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AIRFOIL IN SINUSOIDAL MOTION IN A PULSATING STREAM

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SUMMARY

The forces and moments on a two-dimensional airfoil executing harmonic motions in a pulsating stream are derived on the basis of nonstationary incompressible potential-flow theory, with the inclusion of the effect of the continuous sheet of vortices shed from the trailing edge. An assumption as to the form of the wake is made with a certain degree of approximation. A comparison with previous work applicable only to the special case of a stationary airfoil is made by means of a numerical example and the excellent agreement obtained shows that the wake approximation is quite sufficient. The results obtained are expected to be useful in considerations of forced vibrations and flutter of rotary-wing aircraft for which the lifting surfaces are in air streams of variable velocity.

INTRODUCTION

The problem of an airfoil in arbitrary motion in a pulsating stream arises in connection with rotating blades in forward motion, for example, helicopter blades. Use of two-dimensional theory for this problem leads to deviations from reality with respect to the position of the wake, but, as most of the wake effects arise from that part near the airfoil, the error should not be serious.

Restricted results, applicable to an airfoil at a fixed angle of attack in a pulsating stream, have been obtained by Isaacs (reference 1). In the present paper, under the same linearizing assumptions as are made in the derivation in reference 1 but with explicit consideration and simplification of the form of the wake extending from the rear of the airfoil, the methods of Theodorsen (reference 2) have been extended to obtain the forces on the airfoil not only at a fixed angle of attack but also in arbitrary motion. The result is in closed form as compared with the Fourier series result of reference 1. Inclusion of the effects of arbitrary motion of the airfoil presents the possibility of application to forced vibrations and to flutter of helicopter blades.

SYMBOLS

a'	position of torsion axis of wing measured from center
a	nondimensional position of torsion axis of wing measured from center
b	half chord of wing
α	fixed part of angle of attack measured clockwise from horizontal
β	varying part of angle of attack
h	vertical displacement, positive downward
x'	horizontal coordinate
x	nondimensional coordinate
t	time
v	stream velocity
ν	frequency
ω	circular frequency $(2\pi\nu)$
$k = \frac{\omega b}{v}$	reduced frequency
$C(k) = F + iG$	C function (reference 2)
$\delta(x)$	Dirac delta function (reference 3)
p	local static pressure
ρ	air density
σ	a number determining magnitude of stream pulsations
P	perpendicular force
M_a	pitching moment about $x = a$, positive counter clockwise

φ	noncirculatory velocity potential
φ_Γ	circulatory velocity potential
U	strength of wake discontinuity
ψ	phase angle with respect to stream pulsations
$\Delta\Gamma$	element of vorticity

Subscripts:

α	fixed part of angle of attack
β	varying part of angle of attack
v	stream velocity
h	vertical displacement
0	with β or h, maximum amplitude; with x, wake position; with v, steady part

DERIVATION OF FORCES

Method.- In a derivation parallel to that of reference 2, the forces due to the noncirculatory flow and to the effect of the wake are treated separately, the usual assumptions being made: incompressible potential flow, two-dimensional flat-plate airfoil, small oscillations, and plane wake extending from trailing edge to infinity.

Noncirculatory force and moment.- Consider an airfoil of chord $2b$ at an angle of attack $\alpha + \beta$ with respect to a stream having velocity v directed to the right (fig. 1). Let the angle α be constant so that the only variation in angle of attack is due to a variation in the angle β. If the airfoil is allowed to rotate about the point $x' = a'$ with angular velocity $\dot{\beta}$ (the dot over a symbol denotes differentiation with respect to the time t) and to move downward with a velocity $\dot{h}$, the velocity normal to the airfoil is

$$\epsilon(x') = -\dot{h} - (\alpha + \beta)v + \dot{\beta}(a' - x')$$

The potential for such a normal velocity function is (see reference 2)

$$\varphi = \dot{h}b\sqrt{1 - x^2} + v(\alpha + \beta)b\sqrt{1 - x^2} + \dot{\beta}b^2\left(\frac{1}{2}x - a\right)\sqrt{1 - x^2} \qquad (1)$$

where

$$x = \frac{x'}{b}$$

$$a = \frac{a'}{b}$$

Using the equation of motion for nonstationary flow

$$\frac{\partial w}{\partial t} = -\nabla\left(\frac{p}{\rho} + \frac{1}{2}w^2\right)$$

where

p local static pressure

ρ air density

w local velocity

and substituting

$$w = v + \frac{\partial \varphi}{\partial x}$$

gives the pressure difference at the point x as

$$\Delta p = -2\rho\left(\frac{v}{b}\frac{\partial \varphi}{\partial x} + \frac{\partial \varphi}{\partial t}\right) \qquad (2)$$

Integration of this local pressure difference over the length of the airfoil gives as the total force

$$P = -\pi\rho b^2\left[\ddot{h} + v\dot{\beta} + \dot{v}(\alpha + \beta) - ba\ddot{\beta}\right] \qquad (3)$$

The noncirculatory moment about the point $x = a$ is

$$M_a = b^2 \int_{-1}^{1} \Delta p(x - a)\, dx = -2\rho b^2 \int_{-1}^{1} (x - a) \frac{\partial \varphi}{\partial t} dx + 2\rho v b \int_{-1}^{1} \varphi\, dx$$

which becomes

$$M_a = -\pi \rho b^2 \left[-v^2(\alpha + \beta) - ba\dot{v}(\alpha + \beta) - v\dot{h} + b^2\left(\frac{1}{8} + a^2\right)\ddot{\beta}\right] \qquad (3a)$$

Circulatory force and moment.- The velocity potential of an element of vorticity $-\Delta\Gamma$ at a position x_0 in the wake and its image $\Delta\Gamma$ distributed over the airfoil is (reference 2)

$$\varphi_{xx_0} = \frac{\Delta\Gamma}{2\pi} \tan^{-1} \frac{\sqrt{1 - x^2}\sqrt{x_0^2 - 1}}{1 - xx_0} \qquad (4)$$

The element $-\Delta\Gamma$ moves to the right relative to the airfoil with a velocity v. Thus

$$\frac{\partial \varphi_{xx_0}}{\partial t} = v \frac{\partial \varphi_{xx_0}}{\partial x_0}$$

Substituting this expression and $\frac{\partial \varphi_{xx_0}}{\partial x}$ into equation (2) and integrating the effect of the entire wake on the airfoil yields the force

$$P = -\rho v b \int_{1}^{\infty} \frac{x_0}{\sqrt{x_0^2 - 1}} U\, dx_0 \qquad (5)$$

where $U\,dx_0$ is the element of vorticity $\Delta\Gamma$ at the point x_0. The circulatory moment which is obtained from

$$M_a = b^2 \int_{-1}^{1} \Delta p(x - a)\,dx$$

is, similarly,

$$M_a = -\rho v b^2 \int_{1}^{\infty} \left[\frac{1}{2}\sqrt{\frac{x_0 + 1}{x_0 - 1}} - \left(a + \frac{1}{2}\right)\frac{x_0}{\sqrt{x_0^2 - 1}}\right] U\,dx_0 \tag{5a}$$

The Kutta condition requires that at the trailing edge of the plate the induced velocity be equal to 0; therefore, at the point $x = 1$

$$\left[\frac{\partial}{\partial x'}\left(\varphi_\Gamma + \varphi\right)\right]_{x=1} = 0$$

where

$$\varphi_\Gamma = b\int_{1}^{\infty} \varphi_{xx_0}\,dx_0$$

Introducing the potential from equation (1) results in

$$\frac{1}{2\pi}\int_{1}^{\infty}\sqrt{\frac{x_0 + 1}{x_0 - 1}}\,U\,dx_0 = \dot{h} + v(\alpha + \beta) + \dot{\beta}b\left(\frac{1}{2} - a\right) \tag{6}$$

The type of stream velocity encountered by a helicopter blade section in forward flight is

$$v = v_0\left(1 + \sigma e^{i\omega_v t}\right) \tag{7}$$

where $\frac{2\pi}{\omega_v}$ is the period of pulsation of the stream. The assumption is made that $\sigma < 1$, because if $\sigma > 1$ there would occur a reversal of flow which cannot be treated by existing methods. In usual practice $\sigma > 1$ only for radial positions near the hub of a helicopter blade. For harmonic motions of the airfoil

$$\left.\begin{aligned} \beta &= \beta_0 e^{i(\omega_\beta t + \psi_\beta)} \\ h &= h_0 e^{i(\omega_h t + \psi_h)} \end{aligned}\right\} \tag{8}$$

where $\frac{2\pi}{\omega_\beta}$ and $\frac{2\pi}{\omega_h}$ are the periods of variation in angle of attack and in vertical displacement, respectively, and ψ_β and ψ_h are phase factors.

Equation (6) becomes, after substitution of equations (7) and (8),

$$\begin{aligned} \frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0 + 1}{x_0 - 1}}\, U\, dx_0 &= v_0\alpha + v_0\sigma\alpha e^{i\omega_v t} \\ &+ \left[i\omega_\beta\beta_0 b\left(\frac{1}{2} - a\right) + v_0\beta_0\right] e^{i(\omega_\beta t + \psi_\beta)} \\ &+ i\omega_h h_0 e^{i(\omega_h t + \psi_h)} \\ &+ \sigma v_0\beta_0 e^{i\left[(\omega_v + \omega_\beta)t + \psi_\beta\right]} \end{aligned} \tag{9}$$

It is sufficient to satisfy the Kutta condition separately for each of the terms on the right-hand side of equation (9); therefore, a sheet of discontinuity of the following form is set up:

$$U = \Gamma_\alpha \delta\left(\infty - x_0\right) + U_\alpha e^{i(\omega_v t - k_v x_0)}$$
$$+ U_\beta e^{i(\omega_\beta t - k_\beta x_0)} + U_h e^{i(\omega_h t - k_h x_0)}$$
$$+ U_{v+\beta} e^{i\left[(\omega_v + \omega_\beta)t - k_{v+\beta} x_0\right]} \tag{10}$$

where $\delta(\infty - x_0)$ is the Dirac delta function (reference 3), and where,

$$k_v = \frac{\omega_v b}{v_0}$$

$$k_\beta = \frac{\omega_\beta b}{v_0}$$

$$k_h = \frac{\omega_h b}{v_0}$$

$$k_{v+\beta} = \frac{(\omega_v + \omega_\beta)b}{v_0}$$

Justification for assuming this form for the wake is given in the appendix.

Combining equations (9) and (10) and equating terms of corresponding time variation yields

$$\frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,\Gamma_\alpha \delta(\infty - x_0)\,dx_0 = v_0\alpha$$

or $\Gamma_\alpha = 2\pi v_0\alpha$ (the stationary circulation)

$$\frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,U_\alpha e^{-ik_v x_0}\,dx_0 = \sigma v_0\alpha$$

$$\frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,U_\beta e^{-ik_\beta x_0}\,dx_0 = \left[i\omega_\beta \beta_0 b\left(\frac{1}{2} - a\right) + v_0\beta_0\right] e^{i\psi_\beta}$$

$$\frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,U_h e^{-ik_h x_0}\,dx_0 = i\omega_h h_0 e^{i\psi_h}$$

$$\frac{1}{2\pi}\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,U_{v+\beta} e^{-ik_{v+\beta} x_0}\,dx_0 = \sigma v_0 \beta_0 e^{i\psi_\beta}$$

Introducing these relations and equation (10) into equation (5) gives

$$P = -2\pi\rho v b\left(v_0\alpha + \sigma v_0 \alpha e^{i\omega_v t}\,\frac{\displaystyle\int_1^\infty \frac{x_0}{\sqrt{x_0^2-1}}\,U_\alpha e^{-ik_v x_0}\,dx_0}{\displaystyle\int_1^\infty \sqrt{\frac{x_0+1}{x_0-1}}\,U_\alpha e^{-ik_v x_0}\,dx_0} + \ldots\right)$$

The quotient of integrals occurring in this expression is the C function defined in reference 2 as

$$C(k) = \frac{\int_1^{\infty} \frac{x_0}{\sqrt{x_0^2 - 1}} e^{-ikx_0} dx_0}{\int_1^{\infty} \sqrt{\frac{x_0 + 1}{x_0 - 1}} e^{-ikx_0} dx_0}$$

The force due to the wake is, therefore,

$$\begin{aligned} P = -2\pi\rho vb \Big\{ & v_0\alpha + \sigma v_0 \alpha C(k_v) e^{i\omega_v t} \\ & + \left[i\omega_\beta \beta_0 b\left(\frac{1}{2} - a\right) + v_0\beta_0 \right] C(k_\beta) e^{i(\omega_\beta t + \psi_\beta)} \\ & + i\omega_h h_0 C(k_h) e^{i(\omega_h t + \psi_h)} \\ & + \sigma v_0 \beta_0 C(k_{v+\beta}) e^{i\left[(\omega_v + \omega_\beta)t + \psi_\beta\right]} \Big\} \end{aligned} \tag{11}$$

Applying the same methods to equation (5a) as were applied to equation (5) gives the moment due to the wake as

$$\begin{aligned} M_a = & -\pi\rho vb^2 \left[\dot{h} + v(\alpha + \beta) + \dot{\beta} b\left(\frac{1}{2} - a\right) \right] \\ & + 2\pi\rho vb^2 \left(a + \frac{1}{2}\right) \Big\{ v_0\alpha + \sigma v_0 \alpha C(k_v) e^{i\omega_v t} \\ & + \left[b\left(\frac{1}{2} - a\right)\dot{\beta} + v_0\beta \right] C(k_\beta) \\ & + \dot{h} C(k_h) + \sigma v_0 \beta C(k_{v+\beta}) e^{i\omega_v t} \Big\} \end{aligned} \tag{11a}$$

Adding equations (3) and (11) gives for the total force on the airfoil

$$
\begin{aligned}
P = -\pi\rho b^2 \left[\ddot{h} + v\dot{\beta} + \dot{v}(\alpha + \beta) - ab\ddot{\beta}\right] \\
- 2\pi\rho v b \Bigg\{ v_0\alpha + \sigma v_0 \alpha C(k_v) e^{i\omega_v t} \\
+ \left[b\left(\frac{1}{2} - a\right)\dot{\beta} + v_0\beta\right] C(k_\beta) + \dot{h}C(k_h) \\
+ \sigma v_0 \beta C(k_{v+\beta}) e^{i\omega_v t} \Bigg\}
\end{aligned} \tag{12}
$$

Adding equations (3a) and (11a) gives for the total moment about the point $x = a$

$$
\begin{aligned}
M_a = -\pi\rho b^2 \left[v b\left(\frac{1}{2} - a\right)\dot{\beta} - \dot{v} b a(\alpha + \beta) - ba\ddot{h} + b^2\left(\frac{1}{8} + a^2\right)\ddot{\beta}\right] \\
+ 2\pi\rho v b^2\left(a + \frac{1}{2}\right)\Bigg\{ v_0\alpha + \sigma v_0 \alpha C(k_v) e^{i\omega_v t} \\
+ \left[b\left(\frac{1}{2} - a\right)\dot{\beta} + v_0\beta\right] C(k_\beta) \\
+ \dot{h}C(k_h) + \sigma v_0 \beta C(k_{v+\beta}) e^{i\omega_v t} \Bigg\}
\end{aligned} \tag{12a}
$$

Equations (12) and (12a) are applicable in any case of simple harmonic motions of the airfoil, regardless of the manner in which the motion arises. For instance, change in the angle β may be due either to a twisting of the blade or to the fact that the plane of the helicopter disk is inclined to the direction of motion. An obvious extension to equations (12) and (12a) to include the effect of an aileron can be made by applying the results of reference 2 to the aileron potentials and forces in the same manner as the present application to the potentials and forces of the entire airfoil.

If the variation in displacement or in angle of attack of the airfoil or in stream velocity is made up of a superposition of simple harmonic motions, equations (12) and (12a) must be modified in a manner indicated by the derivation.

Numerical example.- For the purpose of checking the results of the present derivation and in particular the assumption as to the form of the wake, a numerical example is given, and a comparison is made with the example given in reference 1 for the case of the stationary airfoil. A more direct comparison cannot be simply obtained because the form of the wake is not given explicitly in reference 1.

For $\beta \equiv h \equiv 0$, equation (12) becomes

$$P = -\pi\rho b^2\dot{v}\alpha - 2\pi\rho v b\left[v_0\alpha + v_0\sigma\alpha C(k_v)e^{i\omega_v t}\right]$$

If the stream is undergoing sinusoidal pulsations of the form

$$v = v_0(1 + \sigma\sin\omega_v t)$$

$$= R\left[v_0(1 - i\sigma e^{i\omega_v t})\right]$$

then the equation for P becomes

$$P = R(-\pi\rho b^2\dot{v}\alpha) - R(2\pi\rho v b)R\left[v_0\alpha - iv_0\sigma\alpha C(k_v)e^{i\omega_v t}\right] \tag{13}$$

Making use of the definitions

$$k_v = \frac{\omega_v b}{v_0}$$

and (from reference 2)

$$C(k) = F + iG$$

and dividing equation (13) through by $L_0 = -2\pi\rho b v_0^2\alpha$ gives

$$\frac{P}{L_0} = \frac{1}{2}\sigma k_v R(e^{i\omega_v t}) + R(1 - i\sigma e^{i\omega_v t})R\left[1 - i\sigma(F + iG)e^{i\omega_v t}\right]$$

where L_0 is the steady lift due to v_0 alone. From the foregoing equation, after the indicated real parts have been extracted,

$$\frac{P}{L_0} = \left(1 + \frac{\sigma^2 F}{2}\right) + \sigma\left(\frac{k_v}{2} + G\right)\cos\omega_v t + \sigma(1 + F)\sin\omega_v t$$

$$- \frac{\sigma^2 F}{2}\cos 2\omega_v t + \frac{\sigma^2 G}{2}\sin 2\omega_v t$$

The same values of the parameters k_v and σ as those chosen by Isaacs as representative of typical helicopter practice are used; namely, $k_v = 0.0424$ and $\sigma = 0.4$, and values of the functions F and G are taken from a tabulation in reference 1. Then

$$\frac{P}{L_0} = 1.074 - 0.0395 \cos\omega_v t + 0.768 \sin\omega_v t$$

$$- 0.074 \cos 2\omega_v t - 0.0096 \sin 2\omega_v t$$

The value obtained by Isaacs is

$$\frac{P}{L_0} = 1.079 - 0.0376 \cos\omega_v t + 0.770 \sin\omega_v t$$

$$- 0.079 \cos 2\omega_v t - 0.00697 \sin 2\omega_v t$$

$$- 0.00061 \cos 3\omega_v t - 0.0050 \sin 3\omega_v t + \ldots$$

As can readily be seen, these two expressions for the lift are in very good agreement; therefore, the present assumption concerning the form of the wake is quite reasonable. It is also to be noted that this agreement holds for relatively large values of σ and relatively small values of the frequency. Even better results can be expected for higher values of the frequency of pulsation of the stream, as is shown in the appendix.

CONCLUDING REMARKS

The assumption of a sinusoidal wake form behind an airfoil in a pulsating stream has been shown to be adequate for the derivation of the forces and moments. The forces and moments obtained are expected to be useful in considerations of forced vibrations and flutter of rotary-wing aircraft for which the lifting surfaces are in air streams of variable velocity.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX

FORMULATION OF THE WAKE BEHIND AN AIRFOIL IN A PULSATING STREAM

Use of Dirac delta function.- In order to account for the stationary lift on the airfoil, that is, the lift due to the fixed part v_0 of the stream velocity and due to the fixed part α of the angle of attack it is necessary to assume that the vorticity which has been shed at some distant past time has travelled to an infinite distance from the airfoil and is localized there. The delta function has the properties (reference 3)

$$\int_{-\infty}^{\infty} \delta(c - x)\, dx = 1$$

$$\int_{\infty}^{\infty} f(x)\delta(c - x)\, dx = f(c)$$

where $f(x)$ is an arbitrary function. The first of these properties, when applied to the first term on the right-hand side of equation (10), gives

$$\int_{-\infty}^{\infty} \Gamma_\alpha \delta(\infty - x)\, dx = \Gamma_\alpha$$

so that the condition that the total vorticity for the stationary case be preserved is satisfied. The second property has the effect of localizing this total vorticity at the point $x = c$ where c is chosen to be infinite.

Sinusoidal part of wake.- Consider an airfoil moving back and forth harmonically in a uniform stream having a velocity v_0. If σv_0 is the maximum velocity of the airfoil, then the velocity of the airfoil at any time t is given by

$$v_A = \sigma v_0 e^{i\omega_v t}$$

and the position of the trailing edge relative to the equilibrium position of the trailing edge is (fig. 2)

$$\xi_0 = \int_0^t v_A \, dt = \frac{\sigma v_0}{i\omega_v} e^{i\omega_v t} \Big|_0^t = \xi_0(t) \tag{A1}$$

The vortex element at the time t and at the distance ξ from the equilibrium position of the trailing edge of the airfoil is the same as that vortex element which was shed from the airfoil at the time $t - \tau$, where τ is the time necessary for the vortex element to travel with velocity v_0 from the trailing-edge position at time $t - \tau$ to the position ξ. Therefore,

$$\tau = \frac{1}{v_0} \left[\xi - \xi_0(t - \tau)\right] \tag{A2}$$

Suppose that the vorticity at the trailing edge varies sinusoidally with the frequency $\upsilon = \frac{\omega}{2\pi}$. Then

$$\left[\Delta\Gamma(t)\right]_{\xi_0} = \Delta\Gamma_0 e^{i\omega t}$$

But

$$\Delta\Gamma(\xi, t) = \left[\Delta\Gamma(t - \tau)\right]_{\xi_0}$$

Therefore

$$\Delta\Gamma(\xi, t) = \Delta\Gamma_0 e^{i\omega(t-\tau)}$$

$$= \Delta\Gamma_0 e^{i\omega\left\{t - \frac{1}{v_0}\left[\xi - \xi_0(t-\tau)\right]\right\}} \tag{A3}$$

Referring now to a coordinate system the origin of which is at the center of the moving airfoil, that is, reverting to the case of the fixed airfoil in a pulsating stream, requires the transformation

$$x_0 - 1 = \xi - \xi_0(t)$$

Equation (A3) becomes

$$\Delta\Gamma(x_0,t) = \Delta\Gamma_0 e^{i\omega\left\{t-\frac{1}{v_0}\left[x_0-1+\xi_0(t)-\xi_0(t-\tau)\right]\right\}} \tag{A4}$$

From equation (A1) it can be seen that $\xi_0 \rightarrow 0$ as $\omega_v \rightarrow \infty$. Therefore, as $\omega_v \rightarrow \infty$

$$\Delta\Gamma(x_0,t) \sim \Delta\Gamma_0 e^{i\omega\left(t-\frac{x_0}{v_0}+\frac{1}{v_0}\right)} = \Delta\Gamma_0 e^{i(\omega t-kx_0+k)} \tag{A5}$$

where $k = \frac{\omega}{v_0}$. This relation shows that for large values of the frequency of pulsations in the stream, the wake form approaches the sinusoidal form, which was used in the derivation of forces.

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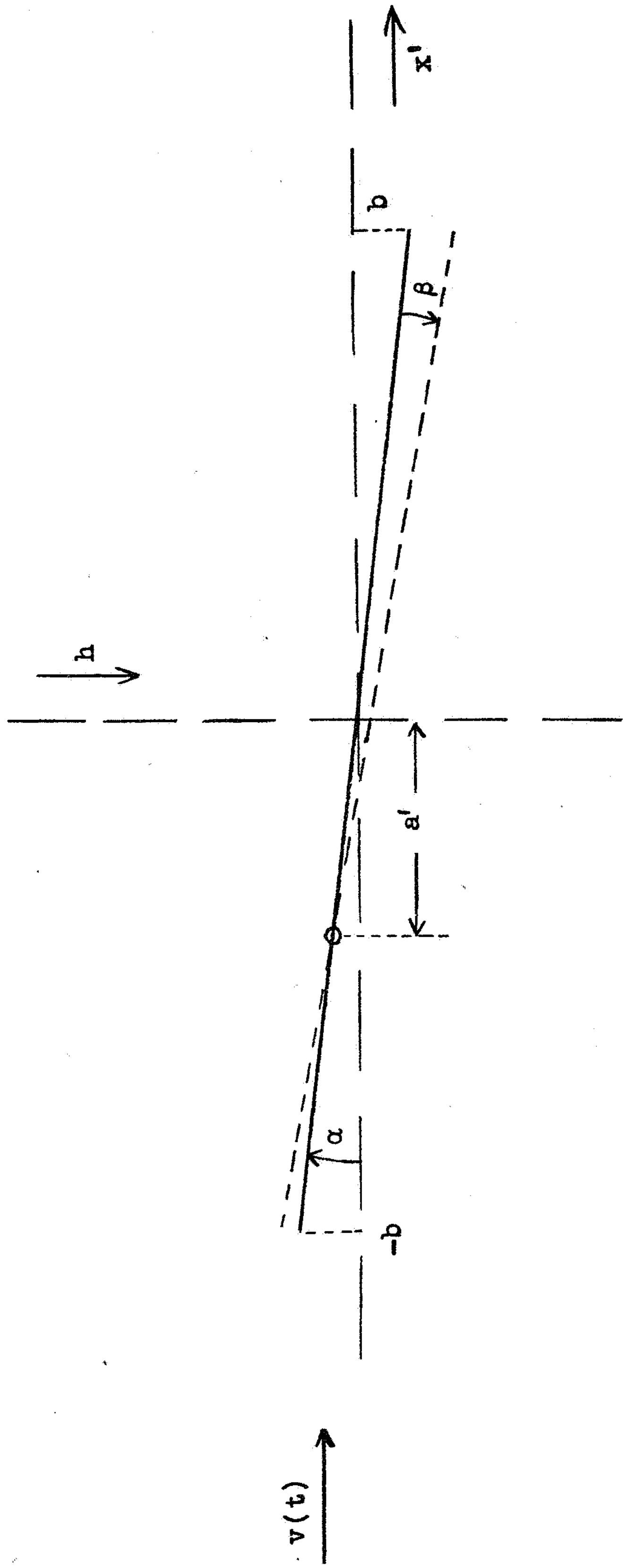

Figure 1.- Diagram showing notation used for oscillating airfoil in nonstationary flow.

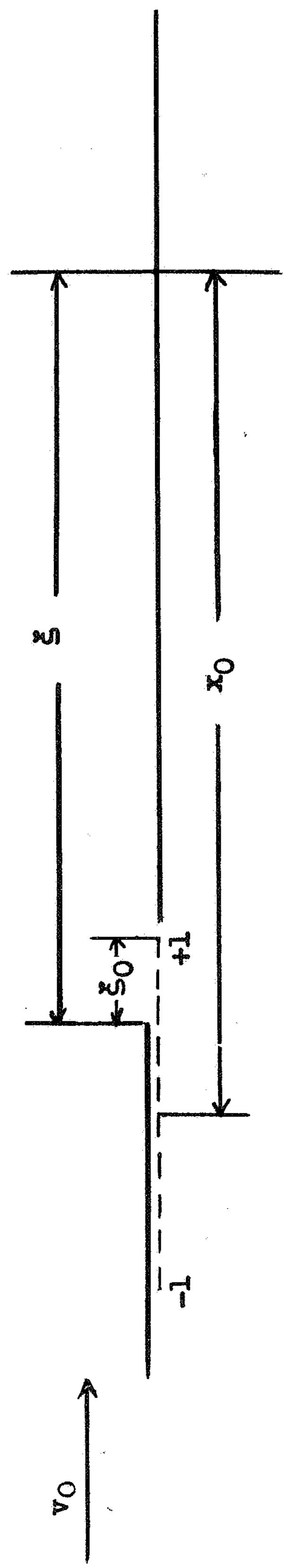

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Figure 2.- Diagram showing notation used in derivation of wake.