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Research Article

Algorithms Based on COPRAS and Aggregation Operators with New Information Measures for Possibility Intuitionistic Fuzzy Soft Decision-Making

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The objective of this paper is to present novel algorithms for solving the multiple attribute decision-making problems under the possibility intuitionistic fuzzy soft set (PIFSS) information. The prominent characteristics of the PIFSS are that it considers the membership and nonmembership degrees of each object during evaluation and their corresponding possibility degree. Keeping these features, this paper presents some new operation laws, score function, and comparison laws between the pairs of the PIFSSs. Further, we define COmplex PRoportional ASsessment (COPRAS) and weighted averaging and geometric aggregation operators to aggregate the PIFSS information into a single one. Later, we develop two algorithms based on COPRAS and aggregation operators to solve decision-making problems. In these approaches, the experts and the weights of the parameters are determined with the help of entropy and the distance measure to remove the ambiguity in the information. Finally, a numerical example is given to demonstrate the presented approaches.

1. Introduction

Multiple attribute decision-making (MADM) is the necessary context of the decision-making science whose aim is to recognize the most exceptional targets among the feasible ones. In real decision-making, the person needs to furnish the evaluation of the given choices by various types of evaluation conditions such as crisp numbers and intervals. However, in many cases, it is difficult for a person to opt for a suitable one due to the presence of several kinds of uncertainties in the data, which may occur due to a lack of knowledge or human error. Accordingly, to quantify such risks and to examine the process, a large-scale family of the theory such as fuzzy set (FS) [1] and its extensions such as intuitionistic FS (IFS) [2], cubic intuitionistic fuzzy set [3], interval-valued IFS [4], and linguistic interval-valued IFS [5] are appropriated by the researchers. In all these theories, an object is evaluated by an expert in terms of their two membership degrees such that their sum cannot exceed one. Since its presence, various researchers have presented their ways to illuminate decision-making problems by using operators or measures [6–12]. For instance, Xu and Yager [6] presented the weighted geometric operators for IFSs. Garg [7, 8] presented interactive weighted aggregation operators with Einstein t-norm. Liu and Li [13] introduced the Muirhead mean aggregation operator to aggregate the information. Liu and Tang [11] presented the intuitionistic fuzzy prioritized interactive Einstein aggregation operators.

The prevailing theories have restrictions because of their inadequacy over the parameterization tool, and consequently, the decision makers cannot give an accurate decision. To overwhelm these disadvantages, Molodtsov [14] collaborated the soft set (SS) theory in which ratings are given on specific parameters. Maji et al. [15, 16] extended this theory by joining it with existing FS and IFS theoretical approach and developed the idea of fuzzy soft set (FSS) and intuitionistic fuzzy soft set (IFSS). The major advantage of the IFSS is that they have considered the information over the set of parameters as compared to the IFS. For instance, if a person wants to buy a laptop, then by utilizing the IFSS theory to access it, an information is represented with the consideration of more than one parameters such as "price," "processor," "memory," "price," and "compatibility." On the other hand, if the judgement is made over the IFS environment, then there is no role of such parameters into the analysis. In other words, IFSS theory deals the information based on more than one parameter which makes it more generalized than IFS theory. Keeping these advantages in mind, several researchers have paid more attention on this theory and applied them to solve the various decision-making problems using different kinds of operations, information measures, and aggregation operators. In that direction, Bora et al. [17] presented the basic operations of IFSS. Petchimuthu et al. [18] defined the generalized products for fuzzy soft matrices and its applications to decision-making process. Jiang et al. [19] presented the entropy measure for the interval-valued fuzzy soft sets. The authors in [20, 21] presented the similarity measures for IFSS and applied them to solve the medical diagnosis problems. The distance measures for the IFSSs have been presented by Khalid and Abbas [22], Athira et al. [23], and Sarala and Suganya [24] and were applied to solve the decision-making problems. In terms of aggregation operators, Arora and Garg [25, 26] presented the weighted averaging/geometric as well as prioritized operators for the intuitionistic fuzzy soft numbers (IFSNs). Later on, Garg and Arora [27] presented the concept of generalized and group-generalized IFSS and presented an algorithm based on the operators to solve the MADM problems. Feng et al. [28] developed an algorithm to deal with decision-making (DM) issues with combined use of the generalized intuitionistic soft set, extended intersectional operation, weighted averaging operator, and other related concepts. Chang [29] defined an intuitionistic fuzzy weighted averaging method to solve the decision-making problem of supplier selection. Garg and Arora [30] developed the Bonferroni mean operators which can reflect the interconnection between two info contentions. Garg and Arora [31] presented the interactive operational laws for IFSNs and defined scaled prioritized averaging aggregation operators for solving the MADM problems. Hu et al. [32] developed the weighted intuitionistic fuzzy soft Bonferroni mean operator for the IFSSs, in which weights are determined by the maximizing deviation method. Garg and Arora [33] presented more generalized aggregation operators for IFSS by using the Archimedean t-norm operations. Alcantud and Muñoz Torrecillas [34] introduced the concept of intertemporal choice of fuzzy soft sets. A survey on different algorithms of parameter reduction of soft sets has been given by Zhan and Alcantud [35].

All the existing studies based on the IFSs, FSSs, IFSSs, etc., are widely used in different environments to solve the decision-making problems. However, under certain cases, these existing approaches fail to classify the objects based on their possibility degrees. In other words, we can say that the existing studies have treated the possibility degree of each element as one. However, in many practical applications, different persons may have their possibility degrees which differ from one related to each object. To address this issue into the decision-making process, Alkhazaleh et al. [36] introduced the concept of the possibility of FSS by assigning a possibility degree to each number of FSS. However, in this set, there is a complete lack of the degree of nonmembership degree during the analysis. To tackle it and address more appropriately, a concept of possibility IFSS (PIFSS) was introduced by Bashir et al. [37]. The PIFSS is more generalized than the existing FSS, IFSS, and other sets. In PIFSS, a degree of possibility of each component is assigned to the degrees of the IFSNs during evaluating the object. For instance, consider a term of "honesty" and three different experts have considered evaluating the candidate. The possibility of the honesty of a candidate according to the first expert can be 0.7, while for others it would differ from the first expert. To evaluate the given candidate, the rating values of it in terms of IFSS are taken as (0.6, 0.3) where 0.6 represents the favorable degree of the expert towards the candidate and 0.3 be against the degree.

Hence, in terms of PIFSS, such information is represented as $(0.6, 0.3; 0.7)$ rather than IFSS or IFS as (0.6, 0.3) only. Therefore, we can conclude that the evaluation of the object by using PIFSS is more reliable and robust than the other existing FSSs or IFSSs. Keeping these features, Bashir et al. [37] gave the similarity measure between the pairs of PIFSSs and applied them to solve the medical diagnosis problems. Later on, Selvachandran and Salleh [38] and Zhang et al. [39] presented the applications of the PIFSS to the decision-making process.

In recent years, a wide variety of methods such as AHP ("Analytic Hierarchy Process"), VIKOR ("VlseKriterijumska Optimizacija I Kompromisno Resenje"), TOPSIS ("Technique for Order Preference by Similarity to Ideal Solution"), and COPRAS ("COmplex PRoportional ASsessment") which can effectively deal with the ranking procedure have been used. The main purpose of these methods is to choose the best alternatives by aggregating the information and hence rank the objective based on their significance. AHP was introduced by Saaty [40], and it provides generally a view of the complex connections and helps the choice maker to survey the relationship between the levels. VIKOR, introduced by Opricovic [41], is a technique to rank the objects based on a specific degree of closeness to the ideal solution and can get a set of compromise solutions when the criterion strikes with each other. On the other hand, the TOPSIS [42] method characterizes the PIS ("Positive Ideal Solution") and the NIS ("Negative Ideal Solution") and selects the best ones whose distance from PIS is less. The COPRAS method introduced by Zavadskas et al. [43] compares each alternative and computes their priorities by taking into account the criteria weights. Among all such methods, COPRAS is one of the most appropriate methods to rank the given alternatives and widely used for both quantitative and qualitative analysis. The COPRAS method considers direct and the proportional reliance of the weights and the utility degree of examined adaptations on a framework of criteria. A comparative analysis of COPRAS and the other AHP, TOPSIS, and VIKOR methods is conducted by Chatterjee et al. [44] and was concluded that the COPRAS method indicates less calculation time, extremely basic, good transparency, and high possibility of graphical understanding of their counterpart strategies. In the literature, there exists many applications of the COPRAS method under the diverse fuzzy environment. For instance, Hajiagha et al. [45] presented the COPRAS method and its applications by taking interval-valued intuitionistic fuzzy information. Turanoglu Bekar et al. [46] created a fuzzy COPRAS technique to evaluate the performance of the total productive maintenance strategy. Garg and Nancy [47] presented the COPRAS method for the possibility linguistic set under the neutrosophic domain. Peng and Dai [48] presented a decision-making approach based on the COPRAS method with hesitant fuzzy soft information. Zheng et al. [49] presented the method for assessing the chronic obstructive diseases based on the hesitant fuzzy linguistic COPRAS method.

Considering the versatility of PIFSS and the quality of the COPRAS method, this paper extends the COPRAS strategy to the PIFSS environment. The essential characteristics of the COPRAS method are (1) it considers the proportions to the ideal solution and the worst solution at the same time during the execution of the process; (2) the method considers the direct and relative dependencies of the importance and the utility degree of the alternatives under the contrary criteria values; (3) this method is compelled to get the decision in a more effectively and sensible way. Thus, by considering the advantages of both the aggregation operators and the COPRAS method, this paper aims to present a novel MAGDM approach to manage the information related to the PIFSSs with some new information measures. Therefore, motivated from the characteristics of PIFSS, COPRAS, information measure, and aggregation operators, the following are the fundamental objectives of the paper:

(1) To develop a new distance measure for PIFSSs to measure the degree of discrimination and similarity among the sets

(2) To propose new weighted averaging and geometric aggregation operators under PIFSS environment, where the information related to each alternative is assessed in terms of possibility intuitionistic fuzzy soft numbers (PIFSNs)

(3) To establish a COPRAS method to rank the given PIFSNs

(4) To build two algorithms, based on COPRAS and aggregation operators, to interpret decision-making concerns

(5) To demonstrate the approach with a numerical example to explore the study

The rest of the text is organized as follows: Section 2 presents basic concepts related to soft sets, FSS, IFSS, and PIFSS. In Section 3, we define the new distance measures as well as the aggregation operators for the PIFSSs. The various desirable features of the proposed measures and operators are investigated in detail. In Section 4, a COPRAS method is presented to rank the alternatives with the PIFSS information. In Section 5, we propose two novel MADM approaches, based on proposed COPRAS and aggregation operators, to solve the MADM problems. The approaches have been facilitated with a numerical example in Section 6. Lastly, a conclusion is summarized in Section 7.

2. Preliminaries

In this section, we discuss some basic terms associated with soft set theory. Let $\mathbf{E}$ be a set of parameters and $\mathbf{U}$ be the set of experts.

Definition 1 (see [14]). A pair $(\mathbf{F}, \mathbf{E})$ is called as the soft set, if $\mathbf{F}$ is a map defined as $\mathbf{F}\colon \mathbf{E} \longrightarrow \mathbf{K}^{\mathbf{U}}$ where $\mathbf{K}^{\mathbf{U}}$ is a set of all subsets of $\mathbf{U}$.

Definition 2 (see [50]). Let $A, B \subset \mathbf{E}$ and $(\mathbf{F}, A)$, $(\mathbf{G}, B)$ be two SSs over $\mathbf{U}$. Then, the basic operations over them are stated as

(1) $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ if $A \subseteq B$ and $\mathbf{F}(e) \subseteq \mathbf{G}(e)$, $\forall e \in A$.

(2) $(\mathbf{F}, A) = (\mathbf{G}, B)$ if $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ and $(\mathbf{G}, B) \subseteq (\mathbf{F}, A)$.

(3) Complement: $(\mathbf{F}, A)^c = (\mathbf{F}^c, A)$, where $\mathbf{F}^c : A \longrightarrow \mathbf{K}^{\mathbf{U}}$ defined as $\mathbf{F}^c(e) = \mathbf{U} - \mathbf{F}(e)$, $\forall e \in A$.

Definition 3 (see [16]). A map $\mathbf{F}\colon \mathbf{E} \longrightarrow \mathbf{F}^{\mathbf{U}}$ is called FSS defined as

$$\mathbf{F}_{u_i}(e_j) = \left\{ \left(u_i, \zeta_j(u_i)\right) \middle| \, u_i \in \mathbf{U} \right\}, \tag{1}$$

where $\mathbf{F}^{\mathbf{U}}$ is a set of all fuzzy subsets of $\mathbf{U}$ and $\zeta_j(u_i)$ is the acceptance degree of an expert u_i over parameter $e_j \in \mathbf{E}$.

Definition 4 (see [16]). For $A, B \subset \mathbf{E}$ and $(\mathbf{F}, A)$, $(\mathbf{G}, B)$ be any two FSSs over $\mathbf{U}$, then

(1) $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ if, $A \subset B$ and $\mathbf{F}(e) \leq \mathbf{G}(e)$ for each $e \in A$.

(2) $(\mathbf{F}, A) = (\mathbf{G}, B)$ is equal if $(\mathbf{F}, A) \subseteq (\mathbf{G}, B)$ and $(\mathbf{G}, B) \subseteq (\mathbf{F}, A)$.

(3) Complement: $(\mathbf{F}^c, A)$ where for each $e \in A$, $\mathbf{F}^c(e) = 1 - \mathbf{F}(e)$.

Definition 5 (see [15]). A mapping $\mathbf{F}\colon \mathbf{E} \longrightarrow \mathbf{IF}^{\mathbf{U}}$ is called IFSS defined as

$$\mathbf{F}_{u_i}(e_j) = \left\{ \left(u_i, \zeta_j(u_i), \vartheta_j(u_i)\right) \middle| \, u_i \in \mathbf{U} \right\}, \tag{2}$$

where $\mathbf{IF}^{\mathbf{U}}$ is the intuitionistic fuzzy subsets of $\mathbf{U}$ and ζ_j and ϑ_j are the "acceptance degree" and the "rejection degree" respectively, with $0 \leq \zeta_j, \vartheta_j, \zeta_j + \vartheta_j \leq 1$ for all $u_i \in \mathbf{U}$. For simplicity, we denote the pair of $\mathbf{F}_{u_i}(e_j)$ as $\beta_{ij} = (\zeta_{ij}, \vartheta_{ij})$ or $(\mathbf{F}, \mathbf{E}) = (\zeta_{ij}, \vartheta_{ij})$ and was called as an intuitionistic fuzzy soft number (IFSN).

Definition 6 (see [26]). For an IFSN $\beta = (\zeta, \vartheta)$, a score function is defined as

$$\mathrm{Sc}(\beta) = \zeta - \vartheta, \tag{3}$$

while an accuracy function is defined as

$$H(\beta) = \zeta + \vartheta. \quad (4)$$

Definition 7 (see [26]). An order relation to compare the two IFSNs β and γ, denoted by $\beta \succ \gamma$, holds if either of the following condition holds:

(1) $\mathrm{Sc}(\beta) > \mathrm{Sc}(\gamma)$,

(2) $\mathrm{Sc}(\beta) = \mathrm{Sc}(\gamma)$ and $H(\beta) > H(\gamma)$.

here "$\succ$" represents "preferred to."

Definition 8 (see [37]). A mapping $\mathbf{F}: \mathbf{E} \longrightarrow \mathbf{IF}^{\mathbf{U}} \times \mathbf{F}^{\mathbf{U}}$ is called PIFSS which is defined as

$$\mathbf{F}_{u_i}(e_j) = \left\{ \left(u_i, \zeta_j(u_i), \vartheta_j(u_i)\right); p_j(u_i) \,\middle|\, u_i \in \mathbf{U} \right\}. \quad (5)$$

It is seen from equation (5) that PIFSS consists of two kinds of information: one is IFSS and the other is the possibility degree, p_j, of existence of the IFSS value for any $u_i \in \mathbf{U}$ over the parameter $\mathbf{E}$ such that $p_j(u_i) \in [0,1]$.

Remark 1. Throughout the text, we represent the pair $\mathbf{F}_{u_i}(e_j)$ or $(\mathbf{F}, \mathbf{E})$ as $\beta_{ij} = (\zeta_{ij}, \vartheta_{ij}; p_{ij})$ and called as possibility IFSN (PIFSN), if the following condition is satisfied:

$$\begin{aligned} &0 \le \zeta_{ij}, \\ &\vartheta_{ij}, p_{ij} \le 1, \\ &\zeta_{ij} + \vartheta_{ij} \le 1. \end{aligned} \quad (6)$$

Remark 2. For a given set, if $p_{ij} = 1$ for all i, j; then PIFSNs reduce to IFSNs.

Remark 3. We denote Γ be the collections of all PIFSNs.

3. Proposed Operational Laws, Distance Measures, and Aggregation Operators for PIFSSs

In this section, we present some operational laws, information measures, and aggregation operators for a collection of PIFSNs.

3.1. Operational Laws. In this section, we define some new operational laws for PIFSNs.

Definition 9. For two PIFSNs $\beta = (\zeta, \vartheta; p)$ and $\gamma = (\theta, \phi; q)$, some operations on it are defined as

(i) $\beta \cup \gamma = (\max(\zeta, \theta), \min(\vartheta, \phi))$;

(ii) $\beta \cap \gamma = (\min(\zeta, \theta), \max(\vartheta, \phi))$;

(iii) Complement: $\beta^c = (\vartheta, \zeta, 1 - p)$;

(iv) $\beta \le \gamma$ if $\zeta \le \theta$, $\vartheta \ge \phi$ and $p \le q$;

and are all again PIFSNs.

Definition 10. For two PIFSNs $\beta = (\zeta, \vartheta; p)$ and $\gamma = (\theta, \phi; q)$ and a real $\lambda > 0$, we define some of their basic operational laws as follows:

(i) $\beta \oplus \gamma = (1 - (1 - \zeta)(1 - \theta), \vartheta\phi; 1 - (1 - p)(1 - q))$;

(ii) $\beta \otimes \gamma = (\zeta\theta, 1 - (1 - \vartheta)(1 - \phi); pq)$;

(iii) $\lambda\beta = (1 - (1 - \zeta)^{\lambda}, \vartheta^{\lambda}; 1 - (1 - p)^{\lambda})$;

(iv) $\beta^{\lambda} = (\zeta^{\lambda}, 1 - (1 - \vartheta)^{\lambda}; p^{\lambda})$.

Theorem 1. *The operations defined in Definition 10 for two PIFSNs are also PIFSNs.*

Proof. Let $\beta = (\zeta, \vartheta; p)$ and $\gamma = (\theta, \phi; q)$ be two PIFSNs which imply that they satisfy the conditions given in equation (6) as

$$\begin{aligned} &0 \le \zeta, \vartheta, p \le 1, \\ &\zeta + \vartheta \le 1, \\ &0 \le \theta, \phi, q \le 1, \\ &\theta + \phi \le 1. \end{aligned} \quad (7)$$

To prove the part (i), we consider $\beta \oplus \gamma = (\zeta_{\beta\oplus\gamma}, \vartheta_{\beta\oplus\gamma}; p_{\beta\oplus\gamma})$, where $\zeta_{\beta\oplus\gamma} = 1 - (1 - \zeta)(1 - \theta)$, $\vartheta_{\beta\oplus\gamma} = \vartheta\phi$, and $p_{\beta\oplus\gamma} = 1 - (1 - p)(1 - q)$. Now, to prove that $\beta\oplus\gamma$ is PIFSN, it is enough to show that it satisfies the conditions as given in equation (6), i.e.,

$$\begin{aligned} &0 \le \zeta_{\beta\oplus\gamma}, \vartheta_{\beta\oplus\gamma}, p_{\beta\oplus\gamma} \le 1, \\ &\zeta_{\beta\oplus\gamma} + \vartheta_{\beta\oplus\gamma} \le 1. \end{aligned} \quad (8)$$

$\zeta, \theta \in [0,1]$ which implies that $1 - (1 - \zeta)(1 - \theta) \le 1$ and hence $\zeta_{\beta\oplus\gamma} \in [0,1]$. Similarly, we can obtain that $\vartheta_{\beta\oplus\gamma} = \vartheta\phi \in [0,1]$ and $p_{\beta\oplus\gamma} = 1 - (1 - p)(1 - q) \in [0,1]$. Now, finally $\zeta_{\beta\oplus\gamma} + \vartheta_{\beta\oplus\gamma} = 1 - (1 - \zeta)(1 - \theta) + \vartheta\phi \le 1 - \vartheta\phi + \vartheta\phi = 1$. Hence, $\beta \oplus \gamma$ is PIFSN. Similarly, we can obtain for the other parts. □

Theorem 2. *Let β and γ be two PIFSNs and $\lambda, \lambda_1, \lambda_2 > 0$ be three real numbers. Then, we have*

(i) $\beta \oplus \gamma = \gamma \oplus \beta$.

(ii) $\beta \otimes \gamma = \gamma \otimes \beta$.

(iii) $\lambda(\beta \oplus \gamma) = \lambda\beta \oplus \lambda\gamma$.

(iv) $(\beta \otimes \gamma)^{\lambda} = \beta^{\lambda} \otimes \gamma^{\lambda}$.

(v) $\lambda_1\beta \oplus \lambda_2\beta = (\lambda_1 + \lambda_2)\beta$.

(vi) $\beta^{\lambda_1} \otimes \beta^{\lambda_2} = \beta^{\lambda_1 + \lambda_2}$.

Proof. We shall proof the parts (iii) and (v) only, while others can be proceeded likewise.

(iii) For any real number $\lambda > 0$,

$$\begin{aligned}\lambda(\beta\oplus\gamma) &= \lambda(1-(1-\zeta)(1-\theta), \vartheta\phi; 1-(1-p)(1-q))\\ &= \left(1-(1-\zeta)^{\lambda}(1-\theta)^{\lambda}, \vartheta^{\lambda}\phi^{\lambda}; 1-(1-p)^{\lambda}(1-q)^{\lambda}\right)\\ &= \left(1-(1-\zeta)^{\lambda}, \vartheta^{\lambda}; 1-(1-p)^{\lambda}\right)\oplus\left(1-(1-\theta)^{\lambda}, \phi^{\lambda}; 1-(1-q)^{\lambda}\right)\\ &= \lambda\beta\oplus\lambda\gamma.\end{aligned} \tag{9}$$

(v) For real numbers $\lambda_1, \lambda_2 > 0$,

$$\begin{aligned}\lambda_1\beta\oplus\lambda_2\beta &= \left(1-(1-\zeta)^{\lambda_1}, \vartheta^{\lambda_1}; 1-(1-p)^{\lambda_1}\right)\oplus\left(1-(1-\zeta)^{\lambda_2}, \vartheta^{\lambda_2}; 1-(1-p)^{\lambda_2}\right)\\ &= \left(1-(1-\zeta)^{\lambda_1}(1-\zeta)^{\lambda_2}, \vartheta^{\lambda_1}\vartheta^{\lambda_2}; 1-(1-p)^{\lambda_1}(1-p)^{\lambda_1}\right)\\ &= \left(1-(1-\zeta)^{\lambda_1+\lambda_2}, \vartheta^{\lambda_1+\lambda_2}; 1-(1-p)^{\lambda_1+\lambda_2}\right)\\ &= (\lambda_1+\lambda_2)\beta.\end{aligned} \tag{10}$$

□

Theorem 3. *For two PIFSNs β and γ and real $\lambda > 0$, we have*

(i) $\lambda(\beta^c) = (\beta^{\lambda})^c$.
(ii) $(\beta^c)^{\lambda} = (\lambda\beta)^c$.
(iii) $\beta^c\otimes\gamma^c = (\beta\oplus\gamma)^c$.
(iv) $\beta^c\oplus\gamma^c = (\beta\otimes\gamma)^c$.

Proof. For PIFSN $\beta = (\zeta, \vartheta; p)$, we have $\beta^c = (\vartheta, \zeta; 1-p)$. Thus, for real $\lambda > 0$, we have

(i) $\lambda(\beta^c) = (1-(1-\vartheta)^{\lambda}, \zeta^{\lambda}; 1-p^{\lambda}) = (\beta^{\lambda})^c$.
(iii) $\beta^c\otimes\gamma^c = (\vartheta, \zeta; 1-p)\otimes(\phi, \theta; 1-q) = (\vartheta\phi, 1-(1-\zeta)(1-\theta); (1-p)(1-q)) = (\beta\oplus\gamma)^c$.

To compare the two or more different PIFSNs, we define score and accuracy functions as follows. □

Definition 11. For a PIFSN $\beta = (\zeta, \vartheta; p)$, the score function is described as

$$S(\beta) = p\left(\frac{1+\zeta-\vartheta}{2}\right), \tag{11}$$

and the accuracy function is

$$H(\beta) = (1-p)\left(\frac{\zeta+\vartheta}{2}\right). \tag{12}$$

Clearly, it is seen that $S(\beta), H(\beta)\in[0, 1]$. A comparison rule between two PIFSNs β and γ denoted by $\beta\succ\gamma$ holds if at least one of the following conditions is met:

(a) $S(\beta) > S(\gamma)$.
(b) $S(\beta) = S(\gamma)$ and $H(\beta) > H(\gamma)$.

3.2. Distance Measure for PIFSS. In this section, we define the distance measure for the distinct PIFSNs. Let $\mathbf{E} = \{e_1, e_2, \ldots, e_m\}$ be the set of parameters and $\mathbf{U} = \{u_1, u_2, \ldots, u_n\}$ be the set of experts.

Definition 12. Let $(\mathbf{F}, \mathbf{E}) = (\zeta_{ij}, \vartheta_{ij}; p_{ij})$ and $(\mathbf{G}, \mathbf{E}) = (\theta_{ij}, \phi_{ij}; q_{ij})$ be two PIFSNs for $i = 1(1)n$; $j = 1(1)m$, and $\lambda\geq 1$ be any real number. Then, the generalized normalized distance measures between them is defined as

$$d_{\lambda}((\mathbf{F}, \mathbf{E}), (\mathbf{G}, \mathbf{E})) = \left(\frac{1}{2mn}\sum_{j=1}^{m}\sum_{i=1}^{n}\left(\left|p_{ij}\zeta_{ij}-q_{ij}\theta_{ij}\right|^{\lambda}+\left|p_{ij}\vartheta_{ij}-q_{ij}\phi_{ij}\right|^{\lambda}\right)\right)^{(1/\lambda)}. \tag{13}$$

Remark 4. From the above definition, we conclude that

(i) If $\lambda = 1$, then equation (13) reduces to normalized Hamming distance.
(ii) If $\lambda = 2$, then equation (13) reduces to normalized Euclidean distance.

Theorem 4. *For two PIFSNs $(\mathbf{F}, \mathbf{E})$ and $(\mathbf{G}, \mathbf{E})$, the measure d_{λ} defined in equation (13) satisfies the following properties:*

(P1) $0\leq d_{\lambda}((\mathbf{F}, \mathbf{E}), (\mathbf{G}, \mathbf{E}))\leq 1$.
(P2) $d_{\lambda}((\mathbf{F}, \mathbf{E}), (\mathbf{G}, \mathbf{E})) = 0$ if $(\mathbf{F}, \mathbf{E}) = (\mathbf{G}, \mathbf{E})$.
(P3) $d_{\lambda}((\mathbf{F}, \mathbf{E}), (\mathbf{G}, \mathbf{E})) = d_{\lambda}((\mathbf{F}, \mathbf{E}), (\mathbf{G}, \mathbf{E}))$.

Proof. For PIFSNs $(\mathbf{F}, \mathbf{E}) = (\zeta_{ij}, \vartheta_{ij}; p_{ij})$ and $(\mathbf{G}, \mathbf{E}) = (\theta_{ij}, \phi_{ij}; q_{ij})$ such that $\zeta_{ij}, \vartheta_{ij}, \theta_{ij}, \phi_{ij}\in[0, 1]$, $\zeta_{ij}+\vartheta_{ij}\leq 1$, and $\theta_{ij}+\phi_{ij}\leq 1$ for all i, j. Then, for $\lambda\geq 1$, we have

(P1) For PIFSNs we have $0 \le \zeta_{ij}, \theta_{ij} \le 1$, $0 \le \vartheta_{ij}, \phi_{ij} \le 1$, and $0 \le p_{ij}, q_{ij} \le 1$; therefore, $-1 \le p_{ij}\zeta_{ij} - q_{ij}\theta_{ij} \le 1$ and $-1 \le p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij} \le 1$. This implies, $0 \le |p_{ij}\zeta_{ij} - q_{ij}\theta_{ij}|^{\lambda} \le 1$ and $0 \le |p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij}|^{\lambda} \le 1$which results that $0 \le |p_{ij}\zeta_{ij} - q_{ij}\theta_{ij}|^{\lambda} + |p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij}|^{\lambda} \le 2$ and hence $0 \le \sum_{j=1}^{m}\sum_{j=1}^{n} |p_{ij}\zeta_{ij} - q_{ij}\theta_{ij}|^{\lambda} + |p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij}|^{\lambda} \le 2mn.$ Thus, $0 \le 1/2mn \sum_{j=1}^{m} \sum_{j=1}^{n} (|p_{ij}\zeta_{ij} - q_{ij}\theta_{ij}|^{\lambda} + |p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij}|^{\lambda}) \le 1$. Hence, $0 \le d_{\lambda}((\mathbf{F},\mathbf{E}), (\mathbf{G},\mathbf{E})) \le 1$.

(P2) If $(\mathbf{F},\mathbf{E}) = (\mathbf{G},\mathbf{E})$ which implies that $\zeta_{ij} = \theta_{ij}$, $\vartheta_{ij} = \phi_{ij}$, and $p_{ij} = q_{ij}$. Thus, equation (13) becomes $d_{\lambda}((\mathbf{F},\mathbf{E}), (\mathbf{G},\mathbf{E})) = 0$.

(P3) For any two positive real numbers h and g, we know that $|h - g| = |g - h|$. Hence, from equation (13), we get $d_{\lambda}((\mathbf{F},\mathbf{E}), (\mathbf{G},\mathbf{E})) = d_{\lambda}((\mathbf{G},\mathbf{E}), (\mathbf{F},\mathbf{E}))$.

Hence, the proposed measure is a valid distance measure.

To demonstrate the working of the stated measure, we explain it with one illustrated example as follows. □

Example 1. Let $\mathbf{U}$ be a set of three houses given by, $\mathbf{U} = \{u_1, u_2, u_3\}$ under the consideration of a person to purchase and $\mathbf{E} = \{e_1, e_2, e_3, e_4\}$ be set of parameters where e_1 stands for "expensive houses," e_2 stands for "wooden houses," e_3 stands for "cheap houses," and e_4 stands for houses which are "in good location." If $\mathbf{F}$ is a mapping from $\mathbf{E}$ to $\mathbf{IF}^{\mathbf{U}} \times \mathbf{F}^{\mathbf{U}}$, then the soft set $(\mathbf{F},\mathbf{E})$ describes the "attractiveness of the houses." An expert gives their preferences in terms of PIFSSs, $(\mathbf{F},\mathbf{E})$, and $(\mathbf{G},\mathbf{E})$, and their rating values are summarized in the form of the following decision matrices:

$$(\mathbf{F},\mathbf{E}) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \\ (0.8, 0.1; 0.3) & (0.5, 0.4; 0.5) & (0.7, 0.2; 0.4) & (0.6, 0.2; 0.7) \\ (0.6, 0.3; 0.5) & (0.8, 0.2; 0.7) & (0.8, 0.2; 0.6) & (0.4, 0.3; 0.6) \\ (0.7, 0.1; 0.6) & (0.6, 0.3; 0.4) & (0.4, 0.5; 0.8) & (0.5, 0.3; 0.8) \end{bmatrix},$$
$$(\mathbf{G},\mathbf{E}) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \end{array} \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \\ (0.9, 0.1; 0.6) & (0.8, 0.1; 0.7) & (0.8, 0.2; 0.5) & (0.7, 0.1; 0.8) \\ (0.6, 0.2; 0.5) & (0.4, 0.2; 0.6) & (0.5, 0.2; 0.8) & (0.5, 0.3; 0.6) \\ (0.5, 0.3; 0.7) & (0.7, 0.2; 0.8) & (0.6, 0.3; 0.7) & (0.6, 0.2; 0.9) \end{bmatrix}. \quad (14)$$

Without the loss of generality, we have taken $\lambda = 1$ in equation (13) to compute the degree of separation between the rating values of $(\mathbf{F},\mathbf{E})$ and $(\mathbf{G},\mathbf{E})$ and obtain

$$\begin{aligned} &d_1((\mathbf{F},\mathbf{E}), (\mathbf{G},\mathbf{E})) \\ &= \frac{1}{24}\sum_{j=1}^{4}\sum_{i=1}^{3}\left(\left|p_{ij}\zeta_{ij} - q_{ij}\theta_{ij}\right| + \left|p_{ij}\vartheta_{ij} - q_{ij}\phi_{ij}\right|\right) \\ &= \frac{1}{24}[(|(0.3)(0.8) - (0.6)(0.9)| + |(0.3)(0.1) - (0.6)(0.1)|) + (|(0.5)(0.5) - (0.7)(0.8)| + |(0.5)(0.4) - (0.7)(0.1)|) \\ &\quad + (|(0.4)(0.7) - (0.5)(0.8)| + |(0.4)(0.2) - (0.5)(0.2)|) + (|(0.7)(0.6) - (0.8)(0.7)| + |(0.7)(0.2) - (0.8)(0.1)|) \\ &\quad + (|(0.5)(0.6) - (0.5)(0.6)| + |(0.5)(0.3) - (0.5)(0.2)|) + (|(0.7)(0.8) - (0.6)(0.4)| + |(0.7)(0.2) - (0.6)(0.2)|) \\ &\quad + (|(0.6)(0.8) - (0.8)(0.5)| + |(0.6)(0.2) - (0.8)(0.2)|) + (|(0.6)(0.4) - (0.6)(0.5)| + |(0.6)(0.3) - (0.6)(0.3)|) \\ &\quad + (|(0.6)(0.7) - (0.7)(0.5)| + |(0.6)(0.1) - (0.7)(0.3)|) + (|(0.4)(0.6) - (0.8)(0.7)| + |(0.4)(0.3) - (0.8)(0.2)|) \\ &\quad + (|(0.8)(0.4) - (0.7)(0.6)| + |(0.8)(0.5) - (0.7)(0.3)|) + (|(0.8)(0.5) - (0.9)(0.6)| + |(0.8)(0.3) - (0.9)(0.2)|)] \\ &= 0.1633. \end{aligned} \quad (15)$$

Similarly, for some other values of λ, say $\lambda = 1.5, 2, 2.5, 3$, we can compute that

$$\begin{aligned} d_{1.5}((\mathbf{F},\mathbf{E}),(\mathbf{G},\mathbf{E})) &= 0.1821,\\ d_{2}((\mathbf{F},\mathbf{E}),(\mathbf{G},\mathbf{E})) &= 0.1978,\\ d_{2.5}((\mathbf{F},\mathbf{E}),(\mathbf{G},\mathbf{E})) &= 0.2110,\\ d_{3}((\mathbf{F},\mathbf{E}),(\mathbf{G},\mathbf{E})) &= 0.2222. \end{aligned} \tag{16}$$

3.3. Aggregation Operator. Let Γ be the collection of PIFSNs $\beta = (\mathbf{F},\mathbf{E})$. Then in the following, we define some operators on Γ named as possibility intuitionistic fuzzy soft weighted averaging and geometric operators denoted by PIFSWA and PIFSWG, respectively.

Definition 13. For the collection of PIFSNs β_{ij}; $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$, a possibility intuitionistic fuzzy soft weighted average (PIFSWA) operator is mapping PIFSWA: $\Gamma^n \longrightarrow \Gamma$ defined as

$$\text{PIFSWA}(\beta_{11},\beta_{12},\ldots,\beta_{nm}) = \bigoplus_{j=1}^{m}\xi_j\left(\bigoplus_{i=1}^{n}\eta_i\beta_{ij}\right), \tag{17}$$

where $\xi_j > 0$, $\sum_{j=1}^{m}\xi_j = 1$ be the weight vector of the parameters and $\eta_i > 0$, $\sum_{i=1}^{n}\eta_i = 1$ be the weight vector of the experts.

Theorem 5. *For PIFSNs $\beta_{ij} = (\zeta_{ij},\vartheta_{ij};p_{ij})$, $i = 1(1)n$, $j = 1(1)m$, the collective value obtained by applying Definition 13 is again PIFSN and given by*

$$\text{PIFSWA}(\beta_{11},\beta_{12},\ldots,\beta_{nm}) = \left(1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\zeta_{ij}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\vartheta_{ij}^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-p_{ij}\right)^{\eta_i}\right)^{\xi_j}\right). \tag{18}$$

Proof. We will prove equation (18) with the help of principle of mathematical induction (PMI) on n, m. For $n = 1$, we get $\eta_1 = 1$. Therefore, by using operational laws of PIFSNs, we have

$$\begin{aligned} \text{PIFSWA}(\beta_{11},\beta_{12},\ldots,\beta_{1m}) &= \bigoplus_{j=1}^{m}\xi_j\left(\bigoplus_{i=1}^{1}\eta_i\beta_{ij}\right)\\ &= \bigoplus_{j=1}^{m}\xi_j\left(\eta_1\beta_{1j}\right)\\ &= \left(1-\prod_{j=1}^{m}\left(\left(1-\zeta_{1j}\right)^{\eta_1}\right)^{\xi_j},\prod_{j=1}^{m}\left(\vartheta_{1j}^{\eta_1}\right)^{\xi_j};1-\prod_{j=1}^{m}\left(\left(1-p_{1j}\right)^{\eta_1}\right)^{\xi_j}\right)\\ &= \left(1-\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1-\zeta_{ij}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\vartheta_{ij}^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{m}\left(\prod_{i=1}^{1}\left(1-p_{ij}\right)^{\eta_i}\right)^{\xi_j}\right). \end{aligned} \tag{19}$$

Similarly, the result holds for $m = 1$.

Assume it holds for $n = k_1 + 1$, $m = k_2$ and for $n = k_1$, $m = k_2 + 1$. For $n = k_1 + 1$, $m = k_2 + 1$, we have

$$\begin{aligned}
&\text{PIFSWA}\left(\beta_{11},\beta_{12},\ldots,\beta_{(k_1+1)(k_2+1)}\right)\\
&=\bigoplus_{j=1}^{k_2+1}\xi_j\left(\bigoplus_{i=1}^{k_1+1}\eta_i\beta_{ij}\right)\\
&=\bigoplus_{j=1}^{k_2}\xi_j\left(\bigoplus_{i=1}^{k_1+1}\eta_i\beta_{ij}\right)\oplus\xi_{k_2+1}\left(\bigoplus_{i=1}^{k_1+1}\eta_i\beta_{i(k_2+1)}\right)\\
&=\left(1-\prod_{j=1}^{k_2}\left(\prod_{i=1}^{k_1+1}\left(1-\zeta_{ij}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{k_2}\left(\prod_{i=1}^{k_1+1}\vartheta_{ij}^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{k_2}\left(\prod_{i=1}^{k_1+1}\left(1-p_{ij}\right)^{\eta_i}\right)^{\xi_j}\right)\\
&\oplus\left(1-\left(\prod_{i=1}^{k_1+1}\left(1-\zeta_{i(k_2+1)}\right)^{\eta_i}\right)^{\xi_{k_2+1}},\left(\prod_{i=1}^{k_1+1}\vartheta_{i(k_2+1)}^{\eta_i}\right)^{\xi_{k_2+1}};1-\left(\prod_{i=1}^{k_1+1}\left(1-p_{i(k_2+1)}\right)^{\eta_i}\right)^{\xi_{k_2+1}}\right)\\
&=\left(1-\prod_{j=1}^{k_2+1}\left(\prod_{i=1}^{k_1+1}\left(1-\zeta_{ij}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{k_2+1}\left(\prod_{i=1}^{k_1+1}\vartheta_{ij}^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{k_2+1}\left(\prod_{i=1}^{k_1+1}\left(1-p_{ij}\right)^{\eta_i}\right)^{\xi_j}\right).
\end{aligned}\tag{20}$$

Hence, the result holds for $n=k_1+1$, $m=k_2+1$. Therefore, by PMI, the result is true for all n,m. □

Definition 14. Let $\beta_{ij}=(\zeta_{ij},\vartheta_{ij};p_{ij})$ be a collection of PIFSNs and let PIFSWG: $\Gamma^n\longrightarrow\Gamma$. If

$$\begin{aligned}
\text{PIFSWG}\left(\beta_{11},\beta_{12},\ldots,\beta_{nm}\right)&=\bigotimes_{j=1}^{m}\left(\bigotimes_{i=1}^{n}\beta_{ij}^{\eta_i}\right)^{\xi_j}\\
&=\left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\zeta_{ij}^{\eta_i}\right)^{\xi_j},1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1-\vartheta_{ij}\right)^{\eta_i}\right)^{\xi_j};\prod_{j=1}^{m}\left(\prod_{i=1}^{n}p_{ij}^{\eta_i}\right)^{\xi_j}\right),
\end{aligned}\tag{21}$$

where $\xi_j>0$, $\sum_{j=1}^{m}\xi_j=1$ and $\eta_i>0$, $\sum_{i=1}^{n}\eta_i=1$ be the weight vector of the parameters and the experts, respectively; then, the function PIFSWG is the possibility intuitionistic fuzzy soft weighted geometric operator.

Let $\mathcal{F}$ be either an PIFSWA or PIFSWG operator. Then, the $\mathcal{F}$ operator satisfies certain properties which are stated as follows.

Theorem 6. *For collection of PIFSNs β_{ij}, we have*

(P1) Idempotency: if all β_{ij} are identical, i.e., $\beta_{ij}=\beta$ for all i, j, then we have $\mathcal{F}(\beta_{11},\beta_{12},\ldots,\beta_{nm})=\beta$.

(P2) Monotonicity: let $\beta_{ij}=(\zeta_{ij},\vartheta_{ij};p_{ij})$ and $\gamma_{ij}=(\theta_{ij},\phi_{ij};q_{ij})$ be two PIFSNs such that $\zeta_{ij}\le\theta_{ij}$, $\vartheta_{ij}\ge\phi_{ij}$ and $p_{ij}\le q_{ij}$, for all i, j then $\mathcal{F}(\beta_{11},\beta_{12},\ldots,\beta_{nm})\le\mathcal{F}(\gamma_{11},\gamma_{12},\ldots,\gamma_{nm})$.

(P3) Boundedness: let $\beta^-=(\min_j\min_i\{\zeta_{ij}\},\max_j\max_i\{\vartheta_{ij}\};\min_j\min_i\{p_{ij}\})$ and $\beta^+=(\max_j\max_i\{\zeta_{ij}\},\min_j\min_i\{\vartheta_{ij}\};\max_j\max_i\{p_{ij}\})$, then

$$\beta^-\le\mathcal{F}\left(\beta_{11},\beta_{12},\ldots,\beta_{nm}\right)\le\beta^+.\tag{22}$$

Proof. Without the loss of generality, we shall prove the result by taking $\mathcal{F}$ as the PIFSWA operator only.

(P1) If $\beta_{ij}=\beta=(\zeta,\vartheta;p)$ for all i, j, then by equation (18), we have

$$\begin{aligned}
\mathcal{F}(\beta,\beta,\ldots,\beta)&=\left(1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(1-\zeta)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\vartheta^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{m}\left(\prod_{i=1}^{n}(1-p)^{\eta_i}\right)^{\xi_j}\right)\\
&=\left(1-\left((1-\zeta)^{\sum_{i=1}^{n}\eta_i}\right)^{\sum_{j=1}^{m}\xi_j},\left(\vartheta^{\sum_{i=1}^{n}\eta_i}\right)^{\sum_{j=1}^{m}\xi_j};1-\left((1-p)^{\sum_{i=1}^{n}\eta_i}\right)^{\sum_{j=1}^{m}\xi_j}\right)\\
&=(1-(1-\zeta),\vartheta;1-(1-p))\\
&=\beta.
\end{aligned}\tag{23}$$

(P2) For PIFSNs β_{ij} and γ_{ij}, we have $\zeta_{ij} \le \theta_{ij}$; therefore, $1-\zeta_{ij} \ge 1-\theta_{ij}$ which gives that $\prod_{i=1}^{n}(1-\zeta_{ij})^{\eta_i} \ge \prod_{i=1}^{n}(1-\theta_{ij})^{\eta_i}$ and hence we get $1-\prod_{j=1}^{m}(\prod_{i=1}^{n}(1-\zeta_{ij})^{\eta_i})^{\xi_j} \le 1-\prod_{j=1}^{m}(\prod_{i=1}^{n}(1-\theta_{ij})^{\eta_i})^{\xi_j}$. Similarly, for $\vartheta_{ij} \ge \phi_{ij}$ implies that $\prod_{j=1}^{m}(\prod_{i=1}^{n}\vartheta_{ij}^{\eta_i})^{\xi_j} \ge \prod_{j=1}^{m}(\prod_{i=1}^{n}\phi_{ij}^{\eta_i})^{\xi_j}$. Further, for $p_{ij} \le q_{ij}$, we can obtain that $1-\prod_{j=1}^{m}\prod_{i=1}^{n}((1-p_{ij})^{\eta_i})^{\xi_j} \le 1-\prod_{j=1}^{m}(\prod_{i=1}^{n}(1-q_{ij})^{\eta_i})^{\xi_j}$. Hence, by Definition 9, we can get $\mathscr{F}(\beta_{11},\beta_{12},\ldots,\beta_{nm}) \le \mathscr{F}(\gamma_{11},\gamma_{12},\ldots,\gamma_{nm})$.

(P3) By the definition of β^-, β_{ij}, and β^+ and Definition 9, we can obtain that $\beta^- \le \beta_{ij} \le \beta^+$. Therefore, by the monotonicity property, we can obtain $\mathscr{F}(\beta^-,\beta^-,\ldots,\beta^-) \le \mathscr{F}(\beta_{11},\beta_{12},\ldots,\beta_{nm}) \le \mathscr{F}(\beta^+,\beta^+,\ldots,\beta^+)$, which implies that $\beta^- \le \mathscr{F}(\beta_{11},\beta_{12},\ldots,\beta_{nm}) \le \beta^+$.

Similarly, we can prove these properties for the PIFSWG operator. □

4. COPRAS Method

This section addresses the COPRAS method by embedding the PIFSS features. The COPRAS method was initially designed by Zavadskas et al. [43] by considering the dependency factor of the priority and the utility degree of the objects under the contrary attributes. To address it, assume decision-making problems whose target is choosen as the most favorable alternative from $\mathscr{V}^{(1)},\mathscr{V}^{(2)},\ldots,\mathscr{V}^{(z)}$ which are evaluated by "n" experts $\{\mathscr{U}_1,\mathscr{U}_2,\ldots,\mathscr{U}_n\}$ under "m" parameters $\{e_1,e_2,\ldots,e_m\}$. Let η_i and ξ_j be the normalized weight vectors of the experts and the parameters, respectively. The rating values given by i^{th} expert $\mathscr{U}_i$ under j^{th} parameter e_j towards the assessment of d^{th} alternative is represented as PISFNs $\beta_{ij}^{(d)} = (\zeta_{ij}^{(d)},\vartheta_{ij}^{(d)};p_{ij}^{(d)})$. Then, the procedure steps which fall under the proposed COPRAS method are presented as follows.

Step 1: compute the weighted decision matrix $\overline{A}^{(d)} = (\overline{\beta}^{(d)})_{n\times m}$, for each alternative $\mathscr{V}^{(d)}$ by using Definition 10, where $\overline{\beta}^{(d)} = (\overline{\zeta}_{ij}^{(d)},\overline{\vartheta}_{ij}^{(d)};\overline{p}_{ij}^{(d)})$ is given by

$$\begin{aligned}\overline{\beta}^{(d)} &= \xi_j\eta_i\mathscr{V}^{(d)}\\ &= \left(1-\left(1-\zeta_{ij}^{(d)}\right)^{\eta_i\xi_j},\left(\vartheta_{ij}^{(d)}\right)^{\eta_i\xi_j};1-\left(1-p_{ij}^{(d)}\right)^{\eta_i\xi_j}\right).\end{aligned} \tag{24}$$

Step 2: assume that out of "m" parameters, "k" are the benefit types and remaining "$m-k$" are the cost types. Now, aggregate the values of these k parameters by using the PIFSWA operator and get preference values as $P^{(d)}$, where

$$\begin{aligned}P^{(d)} &= \mathop{\oplus}_{j=1}^{k}\mathop{\oplus}_{i=1}^{n}\overline{\beta}^{(d)}\\ &= \left(1-\prod_{j=1}^{k}\left(\prod_{i=1}^{n}\left(1-\overline{\zeta}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=1}^{k}\left(\prod_{i=1}^{n}\left(\overline{\vartheta}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j};1-\prod_{j=1}^{k}\left(\prod_{i=1}^{n}\left(1-\overline{p}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}\right).\end{aligned} \tag{25}$$

Step 3: collect the aggregated value of the remaining $m-k$ parameters by using the following equation:

$$\begin{aligned}R^{(d)} &= \mathop{\oplus}_{j=k+1}^{m}\mathop{\oplus}_{i=1}^{n}\overline{\beta}^{(d)}\\ &= \left(1-\prod_{j=k+1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\zeta}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j},\prod_{j=k+1}^{m}\left(\prod_{i=1}^{n}\left(\overline{\vartheta}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j};1-\prod_{j=k+1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{p}_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}\right).\end{aligned} \tag{26}$$

Step 4: compute the minimal value of $R^{(d)}$ as

$$R_{\min} = \min\left\{R^{(1)},R^{(2)},\ldots,R^{(z)}\right\}. \tag{27}$$

Step 5: determine the relative priority values of the alternatives $\mathscr{V}^{(d)}$ by the following equation:

$$Q^{(d)} = S\left(P^{(d)}\right)+\frac{S\left(R_{\min}\right)\sum_{d=1}^{z}S\left(R^{(d)}\right)}{S\left(R^{(d)}\right)\sum_{d=1}^{z}\left(S\left(R_{\min}\right)/S\left(R^{(d)}\right)\right)}, \tag{28}$$

provided $S(R^{(d)}) \ne 0$ and $S(\cdot)$ represents the score function.

Step 6: compute the utility degree for $\mathscr{V}^{(d)}$ as

$$\mathscr{N}^{(d)} = \left(\frac{Q^{(d)}}{Q_{\max}}\right)\times 100\%, \tag{29}$$

where $Q_{\max} = \max_d\{Q^{(d)}\} \ne 0$.

5. Proposed Approaches for Decision-Making Problems

In this section, we addressed two different approaches, based on "COPRAS" and "aggregation operators" with new distance measure, for solving the decision-making problems under the PIFSS environment.

5.1. Model Description. Consider a problem with z alternatives $\mathscr{V}^{(1)}, \mathscr{V}^{(2)}, \ldots, \mathscr{V}^{(z)}$, "$m$" parameters $e_1, e_2, \ldots, e_m$, and "n" experts $\mathscr{U}_1, \mathscr{U}_2, \ldots, \mathscr{U}_n$. The i^{th} expert $\mathscr{U}_i$ evaluates the given alternatives $\mathscr{V}^{(d)}$, $d = 1, 2, \ldots, z$ on j^{th} parameter and represents their preferences in terms of PIFSNs $\beta_{ij}^{(d)} = (\zeta_{ij}^{(d)}, \vartheta_{ij}^{(d)}; p_{ij}^{(d)})$such that $0 \le \zeta_{ij}^{(d)}, \vartheta_{ij}^{(d)} \le 1, 0 \le p_{ij}^{(d)} \le 1$, and $\zeta_{ij}^{(d)} + \vartheta_{ij}^{(d)} \le 1$. The complete information about the given alternatives $\mathscr{V}^{(d)}$ is represented in the following equation:

$$\left(\mathscr{V}^{(d)}, \mathbf{E}\right) = \begin{array}{c} \\ \mathscr{U}_1 \\ \mathscr{U}_2 \\ \vdots \\ \mathscr{U}_n \end{array} \begin{array}{cccc} e_1 & e_2 & \cdots & e_m \\ \left(\zeta_{11}^{(d)}, \vartheta_{11}^{(d)}; p_{11}^{(d)}\right) & \left(\zeta_{12}^{(d)}, \vartheta_{12}^{(d)}; p_{12}^{(d)}\right) & \cdots & \left(\zeta_{1m}^{(d)}, \vartheta_{1m}^{(d)}; p_{1m}^{(d)}\right) \\ \left(\zeta_{21}^{(d)}, \vartheta_{21}^{(d)}; p_{21}^{(d)}\right) & \left(\zeta_{22}^{(d)}, \vartheta_{22}^{(d)}; p_{22}^{(d)}\right) & \cdots & \left(\zeta_{2m}^{(d)}, \vartheta_{2m}^{(d)}; p_{2m}^{(d)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\zeta_{n1}^{(d)}, \vartheta_{n1}^{(d)}; p_{n1}^{(d)}\right) & \left(\zeta_{n2}^{(d)}, \vartheta_{n2}^{(d)}; p_{n2}^{(d)}\right) & \cdots & \left(\zeta_{nm}^{(d)}, \vartheta_{nm}^{(d)}; p_{nm}^{(d)}\right) \end{array}. \tag{30}$$

Consider that the weight of each expert and the parameter are represented as $\eta_1, \eta_2, \ldots, \eta_n$ and $\xi_1, \xi_2, \ldots, \xi_m$, respectively, such that $\eta_i, \xi_j > 0$, $\sum_{i=1}^n \eta_i = 1$, and $\sum_{j=1}^m \xi_j = 1$. If such information is known as "*priori*" then we can use them. On the opposite, if they are unknown, then we can compute them by using the entropy measure, whose procedure is described as follows:

(i) *For determination of* η_i: by taking the information $\beta_{ij}^{(d)}$ and their score function corresponding to each expert $\mathscr{U}_i$, we define the entropy measure as

$$U_i = \frac{1}{z(\sqrt{2}-1)} \sum_{d=1}^{z} \left[\sin\left(\frac{\pi\left(h_{ij}^{(d)}\right)}{2}\right) + \sin\left(\frac{\pi\left(1 - h_{ij}^{(d)}\right)}{2}\right) - 1\right]; \quad i = 1, 2, \ldots, n, \tag{31}$$

where $h_{ij}^{(d)} = 1/2m \sum_{j=1}^m p_{ij}^{(d)} (1 + \zeta_{ij}^d - \vartheta_{ij}^{(d)})$ represents the score values and U_i is the entropy value.

It is quite obvious that the lesser the value of the U_i, the more valuable information is obtained for expert $\mathscr{U}_i$ importance, i.e., smaller entropy value objects possessing higher priority. Based on this policy, the η_is are calculated as

$$\eta_i = \frac{1 - U_i}{n - \sum_{i=1}^n U_i}. \tag{32}$$

(ii) *For determination of*ξ_j: by utilizing the information $\beta_{ij}^{(d)}$ and the Euclidean distance measure between the parameter values and ideal measure $\beta^+ = (\zeta^+, \vartheta^+; p^+)$, we compute the weight vector ξ_j for each parameter e_j by using the entropy measure defined as follows:

$$V_j = \frac{1}{z(\sqrt{2}-1)} \sum_{d=1}^{z} \left[\sin\left(\frac{\pi\left(g_{ij}^{(d)}\right)}{2}\right) + \sin\left(\frac{\pi\left(1 - g_{ij}^{(d)}\right)}{2}\right) - 1\right]; \quad j = 1, 2, \ldots, m, \tag{33}$$

where

$$g_{ij}^{(d)} = \frac{1}{n} \sum_{i=1}^{n} \left(1 - \left(\frac{1}{2}\left(\left|p_{ij}^{(d)}\zeta_{ij}^{(d)} - p^+\zeta^+\right|^2 + \left|p_{ij}^{(d)}\vartheta_{ij}^{(d)} - p^+\vartheta^+\right|^2\right)\right)^{(1/2)}\right). \tag{34}$$

Thus, based on equation (33), the weight vector ξ_j for each parameter e_j is computed as

$$\xi_j = \frac{1 - V_j}{m - \sum_{j=1}^{m} V_j}. \tag{35}$$

Now, we describe two new methods for solving decision-making problems based on COPRAS and aggregation operators by obtaining the collective information $\beta_{ij}^{(d)}$ and the weight vectors η_i and ξ_j. The execution steps of them are represented as follows.

5.2. Approach Based on COPRAS Method. To find the finest alternatives by using the method described in Section 4 for PIFSS environment, the following steps are summarized along with their flowchart in Figure 1:

Step 1: summarize the information about the alternatives $\mathscr{V}^{(d)}$ in the form of equation (30).

Step 2: if weight vectors are completely unknown, then compute η_i and ξ_j by using equations (32) and (35), respectively.

Step 3: by equation (24), calculate the weighted judgement matrix denoted by $\overline{A}^{(d)} = (\overline{\zeta}_{ij}^{(d)}, \overline{\vartheta}_{ij}^{(d)}; \overline{p}_{ij}^{(d)})$ for each alternative.

Step 4: by equation (25), compute the values by using the PIFSWA operator.

Step 5: by equation (26), aggregate the cost type attributes and get $R^{(d)}$.

Step 6: utilize equations (27) and (28) to compute the priority values for $\mathscr{V}^{(d)}$.

Step 7: by equation (29), compute the utility degree $\mathscr{N}^{(d)}$ for each $\mathscr{V}^{(d)}$.

Step 8: arrange the values of $\mathscr{N}^{(d)}$ and select the desired alternatives.

5.3. Approach Based on Operators. This section presents approaches for solving the decision-making problems based on proposed operators. The pictorial representation of this approach is given in Figure 2 while their steps are explained as follows:

Step 1: summarize the information about the alternatives $\mathscr{V}^{(d)}$ in the form of equation (30).

Step 2: if weight vectors are completely unknown, then compute η_i and ξ_j by using equations (32) and (35), respectively.

Step 3: normalize the information $\beta_{ij}^{(d)}$, if needed for the cost type parameters, to $q_{ij}^{(d)}$ by (36) given as

$$q_{ij}^{(d)} = \begin{cases} \left(\zeta_{ij}^{(d)}, \vartheta_{ij}^{(d)}; p_{ij}^{(d)}\right); & \text{for benefit type parameters,} \\ \left(\vartheta_{ij}^{(d)}, \zeta_{ij}^{(d)}; 1 - p_{ij}^{(d)}\right); & \text{for cost type parameters.} \end{cases} \tag{36}$$

Step 4: aggregate the values $q_{ij}^{(d)}$ of each alternative $\mathscr{V}^{(d)}$ into $q^{(d)}$ either by the PIFSWA or PIFSWG operator. The person may utilize the appropriate operator based on their desired goal towards the pessimistic or optimistic decision. For example, if a person utilizes the PIFSWA operator to combine $q_{ij}^{(d)}$ by using η_i and ξ_j information, then the collective one of alternative $\mathscr{V}^{(d)}$ denoted by $q^{(d)} = (\zeta^{(d)}, \vartheta^{(d)}; p^{(d)})$ is computed by equation (37) as

$$q^{(d)} = \left(1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \zeta_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\vartheta_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}; 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - p_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}\right). \tag{37}$$

On the other hand, if a person utilizes the PIFSWG operator, then $q^{(d)}$ is computed by equation (38) as

$$q^{(d)} = \left(\prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\zeta_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}, 1 - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(1 - \vartheta_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}; \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(p_{ij}^{(d)}\right)^{\eta_i}\right)^{\xi_j}\right). \tag{38}$$

Step 5: compute the score values of $\mathscr{V}^{(d)}$ by equation (11) as

$$S\left(q^{(d)}\right) = p^{(d)}\left(\frac{1 + \zeta^{(d)} - \vartheta^{(d)}}{2}\right). \tag{39}$$

If score values are equal for any two indices, then compute the accuracy values for them by using equation (12).

Step 6: by descending values of $S(q^{(d)})$, we rank the given alternatives $\mathscr{V}^{(d)}$ $(d = 1, 2, \ldots, z)$ and select the best ones.

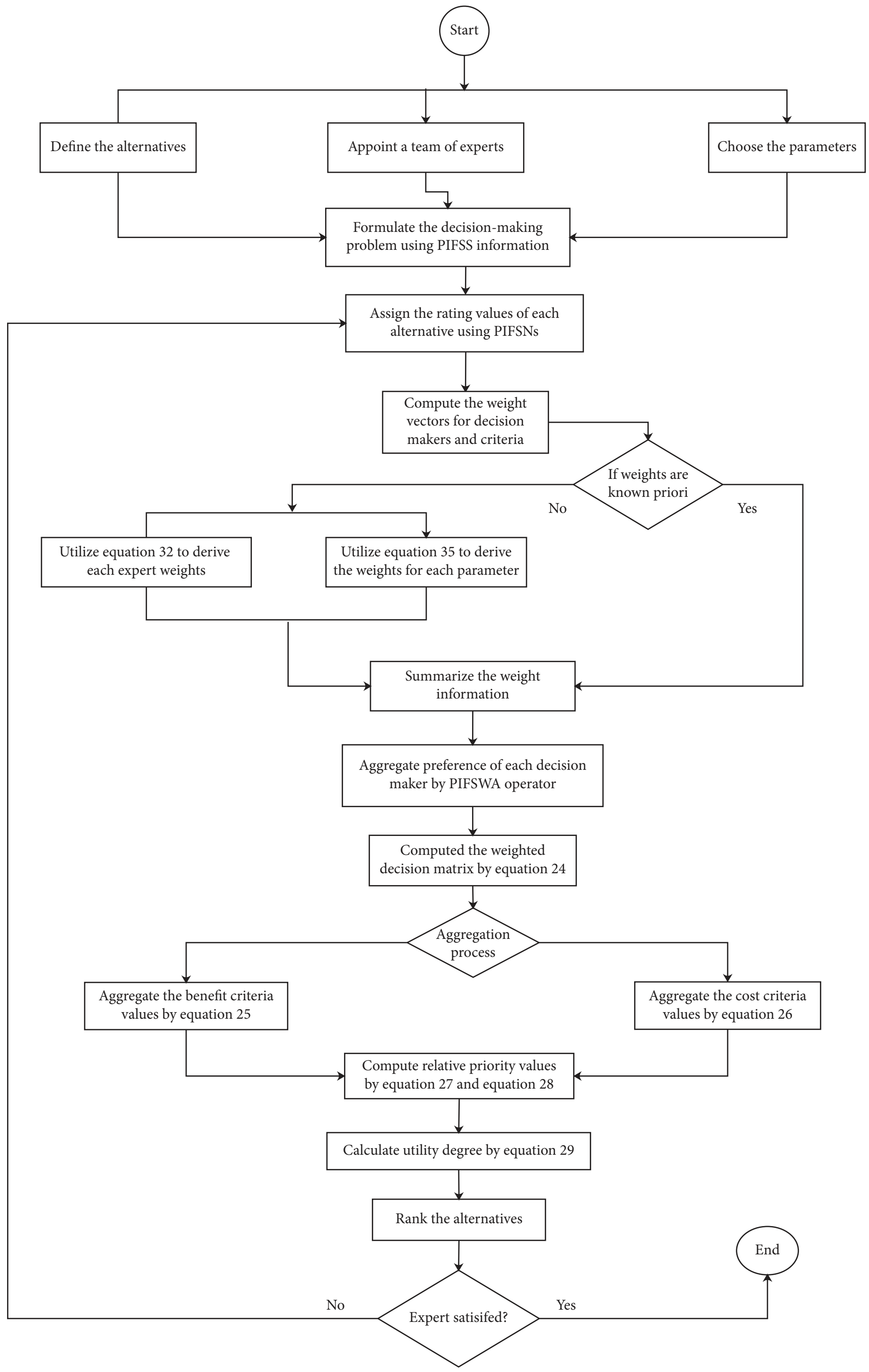

Figure 1: Flowchart of the proposed approach based on the COPRAS method.

6. Illustrative Example

This section demonstrates the abovementioned approaches with a numerical example and compares their results with several existing methods.

6.1. Case Study. "Consider an IT outsourcing provider selection problem as an MADM problem. Millennium Semiconductors (MS), established in October 1995, is an ISO 9001 – 2015 organization with distribution of electronic components as its core expertise. This leading distributor of electronic components in India is synonymous with innovation, and today, it is one of the most reputed names in market. MS has been set up in nearly each locale of India with catering more than 1500 clients in all sections from the recent two decades. It engages in investigation and improvement and production and promoting of items, for example, full shading ultrahigh shine LED epitaxial items,

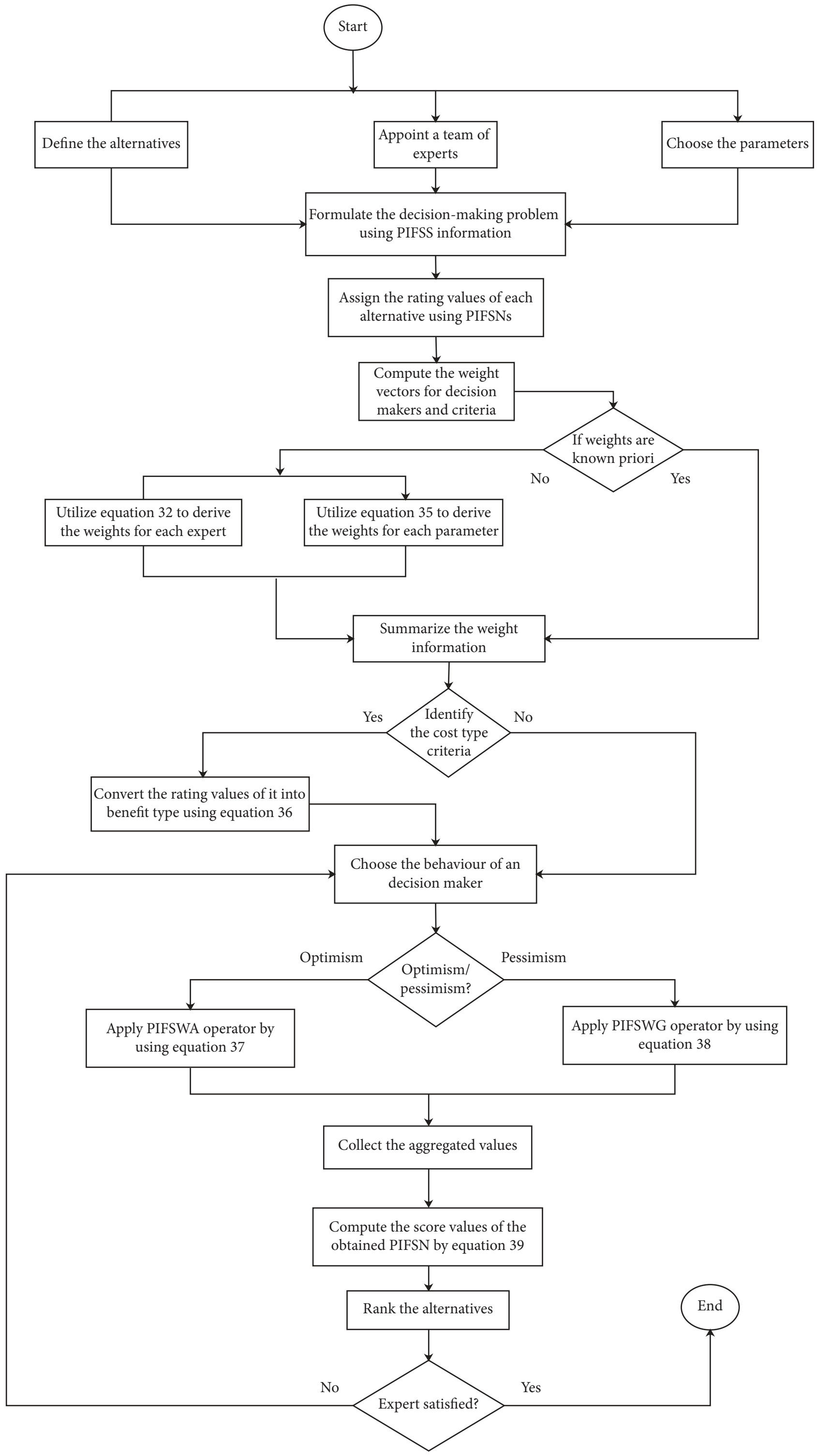

Figure 2: Flowchart of the proposed approach based on aggregation operators.

chips, compound sun oriented cells, and high control concentrating sunlight based items. The branch offices of MS are situated in Delhi, Bangalore, Hyderabad, Ahmedabad, Chennai, and Mumbai in India and Overseas workplaces in Singapore and Shenzhen (China). MS contributes the extraordinary larger part of labor and financial resources to its center competition rather than IT. The outsourcing of IT is a better choice for MS as of its lack of ability to do it efficiently. Therefore, MS selects the following outsourcing providers: Tata Consultancy services ($\mathscr{V}^{(1)}$), Infosys ($\mathscr{V}^{(2)}$), Wipro ($\mathscr{V}^{(3)}$), and HCL ($\mathscr{V}^{(4)}$). Now, to find the more suitable or best outsourcing provider among the above choices, MS hires a team of four experts u_1, u_2, u_3, and u_4. These experts evaluate each provider against the five parameters: design development (e_1), quality of product (e_2), delivery time (e_3), risk factor (e_4), and cost (e_5). Each expert assesses its rating values for each provider over the parameters in terms of PIFSNs which are summarized in Table 1. To compute the importance of each expert, we find their associated weight vector by using equations (31) and (32) and the decision matrices given in Table 1 and can get $\eta_1 = 0.3126$, $\eta_2 = 0.2718$, $\eta_3 = 0.2579$, and $\eta_4 = 0.1578$. Similarity, by utilizing equations (33) and (35), we can get the weights of each parameter which are $\xi_1 = 0.1823$, $\xi_2 = 0.2043$, $\xi_3 = 0.1927$, $\xi_4 = 0.2166$, and $\xi_5 = 0.2041$. Thus, based on such information, we applied the above proposed algorithms to select the finest alternatives.

6.2. Based on COPRAS Method. The steps of the method presented in Section 5.2 are executed on the considered problem as follows:

Step 1: Information about the alternatives is represented in Table 1.

Step 2: By equations (31) and (32), we get $\eta_1 = 0.3126$, $\eta_2 = 0.2718$, $\eta_3 = 0.2579$, and $\eta_4 = 0.1578$. Similarly, the weight vectors for five parameters e_j are computed by using equations (33) and (35) and hence we get $\xi_1 = 0.1823$, $\xi_2 = 0.2043$, $\xi_3 = 0.1927$, $\xi_4 = 0.2166$, and $\xi_5 = 0.2041$.

Step 3: By equation (24), the weighted decision matrix for $\mathscr{V}^{(d)}$ is computed and presented in Table 2.

Step 4: By equation (25), aggregated values $P^{(d)}$ of each alternative $\mathscr{V}^{(d)}$ are computed as $P^{(1)} = (0.3909, 0.5127; 0.2576)$, $P^{(2)} = (0.3070, 0.6036; 0.2472)$, $P^{(3)} = (0.3665, 0.5304; 0.2587)$, and $P^{(4)} = (0.3725, 0.4941; 0.2859)$.

Step 5: The collective values $R^{(d)}$ of each alternative $\mathscr{V}^{(d)}$ by using equation (26) are obtained as $R^{(1)} = (0.3692, 0.4012; 0.4093)$, $R^{(2)} = (0.4255, 0.4217; 0.3738)$, $R^{(3)} = (0.5358, 0.3594; 0.4231)$, and $R^{(4)} = (0.4117, 0.4375; 0.5849)$.

Step 6: By equation (28), the priority values for $\mathscr{V}^{(d)}$ are obtained as $Q^{(1)} = 0.3723$, $Q^{(2)} = 0.3606$, $Q^{(3)} = 0.3145$, and $Q^{(4)} = 0.3058$.

Step 7: The utility degrees for alternative $\mathscr{V}^{(d)}$ are measured by using equation (29) and obtain $\mathscr{N}^{(1)} = 100$, $\mathscr{N}^{(2)} = 96.85$, $\mathscr{N}^{(3)} = 84.47$, and $\mathscr{N}^{(4)} = 82.13$.

Step 8: By the values of $\mathscr{N}^{(d)}$, the ordering of the given objects is $\mathscr{V}^{(1)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(4)}$ and hence $\mathscr{V}^{(1)}$ is the best alternative.

6.3. Computational Results Based on Operators. The steps of algorithm described in Section 5.3 are executed here to get the finest alternatives by using operators:

Step 1: Information about the alternatives is represented in Table 1.

Step 2: The values of η_i and ξ_j are given in Step 2 of Section 6.2.

Step 3: The parameters e_3, e_4, and e_5 are the cost types, and so by equation (36), we compute $q_{ij}^{(d)}$ and results are summarized in Table 3.

Step 4: By taking weight vectors η_i and ζ_j, we apply the PIFSWA operator on the values of $q_{ij}^{(d)}$ by using equation (37). The obtained values of $q^{(d)}$ are $q^{(1)} = (0.4973, 0.3265; 0.4950)$, $q^{(2)} = (0.4368, 0.4129; 0.5058)$, $q^{(3)} = (0.4618, 0.4135; 0.4890)$, and $q^{(4)} = (0.4936, 0.3450; 0.4103)$. On the other hand, if a person utilizes the PIFSWG operator given in equation (38) to obtain the collective values, then we can get $q^{(1)} = (0.3453, 0.4209; 0.4494)$, $q^{(2)} = (0.3380, 0.5043; 0.4759)$, $q^{(3)} = (0.3082, 0.5780; 0.4479)$, and $q^{(4)} = (0.3686, 0.4550; 0.3290)$.

Step 5: By equation (39), the score values of the numbers obtained through the PIFSWA operator are given as $S(q^{(1)}) = 0.2898$, $S(q^{(2)}) = 0.2590$, $S(q^{(3)}) = 0.2563$, and $S(q^{(4)}) = 0.2356$, while these values for the numbers obtained by PIFSWG operators are $S(q^{(1)}) = 0.2077$, $S(q^{(2)}) = 0.1984$, $S(q^{(3)}) = 0.1635$, and $S(q^{(4)}) = 0.1503$.

Step 6: Based on the values obtained from Step 5, we obtain the ranking order as $\mathscr{V}^{(1)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(4)}$ and $\mathscr{V}^{(1)}$ is the best alternative.

6.4. Comparative Studies. To contrast the effects of proposed methods with a few actual strategies under PIFSS and IFSS environment, a correlation analysis has been made with methods given in [25, 26, 37, 51]. The obtained results are reviewed in Table 4. In this table, the first comparison is done with the similarity measure proposed by Bashir et al. [37]. As we can see in Table 4, the ranking order is $\mathscr{V}^{(1)} \succ \mathscr{V}^{(2)} = \mathscr{V}^{(3)} \succ \mathscr{V}^{(4)}$. Here, we cannot distinguish between the choices $\mathscr{V}^{(2)}$ and $\mathscr{V}^{(3)}$. Therefore, the existing similarity measure [37] fails in this case. Moreover, to solve the estimated problem with this method, we need an ideal alternative which enhances the "complexity and computational" overhead, but in proposed approaches, we do not need any ideal resolution. Thus, the recommended methods are more fit for solving DM problems having PIFSS information.

However, the decision maker cannot give an exact opinion due to multifaceted nature and uncertainty in DM process. In the literature, there are numerous techniques proposed without considering the possibility of the assessment esteem to illuminate DM problems with IFSS data, in which they do not consider the decision maker's risk factor during DM process. But, the proposed approaches created in this paper consider the possibility of the data given by the decision maker to an assessed object. Further, in order to contrast the proposed approach result with the existing techniques of IFSS environment, we need to take the degree of possibility equal to 1, i.e., we have to take $p_{ij} = 1 \forall i, j$ for converting PIFSS to IFSS. Thus, the proposed operators PIFSWA and PIFSWG reduce to the existing intuitionistic fuzzy soft weighted averaging and geometric (IFSWA or IFSWG) operators [25], respectively. From Table 4, we can

Table 1: Decision matrices for each alternative in terms of PIFSNs.

	For alternative $\mathscr{V}^{(1)}$					For alternative $\mathscr{V}^{(2)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.5, 0.2; 0.3)	(0.6, 0.3; 0.4)	(0.7, 0.1; 0.5)	(0.3, 0.2; 0.6)	(0.4, 0.5; 0.4)	(0.7, 0.2; 0.5)	(0.5, 0.4; 0.6)	(0.6, 0.3; 0.4)	(0.7, 0.3; 0.5)	(0.6, 0.2; 0.4)
u_2	(0.6, 0.1; 0.4)	(0.8, 0.2; 0.6)	(0.6, 0.2; 0.7)	(0.5, 0.3; 0.7)	(0.4, 0.1; 0.3)	(0.5, 0.3; 0.3)	(0.8, 0.2; 0.5)	(0.8, 0.2; 0.5)	(0.6, 0.1; 0.3)	(0.5, 0.4; 0.6)
u_3	(0.7, 0.3; 0.7)	(0.9, 0.1; 0.7)	(0.8, 0.2; 0.8)	(0.6, 0.2; 0.6)	(0.4, 0.3; 0.5)	(0.3, 0.6; 0.4)	(0.6, 0.3; 0.7)	(0.5, 0.3; 0.6)	(0.3, 0.5; 0.6)	(0.3, 0.4; 0.5)
u_4	(0.8, 0.1; 0.6)	(0.7, 0.2; 0.5)	(0.5, 0.4; 0.4)	(0.5, 0.2; 0.5)	(0.3, 0.5; 0.6)	(0.6, 0.1; 0.5)	(0.7, 0.2; 0.5)	(0.4, 0.3; 0.7)	(0.8, 0.1; 0.7)	(0.7, 0.1; 0.7)
	For alternative $\mathscr{V}^{(3)}$					For alternative $\mathscr{V}^{(4)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.8, 0.2; 0.7)	(0.7, 0.3; 0.5)	(0.9, 0.1; 0.7)	(0.7, 0.2; 0.4)	(0.8, 0.1; 0.7)	(0.9, 0.1; 0.6)	(0.4, 0.3; 0.6)	(0.5, 0.2; 0.7)	(0.6, 0.2; 0.7)	(0.4, 0.3; 0.9)
u_2	(0.7, 0.1; 0.5)	(0.6, 0.2; 0.4)	(0.6, 0.1; 0.5)	(0.6, 0.3; 0.5)	(0.7, 0.2; 0.6)	(0.7, 0.2; 0.5)	(0.6, 0.2; 0.7)	(0.6, 0.4; 0.8)	(0.7, 0.2; 0.6)	(0.5, 0.4; 0.7)
u_3	(0.6, 0.2; 0.6)	(0.8, 0.1; 0.5)	(0.5, 0.5; 0.7)	(0.5, 0.4 0.4)	(0.6, 0.3; 0.5)	(0.6, 0.1; 0.7)	(0.7, 0.1; 0.3)	(0.7, 0.3; 0.6)	(0.7, 0.1; 0.8)	(0.6, 0.3; 0.8)
u_4	(0.5, 0.4; 0.4)	(0.6, 0.3; 0.6)	(0.8, 0.1; 0.4)	(0.6, 0.2; 0.5)	(0.9, 0.1; 0.9)	(0.8, 0.2; 0.5)	(0.7, 0.2; 0.6)	(0.4, 0.6; 0.9)	(0.5, 0.4; 0.6)	(0.5, 0.2; 0.8)

clearly see that the ranking order of the obtained rating values by utilizing proposed PIFSWA is the same as that of the IFSWA operator [25] results. The score values are different because both the techniques consider different score functions. Also, the ranking order obtained by the operators defined in [26] is different. The reason of this difference is that the authors in [26] considered the prioritized relationship between the parameters.

Further, the ranking order of the alternatives by utilizing distance measure [51] is $\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$, which is different from our results because the approach discussed by Selvachandran et al. [51] considers the distances of the alternative from an ideal alternative. However, the determination of the ideal alternative is merely a theoretical concept whose practical equivalency is difficult to attain. In this way, the proposed strategies are more adaptable and viable than the existing strategies.

6.5. Advantages of Proposed Approach. In this section, some advantages of the proposed work are highlighted, which are as follows:

(1) The presented approach took the importance of the concept of possibility along with the IFSS to handle modern decision-making problems. The considered possibility degree reflects the possibility of the existence of the degree of recognition and dismissal; therefore, this organization has huge potential in the true description within the area of computational penetrations.

(2) If we assign possibility value as 1 to each IFSN then the suggested PIFSWA and PIFSWG operators reduce to the existing IFSWA and IFSWG operators [25], respectively. Also, the set defined as PIFSS reduces to IFSS. Hence, IFSS can be exercised as a particular case of PIFSS.

(3) In this manuscript, a new entropy measure has been formed which not only renders the overall knowledge about the amount of uncertainty imbed in the specific structure but also appropriates as an efficient tool in the DM process. As in this process, the allocation of weights to the experts as well as the parameters to signify the preference of both the proposed entropy measures has been utilized. Thus, this paper gives us a way to find the completely unknown weights of the experts and the parameters using entropy measures.

(4) In this manuscript, the COPRAS technique is utilized as a ranking method which is a proper policy to prepare the information sensibly and effectively. The strategy used by the COPRAS method can process the information given over the parameters from distinctive points based on the complex proportional calculation. This method contains more accurate information compared with other strategies dealing with the benefit parameters or the cost parameters.

Table 2: Weighted decision matrix for each alternative.

	For alternative $\mathscr{V}^{(1)}$					For alternative $\mathscr{V}^{(2)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.0387, 0.9124; 0.0201)	(0.0568, 0.9260; 0.0321)	(0.0700, 0.8705; 0.0409)	(0.0239, 0.8968; 0.0602)	(0.0321, 0.9568; 0.0321)	(0.0663, 0.9124; 0.0387)	(0.0433, 0.9432; 0.0568)	(0.0537, 0.9300; 0.0303)	(0.0783, 0.9217; 0.0458)	(0.0568, 0.9024; 0.0321)
u_2	(0.0444, 0.8922; 0.0250)	(0.0855, 0.9145; 0.0496)	(0.0469, 0.9192; 0.0611)	(0.0400, 0.9316; 0.0684)	(0.0279, 0.8801; 0.0196)	(0.0338, 0.9421; 0.0175)	(0.0855, 0.9145; 0.0378)	(0.0808, 0.9192; 0.0357)	(0.0525, 0.8732; 0.0208)	(0.0377, 0.9505; 0.0495)
u_3	(0.0550, 0.9450; 0.0550)	(0.1143, 0.8857; 0.0615)	(0.0769, 0.9231; 0.0769)	(0.0499, 0.9140; 0.0499)	(0.0265, 0.9386; 0.0358)	(0.0166, 0.9763; 0.0237)	(0.0471, 0.9385; 0.0615)	(0.0339, 0.9419; 0.0445)	(0.0197, 0.9620; 0.0499)	(0.0186, 0.9529; 0.0358)
u_4	(0.0452, 0.9359; 0.0260)	(0.0381, 0.9494; 0.0221)	(0.0209, 0.9725; 0.0154)	(0.0234, 0.9465; 0.0234)	(0.0114, 0.9779; 0.0291)	(0.0260, 0.9359; 0.0197)	(0.0381, 0.9494; 0.0221)	(0.0154, 0.9641; 0.0359)	(0.0535, 0.9243; 0.0403)	(0.0380, 0.9286; 0.0380)
	For alternative $\mathscr{V}^{(3)}$					For alternative $\mathscr{V}^{(4)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	(0.0876, 0.9124; 0.0663)	(0.0740, 0.9260; 0.0433)	(0.1295, 0.8705; 0.0700)	(0.0783, 0.8968; 0.0340)	(0.0976, 0.8634; 0.0739)	(0.1229, 0.8771; 0.0509)	(0.0321, 0.9260; 0.0568)	(0.0409, 0.9076; 0.0700)	(0.0602, 0.8968; 0.0783)	(0.0321, 0.9261; 0.1366)
u_2	(0.0579, 0.8922; 0.0338)	(0.0496, 0.9145; 0.0280)	(0.0469, 0.8864; 0.0357)	(0.0525, 0.9316; 0.0400)	(0.0646, 0.9146; 0.0495)	(0.0579, 0.9234; 0.0338)	(0.0496, 0.9145; 0.0647)	(0.0469, 0.9531; 0.0808)	(0.0684, 0.9096; 0.0525)	(0.0377, 0.9505; 0.0646)
u_3	(0.0422, 0.9271; 0.0422)	(0.0813, 0.8857; 0.0359)	(0.0339, 0.9661; 0.0581)	(0.0380, 0.9501; 0.0281)	(0.0471, 0.9386; 0.0358)	(0.0422, 0.8974; 0.0550)	(0.0615, 0.8857; 0.0186)	(0.0581, 0.9419; 0.0445)	(0.0650, 0.8793; 0.0860)	(0.0471, 0.9386; 0.0812)
u_4	(0.0197, 0.9740; 0.0146)	(0.0291, 0.9619; 0.0291)	(0.0478, 0.9324; 0.0154)	(0.0308, 0.9465; 0.0234)	(0.0714, 0.9286; 0.0714)	(0.0452, 0.9548; 0.0197)	(0.0381, 0.9494; 0.0291)	(0.0154, 0.9846; 0.0676)	(0.0234, 0.9692; 0.0308)	(0.0221, 0.9495; 0.0505)

Table 3: Normalized decision matrices for each alternative.

	For alternative $\mathscr{V}^{(1)}$					For alternative $\mathscr{V}^{(2)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	$\begin{pmatrix} 0.5, 0.2; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.3; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.7; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.3; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.4; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.2; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.4; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.6; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.7; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.6 \end{pmatrix}$
u_2	$\begin{pmatrix} 0.6, 0.1; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.8, 0.2; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.5; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.5; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.3; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.8, 0.2; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.8; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.6; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.5; \\ 0.4 \end{pmatrix}$
u_3	$\begin{pmatrix} 0.7, 0.3; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.9, 0.1; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.8; \\ 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.4; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.6; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.3; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.5; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.3; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.3; \\ 0.5 \end{pmatrix}$
u_4	$\begin{pmatrix} 0.8, 0.1; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.2; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.5; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.5; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.3; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.1; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.2; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.4; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.8; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.7; \\ 0.3 \end{pmatrix}$
	For alternative $\mathscr{V}^{(3)}$					For alternative $\mathscr{V}^{(4)}$				
	e_1	e_2	e_3	e_4	e_5	e_1	e_2	e_3	e_4	e_5
u_1	$\begin{pmatrix} 0.8, 0.2; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.3; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.9; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.7; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.8; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.9, 0.1; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.3; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.5; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.4; \\ 0.1 \end{pmatrix}$
u_2	$\begin{pmatrix} 0.7, 0.1; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.2; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.6; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.6; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.7; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.7; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.6; \\ 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.7; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.5; \\ 0.3 \end{pmatrix}$
u_3	$\begin{pmatrix} 0.6, 0.2; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.8, 0.1; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.5, 0.5; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.5; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.6; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.1; \\ 0.7 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.1; \\ 0.3 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.7; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.7; \\ 0.2 \end{pmatrix}$	$\begin{pmatrix} 0.3, 0.6; \\ 0.2 \end{pmatrix}$
u_4	$\begin{pmatrix} 0.5, 0.4; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.3; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.8; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.6; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.1, 0.9; \\ 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.8, 0.2; \\ 0.5 \end{pmatrix}$	$\begin{pmatrix} 0.7, 0.2; \\ 0.6 \end{pmatrix}$	$\begin{pmatrix} 0.6, 0.4; \\ 0.1 \end{pmatrix}$	$\begin{pmatrix} 0.4, 0.5; \\ 0.4 \end{pmatrix}$	$\begin{pmatrix} 0.2, 0.5; \\ 0.2 \end{pmatrix}$

Table 4: Comparison of proposed approaches with existing approaches.

	When equal weights are taken					When the weights obtained by proposed approach are taken				
	$\mathscr{V}^{(1)}$	$\mathscr{V}^{(2)}$	$\mathscr{V}^{(3)}$	$\mathscr{V}^{(4)}$	Ranking order	$\mathscr{V}^{(1)}$	$\mathscr{V}^{(2)}$	$\mathscr{V}^{(3)}$	$\mathscr{V}^{(4)}$	Ranking order
Bashir et al. [37]	0.6439	0.6201	0.6201	0.5250	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(2)} = \mathscr{V}^{(3)} \succ \mathscr{V}^{(4)}$	0.6439	0.6201	0.6201	0.5250	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(2)} = \mathscr{V}^{(3)} \succ \mathscr{V}^{(4)}$
Arora and Garg [25]	0.1961	0.0381	0.0405	0.1759	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(2)}$	0.1709	0.0239	0.0484	0.1487	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(2)}$
Arora and Garg [26]	0.5940	0.5574	0.5864	0.6303	$\mathscr{V}^{(4)} \succ \mathscr{V}^{(1)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(2)}$	0.5940	0.5574	0.5864	0.6303	$\mathscr{V}^{(4)} \succ \mathscr{V}^{(1)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(2)}$
Selvachandran et al. [51]	0.8630	0.9271	0.9611	0.8708	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$	0.8765	0.9289	0.9590	0.8886	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$
Proposed PIFSWA operator with possibility = 1	0.5981	0.5190	0.5203	0.5880	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(3)} \succ \mathscr{V}^{(2)}$	0.5615	0.4843	0.4646	0.5430	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$
Proposed PIFSWG operator with possibility = 1	0.3875	0.4168	0.4429	0.4156	$\mathscr{V}^{(3)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(1)} \succ \mathscr{V}^{(4)}$	0.3753	0.4145	0.4459	0.4131	$\mathscr{V}^{(3)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(1)} \succ \mathscr{V}^{(4)}$
Proposed COPRAS method with possibility = 1	100	86.9067	83.4016	98.2851	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$	100	87.7661	85.4024	99.7088	$\mathscr{V}^{(1)} \succ \mathscr{V}^{(4)} \succ \mathscr{V}^{(2)} \succ \mathscr{V}^{(3)}$

7. Conclusion

The key contribution of the work can be summarized as follows:

(1) The examined study employs the PIFSS to handle the inadequate, vague, and conflicting data by considering membership degree, nonmembership degree, and the possibility degree towards these membership degrees. Thus, a PIFSS expresses the veritable circumstances of the real conditions because it represents the fuzziness and the possibility acquired in genuine issues.

(2) This paper offers new operational laws and distance measures for estimating the degree of discrimination between the two or more PIFSSs. Traditionally, all the measurements are computed without considering the degree of possibility into the analysis, which may not furnish the proper choice to the expert. To succeed it, distance measures are injected in this work which supplies an alternative way to trade with the PIFSS information.

(3) A new COPRAS method is presented for the collection of the PIFSNs to rank the give alternatives. A utility degree has been defined to rank the given numbers.

(4) To aggregate the different collection of PIFSNs, we proposed some series of weighted averaging and geometric operators and investigated their properties. From these stated operators, it can be concluded that by assigning the possibility degree of each rating as one then the operators [25, 26] under the IFSS can be deduced.

(5) Two new algorithms, based on proposed COPRAS and aggregation operators, are presented to solve the multiple attribute decision-making problems with PIFSS information. In these approaches, the weight vector of the experts and the parameters are computed with the help of the entropy and distance measures. The fundamental advantages of proposed techniques as compared to existing ones are that these reflect the decision maker's risk factor in the application fields represented by the possibility of each assessment esteem. Also, the COPRAS method-based approach gives important and valuable data including the extent of goals and the requests accomplished by the choice producers and the amount of proficiency for one elective towards the other. Hence, the proposed work gives a more reasonable picture from dubious perspectives of practical situations. Finally, a numerical example is presented to demonstrate the approach and compare their results with the several existing approaches.

In the future, we shall lengthen the application of the proposed measures to the diverse fuzzy environment as well as different fields of application such as supply chain management, emerging decision problems, brain hemorrhage, and risk evaluation [52–54].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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