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Algorithms of Estimating Reachable Sets of Nonlinear Control Systems with Uncertainty

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Abstract. The problem of estimating reachable sets of nonlinear dynamical control systems with quadratic nonlinearity and with uncertainty in initial states is studied. We assume that the uncertainty is of a set-membership kind when we know only the bounding set for unknown items and any additional statistical information on their behavior is not available. We present approaches that allow finding ellipsoidal estimates of reachable sets. The algorithms of constructing such ellipsoidal set-valued estimates and numerical simulation results are given in two cases, for control systems with classical controls and for measure driven (impulsive) control systems.

Keywords: Nonlinear Control Systems, Uncertainty, State Estimation, Ellipsoidal Calculus, Funnel Equations, Trajectory Tubes, Simulation.

1 Introduction

In this paper we study control systems with unknown but bounded uncertainties related to the case of a set-membership description of uncertainty (Bertsekas and Rhodes[1], Kurzhanski and Valyi[11], Schweppe[13], Walter and Pronzato[15]). The motivation to consider the set-membership approach is that in traditional formulations the characterization of parameter uncertainties requires assumptions on mean, variances or probability density function of errors. However in many applied areas ranged from engineering problems in physics to economics as well as to biological and ecological modeling it occurs that a stochastic nature of the error sequence is questionable. For instance, in case of limited data or after some non-linear transformation of the data, the presumed stochastic characterization is not always valid. Hence, as an alternative to a stochastic characterization a so-called bounded-error characterization, also called set-membership approach, has been proposed and intensively developed in the last decades.

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The solution of many control and estimation problems under uncertainty involves constructing reachable sets and their analogs. For models with linear dynamics under such set-membership uncertainty there are several constructive approaches which allow finding effective estimates of reachable sets. We note here two of the most developed approaches to research in this area. The first one is based on ellipsoidal calculus (Chernousko[2], Kurzhanski and Valyi[11]) and the second one uses the interval analysis (Walter and Pronzato[15]).

Many applied problems are mostly nonlinear in their parameters and the set of feasible system states is usually non-convex or even non-connected. The key issue in nonlinear set-membership estimation is to find suitable techniques, which produce related bounds for the set of unknown system states without being too computationally demanding. Some approaches to the nonlinear set-membership estimation problems and discrete approximation techniques for differential inclusions through a set-valued analogy of well-known Euler's method were developed in Dontchev and Lempio[3], Veliov[14]. In this paper the modified state estimation approaches which use the special quadratic structure of nonlinearity of studied control system and use also the advantages of ellipsoidal calculus (Kurzhanski and Valyi[11], Chernousko[2]) are presented.

2 Preliminaries

In this section we introduce the following basic notations. Let R^n be the n–dimensional Euclidean space and $x'y$ be the usual inner product of $x, y \in R^n$ with prime as a transpose, $\| x \| = (x'x)^{1/2}$. We denote as $B(a,r)$ the ball in R^n, $B(a,r) = \{x \in R^n : \| x - a \| \leq r\}$, I is the identity $n \times n$-matrix. Denote by $E(a,Q)$ the ellipsoid in R^n, $E(a,Q) = \{x \in R^n : (Q^{-1}(x-a),(x-a)) \leq 1\}$ with center $a \in R^n$ and symmetric positive definite $n \times n$–matrix Q.

Consider the following system

$$\dot{x} = Ax + f^{(1)}(x)d^{(1)} + f^{(2)}(x)d^{(2)}, \ x_0 \in X_0, \ t_0 \leq t \leq T, \tag{1}$$

where $x \in R^n$, $\|x\| \leq K$ $(K > 0)$, $d^{(1)}$ and $d^{(2)}$ are n-vectors and $f^{(1)}, f^{(2)}$ are scalar functions,

$$f^{(1)}(x) = x'B^{(1)}x, \quad f^{(2)}(x) = x'B^{(2)}x,$$

with symmetric and positive definite matrices $B^{(1)}, B^{(2)}$. We assume also that $d_i^{(1)} = 0$ for $i = k+1, \ldots, n$ and $d_j^{(2)} = 0$ for $j = 1, \ldots, k$ where k $(1 \leq k \leq n)$ is fixed. This assumption means that the first k equations of the system (1) contain only the nonlinear function $f^{(1)}(x)$ (with some constant coefficients $d_i^{(1)}$) while $f^{(2)}(x)$ is included only in the equations with numbers $k+1, \ldots, n$.

We will assume further that X_0 in (1) is an ellipsoid, $X_0 = E(a,Q)$, with a symmetric and positive definite matrix Q and with a center a.

We will need some auxiliary results.

Lemma 1. *The following inclusion is true*

$$X_0 \subseteq E(a, k_1^2(B^{(1)})^{-1}) \bigcap E(a, k_2^2(B^{(2)})^{-1}) \tag{2}$$

where k_i^2 is the maximal eigenvalue of the matrix $(B^{(i)})^{1/2}Q(B^{(i)})^{1/2}$ $(i = 1, 2)$.

Proof. The proof follows directly from the properties of quadratic forms and from the inclusions

$$E(a,Q) \subseteq E(a, k_1^2 (B^{(1)})^{-1}), \quad E(a,Q) \subseteq E(a, k_2^2 (B^{(2)})^{-1}),$$

which should be fulfilled with the smallest possible values of $k_1 \geq 0$ and $k_2 \geq 0$.

Lemma 2. *The following equalities hold true*

$$\max_{z'B^{(1)}z \leq k_1^2} z'B^{(2)}z = k_1^2 \lambda_{12}^2, \quad \max_{z'B^{(2)}z \leq k_2^2} z'B^{(1)}z = k_2^2 \lambda_{21}^2, \tag{3}$$

where λ_{12}^2 and λ_{21}^2 are maximal eigenvalues of matrices $(B^{(1)})^{-1/2} B^{(2)} (B^{(1)})^{-1/2}$ and $(B^{(2)})^{-1/2} B^{(1)} (B^{(2)})^{-1/2}$ respectively.

Proof. The formulas follow from direct computations of maximal values in (3).

Theorem 1. *(Filippova[6]) For all $\sigma > 0$ and for $X(t_0+\sigma) = X(t_0+\sigma, t_0, X_0)$ we have the following upper estimate*

$$X(t_0+\sigma) \subseteq E(a^{(1)}(\sigma), Q^{(1)}(\sigma)) \bigcap E(a^{(2)}(\sigma), Q^{(2)}(\sigma)) + o(\sigma) B(0,1), \tag{4}$$

where $\sigma^{-1} o(\sigma) \to 0$ when $\sigma \to +0$ and

$$a^{(1)}(\sigma) = a(\sigma) + \sigma k_1^2 \lambda_{12}^2 d^{(2)}, \quad a^{(2)}(\sigma) = a(\sigma) + \sigma k_2^2 \lambda_{21}^2 d^{(1)}, \tag{5}$$

$$a(\sigma) = (I + \sigma A) a + \sigma a' B^{(1)} a d^{(1)} + \sigma a' B^{(2)} a d^{(2)}, \tag{6}$$

$$Q^{(1)}(\sigma) = (p_1^{-1}+1)(I+\sigma R) k_1^2 (B^{(1)})^{-1} (I+\sigma R)' + (p_1+1)\sigma^2 ||d^{(2)}||^2 k_1^4 \lambda_{12}^4 \cdot I, \tag{7}$$

$$Q^{(2)}(\sigma) = (p_2^{-1}+1)(I+\sigma R) k_2^2 (B^{(2)})^{-1} (I+\sigma R)' + (p_2+1)\sigma^2 ||d^{(1)}||^2 k_2^4 \lambda_{21}^4 \cdot I, \tag{8}$$

$$R = A + 2d^{(1)} a' B^{(1)} + 2d^{(2)} a' B^{(2)} \tag{9}$$

and p_1, p_2 are the unique positive solutions of related algebraic equations

$$\sum_{i=1}^{n} \frac{1}{p_1 + \alpha_i} = \frac{n}{p_1(p_1+1)} \ , \quad \sum_{i=1}^{n} \frac{1}{p_2 + \beta_i} = \frac{n}{p_2(p_2+1)} \tag{10}$$

with $\alpha_i, \beta_i \geq 0$ $(i = 1, ..., n)$ being the roots of the following equations

$$\det((I + \sigma R) k_1^2 (B^{(1)})^{-1} (I + \sigma R)' - \alpha \sigma^2 ||d^{(2)}||^2 k_1^4 \lambda_{12}^4 \cdot I) = 0, \tag{11}$$

$$\det((I + \sigma R) k_2^2 (B^{(2)})^{-1} (I + \sigma R)' - \beta \sigma^2 ||d^{(1)}||^2 k_2^4 \lambda_{21}^4 \cdot I) = 0. \tag{12}$$

We may formulate now the following scheme that gives the external estimate of trajectory tube $X(t)$ of the system (1) with given accuracy.

Algorithm 1. Subdivide the time segment $[t_0, T]$ into subsegments $[t_i, t_{i+1}]$ where $t_i = t_0 + ih$ $(i = 1, \ldots, m)$, $h = (T - t_0)/m$, $t_m = T$.

- Given $X_0 = E(a, Q)$, take $\sigma = h$ and define ellipsoids $E(a^{(1)}(\sigma), Q^{(1)}(\sigma))$ and $E(a^{(2)}(\sigma), Q^{(2)}(\sigma))$ from Theorem 2.

- Find the smallest (with respect to some criterion, Kurzhanski and Valyi[11], Chernousko[2]) ellipsoid $E(a_1, Q_1)$ which contains the intersection

$$E(a^{(1)}(\sigma), Q^{(1)}(\sigma)) \bigcap E(a^{(2)}(\sigma), Q^{(2)}(\sigma)) \subseteq E(a_1, Q_1).$$

- Consider the system on the next subsegment $[t_1, t_2]$ with $E(a_1, Q_1)$ as the initial ellipsoid at instant t_1.
- Next steps continue iterations 1-3. At the end of the process we will get the external estimate $E(a(t), Q(t))$ of the tube $X(t)$ with accuracy tending to zero when $m \to \infty$.

3 Main results

3.1 Control system under uncertainty

Consider the following control system in the form of differential inclusion (Kurzhanski and Filippova[10])

$$\dot{x} \in Ax + f^{(1)}(x)d^{(1)} + f^{(2)}(x)d^{(2)} + P, \ x_0 \in X_0 = E(a, Q), \ t_0 \le t \le T, \quad (13)$$

with all previous assumptions being valid. We assume also that P is an ellipsoid, $P = E(g, G)$, with a symmetric and positive definite matrix G and with a center g.

In this case the estimate for $X(t_0+\sigma)$ (the analogy of the formula (4)) takes the form.

Theorem 2. *The following inclusion is true*

$$\begin{aligned} X(t_0 + \sigma) \subseteq\ & E(a^{(1)}(\sigma), Q^{(1)}(\sigma)) \cap E(a^{(2)}(\sigma), Q^{(2)}(\sigma)) \\ & + \sigma E(g, G) \ + \ o(\sigma)B(0, 1), \end{aligned} \quad (14)$$

where $\sigma^{-1}o(\sigma) \to 0$ when $\sigma \to +0$ and the parameters $a^{(i)}, Q^{(i)}$ $(i = 1, 2)$ are described in (5)–(9).

Proof. The inclusion follows from the Theorem 1 and from the properties of the trajectory tubes of related differential inclusions (see also techniques in Filippova[7]).

We should modify now the previous scheme (Algorithm 1) in order to formulate a new procedure of external estimating of trajectory tube $X(t)$ of the system (13).

Algorithm 2. Subdivide the time segment $[t_0, T]$ into subsegments $[t_i, t_{i+1}]$ where $t_i = t_0 + ih$ $(i = 1, \dots, m)$, $h = (T - t_0)/m$, $t_m = T$.

- Given $X_0 = E(a, Q)$, take $\sigma = h$ and define ellipsoids $E(a^{(1)}(\sigma), Q^{(1)}(\sigma))$ and $E(a^{(2)}(\sigma), Q^{(2)}(\sigma))$ from Theorem 2.
- Find the smallest (with respect to some criterion (Kurzhanski and Valyi[11], Chernousko[2]) ellipsoid $E(a^*, Q^*)$ which contains the intersection:

$$E(a^{(1)}(\sigma), Q^{(1)}(\sigma)) \bigcap E(a^{(2)}(\sigma), Q^{(2)}(\sigma)) \subseteq E(a^*, Q^*).$$

- Find the ellipsoid $E(a_1, Q_1)$ which is the upper estimate of the sum (Kurzhanski and Valyi[11], Chernousko[2]) of two ellipsoids, $E(a^*, Q^*)$ and $\sigma E(g, G)$:

$$E(a^*, Q^*) + \sigma E(g, G) \subseteq E(a_1, Q_1).$$

- Consider the system on the next subsegment $[t_1, t_2]$ with $E(a_1, Q_1)$ as the initial ellipsoid at instant t_1.
- Next steps continue iterations 1-3. At the end of the process we will get the external estimate $E(a(t), Q(t))$ of the tube $X(t)$ with accuracy tending to zero when $m \to \infty$.

3.2 Examples

Consider three examples illustrating the techniques of ellipsoidal estimating. For simplicity we take here $d^{(2)} = 0$ so only one quadratic form is present at the right-hand side of the dynamic equations (13).

Example 1. Consider the following control system:

$$\begin{cases} \dot{x}_1 = & 2x_1 + u_1, \\ \dot{x}_2 = & 2x_2 + x_1^2 + x_2^2 + u_2, \end{cases} \qquad x_0 \in X_0, \qquad t \in [t_0, T]. \tag{15}$$

Here we take $t_0 = 0$, $T = 0.35$, $X_0 = B(0, 1)$ and put $P(t) \equiv U = B(0, 0.5)$ in the control constraint. The trajectory tube $X(t)$ and its external ellipsoidal estimate $E(a(t), Q(t))$ are given at Figure 1.

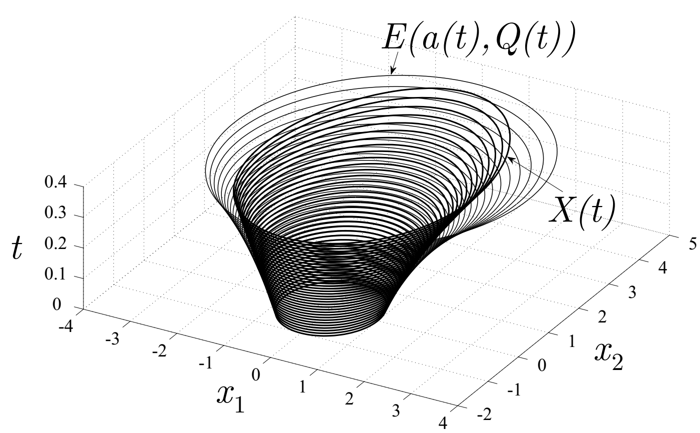

Fig. 1. Trajectory tube X(t) and its ellipsoidal estimate $E(a(t), Q(t))$.

The following example illustrates the case where the reachable set may lose convexity with increasing time $t > t_0$. Nevertheless the related external estimate is also true.

Example 2. Consider the following control system:

$$\begin{cases} \dot{x}_1 = & 2x_1 + u_1, \\ \dot{x}_2 = & 2x_2 + 4x_1^2 + x_2^2 + u_2, \end{cases} \qquad x_0 \in X_0, \qquad t \in [t_0, T]. \qquad (16)$$

Here we take $t_0 = 0$, $T = 0.25$, $X_0 = B(0,1)$ and $P(t) \equiv U = B(0,1)$. The trajectory tube $X(t)$ and its external ellipsoidal estimate $E(a(t), Q(t))$ are given at Figure 2.

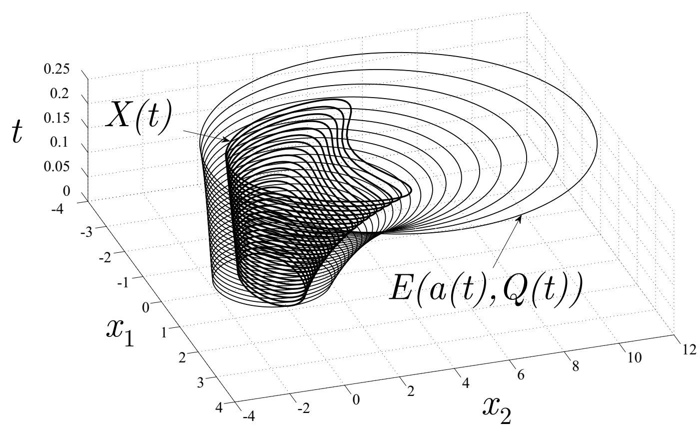

Fig. 2. Nonconvex-valued trajectory tube X(t) and its external ellipsoidal estimate $E(a(t), Q(t))$.

The following example illustrates the main procedure of the new Algorithm 2 of Section 3.1.

Example 3. Consider the following control system with two quadratic forms in its dynamical equations:

$$\begin{cases} \dot{x}_1 = & 1.5x_1 + x_1^2 + 2x_2^2 + u_1, \\ \dot{x}_2 = & 1.5x_2 + 2x_1^2 + x_2^2 + u_2, \end{cases} \qquad x_0 \in X_0, \qquad t \in [t_0, T]. \qquad (17)$$

Here we take $t_0 = 0$, $T = 0.3$, $X_0 = B(0,1)$ and $U = B(0, 0.1)$. Steps of the Algorithm 2 of constructing the external ellipsoidal estimate $E(a(t), Q(t))$ of the reachable set $X(t)$ are shown at Figure 3.

The resulting ellipsoidal estimate $E(a(T), Q(T))$ is shown at Figure 4. A parameter ρ indicated at Fig. 4 depends on a type of the optimality criterion which we use in constructing the external ellipsoid $E(a^*, Q^*)$ at the iterations 2-3 of the first step of Algorithm 2, see also Kurzhanski and Valyi[11] and Chernousko[2]. Here we see that the reachable set $X(T)$ is nonconvex and is contained in the ellipsoid $E(a(T), Q(T))$ for any value of the parameter ρ as shown at Fig. 4.

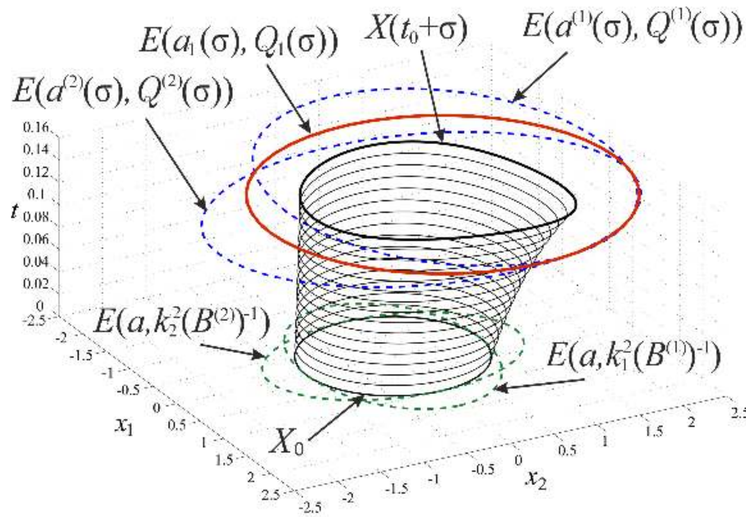

Fig. 3. Reachable set $X(t_0 + \sigma)$ and its estimate $E(a_1(\sigma), Q_1(\sigma))$ at the first step of Algorithm 2 (iterations 1-3).

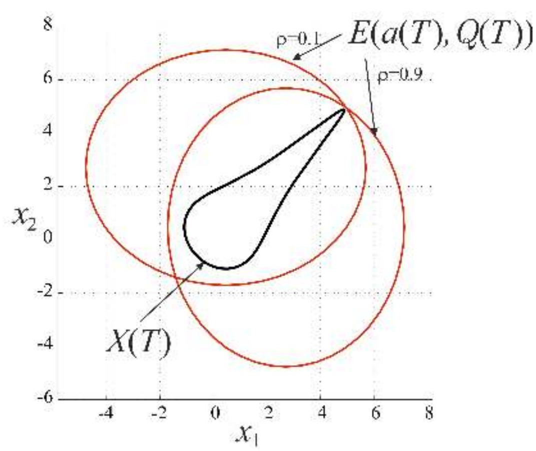

Fig. 4. Reachable set $X(T)$ and its external ellipsoidal estimate $E(a(T), Q(T))$.

3.3 Impulsive systems under uncertainty

Consider the following control system

$$dx(t) = (Ax(t) + \tilde{f}(x)d + u(t))dt + Bdv(t), \quad x \in R^n, \quad t_0 \le t \le T, \qquad (18)$$

where $\tilde{f}(x) = x'\tilde{B}x$ with positive definite and symmetric matrix $\tilde{B}$, parameters d, B are n-vectors, $d, B \in R^n$. Here the function $v : [t_0, T] \to R$ is of bounded variation on $[t_0, T]$, monotonically increasing and right-continuous. We assume that $\mu > 0$ and

$$Var_{t\in[t_0,T]}\, v(t) = \sup_{\{t_i | t_0 \le t_1 \le \ldots \le t_k = T\}} \{\sum_{i=1}^{k} |v(t_i) - v(t_{i-1})|\} \le \mu.$$

We assume also

$$X_0 = E(a, k^2\tilde{B}^{-1}) \ (k \neq 0), \quad U = E(\hat{a}, \hat{Q}). \qquad (19)$$

Consider the following auxiliary equation:

$$\frac{d}{d\eta} \begin{pmatrix} z \\ \tau \end{pmatrix} \in H(\tau, z), \qquad (20)$$

$$z(t_0) = x_0 \in X_0 = E(a, k^2\tilde{B}^{-1}), \quad \tau(t_0) = t_0, \quad t_0 \le \eta \le T + \mu,$$

$$H(\tau, z) = \bigcup_{0\le\nu\le1} \left\{ (1-\nu) \begin{pmatrix} Az + \tilde{f}(z)d + E(\hat{a}, \hat{Q}) \\ 1 \end{pmatrix} + \nu \begin{pmatrix} B \\ 0 \end{pmatrix} \right\}. \qquad (21)$$

Denote the reachable set of the system (20)–(21) as $W(t_0 + \sigma) = W(t_0 + \sigma; t_0, X_0 \times \{t_0\})$.

Theorem 3. *(Filippova[4]) The following inclusion holds true for $\sigma > 0$:*

$$W(t_0 + \sigma) \subseteq \bigcup_{0\le\nu\le1} \begin{pmatrix} E(a^+(\sigma,\nu), Q^+(\sigma,\nu)) \\ t_0 + \sigma(1-\nu) \end{pmatrix} + o(\sigma)B^{n+1}(0,1), \qquad (22)$$

$$\lim_{\sigma\to+0} \sigma^{-1} o(\sigma) = 0.$$

Here

$$a^+(\sigma,\nu) = a(\sigma,\nu) + \sigma(1-\nu)\hat{a} + \sigma\nu B,$$
$$Q^+(\sigma,\nu) = (p^{-1}+1)Q(\sigma,\nu) + (p+1)\sigma^2(1-\nu)^2\hat{Q}, \qquad (23)$$

where $p = p(\sigma,\nu)$ is the unique positive root of the equation

$$\sum_{i=1}^{n} \frac{1}{p+\lambda_i} = \frac{n}{p(p+1)},$$

and $\lambda_i = \lambda_i(\sigma,\nu) \ge 0$ satisfy the equation $|Q(\sigma,\nu) - \lambda\sigma^2(1-\nu)^2\hat{Q}| = 0,$

$$a(\sigma,\nu) = a + \sigma(1-\nu)(Aa + (a'\tilde{B}a)d + k^2 d),$$
$$Q(\sigma,\nu) = k^2(I + \sigma R)\tilde{B}^{-1}(I + \sigma R)', \ R = (1-\nu)(A + 2da'\tilde{B}). \qquad (24)$$

The following lemma explains the construction of the auxiliary differential inclusion (20).

Lemma 3. *(Filippova[4]) The set* $X(T) = X(T, t_0, X_0)$ *is the projection of* $W(T + \mu)$ *at the subspace of variables* z*:*

$$X(T) = \pi_z W(T + \mu).$$

Different variants of algorithms of ellipsoidal estimating for the system similar to (18) basing on the above results are given in Filippova and Matviychuk[9], Matviychuk[12],

Theorem 3 can be generalized to the case of a more complicated form

$$\begin{gathered} dx(t) = (Ax(t) + f^{(1)}(x)d^{(1)} + f^{(2)}(x)d^{(2)} + u(t))dt + Bdv(t), \\ x \in R^n, \quad t_0 \leq t \leq T, \end{gathered} \tag{25}$$

where $B \in R^n$, $Var_{t\in[t_0,T]} v(t) \leq \mu$, $d^{(1)}, d^{(2)} \in R^n$ and

$$f^{(1)}(x) = x'B^{(1)}x, \quad f^{(2)}(x) = x'B^{(2)}x.$$

The above generalization is based on a combination of the techniques described above and the results of Filippova[7].

4 Conclusions

The paper deals with the problems of state estimation for uncertain dynamical control systems for which we assume that the initial system state is unknown but bounded with given constraints.

The solution is studied through the techniques of trajectory tubes of related differential inclusions with their cross-sections $X(t)$ being the reachable sets at instant t to control system.

Basing on the results of ellipsoidal calculus developed earlier for linear uncertain systems we present the modified state estimation approaches which use the special nonlinear structure of the control system and allow to construct the external ellipsoidal estimates of reachable sets.

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