

Algebraic Thinking among Primary Pupils: A Boost for Interest in Mathematics

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ABSTRACT

This study was to help Primary pupils, specifically Basic five (5) pupils to improve their participation and interest in mathematics by promoting their thinking in algebra. It was also to develop the algebraic thinking abilities to promote active learning of Mathematics. Pre-intervention, Post-intervention tests and questionnaire were used in assessing the algebraic thinking abilities of the pupils. The entire class consisting of fifteen pupils was used as the population for the study. Another class of seventy eight pupils was used as a control. It was realised that the pupils performed poorly but for the intervention of Active Learning geared towards Cognitive Reasoning. The pupils though had a low level in developmental thinking with regard to solving algebraic problems and a dread for school because of Mathematics, later proved worthy of increasing their level of algebraic thinking. A new impetus was developed by the pupils as a result of the ability to tackle algebraic problems. This study suggests that Practical Active Learning geared towards Cognitive Reasoning should be used in fostering the algebraic thinking of primary pupils. It also suggests that pupils should be allowed to solve word problems the way they understand the problems by demonstrating with daily life activities and that teachers should keep on motivating pupils to enable them develop interest for Mathematics.

Keywords: practical active learning, algebraic thinking, developmental thinking, Mathematics, primary pupils

INTRODUCTION

Much can be said about ways in which the mathematics education community has narrowed the distance between the vision and the actual practice of reform-based teaching and learning in our schools (Borko et al, 2005). Mathematics is the means of sharpening the mind, shaping reasoning ability and developing personality, hence its immense contribution to the general and basic education of many countries (Asiedu-Addo & Yidana, 2004).

Regardless of the fact that mathematics is one of the basic subjects to build on for higher education, most students shy away from it. Most of them sometimes wish they could be excluded from mathematics lessons.

The study of Mathematics involves the application of everyday life activities to enhance the understanding of the concepts used in the teaching and learning of Mathematics. Mathematics is indisputably essential and its applications in our daily lives are widely evident in theory and practice. The mathematics syllabus in Ghana recommends the use of mathematics in our daily life by recognizing and applying appropriate mathematics problem-solving strategies (Atteh et al., 2017). Algebraic thinking is a tool for learning algebra whereas algebra is a tool box for learning Mathematics.

During an out-segment program at a Public Basic School it was realized that the pupils showed almost no interest in algebra and Mathematics at large. Performing basic operations like addition, subtraction, multiplication and division, was a challenge to the pupils.

This community (of school A) located in the Central region of Ghana has the main work of the indigenous people to be Farming. The community has few literates with most of the literates being foreigners. There is very little motivation for attending school since the parents were mostly about their businesses and gladly indulge pupils even at the expense of going to school.

This study seeks to look at fostering or promoting algebraic thinking among Primary School pupils and thereby boosting their interest in Mathematics. The purpose of this study is therefore to implore teachers and parents to guide and motivate basic school pupils in Ghana to improve their participation in mathematics by promoting their thinking in algebra. It is also to develop the algebraic thinking abilities to promote active learning of Mathematics.

Another community (of school B) located in the southern part of the Western Region of Ghana was used as a control. School B is a private school in a community of highly educated personalities.

In the planning of any curriculum, decisions are made concerning the philosophy of mathematics, the mathematics knowledge appropriate to the phase, the approach to teaching mathematics, and the subsequent assessment. The explicit expression of the underpinning philosophy, the mathematical knowledge, and the related teaching directives vary from country to country. The degree of control exerted centrally by the national education departments also varies. For example, in some education systems, such as that of the Netherlands, 4 broad statements and objectives are provided at the *mega* level for both socio-political and educational purposes, but, at the *micro* level, the details and interpretation of these statements for school purposes and the classroom work scheme are left to the teachers and textbook writers (Thijs & Van den Akker, 2009).

RELATED LITERATURE

Content of Primary Mathematics

The content of primary mathematics is more than just arithmetic. Current content includes number and operations, geometry, measurement, data analysis and beginning experiences with probability. Today, kindergarteners, first, and second graders also engage in algebraic relationship activities. The Madison Metropolitan School District K - 5 Grade Level Mathematics Standards organize math concepts and knowledge into four strands of mathematics content: number, operations and algebraic relationships; geometry; measurement; and data analysis and probability (Madison metropolitan school District, 2006).

In Ghana, the Mathematics curriculum for B1 - B6 has four strands. The strands include number, algebra, geometry & measurement and data. This is a new curriculum which started in September 2019 and it is clear that the emphasis on algebra is prevalent (NaCCA, 2019).

Algebra in Primary School

Algebra is the way we express generalizations about numbers, quantities, relations and functions. For this reason, good understanding of connections between numbers, quantities and relations is related to success in using algebra.

To improve students' learning in mathematics, it is necessary to understand the developmental mode of their thinking and reasoning. With the nature of mathematics that deals with abstract entities, students have difficulty in understanding mathematics concepts, especially those in algebra. Therefore, thinking, particularly algebraic thinking is a tool for understanding abstraction (Russell, 1999). Also to understand algebraic symbols, students have to understand the underlying operations and become fluent with the notational rules. These two kinds of learning, the meaning and the symbol, seem to be most successful when students know what is being expressed and have time to become fluent at using the notation. Students have to learn to recognise the different nature and roles of letters as: unknowns, variables, constants and parameters, and also the meanings of equality and equivalence. These meanings are not always distinct in algebra and do not relate unambiguously to arithmetical understandings, Mapping symbols to meanings is not learnt in one-off experiences (Watson Anne, 2009).

Algebra is an essential part of Mathematics in the primary school. In Ghana, specifically in the new curriculum for basic schools, algebra spans through basic four to basic six (NaCCA, 2019).

Algebraic Thinking

Algebraic thinking or reasoning involves forming generalizations from experiences with number and computation, formalizing these ideas with the use of a meaningful symbol system, and exploring the concepts of pattern and functions (Van de Walle, Karp, & Bay-Williams, 2011).

Algebraic reasoning permeates all of mathematics and is about describing patterns of relationships among quantities - as opposed to arithmetic, which is carrying out calculations with known quantities. In its broadest sense, algebraic reasoning is about generalizing mathematical ideas and identifying mathematical structures. Although formal symbolic algebra is introduced in the Ontario mathematics curriculum in the late junior grades, it should be fostered and nurtured right from Kindergarten. Algebraic reasoning is sometimes thought of as being only symbol manipulation and taught only in secondary grades. However, most educators agree that an emphasis should be placed on students developing algebraic understanding before they are introduced to formal symbolic representation and manipulation. For example, when students in primary grades notice that the order of numbers being added does not change the sum no matter which two numbers are being added, they are focusing on the structure of the relationship rather than the specific numbers that they are working with. This focus on the generalized property of addition (commutative property) is algebraic reasoning ($3 + 4 = 4 + 3$).

Again, Algebraic reasoning is based on our ability to notice patterns and generalize from them. Algebra is the language that allows us to express these generalizations in a mathematical way. Formulas, such as the area of a rectangle = length \times width ($A = l \times w$), are derived through such generalizations. The goal of "algebra for all" has been in place in this country for more than a decade, driven by the need for quantitatively literate citizens and recognition that algebra is a gatekeeper to more advanced mathematics and opportunities (Dudley, 1997; Silver, 1997).

The implication is clear: elementary and middle school mathematics instruction must focus greater attention on preparing all students for challenging middle and high school mathematics programs that include algebra. Thus, "algebraic thinking" has become a catch-all phrase for the mathematics teaching and learning that will prepare students with the critical thinking skills needed to fully participate in our democratic society and for successful experiences in algebra.

In a research by Kriegler, algebraic thinking was put into two major components: the development of mathematical thinking tools and the study of fundamental algebraic ideas. Mathematical thinking tools are analytical habits of mind. They are organized around three topics: problem-solving skills, representation skills, and quantitative reasoning skills. Fundamental algebraic ideas represent the content domain in which mathematical thinking tools develop. They are explored through three lenses: algebra as generalized arithmetic, algebra as a language, and algebra as a tool for functions and mathematical modeling (Kriegler S, 2011).

It was added that there are those who argue that the study of mathematics is important because it helps to develop logical processes and those who express concern about the lack of content and rigor within the discipline itself. Those who argue that the study of mathematics is important because it helps to develop logical processes, may consider mathematical thinking tools as the more critical component of mathematics instruction. On the other hand, those who express concern about the lack of content and rigor within the discipline itself may be focusing greater emphasis on the algebraic ideas themselves. In reality, both are important. One can hardly imagine thinking logically (mathematical thinking tools) with nothing to think about (algebraic ideas).

Early algebra is not about introducing traditional, formal algebra into primary schools, but instead is about developing arithmetical reasoning in conjunction with algebraic reasoning (Warren, 2002). Boero (2001) called this 'pre-algebra'. Some of the core ideas of pre-algebra can be introduced to primary school pupils, for example, recognising patterns and generalising and justifying patterns.

To develop their pre-algebraic thinking, young children must be engaged in generalization and algebraic activities such as making general statements about shapes or geometric patterns. Thus, the study of patterns may be a productive way of developing algebraic reasoning (Femini-Mundy, Lappan, & Phillips, 1997) and the ability to think algebraically in the elementary grades.

The act of developing an awareness of generality and applying it in the mathematical domain is itself an indicator of algebraic thinking (Irwin & Britt, 2005). Driscoll and Moyer (2001) constructed a guideline with five indicators of algebraic thinking, namely (a) systematically searching for a rule, (b) forming a generalized rule, (c) conjecturing a generalised rule, (d) representing this rule in other forms, and (e) connecting the different representations. Thus, pre-algebraic thinking is a process or an action (NCTM, 2004) that may be detected by some indicators.

Based on a review of the literature on how algebraic thinking emerges patterning processes, the nature of pre-algebraic thinking (Boero, 2001) and indicators of algebraic thinking, a conceptual framework for pre-algebraic thinking was constructed. This framework enabled the researchers to infer the pupils' pre-algebraic thinking based on some indicators related to what they said and did while solving the problems.

By using active learning methods, it is hoped that pupils will not only come to a deeper understanding of the issues involved, but also that their motivation and enthusiasm will be heightened. Every pupil and teacher brings with them into the classroom a diversity of skills, experiences, needs and expectations. It is important that you reflect on the dynamics of your class. Classroom surroundings influence how teachers and pupils feel and how they act. The classroom environment must be supportive of practical active learning and teaching.

A research by Anantika Sanchal and Sashi Sharma (2017) shows that researchers have identified important factors contributing to students' attitudes towards learning mathematics. These factors include the students themselves, the school, the teachers' beliefs and attitudes and their teaching methods. The teachers' teaching method mainly influences students' attitudes and interest.

Teachers can do many things to facilitate the classroom learning to heighten students' engagement level and confidence in learning mathematics. Sullivan and McDonough (2007) said that teachers can find ways to encourage student engagement and confidence in learning mathematics. This can be achieved by implementing meaningful activities embedded in real-life contexts.

Attitude towards Mathematics

Mathematics is considered by many individuals as a difficult subject to learn (Fennema, 1992). There are factors that affect pupils' Mathematics achievement positively or negatively. These include psychological and social factors. Also pupils tend to study more, the subject of the teacher they like.

Children have negative attitudes toward mathematics when they are not able to conceptualize or when they keep on getting low marks for a while. It is therefore imperative that teachers make all efforts to keep up the interest of pupils in the learning of Mathematics but more so, the parents. In as much as pupils have different absorption levels, adding up to what pupils' already know should be monitored steadily to make progress in each pupil. The same is true in the education of a child; if he/she didn't understand the previous lesson then he/she can't follow and understand the coming ones. In other words, learning with understanding can be viewed as making connections or establishing relationships either within existing knowledge or between existing knowledge and new information (Hiebert & Carpenter, 1992).

Negative attitude towards Mathematics is also common among most Parents in Ghana but mostly those with their wards in the Public Basic Schools. They try to avoid it for their children as far as possible. However, some parents though they see Mathematics to be hard still provide means of additional tutorials and motivations to get the children to understand and like Mathematics. These parents mostly have their children in the private schools. If students have a positive attitude towards mathematics, it is likely that they will give a large section of their study time to mathematics and try hard to master the knowledge and skill (Sharma Anikta & Verma Richa, 2017).

The role of mathematics in everyday life also contributes to the study of mathematics. It is mostly regarded as necessary for counting therefore the knowledge of counting is enough. In the case of young slow learners the parental attitude towards education is of very considerable significance in the pupil's view of worth (Larcombe, 1985).

METHODOLOGY

The method used to gather information for the study as well as completing the study was in three stages: the design, Selection and Intervention.

Design

The type of research conducted is action research. According to Hackman (2008), an action research aims at finding solution to an organization. The thought for this design was to foster the development of algebraic thinking among Basic five pupils in a Public Basic School in the Central Region of Ghana. Fifty-five Basic five pupils were engaged in the study, fifteen pupils from a public school and forty pupils from a private school.

Selection

The population target is pupils in the Public Basic School (School A). The total number of pupils in the school is one hundred and fifteen (115). This constitutes fifty nine (59) and fifty six (56) girls. However the population of the class under study (Basic 5) is fifteen, four girls and eleven boys. The population of the class used as a control is seventy eight (78). This number is a combination of two primary five (5) classes of school B (a private school) with a total population of 472 pupils in the primary school.

Instruments

Questionnaire and tests were the main instruments used for the study. Pre-intervention test and post-intervention tests were adopted to look into the performance of the pupils. Questionnaire was used in collecting information. The questionnaire used in this research has yes, no idea and no as options for the respondents. The 'no' and 'no idea' referred to as 'others'.

As data gathering devices, tests are among the most useful tools of educational research, for they provide the data for most experimental and descriptive studies in education. The instruments have been designed to describe and measure samples of aspects of human behaviour. These instruments assess variety of human abilities, potentials, achievements and behaviour tendencies. They possess different degrees of validity, reliability and applicability.

Two tests were used in this research: pre-intervention test and post-intervention test. The pre-intervention test provided an actual knowledge of the pupils before the researcher's intervention and the post-intervention test gave the aftermath of the researcher's intervention.

Intervention

The researcher used four days in two weeks to conduct the intervention.

Week 1, Day 1

A lesson was planned on the topic, addition and subtraction to foster the development of algebraic thinking of the pupils. A mini shop was role played in the classroom where buying and selling took place. The pupils played roles as shop keeper and customers. Three customers went to buy from the shop. The first customer bought items at GHC 35 and paid with GHC 50 note. The second customer bought items at GHC 21 and paid with GHC 50 note. The third customer bought items at a cost of GHC 62 and gave to the shop keeper, two GHC 50 notes.

Solution:

The researcher gave each pupil a sheet of paper to calculate the total costs of items the first customer bought. They were then asked to subtract the cost of the items bought from the amount given to the shop keeper ($GHC\ 50 - GHC\ 35 = GHC\ 15$). The researcher asked them to do the same for the second and third customers.

Week 1, Day2

In the second day of week one, the researcher discussed with pupils how to interpret mathematical problems. Pupils were put into 3 groups with each group having 70 bottle tops. The researcher explained and solved an example with the pupils.

The example: Mummy gave little Sally 17 biscuits for scoring 100% in her tests. Her father added nine more and senior brother gave two. Little Sally exchanged five of the biscuits with toffees from her friend. How many biscuits does Sally have now?

Solution:

Pupils were guided to solve the problem. The researcher made pupils count 17 of the bottle tops. They counted 9 more and 2 more. After putting all together, they took out 5 bottle tops to arrive at the solution.

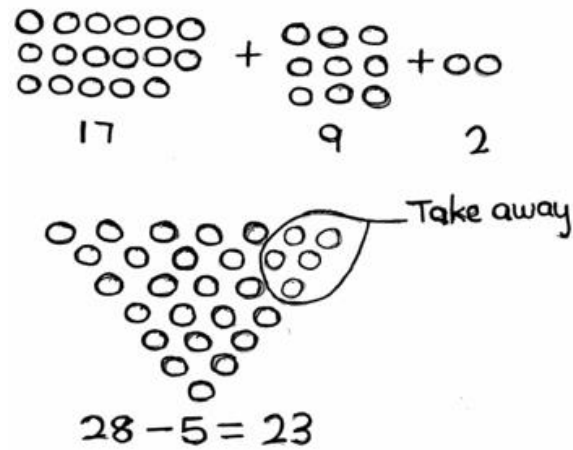


Figure 1. Solution by means of counting bottle tops

Week 2, Day 1

On the first day of the second week, the researcher read out a mathematical problem to the pupils.

Guiding the pupils to solve the problem, they were made to draw a circle with seven divisions. The pupils were then assisted to arrange bottle tops equally on the divisions of the circle.

Problem: Kofi had 7 pieces of pie that cost a total of GHC 35. What is the price of one piece of pie?

Solution:

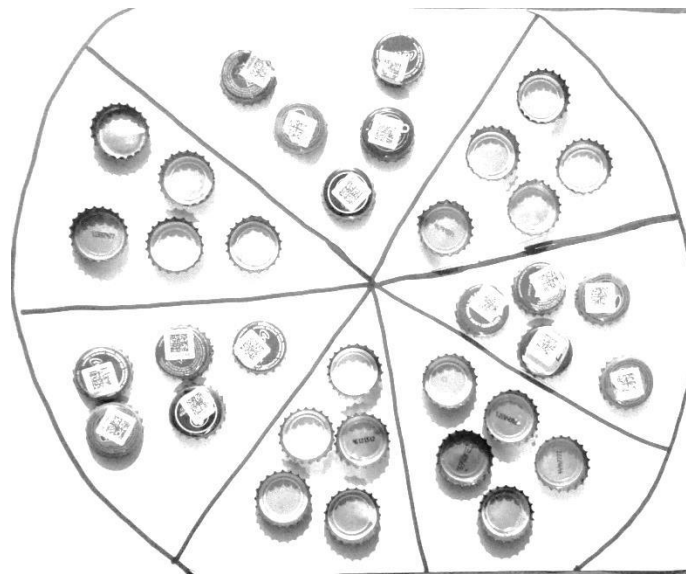


Figure 2. Division of thirty five by seven

Thirty five bottle tops representing GHC 35.00 are distributed one at a time on all the 7 pieces of pie.

Each division had five bottle tops, therefore GHC 5 is the price of one piece of pie.

Week 2, Day 2

On the last day of the intervention, the pupils were taken through a picture chart showing items and values.

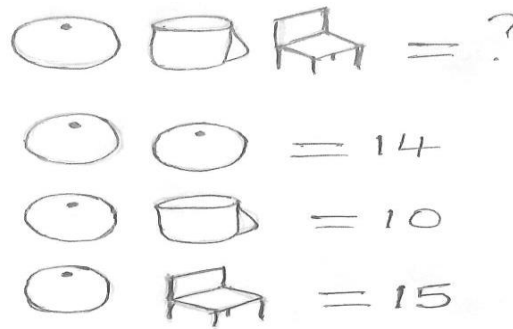


Figure 3. Chart shows the items: orange, cup and chair

The pupils were asked to study the chart to provide the unknown value (See **Figure 3**).

Solution: The pupils were divided into three groups. Each group was given two oranges, a cup and a chair. Each group put two oranges together and tagged them with the value fourteen. They were asked to find the value of one orange by identifying a number that adds to the same number to give fourteen. ($7+7=14$; 1 orange = 7)

Knowing an orange to be seven, a cup was added and together tagged with the number ten.

$7 + \text{----} = 10$; $10 - 7 = 3$. Therefore, a cup = 3.

Again, an orange was picked and added to a chair. Both were tagged together as fifteen.

$7 + \text{----} = 15$; $15 - 7 = 8$. Therefore, a chair = 8.

Hence putting orange, cup and chair together gives $7 + 3 + 8 = 18$.

RESULTS ANALYSIS

Table 1. Pupils' correct answering of each item (question) in pre-test and post-test

Item	Pre-test	Post-test
1	60%	80%
2	33.3%	93%
3	46.6%	100%
4	0%	73.3%

On item one, it sought to demand pupils to make a sum of 5 by knocking down any two bottles with the number written on them. In the pre-test, nine pupils representing 60% were able to make a sum of 5, whereas in the post-test, twelve (12) pupils were able to make this sum representing 80%. On item two, pupils were required to find two different ways of making the following sums: 5, 6, 7 and 9. Examples are, $2 + 3 = 5$ and $4 + 1 = 5$. So the pupils were required to add numbers to make the sums above. In the pretest, five pupils (33.3%) were able to make the sum in two different ways. Unlike the pre-test, the post-test recorded fourteen pupils representing 93% of the pupils who made the sums in two different ways for the values 5, 6, 7, 8, and 9.

On item 3, pupils were to make totals of 9, 10, 13 and 15 by choosing from four cards with numbers on each. Pupils were to select four cards and add them to make the totals. Seven pupils, 46.6% in terms of percentage were able to make the totals but in the post-test fifteen pupils representing 100% were able to make the totals.

The fourth (4th) item, for the pretest and post-test, required pupils to find the value of one guitar from a group of four guitars and one ball. In the pretest, no pupil was able to tackle the item while in the post-test; seven pupils representing a percentage of 73.3 were able to tackle the item. (See **Appendix A**).

DISCUSSIONS

This was measured by a questionnaire that was administered to the pupils. The analysis was performed on the responses pupils gave to the items in the questionnaire (See **Table 2**).

Table 2. Interview questionnaire and responses from school under study (School A)

Statements / reply	YES	OTHERS
1. I think mathematics is important in life.	15 (100%)	0 (0%)
2. My teachers listened to me anytime I had problems in Mathematics.	10 (66.7%)	5 (33.3)
3. I am able to learn more Mathematics on my own.	9 (60%)	6 (40%)
4. My mathematics teachers use cane on us when we are learning mathematics.	2 (13.3%)	13 (86.7%)
5. I don't like mathematics at all.	14 (93.3%)	1 (6.7%)
6. I am happy when I am learning mathematics.	5 (33.3%)	10 (66.7%)
7. I enjoy doing mathematics.	3 (20%)	12 (80%)
8. I do not like school.	9 (60%)	6 (40%)

The Need to Foster Algebraic Thinking through Active Learning of Mathematics

The questionnaire had two scales: 'Yes' and 'Others' ('No' & 'No idea'). In this analysis, the "yes" responses were compared with the 'others'. Trying to know whether they see math as important, 15 participants (100%) responded positively. This means they value the importance of mathematics in their lives. The responses from my teachers listened to me anytime I had problems in Mathematics recorded 10 pupils (66.7%) saying yes, meaning attention was given to anyone who had problems in Math. Also the responses from the 3rd item I am able to learn more Mathematics on my own, had 9(60%). On finding out whether Mathematics teachers used cane on them when learning Mathematics, five (5) pupils, 13.3%, said yes while 86.7% said no idea & no. This means the teachers actually do not use cane during Mathematics lessons but sometimes. On responding to I don't like Math at all, the responses were different. 93.3% of the respondent said yes and 6.7% said otherwise. It is clear here that the pupils' liking Mathematics is negative.

The disparity between the negative responses the pupils gave to the items "I don't like mathematics at all" and "I enjoy doing mathematics" shows that the pupils' not liking mathematics correlates to the pupils not enjoying Mathematics. On the 8th item, 60% of the pupils don't seem to like school as they replied yes. The other, 40% however have interest for school. This was evident in their reply of no to the item, "I do not like school" (See [Table 2](#)).

Effectiveness of Active Learning in Pupils' Mathematics Learning

This section of the analysis attempted to evaluate the effectiveness of the use of active learning in mathematics teaching and learning. Specifically, the impact of active learning usage on pupils' performance was investigated. This employed the use of mean scores and standard deviation of participants in the pretests and post-tests. The result of the analysis of this measurement is summarized in [Tables 3](#) and [4](#).

The results present the pupils' (school A) mean performance in the pretest to be 3.97 with standard deviation 2.43. Pupils were then put on teaching intervention of active learning after which they were retested. The mean score in the post-test was 13.67 with standard deviation, 4.02. It is notable that an average performance, approximately 14/20 (70%) was achieved after the intervention (See [Table 3](#)).

Table 3. Pre-test and Post-test of school A (Public school under study)

TEST	TOTAL FREQUENCY	MEAN	STANDARD DEVIATION
Pre-test	15	3.97	2.43
Post-test	15	13.67	4.02

Also the results of (school B) estimates a mean score which declares an average performance of 9/20 (45%) in the pretest changing to 15/20 (75%) after the intervention of algebraic thinking through active learning (See [Table 4](#)).

Table 4. Pre-test and Post-test of school B (Private school used as control)

TEST	TOTAL FREQUENCY	MEAN	STANDARD DEVIATION
Pre-test	78	9.30	4.88
Post-test	78	14.54	3.69

Comparing their performances in the pretest and post-test, it was realized that their performances after the intervention were far better than their performances before the intervention, in the pretest. It can therefore be said that the gain in their performances was due to the intervention. Hence it is reasonable to think that, algebraic thinking through active learning has an impelling effect of improving pupils' performances in Mathematics.

CONCLUSION

The class five pupils (B5) of the school under study were found to have a negative attitude towards learning Mathematics. They showed a low level of algebraic and developmental thinking with regards to solving algebraic problems. Furthermore, the little interest in attending school on the part of the pupils had to do with Mathematics. The use of cane by teachers during Mathematics lessons cannot be said in this work as a cause for the minimal interest of pupils in going to school.

According to the findings and the discussions of the research, it can be concluded that the pupils performed poorly in the pre-intervention test specifically the fourth item because it demanded more algebraic thinking. The pupils were however very good after the intervention of algebraic thinking through active learning. The pupils proved worthy of increasing their level of algebraic thinking.

The pupils found algebraic reasoning as an everyday activity and therefore developed a new impetus towards learning Mathematics.

RECOMMENDATIONS

The outcome of this work recommends practical active learning in fostering the algebraic thinking of primary pupils. Pupils should be allowed to solve word problems the way they understand the problems by demonstrating with daily life activities.

The teaching and learning of Mathematics will be more interesting if teachers boost the algebraic thinking of pupils and also make it more practical (relating to daily life activities) and activity based. Also, activities when geared toward cognitive reasoning will enhance pupils' algebraic thinking.

The Math-phobia faced by Primary pupils can be cleared by employing demonstrations (relevant to everyday life activities) with teaching materials and more so motivating the pupils.

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APPENDIX A

PRE TEST

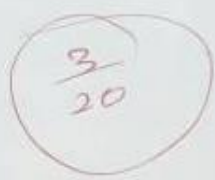
Name: Lucy Obeng

1. Look at the following carefully. Which bottles must you knock down to give you 5 exactly?


 $= 2 + 1 + 2$

2. Find 2 different ways to:

- Score 5 = $1 + 3 + 1$
- Score 6 = $3 + 1 + 2$
- Score 7 = $4 + 2 + 1$
- Score 9 = $5 + 1 + 3$



3. Choose from the 4 cards

3

4


8

6


Make these totals

- 9 = $6 + 3$
- 10 = $4 + 6$
- 13 = $8 + 4 + 1$
- 15 = $8 + 4 + 3$


4. Find the value of one guitar




= 12



= 7



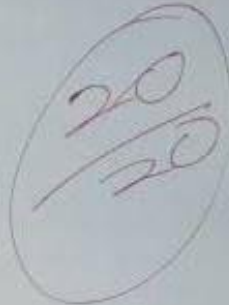
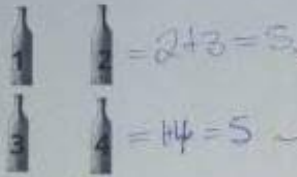
= 19



= 7

Lucy Obeng

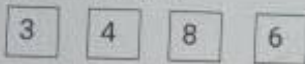
1. Look at the following carefully. Which bottles must you knock down to give you 5 exactly?



2. Find 2 different ways to:

- a. Score 5 = $3 + 2$ and $4 + 1$
- b. Score 6 = $2 + 3$ and $4 + 2$
- c. Score 7 = $2 + 4$ and $3 + 4$
- d. Score 9 = $5 + 4$ and $3 + 6$

3. Choose from the 4 cards



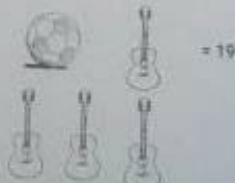
Make these totals

- 9 = $6 + 3$
- 10 = $6 + 4$
- 13 = $6 + 3 + 4$
- 15 = $8 + 3 + 4$

4. Find the value of one guitar



$= 12 \div 4 = 3$



$19 - 3 = 16$
 $16 \div 4 = 4$

QUESTIONNAIRE FOR BASIC FIVE PUPILS

Tick(✓) the appropriate response based on your option	YES	NO IDEA	NO
1. I think mathematics is important in life	✓		
2. My teachers listened to me anytime I had problems in Mathematics	✓		
3. I am able to learn more mathematics on my own	✓		
4. My mathematics teachers used cane on us when we are learning mathematics		✓	
5. I don't like mathematics at all			✓
6. I am happy when I am learning mathematics	✓		
7. I enjoy doing mathematics		✓	
8. I do not like school	✓		