$(2.13) \quad \frac{\partial \mathcal{L}}{\partial t^*} = \frac{\phi(t^*)}{T} \left[ \frac{W(U)}{N} + \lambda - \Omega(f + C) \right]_{t^*} - \mu \phi(t^*) = 0,$$

now turns out to be important. Substituting (2.7), evaluated at the boundary of the city, into (2.13), and using (2.6), we obtain

$$(2.14) \quad \mu = (W'U_T)_{t^*}/N = (\Omega U_T/U_C)_{t^*}.$$

$\Omega$  is the marginal social utility of the private good. Thus,  $\mu$  is the shadow rent on land at the boundary of the city measured in units of social utility. The resulting equation corresponding to (2.11) is

$$(2.15) \quad P = \int_0^{t^*} \frac{U_T}{U_C} \phi dt - \left( N\eta \frac{U_T}{U_C} \right)_{t^*}.$$

From (2.12)  $N\eta$  is the area of the city. The second term in (2.15) is the shadow rent on land at the boundary of the city times the area of the city, so that (2.15) states that expenditure on public goods must equal *differential* land rents (*DLR*). This result carries over to the cases where the boundary of the city is fixed at some  $t$  (a green belt, for instance) and where the shadow rent on land in nonresidential use is greater than zero (land in agricultural use, for instance).

### 2.3. *The Generality of the Henry George Theorem*

In Section I we showed that the Henry George Theorem applies in a competitive economy where residents have identical incomes, and where lots are of fixed and equal size. Earlier in this section we demonstrated that it also applies to an optimal city with identical individuals, independent of the social welfare (provided that it is additively separable and individualistic) or utility function. In this subsection we investigate why the HGT holds in these two cases and the extent to which it generalizes.

We begin by solving an abstract planning problem that has the Henry George result as a feature of its solution. We then investigate the class of economies whose allocation can be attained as the solution to special cases of this planning problem.

The abstract planning problem is as follows. The economy's residents may differ. The characteristic in which they differ is parameterized by  $\theta$ . The city planner is instructed to choose city population  $N$  so as to minimize resource costs per capita, subject to the following constraints:

(i) The distribution of residents over space  $\theta(t)$  is exogenously specified.

(ii) The utility gradient  $U(\theta(t))$  is exogenously specified.

(iii) The relative population density gradient  $D(t)$  is exogenously specified.  $D(t)$  is normalized so that  $\int_0^{t^*} D(t)\phi(t) dt = 1$ , where  $\phi(t)$  is again the shape of the city.

(iv) And the level of public goods  $P$  is exogenously specified.

Note that (i), (iii), and  $\phi(t)$  together imply a frequency distribution for  $\theta, g(\theta)$ . Residents derive utility from other goods  $C$ , lot size  $T$ , and the public good. Production possibilities are linear in  $C, f$ , and  $P$ , there are constant returns in production, and  $C, f$ , and  $P$  are measured so that the production of one unit of each uses up one unit of resources. Thus, resource costs per capita are

$$(2.16) \quad \frac{1}{N} \left( \int_0^{t^*} (C + f)ND\phi dt + P \right).$$

And the planner's problem is

$$(2.17) \quad \min_N \frac{1}{N} \left( \int_0^{t^*} (C + f)ND\phi dt + P \right)$$

subject to

$$U(\theta(t)) = U(t).$$

The urban economies in which the HGT holds have pure public goods, no congestion in transportation ( $\partial f(t)/\partial N = 0$  for all  $t$ ), and constant returns to scale in production. In such economies the first-order condition of (2.17) is

$$(2.18) \quad \int_0^{t^*} \frac{dC}{dN} D\phi dt - \frac{P}{N^2} = 0,$$

where  $dC(t)/dN$  denotes the change in  $C$  at  $t$  from a unit increase in  $N$ , holding the utility at  $t$  fixed. The requirement that utility at each location be unaltered by a change in population implies that

$$(2.19) \quad U_C \frac{dC}{dN} + U_T \frac{dT}{dN} = 0 \quad \text{for all } t.$$

Since  $T = 1/ND$ , then

$$(2.20) \quad \frac{dT}{dN} = \frac{d(1/ND)}{dN} = -\frac{1}{N^2D}.$$

Substituting (2.19) and (2.20) into (2.18) gives the Henry George result. The resource cost of adding "another resident" <sup>13</sup> is

$$\frac{1}{N} \int_0^{t^*} (C + f)ND\phi dt + \frac{1}{N} \int_0^{t^*} \frac{U_T}{U_C} \phi dt,$$

while the average resource cost is

13. "Another resident" means a proportional part of each individual in the economy,  $D(t)$ .

$$\frac{1}{N} \int_0^{t^*} (C + f)ND\phi dt + \frac{P}{N}.$$

Hence, as in subsection 1.2 the HGT is the result of average resource costs equaling marginal resource costs at the population optimum.

Since we made no assumptions about the efficiency of  $\theta(t)$ ,  $U(t)$ ,  $D(t)$ , and  $P$ , it is clear from this formulation that the HGT characterizes the efficient density of economic activity in a wide class of spatial economies, not just those that are completely planned.

Any urban planning problem in which the opportunity rent on land is zero and which involves maximizing social welfare per capita subject to an exogenous frequency distribution of  $\theta$  (without consideration of overall economy population constraints) contains some special case of the abstract planning problem. The primal problem is to maximize social welfare per capita subject to  $P$ ,  $g(\theta)$ , a land availability constraint, and a per capita resource cost constraint. The dual is to minimize resource costs per capita, subject to  $P$ ,  $g(\theta)$ , a land availability constraint, and a per capita social welfare constraint. If  $*$  denotes an optimal value, the solution to this dual problem (and also, of course, the primal) is completely characterized by  $\theta^*(t)$ ,  $U^*(t)$ ,  $D^*(t)$ ,  $N^*$ , and  $P$ . Evidently, minimization of resource costs per capita with respect to  $N$ , with  $\theta^*(t)$ ,  $U^*(t)$ ,  $D^*(t)$ , and  $P$  as constraints, yields the same solution. But this is precisely the form of the abstract planning problem considered earlier in the subsection, whose solution had the Henry George property. Hence, *any optimal*<sup>14</sup> (conditional on  $P$ ) *city with an exogenous frequency distribution of population has the Henry George property.*

If there is an opportunity rent on land in urban use  $\bar{R}$ , then this too would be quoted to the planner. Resource costs per capita would then be

$$(2.21) \quad \frac{1}{N} \left( \int_0^{t^*} (C * f)ND\phi dt + \bar{R} \int_0^{t^*} \phi dt + P \right).$$

Proceeding as above, one obtains that in this case  $P = DLR$  with the optimal population, instead of  $P = ALR$ . When cities are closely packed, the opportunity rent on land varies along the boundary of the city, in which case differential land rents are not well defined, and the

14. Recall that when we refer to an optimal city, we mean one that maximizes a social welfare function, subject to only technological constraints. Thus, a competitive city (which imposes additional behavioral constraints) may not be an optimal city in this sense.

HGT does not hold.<sup>15</sup> We have thus established that differential land rents, when well defined, equal expenditure on the public good in any optimal city with an exogenous frequency distribution of residents.

The above line of argument can be extended to any optimal regional economy in which there are pure public goods, constant returns in production, and no externalities. We could introduce housing, multiple transport modes, employment subcenters, a system of cities, and so on. Having specified the relative density of all economic activity, maximizing with respect to  $N$  would yield the Henry George result. If production costs differed among cities in a region, then the HGT would hold for the region but not separately for each city.

We must be careful in moving from the city or regional economy to the overall economy, since the overall economy has aggregate population constraints that were not considered in the optimal city or regional economy problems. The difficulty can be illustrated by considering an economy of identical individuals. Suppose that maximization of per capita utility in a single city, subject to  $P$  and the land availability and per capita resource cost constraints, yields an optimal city size of  $N^*$ . What happens if the overall population in the economy is only  $3N^*/2$ ? The HGT will not, in general, hold in this economy, whether one or two cities are optimal. If instead the economy population is  $1,000N^* + N^*/2$ , the HGT will nearly hold for the overall economy. We define a large economy to be one in which the number of residents left over after putting residents in cities of optimal size is infinitesimal compared to the overall population of the economy.

15. If cities are hexagonal, their shape is a function of the outer radius of the hexagon  $t^*$ , which will change as the city's population changes. We therefore denote the shape of the city by  $\phi(t, t^*)$ . Proceeding as in subsection 2.2, using  $\phi(t, t^*)$  instead of  $\phi(t)$ , we obtain

$$\mu = \frac{1}{N} \left( \int_0^{t^*} W' U_T \frac{\partial \phi}{\partial t^*} dt \right) / \left( \int_0^{t^*} \frac{\partial \phi}{\partial t^*} dt \right),$$

in the place of (2.14). If we define the opportunity rent of land at a location  $(t, \theta)$ , measured in polar coordinates, to be the land rent at the boundary of the city in the direction  $\theta$ , then expenditure on the public good can be shown to equal differential land rents (defined as aggregate land rents minus the integral of the opportunity rent on land, so defined, over the area of the city). This interpretation of differential land rents is, however, forced and unintuitive. It is more reasonable to define the opportunity rent on land as the rent at the vertices of the hexagon in which case the Henry George Theorem does *not* hold for hexagonal cities, or to say that differential land rents are not well defined. Starrett [1974] overlooked this problem. We have not been able to pinpoint precisely which of his assumptions was responsible for this, but he talks about *the* shadow rent on land at the boundary of the country [p. 432]. With a system of hexagonal cities, however, the shadow rent on land must vary along this boundary. Thus, there must be an implicit assumption in his analysis restricting the shape of cities.

More generally, whenever the shadow rent on land is not the same everywhere along the boundary of the city, the HGT does not hold, since differential land rents are not well defined.

It can be shown that the Henry George Theorem holds exactly in large planned economies in which *DLR* are well defined, but does not, in general, hold in small economies.

#### 2.4. A Characterization of Cities of Non-Optimal Size

Under the conditions for the HGT to hold, resource costs per capita,  $\overline{RC}$  are from (2.16)

$$(2.16') \quad \overline{RC} = \int_0^{t^*} (C + f) D\phi dt + \frac{P}{N}.$$

When  $d\overline{RC}/dN > 0$ , population is superoptimal. Now,

$$\frac{d\overline{RC}}{dN} = \int_0^{t^*} \frac{dC}{dN} D\phi dt - \frac{P}{N^2},$$

and from (2.18) and (2.19),

$$(2.22) \quad \frac{dC}{dN} = \frac{U_T}{U_C} \frac{1}{N^2 D}.$$

Thus,

$$(2.23) \quad \frac{d\overline{RC}}{dN} > 0 \Leftrightarrow N > N^* \Leftrightarrow ALR > P.$$

Similarly, it can be shown that

$$(2.24) \quad \frac{d\overline{RC}}{dN} < 0 \Leftrightarrow N < N^* \Leftrightarrow ALR < P.$$

Consequently, aggregate (differential) land rents exceed expenditure on the public good in a city of greater than optimal size, and are smaller than expenditure on the public good in a city of less than optimal size.

#### 2.5. A Generalization of the Henry George Theorem

When there are other sources of economies and diseconomies of scale, optimal city size can still be characterized by an equality relationship between urban economic aggregates. Consider a city where there are pure public goods and constant returns to scale in production, but where there is an additional source of diseconomies of scale, congestion in transportation, which is modeled as  $f = f(N, t)$  with  $f_N > 0$ . Returning to the general problem treated at the beginning of this subsection, we have that the analog to (2.18) is

$$(2.18') \quad \int_0^{t^*} \left( \frac{dC}{dN} + f_N \right) D\phi dt - \frac{P}{N^2} = 0.$$



Equations (2.19) and (2.20) still hold. Substituting them into (2.18') gives

$$(2.25) \quad \int_0^{t^*} \frac{U_T}{U_C} \phi dt + \int_0^{t^*} (f_{NN})ND\phi dt = P.$$

The second term on the left-hand side may be called the aggregate congestion externality (*ACE*). It is the amount that would be collected in toll revenue in a competitive city if an optimal congestion toll were imposed. Hence, in a city with pure public goods, constant returns to scale in production, and congestion in transportation, the relationship characterizing optimal city size is that  $DLR + ACE = P$  when  $N = N^*$ , and its corollaries are  $DLR + ACE > P \Leftrightarrow N > N^*$ , and  $DLR + ACE < P \Leftrightarrow N < N^*$ .<sup>16</sup> Arnott [1979] has derived the rules characterizing optimal size for residential cities with other sources of economies and diseconomies of scale.

### 2.6. Large, Open, Competitive Economies

In subsection 2.3 we argued that the HGT held at the social welfare optimum in all large planned economies in which differential land rents were well defined, independent of the social welfare function. Since the set of planning optima is coincident with the set of Pareto optimal allocations, to ascertain circumstances under which the HGT holds in large competitive economies, we must identify circumstances under which competition results in Pareto optimality. As we suggested in the introduction, the Pareto optimality of competition depends critically on the definition of competitive behavior in a spatial, urban economy. Stiglitz [1978] has identified one definition that results in the Pareto optimality of competition. The economy he considers has the following characteristics:<sup>17</sup>

- (i) Migration is costless.
- (ii) Each individual is free to form his own city on a separate island. Neither the number of islands nor the amount of land on each island is scarce. He may restrict entry but cannot coerce people to join.
- (iii) The economy is large in the sense that an individual forming a city takes as exogenous the utility of each group in the economy.
- (iv) Economies and diseconomies of scale are such that optimal city sizes are finite and positive.

16. Here and elsewhere we have ignored second-order conditions. The characterization theorems imply, at least for the first-best cases we have considered, that there is a unique interior optimum that is a maximum. The corner solutions of  $N^* = 0$  and  $N^* = \infty$  are of little interest.

17. A formal description of the model is presented in Stiglitz [1978].

(v) And land is homogeneous.<sup>18</sup>

Utility levels are determined in the general equilibrium of the economy by the equality of the supply of and demand for each group. In Section IV we shall present alternative interpretations of competitive behavior in which competitive equilibria are not Pareto optimal.

To summarize: *in all large, Pareto optimal spatial economies in which differential land rents are well defined, the Henry George Theorem holds. Whether the Henry George Theorem holds in a competitive economy depends on, among other things, one's view of what constitutes competitive behavior in a spatial economy.*<sup>19</sup>

### 2.7. Limitations to the Henry George Theorem

So far we have stressed the generality of the HGT. It will be useful to review our results from a different perspective by listing circumstances in which the HGT does *not* hold.

If the government has full controllability (that is, in planned economies), then in an optimal economy the HGT does not hold if

(i) Differential land rents are not well defined, which occurs whenever the opportunity rent on land is not everywhere the same along the boundary of the city; or if

(ii) The overall urban economy is small. In this case, if residents are put into cities of optimal (defined without regard to the overall economy population constraints) size, there may be a significant number of residents, relative to the economy's population, left over.

In competitive economies the HGT does not hold in the above two circumstances and also when competitive behavior leads to a distribution of population over cities that is not Pareto optimal.

18. In this paper we treat a von Thunen economy in which locations differ only in terms of accessibility and land is homogeneous in quality. How are the results affected if, additionally, land varies in terms of its productivity or amenity value? This question has been treated in Flatters, Henderson, and Mieszkowski [1974], and Stiglitz [1977]. They consider a large economy that consists of a group of islands that may differ in fertility. They obtain the result that the Henry George Theorem should obtain for each spatial unit of replication, a group of islands, but not necessarily for each island. For such an economy to be Pareto optimal, it is necessary to effect lump-sum redistribution across islands within each spatial unit of replication. The assumption in Stiglitz [1978] that land is homogeneous was made to circumvent this difficulty. He could alternatively have allowed individuals to form their own *groups of cities* (the spatial unit of replication).

19. In our analysis we have assumed that accessibility can be parameterized by a single variable. We [1978] investigated the circumstances in which it is legitimate to do this. When location must be parameterized by two variables, differential land rents may not be well defined, in which case the HGT does not hold.

If the government can redistribute in lump-sum fashion between individuals and if competition leads to Pareto optimality, then any planning optimum can be attained. However, lump-sum redistribution is typically infeasible, in which case the government must resort to distortionary policies to alter the distribution of utilities. The HGT does not generally hold in such second-best economies. To illustrate this, we consider an economy in which the government has only two policy instruments, a pure public good financed by means of a head tax and the regulation of city size, and in which competitive equilibrium (with government intervention) involves all cities being the same. The latter assumption implies that a change in city size results in an equiproportional change in the population of each group in the city. In note 20 we present an example which indicates that the size of city which maximizes a specific group's utility may vary by group.<sup>20</sup> The reason that different groups may have different optimal populations is that, while all residents receive the same benefit from the addition of a resident, the reduction in the head tax, they do not face the same

20. Suppose that there are two groups in the population. The number of residents in each group is  $N/2$ . The tastes of the residents in the two groups are

$$(i) \quad U_A = \begin{cases} 0 & \text{for } T_A < 1 \\ C_A & \text{for } T_A \geq 1 \end{cases} \quad \text{and} \quad U_B = \begin{cases} 0 & \text{for } T_B < 2 \\ C_B & \text{for } T_B \geq 2. \end{cases}$$

Each resident in group  $A$  has an income of  $Y_A$ , and each resident in group  $B$  an income of  $Y_B$ . We assume that  $Y_A$  and  $Y_B$  are such that  $C_A > 0$  and  $C_B > 0$ , in which case group  $A$  lot sizes are 1, and group  $B$  lot sizes are 2. Both groups' transport costs are 1 per unit distance. The city is long and narrow and one unit wide. The boundary of residential settlement is endogenous, and the opportunity rent on land is zero. Land is owned by the government and auctioned off competitively. The residual revenue to finance the public good is collected using a uniform head tax. Since the absolute value of the slope of the bid-rent curve equals transport costs per unit distance divided by lot size, group  $A$ 's bid-rent curve is steeper than group  $B$ 's. Thus, group  $B$  will live toward the boundary and group  $A$  toward the center. The rent gradient is given by

$$(ii) \quad R(t) = \begin{cases} t^* - t/2 & \text{for } t \geq t^*/3 \\ 2t^*/3 - t & \text{for } t \leq t^*/3 \end{cases} \quad \text{where } t^* = 1.5N.$$

Aggregate land rents, obtained by integrating (ii) over the area of the city, are  $0.625 N^2$ . Since all individuals in group  $A$  have the same utility, to ascertain the population that maximizes  $A$ 's utility, one need determine only the population that maximizes the utility of the individual at the city center. The budget constraint for this individual is

$$(iii) \quad C_A = Y_A - R(0) - H,$$

where  $H$  is the head tax ( $H = P/N - ALR/N$  by assumption). Maximizing  $C_A$  (and therefore  $U_A$ ) in (iii) with respect to  $N$  (where from (ii),  $R(0) = N$ ), gives  $P = 0.6ALR$ . Performing the same exercise for a representative group  $B$  individual, say the individual at the boundary for whom

$$(iv) \quad C_B = Y_B - t^* - H,$$

gives that utility-maximizing population for group  $B$  occurs where  $P = 1.4ALR$ . Since  $ALR$  increases with  $N$ , the group farther out has a lower optimal population.

cost, the increase in lot rent.<sup>21</sup> Let  $N_i^*$  be the city population size that, conditional on  $P$ , maximizes group  $i$ 's utility. We know, given  $P$ , there is a unique population size for which the Henry George Theorem holds  $N^*$ . The optimal second-best population size is

$$\tilde{N}^* = \sum_{i=1}^n \alpha_i N_i^*,$$

where

$$\sum_{i=1}^n \alpha_i = 1$$

and  $\{\alpha_i\}$  reflects the distributional weighting accorded each of the  $n$  groups by the government. Evidently,  $N^*$  will not in general equal  $\tilde{N}^*$ .

### III. ON USING LAND RENTS AS A MEASURE OF THE BENEFITS FROM PUBLIC GOODS

In recent years there have been numerous capitalization studies, one aim of which has been to infer differences in the benefits from public goods across communities or over time from the corresponding differences in land values.

The argument on which this inference is based goes as follows. In an economy with identical residents, utility is a function of land (or housing) rent gross of tax  $R(1 + \tau)$  (where  $R$  is land rent, and  $\tau$  the ad valorem tax rate on land rent), the level of public services  $P$ , and income net of transport costs  $Y$ ; i.e.,  $V = V((1 + \tau)R, Y, P)$ , where  $V$  is the indirect utility function. Consider two individuals who have the same income net of transport costs and live in different communities that have the same tax rate. Where the communities' levels of public services differ by an infinitesimal amount,  $dP$ , then  $dV = V_1(1 + \tau)dR + V_3dP$ , which, using the properties of the indirect utility function, becomes

$$dV = V_2(-T(1 + \tau)dR + (V_3/V_2)dP).$$

21. If the increase in lot rent from the addition of the representative resident is a monotonic function of income, then the regulation of city size may be an efficient means of improving equity. However, if the increase in lot rent is not a monotonic function of income, so that a larger city improves the welfare of the very rich and very poor, while hurting those with intermediate incomes, for instance, the regulation of city size will probably prove to be of only limited efficacy in improving equity. Our tentative conclusion on the basis of some preliminary analysis is that the increase in lot rent is unlikely to be a monotonic function of income.

$V_3/V_2$  is the marginal benefit of the public good to the individual in money terms. When migration is perfect, equalizing individuals' utilities in different communities  $dV = 0$ , so that the difference in the rent gross of tax on corresponding lots in the two communities equals the monetary valuation of the difference in the level of public services provided on those lots. And the difference in lot *values* gross of tax, which equals the discounted present value of the difference in lot *rents* gross of tax, equals the discounted present value of the monetary valuation of the difference in the levels of public services provided.

In this section we shall not discuss empirical applications of capitalization theory, but rather shall extend the theory to investigate whether there is any simple relationship in an open economy between the difference in the *aggregate* land rents between two communities and the corresponding difference in the level of services from the public goods provided. This is of interest for two reasons. First, the analysis will provide further insight into the relationship between aggregate land rents and expenditure on public goods, and second, it will cast some light on the following two aspects of capitalization theory:

- (i) The capitalization argument presented above was partial equilibrium. It does not, for instance, treat the local government budget balance constraint. Do such partial equilibrium capitalization arguments extend to general equilibrium?
- (ii) The argument applies to the marginal individual who is indifferent between living in the two communities being compared. In what ways does it generalize when there are inframarginal individuals who prefer one community to the other?

In subsection 3.1 we consider two cities with identical individuals that differ only in their locational amenities. Subsection 3.2 treats two cases, again with identical individuals, in which communities differ in their fiscal packages. And subsection 3.3 extends the analysis to an economy in which individuals differ.

### 3.1. Locational Amenities

In an economy with identical individuals, consider two communities that have the same fiscal package (head taxes and services) and that are identical in all exogenous respects (e.g., transport costs, shape) except in the level of a locational amenity, such as the quality of microclimate or of a beach. The level of the locational amenity in one of the communities is  $A$  and in the other  $A + dA$ . Both communities are small relative to the whole economy and mobility is perfect, so that the level of residents' utility may be treated as parametric  $\bar{U}$ , as may

the opportunity rent on non-urban land  $\bar{R}$ . Land ownership is external in both communities.

If  $I$  is gross income, an individual's indirect utility function is

$$(3.1) \quad V(R(t), I - f(t), A) = \bar{U},$$

where taxes and services are suppressed to simplify notation. That the services from  $A$  are independent of city population and of location implies that  $A$  has the character of a pure, local public good. Differentiation of (3.1) with respect to  $A$ ,  $\bar{U}$ , and  $t$  fixed gives  $V_1 dR/dA + V_3 = 0$ , or since  $T = -V_1/V_2$ ,

$$(3.2) \quad \frac{dR}{dA} = \frac{V_3}{V_2} \frac{1}{T}.$$

Land rents adjust to offset the difference in amenity levels. Now

$$ALR = \int_0^{t^*} R(t)\phi(t) dt \quad \text{and} \quad V(\bar{R}, I - f(t^*), A) = \bar{U}.$$

The second equation characterizes the location of the boundary of the city. Differentiation of the first equation with respect to  $A$  yields

$$(3.3) \quad \frac{dALR}{dA} = \int_0^{t^*} \frac{dR(t)}{dA} \phi(t) dt + R(t^*)\phi(t^*) \frac{dt^*}{dA}.$$

$\phi(t^*)(dt^*/dA)$  is the amount by which the settled area of the city increases with a unit increase in  $A$ , so that  $R(t^*)\phi(t^*)dt^*$  gives the opportunity rent on the extra land in the community with the higher level of the locational amenity. Hence,

$$\frac{dALR}{dA} - R(t^*)\phi(t^*) \frac{dt^*}{dA} = \frac{dDLR}{dA}$$

Thus,

$$(3.4) \quad \frac{dDLR}{dA} = \int_0^{t^*} \frac{dR(t)}{dA} \phi(t) dt.$$

Substitution of (3.2) into (3.4) gives

$$(3.5) \quad \frac{dDLR}{dA} = \int_0^{t^*} \frac{V_3}{V_2} \frac{\phi}{T} dt.$$

Hence, in an economy with identical individuals and perfect mobility, when the only exogenous difference between two cities is the level of an amenity resource, the difference in differential land rents between

the two cities equals the difference in the aggregate benefits from the amenity resource. The above result generalizes to the case where the boundary of both cities is the same and fixed. However, it does not hold when cities have different shapes or transport cost functions.

### 3.2. Different Fiscal Packages

We now examine the case where a pure local public good is obtained at constant cost, and financed by means of a rent tax (the results to be derived can be shown to hold with a head tax as well). To simplify the analysis, we assume that the opportunity rent on land at the boundary of the city is zero. In this case,

$$(3.6) \quad V(R(t) (1 + \tau), I - f(t), P) = \bar{U},$$

and budget balance requires that

$$(3.7) \quad \tau ALR = P.$$

As  $P$  is varied, population and the tax rate both adjust to satisfy (3.6) and (3.7). Proceeding as in the previous subsection,<sup>22</sup> we obtain

$$(3.8) \quad \frac{dALR}{dP} = \int_0^{t^*} \frac{V_3 \phi}{V_2 T} dt - 1.$$

Now,

$$\int_0^{t^*} \frac{V_3 \phi}{V_2 T} dt > (=, <) 1$$

as the level of the public good is less than (equal to, greater than)

22. Differentiation of (3.6) and (3.7) with respect to  $P$  gives

$$(i) \quad V_1 \left( \frac{dR}{dP} (1 + \tau) + R \frac{d\tau}{dP} \right) + V_3 = 0,$$

and

$$(ii) \quad \frac{d\tau}{dP} ALR + \tau \frac{dALR}{dP} = 1.$$

Then,

$$\begin{aligned} \frac{dALR}{dP} &= \int_0^{t^*} \frac{dR}{dP} \phi dt \\ &= \int_0^{t^*} - \frac{V_3 \phi}{V_1(1 + \tau)} dt - \int_0^{t^*} \frac{R\phi}{1 + \tau} \frac{d\tau}{dP} dt \end{aligned} \quad (\text{using (i)})$$

$$\begin{aligned} &= \int_0^{t^*} \frac{V_3 \phi}{V_2(1 + \tau)T} dt - \frac{ALR}{(1 + \tau)} \left( \frac{1}{ALR} - \frac{\tau}{ALR} \frac{dALR}{dP} \right) \quad (\text{using (ii)}) \\ &= \int_0^{t^*} \frac{V_3 \phi}{V_2 T} dt - 1. \end{aligned}$$

optimal. Hence, with perfect mobility and identical individuals, and when cities differ in no exogenous respect other than  $P$ ,  $dALR/dP = 0$  ( $<0$ ,  $>0$ ) as the level of provision of the public good is optimal (greater than optimal, less than optimal).

Next we compare two communities that spend the same amount on the public good, but one is more efficient in providing the public service because of more efficient administration, for instance. Let  $E$  denote expenditure on the public good,  $P$  the level of service provided, and  $e$  an index of efficiency in the provision of public services defined so that  $P = eE$ . Then,

$$(3.9a) \quad V(R(t) (1 + \tau), I - f(t), eE) = \bar{U}$$

and

$$(3.9b) \quad \tau ALR = E.$$

Straightforward manipulation gives

$$(3.10) \quad \frac{dALR}{de} = E \int_0^{t^*} \frac{V_3 \phi}{V_2 T} dt.$$

Thus, the difference in the aggregate land rents between two open communities with identical individuals that have the same public expenditure but differ in fiscal efficiency equals the difference in the aggregate valuation of public services.

The results of this and the previous subsections indicate that in an open economy with identical individuals, after correct adjustment for other differences between two communities, one can make valid inferences concerning differences in the valuation of their fiscal packages or amenity resources from the difference in their aggregate land rents.

### 3.3. *Heterogeneous Population*

How are the results of the previous two subsections modified when individuals differ? We shall not attempt to provide a full answer to this question, but shall instead consider only the case in which individuals differ solely in terms of their valuation of an amenity resource such as the quality of a beach. We also assume that all individuals' indirect utility functions are separable in  $A$  so that

$$(3.11) \quad V(\beta) = v(R(t), I - f(t)) + \beta g(A),$$

where  $\beta$  indexes increasing valuation of the amenity resource  $A$ , and that the population is continuously distributed over  $\beta$ . We consider two islands in equilibrium. They differ only in that one's amenity



resource level is  $A$ , the other's  $A + dA$ . (3.11) implies that if  $\beta_i > \beta_j$ , then individual  $i$  will reside either on the same island as individual  $j$  or on the island with the higher amenity level. The lower amenity island contains individuals with  $\beta \in [\beta_{\min}, \hat{\beta})$ , and the higher amenity island, individuals with  $\beta \in [\hat{\beta}, \beta_{\max}]$ . The individual with  $\beta = \hat{\beta}$  is marginal in the sense that he is indifferent between living on either island. Individuals with  $\beta \in [\beta_{\min}, \hat{\beta})$  and  $\beta \in (\hat{\beta}, \beta_{\max}^*]$  are inframarginal in that they obtain higher utility on their chosen island than they would on the other island. With the above form of indirect utility function, on each island all residents are indifferent as to where they locate on that island. Thus, if  $V^l(t; \beta)$  and  $V^h(t; \beta)$  denote the utility obtained by individual  $\beta$  at  $t$  on islands  $l$  and  $h$ , respectively,  $V^l(t; \beta) = V^h(t; \hat{\beta})$  for all  $t$ . Hence,

$$\frac{dR(t)}{dA} = -\frac{\hat{\beta}g'}{v_1(t)},$$

which, since  $v_1 = -v_2T$ , gives

$$\frac{dR(t)}{dA} = \frac{\hat{\beta}g'}{v_2(t)T(t)}.$$

Integrating this over all  $t$  gives

$$(3.12) \quad \frac{dALR}{dA} = \int_0^{t^*} \frac{\hat{\beta}g'}{v_2T} \phi dt.$$

(3.12) indicates that the difference in aggregate land rents between the two islands equals what would be the aggregate valuation of the difference in the amenity resource levels if all individuals had the same tastes as the marginal individual. The aggregate income-equivalent benefit of the higher amenity resource level, however, equals the valuation of the difference in the amenity resource levels by residents of the higher amenity community,

$$\int_0^{t^*} \frac{\beta(t)g'\phi}{v_2T} dt,$$

which since  $\beta(t) \geq \hat{\beta}$  for all  $t$  and  $\beta(t) > \hat{\beta}$  for some  $t$ , is greater than  $dALR/dA$ . That intercommunity differences in land rents do not capture inframarginal benefits has important implications for the interpretation of capitalization studies. When individuals are not identical, differences in land rents between communities systematically underestimate the value of their differences in amenities, and

systematically overestimate the cost of their differences in disamenities.<sup>23</sup> By a similar line of argument, it can be shown that when individuals differ, differences in land rents do not reflect the inframarginal benefits of alternative fiscal packages.

This section has shown that, in an open economy, abstracting from other differences between communities, there are systematic relationships across communities between differences in differential land rents, and differences in amenity resources and fiscal packages. These relationships are a consequence of utility-equalizing migration, and are unrelated to the Henry George Theorem, which is a characteristic of large economies with a Pareto optimal distribution of economic activity over space. The conceptual basis of capitalization studies is sound only when marginal individuals are very similar to inframarginal individuals in these communities.

#### IV. COMPETITIVE ATTAINABILITY OF A PARETO OPTIMAL DISTRIBUTION OF ECONOMIC ACTIVITY

The problems associated with the attainability in a free market economy of a Pareto optimal distribution of firms over space, when spatial clustering of firms occurs as a result of agglomerative economies of scale, are familiar. Some are discussed in Starrett [1974]. Here we do not have these problems, because spatial clustering occurs because of pure local goods, but we may have other problems.

Tiebout's classic paper [1956] suggested not only that the problem of preference revelation would be resolved by the local provision of public goods, but also that the resulting spatial distribution of population would be Pareto optimal. Stiglitz [1978] indicates one set of circumstances in which Tiebout's conjectures are correct. Recent papers by Buchanan and Goetz [1972], Flatters, Henderson and Mieszkowski [1974], and Stiglitz [1977], however, present alternative scenarios in which unrestricted migration can result in non-optimality.<sup>24</sup> Here we discuss two other possible sources of market failure not previously treated in the literature.

First, *if city residents do not face the social costs or benefits of an in-migrant, then a Pareto optimum is not competitively sustainable.* To demonstrate this proposition, we treat a simple economy

23. In the case of a public bad such as noise or pollution, intercommunity differences in land rents provide a consistent overestimate of costs. This results, since the cost of the noise to the marginal individual is larger than the cost to inframarginal individuals in the noisy community, who by self-selection are those who are not particularly bothered by the noise.

of identical individuals and cities. The economy is at a Pareto optimum with individuals receiving the same utility. Furthermore, land is so scarce that the cities are hexagonally closest packed. Consider adding an individual  $A$  to city  $i$  from a neighboring city  $j$ . The monetary benefits to residents of city  $i$  from adding  $A$  equal  $I$ , the value of his labor or of the endowment he brings with him. The costs, if  $A$  does not bring his land with him, are the cost of other goods given him  $C$ , plus the cost of transporting him  $f(t)$ , plus the rent on land given him (crowding costs)  $R(t)T$ , where applicable. Optimum population occurs where average resource costs  $I$  equal marginal resource costs  $C + f(t) + R(t)T$  (which is the same wherever the individual is located since incomes are equal). If  $A$  brings his land with him, however, without compensation to city  $j$ , the benefits minus costs to city  $i$  from  $A$  are  $I - C - f(t)$ , while if city  $i$  has to compensate city  $j$ , benefits minus costs from  $A$  are  $I - C - f(t) - R(t)T$ . Thus, if the individual at the border of the city moves without the land that was allocated to him by city  $j$ , or else if compensation for  $A$ 's land need be made by city  $i$  to city  $j$ , then the optimum is stable. However, if  $A$  may annex himself to city  $i$  without compensation to city  $j$ , the optimum is unstable. To put it another way, if local government property rights are unrestrained and if the individual can choose which community to belong to, the optimal allocation cannot be sustained in a competitive market.

Second, *if city residents misperceive the social costs or benefits of an in-migrant, then a Pareto optimum is not competitively sustainable.* In the economy discussed in the above paragraph, city residents, acting as price-takers, may consider the net benefits from a migrant positive if he makes a positive contribution to resources, i.e., if  $I > f(t) + C$ ; that is, they may ignore that his presence will drive up land rents and increase their transport costs. If this is the case, cities of optimal population size will try to bribe residents of other cities to join their city, and the optimum is again unstable.

24. One source of non-optimality is discussed in Buchanan and Goetz [1972], Flatters, Henderson, and Mieszkowski [1974], and Stiglitz [1977]. Migrants consider only average tax levels when making a migration decision and ignore the effect of their move on the tax burdens of existing residents. As mentioned in note 18, correction of this market failure requires lump-sum transfers between islands.

Another source of non-optimality is treated at length in Stiglitz [1977]. Suppose that there are two cities of total population  $2N^*$ , where  $N^*$  is optimal city size, and that initially there are  $2N^*$  people in one city, and none in the other. Let  $U(N)$  refer to the utility level of each individual in a city of population size  $N$ , where all individuals in the city have the same utility. Individuals are utility-takers and migrate if and only if the utility level in the other city is higher. If  $U(2N^*) > U(0)$ , then no individual acting alone has an incentive to migrate. Thus, stable Pareto inferior equilibria are possible.

The central point is that there is no compelling interpretation of competitive behavior in the spatial economy treated in the paper. With some reasonable sets of assumptions concerning residents' knowledge and perception of the operation of the economy, competition neither leads to, nor sustains, cities of optimal size. This suggests that the Henry George Theorem may provide a rather poor explanation of the relationship between urban economic aggregates in a competitive economy.

## V. CONCLUDING COMMENTS

This paper has outlined a general set of relationships between aggregate urban land rents and pure local public goods. The Henry George Theorem, that in cities of optimal size aggregate land rents equal expenditures on public goods, has been established under far more general conditions than in previous studies. It holds (i) for all large economies in which (ii) the spatial distribution of economic activity is Pareto optimal and (iii) in which differential land rents are well defined. All three conditions are required, however; if any one of them is violated, Henry George's single tax on differential land rents may provide too much or too little tax revenue. When, in addition to pure local public goods, there are other sources of economies and diseconomies of scale, e.g., congestion costs, there still exists a simple relationship between differential land rents and a particular set of urban economic aggregates, provided that the three conditions above are still satisfied. Moreover, corollaries of our general Henry George Theorem provide rules indicating whether city population size is greater than or less than optimal.

A quite separate set of relationships between land rents and local public goods is assumed in the capitalization literature, which attempts to infer consumer valuations of differences in city characteristics from differences in land values across cities. If individuals are identical, the theoretical basis of the capitalization literature is sound, and there is a simple relationship between the differences in aggregate land rents across communities and the differences in their characteristics. However, when individuals are not identical, differences in land rents omit inframarginal costs and benefits; the differences in aggregate land rents across communities systematically understate the value of differences in positive characteristics (amenities, local public goods), and overstate the value of differences in negative characteristics (disamenities, tax rates).

Finally, we noted the intimate relationship between the nature

of the land market and the competitive attainability of optimal city size. A system of densely packed cities of optimal size cannot be competitively sustained if individuals are allowed to choose the city to which they belong, and also have the right to determine what city to annex their land to. More generally, we noted a fundamental difficulty in convincingly characterizing competitive behavior in a spatial urban economy; for plausible "competitive" assumptions, even if cities are not densely packed, a system of cities of optimal size may not be competitively sustainable.

This paper has focused on three of the basic hypotheses of urban economics:—(1) the Henry George hypothesis relating aggregate land rents to expenditures on public goods in cities of optimal size; (2) the capitalization hypothesis, relating differences in land rents to differences in public amenities; (3) and the Tiebout hypothesis, that individuals will sort themselves out in such a way as to lead to a Pareto optimal allocation of resources and distribution of population. Though these hypotheses hold far more generally than the simple models in which they were originally established, they are of sufficiently limited generality to warrant caution in their use for purposes of public policy.

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## REFERENCES

- Alonso, W., *Location and Land Use* (Cambridge: Harvard University Press, 1970).
- Arnott, R. J., "Topics in Residential Location Theory," Ph.D. thesis, Yale University, 1975.
- , "Optimal City Size in a Spatial Economy," *Journal of Urban Economics* (1979).
- , and J. G. Riley, "Asymmetrical Production Possibilities, the Social Gains from Inequality, and the Optimum Town," *Scandinavian Journal of Economics*, LXXIX, No. 3 (1977), 301–11.
- , and J. E. Stiglitz, "Aggregate Land Rents and Aggregate Transport Costs," Oxford mimeo, 1978.
- Bollobas, B., and N. Stern, "The Optimum Structure of Market Areas," *Journal of Economic Theory*, IV, No. 2 (April 1972), 174–79.
- Buchanan, J., and C. Goetz, "Efficiency Limits of Fiscal Mobility: An Assessment of the Tiebout Model," *Journal of Public Economics*, I (1972), 25–43.
- Edel, M., and E. Sclar, "Taxes, Spending and Property Values: Supply Adjustment in a Tiebout-Oates Model," *Journal of Political Economy*, LXXXII, No. 5 (1974), 941–54.
- Fishlow, A., *Railroads and the Transformation of the Ante-Bellum Economy* (Cambridge: Harvard University Press, 1965).
- Flatters, F., V. Henderson, and P. Mieszkowski, "Public Goods, Efficiency, and Regional Fiscal Equalization," *Journal of Public Economics*, III, No. 2 (May 1974), 99–112.
- Fogel, R. W., *Railroads and American Economic Growth: Essays in Economic History* (Baltimore: Johns Hopkins Press, 1964).

- Meadows, G. R., "Taxes, Spending, and Property Values: A Comment and Further Results," *Journal of Political Economy*, LXXXIV, No. 4 (1976), 869-81.
- Mirrlees, J. A., "The Optimum Town," *Swedish Journal of Economics*, IV (1972), 114-35.
- Mohring, H., "Land Values and the Measurement of Highway Benefits," *Journal of Political Economy*, LXIX, No. 3 (June 1961), 236-49.
- Oates, W. E., "The Effects of Property Taxes and Local Public Spending on Property Values: An Empirical Study of Tax Capitalization and the Tiebout Hypothesis," *Journal of Political Economy*, LXXVII, No. 6 (1969), 957-70.
- Starrett, D. A., "Principles of Optimal Location in a Large, Homogeneous Area," *Journal of Economic Theory*, IX (1974), 418-48.
- Stiglitz, J. E., "Incentives, Risk and Information: Notes Towards a Theory of Hierarchy," *Bell Journal of Economics and Management Science* (Autumn 1975), 552-79.
- , "The Efficiency Wage Hypothesis, Surplus Labor and the Distribution of Income in L.D.C.'s," *Oxford Economic Papers*, 1976.
- , "The Theory of Local Public Goods," in *The Economics of Public Services*, M. Feldstein and R. Inman, eds. (London: Macmillan Press, 1977).
- , "Public Goods in Open Economies with Heterogeneous Individuals," Oxford mimeo, 1978.
- Tiebout, C. M., "A Pure Theory of Local Expenditures," *Journal of Political Economy*, LXIV (1956), 416-24.
- Vickrey, W., "The City as a Firm," in *The Economics of Public Services*, M. S. Feldstein and R. P. Inman, eds. (London: Macmillan, 1977).