AHP VERSION 5.1

USER'S MANUAL

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MASTER

ABSTRACT

As decisions become more and more complex, decision makers are faced with the challenge of sorting through many variables to arrive at a sound decision. The Analytical Hierarchy Process (AHP) is a tool that allows a systematic, logical approach to reducing complex issues into manageable pieces. The decision maker can then sort through the variables and determine to what degree a particular variable should influence the final decision. The power of the AHP as a management tool comes from the fact that it reduces complex problems to many simple pairwise decisions. Only two items need be compared against one another - a much simpler task than comparing an item to all the others simultaneously. By arranging the items that influence a decision in the form of a matrix and comparing appropriate pairs in this matrix to each other, each item can be compared with every other item. Matrix algebra can then operate on this matrix and rank each item according to its importance to the final decision.

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3. Once the user is satisfied that the input information is correct, it

CONDITIONS OF TRANSMITTAL

This manual and the software described herein are transmitted with the understanding that the recipient will provide feedback that may help improve the user friendliness of future versions of both the manual and the software. The recipient should also note that this is Quality Level C software and has only received the quality assurance checks described in Appendix B.

spaces. (AHP5.1 converts spaces to underscores when saving the input file so that the problem title and the level and item names are contiguous). The order

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negative number to a fraction when manipulating the matrices. Entries made in

The following matrix contains more than 18 parameters

ACKNOWLEDGMENTS

normalized scale where either the sum of the priorities equals one or the largest priority equals one (depending on the normalization scheme selected), and a

C:\AHP
SAATY   .86   SAATY   .98   SAATY   .103   SAATY   .108

C:\AHP
        .   <DIR>   .. <DIR>   TEST.AHP   TESTFILE.IN
SAATY   .86   SAATY   .98   SAATY   .103   SAATY   .108

AHP Version 5.1 User's Manual

1. INTRODUCTION

As decisions become more and more complex, decision makers are faced with the challenge of sorting through many variables to arrive at a sound decision. The Analytical Hierarchy Process (AHP) is a tool, developed by T. L. Saaty [1], that allows a systematic, logical approach to reducing complex issues into manageable pieces. The decision maker can then sort through the variables and determine to what degree a particular variable should influence the final decision. The power of the AHP as a management tool comes from the fact that it reduces complex problems to many simple pairwise decisions. Only two items need be compared against one another - a much simpler task than comparing an item to all the others simultaneously. By arranging the items that influence a decision in the form of a matrix and comparing appropriate pairs in this matrix to each other, each item can be compared with every other item. Matrix algebra can then operate on this matrix and rank each item according to its importance to the final decision.

Section 2 is an overview of the AHP, as presented by T. L. Saaty, so that the reader being introduced to the AHP for the first time can understand the basic concepts. Specific instructions for using AHP5.1 are presented in Section 3. By following these instructions, the user can create an input and an output file. Sections 4 and 5 describe the format of the input and output files, respectively. Finally, Section 6 contains step by step instructions for creating and running a sample problem with AHP5.1. Any error messages that might occur while using AHP5.1 are listed and defined in Section 7.

Appendix A expands on the basic concepts summarized in Section 2. Although the AHP5.1 software was designed to satisfy Quality Level C requirements, the software was subjected to selected validation procedures. These are described in Appendix B. Additional information regarding the accuracy of the AHP methodology itself is available in Reference [2].

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TABLE 1

FUNCTION KEY DEFINITIONS

2. THE ANALYTICAL HIERARCHY PROCESS

As the name suggests, the AHP contains hierarchies or levels. Each level contains items that will be ranked relative to an item in the level above it. By starting at the lowest level - the most fundamental level - the decision maker can rank items with respect to a more general item contained in the next higher level. As the decision maker proceeds through the levels, the items become more general until, finally, the most general item - the goal - is reached. Thus, the decision maker proceeds as if building a pyramid. At the bottom he makes many specific decisions. As he proceeds toward the top he makes fewer and fewer decisions but they become more general. Finally the AHP manipulates the decision makers pairwise decisions to determine how important each of the most specific items are with respect to the most general item, the goal. Section 2.1 describes levels in more detail.

Items in each level are ranked with a tool called a priority matrix. Each entry in a priority matrix compares the _relative_ importance of two items - a pairwise decision only. Each level will contain one or more priority matrices that are filled with these pairwise decisions. Section 2.2 discusses priority matrices further.

In the process of building the priority matrices, the decision maker may inadvertently enter decisions that contradict one another. The amount of contradiction in a priority matrix is measured with a term called the consistency ratio (CR). Consistency ratios from individual priority matrices can be combined to arrive at a measure of contradiction or inconsistency for the entire hierarchy. The consistency ratio is discussed further in Section 2.3 and in Appendix A.

2.1 Levels

The AHP can consist of several levels of decisions with each level representing a different degree of detail in the decision process. There can be as many levels as needed for the problem, but for the following discussion on how

levels and items within these levels interact, we will use a three level hierarchy. The top or first level always contains the goal to be achieved, or the major question being answered. The bottom or lowest level contains the items to be prioritized with respect to their affect on achieving the goal. All the levels in between help the decision maker relate the most specific items in the lowest level to the goal in the top level. Each level contains items that relate to the more general items in the level above it. The more complex the problem is, the more levels it is likely to have.

For example, if one wants to buy a new car but is confused by all the cars and options on the market, one might use a hierarchy with the following levels:

Level 1   Purchasing the car that gives the most satisfaction
Level 2   Items that contribute to satisfaction with the car
Level 3   Specific car models

The decision maker has established a goal (level 1) - to buy a new car he will be satisfied with. The last level (level 3) contains the most specific items, the list of car models under consideration. The decision maker could have used a two level hierarchy consisting only of levels 1 and 3. But deciding which car model gives the most satisfaction can be confusing because several items contribute to satisfaction. Therefore, an additional level is established that will help the decision maker relate the most specific items to the goal. Figure 1 illustrates the hierarchy. By choosing the items related to satisfaction - say comfort, fuel economy, etc., for the intermediate level (level 2), the decision maker can rank the car models on level 3 with respect to the satisfaction items on level 2. Likewise, he can rank the satisfaction items with respect to the goal, satisfaction with the new car. The AHP then uses these pairwise rankings in the form of priority matrices to arrive at an overall ranking of the most specific items with respect to the goal. That is, the various car models will be ranked with respect to overall satisfaction with the new car. The decision maker would then purchase the car model with the highest ranking since it best meets the requirements for satisfaction.

```
        Level 1              Level 2                      Level 3
         Goal            Satisfaction items               Car Models

                     ┌──────────────────────┐   ┌──────────────────────────┐
                     │ 1. Comfort           │   │ Model 1, Model 2, Model 3 │
                     │                      │   ├──────────────────────────┤
                     │ 2. Fuel Economy      │   │ Model 1, Model 2, Model 3 │
      Satisfaction   │                      │   ├──────────────────────────┤
      with the       │ 3. Maintenance Costs │   │ Model 1, Model 2, Model 3 │
      new car        │                      │   ├──────────────────────────┤
                     │ 4. Initial Costs     │   │ Model 1, Model 2, Model 3 │
                     │                      │   ├──────────────────────────┤
                     │ 5. Status            │   │ Model 1, Model 2, Model 3 │
                     └──────────────────────┘   └──────────────────────────┘
```
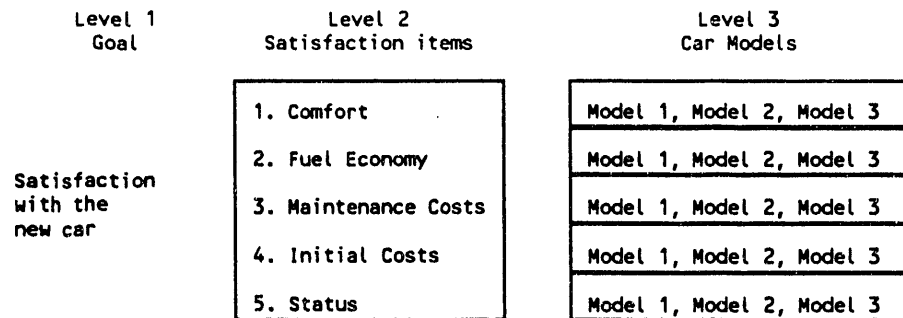
Figure 1.   An Example Hierarchy

2.2 Priority Matrices

Priority matrices, such as the one shown in Figure 2, are key tools for the AHP. It is the priority matrices in which the decision maker enters the pairwise rankings bridging each level. The AHP then uses the priority matrices to determine the ranking of items on each level, and then the ranking of each item in the overall hierarchy (the items on the last level with respect to the top level). The matrix shown in Figure 2 bridges the first and second levels of the hierarchy because it relates satisfaction items from level 2 to the goal that is level 1.

Satisfaction		1.	2.	3.	4.	5.
Comfort	1.	1	1	1/2	1/3	2
Fuel Economy	2.	1	1	2	3	2
Maintenance Costs	3.	2	1/2	1	1/2	2
Initial Costs	4.	3	1/3	2	1	3
Status	5.	1/2	1/2	1/2	1/3	1

Figure 2.   Example Priority Matrix

Each level below level 1 will have a set of matrices (level 1 contains no matrix, only the goal). One matrix will rank items in the current level with respect to an item in the level above it. Thus, each level will have as many matrices as there are items in the level above it. Figure 1 illustrates this.

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Each box in the figure represents a priority matrix, one on Level 2 and five on Level 3.

To construct a priority matrix, the decision maker begins by listing the items in the current level along the left of, and at the top of an empty matrix. Ones are then entered along the diagonal (row 1, column 1 equals one; row 2, column 2 equals one; etc.). These ones indicate that each item is as important as itself. Now, the decision maker is ready to enter the remaining pairwise rankings into the matrix. Pairwise rankings can be entered anywhere in the matrix; however, since the upper right half of the matrix is the reciprocal of the lower left half of the matrix (row 1, column 4 is the reciprocal of row 4, column 1), only half the matrix need be filled in. Figure 2 is an example of a priority matrix that ranks the items on level 2 with respect to level 1, or satisfaction items with respect to the goal, satisfaction with the new car.

Thus, by evaluating this matrix, the decision maker will know which satisfaction item most contributes to his satisfaction with the new car. He doesn't yet know which car to buy; however, he knows what satisfaction items are most important in his selection of a new car. In this example, the decision maker has ranked each satisfaction item using a scale from one to three. A one indicates that the item on the left is of equal importance to the item on the top when considering the goal. A three indicates that the item on the left is much more important than the item on the top whereas one-third indicates the inverse, the item on the top is much more important than the item on the left.

The decisions are pairwise in that the decision maker has made a decision between two items only. He has judged the importance of one item with respect to another item. For example, comfort is somewhat less important than maintenance costs because row 1, column 3 equals one-half. Once the priority matrices are completed, the work for the decision maker is finished. Now AHP5.1 can operate on the matrices and determine the absolute priorities.

2.3 Consistency

Three major items affect the consistency of a matrix and of the resultant hierarchy. Each of these items are discussed below.

2.3.1 The Decision Makers Judgement. It is possible for decisions to contradict one another in a priority matrix. For example, the decision maker might say that item 1 is more important than item 2, that item 2 is more important than item 3; but item 3 is more important than item 1. The measurement of the amount of contradiction in a matrix is called the consistency ratio. Consistency ratios from each priority matrix in the hierarchy can then be used to determine the consistency of the overall hierarchy.

2.3.2 The Scale. The scale used will affect consistency somewhat. A coarse scale will be less consistent than a fine scale. That is, a scale from one to nine will be more consistent than a scale from one to three, provided the decision maker can clearly discern items within the fine scale when assigning pair e rankings. Scales are discussed further in Section 2.4 and in Appendix A. Appendix A also explains how AHP5.1 calculates the consistency parameters. Generally, if the consistency ratio for a matrix is less than 0.1, the consistency is considered acceptable [1].

2.3.3 The Size of the Matrix. The size of the matrix affects consistency because each priority matrix contains redundant information that can be used to estimate its consistency. Unbeknown to the decision maker, the same question is asked, many times and in many different ways, to determine if the answer remains the same. If the answer is always the same, the consistency ratio will be low[1], indicating that the matrix is more consistent. Large matrices contain a larger number of entries that can be used to estimate the consistency of a matrix (see Appendix A for further discussion). Therefore, as a matrix becomes large, a single inconsistent answer becomes less important.

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[1] The term "consistency ratio", in the context of its use herein, is actually a measure of <u>inconsistency</u>.

path. To save a file to another directory or to another disk, include the full

2.4 Choosing the Scale

As discussed in Section 2.3.2, the scale used for pairwise rankings will influence the overall consistency of a matrix. A coarse scale will be less consistent than a fine scale; however, the selection of the scale also depends on other factors. For instance, the decision maker must be able to discern between the discrete steps in the scale when assigning pairwise rankings. A scale from one to three is usually the easiest to apply because the decision maker can easily decide whether an item has equal importance to, somewhat more importance to or much more importance to another item. A scale from one to nine is more difficult to apply because differences between steps in the scale are small. For instance, there are three steps in the scale from one to nine for every single step in a scale from one to three. What was a two in a coarse scale may now be a four, five or six in a fine scale.

Based on experience using the AHP at the Idaho National Engineering Laboratory, the authors recommend using a scale from one to five. Use one to represent equal importance between items, use three to represent that one item is somewhat more important than another item, and use five to represent that one item is much more important than another item. Reserve two and four as compromises to resolve differences of opinions between experts as the priority matrices are being constructed. However, note that regardless of the scale used for the pairwise rankings, the output from AHP5.1 will always be presented on a scale from one to nine. The scale presented in the output has no affect on the consistency of the matrices or of the hierarchy.

3.7 Option 7. Sent a File to the Printer

3. STARTING AHP5.1 AND USING THE MENU SCREENS

This section describes the use of AHP5.1. After invoking AHP5.1 by entering "AHP51" from the DOS prompt at the directory in which AHP5.1 resides, the main menu screen will appear as shown in Figure 3. Each option on this screen is described in a corresponding subsection. Each subsection will in turn explain when an option should be used, and then explain how to use it. Before continuing, a brief introduction to a typical AHP5.1 session will be presented.

```
┌──────────────────────────────────────────────────────────┐
│      ┌────────────────────────────────────────┐          │
│      │    THE ANALYTICAL HIERARCHY PROGRAM     │          │
│      │             VERSION 5.1                 │          │
│      └────────────────────────────────────────┘          │
│                                                            │
│        1. Input new comparison using the keyboard          │
│        2. Load new comparison from an existing file        │
│                                                            │
│        3. Edit comparisons                                 │
│        4. Save comparisons                                 │
│                                                            │
│        5. Calculate - no print file                        │
│        6. Calculate - create print file                    │
│                                                            │
│        7. Send a file to the printer                       │
│                                                            │
│        8. Quit                                             │
│                                                            │
│        Enter desired option:                               │
│ F1:Help                                                    │
└──────────────────────────────────────────────────────────┘
```
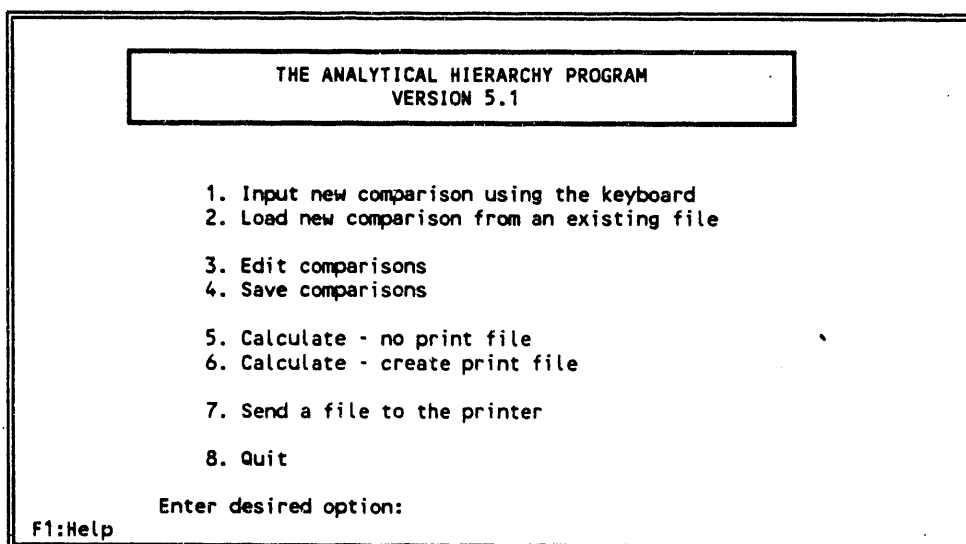
Figure 3. Main Menu Screen

A typical AHP5.1 session will consist of five steps:

1.   Data entry, either interactively (Option 1 from the main menu), or from a preexisting input file (Option 2 from the main menu). The preexisting input file could have been created during a previous AHP5.1 session or using an ASCII text editor.

2.   Correct or modify the input (Option 3 from the main menu). Rankings within any of the priority matrices may be changed. The individual items within any of the priority matrices can also be added or deleted. The problem title and any of the level names may also be changed.

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-hb   Sets the highlighted background color used during editing. The default is "7" (white).

3. Once the user is satisfied that the input information is correct, it can be saved either by entering a new file name (Option 4 from the main menu), or by over writing an existing file.

4. The hierarchy can now be evaluated by choosing one of the two calculation options. One option displays the results of the calculations on the screen only (Option 5 from the main menu) whereas the other option displays the results of the calculations on the screen as well as writing the results to an output file (Option 6 from the main menu). The output file may then be sent to a printer (Option 7 from the main menu).

5. The session can then be terminated by selecting "quit" (Option 8 from the main menu). Note that AHP5.1 does not automatically save or update any of the files prior to quitting. Therefore, be sure to save any of the desired input and output files using Options 4 or 6 before selecting Option 8 to "quit".

```
┌─────────────────────────────────────────┐
│               CAUTION                    │
├─────────────────────────────────────────┤
│ DO NOT SELECT "QUIT" BEFORE SAVING FILES │
└─────────────────────────────────────────┘
```

3.1 Option 1, Input New Comparison Using the Keyboard

3.1.1 Entering Levels. Use Option 1 to start a new problem by entering a "1" from the main menu. You will be directed through the input with help screens. Section 6 presents a sample input session and shows each of the screens that will be displayed while entering a new comparison. You will be asked for the number of levels in the hierarchy, the problem title and then the names of each level. For each level except the first, you will also be asked to enter a list of items that apply to that level. We will refer to this as the master list for that level as it contains every item that will be used to construct the priority matrices. Enter every item as it applies to the level by entering the item name and pressing the "ENTER" key after each entry. Spaces are allowed in the item names; however, the length of each item is limited to 15 characters including

spaces. (AHP5.1 converts spaces to underscores when saving the input file so that the problem title and the level and item names are contiguous). The order in which this list is entered is unimportant, since, later, when constructing the priority matrices, you can specify the order in which each item appears in a matrix. You may also add or delete items when constructing the priority matrices or when editing an input file. End the entry of item names for each level by entering a blank for an item name. AHP5.1 can now determine the number of levels and the items that belong with each level. As such, AHP5.1 also knows how many priority matrices are required for the hierarchy.

3.1.2 Constructing Priority Matrices. Now that the general structure of the hierarchy is established, the priority matrices must be constructed by assigning items from the master list to them. When prompted, enter the item number from the master list that corresponds to the first entry in the matrix. Continue until all items applicable to the current priority matrix are selected. The order in which you enter the item numbers will correspond to the order in which each item appears in the priority matrix. Enter a blank to terminate the entry of items for the current matrix. AHP5.1 will then prompt you for the next matrix to be defined. Enter the number corresponding to the next matrix you wish to define and then select those items from the master list that are part of this matrix. After all the items have been entered for each priority matrix, AHP5.1 will prompt you for the pairwise rankings. If you fail to identify items for each matrix, AHP5.1 will exit to the main menu and all entries will be lost.

After you finish constructing the priority matrices, AHP5.1 will display the first matrix with the list of items along the left hand side in the order they were entered. Note that from this point on, the hierarchy is stored in memory so that if you choose to return to the main menu, you can return to the edit mode without losing any of the data entered to this point. The matrix will initially contain zeros everywhere except along the diagonal that will contain ones. Edit the rankings by moving the cursor with the arrow keys to the desired entry and entering a new number. If you make a mistake, simply reenter a new number. Note that fractional rankings are entered and appear in the priority matrices as a negative integer. In other words, one-third is entered as -3, one-fifth is entered as -5, and so forth. AHP5.1 internally converts each

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negative number to a fraction when manipulating the matrices. Entries made in the matrix will generate an automatic negative entry in the opposite half of the matrix.

When finished editing a matrix, a new matrix can be selected to edit using the function keys. Use the "F5" or "F6" keys to select a matrix to edit on the same level, or use the "F3" or "F4" keys to move to a different level from which to select a matrix to edit. The AHP5.1 editor also has the ability to add or delete items from any matrix, change the name of any item, add new items to the master list for any level, change the title of any level or the title of the problem. A complete list of the function keys to accomplish these features is available by pressing the "F1" key. Editing is discussed in more detail in Section 3.3.

AHP5.1 allows you to initialize matrices of any size. That is, you may assign as many items as you wish to a matrix and AHP5.1 will enter ones on the diagonal and zeros everywhere else. However; if you attempt to edit a matrix with more than 18 items, the screen shown in Figure 4 appears stating that the AHP5.1 editor cannot edit a matrix that contains more than 18 items[2]. Pressing the "F1" key will display the help screen shown in Figure 5. To edit such a matrix, follow the steps listed on the help screen. Briefly, these steps instruct you to continue editing the remaining matrices that contain 18 or fewer items, then save the comparisons to a file using Option 4 from the main menu. AHP5.1 will then generate an input file; matrices with more than 18 items will contain ones on the diagonal and zeros everywhere else. Then, use an ASCII text editor and replace each zero with the proper pairwise rankings, remembering that a fractional entry is input as a negative reciprocal; one-fifth is entered as -5. AHP5.1 will then be able to subsequently read the file and perform the hierarchy calculations.

---

[2] 18 items represent the maximum size matrix that can be graphically displayed on the screen of a common PC monitor.

```
The following matrix contains more than 18 parameters
                    Level1: Itema

              To edit the data, save the matrix comparisons
       Edit with a separate text editor, then reload the edited file

                    Press F1 for additional information
             Select new matrix or Esc to return to main menu.




F1:Help      row: 0  col: 0    level: 2 of  2   F3:Previous Level   F4:Next level
ESC:Return to menu             item: 1 of  3    F5:Previous item    F6:Next item
```

Figure 4. Alert to a matrix with more than 18 items.

```
                              Help Screen

Esc - Return to main menu

        F3 = Previous level             F4 = Next level
        F5 = Previous matrix in level   F6 = Next matrix in level


        F9 = Add item to matrix         F10 = Delete item from matrix


   Comparison matrix is too large to fit on the screen.
        Must edit the large matrices outside this program as follows:

        1 - Move to a different level and finish editing smaller matrices.
        2 - From the main menu, select option 4 and save comparisons.
        3 - Exit this program.
        4 - Using a text editor, edit the saved file.
        5 - Restart this program.
        6 - From the main menu, select option 2 and load the edited
              comparisons.
        7 - Proceed with AHP calculations.

   Press any key to continue                      Free memory:
```

Figure 5. Help Screen that appears when a priority
           matrix contains more than 18 items.

3.1.3 The Scale. You may use any scale when entering rankings as long as,
(1) positive numbers reflect that the item to the left is more important than the
item on the top, (2) negative numbers indicate the inverse, the item on the top
is more important than the item to the left, and (3) the rankings are single
digit integers with either a plus (or no sign) or a minus sign. A one will
always indicate equal importance between the item on the left and the item on the
top of the matrix. Although any scale can be used when assigning the ranking
between two items, the output from AHP5.1 will appear in two scales only; a

normalized scale where either the sum of the priorities equals one or the largest priority equals one (depending on the normalization scheme selected), and a discrete scale ranging from one to nine where nine is the highest rank. See Section 2.4 for a discussion about how the scale used to rank the items affects the consistency of the results.

3.1.4 Exit the Input Session. The input session may be exited any time after constructing the priority matrices. Pairwise rankings need not be entered during a single session. To exit, press the "ESC" key to return to the main menu. The input file should be saved at this time using Option 4 in the event operation of the computer is interrupted and the file is lost. If the priority matrices are completely filled with pairwise rankings, the hierarchy is ready to be evaluated using either Option 5 or Option 6 (see Section 3.5 or 3.6, respectively). If the input is incomplete or incorrect, edit the priority matrices using Option 3 (see Section 3.3).

3.2 Option 2, Load New Comparisons From an Existing File

Use Option 2 to load an existing input file by entering a "2" from the main menu. The input file may have been created and/or edited with either the AHP5.1 editor or with an ASCII text editor. All the files in the default directory will be displayed, as shown in Figure 6, and you will be prompted for the name of the input file. An input file that resides in another directory or disk drive can be selected by specifying the entire path in addition to the file name. No path name is necessary if the input file resides in the default directory. If the file you selected is not in the proper format or is not an AHP5.1 input file, AHP5.1 will display the message "The file does not appear to be an AHP5.1 input file". If the file you selected is an input file, AHP5.1 will read the file and then return you to the main menu. You are now ready to either edit the file (Option 3) or to evaluate the hierarchy (Option 5 or Option 6).

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```
C:\AHP
          .    <DIR>          ..  <DIR>       TEST.AHP        TESTFILE.IN
SAATY   .86          SAATY   .98         SAATY   .103        SAATY   .108
 280567 Bytes free

WARNING: If file exists, it will be overwritten.

Save input file as: _____
```

Figure 6. Prompt Screen for saving an input file.

3.3 Option 3, Edit Comparisons

Use Option 3 to edit an input file, after it has been loaded using
Option 2, by entering a "3" from the main menu. The level 2 matrix will then be
displayed, remembering that Level 2 contains a single matrix only. The function
keys control movement amongst the various matrices and through the editing
process. The definition of several function keys are displayed at the bottom of
the screen, the definition of the other function keys are available by pressing
the "F1" key and viewing the Help screen. Table 1 briefly describes the purpose
of each function key. The features of the AHP5.1 editor are available through
Option 1, Input New Comparisons Using the Keyboard, or Option 3, Edit
Comparisons. These features include:

1. Change pairwise rankings
2. Change an item name, a level title, or the problem title
3. Add an item to a matrix
4. Delete an item from a matrix

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TABLE 1

FUNCTION KEY DEFINITIONS	
F1 Help Screen Alt F1 Ranking Definitions F3 Previous Level F5 Previous Matrix in Level F9 Add New Item to Matrix	F2 Edit Item Names Alt F2 Change Level or Problem Title F4 Next Level F6 Next Matrix in Level F10 Delete Item from Matrix

3.3.1 Change Pairwise Rankings. Upon entering a "3" from the main menu, the matrix for level 2 will be displayed and the rankings can be edited. To move around the matrix, use the arrow keys. The "HOME", "PGUP", "PGDN", and "END" keys will move the cursor to the upper left, upper right, lower right, lower left, corners of the matrix respectively. Change the pairwise rankings by simply entering a new number over the old one. When you are finished editing the matrix, use the function keys to select a different matrix to edit.

3.3.2 Change the Problem or Level Titles, or Item Names. Use the function keys to select the matrix that contains the item whose name you wish to change, then press the "F2" key to edit the item names. The cursor will highlight the first item name at the left of the matrix. Move the cursor up or down with the arrow keys to select the item name you wish to change, then enter a new name over the old one. Any spaces that are entered will be replaced with underscores so that the item names are contiguous. The name is limited to 15 characters including spaces. When you are finished editing the item names for this matrix, press the "ESC" key to return to editing the pairwise rankings.

When an item name is changed in one priority matrix, that change is carried throughout the entire hierarchy. In addition, the corresponding matrix title in the next lower level will also be changed.

The problem title and the level titles can be changed by pressing the "Alt" key and then simultaneously the "F2" key. The problem title and level titles will appear on the screen. Use the arrow keys to select the title you wish to

change, then enter a new title over the old one. Press the "ESC" key to return to editing the pairwise rankings.

3.3.3 Add an Item to the Priority Matrix. There are two methods to add items to a priority matrix.

Method 1:   If the item to be added is in the master list of items for the current level, the item can be selected from the master list.

Method 2:   If the item to be added is not on the master list for the current level, a new item must be defined and added to the master list before it can be added to the matrix.

Each method is discussed below.

Method 1: Choosing an Item from the Master List.

Use the function keys to select the matrix to add the item to, then press the "F9" key to add a new item to the matrix. The master list of items for that level will appear along with a prompt for the item you wish to add to the matrix as shown in Figure 7. Enter the number of the desired item, then press the "ENTER" key. If an item number is entered that does not exist in the master list, or if an item number is entered that already exists in the priority matrix, AHP5.1 will ignore the request. After the new item number is entered, AHP5.1 will return to the current matrix. The matrix will contain the new item as the last entry and the new pairwise rankings will be set to zero. Replace the zeros by editing the rankings as described in Section 3.3.1.

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```
┌─────────────────────────────────────────────────────────────────┐
│ ╔═══════════════════════════════════════════════════════════════╗ │
│ ║      Master list of items that can be included in the matrix rankings: ║ │
│ ║ 1. item1           2. item2           3. item3                ║ │
│ ║                                                               ║ │
│ ║                                                               ║ │
│ ║                                                               ║ │
│ ║ Level2_Title that affects Level1_Title:Level1_item    F9: Add item to list ║ │
│ ║    Enter the item number in the order it appears in the ranking matrix: ║ │
│ ║   1  2 __                                                     ║ │
│ ╚═══════════════════════════════════════════════════════════════╝ │
└─────────────────────────────────────────────────────────────────┘
```

Figure 7. Screen for Adding Item to Matrix

Method 2: Adding an Item to the Master List

Use the function keys to select the matrix to add the new item name to,
then press the "F9" key to add a new item to the matrix. The master list of
items for that level will appear. To add a new item to the master list, press
the "F9" key again. You will then be prompted for a new item name. Enter the
name, no more than 15 characters long including spaces, and press the "ENTER"
key. AHP5.1 will return to the current matrix. The matrix will contain the new
item as the last entry and the new pairwise rankings will be set to zero.
Replace the zeros by editing the rankings as described in Section 3.3.1.

Figure 8 illustrates how adding a item to a level's master list affects the
next level below it by creating a new, but empty, matrix. No items or pairwise
rankings will be associated with the new matrix until the user defines them. To
define the contents of the new matrix, use the function keys to change to the
next lower level and through the matrices until you find the empty matrix with
the title of the new item. The AHP5.1 editor alerts you to an empty matrix with
the screen shown in Figure 9. Press the "F9" key to add items associated with
this matrix as described earlier, the enter the pairwise rankings as described
in Section 3.3.1.

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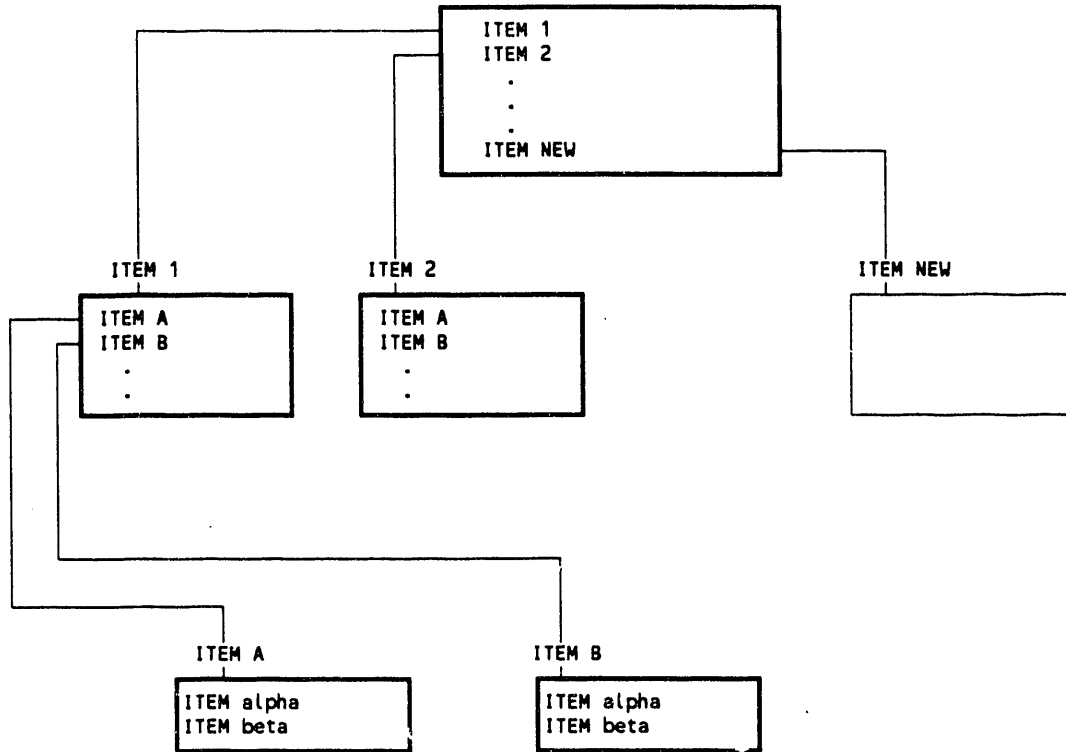
Figure 8. Hierarchy Change as a result of adding a
new item to the master list for the level.


```
                The following matrix contains no rankings
                           component: two

                  Press F9 to add items to this matrix

                  Press F1 for additional information
            Select new matrix or Esc to return to main menu




 F1:Help      row: 0 col: 0   level: 2 of  2   F3:Previous level   F4:Next level
 ESC:Return to menu           item: 1 of  3    F5:Previous item    F6:Next item
```

Figure 9. Alert to an empty matrix

3.3.4 Delete an Item From the Priority Matrix. Use the function keys to select
the matrix from which you wish to delete a item, then press the "F10" key. The
items associated with the matrix will appear along with a prompt for the item you
wish to delete. Enter the number of the desired item, then press the "ENTER"

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key. If an item number is entered that does not exist in the matrix, AHP5.1 will ignore the request. Once an item has been deleted from a matrix, the matrix associated with that item in the next lower level will also be deleted.

3.3.5 Change the Order of the Items in the Priority Matrix. The AHP5.1 editor does not have a convenient way to change the order of items as they appear in the matrix. As such, you must first delete selected items as discussed in Section 3.3.4, then add them in the desired order as discussed in Section 3.3.3. The "Add" feature will add items to the bottom of the matrix whereas the "Delete" feature will delete any item in the matrix. Deleting an item and then adding it will move the item to the bottom of the matrix; however, the pairwise rankings for that item will be reset to zero. By using a series of "Adds" and "Deletes", the matrix can be reordered, albeit the pairwise rankings will have to be reentered, the matrices in the next lower level will need to be redefined, and the rankings reentered. Considering the difficulty associated with reordering the items in a matrix, the desired order should be considered carefully when first entering the items.

3.3.6 Leave the Edit Session. To leave the edit session, press the "ESC" key to return to the main menu. The input file should be saved using Option 4 in the event the operation of the computer is interrupted and the file is lost.

3.4 Option 4, Save Comparisons

Use Option 4 to save the hierarchy by entering a "4" from the main menu. You will be prompted for the name of the file. Enter a legal DOS file name and then press the "ENTER" key.

CAUTION

IF THE FILE NAME YOU ENTERED IS THE SAME AS AN
EXISTING FILE, THE EXISTING FILE WILL BE OVERWRITTEN.

The file will be saved in the default directory that was specified on the command line when AHP5.1 was invoked. See Section 3.9 for specifying the default

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path. To save a file to another directory or to another disk, include the full path name.

```
┌──────────────────────────────────────────────────────────────┐
│                          CAUTION                             │
│                                                              │
│  SAVE THE HIERARCHY BEFORE LOADING ANOTHER INPUT FILE OR QUITTING │
│    AHP5.1.  OTHERWISE, ANY REVISIONS TO THE FILE WILL BE LOST.    │
└──────────────────────────────────────────────────────────────┘
```

3.5 Option 5, Calculate - No Print File

Use Option 5 to evaluate the hierarchy by entering a "5" from the main menu. An input file must have previously been loaded using Option 2 or created using Option 1. Output will be directed to the screen and no output file will be generated. You will be prompted on whether you wish to review the rankings as they are calculated. Answering "yes" causes the output to pause after each matrix is displayed until you press a key to continue. Even though this option does not create an output file, the messages in Section 5 will still appear on the screen.

3.6 Option 6, Calculate - Create Print File

Use Option 6 to evaluate the hierarchy and to generate an output file by entering a "6" from the main menu. An input file must have previously been loaded using Option 2 or created using Option 1. Output will be directed to the screen as well as to an output file. You will be prompted for the name of the output file. Enter a legal DOS name and then press the "ENTER" key. If you used Option 2 to load an input file, AHP5.1 will not allow you to write over that input file. The input file loaded with Option 2 is the only file AHP5.1 will not overwrite.

```
┌──────────────────────────────────────────────────────────────┐
│                          CAUTION                             │
│                                                              │
│  IF THE FILE NAME ENTERED IS THE SAME AS AN EXISTING         │
│    FILE, THE EXISTING FILE WILL BE OVERWRITTEN.             │
└──────────────────────────────────────────────────────────────┘
```

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3.7 Option 7, Sent a File to the Printer

Option 7 is included for user convenience. It simply sends any file you specify to the printer if one is attached to the computer.

3.8 Option 8, Quit

Use Option 8 to quit AHP5.1 and return control to the operating system by entering an "8" from the main menu. This option does not save any files before exiting the program, so be sure to save the input file before selecting this option.

```
┌─────────────────────────────────────────────────────────┐
│                        CAUTION                          │
│                                                         │
│  SAVE HIERARCHY BEFORE QUITTING AHP5.1.  OTHERWISE,     │
│     ANY REVISIONS TO THE FILE WILL BE LOST.            │
└─────────────────────────────────────────────────────────┘
```

3.9 Command Line Options

Options can be specified at the time AHP5.1 is invoked to set the background and foreground colors, the default path, the normalization scheme, and the print file shift. If the defaults are inconvenient to enter, AHP5.1 can be invoked via a batch file. Specify options as follows:

AHP51 [-option1] [parm1] [-option2] [parm2] ...

The command line options are described below and all are optional. If a command line option is not specified, the default value will be used.

-rf   Sets the regular foreground color. The default is "7" (white).

-rb   Sets the regular background color. The default is "0" (black).

-hf   Sets the highlighted foreground color used during editing. The default is "0" (black).

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-hb   Sets the highlighted background color used during editing. The default is "7" (white).

-dp   Sets the default path for reading and writing files. A full path name or a path name relative to the current directory can be entered. The default path is the current directory.

-sp   Sets the left margin of the output file to the specified character. The default is 10 characters.

-os   Selects the normalization scheme used by Saaty [1]. This option has no parameters. Not selecting this option causes Dimenna's normalization scheme to be used instead. The normalization scheme used will be printed at the end of the output. The difference between the two normalization schemes is discussed below.

Once AHP5.1 computes the weights of the individual items in a priority matrix, these weights can be adjusted in one of two ways before they are used to determine the composite priorities for the hierarchy. The first method, which is Saaty's, adjusts the weights of the individual items such that the sum of the weights is one. If this scheme is selected, a disproportionately larger weight will be given to items in a smaller matrix whereas a disproportionately smaller weight will be given to items in a larger matrix. The second method, which is Dimenna's, adjusts the weights of the individual items such that the highest priority item has a value of one. This scheme seeks to offset the matrix size bias by setting the highest priority item in a matrix to one regardless of the size of the matrix. It is recommended that Dimenna's scheme be used when the matrices within a given level are of different sizes. This option is included primarily for verifying the results from AHP5.1 against examples given in Saaty's book [1].

The following is an example of invoking AHP5.1 command line options:

        AHP51 -rf 7 -rb 1 -hf 15 -hb 4 -dp ahp -sp 15

Issuing the above command executes AHP and sets the following program defaults:

        Regular Colors: white letters on a blue background
        Highlighted Colors: bright white letters on a red background
        Default file directory: .\ahp
        Left Margin: 15 columns from the left edge of the page
        Normalization scheme: Dimenna

4. THE INPUT FILE

Creating and editing an input file with the AHP5.1 editor was discussed in Sections 3.1 and 3.3; however, an input file can also be created using an ASCII text editor. In fact, although you can create a priority matrix with more than 18 items, you cannot use the AHP5.1 editor to edit such a large matrix. If an input file is to be created with an ASCII text editor, the file must be in the proper format before AHP5.1 can read it. Figure 10 illustrates the format of such an input file.

```
Problem Title          Problem Title
Dummy Line             Number of items in each level:
                         1  3  4
Level Title            Level1_Title
Mater List                 Level1_Item
Level Title            Level2_Title
Master List                item1
        .                  item2
                           item3
Level Title            Level3_Title
Master List                itema
                           itemb
                           itemc
                           itemd
Dummy Line             Level 2 array. Rank level 2 items with respect to the level 1 item.
Matrix Title           Level1_Item              1.  2.  3.
                           item1          1.    1   2   3
                           item2          2.        1   4
                           item3          3.            1
Dummy Line             Level 3 arrays. Rank level 3 items with respect to the level 2 items.
Matrix Title           item1                    1.  2.  3.
                           itema          1.    1   3   2
                           itemb          2.        1  -2
                           itemc          3.            1
Matrix Title           item2                    1.  2.  3.
                           itemb          1.    1   1   3
                           itemc          2.        1  -2
                           itemd          3.            1
Matrix Title           item3                    1.  2.  3.  4.
                           itema          1.    1   4   3   2
                           itemb          2.        1  -3  -2
                           itemc          3.            1   1
                           itemd          4.                1
```

Figure 10. Typical input file format

Input is free format. That is, the individual parameters may appear anywhere on a line, but must be separated by at least one space. Input in this example is descriptive of the input variable. For example, Level1_Title,

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indicates that a title for level 1 belongs in this location. The problem title is limited to a length of 46 characters, spaces allowed. Level titles, and item names are limited to 15 contiguous characters. Spaces are not allowed directly, but can be input using the underscore character. (When using the AHP5.1 editor to create an input file, spaces are allowed because spaces are automatically converted to underscores to make the entry contiguous). Lines indicated as "Dummy Line" are not used; however, the line, blank or not, must be present. Items that are part of the matrices for a given level must appear on the master list of items for that level. For example, the matrix entitled "item2" in level 3, contains items "itemb", "itemc", and "itemd". These items must appear in the master list beneath "Level3_Title". If an item is found in the matrix that does not appear in the master list, the message "No Match Found For <item name>" will be displayed on the screen. Likewise, matrix titles must appear in the master list for the level above the level containing the matrix. For example, the matrix title "item1" in level 3 must appear in the master list under "Level2_Title". If a matrix title is found that does not appear in the master list, the message "No Match Found For <item name>" will be displayed on the screen.

The matrix indexes or counters (the numbers that appear at the left and top of each matrix) are required input. AHP5.1 does not actually read the number, but looks for groups of characters separated by spaces to verify the size of each matrix. Thus, the "1.", "2.", and "3." could also have been "a.", "b.", and "c." Also, AHP5.1 expects to find the same number of matrices in a level as there are items in the level above it. A matrix may be empty, that is, it may contain no items, but the matrix title must be present. In this example, AHP5.1 will look for three matrices in level 3 because there are three items in level 2. If too few matrices are entered, the message "Past End of File. More Data Expected" will be displayed on the screen.

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5. THE OUTPUT FILE

The following pages discuss the output file from the example problem discussed in the previous section. The first entry in the output file is an echo of the user input hierarchies. Next, the priorities and consistency information is presented. The priorities for each matrix are presented followed by an assessment of the consistency of the matrix. The priorities are presented in one of two scales depending on the normalization scheme being used. If the original Saaty normalization is being used, the priorities for any given matrix will sum to one. If the Dimenna normalization is being used, the largest priority will be set to one and the other priorities will be scaled relative to the adjustment necessary to make the largest priority one. AHP5.1 also examines the consistency ratio of each matrix. If the consistency ratio is greater than 0.1, a message will direct you to a footnote that states "The consistency limit has exceeded 10%. A review of the input assumptions may be necessary."

Following the priority and consistency information for each matrix on the current level, a composite priority is presented. Priorities are presented in two scales relative to level one, a normalized scale where the sum of the priorities sums to one, and a discrete scale from one to nine, where nine represents the highest priority. At the end of the output, the overall consistency of the hierarchy is reported. If the overall hierarchy consistency is greater than 0.1, a message will direct you to the footnote mentioned above.

If AHP5.1 encounters a matrix that contains a zero ranking, a message will be displayed that states "Bad array value" followed by the level number, array name, and the address of the bad value. This message will appear in both the priority and consistency report for the level containing the bad matrix. In addition, the priority and consistency report for the entire hierarchy will contain another message that states "Active item with a zero ranking. Results are not meaningful." Correct the zero entry with either the AHP5.1 editor or with an ASCII text editor and recalculate the problem.

```
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
aaaa      Analytical Hierarchy Process - output file      aaaa
aaaa                  Program version 5.1                 aaaa
aaaa                                                      aaaa
aaaa                    Problem Title                     aaaa
aaaa                                                      aaaa
aaaa      date: 04-13-1992          time:  4:56:20 am     aaaa
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
aaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaaa
```

HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Level1 Title/Level2 Title data arrays

Level1 Item		1.	2.	3.
item1	1.	1	2	3
item2	2.		1	4
item3	3.			1

Level2 Title/Level3 Title data arrays

item1		1.	2.	3.
itema	1.	1	3	2
itemb	2.		1	-2
itemc	3.			1

item2		1.	2.	3.
itemb	1.	1	1	3
itemc	2.		1	-2
itemd	3.			1

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item3		1.	2.	3.	4.
itema	1.	1	4	3	2
itemb	2.		1	-3	-2
itemc	3.			1	1
itemd	4.				1

LEVEL2 TITLE FACTORS RELATIVE TO LEVEL1 TITLE

Factors relative to Level1 Item:
                        weight

item1   0.5171
item2   0.3586
item3   0.1243

        lambda (maximum)   = 3.1078
        consistency index = 0.0539
        consistency ratio = 0.0930

Composite priorities:
                        weight   priority

item1   0.5171   (9)
item2   0.3586   (6)
item3   0.1243   (1)

LEVEL3 TITLE FACTORS RELATIVE TO LEVEL2 TITLE

Factors relative to item1:
                    weight

itema   0.5396
itemb   0.1634
itemc   0.2970

    lambda (maximum) = 3.0092
    consistency index = 0.0046
    consistency ratio = 0.0079

Factors relative to item2:
                    weight

itemb   0.4638
itemc   0.2552
itemd   0.2809

    lambda (maximum) = 3.3674
    consistency index = 0.1837
    consistency ratio = 0.3168   (See footnote below)

Factors relative to item3:
                    weight

itema   0.4760
itemb   0.0969
itemc   0.2157
itemd   0.2114

    lambda (maximum) = 4.0623
    consistency index = 0.0208
    consistency ratio = 0.0231

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Composite priorities:

	weight	priority
itema	0.3382	(9)
itemc	0.2719	(6)
itemb	0.2629	(6)
itemd	0.1270	(1)

Factors relative to item1:

	weight	priority
itema	0.2791	(9)
itemb	0.0845	(3)
itemc	0.1536	(5)

Factors relative to item2:

	weight	priority
itemb	0.1663	(6)
itemc	0.0915	(3)
itemd	0.1007	(4)

Factors relative to item3:

	weight	priority
itema	0.0592	(2)
itemb	0.0120	(1)
itemc	0.0268	(1)
itemd	0.0263	(1)

CONSISTENCY OF THE HIERARCHY = 0.1040   (See footnote below)

Footnote: The consistency limit has exceeded 10%.
           A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

6. AN EXAMPLE PROBLEM

To better understand the AHP5.1 editor, an example problem will be presented that contains step by step instructions for creating an input file. For this exercise, we will consider the problem of choosing a new car to buy. A hierarchical structure might look like Figure 11.

Level 1 Goal	Level 2 Satisfaction items	Level 3 Car Models
Satisfaction with the new car	1. Comfort	Model 1, Model 2, Model 3
	2. Fuel Economy	Model 1, Model 2, Model 3
	3. Maintenance Costs	Model 1, Model 2, Model 3
	4. Initial Costs	Model 1, Model 2, Model 3
	5. Status	Model 1, Model 2, Model 3
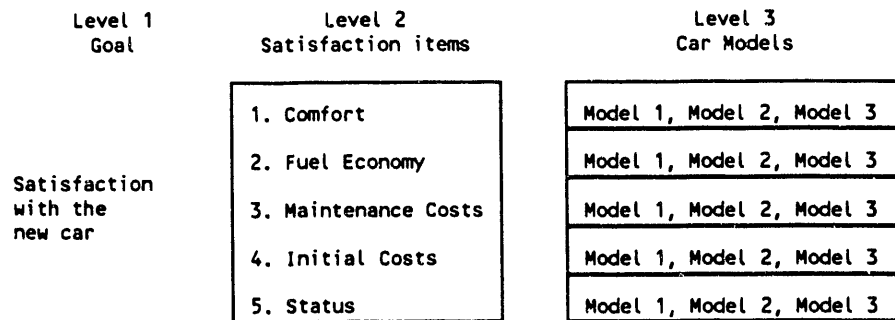
Figure 11. A hierarchy for choosing a new car to buy

Begin entering the hierarchy into AHP5.1 by selecting Option 1 - Input New Comparisons Using the Keyboard - from the main menu. The following screen will be displayed. After reading the screen, press any key to proceed.

```
                            Help Screen

You will be prompted for the following input:
    1. The total number of levels.
    2. A title.  This will appear on the output to identify the problem.
    3. The level name followed by the names of each item associated with it
          until a null (blank) entry is input.  Will start at level 1 and
          continue until each level is supplied.  Errors can be corrected
          later with the editing options in the program.

LEVEL     LEVEL NAME                     ITEM NAMES

  1       Transient                      SB LOCA

  2       Components           Pump    Core    Pressurizer    ..............

  3       Phenomena          Flow    Temperature    Pressure    Speed    ...

    Press any key to continue                    Free memory:
```

There are three hierarchial levels in this example problem, enter a "3" in response to the question and then press the "ENTER" key.

```
Enter number of levels: 3_
```

Next, enter the problem title for the example, "Buy a Car", and then press the "ENTER" key.

```
Enter number of levels: 3

Enter problem title: Buy a Car
```

AHP5.1 will now prompt you for the name of each level and the names of the items associated with each level. Begin by entering "Satisfaction" for the first level and then press the "ENTER" key. After entering the name of the first level, you will not be prompted for the names of items associated with this level. As there is only one item in the first level, the name of the level is used as the name of the only item.

```
Enter name of level  1: Satisfaction
```

Proceed by entering "Sat.Items" for the name of the second level and then press the "ENTER" key.

```
Enter name of level  2: Sat.Items
```

Since level 2 contains several items related to user satisfaction, begin by entering these items as follows. Enter "Comfort" and then press the "ENTER"

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key. Enter "Fuel Econ" and then press the "ENTER" key. Continue by entering the remaining items related to user satisfaction in the above example, which are "Maint.Cost", "Init.Cost" and "Status", each separated by pressing the "ENTER" key. After each item has been entered, terminate entry by pressing the "ENTER" key at an empty field.

```
        Enter name of level  2: Sat.Itams
                Enter names of items associated with this level
1: Comfort_____   2: Fuel Econ_____   3: Maint.Cost_____   4: Init.Cost_____
5: Status_____   6: _____
```

Next, proceed by entering "Models" for the name of the third level and then press the "ENTER" key.

```
        Enter name of level  3: Models_____
```

As was done with those items related to level 2, enter the names of those items related to level 3.   Enter "Model 1", "Model 2", and "Model 3", each separated by pressing the "ENTER" key.   After each item has been entered, terminate entry by pressing the "ENTER" key at an empty field.

```
        Enter name of level  3: Models
                Enter names of items associated with this level
1: Model 1_____   2: Model 2_____   3: Model 3_____   4:_____
```

Now that the level and item names have been entered, the following screen will appear. After reading the screen, press any key to proceed.

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```
                              Help Screen

Esc - Return to main menu

You will now be asked to identify:
     1. Items associated with each matrix and their order in the matrix.  Such
            as which phenomena are relevant to which component and the order
            of the phenomena in the ranking matrix.
     2. The order the matrices are to appear in.  Such as the order the
            component ranking matrices are to appear in when editing the
            rankings and in the output file.
     3. A null (empty) entry ends data input for a particular screen and
            advances to the next input screen.  Input errors such as name
            misspellings and items in the wrong order in the ranking matrix can
            be corrected prior to adding the item to item rankings.



     Press any key to continue                      Free memory:
```

The first hierarchical level to be defined relates the level 2 items to the level 1 item. Each item should be part of this matrix, however this screen allows the items to be reordered if desired. Proceed by identifying those items that belong in the matrix being defined by the order they are to appear in the ranking matrix. After each item has been input, press the "ENTER" key at an empty field.

```
          Master list of items that can be included in the matrix rankings:
     1: Comfort          2: Fuel_Econ       3: Maint.Cost      4: Init.Cost
     5: Status




     Sat.Items that affects Satisfaction
          Enter the item number in the order it appears in the ranking matrix:
          1  2  3  4  5  __
```

The matrices that need to be defined in the next level will now appear. Select the number of the matrix to define. In this example, matrix "1", or the "Comfort" matrix, will be defined first.

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```
┌──────────────────────────────────────────────────────────────────────┐
│║                                                                      ║│
│║                Master list of matrices that can be defined:          ║│
│║       1. Comfort          2. Fuel_Econ       3. Maint.Cost     4. Init.Cost ║│
│║       5. Status                                              .       ║│
│║                                                                      ║│
│║                                                                      ║│
│║                Enter the next matrix to be defined:                  ║│
│║                                                                      ║│
│║       1_                                                             ║│
│║                                                                      ║│
└──────────────────────────────────────────────────────────────────────┘
```

Next, identify those items that belong in the matrix being defined by the order they are to appear in. For this example, enter "1", "2", and "3", each separated by pressing the "ENTER" key.

```
┌──────────────────────────────────────────────────────────────────────┐
│║                                                             .        ║│
│║                                                                      ║│
│║          Master list of items that can be included in the matrix rankings: ║│
│║       1. Model 1          2. Model 2          3. Model 3             ║│
│║                                                                      ║│
│║                                                                      ║│
│║      Models that affects Sat.Items: Comfort                         ║│
│║           Enter the item number ·in the order it appears in the ranking matrix: ║│
│║         1   2   3 __                                                 ║│
│║                                                                      ║│
└──────────────────────────────────────────────────────────────────────┘
```

AHP5.1 will repeat the previous two screens until each of the five matrices have been selected and defined. Define the remaining matrices similar to the first matrix. Next, the following screen will appear. After reading the screen and deciding whether to use a one to nine ranking scale, or a coarser scale, such as one to five, press any key to proceed.

```
                              Help Screen
    Rank the relative importance of one item versus another using the following
    definitions and explanations.  A ranking scale other than 1 to 9 can be used,
    however the output results will be on a 1 to 9 scale.

    RANKING              DEFINITION                          EXPLANATION
       1          Equal importance of both        Two items contribute equally.
                    items.
       3          Weak importance of an           Experience and judgment slightly
                    item over another.              favor one item.
       5          Strong importance of            Experience and judgment strongly
                    an item over another.           favors one item.
       7          Demonstrated importance         Experience and judgment strongly
                    of an item over another.        favors one item, which has
                                                    also been demonstrated.
       9          Absolute importance of an       Evidence favoring one item is
                    two adjacent rankings.
    2,4,6,8       Intermediate values between     Compromise between two judgements.
                    two adjacent rankings.
    Negatives     Inverse of the above.           Above explanations except an item
                                                    disfavored instead of favored.
         Press any key to continue                      Free memory:
```

If the example problem has been entered as described, the following screen contains the first ranking matrix that will appear. However, when it first appears, the matrix will contain ones on the diagonal and zeros everywhere else. Use the arrow keys to move the cursor and enter the rankings as shown.

```
Rankings for SATISFACTION relative to:

                      1. 2. 3. 4. 5.

Comfort      1.      | 1| 1|-2|-3| 2|
Fuel_Econ    2.      | 3| 1| 2| 3| 2|
Maint.Cost   3.      | 3|-2| 1|-2| 2|
Init.Cost    4.      | 3|-2| 1| 1| 3|
Status       5.      |-3|-3|-3|-3| 1|


F1:Help      row: 0 col: 0    level: 2 of 2    F3:Previous level    F4:Next level
ESC:Return to menu            item: 1 of 3     F5:Previous item     F6:Next item
```

When finished entering the above rankings, select a different matrix to rank using the function keys (their functions are listed at the bottom of the computer screen). Enter the following information into each matrix as applicable until each matrix has been defined.

```
COMFORT         1.  2.  3.
Model 1    1.   1   2   3
Model 2    2.  -2   1   2
Model 3    3.  -3  -2   1
```

```
FUEL ECON.      1.  2.  3.
Model 1    1.   1  -2  -3
Model 2    2.   2   1  -2
Model 3    3.   3   2   1
```

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MAINT.COST		1.	2.	3.
Model 1	1.	1	1	2
Model 2	2.	1	1	3
Model 3	3.	-2	-3	1

INIT.COST		1.	2.	3.
Model 1	1.	1	-2	-3
Model 2	2.	2	1	-2
Model 3	3.	3	2	1

STATUS		1.	2.	3.
Model 1	1.	1	2	3
Model 2	2.	-2	1	2
Model 3	3.	-3	-2	1

When finished, press the "ESC" key to return to the main menu and then select Option 4 to save the hierarchy to a file. After saving the file, select Option 6 to calculate the hierarchy and save the results to a file. Both the input file and the output file can now be sent to the printer using Option 7 from the main menu. If the example problem has been correctly entered, the input and output files will look like the input and output files listed on the following pages.

```
Buy_a_Car
Number in each level:
  1  5  3
Satisfaction
    Satisfaction
Sat.Items
    Comfort
    Fuel_Econ
    Maint.Cost
    Init.Cost
    Status
Models
    Model_1
    Model_2
    Model_3
Satisfaction/Sat.Items data arrays
Satisfaction              1.  2.  3.  4.  5.
    Comfort        1.     1   1  -2  -3   2
    Fuel_Econ      2.         1   2   3   2
    Maint.Cost     3.             1  -2   2
    Init.Cost      4.                 1   3
    Status         5.                     1
Sat.Items/Models data arrays
Comfort                   1.  2.  3.
    Model_1        1.     1   2   3
    Model_2        2.         1   2
    Model_3        3.             1
Fuel_Econ                 1.  2.  3.
    Model_1        1.     1  -2  -3
    Model_2        2.         1  -2
    Model_3        3.             1
Maint.Cost                1.  2.  3.
    Model_1        1.     1   1   2
    Model_2        2.         1   3
    Model_3        3.             1
Init.Cost                 1.  2.  3.
    Model_1        1.     1  -2  -3
    Model_2        2.         1  -2
    Model_3        3.             1
Status                    1.  2.  3.
    Model_1        1.     1   2   3
    Model_2        2.         1   2
    Model_3        3.             1
```

THE OUTPUT FILE FOR THE EXAMPLE PROBLEM, BUYING A NEW CAR

```
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@      Analytical Hierarchy Process - output file      @@@@
@@@@                  Program version 5.1                 @@@@
@@@@                                                       @@@@
@@@@                     Buy a Car                         @@@@
@@@@                                                       @@@@
@@@@      date: 04-13-1992         time:  4:56:05 am       @@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
```

HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Satisfaction/Sat.Items data arrays

Satisfaction		1.	2.	3.	4.	5.
Comfort	1.	1	1	-2	-3	2
Fuel Econ	2.		1	2	3	2
Maint.Cost	3.			1	-2	2
Init.Cost	4.				1	3
Status	5.					1

Sat.Items/Models data arrays

Comfort		1.	2.	3.
Model 1	1.	1	2	3
Model 2	2.		1	2
Model 3	3.			1

Fuel Econ		1.	2.	3.
Model 1	1.	1	-2	-3
Model 2	2.		1	-2
Model 3	3.			1

Maint.Cost   1. 2. 3.

    Model 1   1.   1   1   2
    Model 2   2.   1   3
    Model 3   3.   1
_____

Init.Cost   1. 2. 3.

    Model 1   1.   1 -2 -3
    Model 2   2.   1 -2
    Model 3   3.   1
_____

Status   1. 2. 3.

    Model 1   1.   1   2   3
    Model 2   2.   1   2
    Model 3   3.   1

SAT.ITEMS FACTORS RELATIVE TO SATISFACTION

Factors relative to Satisfaction:
                        weight

Comfort   0.1508
Fuel Econ   0.3243
Maint.Cost   0.1723
Init.Cost   0.2614
Status   0.0911

lambda (maximum) = 5.5081
consistency index = 0.1270
consistency ratio = 0.1134   (See footnote below)

Composite priorities:
                     weight   priority

Fuel Econ   0.3243   (9)
Init.Cost   0.2614   (7)
Maint.Cost   0.1723   (4)
Comfort   0.1508   (3)
Status   0.0911   (1)

MODELS FACTORS RELATIVE TO SAT.ITEMS

      Factors relative to Comfort:
                  weight

    Model 1   0.5396
    Model 2   0.2970
    Model 3   0.1634

        lambda (maximum) = 3.0092
        consistency index = 0.0046
        consistency ratio = 0.0079

    Factors relative to Fuel Econ:
                  weight

    Model 1   0.1634
    Model 2   0.2970
    Model 3   0.5396

        lambda (maximum) = 3.0092
        consistency index = 0.0046
        consistency ratio = 0.0079

    Factors relative to Maint.Cost:
                  weight

    Model 1   0.3874
    Model 2   0.4434
    Model 3   0.1692

        lambda (maximum) = 3.0183
        consistency index = 0.0091
        consistency ratio = 0.0158

Factors relative to Init.Cost:
                        weight

Model 1   0.1634
Model 2   0.2970
Model 3   0.5396

        lambda (maximum) = 3.0092
        consistency index = 0.0046
        consistency ratio = 0.0079

Factors relative to Status:
                        weight

Model 1   0.5396
Model 2   0.2970
Model 3   0.1634

        lambda (maximum) = 3.0092
        consistency index = 0.0046
        consistency ratio = 0.0079

Composite priorities:
                        weight   priority

Model 3   0.3848   (9)
Model 2   0.3222   (4)
Model 1   0.2930   (1)

Factors relative to Comfort:
                        weight   priority

Model 1   0.0814   (4)
Model 2   0.0448   (2)
Model 3   0.0246   (1)

Factors relative to Fuel Econ:
                        weight   priority

Model 1   0.0530   (3)
Model 2   0.0963   (5)
Model 3   0.1750   (9)

Factors relative to Maint.Cost:
                        weight   priority

Model 1   0.0667   (4)
Model 2   0.0764   (4)
Model 3   0.0292   (2)

Factors relative to Init.Cost:
                        weight   priority

Model 1   0.0427   (2)
Model 2   0.0776   (4)
Model 3   0.1411   (7)

Factors relative to Status:
                        weight   priority

Model 1   0.0492   (3)
Model 2   0.0271   (2)
Model 3   0.0149   (1)

CONSISTENCY OF THE HIERARCHY = 0.0779

Footnote: The consistency limit has exceeded 10%.
           A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

7. ERROR MESSAGES

"Active item with a zero ranking. . ." AHP5.1 has attempted calculations with a priority matrix containing zeros. Replace the zeroes in the matrix with pairwise rankings and recalculate the problem.

"Bad Array Value. . ." AHP5.1 has attempted calculations with a priority matrix containing zeroes. Along with this message will be the name of the matrix and the address at which the zero(s) were found. Replace the zeros with pairwise rankings and rerun the problem.

"Disk drive not ready." The drive door is open or no floppy in drive. Insert floppy disk and/or close the door and press "r" to retry or "q" to return to the main menu.

"Disk full, cannot save." AHP5.1 is trying to write a file to a disk that has insufficient disk space. Pressing any key will return you to the main menu. Insert a fresh disk and try again. Give a full path name to save a file somewhere other than the default directory.

"Division by Zero." A matrix has probably been defined with dimension zero. This error message is to assist the code development.

"Filename extension limited to 3 characters." The extension part of the filename is too long. Reenter the filename with the proper extension.

"Filename limited to 8 character root and 3 character extension." The root part of the filename is longer than 8 characters and/or the extension exceeds 3 characters. Reenter a correct filename.

"Highlighted foreground and background colors the same. Correct and restart program." The highlighted foreground color is the same as the background color so that highlights will appear invisible. Reissue the command option for background or highlight color and restart the program.

"Input or restart file with this name." Occurs when trying to save an output print file under the same name as the restart or input file that was used to open this session. AHP5.1 will not overwrite it. Enter a different filename. AHP5.1 will overwrite any file, except the input file that was loaded to begin this session, if it has the same name as the output file given by the user.

"Invalid file name." An illegal file name was entered. Reenter a legal filename (maximum of 8 characters plus an extension with a maximum of 3 characters).

"Matrix comparisons do not exist. Must input or load them first." Attempted to edit a nonexistent hierarchy.

"No match found for <item or matrix name>" An item within a priority matrix does not appear in the master list, or, a matrix title does not appear in the master list one level above. This usually occurs when creating an input file with an ASCII text editor as a result of a typing error.

"Out of memory. Cannot Proceed." For some reason, there is not enough memory available for the hierarchy. It could be a result of resident programs, or too large a hierarchy. Delete resident programs or reduce the hierarchy size.

"Past end of file. More data expected." One or more priority matrices are missing. This usually happens when input is created with an ASCII text editor. More items appear in the master list of items than there are matrices in the level below that which contains the master list.

"Print file with same name."   A print file has the same name as the user
attempted to save a restart file with. If allowed to continue, the print
file would be overwritten with the restart file.

"Regular foreground and background colors the same.   Correct and restart
program." Foreground color is the same as the background color so that
text will not be visible on the screen. Reissue the command options with
different background and foreground color numbers.

"Root part of filename limited to 8 characters." The root of the filename is
too long. Reenter the filename with the proper root.

"Subscript out of range." A reference to a matrix calls a nonexistent address.
This error message is to assist code maintenance.

"The file does not appear to be an AHP5.1 input file." Either the format of the
input file is wrong, or, the file is not an input file for AHP5.1. Refer
to the User's Manual for a description of the input file.

8. REFERENCES

1. T. L. Saaty, Decision Making for Leaders, Lifetime Learning Publications, 1982

2. G. E. Wilson, Statistically Based Uncertainty Analysis for Ranking of Component Importance in the Thermal-Hydraulic Safety Analysis of the Advanced Neutron Source Reactor, EGG-NE-10078 (Informal Report), E.G.& G. Idaho, Inc., January 1992.

APPENDIX A

MATRIX THEORY

MATRIX THEORY

This appendix describes the matrix theory that the AHP is built around. Arguments are presented without proof for the sake of brevity. However, references are given to support the arguments. AHP5.1 employs numerical algorithms that mimic the matrix algebra presented here; however, the algorithms themselves are not presented.

1. EVALUATING PRIORITY MATRICES

Priority matrices used throughout the AHP can be classified as non-negative, reciprocal, irreducible, and primitive. A matrix theory textbook, such as references [1] or [2], describe these terms. Basically, the characteristics of a priority matrix that cause it to fall into this classification are (1) the matrix is square, (2) every entry into the matrix is positive (Note: when using AHP5.1, a negative entry is merely a flag, not a value, and these entries are converted internally to the positive reciprocal of the negative number.), and (3) the lower left half of the matrix is the reciprocal of the upper right half of the matrix. An outline of the theory for this classification of matrices is presented here, without proof, to give the user a basic understanding of the numerical process. For rigorous proofs of theorems, consult a matrix theory textbook.

Let's start by examining priority matrix A, which we will consider to be perfectly consistent. Each entry in the matrix is a relative comparison of the absolute weight of one variable to the absolute weight of another variable, the absolute weight being the importance of the variable with respect to all the other variables. Thus, the matrix is filled with ratios as shown.

$$A = \begin{bmatrix} \dfrac{w_1}{w_1} & \dfrac{w_1}{w_2} & \dfrac{w_1}{w_3} & \cdot\cdot & \dfrac{w_1}{w_n} \\[2ex] \dfrac{w_2}{w_1} & \dfrac{w_2}{w_2} & \dfrac{w_2}{w_3} & \cdot\cdot & \dfrac{w_2}{w_n} \\[2ex] \dfrac{w_3}{w_1} & \dfrac{w_3}{w_2} & \dfrac{w_3}{w_3} & \cdot\cdot & \dfrac{w_3}{w_n} \\[2ex] \cdot & \cdot & \cdot & & \cdot \\[1ex] \cdot & \cdot & \cdot\cdot & & \cdot \\[1ex] \dfrac{w_n}{w_1} & \dfrac{w_n}{w_2} & \dfrac{w_n}{w_3} & \cdot\cdot & \dfrac{w_n}{w_n} \end{bmatrix}$$

where the subscripts identify the item, and w is the absolute weight. Thus, w_1/w_2 indicates the relative importance of the first variable with respect to the second variable whereas w_1 is the absolute importance of the first variable with respect to the entire group, 1 through n. Likewise, w_2 is the absolute importance of the second variable with respect to the entire group. Now, define a weighting vector $\underline{w}$ that contains the absolute weights of variables 1 through n.

$$\underline{w} = \left[w_1,\ w_2,\ w_3,\ \ldots,\ w_n \right]$$

Multiplying A by $\underline{w}$ yields the new vector

$$A\underline{w} = \left[nw_1,\ nw_2,\ nw_3,\ \ldots,\ nw_n \right]$$

or,

$$A\underline{w} = n\underline{w}$$

which leads to,

$$\left(A - n\,I \right) \underline{w} = 0$$

where I is the identity matrix. In this form n is the eigenvalue and $\underline{w}$ is the eigenvector of the matrix A. Thus, finding the absolute weights is reduced to a problem of finding the eigenvector as determined by the eigenvalue n.

2. EVALUATING CONSISTENCY

So far, we have assumed matrix A to be perfectly consistent. Hence, the eigenvalue of A is n, or the order of the square matrix (a property of a non-negative, reciprocal, irreducible, primitive matrix). However, matrix A is unlikely to be perfectly consistent since decision makers are not likely to make perfectly consistent judgements as they fill in the matrix. In addition, the scale may be too coarse to allow perfectly consistent entries in the matrix. The arguments for consistent matrices presented in the first section of this appendix can be expanded to account for inconsistency. To do so, replace n by λ_{max}, $\underline{w}$ by $\underline{w}'$, and A by A'; the primes referring to variables associated with inconsistent matrices. Thus, we arrive at an equation similar to the previous equation but for an inconsistent matrix.

$$\left(A' - \lambda_{max} I\right) \underline{w}' = 0$$

Matrix A' is still positive and square. As such, the Perron-Frobenius theorem states that the eigenvalue will be real and positive, as will the eigenvector. In addition, since matrix A' is positive and irreducible, the Wielandt theorem states that the eigenvalue λ_{max} will increase as any element a_{ij} increases. Since matrix A' is also reciprocal, an increase in a_{ij} results in a decrease in a_{ji}, but the net effect is that λ_{max} is always larger than n for any inconsistent priority matrix. This suggests that there must be some way to measure consistency by comparing λ_{max} to n.

2.1 The Consistency Ratio

Several methods may exist to determine, quantitatively, the inconsistency of a priority matrix. One method, recommended by A. A. Girgis, et al. [3] - the one AHP5.1 uses - compares the consistency of the priority matrix to that of a random matrix. Consider the inconsistent matrix A'. Through matrix algebra we can find the eigenvalues. (There will be more than one eigenvalue since matrix A' is not perfectly consistent). The solution for the eigenvalues reduces

to finding the roots of an nth order polynomial. The largest root, λ_{max}, can be compared to n for a consistency index (CI) defined as

$$CI = \left[\frac{\lambda_{max} - n}{n - 1}\right]$$

The consistency index for a perfectly consistent matrix would be 0.0, because λ_{max} would equal n, and would increase as the inconsistency increases. However, since the consistency index depends on the size, or order, of the matrix, it is difficult to compare the consistency of matrices of different orders. That is, if the consistency index of a 3x3 matrix is the same as the consistency index of a 4x4 matrix, it does not mean that the two have the same level of inconsistency. Since it is desirable to compare the inconsistency of matrices of different sizes, it is necessary to derive a scale that is not sensitive to the size of the matrices. As such, we define a random index (RI) as the consistency index of a random matrix. Next, we define the consistency ratio (CR) as

$$CR = \frac{CI}{RI} = \left(\frac{\lambda_{max} - n}{\lambda_{random} - n}\right)$$

as the comparison of the consistency of matrix A' to the consistency of a random matrix of the same order. The random index has been tabulated for matrices of several different sizes in following Table A-1. The random matrix has the same characteristics as a priority matrix, except the rankings in the upper right half of the matrix are random, each entry on the diagonal is one, and the lower left half of the matrix is the reciprocal of the upper right half of the matrix.

TABLE A-1. RANDOM INDEXES

n	random index
1	0.00
2	0.00
3	0.58
4	0.90
5	1.12
6	1.24
7	1.32
8	1.41
9	1.45
10	1.49
11+	1.50

A consistency ratio less than one indicates more consistency than that of the random matrix, whereas a consistency ratio greater than one indicates less consistency than the random matrix. Saaty [4] suggests that a consistency ratio less than 0.10 is an acceptable level of inconsistency.

As such, by calculating the maximum eigenvalue of the priority matrix and comparing it to the eigenvalue of a random matrix of the same order, we can quantify the consistency of the matrix relative to a standard random matrix.

2.2 The Effect of the Scale

The scale used to create the priority matrices, because it is discrete, also contributes to inconsistency. Consider the matrix

$$P = \begin{bmatrix} 1 & 2 & 5 \\ & 1 & 3 \\ & & 1 \end{bmatrix} = \begin{bmatrix} \dfrac{w_1}{w_1} & \dfrac{w_1}{w_2} & \dfrac{w_1}{w_3} \\[2ex] \dfrac{w_2}{w_1} & \dfrac{w_2}{w_2} & \dfrac{w_2}{w_3} \\[2ex] \dfrac{w_3}{w_1} & \dfrac{w_3}{w_2} & \dfrac{w_3}{w_3} \end{bmatrix}$$

In order to specify each entry in this matrix, three decisions must be made; leading to three equations with two unknowns. If the extra equation violates the knowns, a level of inconsistency exists in the matrix. The following equations are based on portions of the above matrix.

$$\frac{w_1}{w_2} = 2$$

$$\frac{w_1}{w_3} = 5$$

$$\frac{w_2}{w_3} = \left(\frac{w_1}{w_3}\right)\left(\frac{w_2}{w_1}\right) = 5/2 = 2.5$$

These equations demonstrate that the pairwise comparison between item 2 and 3 could be found using w_1/w_2 and w_2/w_3. The result indicates that w_2/w_3 is equal to 2.5, not to the matrix entry of 3. Since the scale consists of integers from one to five, an entry of 2.5 is not allowed. Therefore, all the inconsistency in this matrix can be attributed to the scale.

For this matrix, the consistency index is 0.00185 and the consistency ratio is 0.00319. These are very small; but nevertheless, scale does contribute to inconsistency. Note that the total number of decisions needed to complete a priority matrix is $\frac{1}{2}(n^2 - n)$. However, all the information necessary to complete the entire matrix is available in any one row. Thus, if only $(n - 1)$ decisions are made, the equations represented in any row can be used to determine any other element in the matrix.

3. VARIATIONS OF THE AHP

The AHP as developed by Saaty requires that every matrix in a level have the same list of items; hence every matrix is of the same order, and every item is considered in every matrix. For some applications, such as Code Scaling and Applicability, this is impractical because not all items relate to each level matrix. So instead of considering each item to be in every matrix, items that do not contribute to the decision will be omitted from that matrix. This reduces the size of the individual matrices considerably and makes them easier to evaluate; however, this introduces some artificial bias. Because the size of the matrices can be different, items that appear in a large matrix will receive less priority because the priority is shared amongst many items. Similarly, items that appear in a small matrix receive too much priority. To alleviate this problem, AHP5.1 has implemented the Dimenna normalization scheme. See Section 3.9 for more information about the Dimenna normalization scheme.

4. REFERENCES

1. F. R. Gantmacher, The Theory of Matrices, Chelsea Publishing Co., New York, New York, 1960

2. P. M. Lancaster and M. Tismenetsky, The Theory of Matrices, Academic Press, New York, 1980

3. A. A. Girgis et al. Development of Rank Ordering Techniques Based on the Analytical Hierarchy Process for Evaluation of Energy Conversion Devices, International Journal of Energy Research, Vol. 13, 1989.

4. T. L. Saaty, Decision Making for Leaders, Lifetime Learning Publications, Belmont CA, 1982.

APPENDIX B

QUALITY CHECKING

B-2

QUALITY CHECKING

1. TEST PROBLEMS

Appendix B contains output from AHP5.1 for several examples given in Saaty's book <u>Decision Making For Leaders</u> published by Wadsworth, Inc. in 1982. AHP5.1 is Quality Level C, and the ability of AHP5.1 to compute hierarchies correctly has only been verified by running example problems from Saaty's book and comparing the results to Saaty's solutions. The results from the AHP5.1 code are considered acceptable if items are prioritized in the same order as Saaty's examples. That is, if listed in order of decreasing priority, AHP5.1's list of items is in the same order as Saaty's list. Output from AHP5.1 for each test case is attached. Note that all output, except where noted, has been created using the command line option -os to select Saaty's normalization scheme, see Section 3.9.

AHP5.1's results compare very favorably with Saaty's results. Most differences can be attributed to round off error. However, in one case, the comparison on page 86 of Saaty's book, a typographical error was found in the book. When corrected, the results compare well. The first validation problem was run using Saaty's normalization scheme and then rerun using Dimenna's normalization scheme. The results for the normalized scale were slightly different between the two runs. However; the results for the discrete scale remained the same between the two runs.

Additional validation of the software, and evaluation of the accuracy of the AHP methodology has been performed in an application typical to nuclear reactor thermal-hydraulic analysis. This information is reported in Reference [2] of the main body of this report.

Based on the comparisons contained in this Appendix, we conclude that AHP5.1 is capable of calculating hierarchies as presented by Saaty. In addition,

Saaty-type hierarchy (hierarchies in which all matrices on any given level have the same items).

1.1 Page 103, Determining Consumer Preference

Two runs were made, one using Saaty's normalization scheme, and one using Dimenna's normalization scheme. Compare page 7 of the two outputs. The priorities calculated using Saaty's scheme compare exactly with the priorities given in Saaty's book. Dimenna's normalization scheme resulted in somewhat different weights being calculated; however, the priorities on a scale of one to nine remained unchanged.

```
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@      Analytical Hierarchy Process - output file      @@@@
@@@@                  Program version 5.1                 @@@@
@@@@                                                       @@@@
@@@@        AHP sample problem - Saaty page 103            @@@@
@@@@                                                       @@@@
@@@@      date: 04-13-1992          time:  4:53:17 am      @@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
```

             HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Product/Product data arrays

Desirability   1. 2. 3. 4. 5. 6.

      Softness   1.   1 -4 -5 -4   5 -6
      Absorptiveness 2.   1 -3   3   6 -2
      Price   3.   1   4   7   3
      Size   4.   1   5 -5
      Design   5.   1 -7
      Integrity   6.   1

Product/Product subcat data arrays

Softness   1. 2. 3.

      H-Soft   1.   1   5   8
      M-Soft   2.   1   5
      L-Soft   3.   1

Absorptiveness   1. 2. 3.

      H-Absor   1.   1   7   9
      M-Absor   2.   1   7
      L-Absor   3.   1

Price   1. 2. 3.

    H-Price   1.   1 -7 -9
    M-Price   2.   1 -7
    L-Price   3.   1

Size   1. 2. 3.

    H-Size   1.   1   3   5
    M-Size   2.   1   4
    L-Size   3.   1

Design   1. 2. 3.

    H-Design   1.   1 -5   2
    M-Design   2.   1   5
    L-Design   3.   1

Integrity   1. 2. 3.

    H-Integ   1.   1   7   9
    M-Integ   2.   1   7
    L-Integ   3.   1

Product subcat/Product names data arrays

H-Soft   1. 2. 3.

    X   1.   1   5   7
    Y   2.   1   5
    Z   3.   1

H-Absor   1. 2. 3.

    X   1.   1   2   7
    Y   2.   1   8
    Z   3.   1

L-Price		1.	2.	3.
X	1.	1	-4	-7
Y	2.		1	-5
Z	3.			1

H-Size		1.	2.	3.
X	1.	1	2	1
Y	2.		1	1
Z	3.			1

M-Design		1.	2.	3.
X	1.	1	2	1
Y	2.		1	3
Z	3.			1

H-Integ		1.	2.	3.
X	1.	1	4	6
Y	2.		1	4
Z	3.			1

PRODUCT FACTORS RELATIVE TO PRODUCT

Factors relative to Desirability:

	weight
Softness	0.0570
Absorptiveness	0.1679
Price	0.3837
Size	0.1002
Design	0.0269
Integrity	0.2643

$$\text{lambda (maximum)} = 6.6563$$
$$\text{consistency index} = 0.1313$$
$$\text{consistency ratio} = 0.1059 \quad \text{(See footnote below)}$$

Composite priorities:

	weight	priority
Price	0.3837	(9)
Integrity	0.2643	(6)
Absorptiveness	0.1679	(4)
Size	0.1002	(3)
Softness	0.0570	(2)
Design	0.0269	(1)

PRODUCT SUBCAT FACTORS RELATIVE TO PRODUCT

Factors relative to Softness:
                    weight

H-Soft   0.7257
M-Soft   0.2122
L-Soft   0.0621

        lambda (maximum) = 3.1460
        consistency index = 0.0730
        consistency ratio = 0.1259   (See footnote below)

Factors relative to Absorptiveness:
                    weight

H-Absor   0.7608
M-Absor   0.1912
L-Absor   0.0480

        lambda (maximum) = 3.3276
        consistency index = 0.1638
        consistency ratio = 0.2825   (See footnote below)

Factors relative to Price:
                    weight

H-Price   0.0480
M-Price   0.1912
L-Price   0.7608

        lambda (maximum) = 3.3276
        consistency index = 0.1638
        consistency ratio = 0.2825   (See footnote below)

Factors relative to Size:
                       weight

H-Size   0.6267
M-Size   0.2797
L-Size   0.0936

     lambda (maximum) = 3.0858
     consistency index = 0.0429
     consistency ratio = 0.0739

Factors relative to Design:
                       weight

H-Design   0.1786
M-Design   0.7089
L-Design   0.1125

     lambda (maximum) = 3.0536
     consistency index = 0.0268
     consistency ratio = 0:0462

Factors relative to Integrity:
                       weight

H-Integ   0.7608
M-Integ   0.1912
L-Integ   0.0480

     lambda (maximum) = 3.3276
     consistency index = 0.1638
     consistency ratio = 0.2825   (See footnote below)

Composite priorities:

	weight	priority
L-Price	0.2919	(9)
H-Integ	0.2011	(6)
H-Absor	0.1278	(4)
M-Price	0.0734	(3)
H-Size	0.0628	(3)
M-Integ	0.0505	(2)
H-Soft	0.0413	(2)
M-Absor	0.0321	(2)
M-Size	0.0280	(2)
M-Design	0.0190	(1)
H-Price	0.0184	(1)
L-Integ	0.0127	(1)
M-Soft	0.0121	(1)
L-Size	0.0094	(1)
L-Absor	0.0081	(1)
H-Design	0.0048	(1)
L-Soft	0.0035	(1)
L-Design	0.0030	(1)

Factors relative to Softness:

	weight	priority
H-Soft	0.0413	(2)
M-Soft	0.0121	(1)
L-Soft	0.0035	(1)

Factors relative to Absorptiveness:

	weight	priority
H-Absor	0.1278	(4)
M-Absor	0.0321	(2)
L-Absor	0.0081	(1)

Factors relative to Price:

	weight	priority
H-Price	0.0184	(1)
M-Price	0.0734	(3)
L-Price	0.2919	(9)

Factors relative to Size:

	weight	priority
H-Size	0.0628	(3)
M-Size	0.0280	(2)
L-Size	0.0094	(1)

Factors relative to Design:

	weight	priority
H-Design	0.0048	(1)
M-Design	0.0190	(1)
L-Design	0.0030	(1)

Factors relative to Integrity:

	weight	priority
H-Integ	0.2011	(6)
M-Integ	0.0505	(2)
L-Integ	0.0127	(1)

PRODUCT NAMES FACTORS RELATIVE TO PRODUCT SUBCAT

Factors relative to H-Soft:
                    weight

X   0.7147
Y   0.2185
Z   0.0668

        lambda (maximum) = 3.1828
        consistency index = 0.0914
        consistency ratio = 0.1576   (See footnote below)

Factors relative to H-Absor:
                    weight

X   0.5659
Y   0.3727
Z   0.0614

        lambda (maximum) = 3.0764
        consistency index = 0.0382
        consistency ratio = 0.0659

Factors relative to L-Price:
                    weight

X   0.0727
Y   0.2050
Z   0.7223

        lambda (maximum) = 3.1237
        consistency index = 0.0619
        consistency ratio = 0.1066   (See footnote below)

Factors relative to H-Size:
                  weight

X   0.4126
Y   0.2599
Z   0.3275

     lambda (maximum)   = 3.0536
     consistency index = 0.0268
     consistency ratio = 0.0462

Factors relative to M-Design:
                  weight

X   0.4067
Y   0.3695
Z   0.2238

     lambda (maximum)   = 3.3674
     consistency index = 0.1837
     consistency ratio = 0.3168   (See footnote below)

Factors relative to H-Integ:
                  weight

X   0.6817
Y   0.2363
Z   0.0819

     lambda (maximum)   = 3.1078
     consistency index = 0.0539
     consistency ratio = 0.0930

Composite priorities:
                  weight   priority

X   0.3949   (9)
Z   0.3532   (7)
Y   0.2519   (1)

Factors relative to H-Soft:

	weight	priority
X	0.0397	(2)
Y	0.0121	(1)
Z	0.0037	(1)

Factors relative to H-Absor:

	weight	priority
X	0.0972	(4)
Y	0.0640	(3)
Z	0.0105	(1)

Factors relative to L-Price:

	weight	priority
X	0.0285	(2)
Y	0.0804	(3)
Z	0.2834	(9)

Factors relative to H-Size:

	weight	priority
X	0.0348	(2)
Y	0.0219	(2)
Z	0.0277	(2)

Factors relative to M-Design:

	weight	priority
X	0.0104	(1)
Y	0.0095	(1)
Z	0.0057	(1)

Factors relative to H-Integ:

	weight	priority
X	0.1842	(6)
Y	0.0639	(3)
Z	0.0221	(2)

CONSISTENCY OF THE HIERARCHY = 0.1407   (See footnote below)

Footnote: The consistency limit has exceeded 10%.
            A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

```
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@@@@                  Program version 5.1                 @@@@
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```

          HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Product/Product data arrays

Desirability		1.	2.	3.	4.	5.	6.
Softness	1.	1	-4	-5	-4	5	-6
Absorptiveness	2.		1	-3	3	6	-2
Price	3.			1	4	7	3
Size	4.				1	5	-5
Design	5.					1	-7
Integrity	6.						1

Product/Product subcat data arrays

Softness		1.	2.	3.
H-Soft	1.	1	5	8
M-Soft	2.		1	5
L-Soft	3.			1

Absorptiveness		1.	2.	3.
H-Absor	1.	1	7	9
M-Absor	2.		1	7
L-Absor	3.			1

Price   1. 2. 3.

    H-Price   1.   1 -7 -9
    M-Price   2.   1 -7
    L-Price   3.   1

Size   1. 2. 3.

    H-Size   1.   1   3   5
    M-Size   2.   1   4
    L-Size   3.   1

Design   . 1. 2. 3.

    H-Design   1.   1 -5   2
    M-Design   2.   1   5
    L-Design   3.   1

Integrity   1. 2. 3.

    H-Integ   1.   1   7   9
    M-Integ   2.   1   7
    L-Integ   3.   1

Product subcat/Product names data arrays

H-Soft   1. 2. 3.

    X   1.   1   5   7
    Y   2.   1   5
    Z   3.   1

H-Absor   1. 2. 3.

    X   1.   1   2   7
    Y   2.   1   8
    Z   3.   1

L-Price   1. 2. 3.

        X   1. 1 -4 -7
        Y   2.   1 -5
        Z   3.   1

_____

H-Size   1. 2. 3.

        X   1. 1   2   1
        Y   2.   1   1
        Z   3.   1

_____

M-Design   .   1. 2. 3.

        X   1. 1   2   1
        Y   2.   1   3
        Z   3.   1

_____

H-Integ   1. 2. 3.

        X   1. 1   4   6
        Y   2.   1   4
        Z   3.   1

_____

PRODUCT FACTORS RELATIVE TO PRODUCT

Factors relative to Desirability:
                         weight

Softness	0.1484
Absorptiveness	0.4376
Price	1.0000
Size	0.2612
Design	0.0700
Integrity	0.6887

lambda (maximum) = 6.6563
consistency index = 0.1313
consistency ratio = 0.1059   (See footnote below)

Composite priorities:
                    weight   priority

	weight	priority
Price	0.3837	(9)
Integrity	0.2643	(6)
Absorptiveness	0.1679	(4)
Size	0.1002	(3)
Softness	0.0570	(2)
Design	0.0269	(1)

PRODUCT SUBCAT FACTORS RELATIVE TO PRODUCT

Factors relative to Softness:
                        weight

H-Soft   1.0000
M-Soft   0.2924
L-Soft   0.0855

        lambda (maximum) = 3.1460
        consistency index = 0.0730
        consistency ratio = 0.1259   (See footnote below)

Factors relative to Absorptiveness:
                        weight

H-Absor   1.0000
M-Absor   0.2513
L-Absor   0.0632

        lambda (maximum) = 3.3276
        consistency index = 0.1638
        consistency ratio = 0.2825   (See footnote below)

Factors relative to Price:
                        weight

H-Price   0.0632
M-Price   0.2513
L-Price   1.0000

        lambda (maximum) = 3.3276
        consistency index = 0.1638
        consistency ratio = 0.2825   (See footnote below)

Factors relative to Size:
                    weight

H-Size   1.0000
M-Size   0.4463
L-Size   0.1494

        lambda (maximum) = 3.0858
        consistency index = 0.0429
        consistency ratio = 0.0739

Factors relative to Design:
                    weight

H-Design   0.2520
M-Design   1.0000
L-Design   0.1587

        lambda (maximum) = 3.0536
        consistency index = 0.0268
        consistency ratio = 0.0462

Factors relative to Integrity:
                    weight

H-Integ   1.0000
M-Integ   0.2513
L-Integ   0.0632

        lambda (maximum) = 3.3276
        consistency index = 0.1638
        consistency ratio = 0.2825   (See footnote below)

Composite priorities:
```
                weight       priority

L-Price         0.2845       (9)
H-Integ         0.1959       (6)
H-Absor         0.1245       (4)
H-Size          0.0743       (3)
M-Price         0.0715       (3)
M-Integ         0.0492       (2)
H-Soft          0.0422       (2)
M-Size          0.0332       (2)
M-Absor         0.0313       (2)
M-Design        0.0199       (1)
H-Price         0.0180       (1)
L-Integ         0.0124       (1)
M-Soft          0.0123       (1)
L-Size          0.0111       (1)
L-Absor         0.0079       (1)
H-Design        0.0050       (1)
L-Soft          0.0036       (1)
L-Design        0.0032       (1)
```

Factors relative to Softness:
```
                weight       priority

H-Soft          0.0422       (2)
M-Soft          0.0123       (1)
L-Soft          0.0036       (1)
```

Factors relative to Absorptiveness:
```
                weight       priority

H-Absor         0.1245       (4)
M-Absor         0.0313       (2)
L-Absor         0.0079       (1)
```

Factors relative to Price:
```
                weight       priority

H-Price         0.0180       (1)
M-Price         0.0715       (3)
L-Price         0.2845       (9)
```

Factors relative to Size:

	weight	priority
H-Size	0.0743	(3)
M-Size	0.0332	(2)
L-Size	0.0111	(1)

Factors relative to Design:

	weight	priority
H-Design	0.0050	(1)
M-Design	0.0199	(1)
L-Design	0.0032	(1)

Factors relative to Integrity:

	weight	priority
H-Integ	0.1959	(6)
M-Integ	0.0492	(2)
L-Integ	0.0124	(1)

PRODUCT NAMES FACTORS RELATIVE TO PRODUCT SUBCAT

Factors relative to H-Soft:
                       weight

X   1.0000
Y   0.3057
Z   0.0935

    lambda (maximum) = 3.1828
    consistency index = 0.0914
    consistency ratio = 0.1576   (See footnote below)

Factors relative to H-Absor:
                       weight

X   1.0000
Y   0.6586
Z   0.1084

    lambda (maximum) = 3.0764
    consistency index = 0.0382
    consistency ratio = 0.0659

Factors relative to L-Price:
                       weight

X   0.1007
Y   0.2838
Z   1.0000

    lambda (maximum) = 3.1237
    consistency index = 0.0619
    consistency ratio = 0.1066   (See footnote below)

Factors relative to H-Size:
                   weight

X   1.0000
Y   0.6300
Z   0.7937

     lambda (maximum) = 3.0536
     consistency index = 0.0268
     consistency ratio = 0.0462

Factors relative to M-Design:
                   weight

X   1.0000
Y   0.9086
Z   0.5503

     lambda (maximum) = 3.3674
     consistency index = 0.1837
     consistency ratio = 0.3168   (See footnote below)

Factors relative to H-Integ:
                   weight

X   1.0000
Y   0.3467
Z   0.1202

     lambda (maximum) = 3.1078
     consistency index = 0.0539
     consistency ratio = 0.0930

Composite priorities:
                   weight   priority

X   0.4082   (9)
Z   0.3324   (5)
Y   0.2594   (1)

Factors relative to H-Soft:

	weight	priority
X	0.0355	(2)
Y	0.0109	(1)
Z	0.0033	(1)

Factors relative to H-Absor:

	weight	priority
X	0.1047	(4)
Y	0.0689	(3)
Z	0.0114	(1)

Factors relative to L-Price:

	weight	priority
X	0.0241	(2)
Y	0.0679	(3)
Z	0.2392	(9)

Factors relative to H-Size:

	weight	priority
X	0.0625	(3)
Y	0.0394	(2)
Z	0.0496	(3)

Factors relative to M-Design:

	weight	priority
X	0.0167	(1)
Y	0.0152	(1)
Z	0.0092	(1)

Factors relative to H-Integ:

	weight	priority
X	0.1647	(6)
Y	0.0571	(3)
Z	0.0198	(2)

CONSISTENCY OF THE HIERARCHY = 0.1407   (See footnote below)

Footnote: The consistency limit has exceeded 10%.
           A review of the input assumptions may be necessary.

***** Above results produced using the Dimenna normalization

1.2 Page 108, Estimating the Economy's Impact on Sales

On page 3 of the output, in the level "Direction Factors Relative to Economy", the weight of recession differed in the fourth decimal place when compared with the results presented in Saaty's book (0.0666 for Saaty, 0.0667 for AHP5.1). Likewise, on page 4 of the output, in the level "Sales Factors Relative to Direction", the weight for the "0-5" percent sales relative to "Energy Crisis" differed in the fourth decimal place (0.0518 for Saaty, 0.0519 for AHP5.1). Composite priorities on page 4 of the output differed in the fourth decimal place. The order in which the items appear in AHP5.1's priority list remained the same as Saaty's list; therefore, the results from AHP5.1 are acceptable.

```
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@@@@            saaty test problem, page 108               @@@@
@@@@                                                       @@@@
@@@@      date: 04-13-1992           time:  4:52:56 am     @@@@
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HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Economy/Direction data arrays

Future Sales

		1.	2.	3.
Energy crisis	1.	1	7	1
Recession	2.		1	-7
Inflation	3.			1

Direction/Sales data arrays

Energy crisis

		1.	2.	3.	4.
0-5	1.	1	-5	-7	-5
5-10	2.		1	-3	-4
10-15	3.			1	-3
15-20	4.				1

Recession

		1.	2.	3.	4.
0-5	1.	1	2	5	7
5-10	2.		1	3	5
10-15	3.			1	3
15-20	4.				1

Inflation		1.	2.	3.	4.
0-5	1.	1	2	5	7
5-10	2.		1	3	5
10-15	3.			1	3
15-20	4.				1

DIRECTION FACTORS RELATIVE T∩ ECONOMY

Factors relative to Future Sales:
                          weight

Energy crisis   0.4667
Recession   0.0667
Inflation   0.4667

       lambda (maximum) = 3.0000
       consistency index = 0.0000
       consistency ratio = 0.0000

Composite priorities:
                    weight   priority

Energy crisis   0.4667   (9)
Inflation   0.4667   (9)
Recession   0.0667   (1)

SALES FACTORS RELATIVE TO DIRECTION

Factors relative to Energy crisis:
                        weight

0-5   0.0519
5-10   0.1451
10-15   0.2904
15-20   0.5127

        lambda (maximum) = 4.3372
        consistency index = 0.1124
        consistency ratio = 0.1249   (See footnote below)

Factors relative to Recession:
                        weight

0-5   0.5232
5-10   0.2976
10-15   0.1222
15-20   0.0570

        lambda (maximum) = 4.0685
        consistency index = 0.0228
        consistency ratio = 0.0254

Factors relative to Inflation:
                        weight

0-5   0.5232
5-10   0.2976
10-15   0.1222
15-20   0.0570

        lambda (maximum) = 4.0685
        consistency index = 0.0228
        consistency ratio = 0.0254

Composite priorities:

	weight	priority
0-5	0.3032	(9)
15-20	0.2697	(6)
5-10	0.2264	(3)
10-15	0.2007	(1)

Factors relative to Energy crisis:

	weight	priority
0-5	0.0242	(2)
5-10	0.0677	(3)
10-15	0.1355	(5)
15-20	0.2392	(9)

Factors relative to Recession:

	weight	priority
0-5	0.0349	(2)
5-10	0.0198	(2)
10-15	0.0081	(1)
15-20	0.0038	(1)

Factors relative to Inflation:

	weight	priority
0-5	0.2441	(9)
5-10	0.1389	(5)
10-15	0.0570	(3)
15-20	0.0266	(2)

CONSISTENCY OF THE HIERARCHY = 0.0437

Footnote: The consistency limit has exceeded 10%.
           A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

1.3 Page 86, Choosing Among Three Job Offers

Note that on page 87 of Saaty's book there is a typographical error. The pairwise ranking for location versus reputation should be 1/3, not 1 as shown. Reputation versus location is shown as 3 and verifies that there was indeed an error. If the AHP5.1 input is corrected accordingly, AHP5.1's output, when rounded to two decimal places as is Saaty's book, is exactly the same.

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@@@@      Analytical Hierarchy Process - output file      @@@@
@@@@                Program version 5.1                   @@@@
@@@@                                                       @@@@
@@@@            saaty test problem, page 86                @@@@
@@@@                                                       @@@@
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HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Focus/Criteria data arrays

Job satisfactn

Job satisfactn		1.	2.	3.	4.	5.	6.
Research	1.	1	1	1	4	1	-2
Growth	2.		1	2	4	1	-2
Benefit	3.			1	5	3	-2
Colleagues	4.				1	-3	-3
Location	5.					1	-3
Reputation	6.						1

Criteria/Jobs data arrays

Research

Research		1.	2.	3.
a	1.	1	-4	-2
b	2.		1	3
c	3.			1

Growth

Growth		1.	2.	3.
a	1.	1	-4	-5
b	2.		1	-2
c	3.			1

Benefit		1.	2.	3.
a	1.	1	3	-3
b	2.		1	1
c	3.			1

Colleagues		1.	2.	3.
a	1.	1	-3	5
b	2.		1	7
c	3.			1

Location		1.	2.	3.
a	1.	1	1	7
b	2.		1	7
c	3.			1

Reputation		1.	2.	3.
a	1.	1	7	9
b	2.		1	5
c	3.			1

CRITERIA FACTORS RELATIVE TO FOCUS

Factors relative to Job satisfactn:
                      weight

Research   0.1574
Growth   0.1871
Benefit   0.1889
Colleagues   0.0492
Location   0.1221
Reputation   0.2953

        lambda (maximum) = 6.3516
        consistency index = 0.0703
        consistency ratio = 0.0567

Composite priorities:
                      weight   priority

Reputation   0.2953   (9)
Benefit   0.1889   (6)
Growth   0.1871   (5)
Research   0.1574   (5)
Location   0.1221   (3)
Colleagues   0.0492   (1)

JOBS FACTORS RELATIVE TO CRITERIA

Factors relative to Research:
                    weight

a   0.1365
b   0.6250
c   0.2385

        lambda (maximum) = 3.0183
        consistency index = 0.0091
        consistency ratio = 0.0158

Factors relative to Growth:
                    weight

a   0.0974
b   0.3331
c   0.5695

        lambda (maximum) = 3.0246
        consistency index = 0.0123
        consistency ratio = 0.0212

Factors relative to Benefit:
                    weight

a   0.3189
b   0.2211
c   0.4600

        lambda (maximum) = 3.5608
        consistency index = 0.2804
        consistency ratio = 0.4835   (See footnote below)

Factors relative to Colleagues:
                 weight

a   0.2790
b   0.6491
c   0.0719

λ (maximum) = 3.0649
consistency index = 0.0324
consistency ratio = 0.0559

Factors relative to Location:
                 weight

a   0.4667
b   0.4667
c   0.0667

λ (maximum) = 3.0000
consistency index = 0.0000
consistency ratio = 0.0000

Factors relative to Reputation:
                 weight

a   0.7720
b   0.1734
c   0.0545

λ (maximum) = 3.2085
consistency index = 0.1042
consistency ratio = 0.1797   (See footnote below)

Composite priorities:
                 weight   priority

a   0.3986   (9)
b   0.3426   (6)
c   0.2588   (1)

Factors relative to Research:

	weight	priority
a	0.0215	(2)
b	0.0984	(4)
c	0.0375	(2)

Factors relative to Growth:

	weight	priority
a	0.0182	(2)
b	0.0623	(3)
c	0.1066	(5)

Factors relative to Benefit:

	weight	priority
a	0.0602	(3)
b	0.0418	(2)
c	0.0869	(4)

Factors relative to Colleagues:

	weight	priority
a	0.0137	(1)
b	0.0319	(2)
c	0.0035	(1)

Factors relative to Location:

	weight	priority
a	0.0570	(3)
b	0.0570	(3)
c	0.0081	(1)

Factors relative to Reputation:

	weight	priority
a	0.2280	(9)
b	0.0512	(3)
c	0.0161	(1)

CONSISTENCY OF THE HIERARCHY = 0.0876

Footnote: The consistency limit has exceeded 10%.
           A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

1.4 Page 98, Analyzing the Hostage Rescue Operation

On page 5 of the output, the third decimal place causes both composite weights to differ when rounded to two decimal places to match Saaty's results. This could be due to round off error on Saaty's part since his results are presented to only two significant figures. Nevertheless, the results are similar enough to assume that AHP5.1 is working correctly.

```
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
@@@@      Analytical Hierarchy Process - output file      @@@@
@@@@                 Program version 5.1                  @@@@
@@@@                                                      @@@@
@@@@           saaty test problem - page 98               @@@@
@@@@                        -                             @@@@
@@@@      date: 04-13-1992          time:  4:53:41 am     @@@@
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```

HIERARCHY RELATIONSHIPS AND FACTOR RANKS
(a negative rank indicates a reciprocal; e.g., -3 implies 1/3)

Success/Priorities data arrays

Medium   1. 2. 3. 4.

     Hostages lives   1.   1 -3   5 -3
     Political life   2.   1   7   4
     Military costs   3.   1 -6
     US prestige   4.   1

Priorities/Do it? data arrays

Hostages lives   1. 2.

     Go   1.   1   1
     No-Go   2.   1

Political life   1. 2.

     Go   1.   1   3
     No-Go   2.   1

Military costs   1. 2.

     Go   1.   1 -7
     No-Go   2.   1

B-45

US prestige   1. 2.

    Go   1. 1   4
    No-Go ~   2.   1

PRIORITIES FACTORS RELATIVE TO SUCCESS

Factors relative to Medium:
                        weight

Hostages lives   0.1498
Political life   0.5452
Military costs   0.0456
US prestige   0.2594

        lambda (maximum) = 4.3128
        consistency index = 0.1043
        consistency ratio = 0.1158   (See footnote below)

Composite priorities:
                    weight   priority

Political life   0.5452   (9)
US prestige   0.2594   (4)
Hostages lives   0.1498   (3)
Military costs   0.0456   (1)

DO IT? FACTORS RELATIVE TO PRIORITIES

Factors relative to Hostages lives:
                    weight

Go   0.5000
No-Go   0.5000

        lambda (maximum) = 2.0000
        consistency index = 0.0000
        consistency ratio = 0.0000

Factors relative to Political life:
                    weight

Go   0.7500
No-Go   0.2500

        lambda (maximum) = 2.0000
        consistency index = 0.0000
        consistency ratio = 0.0000

Factors relative to Military costs:
                    weight

Go   0.1250
No-Go   0.8750

        lambda (maximum) = 2.0000
        consistency index = 0.0000
        consistency ratio = 0.0000

Factors relative to US prestige:
                    weight

Go   0.8000
No-Go   0.2000

        lambda (maximum) = 2.0000
        consistency index = 0.0000
        consistency ratio = 0.0000

Composite priorities:
                         weight   priority

Go   0.6970   (9)
No-Go   0.3030   (1)

Factors relative to Hostages lives:
                         weight   priority

Go   0.0749   (2)
No-Go   0.0749   (2)

Factors relative to Political life:
                         weight   priority

Go   0.4089   (9)
No-Go   0.1363   (4)

Factors relative to Military costs:
                         weight   priority

Go   0.0057   (1)
No-Go   0.0399   (2)

Factors relative to US prestige:
                         weight   priority

Go   0.2075   (5)
No-Go   0.0519   (2)

CONSISTENCY OF THE HIERARCHY = 0.1158   (See footnote below)

Footnote: The consistency limit has exceeded 10%.
            A review of the input assumptions may be necessary.

***** Above results produced using the Saaty normalization

END

DATE
FILMED
5/27/93