

# ALGORITHM 654

## FORTRAN Subroutines for Computing the Incomplete Gamma Function Ratios and their Inverse

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The FORTRAN subroutines GRATIO and GAMINV given here are the subroutines described in [1] for computing the incomplete gamma function ratios and their inverse. A sample program exercising GRATIO and GAMINV is given. The following functions are used.

$$\begin{aligned}\text{ERF}(x) &= \text{erf } x \\ \text{ERFC1}(i, x) &= \begin{cases} \text{erfc } x & \text{if } i = 0 \\ \exp(x^2)\text{erfc } x & \text{if } i \neq 0 \end{cases} \\ \text{REXP}(x) &= \exp(x) - 1 \\ \text{ALNREL}(a) &= \ln(1 + a) & a > -1 \\ \text{RLOG}(x) &= x - 1 - \ln x & x > 0 \\ \text{RCOMP}(a, x) &= e^{-x} x^a / \Gamma(a) & a > 0, x \geq 0 \\ \text{GAMMA}(a) &= \Gamma(a) & a \neq 0, -1, -2, \dots \\ \text{GAM1}(x) &= 1/\Gamma(1 + a) - 1 & -.5 \leq x \leq 1.5 \\ \text{GAMLN}(A) &= \ln \Gamma(a) & a > 0 \\ \text{GAMLN1}(x) &= \ln \Gamma(1 + x) & -.2 \leq x \leq 1.25\end{aligned}$$

These functions, written by A. H. Morris, are part of the NSWC mathematics subroutine library [3].

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### Machine-Dependent Constants

The function SPMPAR provides the machine-dependent constants needed by GRATIO and GAMINV. It is necessary that SPMPAR be properly defined for the computer arithmetic being used. The constants are defined in the in-line documentation of SPMPAR. Values for these constants are given for many computers in the in-line documentation. SPMPAR, released by Argonne National Laboratory, is an adaptation of the Bell Laboratories function R1MACH [2].

### Transportability

All coding adheres to the 1966 and 1977 ANSI FORTRAN standards. It is assumed that a floating point arithmetic of 6 or more digits is being used. The codes were designed specifically for  $k$ -digit arithmetics where  $k \leq 14$ . If  $k > 14$ , then only 14-digit accuracy will normally be obtained.

### REFERENCES

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3. MORRIS, A. H., JR. NSWC Library of Mathematics Subroutines. Rep. NSWC TR 86-251, Naval Surface Weapons Center, Dahlgren, Va., 1987.

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