

## AIRCRAFT CONTROL LAW RECONFIGURATION

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**Abstract.** The synthesis of MIMO fault-tolerant control systems under control failures is considered. The solution is written in an analytical form, which makes it possible to synthesize simplified circuit implementations and adjust coefficients of the fault-tolerant control system in the case of a non-stationary object. A nonlinear medium-range aircraft model was used to verify the results.

**Keywords:** control systems, aircraft, air incidents, reconfiguration algorithms.

### 1. Introduction

Aircraft control failures are critical since their occurrence leads to aircraft air incidents (Medvedev 2013; Trifonov-Bogdanov *et al.* 2013; Kulčák *et al.* 2000). Control failures can be caused by failure of actuators, damage

of communication lines between actuators and actuator control electronics (ACE), etc. Despite of the causes control failures become evident through their balancing or arbitrary position jamming.

To maintain stability and controllability during such failures redundant actuators, ACE's and communication channels are used. However, multiple redundancies are not feasible for all types of aircraft. In addition, there are off-design situations, such as a fire in the area of control surface, in which redundancy is not effective.

This paper considers aircraft stability and controllability by preserving the reconfiguration control algorithm in the event of control failure. The reconfiguration in this case refers to the redistribution of control action from the damaged surface onto the remaining operable control surfaces in order to maintain similar flight parameters before and after failure. The functional redundancy of the controls forms the basis of the reconfiguration. Aircraft stability and controllability preservation is achieved by the additional off-design deviation of the operable control surfaces (Hajiyev, Caliskan 2004; Dolega 2005; Dolega, Rzucidło 2007).

The algorithm is based on an analytical procedure for the synthesis of multivariable system control laws developed by authors of (Glasov, Kosjanchuk 2010; Kosjanchuk, Zybin 2002; Kosjanchuk *et al.* 2001; Kosjanchuk, Bukov 2001; Kosjanchuk 2002; Misrikhanov *et al.* 2005). The essence of the algorithm is an analytical solution of a matrix equation (Bukov *et al.* 2002) connecting flight parameters, effectiveness of controls before and after the failure, and the special coefficients for control action redistribution to the operable control surfaces. The derived semi-analytical solution forms the aircraft control law with the reconfiguration.

The analytical solution allows considering all inner relations and tuning control law gains in the case of a non-stationary control object.

Finally, the paper presents the results of a simulation which confirms the efficiency and effectiveness of the proposed algorithms. The aim of this study is to apply the algorithm for two simultaneous failures and obtain the testing results of a non-linear model of a medium-range aircraft.

## 2. Problem statement and method of solution

### 2.1. Failure model

Let us extend the results to several simultaneous failures. The motion of a serviceable aircraft is described by a deterministic continuous system linearly depending on control:

$$\dot{x} = f(x, a, t) + B(x, a, t)u, \quad (1)$$

where  $x \in R^n, u \in R^m, f(x, a, t)$  is a vector function of vector arguments  $x$  and  $a$  and scalar  $t$ , and  $B(x, a, t)$  is the matrix of size  $n \times m$ .

Without loss of generality, a linear non-stationary model can be formulated as

$$\dot{x} = A(x, a, t)x + B(x, a, t)u, \quad (2)$$

where  $A(x, a, t)$  and  $B(x, a, t)$  are object and control matrices of sizes  $n \times n$  and  $n \times m$ , respectively. The dependences of elements of these matrices on aircraft parameters are assumed to be known. The transition from model (1) to (2) was performed in order to facilitate the comparison of our results with those of other researchers. In what follows, we prove that the results obtained are applicable to the model (1). In addition, without the loss of generality, we assume that, in the time intervals required for transient processes, matrices  $A(x, a, t)$  and  $B(x, a, t)$  can be replaced by constant matrices, and the results obtained can be extended to non-stationary objects.

The model of a serviceable object is expressed in the following form:

$$\dot{x} = Ax + Bu. \quad (3)$$

The object model after failure is expressed similarly as

$$\dot{x} = Ax_f + B_f u_f, \quad (4)$$

where matrix  $B_f$  takes into account actuator failure and  $u_f$  is the control ensuring solution of the problem of failure compensation (reconfiguration).

If the scope of control system failures by actuator failures as the most critical elements was limited, then:

$$B_f = B \cdot F, \quad (5)$$

where matrix  $F$  represents actuator failures. Matrix  $F$  can be expressed as a diagonal matrix with "one" on the main diagonal and "zero" on the failure surface place.

$$F = \text{diag} \left( 1 \dots \underset{i\text{-cell}}{0} \dots \underset{j\text{-cell}}{0} \dots 1 \dots 1 \right). \quad (6)$$

$F$  represents identity matrices in case of all actuator validity.

$u_f$  is obtained as the sum of control signal  $u$  formed by a conventional control system and compensation signal  $u_k$ ,

$$u_f = u + u_k. \quad (7)$$

We will require a linear dependence of the compensation control  $u_k$  on  $u$ :

$$u_k = Ru. \quad (8)$$

With regard to (8), control in (7) is formulated as:

$$u_f = u + Ru = (I + R)u. \quad (9)$$

Then, the model after failure (4) with regard to (9) takes the following form:

$$\dot{x}_f = Ax_f + B_f(I + R)u. \quad (10)$$

### 2.2. Reconfiguration model

Assuming that failure compensation begins at the moment when the failure is detected, which means

coincidence of initial conditions  $x(t_0) = x_f(t_0)$ , the invariability of the state vector  $x = x_f$  takes place if the following equation holds:

$$B_f(I + R)u = Bu. \quad (11)$$

Let us represent (11) as:

$$(B_f(I + R) - B)u = 0. \quad (12)$$

Clearly, in order for (12) to hold for all control actions, it is necessary that the expression in parenthesis is identically equal to zero:

$$B_f(I + R) - B = 0. \quad (13)$$

Let us reformulate (13) as a linear matrix equation

$$B_f R = B - B_f. \quad (14)$$

As a result, the reconfiguration problem is reduced to solving the matrix equation (14) in an unknown matrix  $R$ . The solution to this equation is found applying the analytical method for solving algebraic equations based on equivalent transformations (Bukov *et al.* 2002). Matrix  $R$  is calculated according to the formula:

$$R = \tilde{B}_f(B - B_f) + \bar{B}_f^R \mu, \quad (15)$$

under the condition that is the solvability condition.

$$\bar{B}_f^L B = 0 \quad (16)$$

holds, where  $\mu$  is an arbitrary matrix of an appropriate size;  $\bar{B}_f^L$  and  $\bar{B}_f^R$  are full-rank left and right zero divisor matrices satisfying the following equations:

$$\bar{B}_f^L B_f = 0, \dots, \bar{B}_f^R, \quad B_f = 0 \quad (17)$$

and  $\tilde{B}_f$  is a generalized inverse matrix satisfying the regularity conditions

$$\tilde{B}_f B_f \tilde{B}_f = \tilde{B}_f, \quad B_f \tilde{B}_f B_f = B_f.$$

Note that the solvability condition (16) follows from the equation below:

$$\bar{B}_f^L (B - B_f) = 0$$

with regard to (17).

Considering matrix:

$$K_p = I + R, \quad (18)$$

it by virtue of (2.15) takes the form of:

$$K_p = I + \tilde{B}_f(B - B_f) + \bar{B}_f^R \mu. \quad (19)$$

Matrix (19) is referred to as the reconfiguration matrix. Due to this matrix the controls are redistributed over serviceable control surfaces; i.e., matrix  $K_p$  relates input vector  $u$  to vector  $u_f$  of the signals that are directly sent to the actuators of the control surfaces in accordance with expression:

$$u_f = K_p u. \quad (20)$$

The calculation of the reconfiguration matrix by (19) ensures compensation of the failure consequences. The analytical representation (19) obtained with the use of only equivalent transformations of matrix  $B_f$  can be extended to models of form (1), where, for the coefficients of matrix functions, one may use symbolic variables, which may be written in different forms and consider the dependence of the coefficients from time  $t$ , states  $x$  and system parameters  $a$ :

$$K_p(x, a, t) = I + \overline{B_f(x, a, t)} (B(x, a, t) - B_f(x, a, t)) + \overline{B_f(x, a, t)}^R \mu. \quad (21)$$

Note that the vector function  $f(x, a, t)$  does not take part in the calculations. The solution above allows us synthesising simplified schemes of implementation of the proposed algorithm and represent reconfiguration as a classical structural scheme.

### 3. Comparative analysis of the results obtained

#### 3.1. Non-linear model of medium-range aircraft linearization

For the reconfiguration algorithm synthesis a medium-range aircraft model was designed. The aircraft is designed as a swept low wing monoplane scheme. The wing is mechanized by single-slotted slats and flaps. The tail unit has a vertical and a horizontal stabilizer. The aerodynamic controls for pitch are controlled stabilizer and two separate elevators, for the roll – ailerons and spoilers on the wing, and a rudder for the yaw.

The linearized system of differential equations (1) obtained from the full system of nonlinear differential equations of motion of the aircraft considered as a solid body was used to study the reconfiguration algorithms. Using the technique of separation of motion on the perturbed and unperturbed, a matrix representation of longitudinal and lateral movement of the aircraft was obtained:

$$\begin{bmatrix} \Delta \dot{V} \\ \Delta \dot{\alpha} \\ \Delta \dot{\theta} \\ \Delta \dot{\omega}_z \\ \Delta \dot{\beta} \\ \Delta \dot{\omega}_x \\ \Delta \dot{\omega}_y \\ \Delta \dot{\gamma} \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{18} \\ \vdots & \ddots & \vdots \\ a_{81} & \dots & a_{88} \end{bmatrix} \times \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta \theta \\ \Delta \omega_z \\ \Delta \beta \\ \Delta \omega_x \\ \Delta \omega_y \\ \Delta \gamma \end{bmatrix} + \begin{bmatrix} b_{11} & \dots & b_{18} \\ \vdots & \ddots & \vdots \\ b_{81} & \dots & b_{88} \end{bmatrix} \times \begin{bmatrix} \delta_{elvL} \\ \delta_{elvR} \\ \varphi \\ \delta_{rud} \\ \delta_{ailL} \\ \delta_{ailR} \\ \delta_{intL} \\ \delta_{intR} \end{bmatrix}, \quad (22)$$

where  $\omega_x, \omega_y, \omega_z$  are the body roll, yaw and pitch rates;  $\alpha, \theta, \gamma, \beta$  – the angle of attack, flight path angle, roll and drag angle;  $V$  – air speed;  $\delta_{elvL}, \delta_{elvR}$  – left and right elevator angle;  $\delta_{ailL}, \delta_{ailR}$  – left and right aileron angle;  $\delta_{intL}, \delta_{intR}$  – left and right interceptor angle;  $\varphi$  – stabilizer angle,  $\delta_{rud}$  – rudder angle.

Using the aircraft aerodynamic characteristics bank and Taylor linearization method from (Lebedev, Chernobrovkin 1973), all coefficients and their derivatives were calculated for the initial conditions of a uniform rectilinear flight at altitude  $H = 3000$  meters, with airspeed  $V = 135$  m/sec.

Linear matrix (22) coefficients are taken as next values for these conditions:

$$A = \begin{bmatrix} -0.0172 & -10.96 & -9.81 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.0015 & -1.529 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0015 & 1.529 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.0005 & -2.191 & 0.0 & -0.401 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0221 & 0.055 & 0.997 & 0.0724 \\ 0.0 & 0.0 & 0.0 & 0.0 & -5.687 & -2.231 & -0.894 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -7.594 & 0.0418 & -1.535 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1 & -0.055 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0317 & 0.0317 \\ -0.0026 & -0.0026 & 0.0 & 0.0 & -0.0013 & -0.0013 & -0.0032 & -0.0032 \\ 0.0026 & 0.0026 & 0.0 & 0.0 & 0.0013 & 0.0013 & 0.0032 & 0.0032 \\ -1.04 & -1.04 & -2.887 & 0.0 & -0.1317 & -0.1317 & 0.1253 & 0.1253 \\ 0.0 & 0.0 & 0.0 & -0.0016 & 0.0 & 0.0 & -0.0001 & 0.0001 \\ 0.593 & -0.593 & 0.0 & -0.572 & 1.2 & -1.2 & -1.841 & 1.841 \\ -0.0266 & 0.0266 & 0.0 & -7.102 & -0.0352 & 0.0352 & 0.0171 & -0.171 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

### 3.1. Left and right elevator failure

Below are the results of the simulation of the left and right elevator failure. The disturbance signal is the pilot side stick deviation of 1/3 of full range during a 6 sec in pitch channel. The reconfiguration matrix has the following implementation for these conditions:

$$K_p = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.2657 & 0.2654 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0013 & -0.0018 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.1501 & 0.9250 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.9099 & 1.1399 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0870 & -0.0897 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \\ -0.0870 & 0.0897 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

Figures 1–5 presents the results of the simulation.

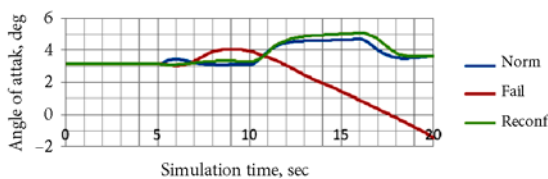


Fig. 1.  $\alpha$  – normal simulation, L/R elevator failure simulation, failure with reconfiguration simulation

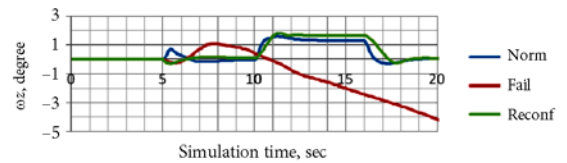


Fig. 2.  $w_z$  – normal simulation, L/R elevator failure simulation, failure with reconfiguration simulation

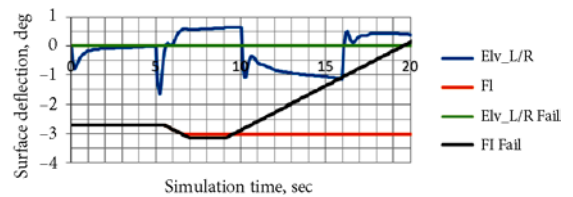


Fig. 3. Elevator and stabilizer deflection – normal simulation and L/R elevator failure simulation

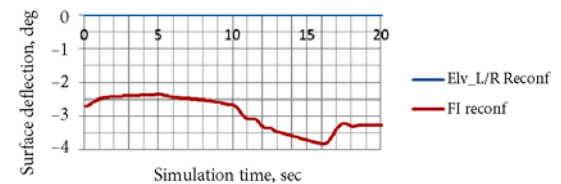


Fig. 4. Elevator and stabilizer deflection – L/R elevator failure with reconfiguration simulation

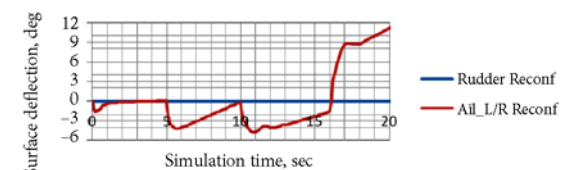


Fig. 5. Rudder and L/R aileron deflection – L/R elevator failure with reconfiguration simulation

As illustrated by the plots, without compensation of consequences of the failure, considerable deviations in the longitudinal channel appear. The use of the reconfiguration allows us to compensate consequences of the failure (plots corresponding to the motion without failures closely related with those with the failure and reconfiguration). Deviations of the controls are depicted in Figures 3–5. It can be seen that the compensation of consequences of the failure is achieved due to deflections of the ailerons and stabilizer. Results of the modeling prove that control reconfiguration in the case of actuator failures results in complete compensation of the failure consequences.

#### 4. Conclusion

In conclusion, it has been found that the use of the reconfiguration algorithms ensures fault-tolerance of control systems in the case of actuator failures and, as a result, improves flight safety and reduces the number of accidents.

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