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Daniel J. Simon
Cleveland State University, d.j.simon@csuohio.edu

D. L. Simon
NASA Glenn Research Center

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Dan Simon
Electrical Engineering Dept.
Cleveland State University
Cleveland, Ohio

Donald L. Simon
US Army Research Laboratory
NASA Glenn Research Center
Cleveland, Ohio

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Outline

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3. Kalman filtering with inequality constraints
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1. Problem statement

- **Estimate the health parameters of turbofan engines**
- **Other approaches:**
  - **Least squares**
  - **Kalman filtering**
  - **Neural networks**
  - **Genetic algorithms**
- **Our approach:**
  - **Assume a good model is available**
  - **Incorporate heuristic knowledge into the analytical Kalman filter solution**

2. Kalman filtering with equality constraints

We are given a linear system:

$$\begin{aligned} x_{k+1} &= \phi x_k + w_k \\ z_k &= H x_k + n_k \end{aligned}$$

In addition, we know from a priori information that the states satisfy s linear constraints:

$$\begin{aligned} D x_k &= d \\ D &= s \times n \text{ full rank matrix} \\ d &= s \times 1 \text{ vector} \end{aligned}$$

We can solve this by introducing s perfect measurements of the state:

$$\begin{aligned} x_{k+1} &= \phi x_k + w_k \\ \begin{bmatrix} z_k \\ d \end{bmatrix} &= \begin{bmatrix} H \\ D \end{bmatrix} x_k + \begin{bmatrix} n_k \\ 0 \end{bmatrix} \end{aligned}$$

Problems:

- **Singular measurement noise may result in numerical problems.**
- **Cannot be extended to inequality constraints.**

Another way to solve the problem is by using the constraints to reduce the dimension of the problem.

$$
\begin{aligned}
x_{k+1} &= \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 4 & -2 & 2 \end{bmatrix} x_k + w_k \\
z_k &= \begin{bmatrix} 2 & 4 & 5 \end{bmatrix} x_k + n_k \\
\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x_k &= 0 \\
x_{3,k} &= -x_{1,k}
\end{aligned}
$$

Put this constraint back in the original state and measurement equations.

$$
\begin{aligned}
x_{1,k+1} &= x_{1,k} + 2x_{2,k} + 3(-x_{1,k}) \\
&= -2x_{1,k} + 2x_{2,k}
\end{aligned}
$$

$$\begin{aligned}
x_{2,k+1} &= 3x_{1,k} + 2x_{2,k} + 1(-x_{1,k}) \\
&= 2x_{1,k} + 2x_{2,k} \\
z_k &= 2x_{1,k} + 4x_{2,k} + 5(-x_{1,k}) \\
&= -3x_{1,k} + 4x_{2,k} \\
x_{k+1} &= \begin{bmatrix} -2 & 2 \\ 2 & 2 \end{bmatrix} x_k + w_k \\
z_k &= \begin{bmatrix} -3 & 4 \end{bmatrix} x_k + w_n
\end{aligned}$$

Advantage: The dimension of the problem is reduced (computational savings).

Problems:

- **The physical meaning of the state variables is not retained.**
- **Cannot be extended to inequality constraints.**

Another way to solve the problem is by returning to a first-principles derivation of the Kalman filter.

1. **Maximum probability approach**

2. **Mean square approach**

3. **Projection approach**

Maximum probability approach

Assuming that x_0, w_k, and n_k are Gaussian, solve the problem

$$\begin{aligned}
\max \mathbf{pdf}(\tilde{x}_k | Z_k) &= \frac{\exp[-(\tilde{x}_k - \bar{\bar{x}}_k)^T P_k^{-1} (\tilde{x}_k - \bar{\bar{x}}_k)/2]}{(2\pi)^{n/2} |P_k|^{1/2}} \\
&\quad \text{such that } D\tilde{x}_k = d \\
Z_k &= \{z_1, z_2, \cdots, z_k\} \\
\bar{\bar{x}}_k &= E(x_k | Z_k) \\
\Rightarrow \tilde{x}_k &= \hat{x}_k - P_k D^T (D P_k D^T)^{-1} (D\hat{x}_k - d)
\end{aligned}$$

Mean square approach

$$\begin{aligned} \tilde{x} &= \arg\min_{\tilde{x}} E(||x - \tilde{x}||^2 | Z) \text{ such that } D\tilde{x} = d \\ \Rightarrow \tilde{x} &= \hat{x} - D^T (DD^T)^{-1} (D\hat{x} - d) \end{aligned}$$

Projection approach

$$\tilde{x} = \arg\min_{\tilde{x}} (\tilde{x} - \hat{x})^T W (\tilde{x} - \hat{x}) \text{ such that } D\tilde{x} = d$$

W is any positive definite weighting matrix.

$$\Rightarrow \tilde{x} = \hat{x} - W^{-1} D^T (D W^{-1} D^T)^{-1} (D\hat{x} - d)$$

The projection approach is the most general approach to the problem.

The maximum probability approach is obtained by setting $W = P^{-1}$.

The mean square approach is obtained by setting $W = I$.

3. Kalman filtering with inequality constraints

Suppose we have inequality constraints instead of equality constraints. Then the preceding approach is modified as follows:

$$\min_{\tilde{x}} (\tilde{x} - \hat{x})^T W (\tilde{x} - \hat{x}) \quad \text{such that} \quad D\tilde{x} \leq d$$

$$\rightarrow \min_{\tilde{x}} (\tilde{x}^T W \tilde{x} - 2\hat{x}^T W \tilde{x}) \quad \text{such that} \quad D\tilde{x} \leq d$$

Assume that t of the s inequality constraints are active at the solution
$\hat{D} = t$ rows of $D \sim$ active constraints
$\hat{d} = t$ elements of $d \sim$ active constraints

$$\min_{\tilde{x}} (\tilde{x}^T W \tilde{x} - 2\hat{x}^T W \tilde{x}) \text{ such that } \hat{D}\tilde{x} = \hat{d}$$

Inequality constrained problem $\equiv$ equality-constrained problem

Properties of the constrained state estimate:

- **Unbiased:** $E(\tilde{x}) = E(x)$
- **If** $W = P^{-1}$ **then Cov**$(x - \tilde{x}) <$ **Cov**$(x - \hat{x})$
- $W = P^{-1}$ **gives the smallest estimation error covariance**
- $||x_k - \tilde{x}_k|| \leq ||x_k - \hat{x}_k||$ **for all** k

4. Aircraft turbofan engine health estimation

- **NASA DIGTEM (Digital Turbofan Engine Model) – Generic nonlinear model of a twin spool low-bypass ratio turbofan engine model**
- **Fortran**
- **16 state variables**
- **6 controls**
- **8 health parameters**
- **12 measurements**

$$\begin{aligned} \dot{x} &= f(x,u,p) + w_1(t) \\ y &= g(x,u,p) + e(t) \end{aligned}$$

States:

1. Low Pressure Turbine (LPT) rotor speed (9200 RPM)
2. High Pressure Turbine (HPT) rotor speed (11900 RPM)
3. Compressor volume stored mass (0.91294 lbm)
4. Combustor inlet temperature (1325 R)
5. Combustor volume stored mass (0.460 lbm)
6. HPT inlet temperature (2520 R)
7. HPT volume stored mass (2.4575 lbm)
8. LPT inlet temperature (1780 R)
9. LPT volume stored mass (2.227 lbm)
10. Augmentor inlet temperature (1160 R)
11. Augmentor volume stored mass (1.7721 lbm)
12. Nozzle inlet temperature (1160 R)
13. Duct airflow (86.501 lbm/s)
14. Augmentor airflow (194.94 lbm/s)
15. Duct volume stored mass (6.7372 lbm)
16. Duct temperature (696 R)

Controls:

1. **Combustor fuel flow (1.70 lbm/s)**
2. **Augmentor fuel flow (0 lbm/s)**
3. **Nozzle throat area (430 in^2)**
4. **Nozzle exit area (492 in^2)**
5. **Fan vane angle (–1.7 deg)**
6. **Compressor vane angle (4.0 deg)**

Health parameters:

1. **Fan airflow (193.5 lbm/s)**
2. **Fan efficiency (0.8269)**
3. **Compressor airflow (107.0 lbm/s)**
4. **Compressor efficiency (0.8298)**
5. **HPT airflow (89.8 lbm/s)**
6. **HPT enthalpy change (167.0 Btu/lbm)**
7. **LPT airflow (107.0 lbm/s)**
8. **LPT enthalpy change (75.5 Btu/lbm)**

Measurements:

1. LPT rotor speed (9200 RPM, SNR = 150)
2. HPT rotor speed (11900 RPM, SNR = 150)
3. Duct pressure (34.5 psia, SNR = 200)
4. Duct temperature (696 R, SNR = 100)
5. Compressor inlet pressure (36.0 psia, SNR = 200)
6. Compressor inlet temperature (698 R, SNR = 100)
7. Combustor pressure (267 psia, SNR = 200)
8. Combustor inlet temperature (1325 R, SNR = 100)
9. LPT inlet pressure (70.0 psia, SNR = 100)
10. LPT inlet temperature (1780 R, SNR = 70)
11. Augmentor inlet pressure (31.8 psia, SNR = 100)
12. Augmentor inlet temperature (1160 R, SNR = 70)

Linearization:

$$
\begin{aligned}
\dot{x} &= f(x,u,p) + w_1(t) \\
y &= g(x,u,p) + e(t) \\
\Rightarrow \delta\dot{x} &= A_1\delta x + B\delta u + A_2\delta p + w_1(t) \\
\delta y &= C_1\delta x + D\delta u + C_2\delta p + e(t) \\
\delta u &= 0 \\
A_1 &= \frac{\partial f}{\partial x} \\
A_1(i,j) &\approx \frac{\Delta\dot{x}(i)}{\Delta x(j)}
\end{aligned}
$$

Similar equations hold for the A_2, C_1, and C_2 matrices.
Use Digtem to numerically approximate A_1, A_2, C_1, and C_2.

Discretization:

$$
\begin{aligned}
\delta x_{k+1} &= A_{1d}\delta x_k + A_{2d}\delta p_k + w_{1k} \\
\delta y_k &= C_1\delta x_k + C_2\delta p_k + e_k
\end{aligned}
$$

Augment the state vector with the health parameter vector:

$$
\begin{aligned}
\begin{bmatrix} \delta x_{k+1} \\ \delta p_{k+1} \end{bmatrix} &= \begin{bmatrix} A_{1d} & A_{2d} \\ 0 & I \end{bmatrix} \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + \begin{bmatrix} w_{1k} \\ w_{2k} \end{bmatrix} \\
\delta y_k &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k
\end{aligned}
$$

w_{2k} is a small noise term that represents model uncertainty and allows the Kalman filter to estimate time-varying health parameter variations.

$$\begin{aligned} \begin{bmatrix} \delta x_{k+1} \\ \delta p_{k+1} \end{bmatrix} &= A \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + w_k \\ \delta y_k &= C \begin{bmatrix} \delta x_k \\ \delta p_k \end{bmatrix} + e_k \end{aligned}$$

Now we can use a Kalman filter to estimate δx_k and δp_k.

Constraint: Engine health does not improve with time.

Constraints:

$\delta p(1)$ = **fan airflow**
$\delta p(2)$ = **fan efficiency**
$\delta p(3)$ = **compressor airflow**
$\delta p(4)$ = **compressor efficiency**
$\delta p(6)$ = **HPT enthalpy change**
$\delta p(8)$ = **LPT enthalpy change**

- **Always less than or equal to zero and always decrease with time.**

- **Vary slowly with time.**

For example,

$$\begin{aligned} \tilde{\delta p}(1) &\leq 0 \\ \tilde{\delta p}_{k+1}(1) &\leq \tilde{\delta p}_k(1) + \gamma_1^+ \\ \tilde{\delta p}_{k+1}(1) &\geq \tilde{\delta p}_k(1) - \gamma_1^- \end{aligned}$$

γ_1^+ allows the estimate to increase (but only slightly) since the estimate may be too low.

γ_1^- prevents the estimate from decreasing too quickly.

$\gamma_1^- > \gamma_1^+$

The γ_1 parameters are heuristic constraints that need to be tuned or optimized.

Constraints:

$\delta p(5) =$ **HPT airflow**
$\delta p(7) =$ **LPT airflow**

- **Always greater than or equal to zero and always increase with time.**

- **Vary slowly with time.**

For example,

$$\begin{aligned} \tilde{\delta p}(5) &\geq 0 \\ \tilde{\delta p}_{k+1}(5) &\leq \tilde{\delta p}_k(5) + \gamma_5^+ \\ \tilde{\delta p}_{k+1}(5) &\geq \tilde{\delta p}_k(5) - \gamma_5^- \end{aligned}$$

γ_5^+ prevents the estimate from increasing too quickly.

γ_5^- allows the estimate to decrease (but only slightly) since the estimate may be too high.

$\gamma_5^+ > \gamma_5^-$

The γ_5 parameters are heuristic constraints that need to be tuned or optimized.

5. Simulation results

- Nonlinear DIGTEM model used to simulate engine sensor measurements
- 30 data points each flight, 500 flights
- Linear + exponential health parameter degradation
- Random initial health parameter degradations
- 30 simulations

- **Matlab code for Kalman filter**

- **Relinearized the Kalman filter every 50 flights around the current estimates**

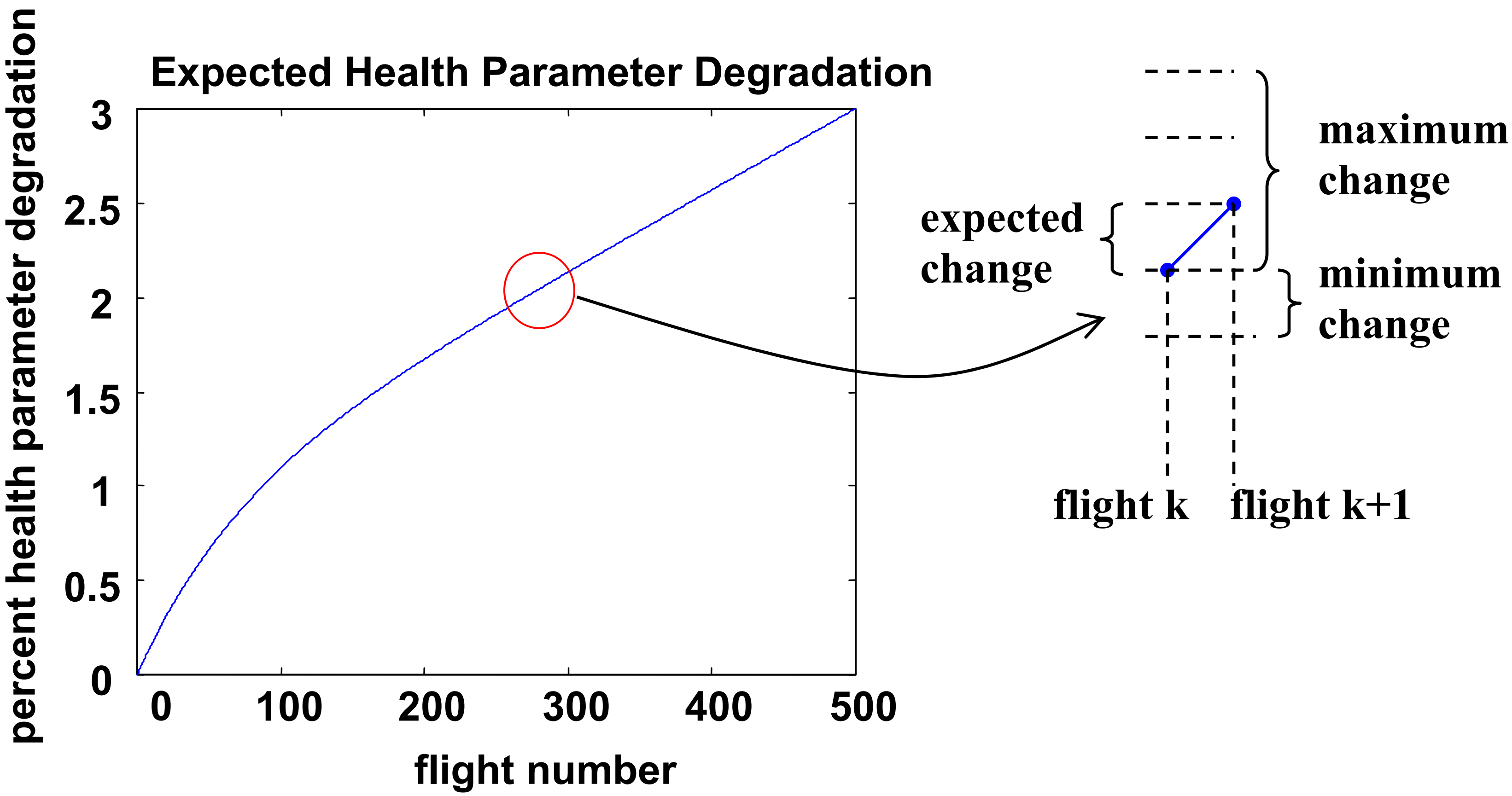
- **One-sigma process noise = 1% of the nominal states**

- **One-sigma process noise = 0.01% of the nominal health parameters**

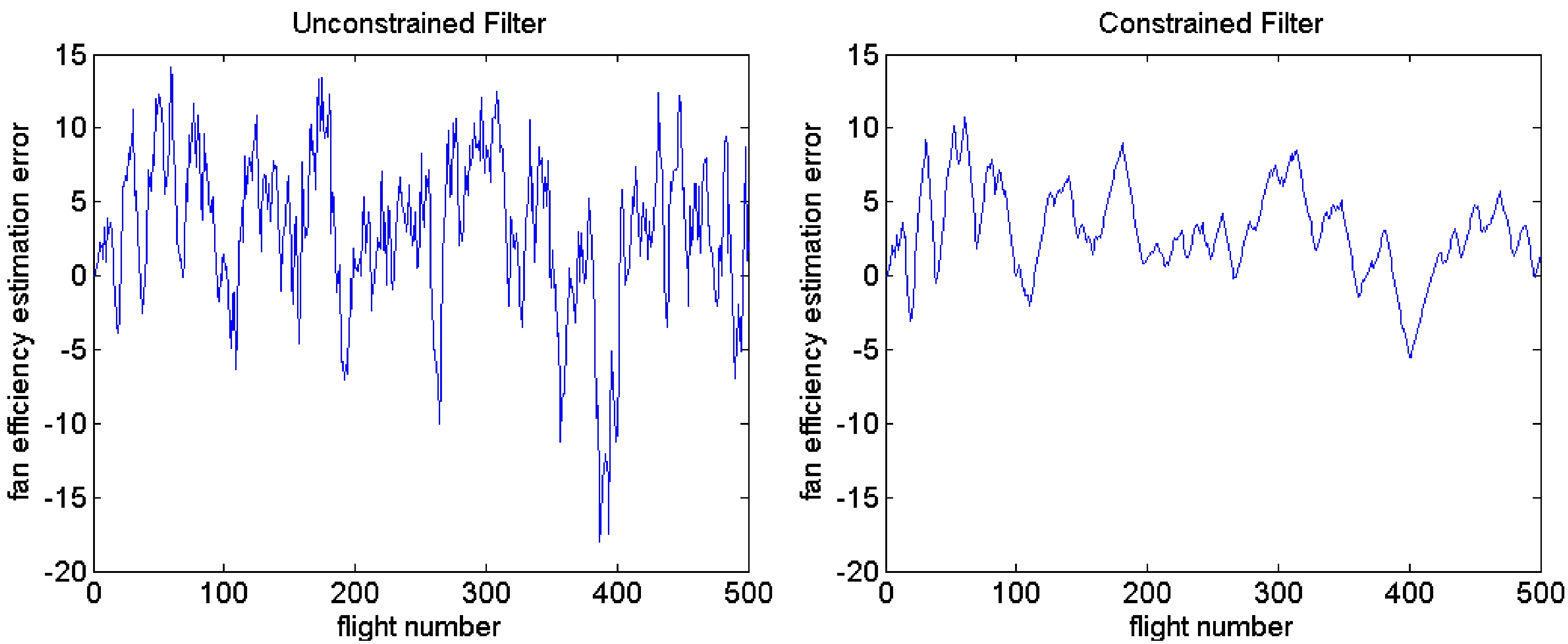
- $W = P^{-1}$

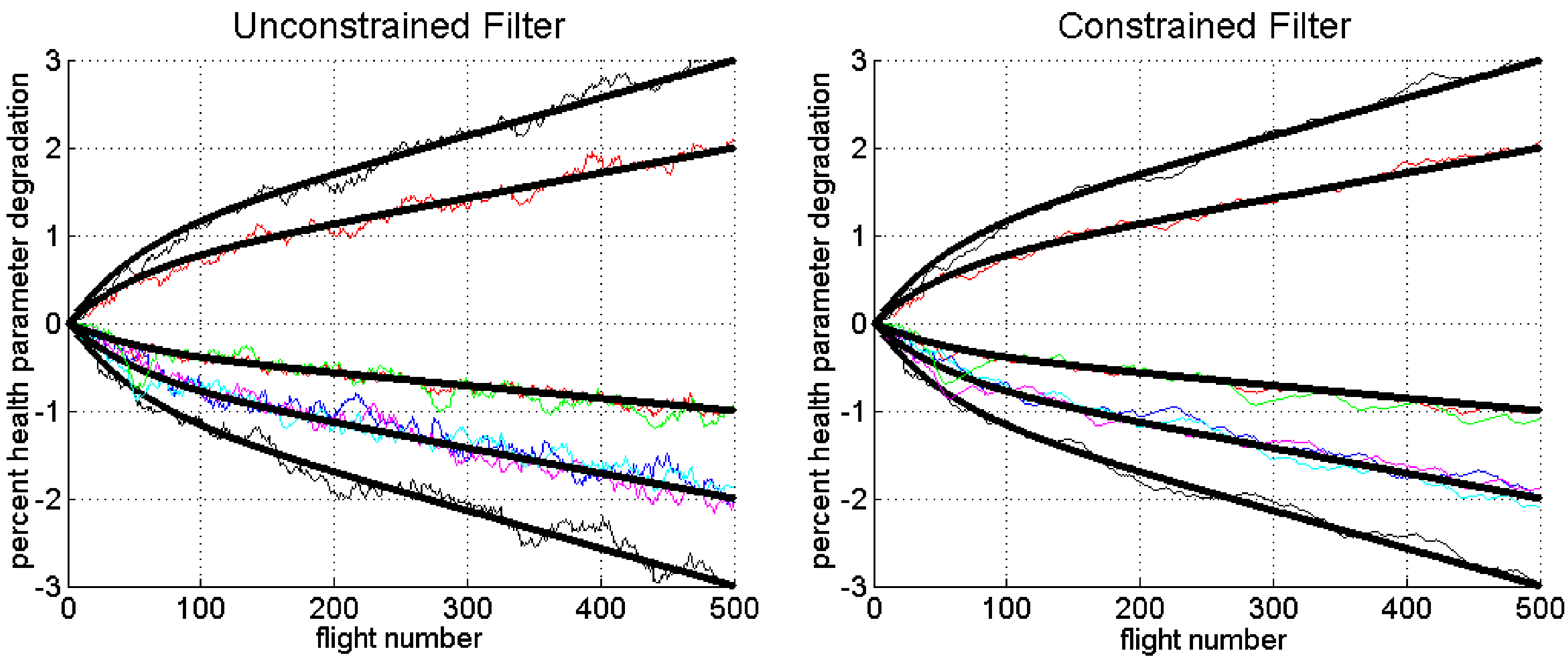
- **For increasing health parameters, the maximum rate of change in $\tilde{\delta p}$ was (–3%, +9%) after 500 flights (very conservative)**

Illustration of Constraint Enforcement



Comparison of Kalman Filter Results





Final degradations:
Compressor airflow, Fan airflow: -1%
Fan eff., Compressor eff., HPT enthalpy change: -2%
LPT enthalpy change: -3%
HPT airflow: $+3\%$
LPT airflow: $+2\%$

	Estimation Error (%)	
Health Parameter	Unconstrained	Constrained
Fan Airflow	4.81	4.41
Fan Efficiency	5.85	4.60
Compressor Airflow	3.43	2.73
Compressor Efficiency	4.82	3.80
HPT Airflow	3.09	2.39
HPT Enthalpy Change	4.48	3.76
LPT Airflow	4.54	4.26
LPT Enthalpy Change	6.28	5.22
Average	4.66	3.90

Average RMS improvement = 0.76 %.

Of the 30 simulations, the smallest RMS improvement was 0.49 %.

6. Conclusion

- Inequality constraints in a Kalman filter
- Application to turbofan engine health estimation
  - Better health parameter estimates for engine control
  - Better trending
  - Better fault isolation
- Constrained estimates have the same general shape as unconstrained estimates
- Computational effort increases by a factor of about four

Future work:

- **Robust Kalman filtering**
- **Optimal constraints - tradeoff in confidence of a priori information**
- **Uncertainties in control inputs**
- **Optimal sensor selection**