Aircraft Trajectory Optimization with Tactical Constraints

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TRITA-AVE.2004:11
ISBN 91-7283-727-6

Typsatt i LaTeX.

TRITA-AVE.2004:11
ISSN 1651-7660
ISRN KTH/AVE/DA–04:11–SE
ISBN 91-7283-727-6

Akademisk avhandling som med tillstånd av Kungliga Tekniska Högskolan fredagen den 11 juni 2004 kl. 10, Salongen, KTH Biblioteket, Osquars backe 31, Stockholm, för teknisk doktorsgrads vinnande framlägges till offentlig granskning av Martin Norsell.

Preface

The work in this doctoral thesis started at the end of the previous millennium at the Department of Aeronautics at the Royal Institute of Technology and was finished at the new Department of Aeronautical and Vehicle Engineering in May 2004. The project has been financially supported by the Swedish Defence Materiel Administration (FMV), monitored by Curt Eidefeldt, Staffan Lundin, Lars Falk, Björn Jonsson, Bertil Brännström and Martin Näsman.

I would like to express my sincere thanks to my supervisor Professor Ulf Ringertz for proposing this study and the invaluable guidance, enthusiasm and support he has provided throughout the work. He is one of the few people I know accomplishing the most remarkable tasks so I look forward to seeing him win the Swedish Soaring Championship.

I would also like to thank the rest of the people at the Department *Far & Flyg*, especially the guys in the Division of Flight Dynamics both past and present. A more diversified and creative 'bunch' of people is not possible to find. There are too many names to mention here, but you know who you are and what you all mean to me!

Furthermore, I appreciate the interaction with the RCS technical group Sigma flyg and other individuals active in this field. In particular, I want to thank Daniel Amann, H-O Berlin, Jakob Bjerkemo, Curt Eidefeldt, Kerstin Fredriksson, Mats Henningsson, Anders Höök, Christer Larsson, Uno Lidvall, P-E Ljung, Staffan Lundin, Tomas Lundin, Jan Melin, J-O Olsson, Jonas Rahm, Jan Rexander and Erik Söderström for sharing their expertise.

I express my deepest gratitude to my family and friends for being so supportive. They say that behind every great man there is a great woman. Well, I don't claim to be a great man, but I certainly have an incredible woman by my side in Madelené. Thank you for always putting up with me and providing an endless amount of support, love and encouragement. Without you this work had simply not been possible.

Stockholm in May 2004

Martin Norsell

Abstract

Aircraft trajectory optimization is traditionally used for minimizing fuel consumption or time when going from one flight state to another. This thesis presents a possible approach to incorporate tactical constraints in aircraft trajectory optimization.

The stealth technology of today focuses on making the tactics already in use more effective. Since tactics and stealth are closely interrelated, new and better results may be obtained if both aspects are considered simultaneously. Simply reducing the radar cross section area in some directions without considering tactical aspects may result in little, if any, improvement.

Flight tests have been performed in cooperation with Ericsson Microwave Systems and the Swedish Air Force Flight Academy. The aircraft used was the subsonic jet trainer Saab 105, designated SK60 by the Swedish Air Force. The results show a decrease of 40% in the time interval between the instant the aircraft was first detected until it could pass above the radar station. This corresponds to a reduced radar cross section (RCS) in the direction from the aircraft to the radar of almost 90%, if classical RCS reduction techniques would have been applied.

If a modern aircraft with stealth properties would be used, the proposed methodology is believed to increase the possible improvements further. This is because the variation of the magnitude of RCS in different directions is greater for a shape optimized aircraft, which is the property exploited by the developed method.

The methods presented are indeed an approach utilizing the ideas of the network centric warfare (NCW) concept. The methodology presented depends on accurate information about the adversary, while also providing up-to-date information to the other users in the information network.

The thesis focuses on aircraft but the methods are general and may be adapted for missiles, ships or land vehicles. The proposed methods are also economically viable since they are useful for existing platforms without costly modifications. The methods presented are not limited to radar threats only. The reasons for using radar in this thesis are the available non-classified data and that radar is known to pose a major threat against aircraft.

Dissertation

This doctoral thesis is based on a short introduction to the area of research and the following appended papers:

Paper A

M. Norsell. Aircraft trajectories considering radar range constraints. *Aerospace Science & Technology,* 6:83-89, 2002.

Paper B

M. Norsell. Flight testing radar detection of the Saab 105 in level flight. *AIAA Journal of Aircraft*, 39(5):894-897, 2002.

Paper C

M. Norsell. Radar cross section constraints in flight-path optimization. Presented at the *41st AIAA Aerospace Sciences Meeting*, Reno, Nevada, January 2003. Published in *AIAA Journal of Aircraft*, 40(2):412-415, 2003 in abbreviated form.

Paper D

M. Norsell. Multistage trajectory optimization with radar range constraints. To appear in *AIAA Journal of Aircraft,* 2004.

Contents

Appended papers:

Introduction

To avoid or delay detection as a strategy for survival is ancient knowledge used long before there were humans on earth. It is also a strategy practiced by both predators and prey. It is interesting to note that to avoid detection it is necessary to have low contrast against the background no matter if detected by the eye, the ears, or by using more modern methods such as radar, infra-red (IR), laser etc.

In a real world setting, scenarios involving aircraft and external threats are very complex. The detection range depends on both the azimuth and elevation angle toward the radar station(s). A generic example is shown in Figure 1. The detection range is here represented by a two-dimensional graph for clarity, although it is three-dimensional in reality.

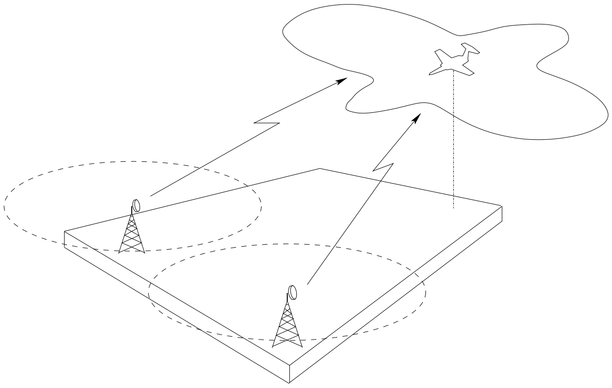

Figure 1: Simplified illustration of detection range and external threats.

In Figure 2, a generic two-dimensional model of the detection range around an aircraft is shown. The circle corresponds to a reference detection range at which the aircraft would be equally easy to detect in all directions. The aircraft is observed from three different directions A, B and C. If the aircraft is observed from direction A, the detection range is 50 % greater than that of the reference circle. However, if the aircraft is observed from direction B or C, the detection range is less than for the comparable reference case. Basically, this thesis is about finding such differences in detection range and using them for tactical advantages.

The methodology in this thesis focuses on aircraft applications considering radar threats but it is not limited hereto. The method can be adapted for missiles and other flying vehicles and also for ships, land

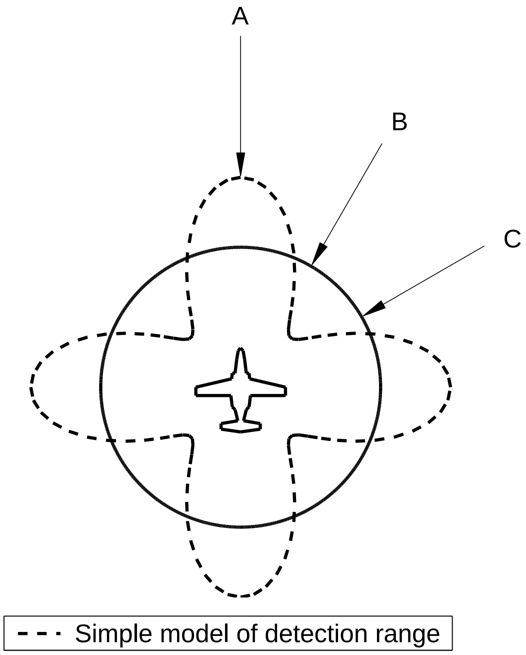

Figure 2: A generic model of radar range in different directions.

vehicles etc. The use of radar range constraints is mainly due to the difficulties to acquire good, open source information for other sensors such as infra-red (IR). Radar is the sensor that traditionally has received most military interest [1].

An illustration of the tactical advantages of using information about the radar range constraints is shown in Figure 3. Two identical aircraft flying straight and level, approach two identical radar stations at a given altitude. The detection ranges for each aircraft are included in the Figure. AC 1 is heading directly toward the radar station. AC 2 uses knowledge about its detection range properties, and the distance when detected by the radar station can hence be significantly reduced. This is a simplified example in two dimensions only, in Paper B a more thorough example is presented and compared to flight tests. A more realistic RCS model is used, and an example showing an aircraft approaching a radar station at different altitudes is also presented.

Aircraft trajectory optimization is traditionally focused on minimizing the time or fuel consumption when going from one flight state to another [2]. Although these considerations are as old as flying itself, the

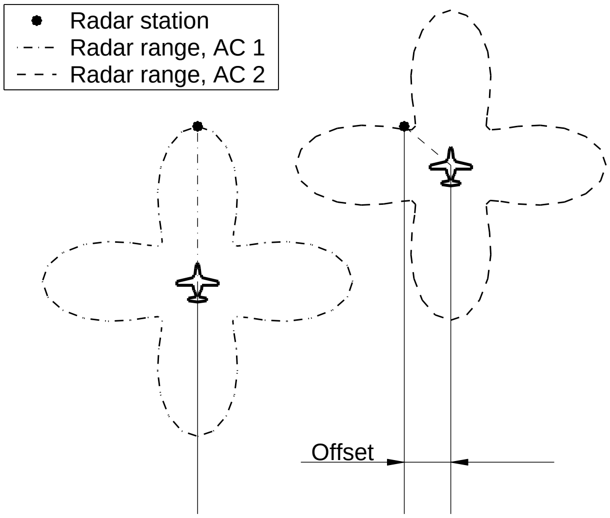

Figure 3: Two identical aircraft approach identical radar stations, with significant difference in detection distances.

concern was significantly increased when the jet-engine came into use. Jet-engines are known to have high fuel consumption, especially when using afterburner [3]. Many of the studies in this field are restricted to two-dimensional models, considering either the vertical or the horizontal plane. The more complicated case involving combined missile and aircraft performance has been investigated by e.g. Järmark [4].

The Department of Aeronautical and Vehicle Engineering at KTH has been involved in trajectory optimization for quite some time. Flight tests have also shown the methods to work in practice [5, 6, 7, 8]. It is important to remember that there is no substitute for flight testing when new methods aiming at being used in practice are developed.

This thesis focuses on the use of aircraft trajectory optimization considering external threats, whereas the stealth technology of today focuses on making the tactics already in use more efficient. Since both tactics and stealth are closely inter-related, new and better results may be obtained if both aspects are considered simultaneously.

The work in this thesis assumes that the location of the threats are known. This makes the proposed methods both dependent on and a part of the Network Centric Warfare concept. The detection limit is modeled as a precise limit excluding the statistics of the threat and atmospheric conditions. Hence, robustness considerations need to be accounted for when determining this limit.

Radar detection

The predecessor to radar (RAdio Detection And Ranging), was patented in 1904 [9], but did not become operative until the 1930's. The military advantages became increasingly important during World War II (WW II). The definitions of the various frequency bands [10] and their use are shown in Table 1. These band designations were developed during WW II and have become standard use. Generally, low frequency radars are used for long-range surveillance, but they are not very effective for accurate determination of aircraft position.

For search and tracking when finer resolution is needed the S-, C-, X- and K_u-bands are used. In these bands, it is possible to obtain narrow beam-widths with much smaller antennas than with the low-frequency radars [10]. However, at higher frequencies the range decreases and the signals are more prone to be affected by atmospheric attenuation and weather.

Band designation	Frequency range, GHz	General usage
HF	0.003-0.03	Over-the-horizon surveillance
VHF	0.03-0.3	Very long-range surveillance
UHF	0.3-1	Very long-range surveillance
L	1-2	Long-range surveillance, en route air traffic control
S	2-4	Medium-range surveillance, terminal traffic control
C	4-8	Long-range tracking
X	8-12	Short-range tracking, missile guidance, airborne intercept
K_u	12-18	High-resolution mapping
K	18-27	High-resolution mapping
K_a	27-40	Very high-resolution mapping
Millimeter	40-300	Very high-resolution mapping

Table 1: Radar frequency bands and their use.

The radar range equation

This section does not aim at being an extensive guide to the radar range equation, but some basic issues will be discussed. For a more thorough analysis see e.g. [11, 12, 13, 14, 15]. If the radar parameters necessary for detection are known, the radar equation can be separated in one part dependent on the aircraft radar cross section (RCS) denoted σ, and one radar dependent part denoted $\hat{\nu}$ resulting in [10, 16]

$$R^4 = \hat{\nu}\sigma, \tag{1}$$

where R denotes the range from the radar to the target. This is the simplified formulation of the radar range equation used in Papers A through D. Although (1) may appear simple, $\hat{\nu}$ models radar properties such as transmitted power, antenna gain, atmospheric attenuation, signal processing etc. Furthermore, σ models, among other things, radar frequency, polarization and orientation of the aircraft.

Common interferences affecting radar detection range are jamming and clutter. There are different forms of jamming, which raises the level of the background noise against which the aircraft radar echo must be detected [16]. Effectiveness of jamming depends, among other issues, on the bandwidth compared to the receiver, the waveform and in which radar-lobe the jamming appears [14].

The radar equation can also be affected by clutter, a term used to denote unwanted radar echoes which originates from the natural environment [15]. Target detection range calculations in a background of clutter is very difficult due to uncertainties in reflectivity, range variation of the clutter power, uncertain statistical distribution and attenuation in the signal processing [14]. Hence, without going into detail, it can be found in for example [13, 15] that the detection range dependency of RCS changes to $R \propto \sqrt{\sigma}$ for volume clutter and $R \propto \sigma$ for area clutter, where $\propto$ denotes a proportional relationship.

The modeling of the radar parameters can be improved, but the described method in this thesis is believed to provide a good starting-point for refined methods. The potential gain using aircraft trajectory optimization increases since the difference between 'good' and 'bad' angular sectors is greater for cluttered or jammed environments. In Figure 2, direction A represents a 'bad' angular sector while direction B represents a 'good' angular sector. It is also interesting to note that discrete clutter sources such as birds and insects may present serious problems for radars trying to detect small targets at low altitude [14]. Birds normally move at the wind velocity ± 15 m/s and these echoes are filtered out by choosing a suitable minimum speed e.g. 25 m/s. This effect may be exploited further by building small and slow flying unmanned aerial vehicles (UAV) in the future.

This leads to another assumption not discussed above, the Doppler effect which is also excluded in (1). A wave radiated from a moving source is compressed in the direction of motion, spread out in the opposite direction, and unaffected in the direction normal to the motion [16]. The most commonly known example is the change in frequency when a police car with sirens passes by. If Doppler effects are modeled and the aircraft utilizes a low closing velocity relative to the radar station, the results of this thesis may be further improved.

Finally, from the fact that shape optimized aircraft, e.g. B-2, F-22 and similar, have a greater difference between good and bad angular sectors, the potential gain of using optimization increases. In summary, an old conventional aircraft in a free space environment, i.e. not too close to the horizon moving directly toward a radar station will show smaller improvements using trajectory optimization with tactical constraints. This compared to a modern shape optimized aircraft operating in a cluttered and jammed environment. Improvement of radar systems performance modeling may include everything from atmospheric attenuation to the physical conditions of the radar operator.

To model the aircraft specific σ in (1) there are two ways of obtaining useful data, measurements and calculations, which will be described in the following.

Measuring radar cross section

Both indoor and outdoor RCS measurement ranges exist. Indoor ranges generally give more accurate and repeatable measurements due to the controlled environment. The main advantages of the outdoor measurement ranges are the possibility to be in the far-field and also to measure large and heavy objects [17]. Large objects can however sometimes be measured indoor using so-called compact ranges. The background noise levels sets the limit of the minimum radar cross section possible to measure. An interesting comparison between indoor and outdoor facilities can be found in [18].

When radar cross section properties are measured, most properties scale intuitively right, e.g. wave-length, length and time scale linearly and permittivity and permeability do not scale [17]. However, the conductivity scales with the length factor, meaning that if a full size object has good conductivity the scaled down model must have even better conductivity. This is the main reason why perfect electric conductors (PEC), where the conductivity almost equals infinity, are frequently used for model measurements.

The best results are obtained measuring the real aircraft, an example of such a measurement is shown in Figure 4 where the Joint Strike Fighter (JSF) is mounted on a pylon. The purpose of the test shown is to test different antenna configurations both for antenna performance and the effect on the RCS [19]. Furthermore, several doors and panels are deliberately damaged and repaired to test possible effects on the RCS when used in rough environments.

Figure 4: The Joint Strike Fighter at the Helendale Measurement Facility, CA (courtesy of Lockheed Martin).

Calculating radar cross section

When calculating the RCS, the underlying Maxwell's partial differential equations [20] are discretized and sometimes simplified since analytical solutions exists only for very special cases. A technique developed before extensive computational power was available is presented by Steyskal [21]. This technique is claimed to work for low and high frequencies, but is less accurate in the resonance region, i.e. when the wave-length is of the same order of magnitude as the size of the object. The method is conceptually fairly simple. First, the object is divided into sub-objects, where each sub-object is chosen to resemble the part of the object as accurately as possible. The RCS of the sub-objects are then calculated individually. Finally, the total RCS is calculated either using both phase and amplitude information of each sub-object, coherent addition, or just using the amplitudes. Apertures are traditionally very complex to model. According to Steyskal, they can be estimated by $\sigma = 4A\cos^2\theta$, were A is the effective aperture area, and θ the angle between the normal to the aperture and the direction to the radar station [21].

The method is based on a certain level of physical insight, and there are three remarkable statements in the report. First, the accuracy compared to measurements was estimated for an aircraft in the S-band to be in the 3-5 dB range (factor 2-3). Secondly it was noted that the differences between individual aircraft in the same series may exceed the above accuracy. Finally, it was concluded that long straight lines, such as the leading edge can be excluded from the analysis since the lobe is narrower than 1 deg, which had been verified by flight tests [21].

Among the computational methods of today, the method of moments (MoM) is the method of choice for accurate calculations involving different frequency bands. Without going into detail, the major computational effort is spent solving a linear system of equations where a dense, complex but symmetric matrix is involved. The method is known to be cumbersome if the object considered is electrically large, i.e. the object is large compared to the wave-length. The method has shown progress lately due to the rapid development of computer power. Computational techniques are developed simultaneously, and an example is the method of panel clustering [22, 23]. Using supercomputers, the RCS for an aircraft in the K-band can now be calculated. However, to be useful for shape optimization involving thousands of function evaluations, substantial improvements are necessary.

Physical optics (PO) is a technique that works for electrically large objects. However, PO is less suitable when the size of the object is similar to the wave-length. The method considers the currents induced on the illuminated portions of the object only. Hence, it is a method which assumes the current on the edges to be zero, i.e. not accounting for the currents from one patch to another [24].

The above mentioned methods are both frequency domain methods. There exist also time-domain methods, in which the finite difference time-domain method (FDTD) is the most commonly used [17]. The method of FDTD is conceptually simple since the problem is discretized in grid points. Boundary conditions representing the illumination are calculated in these grid points and used as a starting point. Then, the differential operators in Maxwell's equations are approximated with finite differences and the fields at the grid points are calculated at discrete time-steps. This is a very versatile method since the grid is easily adapted to complex geometries with different material parameters, i.e. electromagnetic pulse problems, ducts and similar. Furthermore, arbitrary material properties can be assigned to each grid point. However, the time steps needed for accurate results may be prohibitively small, demanding extensive computational effort.

Finally, it should be mentioned that a lot of research of today focuses on so-called hybrid methods using the different methods where appropriate, e.g. MoM for surface geometries, FDTD for cavities and PO for electrically large sub-objects.

In Papers B, C and D the calculated RCS of the Saab 105, designated SK60 by the Swedish Air Force, was used. The RCS calculations were performed by Ericsson Microwave Systems (EMW) using Epsilon$^{\mathrm{TM}}$ [25] which is a program based on physical optics which uses spline elements from the preprocessor PATRAN$^{\mathrm{TM}}$. The model used for the calculations is shown in Figure 5.

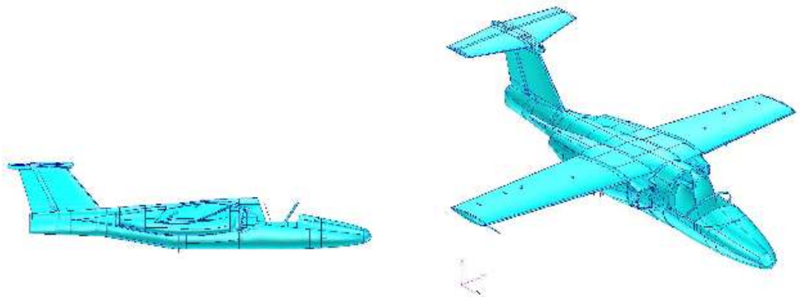

Figure 5: The CAD model used for calculating RCS of the Saab 105 (courtesy of Ericsson Microwave Systems).

During the flight test described in Paper B, some differences in detection range were apparent, but it is not possible to say if they are due to the modeling of the radar station performance, the attitude angles of the aircraft or inaccuracies in the calculated RCS. Hence, it would be very valuable and interesting to compare the calculated RCS with measurements of the Saab 105.

Stealth

According to a standard English Dictionary [26]:

If you use **stealth** *when you do something, you do it in such a slow, quiet and secretive way that other people do not notice what you are doing.*

Sometimes stealth is referred to as the ability to be invisible. That is very different compared to the above definition, since it is important to be invisible against the background or environment where the stealth vehicle operates. This may sound obvious, but it is very important to remember when a vehicle is designed to operate in different environments. An aircraft usually operates in the sky and when observed by ground based radar, the background can be estimated to zero. However, if the aircraft operates close to the ground or sea, it may be in a cluttered surrounding. This is also the reason why ships and tanks designed for stealth share a common design philosophy since they operate in environments with background and the main objective is to minimize the contrast to this background.

Similar methodology as described here may be applied in the acoustic, IR and visual frequency bands. The radar threat was chosen since it is the main threat in the sense that aircraft are almost exclusively first detected by radar although the final control, e.g. of missiles, can be performed by some other means, i.e. IR.

There are mainly two ways to achieve radar cross section reduction, namely shaping and using lossy materials [27]. Knott and Jenn [11, 17] add active and passive cancellation to this list.

The first radar reduction technique using radar absorbing material (RAM) was used by the German Navy on periscopes during World War II. In essence RAM is a resin with embedded carbon or ferrite particles [3]. The incoming energy is bounced around inside the material while transforming the energy into heat. In Figure 2 the use of RAM corresponds to decreasing the distance of detection.

However, lossy materials are traditionally heavy and bulky and not suitable for use in aircraft structures [11]. Therefore, RAM is best used on land and sea vehicles were the weight penalty is less cumbersome. For aircraft purposes, RAM are primarily used where a lot of induced energy is collected and re-radiated and also to reduce the troublesome reflections in ducts such as the engine inlets. Development in the field of nano technology suggests that RAM may be lighter and more easily implemented in the future [28].

Active cancellation is also challenging. Measuring the frequency and angle of the incoming wave usually means pointing a resonant structure (antenna) in the direction of the threat and hence reveal your exact location. However, a lot of research today focuses on developing low RCS antennas [27, 29]. Another emerging field of research is maintainability considerations for RAM and RCS reduction techniques [3, 19].

One of the first aircraft using shaping to achieve low RCS was the F-117A and its, then secret, predecessor Have Blue developed by the Skunk Works. The shaping technique was based on the principles of physical optics (PO), and hence the resulting shape was faceted e.g. F-117A in Figure 6. Aircraft developed later have less facets and a smoother shape to avoid edge diffraction [17], e.g. B-2 in Figure 6. The great care of alignment can be studied where the edges of the B-2 are long straight and aligned in a few principal directions. This is to have very strong but also very narrow 'spikes' in the detection range in a few directions. That is also apparent when the inlets and tail of the F/A-22 and F-35A in Figure 7 are studied. Attention to detail is crucial and to achieve low RCS, the only viable option seems to be internal weapon bays.

Figure 6: The F-117 and the B-2 (courtesy of the Lockheed Martin and US Air Force, photo by Staff Sgt. Rose Reynolds).

It should be emphasized that to achieve really stealth-like performance it is very important to involve signature reduction techniques early in the design phase. It is important to note that also small design changes, e.g. adding aerodynamic devices such as strakes, wing-fences or new sensors - typically done late in the development to correct some problem, may cause large penalties in RCS.

Figure 7: The F/A-22 Raptor and the F-35A Joint Strike Fighter (courtesy of Lockheed Martin).

Optimization with tactical constraints

All optimization starts by defining a scalar termed the objective function, i.e. defining what is good and what is bad. In aircraft trajectory optimization, it may be obvious to minimize the fuel consumption if fuel is a limiting factor. If the objective is to reach a certain altitude as soon as possible, the corresponding objective function is to minimize the final time. However, sometimes it is difficult to find a proper objective function, this is discussed in Paper C and D. It is not obvious that minimizing the time an aircraft is detected by radar during a part of the flight trajectory results in what an experienced pilot calls a ’good’ trajectory for the rest of the flight.

Discretization

Aircraft motion can usually be described by a set of ordinary differential equations (ODE) of the form

$$\dot{x} = f(x, u), \tag{2}$$

where x is a vector of state variables e.g. velocity, altitude, position and u a vector of control variables e.g. thrust setting, flight path angle. Furthermore, additional constraints on e.g. load factor and dynamic pressure are implemented as algebraic constraints of the form

$$\underline{g} \leq g(x, u) \leq \overline{g} \tag{3}$$

where $\underline{g}$ and $\overline{g}$ denote lower and upper bounds on the algebraic constraints. Now, the problem is to determine u such that the best trajectory for a given mission is obtained without violating the constraints.

For solving nonlinear trajectory optimization problems, two classes of methods, namely indirect and direct methods exist. Indirect methods are based on calculus of variations [30]. The indirect methods are, in general, demanding to implement and require good knowledge about the properties of the problem considered [31].

When using a direct transcription method, the problem is discretized and transformed into one large optimization problem, including the control variables, which is then solved in its entirety. The state equations given by (2) are treated as non-linear equality constraints. This means that the state equations are only satisfied at the solution to the optimization problem. The method of direct trajectory optimization using collocation was first suggested by Hargraves and Paris [32] in 1987. This method has been further developed, see Shi *et al.* [33], Gill *et al.* [34] and Betts [35, 36].

Although many different approaches for solving trajectory optimization problems exist, only two will be discussed here. For a more comprehensive overview, see e.g. Betts [31, 37]. The first method can be described as a typical shooting method, where the calculation of the trajectory is performed by applying some standard time-stepping technique such as Runge-Kutta [38]. The other method is maybe less intuitive but not less effective and will be referred to as the Hermite-Simpson (HS) collocation method [39]. This is the method of choice in the appended Papers C and D.

In the shooting method the complete flight trajectory is calculated, or observed, for a given starting guess of the control variables. The control variables are then adjusted using optimization based on the outcome of the trajectory calculation. The shooting method is sufficient for simple and small problems. However, this method is slow for large problems since the whole system of ODEs has to be solved for each set of control variables. If gradient information is needed and finite differences are utilized, the complete flight trajectory has to be calculated for each perturbation [37].

In the HS collocation method, the flight trajectory is divided into segments where the state variables are represented by piecewise smooth functions e.g. cubic polynomials, and the control variables e.g. linear functions. The time is also partitioned into intervals. Furthermore, a vector is formed including the approximate state and control values at each node. Integrating across each segment using Simpson's quadrature rule can be shown to be equivalent to force the derivative at the midpoint of each segment to equal the derivatives of the interpolated midpoint values from the ODEs, see [39].

The vector y is formed, which contains the discretized state- and control-variables and also the final time. An objective function f can be formulated, if for example the flight time should be minimized, as simple as minimizing the appropriate element in y, representing the final time. An optimization problem can now be posed as

$$\begin{array}{ll} \min\limits_{y} & f(y) \\ \text{subject to} & \underline{l} \leq \begin{pmatrix} c(y) \\ Cy \\ y \end{pmatrix} \leq \overline{u}, \end{array} \tag{4}$$

where $c(y)$ denotes the discretized state and algebraic constraints, C defines linear constraints, $\underline{l}$ the lower bounds and $\overline{u}$ the upper bounds. Equality constraints are enforced by setting $\underline{l} = \overline{u}$. If additional constraints such as radar detection have to be considered, it can be observed that for the HS collocation method these constraints are relatively easy to implement as additional algebraic equations.

The HS collocation method may seem more complex to implement and it may be less intuitive than the shooting method. To find a suitable starting point for the HS collocation method can be difficult. Furthermore, if the optimization is interrupted since e.g. no optimal point is found, the latest iteration does not necessarily represent a physically viable aircraft trajectory. For the shooting method, the main drawback is that it is, in general, slower since the whole system of ODEs has to be solved at each function evaluation.

Optimization

In most cases, it is not possible to find the unique global optimum to a nonlinear optimization problem. When numerical optimization is applied, the purpose is to find a local optimum [30]. Another observation appropriate here is that it may not at all be possible to solve the optimization problem. If an aircraft is ready for take-off, and an adversary passes above - is it possible to catch up? When solving such involved problems it can be very difficult, if not impossible, to know if there exists a solution *a priori*.

When linear problems are considered an upper limit on the time it takes to find the solution can be calculated. This is in general, not possible for a non-linear problem [30], since there exists no method that guarantees to find the local optimum and hence, no possibility to estimate an accurate upper bound of the time it may take to find a solution. If the optimization fails or takes too long some fall-back strategy is necessary for practical implementations.

The optimization problem in (4) is solved using a non-linear optimization algorithm, preferably utilizing the sparsity pattern typical for the Jacobian of the constraints when aircraft trajectory optimization is considered. SNOPT by Gill *et al.* [40] was used for solving the large-scale nonlinear optimization problems in Paper B, C, and D. A lot of current research focuses on further development of optimization methods for nonlinear optimization, a promising field is interior methods, see Forsgren *et al.* [41] for an overview.

Modeling and results

The work by Ringertz [6, 7, 8] on computing a trajectory taking an aircraft from one state to another in minimum time or using minimum fuel has formed the basis for the methods presented in this thesis. The trajectories calculated by Ringertz were flight tested by the Swedish Air Force using the supersonic Saab J35 Draken [6] and the subsonic jet trainer Saab 105 [5] with good results.

It is shown in [10, 42] that as a strategy for survival, simply going to lower altitude is not nearly as effective as reducing the radar signature in the target direction. The trajectory optimization methods developed in Paper A through D have been adapted for taking radar threats into account utilizing the RCS properties of the aircraft.

Trajectory optimization for unmanned aerial vehicles (UAV), sometimes involving threat constraints, is often based on graph search methods and using simplistic models for the radar properties [43, 44, 45]. To incorporate threat avoidance in trajectory optimization as given by (1), and maintaining a reasonable computational effort, the RCS representation has to be continuous and differentiable.

In Paper A substantial decrease in detection time for level flight was experienced and this was verified in flight tests presented in Paper B. A more general formulation in three dimensions was developed using a continuous and differentiable B-spline [46] representation of the RCS and combined with a three dimensional performance model in Paper C. Similar work has been performed by Misovec *et al.* [47]. This method works well, numerically, for relatively short total flight distances, i.e. in the 50-100 km range. For large distances many time steps are needed to resolve the rigid body dynamics of the aircraft, which result in very large optimization problems. These obstacles can, at least partially, be overcome by using the multistage strategy presented in Paper D.

Trajectory optimization in network centric warfare

In the current globalization of society focus is changing from national defense toward international peace keeping operations [48]. The first reports mentioned revolutionary military affairs (RMA) which has since been exchanged for "transformation" [49, 50]. According to Moore [49], earlier revolutions in military affairs are described as the ability to take advantage of new technology. An historical example is the use of the longbows in the 15th Century battle between England and France, which gave an advantage for England.

In most literature considering network centric warfare (NCW), a massive change is expected, but much less is known or even discussed about what this change may be [51]. According to Stein [52], the concept of NCW is a derivative of network centric computing, where new communication and information processing technologies have made it possible to use different computers and operating systems, although the underlying architecture is different but the interface is almost identical to the user.

From the official report to the US congress [50]:

Network Centric Warfare (NCW) is no less than the embodiment of Information Age transformation of the DoD (ed. Department of Defense). It involves a new way of thinking about how we accomplish our missions, how we organize and interrelate, and how we acquire and field the system that supports us.

The intention of the NCW is simply to take advantage of this new tool (i.e. rapid information exchange). An example presented by Cebrowski [53], describes a soldier needing supporting fire from a tank placed in his vicinity. It takes about 20-30 minutes with the current structure before the commander of the tank gets the coordinates and permission to fire. In the NCW, the coordinates are supposedly transferred directly to the tank and when the permission to fire is given, the tank is ready. The goal in the context of NCW is to have supporting fire within a few minutes.

It becomes apparent that huge information flows will be available to almost everyone. This may pose challenges both to find only relevant information and to be sure that the information available is correct. According to [53] some military personnel are also concerned about how the chain of command will be affected by too much information.

It is always easy to point out weaknesses of a new methodology, hence some concerns will be presented in the following. The algorithms used to find useful information in the enormous data flows predicted will have to be well protected. If such information is made available to the adversary, it can be used to find tactics utilizing the weakness in the filtering of data processes.

Another delicate matter to resolve is if two sensors viewing the same object feed different information into the network. Which one should be trusted? The time aspect from the instant the information is available until the information is too old to be used, may be a matter of seconds which puts high demand on the decision making process. The described methodology is also expected to work well as long as the situational awareness exceeds that of the adversary. The methods for aircraft trajectory optimization presented in Papers C and D will put high demand on the availability of the network and the security issues.

An approach to deal with the enormous data flows is to use optimization. In order to pose problems possible to solve rapidly, multilevel optimization may be required. An example of research in this area is the Mixed Initiative Control of Automa-Teams (MICA) sponsored by DARPA [54].

Finally, aircraft trajectory optimization as presented in this thesis suits the concept of NCW very well. This thesis focuses on how to use tactics for a given RCS assuming the positions of the radar stations are known. It can be concluded that aircraft trajectory optimization is compatible with the NCW concept, since it is dependent on good intelligence and may provide good feedback to the other users of the information network.

Future work

The only viable way of testing the proposed methodology is to perform flight tests. Flight tests for some simple cases already performed show that the method works but more testing inevitably has to be performed prior to implementing in a practical setting. The flight tests would preferably be made in a controlled environment or at least approaching measurement radar dedicated to this purpose. Robustness is vital and the only possible way to evaluate the methodology is by testing, since radar detection is indeed a statistical measure.

If the results from the flight tests are too complex or the differences compared to the simulations are too large, a possible approach would be to extend the methodology to ship or land vehicles. This may be possible to perform in a more controlled environment, by first measuring the object using an outdoor measuring range and then perform the measurements in a real world setting. An interesting investigation would also be to calculate the dynamic RCS and compare to static RCS tests.

A lot of the current research focuses on UAVs only. To the author's mind this may be a dangerous path to pursuit. If the integrity of the information net in the Network Centric Warfare (NCW) concept is compromised, a new form of very inexpensive warfare may be possible by simply hijacking the enemy's weapon. As long as the vehicles are non-autonomous they may be possible to control by both sides in a conflict.

Today, there are few alternatives suitable to shoot down small UAVs. Using for example AMRAAM missiles is not an economically viable alternative. Hence, small signal seeking missiles may pose a major threat to UAVs. The control and guidance algorithms of such missiles may need to be based on some way of predicting the flight trajectory. This is a field where trajectory optimization may be beneficial.

Another interesting research topic would be to investigate flying with side-slip such that the RCS pointing toward the adversary is decreased. It would also be interesting to incorporate and investigate a missile performance model with suitable RCS description. This could be used for both flight trajectory optimization of the missile and to investigate the combined performance of aircraft and missile in scenarios with hostile radar stations. If terrain following is incorporated in a later stage, better performance can be expected. However, it is very important that this is done after the proposed methodology is validated.

Finally, simplified modeling of both the flight dynamics and the tactical constraints may offer a possible way to find a good starting point for more refined trajectory optimization. This may also form an interface if the methods should be incorporated in multilevel optimization. When accurate radar cross section calculations can be performed quickly and reliably, design optimization combined with tactical considerations would be an interesting research topic with great potential.

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