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ALGORITHMIC ASPECTS OF NEIGHBORHOOD NUMBERS*

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Abstract. In a graph $G = (V, E)$, $E[v]$ denotes the set of edges in the subgraph induced by $N[v] \equiv \{v\} \cup \{u \in V\colon uv \in E\}$. The neighborhood-covering problem is to find the minimum cardinality of a set C of vertices such that $E = \cup\{E[v]\colon v \in C\}$. The neighborhood-independence problem is to find the maximum cardinality of a set of edges in which there are no two distinct edges belonging to the same $E[v]$ for any $v \in V$. Two other related problems are the clique-transversal problem and the clique-independence problem. It is shown that these four problems are NP-complete in split graphs with degree constraints and linear time algorithms for them are given in a strongly chordal graph when a strong elimination order is given.

Key words. neighborhood-covering, neighborhood-independence, clique-transversal, clique-independence, chordal graph, strongly chordal graph, split graph, NP-complete

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1. Introduction. The concept of neighborhood number was first introduced by Sampathkumar and Neeralagi [SN]. Suppose that $G = (V, E)$ is a finite undirected graph with vertex set V and edge set E. The (*open*) *neighborhood* $N(v)$ of a vertex v is the set of vertices adjacent to v, and the *closed neighborhood* $N[v]$ is $\{v\} \cup N(v)$. A *neighborhood-covering set* C is a set of vertices such that $E = \cup\{E[v]\colon v \in C\}$, where $E[v]$ is the set of edges in the subgraph induced by $N[v]$. (This definition is slightly different from the original one in [SN]; we follow the terminology in [LT].) The *neighborhood-covering number* $\rho_N(G)$ of G is the minimum cardinality of a neighborhood-covering set in G. A *neighborhood-independent set* of G is a set of edges in which there are no two distinct edges belonging to the same $E[v]$ for any $v \in V$. The *neighborhood-independence number* $\alpha_N(G)$ of G is the maximum size of a neighborhood-independent set in G. These two parameters are related by a min-max duality inequality: $\alpha_N(G) \leqq \rho_N(G)$ for any graph G. A graph is called *neighborhood-perfect* if $\alpha_N(H) = \rho_N(H)$ for every induced subgraph H of G.

Two other related problems are defined as follows. In a graph $G = (V, E)$, a *clique* is a set of pairwise adjacent vertices. A *maximal clique* is a clique of size $\geqq 2$ that is maximal under inclusion. A *clique-transversal set* of G is a set of vertices that meets all maximal cliques of G. As defined in [T], the *clique-transversal number* $\tau_C(G)$ of G is the minimum cardinality of a clique-transversal set in G. We now introduce the concept of a *clique-independent set*, which means a collection of pairwise disjoint maximal cliques. The *clique-independence number* $\alpha_C(G)$ of G is the maximum size of a clique-independent set in G. There is also a min-max duality inequality: $\alpha_C(G) \leqq \tau_C(G)$ for any graph G. Note that the clique-independence number of a triangle-free graph is equal to its matching number and hence can be computed in polynomial time.

Various properties of $\rho_N(G)$, $\alpha_N(G)$, $\tau_C(G)$, and $\alpha_C(G)$ have been studied in [SN], [LT], [T], [AST], and [EGT]. The aim of this paper is to investigate some problems concerning the algorithmic complexity of determining these four parameters of a given

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graph. Erdös, Gallai, and Tuza [EGT] proved that the problem of finding the clique-transversal number is NP-complete over the class of triangle-free graphs, and more generally over the class of graphs with girth at least g for any fixed $g \geqq 4$. Lehel and Tuza [LT] gave an $O(|V| + |E|)$ algorithm for finding $\rho_N(G)$ and $\alpha_N(G)$ of an interval graph G. Wu [W] gave an $O(|V|^3)$ algorithm for determining $\rho_N(G)$ and $\alpha_N(G)$ of a strongly chordal graph G.

In § 3 we prove that the problems of finding $\rho_N(G)$, $\alpha_N(G)$, $\tau_C(G)$, and $\alpha_C(G)$ are NP-complete over the class of split graphs with degree constraints. Section 4 gives linear time algorithms for determining $\rho_N(G)$, $\alpha_N(G)$, $\tau_C(G)$, and $\alpha_C(G)$ of a strongly chordal graph G if a strong elimination order is available.

2. Terminology.

The concept of chordal graph was introduced by Hajnal and Surányi [HS] in connection with the theory of perfect graphs; see [Go]. A graph is *chordal* (or *triangulated*) if every cycle of length greater than three has a chord (i.e., every induced cycle is a triangle). One of the most important properties of a chordal graph G is that its vertices have a *perfect* elimination order $v_1, v_2, \ldots, v_n$; i.e., for each i $(1 \leqq i \leqq n)$, $N_i[v_i]$ is a clique, where $N_i[x]$ is the closed neighborhood of x in the subgraph G_i of G induced by $\{v_i, v_{i+1}, \ldots, v_n\}$. Note that any maximal clique of a chordal graph G is equal to some $N_i[v_i]$, but $N_i[v_i]$ is not necessarily an maximal clique.

Two interesting subclasses of chordal graphs discussed in this paper are strongly chordal graphs and split graphs. An *s-sun* (or *incomplete s-trampoline*) is a chordal graph with a Hamiltonian cycle $x_1, y_1, x_2, y_2, \ldots, x_s, y_s, x_1$ such that each y_i is of degree two. A *strongly chordal graph* (or *sun-free chordal graph*) is a chordal graph without any s-sun as an induced subgraph for all $s \geqq 3$. It was proved in [F1] that a graph is strongly chordal if and only if its vertices have a *strong elimination order* $v_1, v_2, \ldots, v_n$; i.e., for each i $(1 \leqq i \leqq n)$, $N_i[v_j] \subseteq N_i[v_k]$ when $v_j, v_k \in N_i[v_i]$ and $j < k$. Note that a strong elimination order is always a perfect elimination order. Anstee and Farber [AF] gave $O(|V|^3)$ algorithms; Hoffman, Kolen, and Sakarovitch [HKS] gave an $O(|V|^3)$ algorithm; Lubiw [Lu] gave an $O(|E| \log^2 |E|)$ algorithm; Paige and Tarjan [PT] gave an $O(|E| \log |E|)$ algorithm; and Spinrad [S] gave an $O(|V|^2)$ algorithm for recognizing if a graph $G = (V, E)$ is strongly chordal and for finding a strong elimination order when G is strongly chordal.

A graph $G = (V, E)$ is *split* if its vertex set V can be partitioned into a clique V_1 and an independent set V_2. Every split graph is chordal, and a natural perfect elimination order is given by listing the vertices in V_2 first and then the vertices in V_1. Note that an s-sun in which $\{x_1, x_2, \ldots, x_s\}$ is a clique is a split graph.

3. Split graphs and NP-completeness.

Let us recall the following two problems; see [CN1], [CN2], and [F2]. A *dominating set* D of a graph $G = (V, E)$ is a set of vertices such that every vertex not in D is adjacent to some vertex in D; i.e., $V = \cup\{N[v]: v \in D\}$. The *domination number* $\delta(G)$ of G is the minimum cardinality of a dominating set in G. A 2-*stable set* of G is a set of vertices in which any two distinct vertices are of distance greater than 2. The 2-*stability number* $\alpha_2(G)$ of G is the maximum cardinality of a 2-stable set in G. Note that $\alpha_2(G) \leqq \delta(G)$ for any graph G.

THEOREM 1. *It is* NP-*complete to determine the neighborhood-covering number, the clique-transversal number, and the domination number of a split graph with only degree*-2 *vertices in the independent set.*

Proof. Suppose that $G = (V, E)$ is a split graph without isolated vertices such that V is the disjoint union of a clique V_1 and an independent set V_2. Without loss of generality, we may assume that $N[x]$ is a proper subset of V_1 for any $x \in V_2$ (otherwise, we move x from V_2 to V_1). So the only maximal cliques of G are V_1 and $N[x]$ for all $x \in V_2$.

By the fact that $N[x] \subseteq N[y]$ for any $x \in V_2$ and $y \in N(x)$, we can always find a minimum neighborhood-covering set $C \subseteq V_1$. The same is true for clique-transversal sets and dominating sets. In fact, these three terms are then identical, and so $\rho_N(G) = \tau_C(G) = \delta(G)$.

Note that split graphs are in one-to-one correspondence to hypergraphs in which multiple edges are allowed. Vertices in the clique V_1 of a split graph G correspond to vertices of the hypergraph, and a nonisolated vertex y in the independent set V_2 corresponds to an edge, which is $N_G(y)$, of the hypergraph. It is then clear that $\delta(G)$ is equal to the transversal number of the corresponding hypergraph H_G, which is the minimum number of vertices meeting all edges. Hence the theorem follows from the fact that determining the transversal number of a 2-uniform hypergraph (i.e., a graph) is NP-complete; this problem is called the "vertex cover" problem and also the "hitting set" problem on pp. 190 and 222, respectively, of [GJ]. □

THEOREM 2. *It is* NP-*complete to determine the neighborhood-independence number, the clique-independence number, and the* 2-*stability number of a split graph with only degree*-3 *vertices in the independent set.*

Proof. A neighborhood-independent set of a split graph G must be of the form $\{x'x \in E: x \in S\}$ for some 2-stable set $S \subseteq V_2$. Moreover, a clique-independent set of G is of the form $\{N[x]: x \in S\}$ for some 2-stable set $S \subseteq V_2$. These, together with the fact that any 2-stable set of G is a subset of V_2, imply that $\alpha_N(G) = \alpha_C(G) = \alpha_2(G)$.

Also, $\alpha_2(G)$ is equal to the matching number, which is the maximum number of pairwise disjoint edges, of the corresponding hypergraph H_G as described in the proof of Theorem 1. Hence the theorem follows from the fact that determining the matching number of a 3-uniform hypergraph is NP-complete; a special case of this problem is called "three-dimensional matching" (see [GJ, p. 221]). □

Note that Chang and Nemhauser [CN1] proved that it is NP-complete to determine the domination number and the 2-stability number of a split graph without degree constraints. Moreover, the NP-completeness of the neighborhood-covering/independence problem was first observed by Lehel [L] by a different reduction. Let us note further that Theorems 1 and 2 remain valid under the assumption that the degrees of all vertices in the independent set are equal to k for some $k \geqq 3$.

For any graph $G = (V, E)$, we define the *neighborhood-split graph* $S(G)$ of G in the following way. The vertex set of $S(G)$ is $V \cup E$. In $S(G)$, any two vertices of V are adjacent, E is an independent vertex set, and an $e \in E$ is adjacent to a $v \in V$ if and only if $e \in E[v]$. Note that $S(G)$ has no isolated vertex if G has at least two vertices. The following statement is immediately seen from the definitions.

PROPOSITION 3. *For any graph G with at least one edge*, $\rho_N(G) = \delta(S(G))$ *and* $\alpha_N(G) = \alpha_2(S(G))$.

A structural relation between G and $S(G)$ is given by the following result.

THEOREM 4. *If G is strongly chordal, then so is* $S(G)$.

Proof. Since G is strongly chordal, its vertices have a strong elimination order $v_1, v_2, \dots, v_n$. We order the vertices of $S(G)$ as $e_1, e_2, \dots, e_m, v_1, v_2, \dots, v_n$ in such a way that, for any $e_i = (v_{i_1}, v_{i_2})$, $e_j = (v_{j_1}, v_{j_2})$, $i < j$, $i_1 < i_2, j_1 < j_2$, we have that $i_1 < j_1$ or ($i_1 = j_1$ and $i_2 < j_2$). It is easy to check that this order is a strong elimination order of $S(G)$. Thus $S(G)$ is strongly chordal. □

Note that the strong elimination order of $S(G)$ in the proof of Theorem 4 can be obtained in linear time from a strong elimination order of G. By Proposition 3 and Theorem 4, we can use the linear algorithms [F2], [HKS] for the domination number and the 2-stability number to find the neighborhood-covering number and the neighborhood-independence number of a strongly chordal graph. However, $S(G)$ has $|V| +$

$|E|$ vertices and $O(|V|\,|E|)$ edges. So this method gives an $O(|V|\,|E|)$ algorithm. Actually, the algorithm in [W] is just this method without describing $S(G)$.

4. Efficient algorithms in strongly chordal graphs.

In this section, we derive efficient algorithms for finding $\rho_N(G)$, $\alpha_N(G)$, $\tau_C(G)$, $\alpha_C(G)$, and the corresponding optimum solution sets of a strongly chordal graph G. Suppose that a strong elimination order v_1, $v_2, \ldots, v_n$ of G is given. Note that this is also a perfect elimination order. For technical reasons, we add an isolated vertex v_0 to G.

Recall that $N_i[x]$ (respectively, $N_i(x)$) is the closed (respectively, open) neighborhood of vertex x in the subgraph G_i of G induced by $\{v_i, v_{i+1}, \ldots, v_n\}$. For simplicity, we call $v_i < v_j$ if $i < j$. For each $v_i \in V$, denote by $v_{m(i)}$ the maximum element in $N[v_i]$; i.e., $m(i) = \max\{j: v_j \in N[v_i]\}$.

LEMMA 5. *A clique-transversal set is a neighborhood-covering set for any graph.*

Proof. The lemma follows from the fact that each edge is contained in a maximal clique. □

LEMMA 6. *In a graph, replacing each edge of a neighborhood-independent set by a maximal clique containing it yields a clique-independent set.*

Lemmas 5 and 6, together with the min-max duality inequalities in § 1, give that, for any graph G,

$$\alpha_N(G) \leqq \rho_N(G) \leqq \tau_C(G) \quad \text{and} \quad \alpha_N(G) \leqq \alpha_C(G) \leqq \tau_C(G). \tag{4.1}$$

The idea of our algorithms is to find a clique-transversal set C, which is also a neighborhood-covering set by Lemma 5, a clique-independent set I_C, and a neighborhood-independent set I_N such that $|C| = |I_C| = |I_N|$. If such sets are found, then they are optimum solutions for the four problems, and all inequalities in (4.1) are equalities. This provides an algorithmic proof for a special case of the following result.

THEOREM 7 (see [LT]). $\alpha_C(G) = \alpha_N(G) = \rho_N(G) = \tau_C(G)$ *for any odd-sun-free chordal graph* G.

Algorithm NHD (NHD means NeighborHooD)

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1.   $C \leftarrow \emptyset$;
2.   $I_C \leftarrow \emptyset$;
3.   $I_N \leftarrow \emptyset$;
4.   identify all $i$ such that $N_i[v_i]$ is a maximal clique;
5.   for $i = 1$ to $n$ do
6.      if $N_i[v_i]$ is a maximal clique and $N_i[v_i] \cap C = \emptyset$ then do
7.         $v_p \leftarrow \max\{v_0\} \cup (N[v_i] \cap C)$; { Note that $v_p < v_i$ now. }
8.         $v_j \leftarrow \min(N_i(v_i) - N_p[v_p])$;
9.         $I_N \leftarrow I_N \cup \{v_i v_j\}$;
10.        $I_C \leftarrow I_C \cup \{N_i[v_i]\}$;
11.        $v_{m(i)} \leftarrow \max N[v_i]$;
12.        $C \leftarrow C \cup \{v_{m(i)}\}$;
13.        end if;
14.     end for.
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THEOREM 8. *Algorithm* NHD *gives a minimum clique-transversal set* C, *a maximum clique-independent set* I_C, *and a maximum neighborhood-independent set* I_N *for a strongly chordal graph* G *in linear time when a strong elimination order is given.*

Proof. By steps 6, 11, and 12 of Algorithm NHD, the final C is a clique-transversal set of G.

In step 8, v_j must exist; otherwise, $N_i[v_i] \subseteq N_p[v_p]$ would imply that $N_i[v_i]$ is not a maximal clique. Suppose that v_iv_j and $v_{i'}v_{j'}$ (with $i' < i$) are two distinct edges of I_N that are both in some $E[v_q]$. Consider the set C at the beginning of iteration i, i.e., when step 8 is just done. For the case of $q \leqq i'$, since $q \leqq i' < i < j$, $v_{m(i')} \in N_{i'}[v_{i'}] \subseteq N_q[v_{i'}] \subseteq N_q[v_i] \subseteq N_q[v_j]$; i.e., v_iv_j and $v_{i'}v_{j'}$ both are in $E[v_{m(i')}]$. For the case of $i' < q$, since $i' < q \leqq m(i')$, v_i, $v_j \in N_{i'}[v_q] \subseteq N_{i'}[v_{m(i')}]$; i.e., v_iv_j and $v_{i'}v_{j'}$ both are in $E[v_{m(i')}]$. Note that $v_{m(i')} \in C$, since, in iteration i', we put $v_{i'}v_{j'}$ into I_N and $v_{m(i')}$ into C. By the choice of v_p and v_j (in steps 7 and 8), $v_pv_i \in E$ and $v_pv_j \notin E$, and $v_p \equiv v_{m(i'')}$ for some $v_{i''}v_{j''} \in I_N$ with $m(i') < m(i'') < i$. So $v_p = v_{m(i'')} \in N_{m(i')}[v_i] \subseteq N_{m(i')}[v_j]$, which contradicts $v_pv_j \notin E$. Therefore I_N is a neighborhood-independent set of G.

By Lemma 6, I_C is a clique-independent set of G. Since $|C| = |I_C| = |I_N|$, these three sets are optimum solutions of these four problems.

Next, we show that Algorithm NHD has running time linear in $|V| + |E|$. First, step 4 can be performed by Gavril's linear algorithm; see [G]. In iteration i, step 6 needs $|N_i[v_i]|$ operations to check if $N_i[v_i] \cap C = \varnothing$. This can be done if C is represented by a Boolean function f as follows:

$$f(i) = \begin{cases} 1, & \text{if } i \in C, \\ 0, & \text{if } i \notin C; \end{cases}$$

then we check if $f(q) = 0$ for all $v_q \in N_i[v_i]$. Step 7 can also be done in the same way.

For step 8, we keep an array $g(1\!:\!n)$ whose values are all initially zero. At the beginning of iteration i, $g(1\!:\!n)$ contains values $< i$. To find v_j of step 8, we first set $g(q) \leftarrow i$ for all $v_q \in N_p[v_p]$ and then check if $g(q) < i$ for each $v_q \in N_i[v_i]$ to obtain v_j. Note that $v_p \in N[v_i]$ and $v_p < v_i$ imply that $N_p[v_p] \subseteq N_p[v_i] \subseteq N[v_i]$. So step 8 needs $|N_i(v_i)| + |N_p[v_p]| \leqq 2|N[v_i]|$ operations.

Finally, steps 9, 10, and 12 need constant time, and step 11 needs $|N[v_i]|$ time. So the total running time is $O(\sum_i \deg(v_i) + 1) = O(|V| + |E|)$. □

We can modify Algorithm NHD slightly to get a simpler one as follows. First, we delete step 4 from the algorithm. Then we replace step 6 by step 6′ as follows:

6′. **if** $N_i[v_i] \cap C = \varnothing$ **then do**.

Also, insert step 8.5 between steps 8 and 9, shown below:

8.5. **if** v_j does not exist **then** go to 13.

All results are the same, except that we need not identify all maximal cliques.

THEOREM 9. *The modified algorithm gives a minimum clique-transversal set C, a maximum clique-independent set I_C, and a maximum neighborhood-independent set I_N for a strongly chordal graph G in linear time when a strong elimination order is given.*

Proof. The argument is the same as in the proof of Theorem 8, except that we must prove that, in iteration i, $N_i[v_i]$ is a maximal clique if and only if v_j exists.

Note that if v_j does not exist, then either $N_i(v_i) = \varnothing$, and so $N_i[v_i] = \{v_i\}$ is not a maximal clique; else $N_i[v_i] \subseteq N_p[v_p]$, and so $N_i[v_i]$ is not a maximal clique.

On the other hand, suppose that v_j exists. Then $N_i(v_i) \neq \varnothing$, and so $|N_i[v_i]| \geqq 2$. Suppose that $N_i[v_i]$ is not a maximal clique; i.e., $N_i[v_i]$ is a subset of some maximal clique $N_q[v_q]$, where $v_q < v_i$. Note that $N_q[v_q] \cap C \neq \varnothing$ by the algorithm now, say $v_{m(i')} \in N_q[v_q] \cap C$. Then $v_{i'}v_{j'}$, $v_iv_j \in E[v_{m(i')}]$. By a similar argument as in the proof of Theorem 8, to prove that I_N is neighborhood-independent, we obtain a contradiction. So $N_i[v_i]$ is a maximal clique.

5. Concluding remarks. According to Theorems 1 and 2, we cannot expect a good characterization for the class of graphs G satisfying $\rho_N(G) \leqq k$ (or $\alpha_N(G) \leqq k$) if k is large. We must note here that many graphs G contain some induced subgraph G' in which $\rho_N(G')$ is much larger than $\rho_N(G)$ (and the same holds even for $\alpha_N(G)$). The following problems, however, seem to be easier.

1. Let k by a given natural number. Characterize the graphs G in which $\rho_N(G')$ (and/or $\alpha_N(G')$) is at most k for all induced subgraphs G'. (For $k = 1$, the question is easy; cf. [LT].)
2. Prove that every neighborhood-perfect graph is perfect [LT].
3. Characterize neighborhood-perfect graphs.
4. Determine the algorithmic complexity of finding $\rho_N(G)$ and $\alpha_N(G)$ for planar graphs.
5. Find similar estimates and characterizations for covering and independence, when $E_k[v]$ is defined as the set of edges in the subgraph induced by the vertices of distance at most k from v. (With this notation, $E_1[v] = E[v]$.)

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