Scientific Research Publishing

Open Journal of Philosophy, 2021, 11, 355-357
https://www.scirp.org/journal/ojpp
ISSN Online: 2163-9442
ISSN Print: 2163-9434

Algebraic Structures of Quantum Physics

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How to cite this paper: Jones, R. M. (2021). Algebraic Structures of Quantum Physics. *Open Journal of Philosophy, 11,* 355-357.
https://doi.org/10.4236/ojpp.2021.113024

Received: May 24, 2021
Accepted: June 26, 2021
Published: June 29, 2021

Abstract

We examine a series of developments in mathematics used in quantum physics. These include Hilbert space. We then examine how several developments in mathematics can be used in application to Quantum physics.

Keywords

Algebra, Quantum Physics

1. Introduction

Geometry and other parts of mathematics have significant applications in modern quantum mechanics. These applications have already brought important advances in quantum mechanics. We hope, in a follow-on publication, to explore in detail the contributions to modern quantum mechanics made by the mathematicians Emmy Noether and John Horton.

A very well written account of one of the most important tools of string theory is (Saxe, 2002, FUNC3). It includes bibliographies of mathematicians who have contributed to its development.

(Riesz & Szőkefalvi-Nagy, 1990, FUNC2) is a classic. It includes an extended description of Hilbert and Banach spaces. (Alt2012, FUNC1) has modern mathematical advance.

The theorem of De Moivre shall begin an expository conversation.

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

This result, and others, developed into a theory of imaginary numbers. These came in turn to be called complex numbers. This name change was well justified by the fact that the imaginary numbers were indeed as realistic as the real numbers. A mathematician then appeared who developed this subject to an even more practically applicable system of mathematics for use in formulating physical theory.

2. Applications

David Hilbert was a phenomenally productive mathematician. Hilbert Space is named after him, and uses complex numbers to define a space of great usefulness in quantum mechanics. His accomplishments are described in (Reid, 1996).

Perhaps his most famous publication was a list of 23 problems to be solved by research mathematicians during the 20th Century. To merely describe any of these problems would itself be a challenge. Before turning to an effort in this direction, we may examine a lighter mathematical matter that contains mathematics for entertainment. His name is associated with Hilbert's Hotel. It will lead us on to how we may find the underlying algebra of mathematical proofs.

In the Hilbert Hotel there are infinitely many rooms. They are numbered by the natural number, i.e. the integer numbers greater than, or equal to, Zero. This plentiful supply of rooms is an analogy with the mathematical concepts of sequences and series of numbers. The latter entites are used in calculus and analysis. Hilbert Space generalizes the usual spaces used there with a finite number of dimensions, Hilbert Space considers spaces with an infinite number of dimensions. It also makes extensive use of the Calculus of Variations.

Algebraic Construction of Geometries for Quantum Theory

To examine Hilbert Space we need, firstly, to define some functions of functions.

One of these functions of two functions is called the inner product <◦|◦>.

Another function of one function is called the norn $||F||$.

Let us firstly, define an inner product over a field of complex numbers, C. A complex inner-product space α is a complex vector space α with a function <., .>:. $\chi\chi \rightarrow$ C.

(positive measure)

$$\left(\alpha, \langle f \mid g \rangle\right) :=$$

$$\forall \Gamma \ \ \langle \Gamma | \Gamma \rangle \geq 0$$

(nowhere degenerate in its domain)

$$\forall \Gamma \ \ \langle \Gamma | \Gamma \rangle = 0 \Leftrightarrow \Gamma = 0$$

3. Conclusion

Quantum Physics is potentially the most illuminating theory of the world around us and of the universe in which we occupy a small part. In this paper, we have examined the algebraic foundations of this theory as the development of these was begun in the early twentieth century. We recommend learning modern scientific theory, chronologically, as the scientists themselves did. The reader may, however, skip ahead by reading (Green, 1997). The latter reference includes a bibliography of 119 items of modern research.

Acknowledgements

We wish to express our indebtedness and thanks to two students of Henry Leonard for their numerous recollections of his teachings in logic. They are: Joanne Eicher (University of Minnesota, Twin Cities) and Rolf George (University of Waterloo, Ontario). We are indebted to Henry S. Leonard for his paper (Leonard, 1969). We wish to thank Craig Smorynski, for calling (Hartshorne, 1997) to our attention. It contains an invaluable historical review of the development of axiomatic method in mathematics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

Green, B. (1997). String Theory on Calabi-Yau Maniforlds. In C. Efthimiou, & B. Greene, (Eds.), *Fields, Strings and Duality* (pp. 543-726), Singapore: World Scientific Publishing Co. Ltd.

Hartshorne, R. (1997). *Geometry: Euclid and Beyond*. New York: Springer Verlag.

Leonard, H. S. (1969). The Logic of Existence. *Philosophical Studies*, 7, 49-64. https://doi.org/10.1007/BF02221764

Reid, C. (1996). *Hilbert*. New York: Springer Verlag. https://doi.org/10.1007/978-1-4612-0739-9

Riesz, F., & Szőkefalvi-Nagy, B. (1990). *Functional Analysis*. Mineola, New York: Dover Publications.

Saxe, K. (2002). *Beginning Functional Analysis*. New York: Springer Verlag.. https://doi.org/10.1007/978-1-4757-3687-8