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ALIGNING ENTREPRENEURS, VENTURE CAPITALISTS AND GROWTH OPPORTUNITIES IN ENTREPRENEURIAL FINANCING DECISIONS: A REAL OPTIONS APPROACH

by

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Biographical Note

Miguel de Oliveira Tavares Gärtner was born in Porto, Portugal on 18th July 1985. After completing his undergraduate studies at *Colégio Luso-Francês*, he was admitted to the *School of Economics of the University of Porto* in 2003, where he graduated in *Management* in 2007. He was an Erasmus Exchange student on the academic year 2005/2006, in which he attended the *School of Business and Economics of the University of Jyväskyla* in Finland.

Miguel initiated his work experience in 2006 through a Summer Internship in *Millennium bcp*. He started his first full-time job on September, 2007 as a Corporate Finance consultant at *Deloitte*, where he first got contact with entrepreneurs, corporations, investors, private equity firms, insolvencies and (obviously) consultants, from a wide range of industries. The chance to view investment decisions through the eyes of "real" investors came in 2011, where Miguel joined *Inter-Risco* – one of the major Portuguese independent private equity firms.

Miguel was admitted as a part-time student to the *Doctoral Programme in Business and Management Studies* at the *School of Economics of the University of Porto* in 2009. After completing his coursework in 2012 and presenting his thesis project on January, 2013, his research plan was concluded on April, 2016.

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Abstract

We depict the topic of Entrepreneurial Financing decisions through the lens of real options. We portray Entrepreneurial Financing decisions as an interaction between Entrepreneurs and Venture Capitalists (VCs) mediated by a given Growth Opportunity, which is in turn held by an Entrepreneurial or Start-up Firm. We argue that these three elements should be *aligned* so that an Entrepreneurial Financing decision is settled. We explore this topic through two different perspectives.

In Chapters 2 and 3 we investigate the decision-making process of a capital-constrained Entrepreneur – who owns an Entrepreneurial Firm – and a VC. In Chapter 2, we introduce the real options framework for approaching this setting – under symmetric and asymmetric expectations on profit growth expectations – and showed how an optimum *up-front share premium* might be computed so that Entrepreneurs and VCs may reach an agreement to support a given Growth Opportunity. In Chapter 3, we extend this framework by deriving a set of optimum *contingent payments* to enable such an agreement.

In Chapter 4, we analyse how Public Venture Capitalists (PVCs) and Independent Venture Capitalists (IVCs) may differ on their decision-making processes, with the purpose of identifying the most effective mechanism in anticipating investment in Start-up Firms. Based on a real options framework, we derived a set of empirically testable propositions on the determinants of PVC investment volumes and analyse their prevalence on a sample of European countries, through an ordinary least squares regression. Even though taxation proved not to be correlated with PVC investment volumes, the remaining results provided overall empirical support to our theoretical hypothesis.

As a general conclusion to our research, we highlight that the outcomes of Entrepreneurial Financing decisions may eventually primarily depend on the *relative position* that each of the Entrepreneurs and VCs hold against such Growth Opportunity, rather than on the *intrinsic value* of such Growth Opportunity.

Resumo

Esta tese pretende aprofundar o tópico das decisões de *Entrepreneurial Financing* (EF) através da ótica das opções reais. As decisões de EF são retratadas como o resultado da interação entre *Empreendedores* e *Investidores de Capital de Risco* (ICR), mediadas por uma dada *Oportunidade de Crescimento*, detida por uma *Start-up* ou por uma empresa já estabelecida. Argumentamos que estes três elementos devem *alinhar-se* para que a decisão de EF possa ser concretizada. Exploramos este tópico através de duas perspetivas distintas.

Nos Capítulos 2 e 3 investigamos o processo de tomada de decisão de um Empreendedor com restrições de acesso a capital e detentor único de uma empresa, e um ICR. No Capítulo 2, introduzimos um modelo baseado em opções reais – com expectativas simétricas e assimétricas quanto ao crescimento futuro da rentabilidade – e demonstramos de que forma um *prémio de emissão de ações ótimo* pode ser calculado para que Empreendedores e ICRs possam alcançar um acordo. No Capítulo 3, estendemos este modelo deduzindo um conjunto de *pagamentos contingentes* que permitem que tal acordo possa também ser alcançado.

No Capítulo 4, analisamos de que forma Investidores de Capital de Risco Público (ICRP) e Investidores de Capital de Risco Independentes (ICRI) divergem no seu processo de tomada de decisão de investimento, com objetivo de identificar qual o mecanismo mais eficaz na antecipação da decisão ótima de investimento em *Start-ups*. Com base num modelo de opções reais, deduzimos um conjunto de hipóteses empiricamente testáveis quanto às determinantes do volume de investimento por ICRPs e analisamos a sua prevalência numa amostra de países Europeus. Apesar dos impostos não se encontrarem relacionados com o volume de investimento, os restantes resultados suportam as hipóteses teóricas formuladas.

Como conclusão geral, realçamos que os resultados das decisões de EF poderão eventualmente depender principalmente da *posição relativa* que cada uma das partes detém face à Oportunidade de Crescimento que se encontram a considerar, mais do que do *valor intrínseco* dessa mesma Oportunidades de Crescimento.

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List of Abbreviations

CPM	Contingent Payment Mechanism(s) or Contingent Earn-Out
EF	Entrepreneurial Financing or Entrepreneurial Finance
FEP	Faculdade de Economia do Porto – School of Economics, University of Porto
GBM	Geometric Brownian Motion
GP	General Partner
IVC	Independent Venture Capital or Independent Venture Capitalist
LP	Limited Partner
M&A	Mergers and Acquisitions
ODE	Ordinary Differential Equation
PE	Private Equity
PVC	Public Venture Capital or Public Venture Capitalist
OECD	Organization for Economic Co-Operation and Development
SME	Small and Medium Enterprise(s)
SuF	Start-up Firm(s)
VC	Venture Capital or Venture Capitalist(s)

1. Introduction

The relevance of Entrepreneurial Financing (EF) as a research topic is highlighted by the over 1.4 million employer enterprises that were born worldwide in 2012[1], the over 200,000 existing medium and high growth enterprises as of 2013[2], or the over 60 billion dollars invested by Venture Capital (VCs) funds worldwide in seed, start-ups, early stage and later stage ventures in 2014[3], according to the Organization for Economic Co-Operation and Development (OECD).

In addition to such economic relevance, Small and Medium Enterprises (SMEs) finance has been receiving major political support since the 2007-2008 financial crisis, both from the Europe Union[4], the United States[5] and from a range of different international organizations, such as OECD (2015) or G-20 (International Finance Corporation & World Bank, 2010).

While academia responded to such increasing interest through a growing volume of literature, two major gaps remain and motivated this research: on the one hand, the *demand-side gap* – calling for more Entrepreneur-centric research – and, on the other hand, the gap

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[1] Sample compiled by Organization for Economic Co-Operation and Development (OECD) on the SDBS Business Demography Indicators (ISIC Rev. 4). Sample includes 31 countries. Data on the United States available on the SDBS Business Demography Indicators (ISIC Rev. 3) for 2012, which is not included on the ISIC Rev. 4 figures, records 113,292 employer enterprise births.

[2] Sample compiled by Organization for Economic Co-Operation and Development (OECD) includes 30 countries. Data for New Zealand, Israel, Canada, Brazil, and the United States refer to over 20% growth enterprises. For the rest of countries the criterion used on the sample includes firms with over 10% growth.

[3] Sample compiled by Organization for Economic Co-Operation and Development (OECD) includes 32 countries. Figures for South Africa and Japan refer back to 2013.

[4] Please refer to "*An action plan to improve access to finance for SMEs*" – a communication from the European Commission to the Council, to the European Parliament, to the Committee of the Regions and to the European and Social Committee (2011) and to the "*Programme for the competitiveness of Enterprises and Small and Medium-Sized Enterprises (2013-200) and repealing Decision No 1639/2006/EC*" (2013)

[5] Please refer to the "*SupplierPay pledge*" (2014) issued by The White House and to the announcement of the "*QuickPay Initiative*" on a press-release by The White House entitled "*President Obama announces new partnership with the private sector to strengthen America's Small Business; Renews the Federal Government's QuickPay Initiative*" (2014)

on analytically grounded theoretical developments on the topic. The latter will be presented on the following section, and the former will be highlighted on each of the introductory sections of Chapters 2, 3 and 4.

1.1. Conceptual considerations

This section aims to summarize the most relevant set of conceptual considerations which supported the research outline we will present in section 1.2. We will start by introducing a set of different theoretical perspectives with regards to the EF decision and highlight our intended positioning within the wide range of alternative approaches. Then, we will present some of the supporting literature on the existence of a *demand-side gap*. We will end by going through a set of key concepts to our research and stand out some of the major assumptions which drove our theoretical constructions in Chapters 2, 3 and 4.

1.1.1. Theoretical perspectives on Entrepreneurial Financing

Agency theory stands as one of the main theoretical lens for analysing the VC – Entrepreneur relationship, where VCs are regarded as principals and Entrepreneurs are regarded as agents (Sapienza and Villanueva, 2007). Within the scope of such relationship, agency conflicts are originated by differing goals between Entrepreneurs and VCs, which are intensified when Entrepreneurs act like sole owners of the business after external equity providers joined the Entrepreneurial Firm. According to Arthurs and Busenitz (2003), *perceived goals* and *actual goals* should lead to different types of agency problems on the Entrepreneur – VC relationship (i.e., *perceived*, *hidden* or *visible* agency problems). In addition, agency conflicts are aggravated by information asymmetry, which is expected to be especially latent on early stage ventures, where asset tangibility is low, growth options are significant and asset specificity is high (Gompers, 1995). Overall, agency theory might be regarded as a rational economic framework to depict the behaviour of VCs and Entrepreneurs, in the sense that each of the parties is modelled as an individual agent who maximizes her or his individual payoff.

Stewardship theory takes an opposite view to *agency theory*, assuming that the behaviour of the steward is organization-centred, meaning that it seeks to improve organizational performance by satisfying the principals. Arthurs and Busenitz (2003) argue that agency costs occur mostly during the pre-investment stage, where VCs and Entrepreneurs are more likely to be misaligned. They posit that once an investment is made, Entrepreneurs and VCs had to previously agree on certain milestones which should at least partly alleviate potential goal misalignment.

In addition, goal incongruence might be actually less pronounced in Entrepreneurial Firms, in the sense that (i) Entrepreneurs may retain a substantial fraction of firm ownership and may therefore drive their behaviour towards shareholder value maximization, on a similar way to VCs and (ii) VCs are expected to provide significant non-financial contributions to early stage ventures, which may actually increase the volume of interactions between VCs and Entrepreneurs and minimize potential information asymmetries.

Still, these authors acknowledge that (i) *stewardship theory* fails to understand how can individuals align misaligned interests and (ii) *stewardship theory* assumes that Entrepreneurs will still act like an owner after VCs invests in the new venture. As a consequence, the Entrepreneur will not likely subordinate his or her interests to those of VCs – which in turn brings goal congruence problems.

Other alternative theoretical frameworks and demand-side driven research have provided relevant insights to the debate.

Along with Sapienza and Korsgaard (1996), Busenitz et al. (1998) studied the impact of *procedural justice* on Entrepreneur's receptivity to investors. The authors concluded that VCs (or "investors" in their own terminology) who do not fairly and respectfully treat Entrepreneurs are more prone to face distortions or omissions on key information, while VCs who provide timely feedback to Entrepreneurs may benefit from leveraging positive relations. By examining which factors at the time of first-round funding frame perceptions of fairness in the VC – Entrepreneur relationship, with a special focus on contractual covenants over procedural justice, these authors revealed that Entrepreneurs are willing to accept a certain level of governance in the form of contractual covenants, but, beyond a certain point, they may frame governance as excessive. In these circumstances, such contractual covenants may decrease the perception of fairness formed by Entrepreneurs on VCs.

Sapienza and Korsgaard (1996) highlight that control and information sharing in the venture-building process can create mutual trust and commitment, mitigating fears of opportunism, reducing the costs of delegation of decision making and creating a *cooperative advantage* over competitors. Even though procedural justice provides significant predictors of attitudes of strategic decision makers, it is still taken as a partial driver of their behaviour, suggesting that the theory needs refinement when applied to settings involving high outcome uncertainty and ambiguous or non-hierarchical relationships.

Overall, Sapienza and Villanueva (2007) posit that *agency theory* grounded on a rational economic framework has been the prominent theoretical lens used for analysing the VC – Entrepreneur relationship. These two authors argue that this is the result of the relevance of institutional VCs – in which motivation for venture selection is mostly driven by economic return – and due to the fact that early stage ventures are more prone to high agency costs, due to lower asset tangibility, significant growth options, and greater asset specificity (Gompers, 1995).

Acknowledging the relevant insights from different theoretical perspectives on the EF processes, such as the *agency theory*, *stewardship theory* or *procedural justice* and even admitting some of the criticism to the influence of neoliberal economics on small business research (Parry, 2015), we highlight the role that *profit maximization* holds as a driver of EF decisions. Although several other variables interfere with the EF (as we will point out in section 1.1.3), we believe that *profit maximization* still stands for one of the most significant decision-making factors both for Entrepreneurs and VCs. Moreover, and as described in section 1.1.3, provided we take EF decisions as processes between Entrepreneurs, VCs and Growth Opportunities, in which the latter are essentially potential profit enablers, profits should be regarded as the common ground between each of these drivers of EF processes, without which any decision-making model would be incomplete. Combined with real options perspective in which Entrepreneurs and VCs should support a given Growth Opportunity when having equal optimum investment timing, this standpoint is a major assumption governing the theoretical developments introduced in Chapters 2, 3 and 4.

1.1.2. Demand-side gap

Sapienza and Villanueva (2007) pointed out the need for growing research on the EF process from the perspective of the Entrepreneur (i.e., from the demand-side), given that most research is grounded on a supply-side perspective and is focused on deal selection and portfolio monitoring issues. This *demand-side gap* is not only visible on the dominant area of research in institutional VC, but also on informal VC (Riding et al., 2007) or corporate VC (Zahra and Allen, 2007).

Interestingly, this literature gap prevails even though some of the earliest literature on EF and VC highlighted the importance that the Entrepreneur held on the EF process (Macmillan et al., 1985, Tyebjee and Bruno, 1984, Fried and Hisrich, 1994, Zacharakis and Meyer, 1998, Shepherd et al., 2003, Wright et al., 1997, Gompers et al., 2006, Baron and Markman, 1999, Hsu, 2007).

Some of the first contributions on Entrepreneur-centric research were initiated on early 2000s. Smith (1999) studied the criteria used by Entrepreneurs when evaluating VCs. Ueda (2004) addressed the question of how Start-up Firms (SuFs) decide to raise funds from banks or VCs, having shown that SuFs with little collateral, high growth prospects, high risk and high profitability would tend to primarily seek for VC funding, controlling for information asymmetry and intellectual property rights protection. de Bettignies and Brander (2007) highlight that as a VC funding would always require Entrepreneur's ownership to be diluted, the Entrepreneur would always prefer bank financing unless high value-added managerial skills are expected to be brought in to the venture by the VC. On the finance literature, examples of Entrepreneur focused views come essentially from an agency perspective (Casamatta, 2003, Kirilenko, 2001, Bitler et al., 2005).

The 2007-2008 financial crisis and the constraints it brought to access to finance – especially to SMEs – incentivized academia, especially from the field of entrepreneurship, to foster its research efforts to fill the *demand-side gap*. Contributions from this area are methodologically rich – due to the multi-disciplinary of the entrepreneurship phenomenon – and are grounded from a wide range of disciplines, from economics and finance, to psychology.

In fact, research on the field of entrepreneurship should be increasingly focused on understanding the psychological and cognitive processes that drive entrepreneurial opportunities and on how these mature (Wright and Stigliani, 2013, Fraser et al., 2015, Shepherd, 2015), as exemplified by Orser et al. (2006) and Roper and Scott (2009) who investigated the impact of gender differences on seeking external financing, or by Kon and Storey (2003), who coined the term "discouraged borrowers", i.e., SMEs that refrain from seeking external financing because they feel they will be rejected – even when standing for potential good borrowers.

While we acknowledge that the *demand-side gap* has been narrowing during the past decade, it is our understanding that analytical grounded research is scarce and still provides room for relevant theoretical developments, as we show on the introductory sections of Chapters 2, 3 and 4.

1.1.3. Building Blocks of Entrepreneurial Financing Decisions

Embedded on a given macroeconomic and institutional context exerting influence on the overall behaviour of both supply and demand-side agents and on the size of capital markets (La Porta et al., 1997, Cumming et al., 2010), EF decisions result from the interaction between three interdependent elements: an *Entrepreneur*, a *Growth Opportunity* and a *Financier*, which we jointly name as *the Building Blocks of Entrepreneurial Financing Decisions*. Following McKelvie and Wiklund (2010), Rasmussen and Sørheim (2012), and Chemmanur and Fulghieri (2014), we take the view that no sound theoretical research on EF decisions is possible by separating each of these three elements. As portrayed in Figure 1, we take EF decisions as the outcomes of an alignment process between an *Entrepreneur* and a *Financier*, which is originated by a given *Growth Opportunity*.

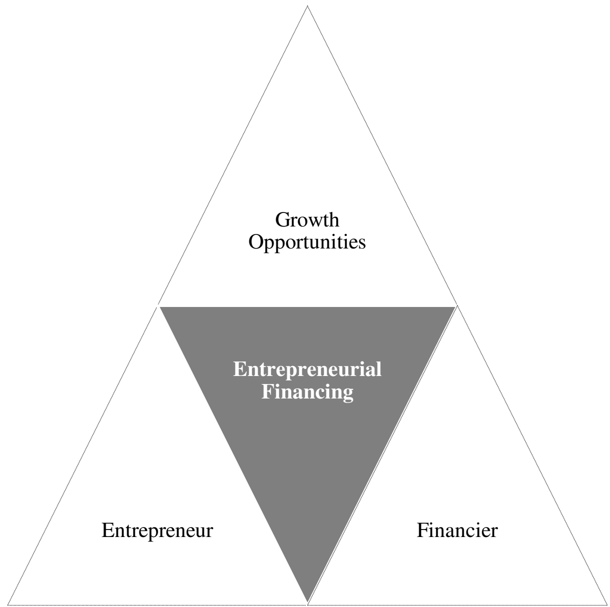

Figure 1. Building Blocks of Entrepreneurial Financing Decisions

1.1.3.1. The Entrepreneur

EF decisions are influenced by a wide range of drivers far beyond the profit-maximization hypothesis. While our theoretical contribution relied heavily on former, the literature reveals that wealth maximization, wealth constraints, risk attitudes and individual goals form the main drivers of an Entrepreneur's behaviour, and should encompass the interpretation of our theoretical results in Chapters 2, 3 and 4.

Rasmussen and Sørheim (2012) suggest that Entrepreneurs' perceptions, preferences, business case, relationship networks and the process of obtaining financing are issues of key importance for understanding the outcomes of an EF process, while Brush et al. (2012) highlight the concept of *venture readiness*. Sapienza et al. (2003) propose a multi-dimensional conceptual framework, in which wealth maximization, and self-determination are the primary motives driving EF choices, from the Entrepreneur's standpoint.

Hvide and Panos (2014) highlight the role that risk tolerance holds as a major driver of entrepreneurial decisions. These authors found evidence that more risk tolerant individuals are more likely to become Entrepreneurs, but perform worse, yielding 25.0% lower sales and 15.0% lower return on assets. Gompers et al. (2010) point out that Entrepreneurs with a track record of success are much more likely to succeed than first-time entrepreneurs and those who have previously failed, as they exhibit persistent behaviour in adequate industry selection and entry timing. Wright et al. (1997) noted that this entrepreneurial profile – named as "serial entrepreneur" – is especially attractive to VCs or Private Equity funds (PEs).

Muzyka et al. (1999) acknowledge that the quality of the Entrepreneur is key to determine the funding decision. Similarly, Baron and Markman (1999) offered support for the hypothesis that the higher Entrepreneurs' social competence is, the greater their financial success. This seems not to be the case for education, as Dimov and Shepherd (2005) found that although general management team human capital has a positive association with the proportion of portfolio companies that went public, specific management team human capital (i.e., MBA, law education, or consulting experience) has not, even though specific management team human capital was negatively associated with the proportion of portfolio companies that went bankrupt.

Hsu (2007) investigated the sourcing and valuation of VC funding among Entrepreneurs with varied levels of prior start-up founding experience, academic training, and social capital, by examining venture valuation, as it reflects enterprise quality and Entrepreneurs' cost of financial capital. Using data from a survey of 149 early stage technology-based SuFs, Hsu (2007) found that (i) prior founding experience (especially financially successful experience) increases both the likelihood of VC funding via a direct tie and venture valuation, (ii) Entrepreneur's ability to recruit executives via their own social network (as opposed to the VC's network) is positively associated with venture valuation and (iii) in the emerging (at the time) Internet industry, entrepreneurial teams with a doctoral degree holder are more likely to be funded via a direct VC tie and receive higher valuations, suggesting a signalling effect.

Chaganti et al. (1995) highlight that the prevailing paradigm ignores factors such as owners' values or goals. Winborg (2000) showed that Entrepreneurs who sought financing to achieve higher growth sought more external funding, and that those who professed a need for external financing also held more positive attitudes towards it. The author talks about a "fear" (beyond economic loss) that Entrepreneurs have regarding external sources of funding.

Although the fear of loss of control or, alternatively, the drive for independence has been frequently mentioned has a key motivator for Entrepreneurs (Ang, 1992), it is still unclear whether observed drive for self-determination is a means to achieve economic ends or a separate end in itself. Chaganti et al. (1995) posit that some Entrepreneurs are motivated by economic gain for themselves or their families and that others are motivated by their "desire" for control or their own affairs and to avoid dependence on others. Those driven by economic gain seek a different mix of external to internal financing mix than those driven by self-determination.

1.1.3.2. Growth Opportunities

Growth Opportunities carried by Entrepreneurial Firms feature significant uncertainty on future cash flow generation, involve considerable irreversible costs and benefit from flexible plans, forming investment opportunities that might be modelled as real options (Schwienbacher, 2007, Li, 2008). In addition, option-games might be useful for modelling EF as an interactive process, as shown by the pricing models for Mergers & Acquisitions (M&A) by Lukas et al. (2012) and Yu and Xu (2011).

Debt financing is usually not an option for backing such Growth Opportunities (Revest and Sapio, 2012), as Entrepreneurial Firms typically bear losses (or record low initial profits) and lack tangible assets, driving Entrepreneurs to choose *between different sources of equity* (Fairchild, 2011, Andrieu and Groh, 2012, Schwienbacher, 2013, Chemmanur and Chen, 2014, Kim and Wagman, 2016), rather than deciding *between equity and debt* financing as in Winton and Yerramilli (2008) or simply *seeking for a debt provider* (Adelino et al., 2015, Chatterji and Seamans, 2012).

Accordingly, similarly to Parhankangas (2007) and following the results obtained by Hechavarría et al. (2015) when testing the prevalence of the pecking-order hypothesis on early-stage firm, we consider that within an EF context, retained earnings are insignificant, Entrepreneurs are equity constrained (Elston and Audretsch, 2009) and, therefore, Financiers are mostly external equity providers and, particularly VCs in the broad sense – either institutional, captive, informal, public or corporate VCs. While in Chapters 2 and 3, we take VCs as a homogenous class of investors, in Chapter 4 we will point out the differences on investment behaviour of Independent Venture Capitalists (IVCs) and Public Venture Capitalists (PVCs).

1.1.3.3. Venture Capitalists

PE and VC are essentially portrayed as financial intermediaries that invest in private companies for a limited time period and through equity or quasi-equity instruments. However, we should further distinguish these two concepts and highlight how our research is positioned between these two. Following the approach by Landström (2007), we take VC as "primarily devoted to equity or equity-linked investments in *young growth-oriented ventures*", and PE to "investments that *go beyond venture capital* – covering a range of other stages and established businesses". From this definition, it becomes clear that if EF decisions involve *young* and *growth-oriented ventures*, equity providers for such Entrepreneurial Firms stand for VCs and not for PEs. This allows us to exclude from our theoretical contributions some of the specific characteristics and types of PE investment, such as deal leverage (Axelson et al., 2009), replacement capital, turnarounds (Cuny and Talmor, 2007) or management buyouts (Kaplan, 1989).

VCs might be regarded as profit maximisers with specific return on investment thresholds, which make use of a set of mechanisms to deal with information asymmetry and potential agency conflicts, as highlighted by Reid (1996) or from an entrepreneurial perspective by Zou et al. (2015). Examples of these mechanisms include anti-dilution clauses, non-compete clauses, liquidation rights (Leisen, 2012), preferred and convertible stock (Cumming, 2008, Sahlman, 1990, Gompers, 1997, Kaplan and Stromberg, 2001, Bascha and Walz, 2000), staged capital infusion and rights of first refusal (Dahiya and Ray, 2012, Lukas et al., 2016, Sahlman, 1990, Gompers, 1995, Gompers and Lerner, 2001, Wang and Zhou, 2004), tag-along and drag-along rights (Carter and Van Auken, 1994, Kaplan and Stromberg, 2001, Cumming and Johan, 2007) and compensation schemes involving equity and options (Gompers and Lerner, 2001).

In addition, and as argued by Hellmann and Puri (2002), VCs may provide non-financial contributions to Growth Opportunities and Entrepreneurial Firms, such as value-adding monitoring (Bernstein et al., 2015), professionalization (Kaplan and Stromberg, 2001) and firm certification and reputation (Hsu, 2004, Nahata, 2008). These may in turn influence how Entrepreneurs may choose between different sources of equity.

While financial contributions, non-financial contributions and contractual mechanisms form the grounds for understanding how VCs drive EF processes, the existing literature points out that fund demography (defined as fund size, fund location, fund age and fund experience) may also exert significant influence on their behaviour (Tian, 2011). Following this perspective, Ewens and Rhodes-Kropf (2015) revealed that the quality of the human capital of the VC partners seems to be two to five times more important than organizational capital in explaining consistency in VC fund performance.

In fact, assessing the determinants and the actual performance – properly adjusted for risk and liquidity – of PE and VC funds has been highlighting an area of vast research (Korteweg and Nagel, 2016, Sorensen and Jagannathan, 2015, Harris et al., 2014, Braun et al., 2015, Franzoni et al., 2012, Phalippou and Gottschalg, 2009, Cochrane, 2005, Benson and Ziedonis, 2010), which has highlighted the debate on the actual economic contribution of PE and VC sponsored firms (Popov and Roosenboom, 2013, Alperovych and Hübner, 2012, Inderst and Mueller, 2009) and to a discussion on whether VCs and PEs actually provide a *value-enhancing* contribution to investees or they *cherry-pick* best-in-class firms instead (Croce et al., 2013, Bertoni et al., 2011, Bertoni et al., 2016).

1.1.4. Real options

Myers (1977) coined the term *real options* by noticing that several of the features of investment and growth opportunities resemble those of a call option, standing for rights – and not obligations – to invest or to expand a certain business in the future. Moreover, Myers (1977) conceptualized the value of the firm as the combined value of “real assets” – whose market value should not depend on the firm's investment strategy – and of "real options", which are "*opportunities to purchase real assets on possibly favourable terms*".

Following this conceptual view, some of the earliest analytical contributions on real options include Tourinho (1979), Brennan and Schwartz (1985), McDonald and Siegel (1985) and McDonald and Siegel (1986). These authors explored a wide range of different types of real options – having emphasized the role played by *uncertainty* and *flexibility* on valuing at least partially *irreversible* investment opportunities – and which were later summarized by Dixit and Pindyck (1994). These authors presented a comprehensive review of a wide range of growth options which form a range of different backgrounds for the setting in which EF processes takes place. These include combined entry and exit strategies, growth options with lay-up reactivation and scrapping options, multistage projects, or growth options in competitive industry settings, covering distinct sources of uncertainty, from prices, to costs, to volumes.

Why did we rely on the real options framework to approach the EF phenomenon? First, because Growth Opportunities are taken as a nuclear component to EF decisions, as we described in section 1.1.3.2 above. Without a Growth Opportunity, the EF decision becomes a capital structure decision and might be explained by the trade-off (Kraus and Litzenberger, 1973), market-timing (Baker and Wurgler, 2002), neutral mutations (Miller, 1977), stakeholder (Cornell and Shapiro, 1987), and managerial over-optimism (Heaton, 2002) perspectives.

Second, because Growth Opportunities held by Entrepreneurial Firms resemble the three key features of real options (Dixit and Pindyck, 1994), covering at least a partially *irreversible investment* (most of the times, involving intangible assets or working capital requirements), a significant level of *uncertainty* on future cash flow generation and *flexible* strategic development options – covering waiting, expansion, scrapping or switching options. While in Chapters 2 and 3, we combine the both expansion and waiting options, as our setting is focused on established Entrepreneurial Firms – in Chapter 4, we highlight the waiting feature of such investment opportunities – as our setting relies on SuFs.

Third, because real options are an analytical tool that explicitly derives optimal investment timing. We take the view that this is crucial for investigating EF decisions, where Entrepreneurs and VCs should only be willing to jointly support the investment opportunity held by each of the parties if their optimum investment timing is the same. This view regards EF as an *alignment* process between Entrepreneurs and VCs, mediated by the underlying Growth Opportunity. Morellec and Zhdanov (2005) and Hackbarth and Morellec (2008) took a similar approach within a M&A context.

Fourth, Entrepreneurial Firms hold monopolistic options to invest in their Growth Opportunities, in the sense that no other firm should hold that same investment opportunity or should be able to replicate it. As a result, we find no need to augment our real options framework to account for competition issues in order to generate sound theoretical developments. We will portray Entrepreneurial Firms as small scale profit-makers that are intended to be scaled up through its EF decision, seizing a previously generated proprietary technology or competitive advantage.

This is not to say that competition is irrelevant for EF decisions, but rather that it is softened within the context of EF decisions. We acknowledge that our theoretical contribution could be extended to account for competition (either between Entrepreneurs, or between VCs) through the use of options-games. Some examples of analytical models on EF with competition include Kanniainen and Keuschnigg (2004), who highlight how VC industry structure and managerial expertise can create rents which eventually attract new VCs who compete them away in the long-run and Elitzur and Gavious (2011), who developed a model of EF where Entrepreneurs compete in an auction-like setting for VC funding in an information asymmetry context.

1.2. Research outline

We organized our research in two different stages standing for two different perspectives on the EF process. On the first stage, featuring Chapters 2 and 3, we focused on EF decisions involving Entrepreneurs and VCs at a micro-level. We started by asking "*how do Entrepreneurs and VCs individually screen their option to invest in Growth Opportunities* *held by Entrepreneurial Firms?*". The answer to this question drove us to design the real options framework presented in Chapter 2, in which most of our theoretical developments are grounded.

Then we asked "*how can Entrepreneurs and VCs be willing to simultaneously support the Growth Opportunity?*". We answered this question on Chapters 2 and 3. On Chapter 2, we analysed whether there is a certain profit growth expectation that would allow parties to align their optimal investment timing. However, given that in our framework profit growth expectations are exogenously set by parties, these could not be taken as a way to line up Entrepreneurs and VCs. Therefore, we focused our research efforts into two different mechanisms: *share premiums (or discounts)*, which are set up-front (i.e., at the moment in which the EF decision is agreed between Entrepreneurs and VCs) and are derived in Chapter 2 and *contingent payments (or earn-outs*), which are deferred payments due to Entrepreneurs by VCs, dependant on future performance of the Entrepreneurial Firm. This is the focus of Chapter 3.

The second research stage is featured in Chapter 4 and shows a different perspective on EF decisions. Unlike Chapters 2 and 3, in which we depicted the EF decision covering Entrepreneurs and VCs, in Chapter 4 we compared the investment behaviour of independent VCs (IVCs) and PVCs. We therefore answered the question "*how do IVCs and PVCs screen their options to invest in SuFs?*". As a result, while in Chapters 2 and 3 we took VCs as homogenous – meaning that they are assumed to be either IVCs, PVCs, business angels or corporate VCs – in Chapter 4 we investigated the differences on investment screening between IVCs and PVCs, by taking into account some of their specific features, such as taxation, relative efficiency and performance compensation. In addition, as several PVC initiatives have been launched throughout the world with the purpose of fostering investment volumes on the early stage firms, we scrutinized the question of "*which is the most effective investment strategy to be set by Governments to anticipate optimal investment timing in SuFs?*". We compared four different approaches that might be put in place for this purpose: (i) directly investing in SuFs through a PVC initiative, (ii) co-investing in SuFs alongside

IVCs, (iii) letting IVCs invest in SuFs and providing a subsidy and (iv) refraining from intervening on the SuF financing market, by letting IVCs invest in SuFs.

As co-investing requires an *alignment* between PVCs and IVCs similarly to EF decisions between Entrepreneurs and VCs, *alignment* on EF decisions forms the common ground for our research topic. Our research outline is then summarized in Figure 2.

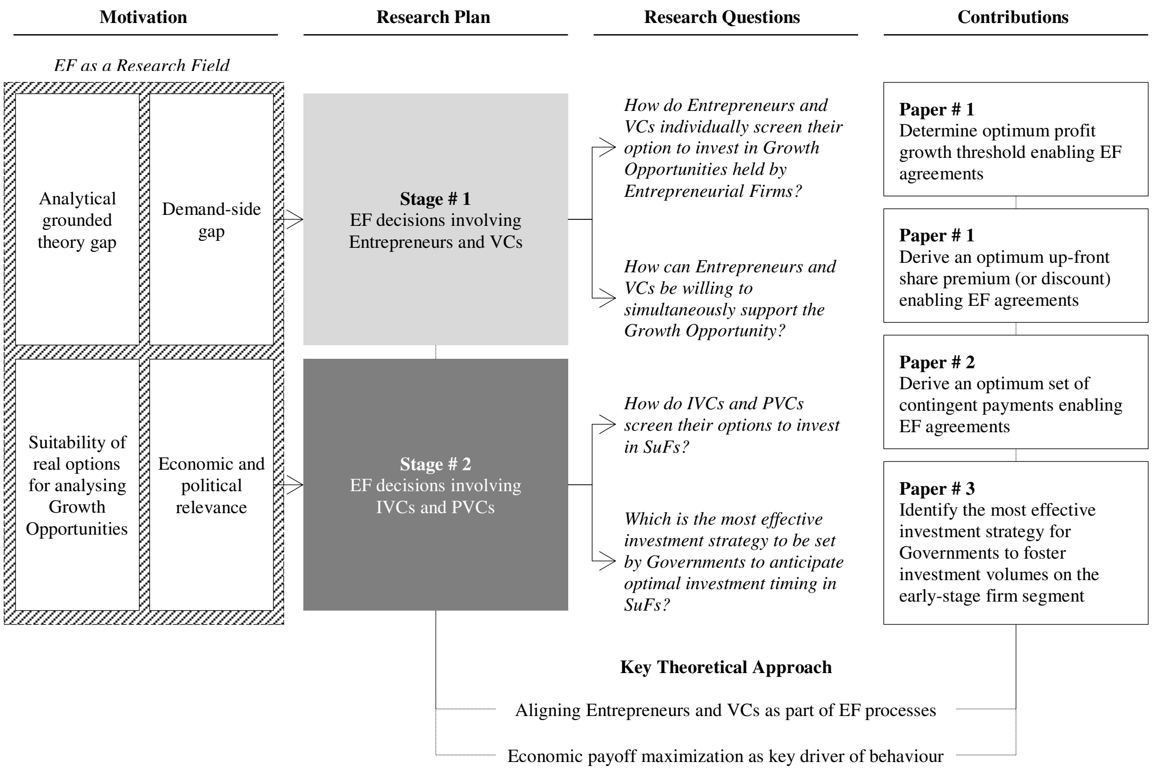

Figure 2. Research outline

From a methodological perspective, theoretical developments rely on the real options framework, by taking a profit-maximization perspective on the investment behaviour of Entrepreneurs and VCs. Provided that no empirical data is available to test our theoretical propositions from Chapters 2 and 3, these were illustrated with numerical examples and through analytical sensitivity analysis. In Chapter 4, we were able to take a different approach. Since solutions for optimal investment timing are numerically derived, a set of theoretical hypothesis were obtained from a numerical example and then empirically assessed through a multivariate ordinary least squares linear regression.

Wolfram Mathematica 10© and IBM SPSS Statistics 22© were used as supporting software packages, respectively, for the analytical and econometric tasks performed throughout this research.

2. A framework for understanding Entrepreneurial Financing decisions with profit growth expectations and up-front share premiums or discounts

Even though there is a vast stream of conceptual research taking Entrepreneurial Financing (EF) as an interactive process between Entrepreneurs and Financiers (Sapienza and Korsgaard, 1996, Cable and Shane, 1997, De Clercq and Sapienza, 2001, De Clercq and Fried, 2005), the existing contributions on decision-making models with similar approaches come mostly from a mergers and acquisitions context with interacting bidding and target firms (Lukas et al., 2012, Lukas and Heimann, 2014, Yu and Xu, 2011, Morellec and Zhdanov, 2005, Hackbarth and Morellec, 2008) and not from an Entrepreneurial Financing one. We now point out some of these notable exceptions.

Elitzur and Gavious (2003) discuss optimum EF contract design through a game-theoretic model, comprising one Entrepreneur and one Venture Capitalist (VC). Parties intend to draft an incentive contract for a given investment period, in which fixed payments to the Entrepreneur are set in advance and the Entrepreneur incurs in a cost of effort at each stage. In turn, if the investment by the VC does not exceed its gross benefits before the end of the investment period, the exit stage takes place and the game ends.

Schwienbacher (2007) analyses two different financing strategies that Entrepreneurs may choose between: either waiting until raising enough money to complete the project, or using limited resources to achieve some intermediate milestone before reaching a VC to proceed with its growth strategy. Entrepreneurial types ("profit-maximizing", "life-style" and "serial"), alongside entrepreneurial choice on which type of uncertainty to be first resolved

– technological or financial – are the primary drivers on determining the choice on the best EF strategy.

Fairchild (2011), Andrieu and Groh (2012), Schwienbacher (2013) and Chemmanur and Chen (2014) examined how Entrepreneurs choose between different types of equity investors to support their ventures. Fairchild (2011) considered a setting in which Entrepreneurs are deciding between two different sources of equity (VCs or business angels) and in which there is a double-sided moral hazard, with *ex-ante* effort-shirking and *ex-post* project expropriation. VCs are taken as having higher value-creating abilities than business angels, but Entrepreneurs anticipate to establish a closer and more trusting relationship with the former. This choice between VCs and business angels is also examined by Chemmanur and Chen (2014) and Kim and Wagman (2016). Chemmanur and Chen (2014) conceived a theoretical model with a setting where VCs are assumed to be scarcer, but may exert effort, which, together with the Entrepreneur's effort, are assumed to increase the probability of success of the Entrepreneurial Firm. The equilibrium VC financing contract ensures optimal effort-exertion by both Entrepreneur and VC. Kim and Wagman (2016) discuss the choice between VCs and business angels in a setting where a negative signal is inferred by the market when inside investors do not to follow on a subsequent investment. As a result, given that business angels are unlikely to follow on throughout subsequent equity rounds, these are expected to be chosen by Entrepreneurs to finance lower quality ventures than VCs. Andrieu and Groh (2012) considered Entrepreneur's choice between independent VCs or bank-affiliated VCs, where the latter are assumed to provide better support quality and the former are assumed to be less financially constrained. Entrepreneur's decision-making is then affected by this trade-off. In Schwienbacher (2013), Entrepreneurs seek early-stage financing from either a specialist or a generalist investor in a context of staged financing. Early-stage specialists are assumed to be less efficient in assisting a venture beyond the early-stage round than generalists, creating a trade-off on the Entrepreneur's decision-making process.

This topic was further explored by Hsu (2010), Dahiya and Ray (2012) and Lukas et al. (2016), who investigated how investment staging may influence EF decisions. Hsu (2010) introduced a real options based principal-agent framework, where a VC acts as a principal and as a value-maximizer who may choose between providing staged financing and lump-sum financing, and an Entrepreneur acts as an agent, who can in turn choose the riskiness borne by the Entrepreneurial Firm with the purpose of maximizing the probability of obtaining subsequent financing. Dahiya and Ray (2012) designed a setting in which VCs make use of staging to sort investments, i.e., to decide those that shall be abandoned and those to hold, highlighting the contribution that staging may provide to cope with uncertainty. Lukas et al. (2016) developed a dynamic model of EF based on option exercise games between an Entrepreneur and a VC, with multi-staged financing, and economic and technological uncertainty. In this context, and by combining compound option pricing with sequential non-cooperative contracting, uncertainty is posited to positively influence the equity share held by the VC on the Entrepreneurial Firm, while renegotiation may actually lead to a shift of control on the Entrepreneurial Firm, eventually preventing it from failure.

Our approach differs from its related literature in the sense that we provide a tractable framework for understanding the foundations of Entrepreneurial Financing decisions, by analysing the conditions under which any of the parties will demand for entering into a negotiation process, and by analysing the conditions which will actually ease an agreement between Entrepreneurs and Venture Capitalists. In addition, we highlight how Entrepreneurs and VCs may settle an agreement when facing different expectations on Entrepreneurial Firm profit growth, in an information symmetric but potentially expectation asymmetric context. Within our framework, parties know which stochastic process governs the profit flow of the Entrepreneurial Firm and its Growth Opportunity, even though holding different expectations on future profit growth. Therefore, we take the view that, prior to initiating a negotiation process, parties define their own perspectives on the Growth Opportunity, and that such expectations might diverge, even when parties have access to the same level and amount of information. We therefore emphasize the role that cognition may have on generating the concept of opportunity which underlies any Entrepreneurial Financing process.

Moreover, most of the related literature deals with the topic of how Entrepreneurs choose from alternative sources of equity. We believe that our contribution does not challenge the relevance of this topic, but rather introduces a set of conditions – grounded on profit-maximizing agents – which should be relevant irrespective of their nature (i.e., to whatever "type" of Entrepreneur or VC), and over which additional trade-offs could be grounded.

Our contribution to the existing literature is then essentially twofold. First, we designed a framework for analysing EF decisions, in which the key three elements of any EF process − the Entrepreneur, the Growth Opportunity (defined by a given capital outlay and an expected profit growth) and the Financier (taken as a VC) – interact, serving as a ground for further theoretical research. Second, within this framework, we showed how Entrepreneurs and VCs may be willing to jointly support the Growth Opportunity, by having the same optimum investment timing. We highlight how asymmetric expectations on profit growth, usually taken as a blocking force, may actually be reconciled to support EF decisions and show how intrinsic firm valuation may not entirely drive the outcomes of EF decisions. Taking the view that more demand-side research is required for further theoretical and empirical developments (Sapienza and Villanueva, 2007, Rasmussen and Sørheim, 2012), we highlight throughout the paper the conditions which, from the Entrepreneur's perspective, should be verified so that an agreement with a prospective VC might be reached.

This Chapter is structured as follows. Section 2.1 outlines the base case where Entrepreneurs and VCs hold symmetric expectations on profit growth, while section 2.2 analyses the case for asymmetric expectations. Section 2.3 presents a numerical example, while section 2.4 summarizes some the major findings arising from this theoretical framework.

2.1. The case with symmetric profit growth expectations

In this section, we describe the base case, comprising a setting in which Entrepreneurs and VCs hold symmetric expectations on the Entrepreneurial Firm's profit growth. First, we will derive the conditions under which Entrepreneurs and VCs would be *individually* willing the support the Growth Opportunity. We then depict how share premiums or discounts might be computed with the purpose of aligning Entrepreneurs and VCs to back the Entrepreneurial Firm and seize the envisaged Growth Opportunity.

2.1.1. Setup

The base case comprises one Entrepreneurial Firm, owned by a single Entrepreneur (named as E), which generates a flow of positive profits (π). This Entrepreneurial Firm, in which the Entrepreneur invested an initial capital of $k^i > 0$, holds a Growth Opportunity, defined by an expansion of its current profit flow (given by $e > 1$) and a given capital expenditure ($k > 0$). Assuming that neither the Entrepreneurial Firm nor the Entrepreneur have access to debt[6], such capital expenditure should be funded through an equity round backed by the Entrepreneur, who is assumed to own limited resources (given by $0 < k^a < k$), and by an external financier, who is assumed to be a VC with no funding constraints. VCs are then assumed to fund the part of the required equity that the Entrepreneur is not able to provide (i.e., $k - k^a$). Post-equity round firm ownership held by the Entrepreneur is denoted by $0 < Q^E < 1$ while post-equity round firm ownership held by the VC is denoted by $0 < Q^{VC} < 1$ and $Q^{VC} = 1 - Q^E$.

2.1.2. Individual assessment of the Growth Opportunity

Firstly, each of the parties shall *individually* analyse the investment opportunity held by the Entrepreneurial Firm, assuming that post-equity round firm ownership will be split according to the capital contributions made by each of the parties to the Entrepreneurial Firm. Therefore, $Q^E = \frac{k^i + k^a}{k^i + k}$ and $Q^{VC} = \frac{k - k^a}{k^i + k} = 1 - Q^E$.

The Entrepreneurial Firm generates a continuous-time profit flow (π), which is assumed to follow a Geometric Brownian Motion (GBM) diffusion process given by:

[6] We find this as a reasonable assumption, considering that these firms typically (i) present a limited or inexistent historical firm performance, whereby banks can accurately assess credit risk for the Entrepreneurial Firm, (ii) do not own tangible assets which could serve as a collateral to debt financing, or would have to bear prohibitive interest costs otherwise, and (iii) debt financing could potentially lead to inadequate capital structures, with debt repayment schedules causing major cash-flow constraints to small and rapidly growing firms facing significant uncertainties. In addition, there could exist major credit restrictions due to macroeconomic and other exogenous factors that can exclude any debt financing alternatives.

$$d\pi = \alpha\pi dt + \sigma\pi dz \tag{2.1}$$

where $\pi > 0$, α and σ stand for the trend parameter (i.e., the drift) and to the instantaneous volatility, respectively. Additionally, assuming that Entrepreneurs and VCs are risk neutral, $\alpha = r - \delta$, where $r > 0$ is the risk-free rate and $\delta > 0$ stands for the asset yield. Finally, dz is the increment of a Wiener process. We assume that both Entrepreneurs and VCs understand that the continuous profit flow (π) follows the same stochastic process.

Two important considerations should be taken into account at this point. First, by considering that the continuous-time profit flow (π) is governed by a Geometric Brownian Motion diffusion process, we implicitly assume that profits present percentage returns that are normally distributed and that such profits are always greater than zero. From our perspective, this stands for a reasonable assumption either in the current Chapter, but also in Chapter 3 – where we deal with established and profit-making Entrepreneurial Firms – and in Chapter 4, where even though we deal with Start-up Firms (SuFs), we are particularly interested in determining optimum investment timing for Independent Venture Capitalists (IVCs) and Public Venture Capitalists (PVCs) and it seems plausible to admit that such entities would only be willing to support a SuF when the present value of its profits is expected to be positive.

However, given that Entrepreneurial Firms and SuFs may present operating losses, some authors make use of Arithmetic Brownian Motions to portray their profitability (Lukas et al., 2012). While the prospects of bearing operating losses is plausible, we understand that its assumption of normally distributed absolute changes in profits disregards current profitability of a determinant of its expected change. In fact, in an Arithmetic Brownian Motion, the expected amount of change in profits is the same whether the Entrepreneurial Firm yields a one euro or a one million euro profit. Taking into account that within our setting the value of Growth Opportunities stand for a proxy of firm value, using Arithmetic Brownian Motions implies that agents considering to invest in such Growth Opportunities would be assuming that relative return of firm value would decrease with its level of profitability. We understand that this would violate an important feature of investment decision-making processes by profit maximizing agents.

Overall, we believe that more than *which* stochastic process should govern firm profitability, the assumption that Entrepreneurs and VCs accept that profits are driven by the *same* stochastic process is of much more relevance to our findings, in the sense that we are systematically interested in understanding how two parties interact within the course of an EF process. Accordingly, we take a GBM as a proper stochastic process for the purposes of our research as its tractability will highlight the impact that the specific variables that distinguish the decision-making process of Entrepreneurs and VCs hold on the outcomes of EF processes. In addition, the choice for GBM allows a vast range of previously developed option value formulations, comprising all derivations from the seminal Black-Scholes model (Black and Scholes, 1973), to be consistently used along the real options frameworks we conceived, as their stochastic variables are also assumed to follow GBMs.

Second, we assume that Entrepreneurs and VCs are equally diversified and require the same asset yield, given by δ. However, one may argue that Entrepreneurs are not as diversified as VCs and should therefore seek a differentiated return profile when screening the option to invest in the Growth Opportunity when compared to VCs. We claim there are reasons for believing that neither VCs *are as diversified* as other investors and that neither Entrepreneurs *are as undiversified* as one may imagine.

On a sample of 865 Private Equity (PE) funds with fund vintages from 1974 to 2010 and an average fund size of 1,045 million dollars, Braun et al. (2015) computed an average of 15.6 deals per fund, which is significantly below the range of 30 to 40 stocks required by a well-diversified portfolio of randomly chosen stocks (Statman, 1987). This means that even though Limited Partners (LPs) may benefit from diversification by deploying capital into a set of different PE and VC funds, General Partner (GPs) – those who actually manage each of those funds and drive their investment and divestment decisions – do not. In addition, diversification benefits to GPs are restrained by contractual covenants typically imposed by LPs to fund managers with the purpose of limiting their investment behaviour or even their outside activities (Gompers and Lerner, 1996), meaning, for example, that they may not be allocated to more than a given number of funds and benefit from the diversification effects on their performance-based compensation. Still on the fund perspective, diversification benefits are also limited by covenants restricting the types of investment that might be carried by the funds managed by GPs, including other PE or VC funds, public securities, leveraged buyouts, foreign securities and other asset classes (Gompers and Lerner, 1996).

In turn, and differently to VCs, Entrepreneurs are not restrained to certain asset classes or investment strategies on managing their personal wealth. As a result, they may not fully deploy their entire wealth on her or his Entrepreneurial Firm or SuF. For example, several authors point out that "hybrid entrepreneurs" remain as employees while proceeding with a part-time entrepreneurial activity (Folta et al., 2010, Raffiee and Feng, 2014). If "hybrid entrepreneurs" do not entirely allocate their time to their Entrepreneurial Firm, we may also argue that their wealth is also not fully allocated to their Entrepreneurial Firm. In addition, "serial entrepreneurs" hold a track-record of success in setting up and selling companies and they are also not likely to deploy most of their wealth on an entrepreneurial play (Wright et al., 1997, Gompers et al., 2006). Finally, we argue that Entrepreneurs and VCs face the same liquidity risk when investing in an Entrepreneurial Firm or SuF, as these are not publicly listed and they cannot be readily traded irrespective of any capital demands held by Entrepreneurs and VCs (Lerner and Schoar, 2004).

2.1.2.1. The decision to invest in the Growth Opportunity for the Entrepreneur

Following the contingent-claim approach used by Dixit and Pindyck (1994), the value of the option held by the Entrepreneur to invest in the growth opportunity of the Entrepreneurial Firm, $E(\pi)$, must satisfy the following ordinary differential equation (ODE):

$$\frac{1}{2}\sigma^2 \, \pi^2 \, E''(\pi) + (r-\delta)\pi \, E'(\pi) - r \, E(\pi) + \pi = 0 \qquad (2.2)$$

where the last term on the left hand side of equation (2.2) refers to the current profit flow of the Entrepreneurial Firm and the remaining terms refer to the growth option held by the Entrepreneurial Firm. The general solution for (2.2) comes:

$$E(\pi) = A\pi^{\beta_1} + B\pi^{\beta_2} + \frac{\pi}{\delta} \tag{2.3}$$

where A and B are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic Q_E, given by:

$$Q_E(\beta) = \frac{1}{2}\sigma^2\,\beta\,(\beta-1) + (r-\delta)\beta - r = 0 \tag{2.4}$$

i.e.

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{2.5}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{2.6}$$

Assuming that π_E^* stands for the optimal profit trigger to obtain Entrepreneur's support to the Growth Opportunity, and considering that, in order to execute the growth strategy, $Q^E < 100\%$, the problem must be solved by considering the following boundary conditions:

$$E(0) = 0 \tag{2.7}$$

$$E(\pi_E^*) = \frac{e.\pi_E^*}{\delta}.Q^E - k^a \tag{2.8}$$

$$E'(\pi_E^*) = \frac{e}{\delta}.Q^E \tag{2.9}$$

Respecting condition (2.7) and noting that $\beta_2 < 0$, then B on the equation (2.3) must be equal to zero and $\beta \equiv \beta_1$. The unknowns A and π_E^* are obtained by combining conditions (2.8) and (2.9). Solutions for the optimal profit trigger and for the option to invest on the growth opportunity then come:

$$\pi_E^* = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e.Q^E - 1} k^a \tag{2.10}$$

$$E(\pi) = \begin{cases} \dfrac{\pi}{\delta} + \left(\dfrac{(e.Q^E - 1).\pi_E^*}{\delta} - k^a\right)\left(\dfrac{\pi}{\pi_E^*}\right)^\beta, for\ \pi < \pi_E^* \\ \dfrac{e.\pi.Q^E}{\delta} - k^a, for\ \pi \geq \pi_E^* \end{cases} \tag{2.11}$$

Notice that as $e.Q^E \rightarrow 1$, $\pi_E^* \rightarrow +\infty$, meaning that for a given k and k^a, the lower the level of profit growth, the profit trigger will be prohibitive and no Entrepreneur would be willing to obtain support from an external equity provider for the growth strategy held by the Entrepreneurial Firm. This forms an *ownership dilution condition* for the Entrepreneur to consider an equity round to support a given Growth Opportunity: no Entrepreneur would be willing to seek for external equity unless the profit expansion of the Entrepreneurial Firm offsets the loss coming from a lower ownership on the Entrepreneurial Firm. Formally, this will lead to an asymptote on π_E^* when $e = \frac{1}{Q^E}$.

Model outcomes reveal that π_E^* is smaller, (i) the larger the profit growth is $\left(\frac{\partial \pi_E^*}{\partial e} < 0\right)$ and (ii) the higher the post-project firm ownership retained is $\left(\frac{\partial \pi_E^*}{\partial Q^E} < 0\right)$, while this profit trigger π_E^* becomes higher, the higher the overall capital outlay for deploying the growth strategy k is $\left(\frac{\partial \pi_E^*}{\partial k} = \frac{\partial \pi_E^*}{\partial Q^E}\frac{\partial Q^E}{\partial k} > 0\right)$. In turn, the value of the growth opportunity increases both with the underlying profit expansion factor e and post-project firm ownership Q^E.

2.1.2.2. *The decision to invest in the Growth Opportunity for the VC*

Similarly to the Entrepreneur, the value of the option to invest on the Growth Opportunity held by the VC, given by $VC(\pi)$, should also satisfy an ODE, as presented in equation (2.12). However, unlike the Entrepreneur, this option does not include the current profit flow π of the Entrepreneurial Firm. VCs can only profit by undertaking the Growth Opportunity, and not from existing firm profitability, when they decide not to participate in this growth strategy.

$$\frac{1}{2}\sigma^2 \pi^2 VC''(\pi) + (r - \delta)\pi VC'(\pi) - r VC(\pi) = 0 \tag{2.12}$$

The general solution for (2.12) is:

$$VC(\pi) = C\pi^{\beta_1} + D\pi^{\beta_2} \tag{2.13}$$

where C and D are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic, according to equations (2.5), (2.6) and (2.4), respectively. Similarly to the Entrepreneur, the boundary conditions are as follows:

$$VC(0) = 0 \tag{2.14}$$

$$VC(\pi_v^*) = \frac{e.\pi_{VC}^*}{\delta}.Q^{VC} - (k - k^a) \tag{2.15}$$

$$VC'(\pi_{VC}^*) = \frac{e}{\delta}.Q^{VC} \tag{2.16}$$

where $\pi\,^*_{VC}$ stands for the optimal profit trigger to support the growth strategy for the VC firm. Respecting condition (2.14) and noting that $\beta_2 < 0$, then D on equation (2.13) must be equal to zero. Therefore, and assuming that Entrepreneurs and VCs consider that the profit flow of the Entrepreneurial Firm is governed by the same stochastic process and underlying variables, $\beta \equiv \beta_1$ for the remaining of this paper. Solutions to the unknowns underlying the option value are given by:

$$\pi_{VC}^{*} = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e.Q^{VC}} (k - k^{a}) \tag{2.17}$$

$$VC(\pi) = \begin{cases} \left(\frac{e.Q^{VC}.\pi_{VC}^{*}}{\delta} - (k - k^{a})\right)\left(\frac{\pi}{\pi_{VC}^{*}}\right)^{\beta}, for\ \pi < \pi_{VC}^{*} \\ \frac{e.\pi}{\delta}.Q^{VC} - (k - k^{a}), for\ \pi \geq \pi_{VC}^{*} \end{cases} \tag{2.18}$$

It is implicitly assumed that the VC does not burden any additional opportunity costs from foregoing other potential investments in other companies or equivalently, it is assumed that the current investment opportunity is the best opportunity in which the VC fund may invest in.

2.1.2.3. Aligning optimum investment timing through a share premium or discount

So far, we have considered that parties assumed that post-equity round ownership on the Entrepreneurial Firm should be split according to the equity contributions each committed to the Entrepreneurial Firm. However, this would not enable the Entrepreneurial Firm to secure financing to its Growth Opportunity, by leading both Entrepreneurs and VCs to hold the same optimum investment timing, i.e., $\pi_E^{*} = \pi_{VC}^{*}$[7].

In addition, given that the value of the existing assets in place held by the Entrepreneurial Firm might be greater than the face value of the equity contributions made by the Entrepreneur previous to the growth opportunity (i.e., k^{i}), the former could legitimately

[7] We could obtain solutions for k and e that would impose $\pi_E^{*} = \pi_{VC}^{*}$, by equating (2.10) and (2.17) and solving for k and e, respectively. However, we argue that these two variables are exogenous to the model and that any of the parties would not revise their profit growth expectations or capital outlay required by the Growth Opportunity with the purpose of reaching a joint agreement.

argue that if there is no share premium, then, the VC would be having an arbitrage gain equal to the difference between the fair value of the equity stake that the VC obtained in the company, and the amount of the actual equity contribution made by the VC.

Conversely, if the value of the existing assets is low or if the expected profit growth by the Entrepreneur is very high, the Entrepreneur could be willing to offer a discount on equity issuance, in order to secure the necessary funding for executing the envisaged growth strategy. In fact, VCs might argue that without their equity contribution, the value of the Growth Opportunity held by the Entrepreneurial Firm is equal to zero, since neither the firm, nor the Entrepreneur, hold enough resources to proceed with its execution.

As a result, and given the relevance of this topic within the negotiation context between Entrepreneurs and VCs, we are interested in understanding how this premium (or discount) might be computed and actually work as an alignment mechanism between the two parties. In this setting, this share premium (or discount) is named as p and it is introduced as an additional ownership dilution effect over the VC[8], subject to $p > -(k^i + k^a)$. Therefore, in the presence of this share premium (or discount) we denote the post-equity round firm ownership held by the Entrepreneur as $Q^{EP} = \frac{k^i+k^a+p}{k^i+k}$ and the post-equity round firm ownership held by the VC as $Q^{VCP} = \frac{k-k^a-p}{k^i+k}$.

As no further changes are introduced, we obtain the new boundary conditions and profit triggers simply by replacing Q^E and $Q^{VC}$by Q^{EP} and Q^{VCP} respectively on equations (2.9) to (2.11) and (2.16) to (2.18) as shown in Table 1.

[8] Share discounts might be subject to legal constraints, whose impacts on the model outcomes are far beyond the scope of this paper. Our intention is to focus on the purely economic drivers that, within the framework herein presented, would induce both Entrepreneurs and VCs to become willing to accept a share premium or discount on the equity issuance to be carried by the Entrepreneurial Firm.

For the Entrepreneur		For the VC	
$E(\pi_E^*) = \frac{e.\pi_E^*}{\delta}.Q^{EP} - k^a$	(2.19)	$VC(\pi_{VC}^*) = \frac{e.\pi_{VC}^*}{\delta}.Q^{VCP} - (k - k^a)$	(2.20)
$E'(\pi_E^*) = \frac{e}{\delta}.Q^{EP}$, leading to	(2.21)	$VC'(\pi_{VC}^*) = \frac{e}{\delta}.Q^{VCP}$, leading to	(2.22)
$\pi_E^* = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e.Q^{EP} - 1} k_\alpha$	(2.23)	$\pi_{VC}^* = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e.Q^{VCP}} (k - k_\alpha)$	(2.24)

Table 1. Boundary conditions and profit triggers with a share premium (or discount)

Equating the resulting profit triggers for the Entrepreneur and the VC – i.e., equating (2.23) and (2.24) – and solving this for the premium p, we obtain the following result:

$$p = \frac{(k-k^a)[k-k^i(e-1)]}{ek}, \text{ or equivalently} \tag{2.25}$$

$$Q^{EP} = \frac{k + k^a(e-1)}{ek} \tag{2.26}$$

Referring to equation (2.25), these results show that the share premium (or discount) that would lead Entrepreneurs and VCs to support the growth opportunity:

- Is positively influenced by the amount of capital to be deployed by the VC to support the growth strategy (i.e., $k - k^a$);

- Is equal to zero when $k - k^i(e-1) = 0$ (i.e., when $e = 1 + \frac{k}{k^i}$[9]), highlighting that if $e > 1 + \frac{k}{k^i}$ a share discount on equity issuance is obtained. This term also

[9] In fact, this is the solution we obtain by combining equations (2.23) and (2.24), and solving for e, i.e., when we would allow parties to change their profit growth expectations to ensure that they would both support the Growth Opportunity held by the Entrepreneurial Firm.

shows that the lower is the capital initially deployed by the Entrepreneur (k^i), the greater should be the share premium required by the Entrepreneur, as the greater is the ownership dilution loss caused by the growth opportunity;

- Is negatively influenced by the expected profit growth e, given that $\frac{dp}{de} = -\frac{(k-k^a)k^i}{ek} - \frac{(k-k^a)[k-k^i(e-1)]}{e^2 k} < 0, \forall\, e > 1$. This results also holds for the post-deal Entrepreneur's ownership stake on the Entrepreneurial Firm, as $\frac{dQ^{EP}}{de} < 0, \forall\, e > 1$. This reveals that the greater the profit growth expectations, the more is the Entrepreneur willing to trade-off ownership on the Entrepreneurial Firm for the profit generated by the growth strategy. This means that, if the Entrepreneur is growth-oriented and not adverse to control risk, the greater her or his profit growth expectations are, the more is she or he willing to have a "smaller share" of a "bigger venture".

Equations (2.25) and (2.26) also reveal that Entrepreneurs attain a benefit when obtaining external equity to support a given Growth Opportunity, since they retain Q^{EP} of its profits, but by spending a fraction $\frac{k^a}{k}$ of its required capital outlay. Such benefit is formally derived as $Q^{EP}.k - k^a$ (i.e., the difference between the capital outlay k that the Entrepreneur should have made considering the firm ownership she or he retained, and the capital outlay that the Entrepreneur actually made), and is equal to $\frac{k-k^a}{e}$.

Interestingly, equation (2.25) and (2.26) show that the optimum share premium (or discount) set by the Entrepreneur and the VC does not depend on any of the parameters of the underlying stochastic process that governs the profit flow of the Entrepreneurial Firm, provided that both the Entrepreneur and the VC regard its profit flow to follow the same Geometric Brownian Motion.

2.2. The case with asymmetric profit growth expectations

Taking the general view from de Meza and Southey (1996) that Entrepreneurs and Financiers hold different prospects on future firm performance, we will now relax the assumption that Entrepreneurs and VCs share the same perspectives on profit growth, given by e. In fact, Entrepreneurs and VCs often hold distinct prospects over the same growth strategy, which could in turn lead to long discussions during negotiation stages. We argue that, even in these conditions, an agreement might be reached.

In this new setting, we take the profit growth envisaged by the Entrepreneur as e_E and the profit growth envisaged by VC as e_{VC} and re-write the boundary conditions and profit triggers for each of the options to invest on the growth strategy held by the Entrepreneur and the VC. Similarly to the previous section, we derive the conditions for the share premium (or discount) that, in the presence of asymmetric expectations on profit growth, would allow Entrepreneurs and VCs to optimally and simultaneously invest on the Growth Opportunity held by the Entrepreneurial Firm.

In this setting, Q^{EP} and Q^{VCP} are computed as before. The new boundary conditions and profit triggers are obtained by replacing by replacing e by e_E and e_{VC}, respectively, on equations (2.19), (2.21), (2.20) and (2.22), as shown in Table 2.

For the Entrepreneur		For the VC	
$E(\pi_E^*) = \frac{e_E . \pi_E^*}{\delta} . Q^{EP} - k^a$	(2.27)	$VC(\pi_{VC}^*) = \frac{e_{VC} . \pi_{VC}^*}{\delta} . Q^{VCP} - (k - k^{\alpha})$	(2.28)
$E'(\pi_E^*) = \frac{e_E}{\delta} . Q^{EP}$, leading to	(2.29)	$VC'(\pi_{VC}^*) = \frac{e_{VC}}{\delta} . Q^{VCP}$, leading to	(2.30)
$\pi_E^* = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e_E . Q^{EP} - 1} k_{\alpha}$	(2.31)	$\pi_{VC}^* = \frac{\beta}{\beta - 1} \cdot \frac{\delta}{e_{VC} . Q^{VCP}} (k - k_{\alpha})$	(2.32)

Table 2. Boundary conditions and profit triggers with different prospects on profit growth and with an up-front share premium (or discount)

Equating the resulting profit triggers for the Entrepreneur and the VC – i.e., equating (2.31) and (2.32) – and solving this for the premium p, we obtain the following result:

$$p = \frac{(k-k^a)[k-k^a(e_E-e_{VC})-k^i(e_E-1)]}{e_E k-k^a(e_E-e_{VC})}, \text{ or equivalently} \quad (2.33)$$

$$Q^{EP} = \frac{k + k^a(e_{VC} - 1)}{e_E k - k^a(e_E - e_{VC})} \quad (2.34)$$

Provided that the general intuition of these results was presented on the previous section, we now focus on how asymmetric expectations on profit growth (i.e., $e_E - e_{VC}$) affect the optimum share premium or discount on equity issuance given by p. On the one hand, the numerator in equation (2.33) shows that profit growth expectations held by the Entrepreneur negatively influence the optimum share premium or discount, given the negative coefficients that drive the terms on $k^a\ (e_E - e_{VC})$ and $k^i\ (e_E - 1)$ and that essentially capture the growth effect on the capital deployed by the Entrepreneur (k^i and k^a) on the Entrepreneurial Firm. Interestingly, the term on k^i does not depend on e_{VC} but only on e_E, reflecting the fact the capital initially deployed by the Entrepreneur (k^i) fully benefits from the outcomes of the Growth Opportunity.

On the other hand, the denominator in equation (2.33) shows that the profit growth expectations held by the Entrepreneur negatively influence the optimum share premium or discount, as it may be written as $e_E\ (k - k^a) + k^a\ e_{VC}$. This term should be jointly interpreted with the coefficient $(k - k^a)$ presented on the numerator, as it grosses down the overall ownership dilution effect caused by the growth opportunity $(k - k^a)$ through the expected profit growth effects (e_{VC} and e_E). However, the greater the profit growth expectations (e_E) are, the lower the overall ownership dilution effect is she or he willing to accept and the higher optimum share premium is.

Overall, given that $\frac{dp}{de^E} < 0, \forall\ e^E > 1$, the growth effects underlying the investment opportunity dominate over the ownership dilution and, therefore, the greater the profit growth expectations held by the Entrepreneur, the more she or he is willing to trade-off ownership on the Entrepreneurial Firm for the profit generated by the growth strategy.

Equation (2.33) also shows that the Entrepreneur would accept an equity issuance with no share premium (i.e., $p = 0$), when $e_E = \frac{1}{Q^E} + e_{VC} \frac{k^a}{k^i + k^a}$. As a result, when $e_E > \frac{1}{Q^E} + e_{VC} \frac{k^a}{k^i + k^a}$, the Entrepreneur would be in fact willing to accept a discount on the equity issuance in favour of the VC.

2.3. Numerical example

After having formally derived our framework for discussing Entrepreneurial Financing decisions, we will now introduce a numerical example to illustrate our findings and highlight their underlying economic intuition. We will first present an example for the case for symmetric expectations on profit growth, and then move to the case in which Entrepreneurs and VCs hold different prospects on profit growth.

2.3.1. Numerical assumptions

In this example, and following de Meza and Southey (1996) and Hmieleski and Baron (2009), we will take the view that Entrepreneurs are usually more optimistic than financiers and, therefore, e_E is assumed to be greater than e_{VC}. The rest of the parameters are standard.

Variable	Numerical Assumption	Variable	Numerical Assumption	Variable	Numerical Assumption
k_i	150	r	0.04	e	3.00
k_a	275	σ	0.30	e_E	3.00
k	500	δ	0.08	e_{VC}	2.00

Table 3. Numerical assumptions

2.3.2. The optimum share premium (or discount) with symmetric profit growth expectations

In this setting, we will have $Q^E = 65.4\%$, $Q^{VC} = 34.6\%$, and no joint support from both Entrepreneurs and VCs to this Growth Opportunity, if $e = 3.00$. In the absence of a share premium or discount, this Growth Opportunity would not obtain joint support from Entrepreneurs and VCs – as shown in Figure 3 – and would then become valueless.

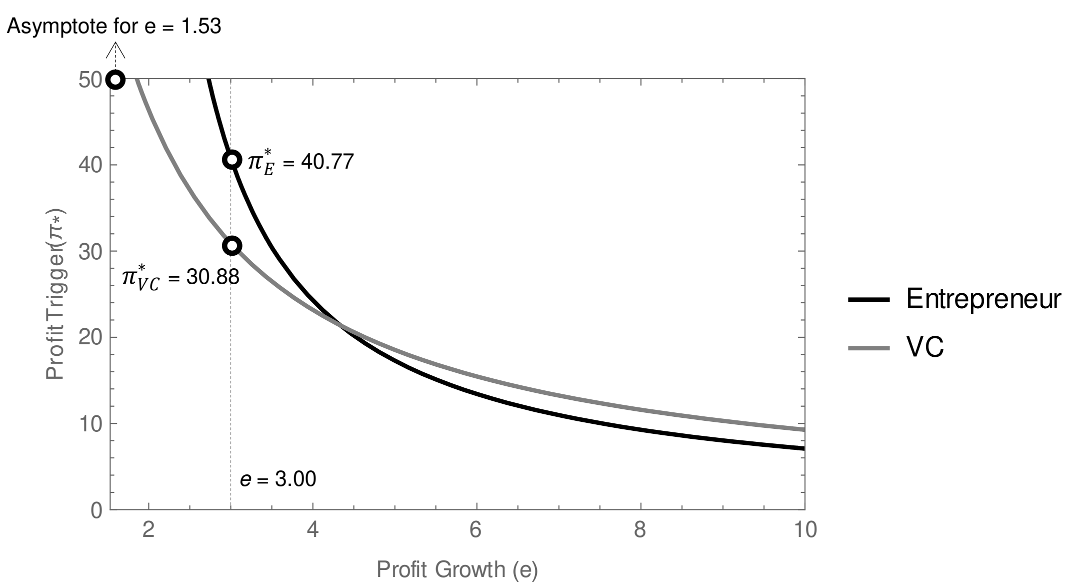

Figure 3. Profit triggers for Entrepreneurs and VCs with symmetric expectations on profit growth with no share premium or discount

By combining equations (2.23) and (2.24), we obtain the optimum share premium $p = 30.00$ that would allow Entrepreneurs and VCs to engage on the growth strategy, as shown in Figure 4. A share premium ($p > 0$) is consistent with the example shown in Figure 3, given that e is below the optimum profit growth that allows parties to jointly support the investment opportunity ($e = 4.33$). As a result and according to equation (2.26), $Q^{EP} = 70.0\%$, instead of $Q^E = 65.4\%$.

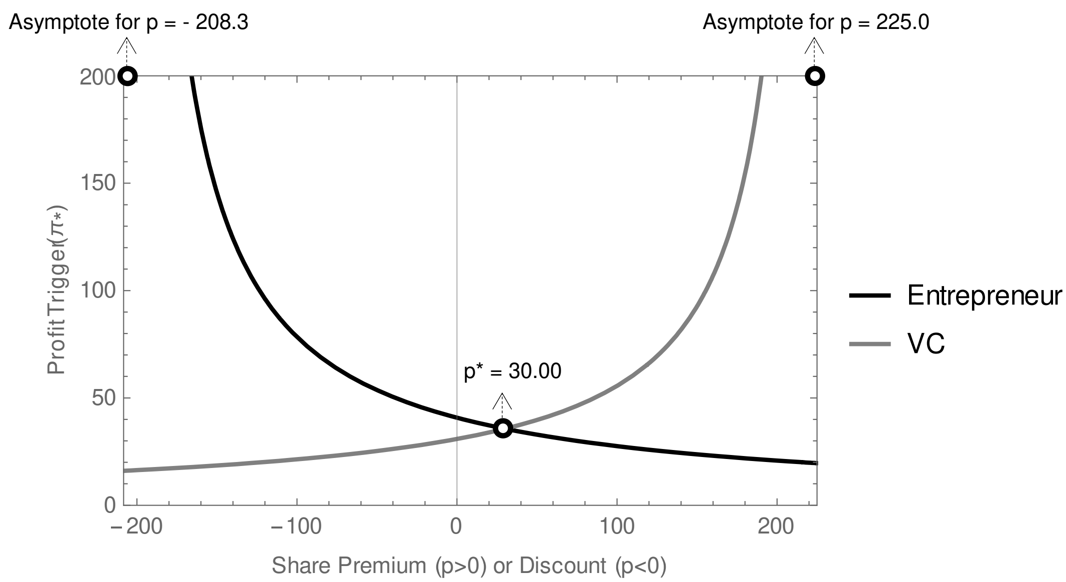

Figure 4. Profit triggers for Entrepreneurs and VCs with symmetric expectations on profit growth

2.3.3. The optimum share premium with asymmetric expectations on profit growth

In this setting, Q^{E} and Q^{VC} are again affected by the share premium or discount given by p and are therefore replaced by Q^{EP} and Q^{VCP}. We are interested in obtaining the optimum share premium that would allow Entrepreneurs and VCs to reach an agreement towards supporting the growth opportunity and visualizing the impact of different profit growth expectations held by the Entrepreneur on the optimum share premium or discount.

The optimum share premium is given by equation (2.33). Considering $e_E = 3.0$ and $e_{VC} = 2.0$, we obtain $p = -13.77$, as shown on Figure 5.

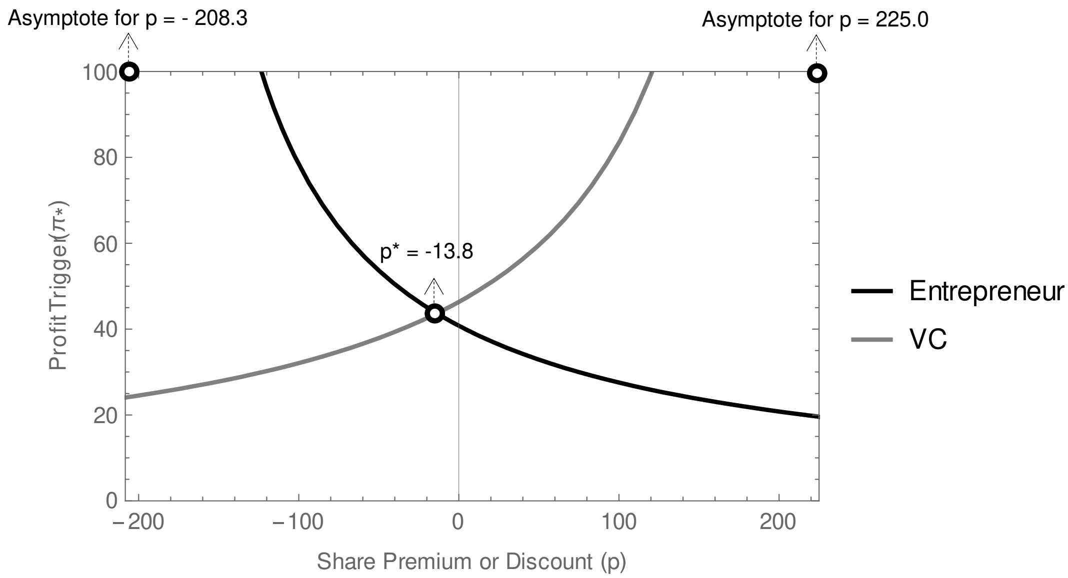

Figure 5. Profit triggers for Entrepreneurs and VCs with asymmetric expectations on profit growth and a share premium (p > 0) or discount (p < 0) on equity issuance

As a result, Q^{VCP} would equal 36.7%, increasing from $Q^{VCP} = 34.6\%$, as the Entrepreneur would actually be willing to trade a lower shareholding for the chance of entering into the Growth Opportunity. In this case, there is also an asymptote on the Entrepreneur's profit triggers given by $p = -208.33$, meaning that for discounts above this threshold, no agreement is possible to be settled[10]. Similarly, there is an asymptote on the VC's profit trigger, given by $p = 225.00$, as the share premium cannot be greater than the amount of capital that the VC is deploying on the Entrepreneurial Firm (i.e., $k - k^a$) and $Q^{VC} > 0$.

Taking a look at how different Entrepreneur's profit growth expectations e_E drive the optimum share premium p, on Figure 6 we may observe that p is decreasing with e_E, and

[10] This asymptote captures the ownership dilution condition, which was introduced in section 2.1.2 and it is given by solving the condition $e^E \left(\frac{k^i+k^a+p}{k^i+k}\right) - 1 = 0$ on the denominator of the Entrepreneur's profit trigger, given by equation (2.31).

that for $e^E > \frac{1}{Q^E} + e^{VC}\frac{k^a}{k^i+k^a}, i.e., e^E > 2.82$, the Entrepreneur would actually be willing to accept a discount on its shareholding to allow the VC to support the Growth Opportunity. This is the same trigger on e_E that would allow Entrepreneurs and VCs to jointly support the growth strategy, in the absence of any share premium or discount.

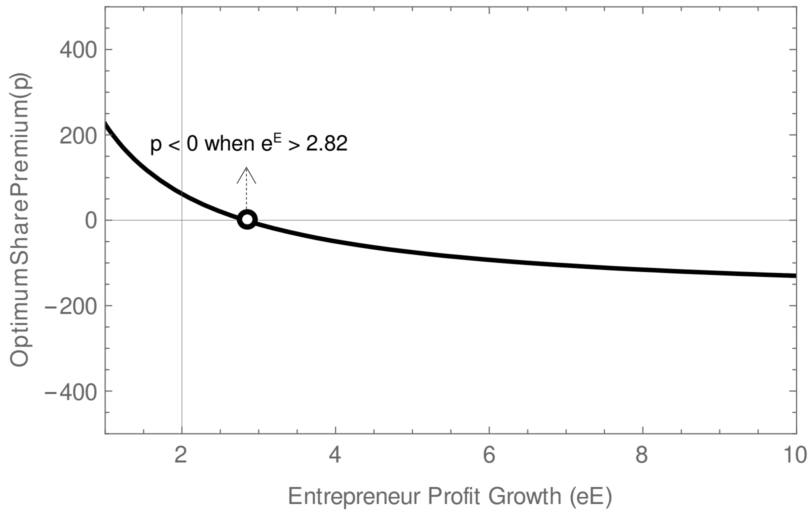

Figure 6. Optimum share premium or discount and the Entrepreneur's profit growth expectations

The numerical example shows that the greater the Entrepreneur's expectations on profit growth, the more is she or he willing to trade-off firm ownership for the profit growth underlying the Growth Opportunity.

2.4. Chapter summary

In this Chapter we developed a real options framework for determining alignment conditions between Entrepreneurs and Venture Capitalists (VCs) on EF decisions under symmetric and asymmetric profit growth expectations, for a given Growth Opportunity.

We showed that Entrepreneurs would only be willing to seek for external equity if value creation effects rising from the growth opportunity offset the loss caused by her or his ownership dilution, while, on the other hand, VCs would only be willing to support the Growth Opportunity if its value creation effects are such that will offset the loss from retaining a lower share of the Entrepreneurial Firm than the one for which they provided to the Growth Opportunity.

In addition, we presented how a share premium (or discount) mechanism could be computed so that the Growth Opportunity obtains joint support from Entrepreneurs and VCs and showed how asymmetries on future profit growth expectations may actually contribute to aligning Entrepreneurs and VCs in supporting Growth Opportunities.

On the next Chapter, we will introduce contingent payments as alternative enablers of EF decisions. In that setting – and differently from the framework we revealed on this Chapter – instead of settling up-front the definite financial terms of the EF decision, VCs agree to pay Entrepreneurs an extra amount of cash, subject to the accomplishment by the Entrepreneurial Firm of certain performance goals within a pre-determined future time horizon.

3. Contingent Payment Mechanisms and Entrepreneurial Financing decisions

Entrepreneurial Financing (EF) decisions cover a distinct range of financial and non-financial terms that Entrepreneurs and Venture Capitalists (VCs) negotiate, which should as a whole trigger their willingness to forego firm ownership, provide funds to support a given growth strategy or get access to a range of financial and managerial skills (Croce et al., 2013, Kaplan and Strömberg, 2001, Hsu, 2004).

From a financial perspective, and leaving aside post-deal compensation and interest alignment mechanisms which may be set between the parties, EF decisions usually involve discussions on valuing the Entrepreneurial Firm, and on how parties will split firm ownership. Considering how uncertainty surrounds the prospects of Entrepreneurial Firms, parties may choose to solve discussions on valuation and firm ownership by engaging into an up-front share or cash premium (or discount) or, alternatively, by setting a deferred and Contingent Payment Mechanism (CPM) subject to a given performance benchmark or strategic milestone of the Entrepreneurial Firm.

While up-front cash and shares largely dominate as deal currency mechanisms, accounting for 80.8% and 24.9%[11] of the total transactions that took place between 2000 and 30th June, 2015 according to Zephyr[12], deals involving CPMs – also known on the literature as *Earn-*

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[11] Bear in mind that one given deal may have more than one deal currency mechanism (for example, a combination of cash and shares). Therefore, summing up the share of different deal currency mechanisms on total deal volumes leads to over 100.0%.

[12] Estimates based on a sample extracted from Zephyr comprising completed deals from 1st January, 2000 to 30th June, 2015, including acquisitions, institutional buy-outs, capital increases, management buy-ins, management and buy-outs, involving targets located in the Baltic States, Eastern Europe, North America, Oceania, Scandinavia, and Western Europe and acquisitions with, at least, 15.0% stakes on target firms, totaling 331,419 transactions. Deals for which no payment terms are available are excluded from the statistics of deal payment terms mentioned throughout the paper.

Outs or *Contingent Earn-Outs* – stood for 7.9% of total deal volumes during this period, and are being increasingly used, standing for 11.3% of total deal volumes from January to June, 2015 against 4.5% in 2000. CPMs seem to be more popular in industries especially reliant on intangible assets, such as "*Computer, IT and Internet Services*" (where 14.2% of deal volumes between 2000 and 30th June, 2015 used CPMs) or "*Biotechnology, Pharmaceuticals and Life Sciences*" (11.4%), or industries featuring significant volatility on cash flow generation, such as "*Construction*[13]" (10.3%). As they introduce additional complexity on deal terms, CPMs are more popular on professional investors, such as deals involving Private Equity or VCs divestment (12.6%) or on deals involving Sovereign Wealth Funds (6.7%). In addition, CPMs are more frequent on smaller deals in terms of deal value, as only 23.7% of the deals including CPMs involve deal values on the top quartile of our sample (i.e., deals above 34 million euros).

Such evidence on CPMs is broadly consistent with previous literature findings. Cain et al. (2011) posited that higher contingent payments are observed when targets possess high growth opportunities and are exposed to greater uncertainty, while Datar et al. (2001) found that acquisitions of high technology, service intensive or small private companies are more prone to use CPMs. Barbopoulos and Sudarsanam (2012) points out that contingent payments are more likely in the "*Media and Entertainment*", "*Consumer Products*", "*High Technology*", "*Healthcare*" and "*Telecommunications*" industries, which hold large intangible assets, and are surrounded by greater volatility on their cash flow generation, taking prospective bidders to higher value at risk alongside information asymmetries.

Overall, both literature and empirical findings support the idea that CPMs should be particularly relevant within an Entrepreneurial Financing context, in which Entrepreneurial Firms also face valuable growth opportunities, and major uncertainties on future cash flow generation and business prospects.

In spite of such conceptual argument, literature on decision-making models for EF decisions involving CPMs – or, even more broadly, within a Mergers & Acquisitions (M&A)

[13] Industry taxonomy as provided by Zephyr database according to Zephus classification.

context – is scarce (Lukas et al., 2012) and has not still, at the moment, comparatively discussed the design of different types of CPMs. Therefore, we expect to provide a contribution to fill this gap, (i) by presenting a taxonomy for classifying different CPMs based on their payment term and amount, (ii) by introducing an options-based approach to value each of the four major different CPMs we identified, and (iii) by demonstrating how the key terms on each of the four major CPMs should be computed so that Entrepreneurs and VCs would be jointly willing to support a given Entrepreneurial Firm and its growth strategy. With this purpose, we extend the real options based framework for analyzing Entrepreneurial Financing decisions introduced on the previous Chapter.

This Chapter is structured as follows. In section 3.1 we go through the existing literature on the topic. In section 3.2, we propose a taxonomy for classifying CPMs. In section 3.3 we value each of the four major CPMs we identified and derive optimum CPM terms that enable Entrepreneurs and VCs to jointly support a given Entrepreneurial Firm with a growth opportunity. In section 3.4 we illustrate the different CPMs with a numerical example. In section 3.5 we discuss our findings and present a summary of this Chapter in section 3.6.

3.1. Literature review

The concept of Contingent Payment Mechanisms is often defined on the literature. For example, Bruner and Stiegler (2014) highlight that CPMs are "*contingent on achievement of financial or other performance targets after the deal close*"[14], while Reuer et al. (2004) underline that they are "*deferred variable payments (...) within a certain time frame after the deal has been consummated*"[15]. We understand that a more general definition is required for

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[14] Bruner and Stiegler (2014) define an Earn-Out as "*an arrangement under which a portion of the purchase price in an acquisition is contingent on achievement of financial or other performance targets after the deal closes*".

[15] Reuer et al. (2004) define Contingent Earn-Outs as "*deferred variable payments tied to the target's ability to meet pre-specified performance goals within a certain time frame after the deal has been consummated*".

a better understanding of CPMs, as these (i) may either comprise fixed or variable amounts (for example, when it equals a given multiple over revenues, EBITDA or profits above a certain threshold, measured at a given date), (ii) may either be paid on a pre-determined date or at any date within a given period (i.e., when a certain financial or business milestone is met) or (iii) may either require or not require a pre-specified goal to be met (for example, when a contingent payment equals a given multiple of all incremental EBITDA generated post-deal against the one on the deal completion accounts). Therefore, we follow the more general approach introduced by Datar et al. (2001) in which CPMs are defined as "*a method of acquisition where the final consideration received by the seller is based on the future performance of his business*"[16].

Some general insights on the pros and cons of CPMs from the M&A literature are extendable to an EF context, even though, differently from an M&A process, EF decisions do not involve a sale and purchase agreement of part of the whole firm ownership[17].

On the one hand, CPMs may reduce the risk of adverse selection and overpayment on the existence of private information on the business of the target firm (Kohers and Ang, 2000, Datar et al., 2001, Ragozzino and Reuer, 2009) or, conversely, reduce the risk of inverse adverse selection and underpayment, where bidders are potentially more informed than vendors, by allowing the latter to benefit from post-deal value creation (Ragozzino and Reuer, 2009). In addition, CPMs mitigate potential moral hazard risks on the post-deal stage, by providing incentives for vendors and/ or target management to adjust their behavior with the purpose of maximizing the probability of obtaining a contingent payment in the future (Krug and Hegarty, 2001, Kohers and Ang, 2000). Finally, CPMs may also be regarded as a

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[16] Usually CPMs do not require vendors to return part of the initial consideration to acquirers whether a certain future performance is not achieved by the target firm. This would lead to the introduction of "*Contingent Earn-Ins*", which are rarely found on the M&A market. This kind of purchase price adjustment mechanism is more frequent for addressing potential liabilities of the target firm, rather than for establishing some kind of performance-based compensation.

[17] In Entrepreneurial Financing decisions, an outside investor financially supports a given growth opportunity held by an Entrepreneurial Firm, through an equity round. The Entrepreneurial Firm is in turn owned by a wealth constrained shareholder (or set of shareholders), taken as the Entrepreneur(s), who is not able to provide the Entrepreneurial Firm with all the necessary financial resources to execute its growth strategy.

financial leverage enhancer, by providing the acquirer with an option to fund total deal consideration with the underlying cash flow of the target firm through a deferred payment, or by providing the acquirer with the benefits of a staged investment process, given that it may be required to commit additional capital to support the growth opportunities of the target firm (Del Roccili and Fuhr Jr, 2001).

On the other hand, CPMs may incentivize acquirers, vendors or target managers to influence the performance of the target firm with the purpose of maximizing or minimizing the amount of the future contingent payment, and therefore influencing the firm towards short-term rather than long-term goals (Lukas et al., 2012). Moreover, CPMs may introduce complexities on performance measurement, which may slow down post-deal integration and value creation effects, and may consequently introduce significant contracting and monitoring costs (Caselli et al., 2006, Datar et al., 2001). Barbopoulos and Sudarsanam (2012) argues that CPMs do not, in fact, provide a superior benefit against stock offers, since these may present similar contingency and value mitigation characteristics to CPMs, especially when entities have comparable sizes, a comparable contribution to post-deal value creation or when the stock of the acquirer is publicly traded and therefore allows the vendor to easily transform her or his ownership in cash. In fact, and by making use of a logistic model, these authors empirically concluded that acquisitions of privately owned firms or of subsidiaries of public firms are more prone to involve contingent payments. Lastly, one may also argue that, specifically within an Entrepreneurial Financing context, the risk-return profile of Entrepreneurial Firms, where failure rates are high, may advise investors not to reduce their potential upsides on the few successful Entrepreneurial Firms they support through CPMs or other similar mechanisms.

Empirical research on designing CPMs reveals that contingent payments may stand from 15.0% to 80.0% of total deal consideration (Bruner and Stiegler, 2014), with an average of 33.0% according to Cain et al. (2011) and acquisitions involving privately owned firms recording a 44.0% higher average contingent payment (Kohers and Ang, 2000). The CPM period ranges from one month to twenty years, with an average of 2.57 years (Cain et al., 2011), but more frequently laying on the two to five years range (Kohers and Ang, 2000).

In general, empirical research on acquirer stock returns provides support to a more widespread use of CPMs, at least from the bidder perspective. Supporting evidence includes Kohers and Ang (2000), who revealed that acquirers using CPMs recorded an abnormal return of 1.356% on the date of announcement against those that did not employ CPMs, on a sample comprising 938 deals with 82.1% Anglo-American bidders. Lukas and Heimann (2014) recorded an average 1.439% abnormal return at the date of announcement, and an average abnormal return on a three days window around the announcement date of 2.036%, in a sample exclusively involving deals in Germany. On a sample of bids announced by UK firms, Barbopoulos and Sudarsanam (2012) found that overall earn-out bids yield significantly higher returns than non earn-out bids (i.e., 1.48% against 1.07%) on a two days window around the announcement date, and that the benefits of optimal CPM use are exhausted by the second year after deal completion. In turn, unclear evidence is presented by Mantecon (2009), who found an average cumulative return of 1.01% for a three days window around the announcement date with a sample involving 2/3 of Anglo-American bidders, but weekly positive for domestic transactions and even insignificant for cross-border deals.

From an analytical point-of-view, several authors argue the existence of an analogy between CPMs and real options (Bruner and Stiegler, 2014, Caselli et al., 2006, Lukas et al., 2012), as shown in Table 4. Even though contingent payments do not hold an optionality feature, they provide payoffs that mirror those of real options and might be specified as call options, as argued by Bruner and Stiegler (2014).

Notwithstanding, there are only a few analytical papers discussing how CPMs may drive acquisitions.

Lukas et al. (2012) took a two-stage option-game approach to CPMs to examine the impact of uncertainty and of contingent payment terms on optimal M&A timing. The resulting model allowed the authors to specify a set of three empirically testable hypothesis, regarding Earn-Out ratios, Earn-Outs premiums[18] and initial deal consideration. In particular,

[18] "*Earn-Out Premium*" is defined as the amount of the contingent payment itself that was set by the parties, while the "*Earn-Out Ratio*" is defined as the ratio of all contingent payments in relation to the maximum price paid.

uncertainty is argued to positively drive the initial deal consideration, the Earn-Out premium and the Earn-Out ratio, while the Earn-Out period negatively influences the initial payment and positively affect the Earn-Out premium and the Earn-Out ratio. In turn, higher performance benchmarks positively drive initial payments and negatively drive both Earn-Out premium and ratio. On this model, the contingent payment is fixed and paid at a given pre-determined date.

	Call Options on Common Stock	CPMs
Underlying Asset	Shares of common stock	Some index or measure of financial or operating performance (sales, earnings, cash flow, awards)
Exercise Price	The stated strike price of the options contract	Any benchmark, hurdle or triggering event, beyond which the CPM starts paying off
Price of the Underlying Asset	Share price of the underlying common stock	The level of the index or measure of performance (sales, earnings, cash flow, etc.)
Interim Payouts	Dividends	Dividends and any interim cash flows associated with the CPM
Term of the Option	On a pre-specified date, typically from 3 to 9 months from original issue	On a pre-specified date, or when a given event takes place during an certain period set by the parties
Uncertainty	Volatility of returns on the underlying asset	Uncertainty about the performance of the underlying asset to which the CPM is pegged

Adapted from Bruner and Stiegler (2014)

Table 4. Comparison of Contingent Payment Mechanisms and call options on stock

Lukas and Heimann (2014) also derived a set of testable empirical propositions on CPMs through a theoretical model set for an M&A context featuring information asymmetries. Grounded on a classic principal-agent model, the authors conceived a utility model in which the target firm envisages no uncertainty on product launch, while the bidder computes expected target performance through a uniform distribution. Model outputs reveal that CPMs increase the utility of buyers, by transferring some of the acquisition risk to vendors, especially when the volatility of cash flow generation of the target firm increases. Shorter

Earn-Out periods are also argued to increase the utility of buyers and extremely high levels of information asymmetry may actually impede deals from taking place, by lowering the utility of both bidders and vendors. Overall, in the absence of technological information asymmetry, as CPMs allow buyers to improve their utility, such deal currency should be preferred over a classical lump sum. On this setting, the contingent payment equals a fraction (from 0.0 to 1.0) of the cash flow generated by the target firm on a pre-specified date.

Choi (2015) developed a two stage game-theoretic model for an M&A context with the purpose of addressing the question of how the post-closing stage influences the design of optimal CPMs. The authors drafted two different settings (one in which vendors holds private information, and one where bidders and sellers hold different expectations on future profit generation by the target firm) and showed that CPMs will be structured with the purpose of minimizing the deadweight loss resulting from a smaller incentive component and that, when there is a small valuation gap between acquirer and vendors, parties may actually forego from using a CPM. Similarly to Lukas et al. (2012), the contingent payment is also fixed and paid at a given pre-determined date on this model.

Overall, the literature reveals that there is still room for progressing analytical research on CPMs in two different ways. Firstly, by cataloguing and valuing alternative designs for CPMs, and secondly by expanding such analytical tools to an EF context. We intend to address these two issues on the coming sections.

3.2. A taxonomy for Contingent Payment Mechanisms

Recent news on M&A deals reveal that CPMs may present several distinctive features. For example, on the recent acquisition of GlaxoSmithKline’s oncology products unit by Novartis, completed on March, 2015, whose total consideration amounted to 16 billion dollars in cash, Zephyr reported that up to 1.5 billion dollars are contingent on the results of the Combi-D trial, a Phase III study evaluating the safety and efficacy of the combination of Tafinlar and Mekinist against BRAF monotherapy. Differently, on the acquisition of the Portuguese assets of Portugal Telecom from Oi, which was completed on June, 2015, Altice offered a total consideration of 7.03 billion euros, including 800 million euros contingent on revenue milestones being met. Moreover, on the acquisition of the price comparison site uSwitch that was completed on May, 2015, the property search portal Zoopla offered a total consideration of 130 million British pounds, including a "*potential payment of up to 30 million British pounds for uSwitch's management dependent on achievement of certain financial performance targets for fiscal 2016*", according to Zephyr.

Such cases illustrate that different types of CPMs should be put in place for different circumstances. On the sale of the oncology business unit held by GlaxoSmithKleine to Novartis, a variable payment up to 1.50 billion dollars was put in place contingent on the accomplishment of a relevant research and development initiative, which is due when the results of such ongoing research are known. On the sale of the Portuguese assets held by Portugal Telecom to Altice, one might argue that the 800 million euros contingent payment served as a mechanism for narrowing a potential valuation gap. Finally, on the acquisition uSwitch by Zoopla, the contingent payment is explicitly aimed at providing an incentive to uSwitch's management, who also sold their stakes to Zoopla, to achieve certain financial targets.

We propose a simple, but still not exhaustive, taxonomy for CPMs, which is aimed to cover the most common cases and which is essentially defined by the key financial terms of a contingent payment: its *amount* and its *due date*. On the one hand, we understand that the amount of the contingent payment might be *fixed*, i.e., irrespective of the completion rate of the performance benchmark that triggers the contingent payment (e.g., a contingent payment of one million euros if revenues by the end of the first twelve months after deal completion reach or surpass five million euros) or *variable*, i.e., dependent on the completion rate of the performance benchmark (i.e., a given multiple on the excess revenue between the first twelve months after deal completion and the last twelve months prior to deal completion). On the other hand, we understand that contingent payments might be due at the *term* of the CPM period (as in the case for uSwitch, i.e., subject to the performance of the 2016 fiscal year) or at *hit* (as in the case for GlaxoSmithKleine), i.e., at the moment in which the performance benchmark is achieved[19]. Our taxonomy is then summarized on Table 5.

Amount \ Due Date	At Hit	At Term
Variable	**Variable at Hit CPM**	**Variable at Term CPM**
Fixed	**Fixed at Hit CPM**	**Fixed at Term CPM**

Table 5. A taxonomy for Contingent Payment Mechanisms

We understand that these two CPM segmentation variables pose different challenges from a valuation and analytical perspective – as we will depict in section 3.3 – and primarily serve two different purposes within the context of an M&A or Entrepreneurial Financing process.

The *due date* essentially addresses valuation gaps when parties set the contingent payment to be due at *term*, as at the moment of deal completion they may diverge on expectations regarding future performance of the target firm within a specific time-frame, yielding a direct impact on settling an agreement regarding firm valuation. In this setting, parties may "*agree to disagree*" and wait for time to resolve their gaps on firm performance and firm valuation (Kohers and Ang, 2000). When set at *hit*, CPMs foremost privilege the provision of incentives to vendors or target management to pursue a certain goal. Alternatively, CPMs which are due *at hit* may also be suitable when the moment in which the attainment of given milestone or performance benchmark does not depend on the willingness or effort of the

[19] Notwithstanding, we argue that even when due at *hit*, CPMs require parties to define a given time period under which they allow the contingent payment to take place.

parties to pursue that goal (e.g., a license to operate a given plant, to which all the requirements were fulfilled by the target firm, and whose issuance is currently pending by public authorities).

The *amount* primarily addresses the perception of parties regarding the attainment of the contingent payment triggers and their willingness to benefit (or lose) from potential performance upsides or downsides. When they are set in a way in which they do not replicate *fixed* contingent payments[20], *variable* contingent payments may allow vendors to start profiting from the contingent payment when lower levels of performance are achieved by the firm (i.e., minimizing the down-side risk, if a CPM is put in place), and to exceed the payoff of a *fixed* contingent payment, when performance benchmarks are beaten by far (i.e., maximizing the upside potential, if a CPM is put in place).

Other potential variables for classifying CPMs are grounded on the underlying asset (for example, whether the CPM is based on a financial or on an operating measure or event, such as obtaining a pending license), on the underlying method of payment (for example, cash, shares of the acquiring firm or shares of the target firm), on the number of contingent payments to take place (for example, parties can set one single contingent payment, or a set of contingent payments to take place throughout several years), or on the different types of performance benchmarks – fixed, moving or cumulative – as proposed by Reum and Steele (1970).

We argue that within an Entrepreneurial Financing context, CPMs stand for a valuable framework to engage Entrepreneurs and VCs to support an investment opportunity held by a previously established Entrepreneurial Firm.

On the one hand, if parties accept to split firm ownership according to the face value of their equity contributions, Entrepreneurs may argue that VCs would be benefiting from an

[20] For example, when a contingent payment, due on a pre-determined moment of time, equals a given multiple on the revenues in excess of a given performance benchmark, there is no difference between variable and fixed contingent payments.

arbitrage gain equal to the difference between the fair value of the equity stake that the VC obtained in the company, and the amount of the actual equity contribution made by the VC.

On the other hand, the volatility of the underlying profit flow of the Entrepreneurial Firm and its growth opportunity advises VCs to be cautious on valuing Entrepreneurial Firms and accepting up-front share or cash premiums. CPMs may then allow these two parties to "agree to disagree" (Kohers and Ang, 2000) on firm valuation, by letting time resolve part of the uncertainty that the VC faces when investing on the Entrepreneurial Firm and letting *actual* (instead of *expected*) firm performance set the amount of the cash premium that the Entrepreneur should be entitled to.

3.3. Contingent Payment Mechanisms and Entrepreneurial Financing decisions

In this section, we extend the analytical framework to support decision-making and determine investment timing in EF processes introduced on the previous Chapter, with the purpose of establishing the grounds for investigating how CPMs may influence their outcomes. Therefore, we first briefly present the real options model which we will make use throughout the paper and show how CPMs might be valued and introduced on such framework. We then derive the optimum investment timing conditions that allow Entrepreneurs and VCs to jointly and simultaneously support the execution of a given growth strategy by an Entrepreneurial Firm.

3.3.1. A real options framework for Entrepreneurial Financing decisions with Contingent Payment Mechanisms

Building on the previous Chapter, the setting comprises one Entrepreneurial Firm, owned by a single Entrepreneur, which generates positive profits and holds a Growth Opportunity, defined by an expansion of its current profit flow (named as $e_{EXP} > 1$) and a given capital expenditure (named as $k > 0$). Assuming that neither the Entrepreneur nor the Entrepreneurial Firm have access to debt financing, such capital expenditure should be funded through an equity round backed by the Entrepreneur, who is assumed to own limited resources, and by an external financier, who is assumed to be a VC with no funding constraints. VCs are then assumed to provide the part of the required equity that the Entrepreneur is not able to provide.

In this setting, $k^i > 0$ stands for the amount of capital initially invested by the Entrepreneur on the Entrepreneurial Firm, $k^a < k$ stands for the amount of additional capital that the Entrepreneur is willing to deploy on the Entrepreneurial Firm, $k > 0$ is the amount of the total capital expenditure required for executing the growth strategy and $(k - k^a)$ is the amount of capital to be deployed by the VC on the Entrepreneurial Firm. Parties are assumed to split firm ownership after carrying the equity round according to the amount of capital that each of the parties contributed to the Entrepreneurial Firm. As a result, post-equity round firm ownership held by the Entrepreneur is denoted by $0 < Q^E < 1$ and $Q^E = \frac{k^i+k^a}{k^i+k}$, while post-equity round firm ownership held by the VC is denoted by $0 < Q^{VC} < 1$ and $Q^{VC} = \frac{k-k^a}{k^i+k} = 1 - Q^E$.

The Entrepreneurial Firm generates a continuous-time profit flow (π), which is assumed to follow a Geometric Brownian Motion (GBM) diffusion process given by:

$$d\pi = \alpha\pi dt + \sigma\pi dz \tag{3.1}$$

where $\pi > 0$, α and σ stand for the trend parameter (i.e., the drift) and to the instantaneous volatility, respectively. Additionally, assuming that agents are risk neutral, $\alpha = r - \delta$, where $r > 0$ is the risk-free rate and $\delta > 0$ stands for the asset yield. Finally, dz is the increment of a Wiener process. Entrepreneurs and VCs are assumed to understand that the continuous profit flow (π) follows the same stochastic process.

3.3.1.1. The option to invest on the Growth Opportunity held by the Entrepreneur

Following the contingent-claim approached used by Dixit and Pindyck (1994), the value of the option held by the Entrepreneur to invest in the growth opportunity of the

Entrepreneurial Firm, $E(\pi)$, must satisfy the following Ordinary Differential Equation (ODE):

$$\frac{1}{2}\sigma^2\,\pi^2\,E''(\pi) + (r-\delta)\pi\,E'(\pi) - r\,E(\pi) + \pi = 0 \tag{3.2}$$

where the last term on the left hand side of equation (3.1) refers to the current profit flow of the Entrepreneurial Firm and the remaining terms refer to the growth option held by the Entrepreneurial Firm. The general solution for (3.1) comes:

$$E(\pi) = A\pi^{\beta_1} + B\pi^{\beta_2} + \frac{\pi}{\delta} \tag{3.3}$$

where A and B are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic, given by:

$$Q_E(\beta) = \frac{1}{2}\sigma^2\,\beta\,(\beta-1) + (r-\delta)\beta - r = 0 \tag{3.4}$$

i.e.

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{3.5}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{3.6}$$

Assuming that π_E^* stands for the optimal profit trigger to obtain Entrepreneur's support to the growth opportunity, and considering that, in order to execute the growth strategy, $Q^E < 100\%$, and naming $CPM(\pi)$ as the contingent payment mechanism set between parties, the problem must be solved by considering the following boundary conditions:

$$E(0) = 0 \tag{3.7}$$

$$E(\pi_E^*) = \frac{e_{EXP}.\pi_E^*}{\delta}.Q^E - k^a + CPM(\pi_E^*) \tag{3.8}$$

$$E'(\pi_E^*) = \frac{e_{EXP}}{\delta}.Q^E + CPM'(\pi_E^*) \tag{3.9}$$

Respecting condition (3.7) and noting that $\beta_2 < 0$, then B on the equation (3.3) must be equal to zero. Therefore, for the remaining of this paper, $\beta \equiv \beta_1$. The unknowns A and π_E^* are obtained by combining conditions (3.8) and (3.9), i.e., the value matching and the smooth pasting conditions, respectively. Notice that the value matching condition held by the Entrepreneur is positively influenced by the CPM that was set between the parties. Solutions for the optimal profit trigger and for the option to invest on the growth opportunity depend on the specification defined for $CPM(\pi)$, and shall be presented on the following sections for each of four major different types of CPMs we previously introduced.

The economic interpretation of condition (3.8) is straightforward. When the Entrepreneur invests in such growth opportunity, she or he is entitled to a Q^E fraction of the value of the Entrepreneurial Firm after exercising the option to expand (i.e, $\frac{e_{EXP}.\pi_E^*}{\delta}$), and to a future contingent payment given by $CPM(\pi)$, at the expense of a cash outflow of k^a. Condition (3.13) will resemble this same interpretation by taking the perspective of the VC, which shall not only bear a cash outflow of $(k - k^a)$ but will also be liable on a future contingent payment given by $CPM(\pi)$.

3.3.1.2. The option to invest on the Growth Opportunity held by the VC

The value of the option to invest on the growth opportunity held by the VC, given by $VC(\pi)$, should also satisfy an ODE, as shown in equation (3.10) below. However, unlike the Entrepreneur, this option does not include the current profit flow π of the Entrepreneurial Firm, as VCs can only profit by undertaking the Growth Opportunity, and not from existing firm profitability, when they decide not to participate in this growth strategy.

$$\frac{1}{2}\sigma^2\,\pi^2\,VC''(\pi) + (r-\delta)\pi\,VC'(\pi) - r\,VC(\pi) = 0 \tag{3.10}$$

The general solution for (3.10) is:

$$VC(\pi) = C\pi^{\beta_1} + D\pi^{\beta_2} \tag{3.11}$$

where C and D are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic, as presented in equations (3.5) and (3.6). Similarly to the Entrepreneur, the boundary conditions are as follows:

$$VC(0) = 0 \tag{3.12}$$

$$VC(\pi_v^*) = \frac{e_{EXP}.\pi_{VC}^*}{\delta}.Q^{VC} - (k - k^a) - CPM(\pi_{VC}^*) \tag{3.13}$$

$$VC'^{(\pi_{VC}^*)} = \frac{e_{EXP}}{\delta}.Q^{VC} - CPM'(\pi_{VC}^*) \tag{3.14}$$

where π_{VC}^* stands for the optimal profit trigger to support the growth strategy for the VC firm. Respecting condition (3.12) and noting that $\beta_2 < 0$, then D on equation (3.11) must be equal to zero and, as before, $\beta \equiv \beta_1$. Differently to the Entrepreneur case, notice that the value matching condition stated on equation (3.13) will be negatively affected by the CPM.

The unknowns C and π_{VC}^* are obtained by combining conditions (3.13) and (3.14), i.e., the value matching and the smooth pasting conditions, respectively. Solutions for the optimal profit trigger and for the option to invest on the growth opportunity depend on the specification defined for $CPM(\pi)$.

3.3.2. Aligning Entrepreneurs, VCs and Growth Opportunities through Contingent Payment Mechanisms

We are interested in determining the conditions under which Entrepreneurs and VCs would be willing to jointly support the growth opportunity held by the Entrepreneurial Firm, i.e., the conditions under which $\pi_E^* = \pi_{VC}^*$. From a deal structuring perspective, parties can reach such an agreement either by pre-determining all deal terms – including firm ownership and any eventual up-front share consideration or premium, as in the previous Chapter – or by choosing to let part of definite deal terms be contingent on future performance benchmarks of the Entrepreneurial Firm.

The relevance of this issue within an EF context is highlighted by the fact that post-equity round firm ownership is assumed to be split according to the equity contributions made by the Entrepreneur and the VC, even though the value of the assets in place held by the Entrepreneurial Firm might be greater than the face value of his equity contributions prior to executing the growth strategy (i.e., $\frac{\pi}{\delta}$ might be greater than or, more generally, might be different from k^i).

On the other hand, and differently from a typical M&A context, Entrepreneurial Financing decisions allow the Entrepreneur to retain a portion of the ownership of the Entrepreneurial Firm and, therefore, significantly profit from the value creation effects generated by the growth opportunity. In fact, without an outside investor that would allow Entrepreneurs to obtain the indispensable resources to execute the envisaged growth strategy, the value of their option to invest in the Growth Opportunity would be equal to zero.

These two forces shall drive how CPMs are set, and we are specifically interested in understanding how CPMs can be designed in such a way that $\pi_E^* = \pi_{VC}^*$. With this purpose, we now analytically define each of the four major CPMs we previously introduced.

Broadly, and following the option analogy, we regard CPMs as binary call options and not as common call options on stock (Bruner and Stiegler, 2014), since their payoffs are actually discontinuous, i.e., either a fixed amount, or a variable amount linearly dependent on the value of its underlying asset.

Therefore, we analytically define *fixed* amount CPMs as *cash-or-nothing call binary options*, as the Entrepreneur is entitled to obtain a fixed amount of cash if the Entrepreneurial Firm achieves or exceeds a given performance benchmark. Concerning *variable* amount CPMs, we introduce two relevant assumptions. First, taking into account that $e_{EXP} > 1$

stands for the expected expansion of the profit flow generated by the Entrepreneurial Firm following the exercise of the option to invest in the growth opportunity, we assume that performance benchmarks (which we will name as $e_{BEN} > e_{EXP} > 1$) are grounded on the profitability of the Entrepreneurial Firm – and not on an operating measure or other financial measure different from profits – given by π. Second, we assume that the Entrepreneur will be entitled, in this case, to a multiple ($m > 0$) on the excess profit that the Entrepreneurial Firm generates over a given benchmark. Considering these two assumptions, we define *variable* amount CPMs as *asset-or-nothing call binary options*, as the underlying asset of this binary option is the profitability of the Entrepreneurial Firm itself.

As a result, when computing the conditions under which $\pi_E^* = \pi_{VC}^*$, *fixed* amount CPMs require determining the amount of cash (named as θ) that might be due to Entrepreneurs while *variable* amount CPMs require determining the multiple (named as m) on the excess profitability that will determine the amount of the contingent payment.

Concerning *due date*, and taking into account the considerations for valuing *fixed* and *variable* amount CPMs we presented, when CPMs are due *at term*, contingent payments might be modelled as traditional *binary options* (Hull, 2012), while when CPMs are due *at hit*, contingent payments should be modelled as *binary barrier options* (Rubinstein and Reiner, 1991, Rubinstein, 1992). In the case of CPMs due *at term*, we will assume this term is exogenously determined by parties and given by $t > 0$. In the case of CPMs due *at hit*, we will assume that parties exogenously set a time period under which the Entrepreneur might be entitled to the CPM (given by $tmax$), i.e., a time period under which the parties agree that if the performance benchmark is set, the Entrepreneur is entitled to the contingent payment.

Throughout the Chapter, subscripts will be used to indicate the type of CPM to which a given function or variable refers to, by first indicating the acronym for the CPM amount, using an F for *fixed* amount CPMs and a V for *variable* amount CPMs, and then by indicating the acronym for the CPM *due date*, using a T for CPMs due *at term* and a H for CPMs due *at hit*. Subscripts are not used when such function or variable is not affected by the type of CPM it may refer to.

In Table 6, we summarize the key CPM specifications that will be used for valuing each of the four major types of CPMs we introduced, and for analysing the conditions under which $\pi_E^* = \pi_{VC}^*$.

		Due Date		
		At Hit (H)	At Term (T)	*Alignment Variable*
Amount	Variable (V)	CPM_{VH} **Asset-or-Nothing Binary Barrier Option**	CPM_{VT} **Asset-or-Nothing Binary Option**	***m*** i.e., the multiple on the excess profit over a given benchmark
	Fixed (F)	CPM_{FH} **Cash-or-Nothing Binary Barrier Option**	CPM_{FT} **Cash-or-Nothing Binary Option**	***θ*** i.e., the amount of the contingent payment

Table 6. Alternative specifications for Contingent Payment Mechanisms

Four relevant considerations should be highlighted at this point. First, based on the seminal work by Black and Scholes (1973), each of the four specifications is consistent with the underlying stochastic process that governs the profit flow of the Entrepreneurial Firm, i.e., a Geometric Brownian Motion. Second, with our approach we intend to highlight the role that uncertainty may hold on determining how parties value CPMs and not on how CPMs may influence the behavior and effort of Entrepreneurs and VCs towards the accomplishment of the performance benchmarks, as in Lukas et al. (2012). Third, and unlike the previous Chapter, we do not intend to point out how asymmetric expectations on profit growth may govern the agreement between Entrepreneurs and VCs with the purpose of supporting the growth opportunity held by the Entrepreneurial Firm. Notwithstanding, the closed-form solutions which we will derive on the following sub-sections should also exist when parties hold different expectations on profit growth. Finally, within the real options framework previously presented for analyzing EF decisions, it is assumed that, when obtaining support to the growth strategy, the Entrepreneurial Firm immediately spends the total capital expenditure requirements (named as k) and immediately records an increase in its profitability to $\pi^*. e_{EXP}$. As a result, model outcomes should be carefully interpreted when the underlying growth strategy is expected to be put in place throughout a long period or when its payoffs shall only become visible on a far future.

On the following sections we will present, for each of the four major CPMs we introduced, how each contingent payment instrument is valued, the option to invest on the growth opportunity for Entrepreneurs and VCs, their underlying profit triggers and the optimum contingent payment θ or optimum contingent payment multiple m that would allow Entrepreneurs and VCs to jointly support the Growth Opportunity held by the Entrepreneurial Firm.

3.3.2.1. Fixed Contingent Payment at Term

The value of this CPM is taken as a *cash-or-nothing call* (Hull, 2012) as follows:

$$CPM_{FT} = \theta_{FT}\, e^{-rt} N(d2_{FT}) \tag{3.15}$$

where

$$d2_{FT} = \frac{log\left(\frac{e_{EXP}.\pi}{e_{BEN_{FT}}\pi^*}\right) + t(r - \delta - \frac{\sigma^2}{2})}{\sigma\sqrt{t}} \tag{3.16}$$

$N(z)$ stands for the cumulative normal density function, π^* stands for the profit trigger, π stands for the current profit, $e_{BEN_{FT}}$ stands for the profit growth expansion benchmark for triggering the contingent payment and θ_{FT} stands for the amount of the contingent payment.

Note that in the moment in which parties exercise their option to invest in the growth opportunity (i.e., when $\pi_E = \pi_E^*$ and $\pi_{VC} = \pi_{VC}^*$), the current profitability of the Entrepreneurial Firm is $e_{EXP}.\pi^*$, and the profitability benchmark is given by $e_{BEN_{FT}}\pi^*$. Therefore, when computing π_E^* and π_{VC}^*, $log\left(\frac{e_{EXP}\,.\,\pi}{e_{BEN_{FT}\,.\,\pi^*}}\right) = log\left(\frac{e_{EXP}\,.\,\pi^*}{e_{BEN_{FT}\,.\,\pi^*}}\right)$ in equation

(3.16), implying that $N(d2_{FT})$ does not depend on π and that $\frac{\partial N(d2_{FT})}{\partial \pi} = 0$. This will allow closed-form solutions to be derived for each of the value functions of the option to invest in the growth opportunity, for each of the profit triggers and for the optimum CPM that will allow parties to jointly support the Growth Opportunity[21].

As a result, by combining equations (3.8) and (3.15) and accordingly with (3.9) at this stage, we obtain the value matching and smooth pasting conditions that allow us to derive the option to invest on the growth opportunity held by the Entrepreneur in the presence of a fixed amount CPM due at term, as well as the profit trigger held by the Entrepreneur to invest in the Growth Opportunity given by $\pi^*_{E_{FT}}$, i.e.

$$E_{FT}(\pi) = \begin{cases} \left((e_{EXP}Q^E - 1)\frac{\pi^*_{E_{FT}}}{\delta} + CPM_{FT} - k^a\right)\left(\frac{\pi}{\pi^*_{E_{FT}}}\right)^\beta + \frac{\pi}{\delta}, for\ \pi < \pi^*_{E_{FT}} \\ e_{EXP}Q^E\frac{\pi}{\delta} + CPM_{FT} - k^a, for\ \pi \geq \pi^*_{E_{FT}} \end{cases} \quad (3.17)$$

where

$$\pi^*_{E_{FT}} = \frac{e^{-rt}\left(k + k^i\right)\beta\delta\left(\theta_{FT}\, N(d2_{FT}) - e^{-rt}k^a\right)}{(k + k^i - e_{EXP}(k^a + k^i))(\beta - 1)} \quad (3.18)$$

Similarly, by combining equations (3.13) and (3.15) alongside with (3.14), we obtain the value matching and smooth pasting conditions that allow us to derive the option to invest on the Growth Opportunity held by the VC in the presence of a *fixed* amount CPM due *at term*, as well as the profit trigger held by the VC to invest in the Growth Opportunity given by $\pi^*_{VC_{FT}}$, i.e.

$$VC_{FT}(\pi) = \begin{cases} \left(e_{EXP}Q^{VC}\frac{\pi^*_{VC_{FT}}}{\delta} - CPM_{FT} - (k - k^a)\right)\left(\frac{\pi}{\pi^*_{VC_{FT}}}\right)^\beta, for\ \pi < \pi^*_{VC_{FT}} \\ e_{EXP}\frac{\pi}{\delta}Q^{VC} - CPM_{FT} - (k - k^a), for\ \pi \geq \pi^*_{VC_{FT}} \end{cases} \quad (3.19)$$

[21] Note that if the profitability benchmark is exogenously determined and not dependent on π^* – meaning that $e_{BEN_{FT}}\pi^*$ would be replaced by a constant in equation (3.16) – $\partial N(d2_{FT})/\partial \pi$ would be have to be numerically determined. As a result, profit triggers both for Entrepreneurs and VCs alongside optimum CPM design would be obtained by numerical procedures. The same reasoning applies to the remaining CPMs presented on the following sections.

where

$$\pi^*_{VC_{FT}} = \frac{e^{-rt}\left(k + k^i\right) \beta\delta \left(\theta_{FT} N(d2_{FT}) + e^{rt}(k - k^a)\right)}{e_{EXP}(k - k^a)(\beta - 1)} \tag{3.20}$$

By equating $\pi^*_{E_{FT}} = \pi^*_{VC_{FT}}$ – i.e., equations (3.18) and (3.20) – and solving for θ_{FT}, we obtain the optimum *fixed* contingent payment *at term* that would enable both Entrepreneurs and VCs to jointly support the Growth Opportunity.

$$\theta^*_{FT} = \frac{e^{rt}(k - k^a)\,(k - k^i(e_{EXP} - 1))}{(e_{EXP} - 1)\,(k + k^i)\,N(d2_{FT})} \tag{3.21}$$

3.3.2.2. Variable Contingent Payment at Term

For valuing this CPM, we will assume that the amount of the contingent premium equals a given multiple m over all the profit in excess of the existing profitability prior to executing the growth strategy. This is the reason why on equation (3.22), we not only introduce the variable m to account for the contingent payment multiple but also the term $(e_{EXP} - 1)$ to account for the fact that the contingent payment will be computed on the excess profitability prior to the growth strategy.

This CPM is then taken as an *asset-or-nothing call* (Hull, 2012), as that the Entrepreneur will be entitled to a fraction or a multiple of the underlying asset, i.e., the profit generated by the Entrepreneurial Firm.

$$CPM_{VT} = m_{VT}\,\pi\,(e_{EXP} - 1)e^{-\delta t}N(d1_{VT}) \tag{3.22}$$

where

$$d1_{VT} = \frac{Log\left(\frac{e_{EXP}\,.\pi}{e_{BEN_{VT}}.\pi^*}\right) + t(r - \delta + \frac{\sigma^2}{2})}{\sigma\sqrt{t}} \tag{3.23}$$

Following the same approach of the previous section, whereby we consider that at the moment in which parties decide to invest in the growth opportunity $log\left(\frac{e_{EXP}\,.\,\pi}{e_{BEN_{VT}}.\,\pi^*}\right) =$

$log\left(\frac{e_{EXP}}{e_{BEN_{VT}}}\right)$, we may obtain the value of the option to invest in the Growth Opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (3.8), (3.9) and (3.22) for the Entrepreneur, and by combining equations (3.13)(3.14) and (3.22) for the VC, as follows:

$$E_{VT}(\pi) = \begin{cases} \left((e_{EXP}Q^E - 1)\frac{\pi^*_{E_{VT}}}{\delta} + CPM_{VT} - k^a\right)\left(\frac{\pi}{\pi^*_{E_{VT}}}\right)^\beta + \frac{\pi}{\delta}, for\ \pi < \pi^*_{E_{VT}} \\ e_{EXP}Q^E\frac{\pi}{\delta} + CPM_{VT} - k^a, for\ \pi \geq \pi^*_{E_{VT}} \end{cases} \quad (3.24)$$

where

$$\pi^*_{E_{VT}} = \frac{e^{t\delta}k^a(k+k^i)\,\beta\delta}{\left(-e^{t\delta}\left(k+k^i - e_{EXP}(k^a+k^i)\right) + (e_{EXP}-1)(k+k^i)\,m_{VT}\,\delta\,N(d1_{VT})\right)(\beta-1)} \quad (3.25)$$

and

$$VC_{VT}(\pi) = \begin{cases} \left(e_{EXP}\frac{\pi^*_{VC_{VT}}}{\delta}Q^{VC} - CPM_{VT} - (k-k^a)\right)\left(\frac{\pi}{\pi^*_{VC_{VT}}}\right)^\beta, for\ \pi < \pi^*_{VC_{VT}} \\ e_{EXP}\frac{\pi}{\delta}Q^{VC} - CPM_{VT} - (k-k^a), for\ \pi \geq \pi^*_{VC_{VT}} \end{cases} \quad (3.26)$$

where

$$\pi^*_{VC_{VT}} = \frac{e^{t\delta}(k-k^a)(k+k^i)\,\beta\delta}{\left(e^{t\delta}e_{EXP}(k-k^a) - (e_{EXP}-1)(k+k^i)\,m_{VT}\,\delta\,N(d1_{VT})\right)(\beta-1)} \quad (3.27)$$

Both $\pi^*_{E_{VT}}$and $\pi^*_{VC_{VT}}$ present asymptotes dependent on m_{VT}, which are obtained by finding the roots on the denominator of equations (3.25) and (3.27). For such values of m_{VT}, there is no possible agreement between Entrepreneurs and VCs to support a given Growth Opportunity.

For the Entrepreneur, the asymptote on $\pi^*_{E_{VT}}$ is given by:

$$m_{VT} = \frac{2\,e^{t\delta}\left(k+k^i - e_{EXP}(k^a+k^i)\right)}{(e_{EXP}-1)(k+k^i)\,\delta\,N(d1_{VT}))} \quad (3.28)$$

For the VC, the asymptote on $\pi^*_{VC_{VT}}$ is given by:

$$m_{VT} = \frac{e^{t\delta} e_{EXP}\, (k - k^a)}{(e_{EXP} - 1)(k + k^i)\, \delta\, N(d1_{VT}))} \qquad (3.29)$$

By equating $\pi^*_{E_{VT}} = \pi^*_{VC_{VT}}$ – i.e., equations (3.25) and (3.27) – and solving for m_{VT}, we obtain the optimum multiple on the CPM due at term that would enable both Entrepreneurs and VCs to jointly support the Growth Opportunity.

$$m^*_{VT} = \frac{e^{t\delta}\, (k - k^a)\left[k - k^i(e_{EXP} - 1)\right]}{(e_{EXP} - 1)\, k\, (k + k^i)\, \delta\, N(d1_{VT})} \qquad (3.30)$$

3.3.2.3. Fixed Contingent Payment at Hit

Following the approach by Rubinstein and Reiner (1991), this CPM is derived as an *up-and-in cash-or-nothing binary barrier option*, assuming that the performance benchmark that will trigger the contingent payment is greater than or equal to its current level, as follows:

$$CPM_{FH} = \theta_{FH}\left(\left(\frac{e_{BEN_{FH}}.\, \pi^*}{e_{EXP}\, .\, \pi}\right)^{a+b} N(-z) + \left(\frac{e_{BEN_{FH}}.\, \pi^*}{e_{EXP}\, .\, \pi}\right)^{a-b} N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right) \qquad (3.31)$$

where

$$a = \frac{r - \delta}{\sigma^2} \qquad (3.32)$$

$$b = \sqrt{\frac{(r - \delta)^2 + 2\log(1 + r)\, \sigma^2}{\sigma^2}} \qquad (3.33)$$

$$z = \frac{log(\frac{e_{BEN_{FH}}.\pi^*}{e_{EXP}\, .\pi})}{\sigma\sqrt{tmax}} + b\, \sigma \sqrt{tmax} \qquad (3.34)$$

Similarly to the previous sub-sections, as $\frac{e_{BEN_{FH}}\, .\, \pi^*}{e_{EXP}\, .\, \pi} = \frac{e_{BEN_{FH}}\, .\, \pi^*}{e_{EXP}\, .\, \pi^*}$ and $\log\left(\frac{e_{BEN_{FH}}\, .\, \pi^*}{e_{EXP}\, .\, \pi}\right) = \log\left(\frac{e_{BEN_{FH}}}{e_{EXP}}\right)$, we obtain the value of the option to invest in the growth opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (3.8), (3.9) and (3.31) for the Entrepreneur, and by combining equations (3.13), (3.14) and (3.31) for the VC, as follows:

$$E_{FH}(\pi) = \begin{cases} \left((e_{EXP} Q^E - 1) \dfrac{\pi^*_{E\,FH}}{\delta} + CPM_{FH} - k^a \right) \left(\dfrac{\pi}{\pi^*_{E\,FH}} \right)^{\beta} + \dfrac{\pi}{\delta}, for\ \pi < \pi^*_{E\,FH} \\ e_{EXP} Q^E \dfrac{\pi}{\delta} + CPM_{FH} - k^a, for\ \pi \geq \pi^*_{E\,FH} \end{cases} \tag{3.35}$$

where

$$\pi^*_{E\,FH} = \frac{\left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{-b} (k + k^i)\, \beta\delta \left[\theta_{FH} \left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{a} \left[N(-z) \left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{2b} + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right] - k^a \left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{b}\right]}{\left(k + k^i - e_{EXP}(k^a + k^i)\right)(\beta - 1)} \tag{3.36}$$

and

$$VC_{FH}(\pi) = \begin{cases} \left(e_{EXP} \dfrac{\pi^*_{VC\,FH}}{\delta} Q^{VC} - CPM_{FH} - (k - k^a) \right) \left(\dfrac{\pi}{\pi^*_{VC\,FH}} \right)^{\beta}, for\ \pi < \pi^*_{VC\,FH} \\ e_{EXP} \dfrac{\pi}{\delta} Q^{VC} - CPM_{FH} - (k - k^a), for\ \pi \geq \pi^*_{VC\,FH} \end{cases}, \tag{3.37}$$

where

$$\pi^*_{VC\,FH} = \frac{\left(k + k^i\right) \beta\delta \left[1 + \dfrac{\theta_{FH} \left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{a-b} \left(\left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right)}{k - k^a}\right]}{e_{EXP}(\beta - 1)} \tag{3.38}$$

By equating $\pi^*_{E\,FH} = \pi^*_{VC\,FH}$ – i.e., equations (3.36) and (3.38) – and solving for θ_{FH}, we obtain the optimum *fixed* contingent payment *at hit* that would enable both Entrepreneurs and VCs to jointly support the Growth Opportunity.

$$\theta^*_{FH} = \frac{\left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{b-a} (k - k^a) \left[k - k^i(e_{EXP} - 1)\right]}{(e_{EXP} - 1)(k + k^i) \left[\left(\frac{e_{BEN_{FT}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right]} \tag{3.39}$$

3.3.2.4. Variable Contingent Payment at Hit

Taking this CPM as an *up-and-in asset-or-nothing binary barrier option*, we follow Rubinstein (1992) to analytically derive the value of this contingent asset, considering that the profit benchmark is equal or greater than its current value, as follows:

$$CPM_{VH} = m_{VT}\, \pi\, (e_{EXP} - 1)\left(\left(\frac{e_{BEN_{VH}}.\pi^*}{e_{EXP}.\pi}\right)^{a+b} N(-z) + \left(\frac{e_{BEN_{VH}}.\pi^*}{e_{EXP}.\pi}\right)^{a-b} N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right) \quad (3.40)$$

where a, b and z are defined in equations (3.32), (3.33) and (3.34), respectively.

Considering that $\left(\frac{e_{BEN_{VH}}.\pi^*}{e_{EXP}.\pi}\right) = \left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)$ as before, we obtain the value of the option to invest in the growth opportunity to the Entrepreneur and to the VC, alongside each of their optimum investment profit triggers, by combining equations (3.8), (3.9) and (3.40) for the Entrepreneur, and by combining equations (3.13), (3.14) and (3.40) for the VC, as follows:

$$E_{VH}(\pi) = \begin{cases} \left((e_{EXP}Q^E - 1)\dfrac{\pi^*_{E_{VH}}}{\delta} + CPM_{VH} - k^a\right)\left(\dfrac{\pi}{\pi^*_{E_{VH}}}\right)^{\beta} + \dfrac{\pi}{\delta}, for\ \pi < \pi^*_{E_{VH}} \\ e_{EXP}Q^E\dfrac{\pi}{\delta} + CPM_{VH} - k^a, for\ \pi \geq \pi^*_{E_{VH}} \end{cases}, \quad (3.41)$$

where

$$\pi^*_{E_{VH}} = \frac{\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^b k^a\,(k + k^i)\,\beta\delta}{\left(m_{VH}\,\delta\,(e_{EXP} - 1)(k + k^i)\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^a\left[\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right] - \left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^b (k + k^i - e_{EXP}(k^a + k^i))\right)(\beta - 1)} \quad (3.42)$$

and

$$VC_{VH}(\pi) = \begin{cases} \left(e_{EXP}\dfrac{\pi^*_{VC_{VH}}}{\delta}Q^{VC} - CPM_{VH} - (k - k^a)\right)\left(\dfrac{\pi}{\pi^*_{VC_{VH}}}\right)^{\beta}, for\ \pi < \pi^*_{VC_{VH}} \\ e_{EXP}\dfrac{\pi}{\delta}Q^{VC} - CPM_{VH} - (k - k^a), for\ \pi \geq \pi^*_{VC_{VH}} \end{cases}, \quad (3.43)$$

where

$$\pi^*_{VC_{VH}} = \frac{\beta}{\left[\dfrac{e_{EXP}}{(k + k^i)\delta} - \dfrac{m_{VH}\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{a-b}(e_{EXP} - 1)(\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)}{k - k^a}\right](\beta - 1)} \quad (3.44)$$

As in the *variable* amount due *at term* CPM, there are both asymptotes on $\pi^*_{E_{VH}}$ and $\pi^*_{VC_{VH}}$, which are, respectively, given by:

$$m_{VH} = \frac{\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{b-a}\left[k + k^i - e_{EXP}\left(k^a + k^i\right)\right]}{\delta\,(e_{EXP} - 1)(k + k^i)\left(\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\, b\, \sigma \sqrt{tmax}\right)\right)} \quad (3.45)$$

$$m_{VH} = \frac{\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{b-a} e_{EXP}(k - k^a)}{\delta\,(e_{EXP} - 1)(k + k^i)\left(\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\,b\,\sigma\,\sqrt{tmax}\right)\right)} \quad (3.46)$$

By equating $\pi^*_{E_{VH}} = \pi^*_{VC_{VH}}$ – i.e., equations (3.42) and (3.44) – and solving for m_{VH}, we obtain the optimum multiple on the CPM due *at hit* that would enable both Entrepreneurs and VCs to jointly support the Growth Opportunity.

$$m^*_{VH} = \frac{\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{b-a} (k - k^a)\left[k - k^i(e_{EXP} - 1)\right]}{\delta\,(e_{EXP} - 1)\,k\,(k + k^i)\left(\left(\frac{e_{BEN_{VH}}}{e_{EXP}}\right)^{2b} N(-z) + N\left(-z + 2\,b\,\sigma\,\sqrt{tmax}\right)\right)} \quad (3.47)$$

3.4. Numerical example

We now illustrate the economic intuition behind the results introduced on the previous section through a numerical example. We start by listing the numerical assumptions we use in Table 7, and by summarizing the model outputs regarding investment timing and the design of CPMs. We conclude this section by presenting a set of sensitivities on some of the key value drivers.

Risk Parameters		**Capital and Growth Opportunity**		**Contingent Payment Mechanism**	
Variable	Numerical Assumption	Variable	Numerical Assumption	Variable	Numerical Assumption
r	0.04	k^i	150.00	$e_{BEN_{VT}}$	2.50
σ	0.30	k^a	275.00	$e_{BEN_{FT}}, e_{BEN_{FH}}, e_{BEN_{VH}}$	2.75
δ	0.08	k	500.00	t	2.00 years
		e_{EXP}	2.50	$tmax$	5.00 years

Table 7. Numerical assumptions

By combining the risk parameters r, δ and σ, we obtain $\beta = 2.28$ through equation (3.5), and by combining k^i, k^a and k, we obtain $Q^E = 65.4\%$ and $Q^{VC} = 34.6\%$, following the approach introduced in section 3.3.1. We have also set the profit benchmark for the *variable* amount *at term* CPMs ($e_{BEN_{VT}}$) equal to the expected profit growth (e_{EXP}), assuming that the amount of this contingent payment would be grounded on all excess profit above the expected growth.

3.4.1. Contingent Payment Mechanisms and optimum investment timing

Results on the optimum CPM design – comprising both optimum *amount* for fixed contingent payments and optimum *multiple* for variable contingent payments – alongside their underlying profit triggers (π^*) are presented in Table 8.

Our numerical example shows that CPMs which are due *at hit* should present lower multiples or fixed amounts then those due *at term*, when parties understand that profit benchmarks should be attainable in the short-term or, equivalently, when they understand that the probability of profits staying below the benchmark (or reverting to levels below the benchmark) *at term* is significant, considering the underlying uncertainty on the profit flow of the Entrepreneurial Firm.

		Key Assumptions			Optimum CPM Design		
CPM	Supporting Equations	e_{EXP}	e_{CUR}	e_{BEN}	θ^*	m^*	$\boldsymbol{\pi}^*$
CPM_{FT}	(3.18), (3.20), (3.21)	2.50	2.50	2.75	258.57	-	47.52
CPM_{VT}	(3.25), (3.27), (3.28)	2.50	2.50	2.50	-	3.65x	47.52
CPM_{FH}	(3.36), (3.38), (3.39)	2.50	2.50	2.75	74.81	-	47.52
CPM_{VH}	(3.42), (3.44), (3.45)	2.50	2.50	2.75	-	1.87x	47.52

Table 8. Illustration of optimal Contingent Payment Mechanism design

In fact, results reveal that θ^*_{FT} is more than triple than θ^*_{FH}, while m^*_{VT} is slightly more than double than m^*_{VH}, even when the latter comprises a higher profit benchmark (2.75 against 2.50). The practical implication of this result is that, CPMs which are due *at term* should lead to higher contingent payments than CPMs which are due *at hit*, when their underlying performance benchmarks are likely to be achieved before the term[22].

3.4.2. Contingent Payment Mechanisms and the value of the investment opportunity to the Entrepreneur

In the previous section, we showed that, when CPMs are optimally set, optimum investment timing is the same for each of the four alternative CPMs. However, as different CPMs are differently valued, their underlying value of the investment opportunity should differ. This argument also holds from Table 7, since the same optimum *fixed* contingent payments present different amounts when due *at term* or *at hit*, and same applies to optimum variable contingent payments, whose multiples differ when payments are due *at term* or *at hit*.

From the perspective of the Entrepreneur, we posit that CPMs which are due *at hit* should be more valuable than those which are due *at term*, controlling for the likelihood of the profit benchmark to be achieved (i.e., the closer $\frac{e_{EXP}}{e_{BEN}}$ is to 1) and that *variable* contingent payments should be more valuable than their *fixed* counterparties, as they may generate a positive payoff for lower levels of profitability and may generate a higher payoff than fixed amount contingent payments when profitability exceeds its underlying threshold. From the perspective of the VC, the converse argument should hold.

[22] Moreover, we find that optimum investment timing is independent of CPM selection, as $\pi* = 47.52$ for each of the different alternative CPMs. In fact, this would also stand for the profit trigger of a central planner who would look into the option to invest in this growth opportunity by aggregating the option to invest in this growth opportunity, which is held by the Entrepreneur and the VC. As CPMs stand for an asset on the option to invest in the growth opportunity held by the Entrepreneur and for an equal liability on the option to invest in the growth opportunity held by the VC, these offset each other from the aggregate perspective of a central planner, whose optimum investment triggers would then remain unchanged to those computed by Entrepreneurs and VCs.

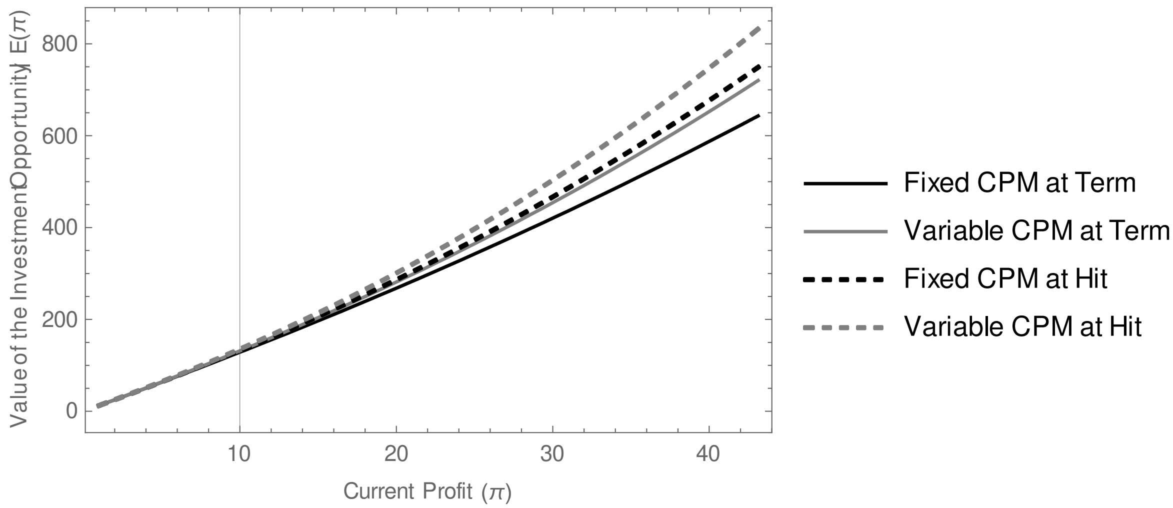

Figure 7. Value of the investment opportunity held by the Entrepreneur for each type of Contingent Payment Mechanism

In Figure 7, and further assuming that $m = 4.0$ and that $\theta = 150.00$, we illustrate this intuition on the value of the investment opportunity held by the Entrepreneur for the range $\pi < \pi^*$. Dashed lines stand for CPMs which are due *at hit* while normal lines stand for CPMs which are due *at term*.

3.4.3. Profit growth expectations and profit triggers

More aggressive profit growth expectations are expected to decrease profit triggers, making the investment opportunity more attractive both to the Entrepreneur and to the VC. We may observe such relationship in Figure 8, as we plot the profit trigger to invest on the growth opportunity (π^*) against the expected profit growth (e_{EXP}), when parties engage into optimum CPM design. For simplicity, we plot a single curve for all the different CPMs, since all of their profit triggers are the same.

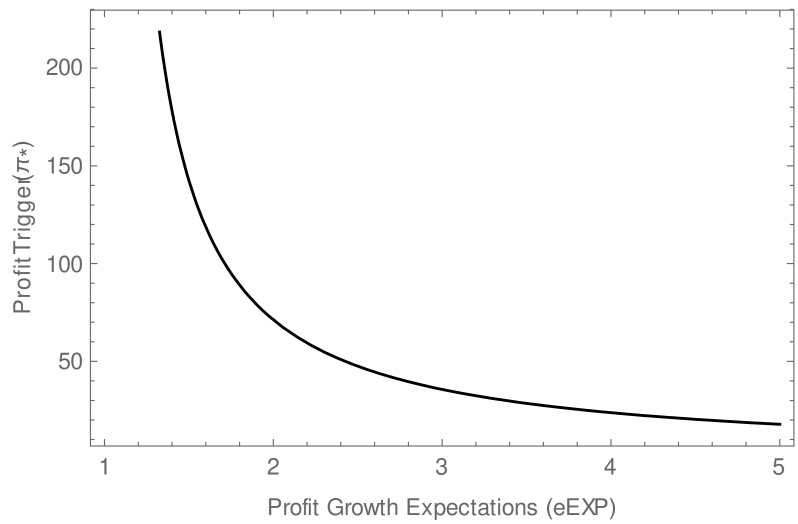

Figure 8. Profit growth expectations and profit triggers when Contingent Payment Mechanisms are optimally designed

3.4.4. Volatility, value of the investment opportunity to the Entrepreneur and profit triggers

Uncertainty of future business performance is taken as one of the main reasons behind the use of CPMs, as mentioned in section 3.1. We will now illustrate, within the framework we introduced, how uncertainty influences investment timing and the value of option to invest in the growth opportunity for each of the CPMs we derived. For this purpose, we will keep the assumptions that $m = 4.0$, that $\theta = 150.00$ and further assume that the current profit of the Entrepreneurial Firm is $\pi_0 = 30.00$. In Figure 9, we show how profit triggers are affected by uncertainty from the perspective of the Entrepreneur, plotting a single curve for all the different CPMs, as before. In Figure 10, we plot volatility against the value of the investment opportunity held by the Entrepreneur, for each of the four major CPMs we derived.

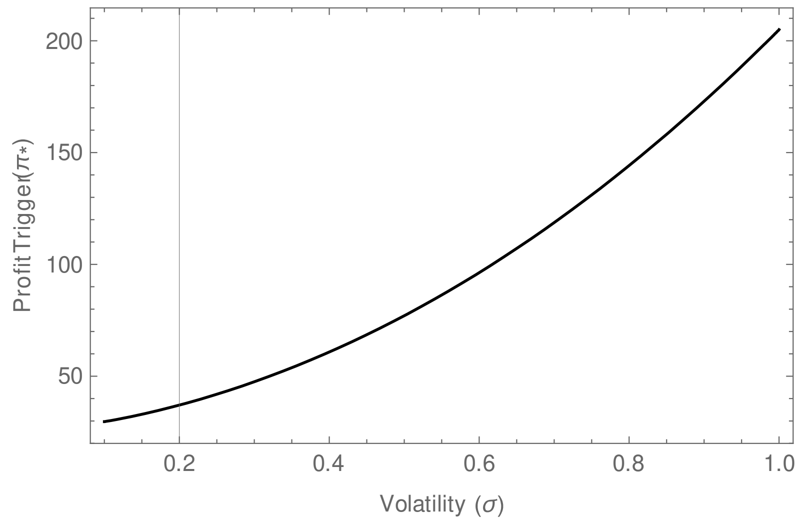

Figure 9. Volatility and profit triggers when Contingent Payment Mechanisms are optimally designed

As expected, Figure 9 reveals that additional volatility increases the profit triggers required by Entrepreneurs and VCs to support the investment opportunity, while, from the Entrepreneur's perspective, Figure 10 reveals that, in the presence of CPMs, growing volatility generates additional value to the investment opportunity. For lower levels of volatility, results similar to those in section 3.4.2 hold, implying that CPMs which are due *at hit* are more valuable to those which are due *at term*, and *variable* amount CPMs are more valuable than *fixed* amount CPMs.

However, for higher levels of volatility, we observe that the value of the investment opportunity with a *fixed* amount CPM which is due *at hit* actually converges to the value *fixed* amount CPM which is due *at term*, since for very high levels of uncertainty, the probability of the profit threshold to be hit *before* the term, converges to the probability of the profit threshold to be hit *at* the term.

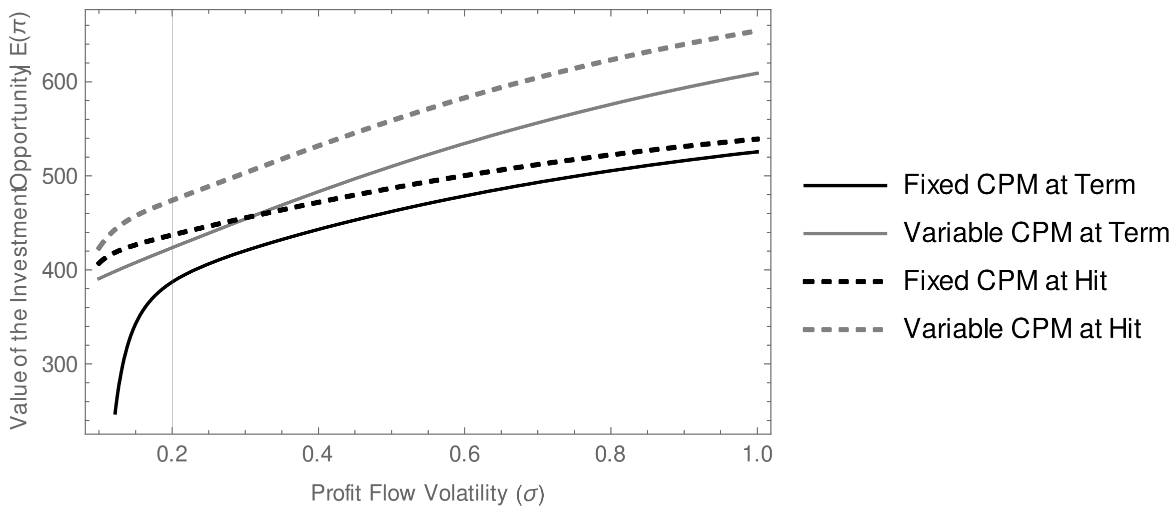

Figure 10. Profit flow volatility and value of the investment opportunity held by the Entrepreneur

3.4.5. Profit benchmarks and value of the investment opportunity to the Entrepreneur

As profit benchmarks affect the likelihood of a contingent payment to become firm, a negative relationship between both is expected to be found. Keeping the assumptions that $m = 4.0$, $\theta = 150.00$ and $\pi_0 = 30.00$ we illustrate this in Figure 11, where we plot the value of the option held by the Entrepreneur to invest in the Growth Opportunity against a range of profit benchmarks above the expected profit growth (i.e., $e_{BEN} > e_{EXP}$), and in Figure 12, where we plot the relationship between a range of profit benchmarks and the optimum multiple for a variable CPM due *at term*. A similar relationship on optimum multiple behaviour would be visible for a variable CPM due *at hit*.

Figure 11 shows that profit benchmarks negatively influence the value of the investment opportunity to the Entrepreneur, while, conversely, Figure 12 reveals that profit benchmarks positively affect the optimum contingent payment multiple on a *variable* amount CPM due *at term*.

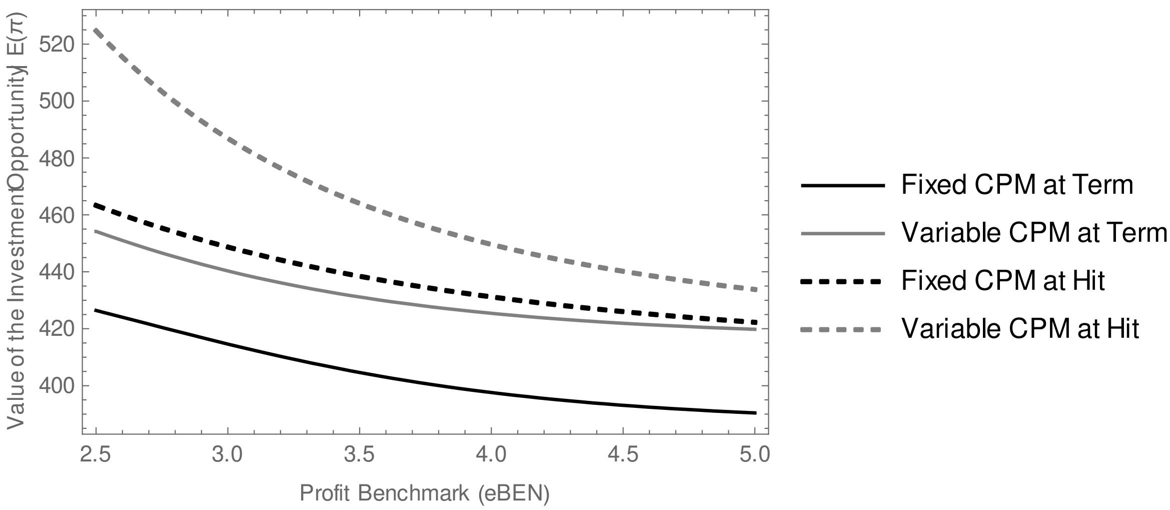

Figure 11. Profit benchmarks and value of the investment opportunity held by the Entrepreneur

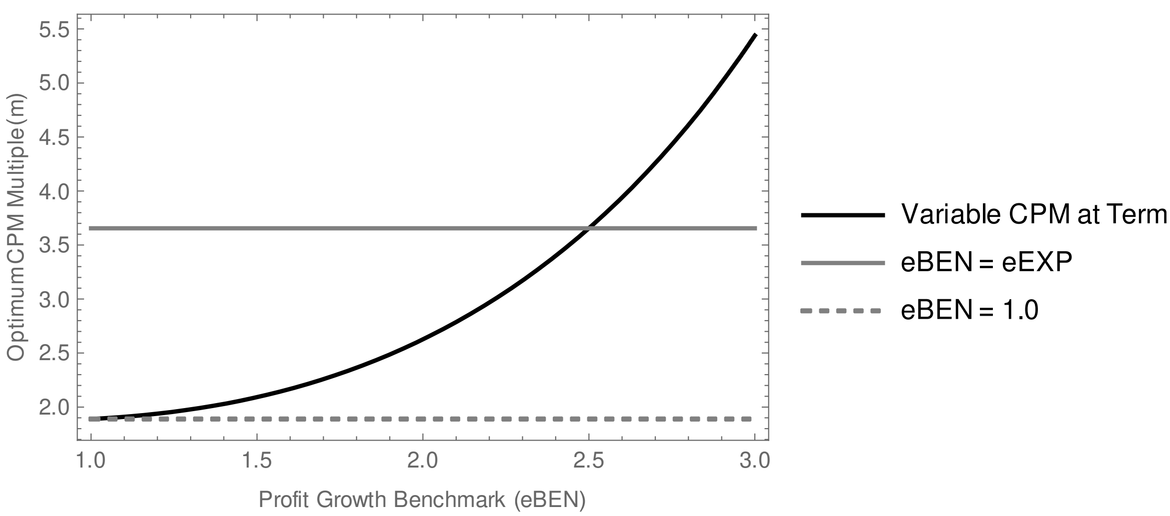

Figure 12. Profit growth benchmarks and optimum multiples in variable amount Contingent Payment Mechanisms due at term

3.4.6. Due dates and optimum contingent payments

By analysing *fixed* amount CPMs, we now intend to illustrate how due dates may affect contingent payment design and, particularly, the amount of the contingent payment that should be set by Entrepreneurs and VCs so that the Entrepreneurial Firm obtains their joint support to proceed with the envisaged growth strategy. Similar results would hold to *variable* amount CPMs, in which we would observe analogous outputs on optimum contingent payment multiples.

Assuming that $tmax = t$, Figure 13 reveals that the contingent payment period holds a different effect for CPMs which are due *at hit* and CPMs which are due *at term*. While for CPMs due *at term*, longer contingent payment periods lead to higher optimum amounts, CPMs which are due *at hit* present lower optimum amounts.

The intuition behind this result lays on the fact that longer payment periods increase the probability of the profit benchmark to be achieved at any moment within the payment period, therefore making more valuable CPMs which are due *at hit*. When CPMs are due *at term*, longer payment periods actually stand for a longer deferred payment whose present value is inferior. In addition, in CPMs due *at term*, the underlying performance measurement is made at a specific moment of time and, therefore, in this sense less probable than for CPMs which are due *at hit*.

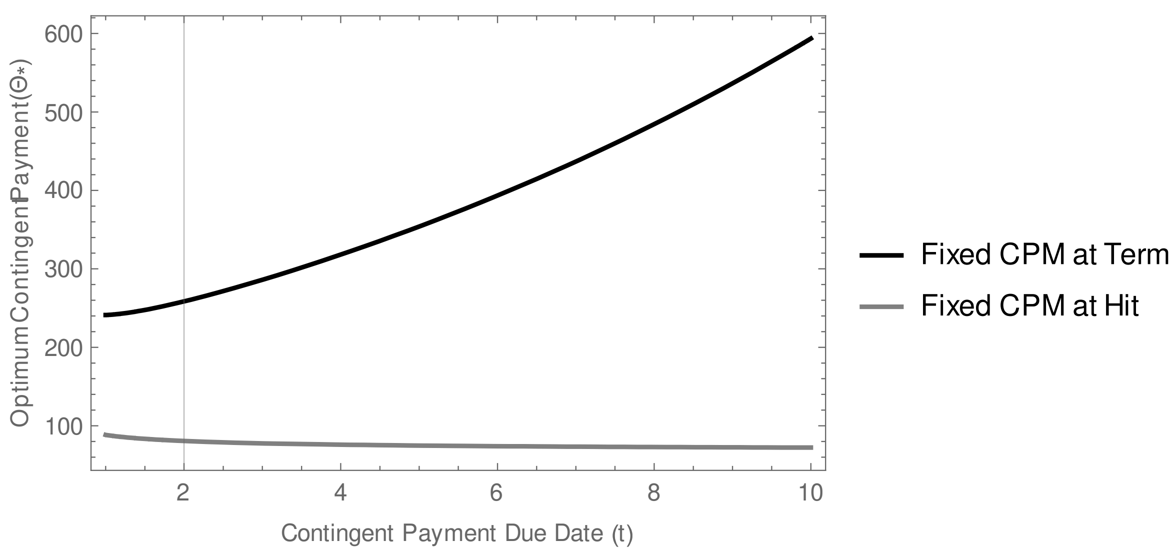

Figure 13. Due dates and optimum θ (assuming $tmax = t$)

3.5. Discussion of the results

Model outputs illustrated on the previous section are generally consistent with those in prior literature. The impact of uncertainty on CPM design and optimum investment timing shown in section 3.4.4 and the impact of the contingent payment period on the amount of the optimum contingent payment illustrated in section 3.4.6 is broadly similar to the hypothesis derived by Lukas et al. (2012). As an increase in uncertainty leads to an increased value of the option to invest in the growth opportunity held by the Entrepreneur, results in section 3.4.6 are also consistent with Mantecon (2009), Ragozzino and Reuer (2009) and Lukas and Heimann (2014), in the sense that these authors predict that an increase in uncertainty increase the attractiveness of CPMs.

The impact of the profit benchmark on CPM design we introduced in section 3.4.5 differs from Lukas et al. (2012), as these authors developed a framework for M&A decisions which involve an initial payment to the vendor of the target firm, instead of an equity issuance. Therefore, Lukas et al. (2012) argue that a trade-off might exist between such initial payment and the amount of a fixed contingent payment, when profit benchmarks increase. We conjecture that Lukas et al. (2012) would obtain a similar result to ours in a setting without initial payment.

Our results show that, when optimally designed, different CPMs are equivalent when it comes to determining optimum investment timing, as profit triggers revealed to be the same for each of the four alternative CPMs we investigated[23]. As a result, the choice of which CPM should be set between Entrepreneurs and VCs should actually be driven by variables which are exogenous to the framework we designed, such as:

- *Liquidity constraints* on the VC side could limit the amount of funds available for deploying on a given investment opportunity or condition the timing within which

[23] By following the approach from the previous Chapter, it could also be shown that an optimum up-front share premium would also have the same profit triggers than those presented for the different CPMs.

such funds are available (Faccio and Masulis, 2005). Such variable could lead a preference for a mechanism whose expected amount is lower or for a mechanism that would increase the chance of its underlying liability to be financed through the Entrepreneurial Firm itself;

- *Time constraints* on the VC side, given that the underlying VC cycle may condition the amount and the timing of the contingent liabilities that it may be able to accept at the moment in which the investment opportunity is being screened. In particular, a VC would not be allowed to bear a contingent liability which might be due after the fund term;

- *Liquidity preferences* on the Entrepreneur side, which may favor a deal structuring mechanism in which she or he would be entitled to an *up-front cash in*, instead of a *contingent* payment or even an *up-front share premium*;

- *Risk preferences* on the Entrepreneur side may drive the choice between an up-front cash in, and a fixed or a variable contingent payment. Risk-averse Entrepreneurs should prefer up-front payment mechanisms to contingent payments, settle lower benchmarks for triggering contingent payments and may reveal a preference for *fixed* amount CPMs, that protect them against down-side performance;

- *Credit risk* may play a role in analyzing a potential CPM. Entrepreneurs may regard CPMs as deferred payment mechanisms (such as vendor loans) and may therefore subject the acceptance of this contingent asset to a proper assessment of the credit risk of the VC and of the CPM that may minimize such risk;

- *Post-deal performance measurement and integration* may restraint the settlement of CPMs, as deal terms may reduce the perception of decision-control held by the Entrepreneur – meaning that potential performance might be influenced by decisions taken by the VC and affect the probability of a contingent payment to take place – and lead to the establishment of discretionary expenses, profit decisions or a new corporate organization that may affect the ability of the parties to properly measure the future performance of the Entrepreneurial Firm (Bruner and Stiegler, 2014);

- *Overall deal terms* require both Entrepreneurs and VCs to agree on a wide set financial and non-financial terms (including compensation, performance bonuses, value-adding roles by VCs, and corporate governance), which generate a set of negotiation trade-offs and lead to different choices of CPMs.

3.6. Chapter summary

In this Chapter we discussed how Contingent Payment Mechanisms (also known as Contingent Earn-Outs) enable of EF decisions. First, we presented a taxonomy of contingent payment mechanisms, by combining features regarding their term and amount. Second, we introduced each of these alternative mechanisms on the real options framework for analysing Entrepreneurial Financing decisions derived on the previous Chapter.

We concluded that, when optimally designed, different contingent payment mechanisms are equivalent in obtaining joint support from Entrepreneurs and VCs regarding optimum investment timing and, therefore, that the choice on the optimum mechanism to use depends on variables which are exogenous to the model, such as liquidity preferences or constraints, timing requirements, post-deal integration or overall deal terms.

On the following Chapter, and leaving aside the debate on how Entrepreneurs and VCs may be supportive of EF decisions, we will take a public policy perspective on the Start-up Firm (SuF) financing segment, with the purpose of discussing which might be the most effective mechanism that Governments may put in place in order to promote investment volumes on this specific firm segment.

4. Should Public Venture Capitalists invest, co-invest, or not invest in Start-up Firms?

Public Venture Capital (PVC) initiatives are popular amongst Governments worldwide. In Europe, and according to the European Venture Capital Association (EVCA[24]), PVC investment stood for 1.1% of total investment amounts in Private Equity (PE) and Venture Capital (VC) and 8.0% of total investment volumes between 2007 and 2014, with a record high being reached in 2014, both in terms of investment amounts (2.6%) and investment volumes (10.3%)[25]. In the United States, the Government created the first PVC programs in the 1970s (Leicht and Jenkins, 1998, Jenkins and Leicht, 1996) and launched the Small Business Innovation Research (SBIR) program in the 1980s, which provided over 7 billion dollars to small high-technology firms between 1983 and 1997 (Lerner, 1999), and has been extended by the Congress since 2000 and until 2017. Argentina (Butler et al., 2015), Australia (Lerner and Watson, 2008), Canada (Brander et al., 2008, Ayayi, 2004), Chile (Murray, 2007, Avnimelech and Teubal, 2008), New Zealand (Murray, 2007), Israel (Avnimelech et al., 2010, Avnimelech and Teubal, 2008), Portugal (Jeng and Wells, 2000), Spain (del-Palacio et al., 2012) and Taiwan (Chen et al., 2012) also established similar PVC initiatives, most of which targeting Entrepreneurial Firms on the early stage/ high-tech segment.

Why should Governments sponsor such PVC initiatives aimed at funding young and innovative Entrepreneurial Firms? On the one hand, investment opportunities held by such

[24] EVCA was recently renamed as "Invest Europe".

[25] Figures from "*EVCA Yearbook – European Private Equity Activity in 2014*". Sample includes Belgium, Bulgaria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Romania, Spain, Sweden, Switzerland, Ukraine, and the United Kingdom. It is worth mentioning that Bulgaria, Czech Republic, Greece, Italy, Luxembourg, Poland, Romania and Ukraine do not record any PVC volumes or amounts during the 2007-2014 period.

Entrepreneurial Firms are deemed to hold *positive externalities*, in the sense that returns from their R&D expenditures may actually exceed their private returns (Lerner, 1999, Lerner, 2002). Knowledge generated within such context is hardly appropriable and intellectual property protection is limited in Entrepreneurial Firms. As a result, private returns lag behind social returns leading to under-investing on this firm segment (Peneder, 2008)[26]. On the other hand, investing in Entrepreneurial Firms requires significant *information asymmetry risks* to be mitigated by Independent Venture Capitalists[27] (IVCs[28]). By investing in such Entrepreneurial Firms, PVCs provide a signaling or certification effect (Lerner, 2002, Guerini and Quas, 2016) to private investors which might be relevant for IVCs to assess their willingness to participate in future equity rounds. Additionally, the establishment of PVC initiatives might be particularly relevant on the nascent stages of the VC industry, by allowing investment professionals to be trained and then moved to private VC organizations (Lerner, 1999)[29]. This not only contributes to a lower informational gap on VC as an asset class itself, but also to promote the investment on the Entrepreneurial Firms segment, as such investment professionals become acquainted with this early-stage market segment both from a demand and supply side perspective. Finally, as a result of its vast economies of scale (Murray, 1998), *VC fund economics* feature strong incentives for General Partners (GPs) to abandon early-stage equity finance as their track-record allows them to sustain their presence on the market.

[26] Peneder (2008) lists a set of institutional mechanisms to overcome the under-investing phenomenon in Entrepreneurial Firms, including (i) the establishment of an effective system of intellectual property rights, (ii) the existence of public research centers that favor industrial applications over long run and (iii) the provision of public subsidies to innovation.

[27] Independent Venture Capitalists (IVCs) should be more accurately mentioned as Private Venture Capitalists, as this category comprises the broader range of VCs, such as Captive Venture Capitalists, Corporate Venture Capitalists, Institutional Venture Capitalists and Informal Venture Capitalists (Sapienza & Villanueva, 2007). However, throughout this paper we are particularly interested in analyzing the behavior of non-captive (i.e., independent) VC organizations and, for the sake of convenience, we will adopt acronym IVC with the purpose of more easily distinguish from PVC.

[28] We will use the acronym IVC throughout the Chapter interchangeably either to refer to "Independent Venture Capital" or "Independent Venture Capitalists", depending on context.

[29] Please refer to the Small Business Investment Company (SBIC) example which we refer on this section.

Independent GPs have the incentive to focus on significantly increasing the amount of funds under management – with the purpose of maximizing their management fees – and on minimizing monitoring and portfolio risk (Murray, 2007) – in order to increase the probability of providing consistent returns on investment. The investment record of early-stage funds is poor (EVCA, 2014) – with 10-year internal rate of returns standing for 1.32% on the VC segment against 9.63% on the buyout segment as of 31st December 2013[30]. Moreover, small, early-stage funds have a series of structural weaknesses that hinder their potential risk-adjusted profitability. These funds face higher due diligence and monitoring costs, need to provide high levels of management support and guidance to investees, have a more limited ability to fully benefit from diversification, bear higher ownership dilution risks, and present a skewed risk/ return profile which requires exceptional successes to be generated within the portfolio so that an attractive risk-adjusted return is provided to their Limited Partners (LPs). In addition, they are unable to attract the largest and most professional LPs[31] and hold limited ability in recruiting experienced professional investment executives (Murray, 2007, Lawton, 2002, Hood, 2000).

The combination of *positive externalities*, *information asymmetry risks* and *VC fund economics* lead to a *market failure* on the Entrepreneurial Financing segment (Murray, 2007, van der Schans, 2015, Giacomo, 2004), which motivates public authorities to intervene on the VC market. Such intervention should target the provision of equity – in order to close this

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[30] This result holds a wide range of possible interpretations, which should be put into perspective. First, the focus of PVC initiatives on the early-stage segments and their presumable inability to generate adequate risk-adjusted returns might be driving down returns on the VC segment as a whole. Still, further data on returns on PVC funds and their investment strategy would be required to assess such assertion. Second, the persistence of low returns on the VC segment might contradict the *market failure* hypothesis, in the sense that low returns stand for a sign that IVCs are actually allocating excessive amounts of cash to this firm segment. IVCs might be accepting excessive share premiums on the equity rounds they support, or might be unable to find Entrepreneurs willing to accept the appropriate share premiums for the risk that IVCs accept to bear when investing in this firm segment.

[31] Large institutional LPs typically seek for bigger-ticket fund allocations, so that these may have an influential impact on their overall fund performance. As early-stage fund sizes tend to be small when compared to other PE and VC segments, they become less attractive to large institutional LPs.

equity gap[32, 33] – when efficient IVCs avoid the Entrepreneurial Firm segment due to an unattractively priced investment proposal[34], or should target the improvement of entrepreneurial quality and context (Lerner, 2010), when IVCs avoid this segment due to an insufficient supply of investment opportunities[35] (Murray, 2007, Mason and Brown, 2013, Mason and Harrison, 2001).

Criticism on PVC initiatives highlight that such instruments may primarily serve interest groups or politicians seeking for private benefits. As a result, Entrepreneurial Firms may seek transfer payments that increase profits, by conducting transfers to politically connected companies, while Governments may seek to select firms based on their likely success, regardless of their actual need for capital. In this case, Governments launch PVC initiatives so that they can claim credit on the success of the investees, even when their marginal contribution to value creation is low (Lerner, 2002, Lerner, 1999).

According to Harrison and Mason (2000), Giacomo (2004) and Murray (2007), Governments may intervene on the VC market either through a *direct intervention*[36] – where

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[32] Murray (1995) distinguishes two different types of equity gaps: the *initial equity gap*, which describes the financing problems faced by Start-up Firms looking for seed capital and *the second equity gap*, which happens at a stage when initial seed or start-up capital is exhausted and no additional equity providers are prepared to provide follow-on financing.

[33] Interestingly, research carried by Da Rin et al. (2006) on a set of 14 European countries found no evidence of a shortage of supply of VC funds in Europe.

[34] This seems to be the case of Italy, where Colombo et al. (2007) carried an empirical research on a net of 550 NTBFs, whose results "only partially confirm the relevance of founders' competencies as important drivers of VC investment decisions", suggesting the presence of other inefficiencies in the VC market.

[35] Murray (2007) illustrates the inexistence of attractive investment opportunities by citing a set of statistics relating to the proportion of small business owners seeking for external financing, and the proportion of Small and Medium Enterprises (SMEs) that point easy access to finance as their primary concern.

[36] One of the most original forms of public intervention on the Entrepreneurial Financing markets is pictured by Ayayi (2004), who assessed the performance of the Canadian LSVCFs (Labor-Sponsored Venture Capital Funds). These operated similarly to open-ended mutual funds, were capitalized by individual investors and had a regional focus. Significant tax benefits were awarded to investors with the purpose of committing their capital for eight years to inherently risky SMEs. LSVCFs were managed by labor unions or federations and had the

a PVC organization is established and the Government simultaneously assumes the role of a GP and of a LP – or through an *indirect intervention*, where IVCs act on behalf of the Government. In this case, Governments essentially act as LPs and may establish *equity enhancement schemes*. Within these, Governments may subsidize the operating costs of VC funds, co-invest with private LPs matching their requirements, be willing to accept a capped return, concede a buyout option for private LPs, underwrite part of all of the risk of financial loss borne by GPs and LPs and enhance the internal rates of return earned by private LPs by accepting a reordering of cash-flows, so that public LPs are the first to provide capital and the last to have their capital reimbursed[37].

Given that Governments may let their decision-making process be influenced by the dynamics of political conditions, *indirect* or *hybrid* interventions on the Entrepreneurial Financing markets are seen as a best practice (Gilson, 2003, Lerner, 2010). In addition, PVC organizations might be hindered by their inability of hiring and retaining appropriate investment professionals or their incapacity of learning and benefiting from their own investment experience. As a result, PVCs become less capable of providing valuable advice[38]

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purpose to fulfill a set of social goals, including job creation and regional economic development. Results reveal that even though LSVCFs were able to attract significant amounts of capital due to its tax benefits, their performance was significantly poor when compared to a wide range of benchmarks (from sectorial mutual funds to stock indexes) due to poor management and fund regulations. Still, Ayayi (2004) found evidence that LSVCFs did not use their funds to aggressively price their deals and that were able to deploy their capital into local communities.

[37] Governments may also engage on a *fund of funds* initiative, which may specifically target Entrepreneurial Firms or other firms of policy interest. Examples include the *Fonds de Promotion pour le Capital Risque – FPCR* established in France with 150 million euros under management in 2001 and the *UK High Technology Fund*, established in 2000 by the British Government with £ 126 million under management, of which £ 20 M were provided by Public Authorities.

[38] Value creation effects from IVCs on investee led academia to discuss whether IVCs provide a *treatment effect* – i.e., whether their monitoring and advice activities actually contribute to improve their performance (Hellmann and Puri, 2002) – or benefit from a *selection effect*, in which experienced IVCs evidence better performance since they are able to select the best deals (Bertoni, 2011; Croce et al., 2013). Still, Cumming and Fischer (2012) analyzed the contribution of a non-profit business advisory center in Canada, having found that, in this case, public funded business advisory services to Entrepreneurial Firms are positively associated with firms' sales growth, patents, finance and alliances.

to their portfolio firms and are therefore less prone to generate benchmark returns. Moreover, and depending on their size and on how the industry matures, PVC organizations might negatively influence the institutional framework of the local VC industry – especially when it is still nascent – by crowding out private investors (Munari and Toschi, 2015) and offering finance below an appropriate risk premium (Jääskeläinen et al., 2007). Hence, taking into account the previously described features of *VC fund economics* and the risk/return profile of early stage investing, only by providing more attractive profit expectations to private LPs may Governments actually contribute to narrow the *equity gap* (Gilson, 2003, Hirsch, 2006, Jääskeläinen et al., 2007). By co-investing *pari passu* with other private LPs, Governments merely help VC funds reaching their minimum efficient scale (Murray, 2007)[39].

Several studies attempt to empirically assess the impact and the performance of the wide range of PVC initiatives worldwide. Beuselinck and Manigart (2007) analyzed the relevance of PVC investment on a set of ten European countries for the 1989 to 2003 period, having concluded that early stage and high-tech VC investments are higher in countries where PVC initiatives are more important, that PVC programs are less dependent on economic cycles than IVC investments – providing a contribution to stabilize a traditionally cyclical industry – and that PVC investment is negatively correlated with the number of listings on the local stock market[40]. Similar results were obtained by Jeng and Wells (2000). Leleux and Surlemont (2003) found that PVCs do not favor labor-intensive industries, nor even that

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[39] Lerner (2010) points out other policy principles that should govern the functioning of PVC initiatives, including (i) the establishment of efficient technology transfer offices, (ii) the acceptance of the generalized accepted standards regarding partnerships and preferred stock structures, (iii) the establishment of shorter lead times on the public-side regarding shared decision-making processes with IVCs, and (iv) the establishment of evaluation initiatives of public programs.

[40] Michelacci and Suarez (2004) and Keuschnigg and Nielsen (2007) show that the presence of liquid stock markets allow VCs to exit from their portfolio companies more easily and allow VC financing to become more attractive to Entrepreneurs, since VC control ownership is expected to last for a more limited period of time. By enabling a faster turnover of portfolio firms, liquid portfolio markets allow a larger rate of firms to be backed by IVCs. Carpentier et al. (2010) analyze the Canadian experience on the TSX Venture Exchange – a public equity market for newly created companies with no history or sales are allowed to list – with the ultimate goal of "graduating" its best performers to the main exchange, the Toronto Stock Exchange (TSX). These authors found that the returns of this "public VC market" are able to compete with the private ones.

PVCs co-invest their investments less than IVCs. Alongside del-Palacio et al. (2012) whose study focused on the Spanish market, as well as Jeng and Wells (2000) and Brander et al. (2014) whose samples compare developed countries worldwide, Leleux and Surlemont (2003) found no support to the *crowding-out* hypothesis of PVC initiatives, having in turn obtained empirical evidence to support the opposite assertion. The authors showed that PVCs actually lead to a spillover on the flow of private investors into the VC markets and found a positive association between PVC volumes and later-stage deals, in general[41]. Differently, Munari and Toschi (2015) empirically investigated the performance of PVC funds according to three performance measures – exit rate and IVC attraction rate, through staging and syndication – having found support for the *crowding-out* hypothesis and having showed that PVCs exhibit lower performance than their private peers[42,43]. Similar results on performance and on the *crowding-out* hypothesis were obtained by Cumming and MacIntosh (2006), when studying the impact of the LSVCFs established in Canada.

[41] Lerner (1999) also provides positive guidance on the impact of PVC initiatives performed by the SBIR program in the United States, as well as Cumming (2007), who observed that the Innovation Investment Fund (IIF) set up by Australia facilitated investment in start-up, early stage and high tech firms, alongside the provision of monitoring and value-added advice to investees. On a set of seven European countries, Luukkonen et al. (2013) found no statistically significant difference between the performance of firms held by PVCs and IVCs. However, the contributions of IVCs funds proved to be significantly higher than those of PVCs regarding the development of the business idea, professionalization and exit orientation.

[42] Munari and Toschi (2015) argue that the reason behind the poor performance of the PVC initiatives is grounded on the existence of unclear, multiple and conflicting goals for PVCs – from local development, to employment, entrepreneurial culture or industry restructuring – which incentivize self-serving behaviors and low PVCs accountability for selecting and monitoring high-quality firms (Fisher, 1998; Hood, 2000). In addition, PVCs are not able to attract and retain talented investment professionals – when compared to IVCs – and are therefore unable to positively influence the performance of their portfolio companies. This restraints the ability of PVCs to attract private LPs as investors or private GPs as co-investors, since these form limited profit expectations. Finally, PVCs frequently support regional investment initiatives, driving investment to less innovative regions and underachieving companies, subordinating financial objectives to policy goals (Mason and Pierrakis, 2013). In fact, such regional VC funds tend to establish contacts in their neighborhoods, whose value creation resources might be less valuable when compared to other regions, leading to unintended effects that may contradict the aims of closing regional disparities in risk finance and entrepreneurship (Harrison and Mason, 2000; Sunley et al., 2005).

[43] Munari and Toschi (2015) based their research on a sample of 628 VC-backed companies in the United Kingdom, from 1998 to 2007, in which they are able to distinguish PVCs between regional and governmental types.

Munari and Toschi (2015) also point out that PVC initiatives may actually serve the purpose of encouraging the involvement of qualified GPs and LPs in depressed regions (Sunley et al., 2005, Mason and Harrison, 2003, Murray, 1998). Such catalyst effect is assessed by Brander et al. (2014) and Grilli and Murtinu (2015). The former found that enterprises funded both by PVCs and IVCs obtain more investment than enterprises uniquely funded by PVCs or by IVCs. The latter studied the impact of PVC on sales of new-technology-based firms (NTBFs). Results revealed that PVC-backed NTBFs underperformed IVC-backed firms, but when PVCs co-invested alongside IVCs, the former are able to inflict a positive and statistically significant impact on sales. This is consistent with Grilli and Murtinu (2014), who showed that, on a dataset of firms based on the European Union, firms held by IVC investors generate a significant and positive performance on sales growth, firms held by PVCs record a negligible change on sales, and firms in which IVCs and PVCs co-invested – while letting IVCs retaining the lead – also had a positive and significant performance on sales. Co-investing exemplifies the establishment of close relationships with the IVC community with the purpose of improving the performance of PVC initiatives, which is advocated by Hood (2000) and Lerner (2002).

Overall, the literature provides multiple and dissimilar results on the success of PVC programs, leading to the idea that there are no "one-size-fits-all" impacts and that their performance is significantly mediated by contextual conditions, not only on the supply-side but also on the demand-side.

In spite of its growing prominence, several authors highlight that public intervention on the VC and Entrepreneurial Financing (EF) markets lacks theoretical guidance (Lerner, 1999, Avnimelech and Teubal, 2008). Therefore, Governments often undertake programs without a clear understanding on the full consequences of their actions and without synchronizing political and investment cycles (Gilson, 2003). Murray (2007) understands that a standardized evaluation method is "urgently required" for better understanding the impact of PVC initiatives, alongside a set of best-in-class programs from which performance benchmarks could be derived (Jeng and Wells, 2000).

Avnimelech and Teubal (2008) provided a theoretical approach to guide public intervention on the Entrepreneurial Financing markets. Grounded on the successful initiatives by Israel, Chile and Korea, the authors propose a conceptual three stage model, where Government support is crucial on transiting countries from the first stage – where there is a *direct* public support to research and development initiatives – to the third stage, where they should essentially provide *indirect* support to the existing ecosystem of IVCs and Entrepreneurial Firms. During the second stage, Governments should simultaneously carry a *direct*, *indirect* or *hybrid* intervention, as the *privatization* of finance is taken as a gradual process that should be adapted to the variety of countries' institutional, cultural and political contexts. Such evolutionary perspective was further described in Avnimelech et al. (2010) and Lerner and Tag (2013).

Still, in spite of such conceptual contributions, analytical grounded literature on the topic is scarce and, to the best of our knowledge, limited to Keuschnigg and Nielsen (2001), Keuschnigg and Nielsen (2007) and Wong (2014). However, these do not deal with the topic of direct intervention of public authorities on the VC and Entrepreneurial Financing markets, but rather with the instruments that public authorities may use in order to indirectly stimulate equity financing markets. Other relevant existing theoretical contributions are focused on understanding the behavior and performance of private GPs and LPs, but not from a public intervention perspective (Liu and Yang, 2015, Sorensen et al., 2014).

Keuschnigg and Nielsen (2001) designed a partial equilibrium model for analyzing how government spending on entrepreneurial training, subsidies for investing in equipment and output subsidies may influence VC activity. Model outputs reveal that a subsidy for investing in equipment induce GPs to reduce their managerial support to investee firms, while an output subsidy should be neutral regarding professional advice. However, only entrepreneurial training provides welfare gains, by decreasing costs to market, and favoring industry output alongside firm creation. Still, improving entrepreneurial training holds the downside effect of crowding out managerial support from GPs to Entrepreneurial Firms.

Keuschnigg and Nielsen (2007) set up a two-period model in industry equilibrium with the purpose of analyzing how taxes and subsidies may contribute to foster the VC market, in terms of quality and quantity of investees. Corporate income taxes reduce firm creation, even when the tax is paid by mature companies, as lowering the value of mature firms diminishes the gains from setting up new companies as well. Corporate income taxes will postpone and impair the reward to effort in Start-up Firms (SuFs). Since subsidies are independent from firm performance, they do not strengthen incentives towards improved entrepreneurship quality and even reduce social welfare. Capital gains tax has an ambiguous effect on firm creation, while personal income tax holds a positive effect[44]. From a policy making perspective, and alongside Murray (1998), the authors highlight that public policy should be designed with the purpose of generating higher quality entrepreneurship, rather than higher firm creation volumes. Such theoretical findings are close to those of Holtz-Eakin (2000), whose empirical findings showed that introducing preferential tax treatments for VCs would not be enough to eliminate the *equity gap*, but somehow contradict Gompers and Lerner (2004), who found evidence of a moderately negative effect on the capital gains tax on VC investing and fund raising.

In a setting where Entrepreneurs are assumed to be risk averse and VCs are assumed to be risk-neutral, Wong (2014) highlights how Government intervention on the EF markets through tax incentives and regulatory measures may contribute to ameliorate the asymmetric information problem between Entrepreneurs and VC. In this model, Governments are then able to promote investment volumes, by allowing risk sharing to be shifted between Entrepreneurs and VCs. In particular, when tax incentives and regulations are uncertain, and efforts by VCs are hidden, Entrepreneurs are shown to transfer part of their risk to VCs, which will bear most of the risk of the project. In turn, when tax incentives and regulations are available, and entrepreneurial effort is hidden, risk is shifted towards risk averse Entrepreneurs, even though optimal entrepreneurial effort is not achieved. Overall, Wong

[44] Some of these results are not consistent with the empirical evidence provided by Da Rin et al. (2006) on a dataset of 14 European countries. The opening of stock markets provided a positive effect on the development of active VC markets, while reductions in the corporate capital gains tax rate increased the share of both high-tech and early stage investment. Additionally, a reduction in labor regulation also led to a higher share of high-tech investments and increased public spending on R&D was found not to have significant influence on the early stage investment.

(2014) reveals that when inputs from one party become hidden, Government intervention may drive risks to move to that party, therefore alleviating information asymmetry risks.

We intend to provide a contribution to fill this literature gap, by taking a real options perspective on the decision-making process faced by IVCs, PVCs and Governments in general, when considering investing in or subsidizing a certain SuF. In particular, we are interested in assessing (i) how optimum investment timing may change between IVCs and PVCs, (ii) how a co-investment by IVCs and PVCs might be put in place and how such co-investment will affect optimum investment timing, and (iii) how a subsidy might be set up with the purpose of anticipating optimum investment timing by IVCs.

On the side of IVCs, we will properly account for the partnership structure that features typical VC investing. In this setting, GPs will be compensated by the value creation effect of the investment they will back (essentially, the net present value from the investment on the SuF) – net of expected taxation on capital gains– and by the performance compensation GPs will earn if return on investment exceeds a certain benchmark (i.e., the carried interest).

When scrutinizing the decision-making process of PVCs, we will take into account that taxation is neutral for their capital budgeting process (i.e., tax expenditures borne by SuFs stand for tax revenues to the Government), and, based on the literature findings, that investments undertaken by PVCs will generate a lower value creation effect than those carried by IVCs. In addition, assuming that optimum investment timing by IVCs is observed, we consider that PVCs take into account the opportunity cost they bear, as when investing on the SuF, PVCs forego the option of not undertaking the investment, waiting for the profitability of SuFs to reach the trigger required by IVCs, and providing Governments with the benefits from incremental tax revenues without carrying any investment, either through a PVC organization or through a subsidy to the Entrepreneurial Firm. Our model explicitly deals with such waiting option.

We will also investigate the co-investment case, where IVCs invest alongside PVCs. We show that – on the absence of portfolio risk mitigation procedures, financing constraints, perceived investment performance contributions or other drivers of co-investment decisions – IVCs would only be willing to co-invest with PVCs if they are able to benefit from an *equity enhancement* effect, which we will model as a share premium to be offered to IVCs by PVCs. Within this context, PVCs would be willing to co-invest and offer a share premium to IVCs with the purpose of benefiting from the incremental efficiency on investment outcomes and avoiding the opportunity cost borne when fully investing on SuFs by themselves.

Finally, we compare the co-investment case with the subsidy case, in which Governments, instead of acting as PVCs, provide a subsidy to SuFs with the purpose of supporting their investment, anticipating optimum investment timing from IVCs and benefiting from the incremental tax revenues brought by the exercise of such option to invest by IVCs.

Through a numerical example with representative assumptions on its underlying variables, model reveals that co-investment will actually be more effective in anticipating optimum investment timing than subsidies. This will allow us to derive a set of empirically testable propositions regarding the determinants of PVC, which we will investigate through a dataset of European countries.

This Chapter is structured as follows. In section 4.1, we setup the real options framework that will support our investigation and present its underlying economic intuition through a numerical example. Based on such insights, we outline the major public policy and managerial implications in section 4.3, while in section 4.4, we point out the theoretical hypothesis that may govern the relevance of PVC initiatives and render an empirical study on their prevalence. We summarize this Chapter in section 4.5.

4.1. Model

Our setting comprises one SuF which is expected to generate a flow of profits before payroll and taxes named as x, following a Geometric Brownian Motion (GBM) diffusion process given by:

$$dx = \alpha x dt + \sigma x dz \tag{4.1}$$

where $x > 0$, α stands for the trend parameter (i.e., the drift) and σ to the instantaneous volatility. Considering that agents shall be risk neutral within this setting, $\alpha = r - \delta$, where

$r > 0$ is the risk-free rate and $\delta > 0$ stands for the asset yield. Lastly, dz is the increment of a Wiener process.

Assuming that no debt is available to support such investment opportunity, and that the Entrepreneur will support an exogenously determined $Q^E \in (0; 1)$ fraction of the total capital expenditures, the SuF will be seeking for an equity investor – taken as a IVC and/ or a PVC – to obtain a capital commitment of $k > 0$. Entrepreneurs and investors are then assumed to share firm ownership according to the amount of capital that each party provided to the SuF[45].

The SuF will bear payroll costs amounting to $w.x$, where $w \in (0; 1)$ and corporate income tax given by $T_C \in (0; 1)$. Investors will be liable on capital gains tax – named as $T_G \in (0; 1)$ – at the moment in which they will divest from the SuF – given by $ED > 0$ – while wages are subject to personal income tax, named as $T_P \in (0; 1)$.

If the SuF is subsidized by the Government to support the investment, it will get a subsidy named as $sub \in (0; k)$. When $sub > 0$, Entrepreneurs need not to seek for IVCs or PVCs to provide k, but rather $k - sub$. As a result, Entrepreneurs retain a q^E ownership in the SuF, given by:

$$q^E = Q^E \frac{k}{k - sub(1 - Q^E)} \tag{4.2}$$

Notice that when $sub = 0$, then $q^E = Q^E$. In this case, external investors will deploy a total capital commitment amounting to k, in exchange for a $(1 - Q^E)$ ownership in the SuF. For the rest of the paper, and for generalization purposes, we will use q^E to refer to the ownership in the SuF held by the Entrepreneur, irrespective of $sub = 0$ or $sub > 0$.

In the co-investment case, IVCs and PVCs will, respectively, hold a fraction q^{IVC} and $(1 - q^{IVC})$ of the remaining firm ownership $(1 - q^E)$, while these two parties may agree on

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[45] Therefore, we implicitly assume that the Entrepreneur is able to offer the Start-up Firm an amount of capital equal to $Q^E \times \frac{k}{(1-Q^E)}$.

a share premium between them, named as p[46,47]. Therefore, when illustrating the results on the investment case performed by the IVC where no co-investment takes place, we will assume that $q^{IVC} = 1$, $p = 0$ and $sub = 0$, while when going through the investment case performed by the PVC without co-investment, we will assume that $q^{IVC} = 0$, $p = 0$ and $sub = 0$. In section 4.1.3, we analyze the co-investment case, in which q^{IVC} will let be exogenously determined and p will be computed such that IVCs and PVCs hold the same optimum investment trigger on x.

4.1.1. The option to invest in the Start-up Firm held by the Independent Venture Capitalist

Taking into account the typical structuring of venture partnerships, we assume that this investment opportunity shall be screened by an established IVC fund, whose GP is entitled to decide whether to invest or not. GPs usually hold a minor stake $i \in (0; 1)$ on the IVC fund and benefit from a performance-based compensation mechanism (i.e., the carried interest). Considering that the GP and its IVC fund are already established during investment screening, we assume that there are no incremental revenues or costs rising from management fees or increasing monitoring costs when GPs exercise the option to invest[48].

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[46] On the co-investment case, $sub = 0$ and therefore the Entrepreneur will retain a $Q^E = q^E$ ownership in the SuF.

[47] As a result, IVCs will retain a $(1 - q^E)\times(q^{IVC} + p)$ share on the Start-up Firm, while PVCs will retain a $(1 - q^E)\times(1 - q^{IVC} - p)$ share on the SuF. Bear in mind that, in the co-investment case, $sub = 0$ and therefore $q^E = Q^E$.

[48] Typically, management fee structure changes throughout fund life. During the investment period, management fees are charged over total committed capital, while during the divestment period, they are charged over the acquisition cost of portfolio firms. In this sense, it could be argued that deploying a greater amount of capital on the acquisition of portfolio firms would lead GPs to perceive an incremental revenue on the divestment period, which should be introduced on their decision-making model. However, if GPs are profit-maximizers, they have strong incentives to fully deploy the committed capital during the investment stage with the purpose of maximizing their management fees during the divestment period. As a result, regardless of the amount k, GPs will always be seeking for additional investment opportunities to exhaust the committed capital and benefit from management-fee maximization. This understanding allows us to consider that management fees may not be regarded as incremental revenues during the screening stage of one given investment opportunity. Still, and even though that is not the focus of this paper, we acknowledge that when comparing

Following the contingent-claim approach presented by Dixit and Pindyck (1994), the value of the option to invest in the SuF – $IVC(x)$ – must satisfy the following Ordinary Differential Equation (ODE):

$$\frac{1}{2}\,\sigma^2\,x^2\,IVC''(x) + (r-\delta)\,x\,IVC'(x) - r\,IVC(x) = 0 \tag{4.3}$$

The general solution for (4.3) is:

$$IVC(x) = A\,x^{\beta_1} + B\,x^{\beta_2} \tag{4.4}$$

where A and B are constants to be determined, while β_1 and β_2 are the roots of the fundamental quadratic, given by:

$$Q = \frac{1}{2}\sigma^2\beta(\beta-1) + (r-\delta)\beta - r = 0 \tag{4.5}$$

i.e.

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \tag{4.6}$$

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{r-\delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \tag{4.7}$$

Assuming that x^*_{IVC} stands for the optimal trigger for the IVC to exercise the option to invest in this investment opportunity, and naming $NPV_{IVC}(x)$ as its pre-capital gains tax net present value, $i.CGT_{IVC}(x^*_{IVC})$ as the value of the expected capital gains tax and $CI(x)$ as the value of the carried interest earned by the GP (i.e., its performance based compensation), the problem must be solved by considering the following boundary conditions:

$$IVC(0) = 0 \tag{4.8}$$

$$IVC(x^*_{IVC}) = NPV_{IVC}(x^*_{IVC}) - i.CGT_{IVC}(x^*_{IVC}) + CI(x^*_{IVC}) \tag{4.9}$$

two different investment opportunities, GPs hold an economic incentive to prefer the one requiring the greatest amount of capital *k*.

$$IVC'(V_{IVC}^*) = NPV_{IVC}'(x_{IVC}^*) - i.CGT_{IVC}'(x_{IVC}^*) + CI'(x_{IVC}^*) \quad (4.10)$$

Respecting condition (4.8) and noting that $\beta_2 < 0$, then B on equation (4.4) must be equal to zero. Therefore, $\beta \equiv \beta_1$. The unknowns A and x_{IVC}^* are obtained by combining conditions (4.9) and (4.10), i.e., the value matching and the smooth pasting conditions, respectively[49]. The interpretation of the value matching condition is straightforward: at the moment in which GPs exercise an option to invest in a SuF, these are entitled to the net present value of such investment given by $NPV_{IVC}(x_{IVC}^*)$, and to the contribution to its carried interest that such investment opportunity will generate – given by $CI(x_{IVC}^*)$ – net of the capital gains tax GPs are expected to be liable on at the moment in which it divests, given by $i.CGT_{IVC}(x_{IVC}^*)$. We now analytically define each of these three components.

First, $NPV_{IVC}(x)$ shall essentially portray the fraction of the payoff on the investment on the SuF which is attributable to the GP – given by i. The LPs of the VC fund shall then provide $(1-i)(1-q^E)$ fraction of the total capital required by the SuF.

$$NPV_{IVC}(x) = \left(\frac{x(1-w)(1-T_C)}{\delta}\right)(1-q^E)(q^{IVC}+p)i - (k-sub)q^{IVC}i \quad (4.11)$$

Second, carried interest is contingent on the realized payoff of the investment at the exit date. If such a realized payoff exceeds a given hurdle – determined by the hurdle rate $h > 0$ – GPs are entitled to a fraction $s > 0$ of that excess payoff. Such divestment is expected to occur on the moment $ED > 0$[50] after the investment is made[51]. Considering that the payoff

[49] Notice that we do not provide closed-form solutions to this system of equations since it is not possible to obtain closed-form derivatives for $CGT_{IVC}(V_{IVC}^*)$ and $CI(V_{IVC}^*)$, as these will depend on the integral of the cumulative normal distribution density function in order to *t*.

[50] "ED" stands for exit date.

[51] With the purpose of keeping results as tractable as possible, we assume that when screening a given investment opportunity, GPs will take into account the expected value of such performance based compensation. Still, the actual carried interest attributable to GP might be pooled against the overall performance of the VC fund and not awarded to GPs individually, on each of the realized investments. This is more frequent in Europe than in the USA, where carried interest is usually computed and paid to GPs on a deal by deal basis. We will briefly discuss the impact of the pooling effect in section 4.3.

from such variable compensation resembles that of a call option, we compute $CI(x)$ similarly to Sorensen et al. (2014)[52] following the classic approach by Black and Scholes (1973) and Scholes (1976). Taking $GR_{IVC}(x)$ as the gross return of the IVC fund on the investment and $H(k)$ as the hurdle, we have:

$$CI(x) = \left[GR_{IVC}(x)e^{-\delta\, ED}N(d1_{CI}) - H(k)e^{-r\, ED}N(d2_{CI})\right](1-i)(1-T_C)s \quad (4.12)$$

where $N(z)$ stands for the cumulative normal distribution density function, and

$$GR_{IVC}(x) = \left(\frac{x(1-w)}{\delta}\right)(1-T_C)(1-q^E)(q^{IVC}+p) \quad (4.13)$$

$$H(k) = \int_0^{ED} (k - sub)\, e^{h\, t} dt \quad (4.14)$$

$$d1_{CI} = \frac{log\left(\frac{GR_{IVC}(x)}{H(k)}\right) + ED(r - \delta - \frac{\sigma^2}{2})}{\sigma\sqrt{ED}} \quad (4.15)$$

$$d2_{CI} = d1_{CI} - \sigma\sqrt{ED} \quad (4.16)$$

Finally, and similarly to carried interest, we define $CGT_{IVC}(x)$ as a short position on a call option, given that taxation on capital gains is contingent on x at the moment in which the divestment takes place – since taxation will only occur if divestment consideration exceeds the invested capital k. This is similar to a hurdle on carried interest when $h = 0$, as shown below:

$$CGT_{IVC}(x) = \left[GR_{IVC}(x)e^{-\delta\, ED}N\left(d1_{CGT_{IVC}}\right) - q^{IVC}(k - sub)e^{-r\, ED}N\left(d2_{CGT_{IVC}}\right)\right]T_G \quad (4.17)$$

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[52] Unlike Sorensen et al. (2014) and for tractability purposes, we did not include the “catch-up” region, in which GPs are awarded with a fraction (typically, 100%) of the payoffs above the hurdle until they are entitled with the same share $s > 0$ of the payoffs which are below the hurdle. For similar reasons, we did not account for transaction costs which might be borne by IVCs and PVCs both on investment and divestment, including potential performance fees due to the management of the portfolio firm (commonly mentioned as “ratchet”). Estimates presented by Hunter and Jagtiani (2003) reveal that, on a sample of 635 mergers and acquisitions involving public companies from 1985 to 2004, mean advisory fees stand for 0.524% and 0.659% of the deal value for acquirers and targets, respectively. We argue that these should be greater on the PE and especially on the VC segment, as deals with private firms are smaller and information asymmetries are more severe, driving acquirers to undertake relatively more time consuming due diligence procedures.

where $GR_{IVC}(x)$ is the same as before, and

$$d1_{CGT_{IVC}} = \frac{log\left(\frac{GR_{IVC}(x)}{q^{IVC}(k-sub)}\right) + ED(r-\delta-\frac{\sigma^2}{2})}{\sigma\sqrt{ED}} \tag{4.18}$$

$$d2_{CGT_{IVC}} = d1_{CGT} - \sigma\sqrt{ED} \tag{4.19}$$

For generalization purposes – and in order to avoid redundancies with section 4.1.3, where we will handle the co-investment case – we kept q^{IVC}, p, and sub on the above formulation. However, please bear in mind that, in this case, we will set $q^{IVC} = 1, p = 0$ and $sub = 0$.

4.1.2. The option to invest in the Start-up Firm held by the Public Venture Capitalist

PVCs might be regarded as a specific type of VC organization where Governments simultaneously act as GPs and LPs (Murray, 2007). Therefore, from a public policy perspective, we take PVCs as neutral to carried interest, management fees and any other form of GP compensation that might be set on the PVC organization. For an analogous reason, we take PVCs as neutral to taxation, since Governments offset the tax expenditures borne by portfolio firms through their own tax collection[53]. In fact, one may regard such incremental tax revenues as a proxy of the social welfare generated by the investment opportunity or some kind of spillover effect from Government spending.

Based on the recent literature findings by Grilli and Murtinu (2015) and on the more general assessment of public sector inefficiency in output generation (Afonso et al., 2005, Afonso et al., 2010, Adam et al., 2011, Gao, 2015), which was also followed by Barbosa et al. (2016) on a real options approach to analyze public stimulus for private investment, we assume that the value generation effect of the investment undertaken by PVCs will be lower

[53] We excluded Social Security charges as an incremental tax revenue in the sense that, unlike other income taxes, such incremental revenue leads to an incremental liability to the Government rising from increasing state pensions or costs with social protection. Still, Social Security charges stand for an additional cost bore by SuFs which hinders their profitability, whose impact on our framework is portrayed by the variable w. In section 4.2.2. we present a sensitivity analysis on w, which might be understood to address the impact of Social Security charges on the theoretical hypothesis we derive.

than the one by IVCs by $(1 - \phi)$ and $\phi \in (0; 1)$. This parameter ϕ then stands for an efficiency gap between PVCs and IVCs.

In addition, assuming that Government is able to observe the investment behavior of IVC, when exercising the option to invest on the SuF, Governments forego the option to wait for its profitability x to reach x^*_{IVC}, let IVC perform the investment on the SuF and obtain an incremental tax revenue, without having to carry any expenditure on the SuF. Therefore, when valuing the option to invest in the SuF, Governments take into account this short position on this contingent asset, which we value as a *cash-or-nothing binary barrier option* (Rubinstein and Reiner, 1991). These options pay a pre-determined amount of cash – in our case, the incremental tax revenues – whether during a certain period of time – given by ED – the underlying asset – given by x – reaches a given barrier, which stands for the optimum investment trigger held by the IVC, and named as x^*_{IVC}. This opportunity cost is then modelled as a short position on a binary barrier option and shall be named as $\Psi(x)$.

For similar reasons to the previous section, and except otherwise mentioned, we kept q^{IVC}, p, and sub on the above formulation. However, please bear in mind that, in this case, we will set $q^{IVC} = 0$, $p = 0$ and $sub = 0$.

Following the contingent-claim approach as before, the value of the option held by the PVC to invest in the SuF – $PVC(x)$ – must satisfy the following ODE:

$$\frac{1}{2}\,\sigma^2\,x^2\,PVC''(x) + (r - \delta)\,x\,PVC'(x) - r\,PVC(x) = 0 \tag{4.20}$$

The general solution for (4.20) is:

$$PVC(x) = C\,x^{\beta_1} + D\,x^{\beta_2} \tag{4.21}$$

where C and D are constants to be determined, and β_1 and β_2 are the previously derived roots of the fundamental quadratic given by (4.5).

Considering that x^*_{PVC} stands for the optimal trigger for the PVC to exercise the option to invest in the SuF, the problem must be solved by considering the following boundary conditions, in which $NPV_{PVC}(x)$ is net present value of the investment undertaken by the PVC, $TAX(x)$ stands for the incremental tax revenues that the Government benefits from, and $\Psi(x)$ stands for the opportunity cost borne by Governments from not benefiting from the incremental tax revenues that would be obtained if the IVC invested on the SuF instead:

$$PVC(0) = 0 \tag{4.22}$$

$$PVC(x^*_{PVC}) = NPV_{PVC}(x^*_{PVC}) + TAX(V^*_{PVC}) - \Psi(x^*_{PVC}) \tag{4.23}$$

$$PVC'(x^*_{PVC}) = NPV'_{PVC}(x^*_{PVC}) + TAX'(x^*_{PVC}) - \Psi'(x^*_{PVC}) \tag{4.24}$$

Respecting condition (4.22) and noting that $\beta_2 < 0$, then D on equation (4.21) must be equal to zero and, as before, $\beta \equiv \beta_1$. The unknowns C and x^*_{PVC} are obtained by combining equations (4.23) and (4.24), i.e., the value matching and smooth pasting conditions, respectively. Again, the interpretation of the value matching condition is direct. Similarly to IVCs, at the optimum investment timing, PVCs are entitled to the net present value of the investment on the SuF, given by $NPV_{PVC}(x^*_{PVC})$. However, unlike IVCs, PVCs benefit from additional corporate and personal tax revenues rising from exercising the option to invest on the SuF – $TAX(x^*_{PVC})$ – lose the contingent asset $\Psi(V^*_{PVC})$, which stands for the incremental tax revenues that Governments would obtain if profitability of the SuF reaches x^*_{IVC} and IVCs exercise their option to invest, and are not liable on capital gains tax[54]. We will now depict each of these individual components.

$NPV_{PVC}(x)$ is similar to $NPV_{IVC}(x)$, except for the holding on the SuF – which will be $(1 - q^E)(1 - q^{IVC} - p)$ instead of $(1 - q^E)(q^{IVC} + p)i$ – and for the inefficiency factor on the value creation effect of the investment in the SuF. Therefore, we introduce the $(1 - \phi)$ coefficient, with $\phi > 0$. $NPV_{PVC}(x)$ is then defined as follows:

$$NPV_{PVC}(x) = \left(\frac{x(1 - \phi - w)(1 - T_C)}{\delta}\right)(1 - q^E)(1 - q^{IVC} - p) - k(1 - q^{IVC}) \tag{4.25}$$

[54] We do not include capital gains tax on $TAX(x^*_{PVC})$, since it would simultaneously stand for a liability at divestment date (given by ED) to the the PVC and an asset (i.e., a tax revenue) to the Government at the divestment date.

The tax collection brought to Governments and rising from the exercise of the option to invest in the SuF is given by $TAX(x)$, which will account for either corporate income tax – given by $C_{TAX}(x)$ – and personal income tax – named as $P_{TAX}(x)$ – as follows:

$$TAX(x) = C_{TAX}(x) + P_{TAX}(x) \tag{4.26}$$

where

$$C_{TAX}(x) = \left(\frac{x(1-\phi-w)}{\delta}\right) T_C \tag{4.27}$$

$$P_{TAX}(x) = \frac{xw}{\delta} T_P \tag{4.28}$$

Assuming that PVC is able to observe x^*_{IVC}, the contingent asset $\Psi(x^*_{PVC})$ stands for a set of potential incremental tax revenues to the Government – named $\Delta_{TAX}(x)$ – that will be valued as a *cash-or-nothing binary barrier option* (Rubinstein and Reiner, 1991), provided that such contingent benefit will only be earned by the Government if x hits a certain barrier given by x^*_{IVC}. Such incremental tax revenues are obtained by comparing tax revenues when $qIVC = 1$ and $qIVC = 0$, as shown in equations (4.33) and (4.34). Incremental tax revenues are caused by the higher taxable income generation rate of the IVC on the investee firm (which we take as ϕ) – bringing both incremental corporate income tax and capital gains tax revenues to the Government – and by the incremental corporate income tax revenues over the carried interest – set as $CI(x)$ – earned by the GP of the IVC fund. We then have:

$$\Psi(x) = \Delta_{TAX}(x)\left[\left(\frac{x^*_{IVC}}{x}\right)^{a+b} N(-z) + \left(\frac{x^*_{IVC}}{x}\right)^{a-b} N\left(-z + 2\, b\, \sigma \sqrt{ED}\right)\right] \tag{4.29}$$

where

$$a = \frac{r-\delta}{\sigma^2} \tag{4.30}$$

$$b = \sqrt{\frac{(r-\delta)^2 + 2\log(1+r)\,\sigma^2}{\sigma^2}} \tag{4.31}$$

$$z = \frac{\log\left(\frac{x^*_{IVC}}{x}\right)}{\sigma\sqrt{ED}} + b\,\sigma\,\sqrt{ED} \tag{4.32}$$

$$\Delta_{TAX}(x) = \Delta C_{TAX}(x) + \overbrace{CGT_{IVC}(x)}^{qIVC=1}, \text{ and} \tag{4.33}$$

$$\Delta C_{TAX}(x) = \frac{x\phi}{\delta}T_C + \frac{T_C}{1-T_C}\overbrace{CI(x)}^{qIVC=1} \tag{4.34}$$

4.1.3. The option to co-invest in the Start-up Firm held by the Public Venture Capitalist

On the co-investment case, we let the proportion of capital to be provided by IVCs and PVCs – given by q^{IVC} and $(1-q^{IVC})$ – to be exogenously determined, and let IVCs and PVCs set the share premium p[55], which will enable both parties to have the same optimum investment timing, and co-investment to take place. Such share premium p is obtained by equating their respective triggers $x^*$and solving for p, and stands for an *equity enhancement* effect that the Government is willing to provide to IVCs, in the sense of Murray (2007).

From the perspective of the IVC, no changes occur to the setting presented in section 4.1.1, where no co-investment took place, except that $qIVC \in (0;1)$, instead of $qIVC = 1$ and we do not impose that $p = 0$, as p shall be endogenously determined[56]. Therefore, and as before, we obtain the relevant unknowns A and x^*_{IVC} by combining conditions (4.9) and (4.10), while $IVC(x)$ remains as the relevant function for portraying the value of the option to invest in the SuF held by the IVC.

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[55] One could argue that q^{IVC} could also serve the purpose of aligning optimum investment timing. However, in the absence of capital constraints or any other effects not explicitly derived by our framework, changing q^{IVC} will not affect x^*_{IVC} or x^*_{PVC}, since it only lead the whole investment opportunity and its value matching conditions to become proportionately smaller or larger, yielding no effect on optimum investment timing.

[56] In addition, $sub = 0$ is maintained.

Things change a little from the perspective of the PVC for three reasons. First, we assume that co-investing allows the inefficient factor ϕ to be eliminated, which is equivalent to set $\phi = 0$ in the no co-investment case presented in section 4.1.2. Second, and for the same reason, when exercising the option to co-invest, the Government does not forego the contingent asset $\Psi(x)$, since that there are no incremental tax revenues lost. Third, total tax revenues generated to the Government in this setting – to be named as $TAX_{CO}(x)$ instead of $TAX(x)$ – should include capital gains tax arising from the investment performed by IVCs – named as $CGT_{IVC}(x)$ – and should include corporate income tax on the carried interest, which is expected to be earned by the GP of the IVC fund. With this purpose, we set $C_{TAX_{CO}}(x)$ to capture the amount of corporate income tax revenues which are generated in this setting.

We define $PVC_{CO}(x)$ as the function of the option to co-invest in the SuF, which is obtained similarly to $PVC(x)$, but considering the following alternative set of boundary conditions:

$$PVC(0) = 0 \tag{4.35}$$

$$PVC_{CO}\left(x^*_{PVC_{CO}}\right) = \overbrace{NPV_{PVC}\left(x^*_{PVC_{CO}}\right)}^{\phi=0} + TAX_{CO}\left(x^*_{PVC_{CO}}\right) \tag{4.36}$$

$$PVC_{CO}'\left(x^*_{PVC_{CO}}\right) = \overbrace{NPV_{PVC}'\left(x^*_{PVC_{CO}}\right)}^{\phi=0} + TAX_{CO}'\left(x^*_{PVC_{CO}}\right) \tag{4.37}$$

where

$$TAX_{CO}\left(x^*_{PVC_{CO}}\right) = C_{TAX_{CO}}\left(x^*_{PVC_{CO}}\right) + P_{TAX}\left(x^*_{PVC_{CO}}\right) + CGT_{IVC}\left(x^*_{PVC_{CO}}\right) \tag{4.38}$$

and

$$C_{TAX_{CO}}\left(x^*_{PVC_{CO}}\right) = \overbrace{C_{TAX}\left(x^*_{PVC_{CO}}\right)}^{\phi=0} + \frac{T_C}{1-T_C}\overbrace{CI\left(x^*_{PVC_{CO}}\right)}^{qIVC=\overline{qIVC}} \tag{4.39}$$

Analogously to the previous section, considering condition (4.35) and noting that $\beta_2 < 0$, D on equation (4.21) must be equal to zero and $\beta \equiv \beta_1$ as before. The unknowns C and

$x^*_{PVC_{CO}}$ are obtained by combining equations (4.36) and (4.37), i.e., the value matching and smooth pasting conditions, respectively. By equating $x^*_{PVC_{CO}}$ and x^*_{IVC} and solving for p, we obtain the optimum share premium p^*, that would allow IVCs and PVCs to jointly co-invest on the SuF.

4.1.4. The option to subsidize the Start-up Firm held by the Government

In this setting, we assume that instead of investing or co-investing in the SuF through its PVC organization, Governments may choose to provide a subsidy to the SuF, which we name as $sub > 0$. The decision to provide or not such subsidy may also be pictured as a real option as in Dixit and Pindyck (1994) for two reasons. On the one hand, by subsidizing the SuF, the Government is entitled to benefit from incremental tax revenues from corporate income tax, personal income tax and capital gains tax and, in this sense, the subsidizing decision might be regarded as an investment decision. On the other hand, the flexibility held by the Government in deciding when to support the SuF, allows the Government to determine the profitability trigger of the SuF for which it will be willing to subsidize it. Finally, the return on the subsidy is actually uncertain as it is dependent on a stochastic variable, given by x.

Notwithstanding, the amount of the subsidy that Governments may be willing to offer, may not be enough for the SuF to be able to perform the envisaged investment on its own. As a result and in any case, we assume that the SuF will still have to seek for an external equity provider, which we will assume to be an IVC fund with the decision-making process we described in section 4.1.1. We will then consider that $q^{IVC} = 1$ and that $sub > 0$[57].

Thus the option to subsidy the SuF also becomes a "co-investment" decision, in the sense that both the Government and the IVC (managed by its GP) should be simultaneously willing to fund the SuF so that the investment opportunity might be undertaken. The former should be willing to provide a subsidy, while the latter should be willing to underwrite an equity

[57] We also maintain that $p = 0$ and $\phi = 0$ in this setting.

issuance on $k - sub$. We shall now formulate $\mathcal{S}(x)$ – the option to subsidy the SuF held by the Government – so that the optimum subsidy can be derived.

Following the contingent-claims approach, $\mathcal{S}(x)$ must satisfy the ODE described in equation (4.40), whose general solution is presented in (4.41):

$$\frac{1}{2}\,\sigma^2\,x\,\mathcal{S}''(x) + (r-\delta)\,x\,\mathcal{S}'(x) - r\,\mathcal{S}(x) = 0 \tag{4.40}$$

$$\mathcal{S}(x) = E\,x^{\beta_1} + F\,x^{\beta_2} \tag{4.41}$$

E and F are constants to be determined, while β_1 and β_2 are the previously derived roots of the fundamental quadratic given by (4.5).

Naming $x_{\mathcal{S}}^*$ as the optimal trigger for the Government to exercise the option to subsidy the SuF, the problem is solved by introducing the following boundary conditions, in which $TAX_{\mathcal{S}}(x)$ is the incremental tax revenues that the Government benefits from and *sub* is the amount of the subsidy to be awarded to the SuF:

$$\mathcal{S}(0) = 0 \tag{4.42}$$

$$\mathcal{S}(x_{\mathcal{S}}^*) = TAX_{\mathcal{S}}(x) - sub \tag{4.43}$$

$$\mathcal{S}'(x_{\mathcal{S}}^*) = TAX_{\mathcal{S}}{}'(x) \tag{4.44}$$

where

$$TAX_{\mathcal{S}}(x) = C_{TAX_{\mathcal{S}}}(x) + P_{TAX}(x) + \overbrace{CGT_{IVC}(x)}^{qIVC\,=\,1} \tag{4.45}$$

and

$$C_{TAX_{\mathcal{S}}}(x) = \overbrace{C_{TAX}(x)}^{\phi=0} + \frac{T_C}{1-T_C}\,\overbrace{CI(x)}^{qIVC\,=\,1} \tag{4.46}$$

Regarding condition (4.42) and considering that $\beta_2 < 0$, then F on equation (4.41) must be equal to zero and, as before, $\beta \equiv \beta_1$. The unknowns E and $x_{\mathcal{S}}^*$ are obtained by combining equations (4.43) and (4.44), i.e., the value matching and smooth pasting conditions, respectively. Finally, we obtain the optimum subsidy sub^* by equating x_S^* and x_{IVC}^* and solving for sub.

4.2. Numerical example

This illustration aims to demonstrate the numerical solutions for the framework we derived on the previous section and highlight the economic intuition that governs its results. With this purpose, we will render a set of sensitivities on the base case for all of its relevant variables. A summary of key variables and numerical parameters is presented in Table 9. Where possible, we use parameters from Sorensen et al. (2014).

Our assumption for w was computed through a sample of 4,523,712 companies obtained in Bureau van Dijk – Amadeus (Amadeus) database for the 2011 to 2014 period, covering firms located in Austria, Belgium, Cyprus, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Liechtenstein, Luxembourg, Malta, Monaco, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, and United Kingdom, and excluding companies from the financial services, real estate and insurance industries. As costs with payroll available in Amadeus include Social Security charges and w does not, we grossed-down those through an estimate of the average employer Social Security contributions obtained on the OECD Tax Database, for the same period, whose sample included European-only Organization for Economic Co-Operation and Development (OECD) members.

Our assumption for ϕ was based on an arithmetic average from the estimates of the output efficiency gap on public spending for European countries, presented in Afonso et al. (2005), Afonso et al. (2010), Adam et al. (2011) and Gao (2015), amounting to 16.0%, 22.0%, 27.0% and 6.0%, respectively. Excluding the lower estimate from Gao (2015) would lead to $\phi = 22.0\%$ instead of $\phi = 18.0\%$, which we would not materially change results. Still, we will run sensitivities on ϕ and show that the economic intuition of our results remains unchanged.

Variable	Symbol
Earnings before taxes and gross payroll	x
IVC's investment trigger on $\boldsymbol{x}$	x^*_{IVC}
PVC's investment trigger on $\boldsymbol{x}$	x^*_{PVC}
Co-investment trigger on $\boldsymbol{x}$	x^*_{CO}
Optimum share premium	p^*
Subsidy trigger on $\boldsymbol{x}$	x^*_S
Optimum subsidy	sub^*
IVC's SuF investment value to investment opportunity value at trigger	ωNPV_{IVC}
IVC's capital tax liability to investment opportunity value at trigger	ωCGT
IVC's carried interest value to investment opportunity value at trigger	ωCI
PVC's SuF investment value to investment opportunity value at trigger	ωNPV_{PVC}
PVC's incremental tax revenues to investment opportunity value at trigger	ωTAX
PVC's contingent asset value to investment opportunity value at trigger	$\omega\psi$

Output Parameters					
β =	3.39	a =	0.96	b =	0.39

Parameter	Symbol	Value	Source
Risk-free rate	r	0.05	Sorensen et al. (2014)
Volatility on x	σ	0.25	Liu and Yang (2015)
Expected SuF return	δ	0.11	Sorensen et al. (2014)
Amount of the investment	k	150	-
Gross payroll on x	w	0.60	Bureau van Dijk Amadeus
Ownership retained by the Entrepreneur	Q^E	0.20	-
Co-investor ownership allocated to IVC	q^{IVC}	0.50	-
PVC efficiency gap	ϕ	0.18	Afonso et al. (2005), Afonso et al. (2010), Adam et al. (2011), Gao (2015)
Average personal income tax	T_P	0.19	OECD
Average effective corporate income tax	T_C	0.25	Eurostat, European Commission and DG Taxation & Customs (2015)
Average capital gains tax at fund level	T_G	0.15	EVCA (2013)[58]
Expected holding period (years)	ED	4.00	Braun et al. (2015)
Hurdle rate	h	0.08	Metrick and Yasuda (2011), Sorensen et al. (2014), Liu and Yang (2015)
Carried interest	s	0.20	Sorensen et al. (2014)
GP stake on the IVC fund	i	0.01	-

Table 9. Baseline variables and assumptions

The list of variables resembles the set of decision-making criteria that were introduced in the previous sections – namely, x^*_{IVC}, x^*_{PVC}, x^*_{CO} and x^*_S – while introducing ωNPV_{IVC}, ωCGT, ωCI, ωNPV_{PVC}, ωTAX and $\omega\psi$. This group of variables is intended to capture the weight that each of these components holds on the value of the investment opportunity when IVCs

[58] We assumed $T_G = 15.0\%$ as the average minimum and maximum capital gains tax at fund level reported for Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, Switzerland, the United Kingdom and the United States.

and PVCs exercise their option to invest in the SuF, i.e., when $x = x^*_{IVC}$ and $x = x^*_{PVC}$. Formally, and for example,

$$\omega NPV_{IVC} = \frac{NPV_{IVC}(x^*_{IVC})}{IVC(x^*_{IVC})} \tag{4.47}$$

$$\omega TAX = \frac{TAX(x^*_{PVC})}{PVC(x^*_{IVC})} \tag{4.48}$$

Analyzing these output variables will also allow us to depict how value creation is governed when IVCs and PVCs exercise their option to invest in the SuF.

Following our discussion from the introductory section of this Chapter, we take the view that investing in early stage SuFs generates *positive externalities* (Lerner, 1999, Lerner, 2002). As a result, from a public policy perspective, we assume that Governments will be interested in fostering investment volumes in such firm segment. We will then compare how each of the four different deal arrangements anticipate optimum investment timing or, equivalently, minimize optimum profitability triggers.

4.2.1. Baseline results

Our baseline results show that $x^*_{IVC} = 99.12$, $x^*_{PVC} = 75.32$, $x^*_{CO} = 51.80$ with $p^* = 45.7\%$ and $x^*_S = 57.31$ with $sub^* = 79.08$, meaning that $x^*_{IVC} > x^*_{PVC} > x^*_S > x^*_{CO}$.

Taxation on personal income, taxation on corporate income, as well as taxation on capital gains that would be borne by PVCs, allow Governments to generate a set of incremental revenues from exercising the option to invest, when compared to IVCs, enabling them to offset their efficiency gap – measured by ϕ – and still significantly exceed the return on investment obtained by IVCs. As a result, we find that $x^*_{IVC} > x^*_{PVC}$.

Our results also reveal that $x^*_S > x^*_{CO}$. On each of these two scenarios, Governments benefit from incremental revenue taxes, since by letting IVC co-invest alongside PVC or by letting IVC invest on the SuF and then partly subsidizing k, they do not bear the efficiency gap when PVCs invest on their own. From the perspective of the IVC, the subsidy provides a one-time effect on the value of the investment opportunity, while co-investing generates an actual *equity enhancement* effect (Murray, 2007), as IVCs provide $(1-q^E)\,q^{IVC} = (1-20.0\%)\times 50.0\% = 40.0\%$ of the required capital, but retain a $(1-q^E)\,(q^{IVC}+p^*) = (1-20.0\%)\times(50.0\%+45.7\%) = 76.5\%$ share on the SuF. In the absence of capital constraints, portfolio risk mitigation and value creation policies held by IVCs, we show that a mere allocation of money from PVCs on the co-investment is not enough for anticipating their optimum investment timing.

As the baseline example illustrated some of the intuition governing the framework derived throughout the previous sections, we now carry a set of sensitivity analysis with the purpose of further depicting results and highlighting further insights to our research.

4.2.2. Efficiency gap (ϕ) and gross payroll on x (*w*)

The results of the sensitivity analysis on ϕ and w are presented in Table 10. As expected, an increase in w leads to an increase in x^*_{IVC}, x^*_{PVC}, x^*_{CO} and x^*_S, as it decreases the profitability of the SuF. In turn, the greater the efficiency gap measured by ϕ is, the greater x^*_{PVC} is, as the value of the investment on the SuF becomes lower for PVCs. In addition, we observe that the greater the efficiency gap is, the greater is the gap between $x^*_{PVC}$and $x^*_{CO}$is or, equivalently, the lower the ratio $\frac{x^*_{CO}}{x^*_{PVC}}$ is. For example, when $\phi = 0.12$ and $w = 0.45$, $\frac{x^*_{CO}}{x^*_{PVC}} = 83.1\%$, but when ϕ increases to $\phi = 0.24$, $\frac{x^*_{CO}}{x^*_{PVC}} = 65.8\%$.

Therefore, we conjecture that more inefficient Governments should be more likely to make use of co-investing, as incremental tax revenues become more significant. x^*_{IVC}, x^*_{CO} and x^*_S remain unchanged to changes in ϕ as they do not depend on this parameter. When the investment is carried by IVCs, whether it is subsidized or not, or when co-investment takes place, $\phi = 0$.

	$\phi = 0.12$		$\phi = 0.18$		$\phi = 0.24$	
$w = 0.45$	$x^*_{IVC} = 72.09$	$x^*_{PVC} = 51.18$	$x^*_{IVC} = 72.09$	$x^*_{PVC} = 57.04$	$x^*_{IVC} = 72.09$	$x^*_{PVC} = 64.65$
	$x^*_{CO} = 42.53$ $p^* = 34.7\%$	$x^*_S = 46.39$ $sub^* = 66.85$	$x^*_{CO} = 42.53$ $p^* = 34.7\%$	$x^*_S = 46.39$ $sub^* = 66.85$	$x^*_{CO} = 42.53$ $p^* = 34.7\%$	$x^*_S = 46.39$ $sub^* = 66.85$
$w = 0.60$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 65.47$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 89.20$
	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$
$w = 0.75$	$x^*_{IVC} = 158.59$	$x^*_{PVC} = 91.11$	$x^*_{IVC} = 158.59$	$x^*_{PVC} = 111.22$	$x^*_{IVC} = 158.59$	$x^*_{PVC} = 144.58$
	$x^*_{CO} = 66.25$ $p^* = 69.69\%$	$x^*_S = 74.97$ $sub^* = 98.86$	$x^*_{CO} = 66.25$ $p^* = 69.69\%$	$x^*_S = 74.97$ $sub^* = 98.86$	$x^*_{CO} = 66.25$ $p^* = 69.69\%$	$x^*_S = 74.97$ $sub^* = 98.86$

Table 10. Sensitivity analysis on ϕ and w

4.2.3. Amount of investment (k) and volatility on x (σ)

Profit triggers x^*_{IVC}, x^*_{PVC}, x^*_{CO} and x^*_S increase with k as shown in Table 11, in order to sustain return on investment. Similarly, as σ increases the value of the option to invest, x^*_{IVC}, x^*_{PVC}, x^*_{CO} and x^*_S increase with σ. However, σ leads to asymmetric effects between $x^*_{IVC}$and x^*_{PVC}, with changes in x^*_{IVC} being much greater than those in x^*_{PVC}, especially when $\sigma > 0.25$.

This behaviour points out the existence of a gap on the financing market for high risky projects that might be filled by PVCs, even when comparing with the option to co-invest. This would be the case on the early stage segment, where we then expect PVCs to have a higher share in deal volumes. Even though results from this numerical example are not illustrative, we may conjecture that that for very high degrees of uncertainty ($\sigma > 0.25$) subsidizing might be preferable to co-investing, as it may act as a remedy to the impact of uncertainty on the profitability of the SuF.

	$k = 50$		$k = 150$		$k = 250$	
$\sigma = 0.15$	$x^*_{IVC} = 26.82$	$x^*_{PVC} = 21.17$	$x^*_{IVC} = 80.46$	$x^*_{PVC} = 63.51$	$x^*_{IVC} = 134.10$	$x^*_{PVC} = 105.86$
	$x^*_{CO} = 14.14$ $p^* = 44.81\%$	$x^*_S = 15.62$ $sub^* = 26.09$	$x^*_{CO} = 42.43$ $p^* = 44.81\%$	$x^*_S = 46.87$ $sub^* = 78.28$	$x^*_{CO} = 70.72$ $p^* = 44.81\%$	$x^*_S = 78.12$ $sub^* = 130.46$
$\sigma = 0.25$	$x^*_{IVC} = 33.04$	$x^*_{PVC} = 25.11$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 165.20$	$x^*_{PVC} = 125.53$
	$x^*_{CO} = 17.27$ $p^* = 45.67\%$	$x^*_S = 19.10$ $sub^* = 26.36$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$	$x^*_{CO} = 86.33$ $p^* = 45.67\%$	$x^*_S = 95.52$ $sub^* = 131.80$
$\sigma = 0.35$	$x^*_{IVC} = 91.82$	$x^*_{PVC} = 31.41$	$x^*_{IVC} = 275.45$	$x^*_{PVC} = 94.22$	$x^*_{IVC} = 459.08$	$x^*_{PVC} = 157.04$
	$x^*_{CO} = 33.81$ $p^* = 85.79\%$	$x^*_S = 45.03$ $sub^* = 50.00$	$x^*_{CO} = 101.42$ $p^* = 85.79\%$	$x^*_S = 102.84$ $sub^* = 117.49$	$x^*_{CO} = 169.04$ $p^* = 85.79\%$	$x^*_S = 171.41$ $sub^* = 195.82$

Table 11. Sensitivity analysis on k and σ

4.2.4. Taxation (T_C and T_G)

Growing effective taxation rates lead to different effects on IVCs and PVCs, as we show in Table 12. The latter will demand an increasing x^*_{IVC}, since the profitability of the SuF is hindered by T_C and T_G lowers the net proceeds from divesting to the IVC. The former will benefit from a greater tax collection that will lead x^*_{PVC} to decrease. Equivalently, for very low taxation levels, and depending on the efficiency gap ϕ, we would observe $x^*_{PVC} > x^*_{IVC}$. We then conjecture that Governments whose overall taxation are greater might be more supportive of PVC initiatives.

Similarly to Keuschnigg and Nielsen (2007), x^*_{IVC} reveals a higher sensitivity to T_C than to T_G, as the impact of T_G is contingent on x at divestment, while T_C negatively affects NPV_{IVC}, alongside CGT_{IVC}. We would also highlight that the greater taxation is, the more effective co-investing becomes comparing to subsidizing in what regards anticipating optimum investment timing. This is because the share premium mechanism introduced in x^*_{CO} is neutral to taxation at the IVC fund level, differently than subsidization which is subject to T_C.

On the lower panel of Table 12, we depict how ωNPV_{IVC}, ωCGT, ωCI, ωNPV_{PVC}, ωTAX and $\omega\psi$ change with T_C and T_G. As for IVCs, increasing T_C negatively affects ωCI as this component is the only one which is exclusively affected by T_C. As the value of investing on the SuF is simultaneously affected by T_C and T_G, IVCs are willing to trade-off ωCI in favor of $\omega NPV_{IVC}$and ωCGT. Conversely, increasing T_G makes the value of investing on the SuF less profitable than carried interest, which is not affected by T_G. As a result, IVCs are willing to trade-off $\omega NPV_{IVC}$and ωCGT for ωCI in this case.

	$T_C = 0.15$		$T_C = 0.25$		$T_C = 0.35$	
$T_G = 0.05$	$x^*_{IVC} = 87.18$	$x^*_{PVC} = 77.53$	$x^*_{IVC} = 98.66$	$x^*_{PVC} = 75.60$	$x^*_{IVC} = 113.68$	$x^*_{PVC} = 74.23$
	$x^*_{CO} = 52.76$ $p^* = 32.61\%$	$x^*_{S} = 57.35$ $sub^* = 64.15$	$x^*_{CO} = 51.79$ $p^* = 45.24\%$	$x^*_{S} = 57.25$ $sub^* = 78.69$	$x^*_{CO} = 50.86$ $p^* = 61.75\%$	$x^*_{S} = 57.17$ $sub^* = 93.21$
$T_G = 0.15$	$x^*_{IVC} = 87.59$	$x^*_{PVC} = 76.96$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 114.20$	$x^*_{PVC} = 74.13$
	$x^*_{CO} = 52.77$ $p^* = 32.99\%$	$x^*_{S} = 57.41$ $sub^* = 64.60$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_{S} = 57.32$ $sub^* = 79.08$	$x^*_{CO} = 50.87$ $p^* = 62.24\%$	$x^*_{S} = 57.22$ $sub^* = 93.54$
$T_G = 0.25$	$x^*_{IVC} = 88.02$	$x^*_{PVC} = 76.43$	$x^*_{IVC} = 99.60$	$x^*_{PVC} = 75.05$	$x^*_{IVC} = 114.75$	$x^*_{PVC} = 74.04$
	$x^*_{CO} = 52.78$ $p^* = 33.39\%$	$x^*_{S} = 57.48$ $sub^* = 65.07$	$x^*_{CO} = 51.81$ $p^* = 46.12\%$	$x^*_{S} = 57.37$ $sub^* = 79.49$	$x^*_{CO} = 50.88$ $p^* = 62.77\%$	$x^*_{S} = 57.29$ $sub^* = 93.89$

	$T_C = 0.15$		$T_C = 0.25$		$T_C = 0.35$	
$T_G = 0.05$	$\omega NPV_{IVC} = 100.41\%$ $\omega CGT = -2.65\%$ $\omega CI = 2.24\%$	$\omega NPV_{PVC} = -33.39\%$ $\omega TAX = 155.28\%$ $\omega\psi = -21.89\%$	$\omega NPV_{IVC} = 100.69\%$ $\omega CGT = -2.66\%$ $\omega CI = 1.97\%$	$\omega NPV_{PVC} = -41.14\%$ $\omega TAX = 161.22\%$ $\omega\psi = -20.08\%$	$\omega NPV_{IVC} = 100.96\%$ $\omega CGT = -2.67\%$ $\omega CI = 1.70\%$	$\omega NPV_{PVC} = -44.74\%$ $\omega TAX = 158.41\%$ $\omega\psi = -13.68\%$
$T_G = 0.15$	$\omega NPV_{IVC} = 105.96\%$ $\omega CGT = -8.37\%$ $\omega CI = 2.41\%$	$\omega NPV_{PVC} = -35.05\%$ $\omega TAX = 159.01\%$ $\omega\psi = -23.96\%$	$\omega NPV_{IVC} = 106.3\%$ $\omega CGT = -8.4\%$ $\omega CI = 2.1\%$	$\omega NPV_{PVC} = -41.82\%$ $\omega TAX = 162.34\%$ $\omega\psi = -20.52\%$	$\omega NPV_{IVC} = 106.60\%$ $\omega CGT = -8.43\%$ $\omega CI = 1.83\%$	$\omega NPV_{PVC} = -44.89\%$ $\omega TAX = 158.52\%$ $\omega\psi = -13.63\%$
$T_G = 0.25$	$\omega NPV_{IVC} = 112.11\%$ $\omega CGT = -14.72\%$ $\omega CI = 2.60\%$	$\omega NPV_{PVC} = -36.65\%$ $\omega TAX = 162.54\%$ $\omega\psi = -25.89\%$	$\omega NPV_{IVC} = 112.49\%$ $\omega CGT = -14.78\%$ $\omega CI = 2.29\%$	$\omega NPV_{PVC} = -42.46\%$ $\omega TAX = 163.37\%$ $\omega\psi = -20.91\%$	$\omega NPV_{IVC} = 112.86\%$ $\omega CGT = -14.84\%$ $\omega CI = 1.98\%$	$\omega NPV_{PVC} = -45.03\%$ $\omega TAX = 158.60\%$ $\omega\psi = -13.57\%$

Table 12. Sensitivity analysis on T_C and T_G

As for PVCs, and before interpreting how T_C and T_G affect its value drivers, we should first highlight two important results. First, the opportunity cost borne by the Government when PVC invests in the SuF given by $\Psi(x)$ may stand for over 20.0% of the value of the option to invest, having a material impact on the decision-making process of PVCs. Second, and unlike IVCs, PVCs should be willing to exercise their option to invest on the SuF even when $NPV_{PVC}(x^*_{PVC})$ is negative, as such loss is offset by a set of incremental tax revenues, even in the presence of an efficiency gap.

The influence of T_C and T_G in $\omega\psi$ is mediated by the impact that these two variables inflict in x^*_{IVC}. As increasing T_C and T_G restrains return on investment, they exert positive impact in x^*_{IVC}. Therefore, the gap between x^*_{IVC} and x^*_{PVC} becomes greater and the less likely it becomes for x to reach x^*_{IVC}. As a result, the opportunity cost that Governments incur from readily investing in the SuF becomes lower, as perceived incremental tax revenues from letting IVCs carry the investment are also lower. In addition, as T_C and T_G anticipate optimum investment timing for PVCs, ωNPV_{PVC} and ωTAX decrease their share on value creation.

4.2.5. Profit sharing (*s*) and holding period (*ED*)

In Table 13 we are particularly interested in understanding how x^*_{IVC} is affected by changes in profit sharing and in holding period, since x^*_{PVC} is only indirectly affected by these components through $\psi(x)$. These two variables influence the value of the carried interest component – given by $CI(x)$ – while the expected capital tax gain liability – given by $CGT(x)$ – is not affected by *s*.

Common sense would suggest that more aggressive profit sharing mechanisms would lead IVCs to anticipate their optimum investment timing, as they would be entitled to retain a greater share of the payoffs generated above the hurdle. However, our results show exactly the opposite result. In fact, and consistently with Liu and Yang (2015), when increasing *s*, $CI(x)$ becomes relatively more valuable than $NPV_{IVC}(x)$, only if the hurdle is achieved at divestment date. The likelihood of the hurdle to be achieved at this moment is greater, the greater x^*_{IVC} is. Therefore, performance-based compensation schemes such as carried interest, may actually lead to the counter-intuitive result of postponing optimum investment timing.

	$s = 0.15$		$s = 0.20$		$s = 0.25$	
$ED = 3$	$x^*_{IVC} = 101.22$	$x^*_{PVC} = 74.62$	$x^*_{IVC} = 102.69$	$x^*_{PVC} = 74.69$	$x^*_{IVC} = 104.54$	$x^*_{PVC} = 74.78$
	$x^*_{CO} = 52.35$ $p^* = 46.67\%$	$x^*_S = 58.01$ $sub^* = 80.05$	$x^*_{CO} = 52.72$ $p^* = 47.39\%$	$x^*_S = 58.48$ $sub^* = 80.73$	$x^*_{CO} = 53.19$ $p^* = 48.81\%$	$x^*_S = 59.05$ $sub^* = 81.59$
$ED = 4$	$x^*_{IVC} = 98.85$	$x^*_{PVC} = 75.31$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 99.40$	$x^*_{PVC} = 75.32$
	$x^*_{CO} = 51.73$ $p^* = 45.53\%$	$x^*_S = 57.22$ $sub^* = 78.95$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$	$x^*_{CO} = 51.87$ $p^* = 45.81\%$	$x^*_S = 57.40$ $sub^* = 79.22$
$ED = 5$	$x^*_{IVC} = 98.41$	$x^*_{PVC} = 75.98$	$x^*_{IVC} = 98.51$	$x^*_{PVC} = 75.98$	$x^*_{IVC} = 98.61$	$x^*_{PVC} = 75.98$
	$x^*_{CO} = 51.62$ $p^* = 45.33\%$	$x^*_S = 57.07$ $sub^* = 78.76$	$x^*_{CO} = 51.64$ $p^* = 45.38\%$	$x^*_S = 57.11$ $sub^* = 78.80$	$x^*_{CO} = 51.67$ $p^* = 45.43\%$	$x^*_S = 57.14$ $sub^* = 78.85$

Table 13. Sensitivity analysis on s and ex

Additionally, longer holding periods measured by ED lead to a lower present value of both $CI(x)$ and $CGT(x)$. Therefore, IVCs trade-off $CI(x)$ for $NPV_{IVC}(x)$ in their decision-making process, leading x^*_{IVC} to decrease with ED.

4.2.6. Hurdle rate (h) and stake in IVC held by the GP (i)

Our sensitivity analysis on h describes the opposite effect from the one described for s in the previous section, as presented in Table 14. Increasing the hurdle decreases the likelihood of x exceeding such threshold at divestment date and, as a result, $CI(x)$ becomes less attractive than $NPV_{IVC}(x)$. This is also visible on the lower panel of Table 14, where we observe that ωCI decreases with h.

In turn, i holds a significant effect on decreasing x^*_{IVC}. The greater is a GP "skin in the game", the less valuable $CI(x)$ is, and the more $NPV_{IVC}(x)$ driven becomes its decision-making process, leading to lower x^*_{IVC} levels. This effect is particularly strong for very low levels of i, where x^*_{IVC} significantly lowers from $i = 0.005$ to $i = 0.010$, but reveals a more gradual change when $i > 0.010$.

	$h = 0.00$		$h = 0.08$		$h = 0.16$	
$i = 0.005$	$x^*_{IVC} = 104.51$	$x^*_{PVC} = 75.42$	$x^*_{IVC} = 100.33$	$x^*_{PVC} = 75.34$	$x^*_{IVC} = 98.84$	$x^*_{PVC} = 75.31$
	$x^*_{CO} = 53.17$ $p^* = 48.28\%$	$x^*_S = 59.04$ $sub^* = 81.58$	$x^*_{CO} = 52.11$ $p^* = 46.27\%$	$x^*_S = 57.71$ $sub^* = 79.65$	$x^*_{CO} = 51.73$ $p^* = 45.53\%$	$x^*_S = 57.22$ $sub^* = 78.95$
$i = 0.010$	$x^*_{IVC} = 100.68$	$x^*_{PVC} = 75.34$	$x^*_{IVC} = 99.12$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 98.45$	$x^*_{PVC} = 75.31$
	$x^*_{CO} = 52.20$ $p^* = 46.44\%$	$x^*_S = 57.82$ $sub^* = 79.82$	$x^*_{CO} = 51.80$ $p^* = 45.67\%$	$x^*_S = 57.31$ $sub^* = 79.08$	$x^*_{CO} = 51.63$ $p^* = 45.34\%$	$x^*_S = 57.09$ $sub^* = 78.77$
$i = 0.020$	$x^*_{IVC} = 99.27$	$x^*_{PVC} = 75.32$	$x^*_{IVC} = 98.58$	$x^*_{PVC} = 75.31$	$x^*_{IVC} = 98.26$	$x^*_{PVC} = 75.31$
	$x^*_{CO} = 51.84$ $p^* = 45.74\%$	$x^*_S = 57.36$ $sub^* = 79.15$	$x^*_{CO} = 51.67$ $p^* = 45.40\%$	$x^*_S = 57.14$ $sub^* = 78.83$	$x^*_{CO} = 51.58$ $p^* = 45.25\%$	$x^*_S = 57.03$ $sub^* = 78.68$

	$h = 0.00$	$h = 0.08$	$h = 0.16$
$i = 0.005$	$\omega NPV_{IVC} = 94.99\%$ $\omega CGT = -7.36\%$ $\omega CI = 12.37\%$	$\omega NPV_{IVC} = 103.78\%$ $\omega CGT = -8.15\%$ $\omega CI = 4.37\%$	$\omega NPV_{IVC} = 107.12\%$ $\omega CGT = -8.48\%$ $\omega CI = 1.35\%$
$i = 0.010$	$\omega NPV_{IVC} = 102.23\%$ $\omega CGT = -8.01\%$ $\omega CI = 5.79\%$	$\omega NPV_{IVC} = 106.28\%$ $\omega CGT = -8.40\%$ $\omega CI = 2.12\%$	$\omega NPV_{IVC} = 107.89\%$ $\omega CGT = -8.55\%$ $\omega CI = 0.67\%$
$i = 0.020$	$\omega NPV_{IVC} = 105.51\%$ $\omega CGT = -8.33\%$ $\omega CI = 2.82\%$	$\omega NPV_{IVC} = 107.48\%$ $\omega CGT = -8.52\%$ $\omega CI = 1.04\%$	$\omega NPV_{IVC} = 108.27\%$ $\omega CGT = -8.59\%$ $\omega CI = 0.33\%$

Table 14. Sensitivity analysis on h and i

4.3. Policy implications

The theoretical model and the numerical example introduced on the previous section allow us to outline a set of policy implications, which may be relevant not only for public decision-makers, but also for private LPs investing in IVC funds.

First, our results reveal that, in general, co-investment between IVCs and PVCs is more efficient than direct subsidization of the SuF in anticipating optimum investment timing, if

Governments are willing to provide an *equity enhancement* effect to IVCs through a share premium, or any other equivalent mechanism that leverages its value appropriation. This follows from our baseline case, but also from the sensitivity analysis we rendered in sections 4.2.2 to 4.2.6.

Second, more inefficient Governments should be more willing to allow PVCs to co-invest alongside IVCs as this deal structure may eliminate the efficiency gap between these two types of investors, and the benefits for Governments from its elimination directly increase with their inefficiency. Following the same argument, direct subsidization might be a more effective mechanism in anticipating optimum investment timing than direct PVC investment, as it also enables such efficiency gap to be eliminated, by letting IVCs lead the investment. This was highlighted in section 4.2.2.

Third, volatility is determinant in establishing a *market failure* on financing the segment of early stage companies by IVCs, whose volatility on their future profit generation is higher. Therefore, PVCs should be focused on this deal segment according to results from section 4.2.3. In addition, decreasing perceived volatility should be an explicit focus of public policy intended to foster investment volumes. This may encompass either long-term oriented economic and fiscal policies as well as specific policies intended to decrease operating costs, improve entrepreneurial quality or minimizing investment downside risk. Conversely, we may conjecture that subsidies, as uncertainty-free payoffs, might be more effective in anticipating optimum investment timing for IVCs, when volatility on profit generation is extreme.

Forth, our results from section 4.2.6 reveal that the stake held by the GP on its IVC fund stands as one of the most effective mechanisms to foster investment volumes by IVCs and anticipate their optimum investment timing, while profit sharing mechanisms grounded on carried interest instruments hold the opposite effect. Therefore, public policies aimed at increasing ι might be regarded as a positive contribution to promoting investment volumes. An example of such instruments includes multi-fund commitments, where public LPs could commit funds to subsequent funds, subject to certain performance benchmarks to be verified on the initial fund and to an increase in commitments by GPs on the subsequent fund, controlling for its investment strategy.

Finally, results from section 4.2.5 suggest that public policy should be more or less supportive of PVC initiatives, depending on the average fund age of their IVC peers and on their performance since inception. These two variables influence how GPs value $CI(x)$, which in turn affects x^*_{IVC}. In particular, if IVC funds are newborn, they envisage $CI(x)$ as valuable and will therefore demand greater x^*_{IVC}, further deepening the gap between the optimum investment timing held by IVCs and PVCs. As the investment period progresses, GPs are informed of the pooled performance of the portfolio firms on its IVC fund and, therefore, of the probability of obtaining a positive carried interest after fund liquidation. Two different scenarios might be in place at this point. First, the pooled performance of the IVC fund is such that GPs understand they will not benefit from carried interest and then will assume that $CI(x) = 0$ for the remaining investment decisions of the IVC fund. Hence, x^*_{IVC} shall decrease, meaning that IVCs might be either more willing to directly invest in target companies, or co-invest alongside PVCs, who will have to provide a lower *equity enhancement*. Second, if the pooled performance of the IVC fund is such that the GP admits that it will be entitled to the carried interest, it will assume that $CI(x) > 0$ for the remaining investment decisions of the IVC fund. As a result, as the carried interest becomes more valuable and the holding period becomes shorter, x^*_{IVC} increases. Again, the gap between x^*_{IVC} and x^*_{PVC} will be greater, and PVCs should be more willing to directly intervene on the equity financing market. Such behavior should however be also influenced by progress on investment execution *vis-à-vis* the investment period, as this may be determinant in securing management fees for the GP during the divestment period. If execution is low and the investment period is about to end, then x^*_{IVC} should decrease regardless of prospects on carried interest, as GPs are incentivized to maximize the amount of management fees to be earned during the divestment period, and which are usually a function of the acquisition cost of portfolio firms. In addition, fund raising plans or initiatives by GPs may also exert influence on current investment decision-making. These two effects are not captured in our framework from section 4.1.

4.4. Empirical evidence on the determinants of investment by Public Venture Capitalists

In this section we investigate whether some of the outcomes from our theoretical model introduced in section 4.1 and illustrated in section 4.2 are empirically supported. We will be specifically interested in examining the determinants of investment volumes carried by PVCs, taking into account that volumes stand for a more appropriate measure of earlier optimum investment timing than investment amounts.

4.4.1. Hypothesis

Grounded on the relevant literature contributions and on the underlying intuition of the framework we introduced in section 4.1, we posit the following set of theoretical propositions:

H1: *Co-investment volumes are positively correlated with the share of PVC investment in total PE and VC investment volumes in countries in which Governments are more inefficient.*

This follows our results from section 4.2.2, where we showed that the gap between x^*_{PVC} and x^*_{CO} was followed by a greater efficiency gap, measured by ϕ. This view is also grounded on Murray (2007) and Munari and Toschi (2015), who pointed out that one of the drivers for the establishment of PVC initiatives lays on leveraging private financing through co-investment on Leleux and Surlemont (2003), who revealed that PVCs do co-invest less than IVCs, and on Grilli and Murtinu (2014) and Grilli and Murtinu (2015), who revealed that firms in which PVCs and IVCs co-invested recorded better performance than those in which only PVCs invested.

H2: *Higher shares of PVC investment in total PE and VC investment volumes occur in countries in which Governments are more efficient.*

This is a corollary from H1 and it is also derived from our results in section 4.2.2. As we expect more inefficient Governments to have a greater share on PVC investment volumes through co-investment practices, we expect more efficient Governments to have a greater share of direct PVC investments.

H3: *Higher shares of PVC investment in total PE and VC investment volumes occur in countries in which a greater proportion of seed stage deals are observed.*

This follows our discussion in sections 4.2.3 and 4.3 on the impact of profit volatility over the x_{IVC}^{*} and x_{PVC}^{*}, especially for very high levels of volatility – as in the case of seed investments, which face a hyper-uncertain environment faced by early stage firms (Venkataraman, 1997).

H4: *Higher shares of PVC investment in total PE and VC investment volumes occur in countries in which taxation is greater.*

Governments with higher tax levels should be able to more easily profit from investment activities carried by PVCs and, therefore, as illustrated in 4.2.4, present a lower x_{PVC}^{*} which incentivizes direct PVC investment. Analogously, in countries where greater effective taxation rates are observed, we expect x_{IVC}^{*} to increase and IVCs to become more focused on opportunities yielding a more aggressive return profile, leaving room for PVC initiatives. This follows Keuschnigg and Nielsen (2007) who argued that corporate income taxes would postpone effort in SuFs.

In addition to H1, H2, H3 and H4, we introduce a set of other hypothesis, which essentially stand for control variables on examining the determinants of PVC volumes.

H5: *Higher shares of PVC investment in total PE and VC investment volumes occur in countries in which the macroeconomic environment is more positive.*

This theoretical hypothesis might be either regarded as a control variable, or as a corollary from H3, in the sense that it may also depicts the role that volatility inflicts on the PVC investment decision-making. Following the approach by the World Economic Forum, we take the *macroeconomic environment* as a measure of economic stability, given by Government budget balance, gross national savings, inflation, public debt and country credit rating.

H6: *Lower shares of PVC investment in total PE and VC investment volumes occur in countries in which companies have easier access to equity financing.*

This hypothesis controls for the *equity gap* that Governments may fill by launching PVC initiatives, when perceiving that companies find important restraints on raising capital (Beuselinck and Manigart, 2007). In fact, this hypothesis will allow us to depict whether perceived *market failures* on the equity financing market (Giacomo, 2004, Murray, 2007, van der Schans, 2015) actually motivate Governments to launch PVC initiatives.

H7: *Shares of PVC investment in total PE and VC investment volumes are uncorrelated to macroeconomic business cycles.*

We replicate this hypothesis introduced by Beuselinck and Manigart (2007), arguing that Governments intend to close potential *equity gaps* through PVC initiatives and therefore their investment pattern should not be driven by business cycles or return opportunities. Following their results and following the fact that their sample broadly replicates our sample of European countries, we do expect a similar outcome.

4.4.2. Data and method

Our dataset comprises a sample of European countries for which data on PVC investment is available on the annual statistical yearbook published by EVCA, including Austria, Belgium, Denmark, Finland, France, Germany, Hungary, Ireland, the Netherlands, Portugal, Spain, Sweden and the United Kingdom. As data covered the 2007 to 2014 period, a total number of 104 observations was collected. This subset of countries stands for 90.4% of total investment amounts and 89.6% of total investment volumes in Europe for the 2007-2014 period according to EVCA, and allowed us to compute the share of PVC investment volumes on the total PE and VC investment volumes for each the countries in our sample. This will stand for our dependent variable which we will name as $PVCVOL$. Data for the independent variables that follow our theoretical hypothesis was obtained through a range of different sources, which we summarized in Table 15.

Variable Name	Variable Description	Source	Supporting	Sign
Research Hypothesis				
COINV	Share of co-investment deals in total PE and VC deals, per country per year	EVCA Yearbook	H1	(+)
GOVINEFF	This is the inverse of the Government Efficiency indicator (*GOVEFF*), ranging from a scale from 1 to 7, per country and per year. This is computed by dividing 7 by the result obtained in *GOVEFF*.	World Economic Forum, Executive Opinion Survey		
GOVEFF	Annual composite indicator of Government Efficiency available for each of the countries in our dataset, ranging from a scale from 1 to 7, assessing (i) the wastefulness of government spending, (ii) the burden of government regulation, (iii) the efficiency of legal framework in setting disputes, (iv) the efficiency of legal framework in challenging regulations, and (v) the transparency of government policymaking	World Economic Forum, Executive Opinion Survey	**H2**	(+)
SEED	Share of seed deals in total PE and VC deals, per country and per year	EVCA Yearbook	**H3**	(+)
TAXGDP	Tax and Social Security contributions as a percentage of gross domestic product (GDP), per country and per year	Eurostat	**H4**	(+)
Control Variables				
MACRO	Annual composite indicator of Macroeconomic Environment, available for each of the countries in our dataset, summarizing (i) public budget balance, (ii) gross national savings, (iii) inflation, (iv) government debt and (v) country credit rating	World Economic Forum, Global Competitiveness Report	**H5**	(+)
VCACCESS	Annual indicator available for each of the countries in our dataset, ranging from 1 (extremely difficult) to 7 (extremely easy), picturing how easy it is for Entrepreneurs with innovative projects to find venture capital	World Economic Forum, Executive Opinion Survey	**H6**	(-)
$YEARS_{2007\text{-}2014}$	Dummy variables for each of the years on the sample, equal to 1 if data refers to a given year and equal to 0 otherwise	-	**H7**	**Not significant**

Table 15. Variables definition

As for Government efficiency and access to equity financing, we relied on survey data rendered by the World Economic Forum, which provides annual records for each of the countries included in our sample. While there are several quantitative measures of Government efficiency on the literature (Afonso et al., 2005, Afonso et al., 2010, Gao, 2015, Adam et al., 2011), no consistent data and methodology exists on this indicator for a time series ranging from 2007 to 2014. Differently, ease of access to equity financing is not an observable variable and, as a result, proxies or survey data had to be introduced in our empirical investigation.

Similarly to Beuselinck and Manigart (2007), given that our sample covers eight observation years across thirteen different European countries, we set up a panel data regression, with the purpose of mitigating problems of biased and inconsistent parameters in cross-sectional time-series models (Baltagi, 2008). We run three linear regression models, specified as follows[59]:

(I) $$PVCVOL_{ct} = \beta_{0ct} + \beta_{1ct}COINV \times GOVINEFF + \beta_{2ct}GOVEFF + \beta_{3ct}SEED + \beta_{4ct}TAXGDP + \beta_{5ct}MACRO + \beta_{6ct}VCACCESS + \beta_{7ct}2007 + \cdots \beta_{8ct}2014 + \varepsilon_{ct} \quad (4.49)$$

(II) $$PVCVOL_{ct} = \beta_{0ct} + \beta_{1ct}COINV \times GOVINEFF + \beta_{2ct}GOVEFF + \beta_{3ct}SEED + \beta_{4ct}TAXGDP + \beta_{5ct}MACRO + \beta_{6ct}VCACCESS + \varepsilon_{ct} \quad (4.50)$$

(III) $$PVCVOL_{ct} = \beta_{0ct} + \beta_{1ct}COINV \times GOVINEFF + \beta_{2ct}SEED + \beta_{3ct}TAXGDP + \beta_{4ct}MACRO + \beta_{5ct}VCACCESS + \varepsilon_{ct} \quad (4.51)$$

Regression (I) reflects the whole set of theoretical hypothesis described in section 4.3 Considering that PVC investment volumes are expected not to be correlated with economic cycle, we run regression (II), in which we exclude the dummy variables on investment years. Finally, as H2 might be conceived as a corollary of H1, in regression (III) we exclude its underlying variable $GOVEFF$.

4.4.3. Results

We show regression results in Table 16[60]. As expected, macroeconomic cycle depicted by dummy variables on investment years does not have any significant statistical impact over

[59] For ease of implementation, variables *PVCVOL*, *COINV*, *SEED* and *TAXGDP* have been multiplied by 100 in the four models. This should be taken into account when interpreting the regression coefficients.

[60] We ran multicollinearity tests on regression results, which revealed that independent variables are not significantly correlated. VIF levels among independent variables in all regressions stood below the 10.0 threshold suggested by Hair et al. (2010). In addition, we performed robustness tests with transformations on $TAXGDP$, $COINV \times GOVINEFF$, $MACRO$, $SEED$and $PVCVOL$. Regression results were essentially confirmed,

$PVCVOL$ in regression (I). Coefficients for $COINV{\times}GOVINEFF$, $GOVEFF$, $SEED$, $MACRO$, $TAXGDP$ and $VCACCESS$ hold the predicted signs, although coefficient on $TAXGDP$ only reveals statistical significance at 87.4%. Therefore, we found no support for H4. In addition, regression coefficient on $COINV{\times}GOVINEFF$ was proved to be significant at 15.2%, which does not allow us to obtain strong statistical evidence on H1.

	Regression (I)			**Regression (II)**			**Regression (III)**			**Regression (II)** *ex-TAXGDP*		
Variables	*β*	Standard Error	*t-value*	*Β*	Standard Error	*t-value*	*β*	Standard Error	*t-value*	*β*	Standard Error	*t-value*
(Constant)	-23.85***	8.45	-2.82	-24.89***	7.86	-3.17	-23.50***	8.03	-2.93	-24.32***	6.95	-3.50
Research Hypothesis												
COINV x *GOVINEFF*	0.04♯	0.03	1.45	0.03	0.03	1.19	0.05*	0.03	1.71	0.03	0.03	1.19
GOVEFF	4.37**	2.01	2.18	3.78**	1.57	2.41	-	-	-	3.78**	1.56	2.42
SEED	0.67***	0.08	7.99	0.68***	0.08	8.63	0.73***	0.08	9.31	0.68***	0.08	8.68
TAXGDP	0.01	0.17	0.07	0.03	0.16	0.16	0.02	0.16	0.10	-	-	-
Control Variables												
MACRO	4.84***	1.90	2.56	4.17**	1.82	2.29	5.72***	1.74	3.29	4.28***	1.67	2.56
VCACCESS	-4.12*	2.26	-1.82	-2.87**	1.35	-2.13	-1.19	1.18	-1.01	-2.88**	1.34	-2.16
2008	-2.78	3.02	-0.92	-	-	-	-	-	-	-	-	-
2009	-1.54	3.60	-0.43	-	-	-	-	-	-	-	-	-
2010	-5.22	4.07	-1.28	-	-	-	-	-	-	-	-	-
2011	-4.84	3.95	-1.23	-	-	-	-	-	-	-	-	-
2012	-1.00	4.02	-0.25	-	-	-	-	-	-	-	-	-
2013	-2.17	4.13	-0.53	-	-	-	-	-	-	-	-	-
2014	-2.67	3.91	-0.68	-	-	-	-	-	-	-	-	-
Observations		*104*			*104*			*104*			*104*	
R^2		*0.630*			*0.613*			*0.590*			*0.613*	
Adjusted R^2		*0.577*			*0.589*			*0.573*			*0.593*	

***, **, * and ♯ denote statistical significance at the 1%, 5%, 10% and 15% levels respectively

Table 16. Regression results

This pattern is confirmed in regression (II), where $TAXGDP$ remained statistically insignificant and $COINV{\times}GOVINEFF$ lost statistical significance to 23.7%, even though maintaining the expected sign. Regression (II) without $TAXGDP$ resembles the same results,

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with $TAXGDP$ and $COINV{\times}GOVINEFF$ proving to be statistically insignificant. In addition, residuals statistics revealed that outliers are not impacting regression results. Centered leverage values range between 0.006 and 0.207 with a mean of 0.048, while Cook's distance mean was 0.011 on a 0.000 to 0.142 range.

providing no clear evidence on the impact of $COINV \times GOVINEFF$ on $PVCVOL$. Still, we obtain supporting evidence for H2, H3, H5, H6 and H7.

By excluding $GOVEFF$ in regression (III) we attempt to highlight the effect of $COINV \times GOVINEFF$ over $PVCVOL$. In this case, $COINV \times GOVINEFF$ maintains the expected sign and becomes statistical significant at 10.0%, providing some supporting evidence for H1. As $VCACCESS$ becomes insignificant in this specification, we argue that these results may reveal that PVC-sponsored co-investment initiatives may actually eliminate restrictions on the access to equity financing.

Four important considerations should be summarized on regression results. Firstly, we acknowledge that our data poses four relevant restraints to our investigation, providing room for further research on the field: it depicts a relative short period of time which is largely affected by the 2007-2008 financial crisis, it is restricted to the European market, it is only able to capture Government intervention on the equity financing market through PVC (i.e., *direct* and not *indirect*) initiatives and it simultaneously comprises deals on the PE (or *later-stage*) and on the VC (or *early-stage*) segments. Depicting whether the determinants of PVC investment between these two segments differ stands for a relevant empirical assessment of the theoretical hypothesis we derived. Second, our results clearly show that taxation is not a relevant variable in determining PVC investment volumes, which is in contrast with H4. This means that Governments do not define their policy for intervening on the equity financing markets based on taxation, nor that they make use of PVC initiatives grounded on lower pre-tax return on investment requirements. Third, we found no strong evidence supporting H1, even though we obtained supporting evidence for H2 and the coefficient on $COINV \times GOVINEFF$ revealed the expected sign. This suggests that the most inefficient Governments do not make use of co-investment mechanisms to overcome their efficiency gap, which may, in fact, be regarded as a feature of the most inefficient Governments. In this sense, even though it may exist an economic rationale for the most inefficient Governments to support co-investment, this may not be empirically observed, since these shall not to choose such efficiency enhancing strategies. Moreover, and to the best of our knowledge, when co-investing alongside IVCs, PVCs do not usually offer any *equity enhancement* effects.

According to our findings from sections 4.2 and 4.3, this *equity enhancement* – given by a share premium – should be significant so that co-investment becomes possible and actually anticipates optimum investment timing. Forth, we call attention to the role that the *macroeconomic environment* – portrayed by H5 – plays over $PVCVOL$ and most likely on the investment volumes of IVCs as well. Economic stability – measured by public debt, balance and rating, alongside inflation and saving rate in our regression – then seems to form a common ground for the establishment of relevant PVC initiatives.

Overall, regression results provide empirical support to H2, H3, H5, H6 and H7, of which H2 and H3 are directly derived from our theoretical framework, while H5, H6 and H7 confirmed previous literature findings or inferences. No clear statistical support was obtained for H1.

4.5. Chapter summary

In this Chapter we depicted the decision faced by PVCs and Governments on the best mechanism to promote the investment in SuFs: should PVCs invest directly in SuFs, should they co-invest alongside IVCs, should they let IVCs invest, or should Governments directly subsidize SuFs with the purpose of triggering investment from IVCs, instead?

Focusing on the economic payoffs earned by PVCs and IVCs, we modelled each of the alternative investing mechanisms as a real option and illustrate that co-investing might be the most effective mechanism in anticipating optimum investment timing. Grounded on this theoretical framework, we listed a set of both public policy and managerial implications, derive a set of empirically testable propositions on the determinants of PVC investment volumes and analyse their prevalence on a sample of European countries.

Even though taxation proved not to be correlated with PVC investment volumes, the remaining results provided overall empirical support to our theoretical hypothesis.

5. Conclusion

In this thesis, we introduced a set of theoretical models to support decision-making processes in Entrepreneurial Financing (EF) decisions.

In the first model introduced in Chapter 2, we developed a real-options framework with the purpose of understanding EF decisions. From a rationale economic and profit-maximization perspective, we discussed the foundations of EF decisions and how expectations on profit growth influence their outcomes. In addition, we demonstrated how a share premium (or discount) should be computed so that the Entrepreneurial Firm is able to obtain joint support from Entrepreneurs and Venture Capitalists (VCs) to proceed with a given Growth Opportunity and showed how asymmetries on future profit growth expectations may actually contribute to aligning Entrepreneurs and VCs in supporting them. Based on these findings, we are able to design a set of theoretical propositions, which may serve as a ground for further empirical research.

First, we point out that Entrepreneurs would never be willing to bring in an external equity provider to the Entrepreneurial Firm, unless profit growth expectations are such that would allow them to offset losses arising from ownership dilution. This is a straight forward explanation to why VC-backed firms may evidence higher growth rates than non-VC-backed firms: only Entrepreneurial Firms with high growth prospects seek for external equity – all the other would prefer to fund their growth plans internally. Therefore, we suggest that growth patterns of VC-backed firms might be intrinsically affected by a *selection effect* rather than a *treatment effect*, as discussed by Bertoni et al. (2011).

Second, we conjecture that VCs are not willing to underwrite an equity issuance on the Entrepreneurial Firm unless profit growth expectations are such that allow them to offset the capital dilution loss they bear, from potentially retaining a lower share of the Entrepreneurial Firm than of the total capital outlay required by the Growth Opportunity. This may eventually explain a driving force of venture valuation on subsequent funding rounds: VCs may only be willing to back a given Entrepreneurial Firm and accept a given ownership, if their profit growth expectations are greater than those of the investors on previous rounds, or if their perception on business risk of the Entrepreneurial Firm is lower than the one held by investors on the previous equity rounds.

Third, we highlighted that Entrepreneurs and VCs may reach an agreement even when holding different expectations on future profit growth. On the one hand, and within the framework we introduced, such an agreement is feasible since the Entrepreneur is also able to dilute the VC through the pre-existent assets in place. On the other hand, this supports the idea that Growth Opportunities and Entrepreneurial Firms are not worth by themselves, but that they are instead worth something to someone, whose approach to venture valuation may not exactly mimic the intrinsic value of the underlying assets. In this sense, one could argue that deal prices are just mere meeting points between buyers and sellers, rather than an intrinsic expression of fair value of the underlying assets.

Lastly, we showed that the greater the gap between Entrepreneurial and VC expectations on profit growth, the more likely an agreement is expected to be reached. The higher the Entrepreneur's profit growth expectations are, the more is she or he willing to trade-off a lower shareholding, for the profit brought by the envisaged Growth Opportunity and, in addition, the more valuable the initial capital she or he deployed in the Entrepreneurial Firm is understood to be.

We reckon that there are several rooms for improving and extending the framework we designed. First, we are taking a pure profit-maximizing view of the EF process, even though there is a vast set of deal drivers which were not accounted for. In particular, we would be interested in deepening our understanding on the Entrepreneur's preferences for ownership control and liquidity (Hellman, 1998), through which we could explore a trade-off between the two, in a framework where we would allow VCs not only to underwrite equity but also to acquire ownership to the Entrepreneur. Self-determination, decision control risk and perceived decision control risk are also cognitive drivers of the Entrepreneurial decision-making process, which might be introduced in our framework. Secondly, given that we showed that an external equity provider may join the Entrepreneurial Firm even if revealing different expectations on profit growth, we argue that this might be a source for potential conflicting goals on the post-deal stage, which could be studied within the framework we derived. Thirdly, we analysed how share premiums or discounts may contribute to an agreement between Entrepreneurs and VCs on a single stage negotiation process and instantaneous Growth Opportunity execution. However, by extending the framework over additional stages, we may introduce into our framework additional contractual mechanisms (such as share repurchases) or alternative funding strategies, comprising staged capital infusions by VCs (Lukas et al., 2016), or a staged growth strategy execution by the Entrepreneur (Schwienbacher, 2007), in addition to the Contingent Payment Mechanisms (CPMs) we investigated in Chapter 3.

In the second theoretical model, which we presented in Chapter 3, we showed that CPMs are becoming more relevant on Mergers & Acquisitions (M&A) deal volumes and that may also be a relevant tool for enabling EF decisions. Our novelty approach starts by acknowledging that there are different types of CPMs, which we propose to distinguish according to their amount and due date. We then identify four major types of CPMs and separately value each one of them, following the option analogy. After extending the real options framework from Chapter 2, we derived the optimum CPM design that would allow Entrepreneurs and VCs to jointly support a given Growth Opportunity and illustrated model outcomes through a numerical example, whose results allowed us to compare different CPMs and revealed to be consistent with previous literature findings.

While optimum investment timing is unaffected by CPM design, the value of each of these mechanisms is not the same. From the perspective of the Entrepreneur, and controlling for the likelihood of the profit benchmark to be reached by the Entrepreneurial Firm, *variable* amount CPMs revealed to be more valuable than *fixed* amount CPMs, as CPMs due *at hit* revealed to be more valuable than those which are due *at term*.

Regarding future research paths and following the work by Cain et al. (2011), Barbopoulos and Sudarsanam (2012) and Lukas and Heimann (2014), we understand that further empirical research is needed for testing some of the theoretical propositions on CPMs within an EF context, instead of a M&A context. Alternatively, and even within an M&A

setting, empirical research could investigate how differently *variable* amount and *fixed* amount CPMs are set and assess whether they are differently perceived by financial markets as, to the best of our knowledge, literature has not yet explored how differently these two alternative mechanisms may perform. The same empirical approach could follow for CPMs which are due *at hit* and for CPMs which are due *at term*. From an analytical perspective, the range of alternative CPMs extends beyond the four major types we introduced, as shown by Reum and Steele (1970), providing room for additional extensions of the framework we introduced.

On Chapter 4, we derived the third theoretical model, comprising a real options framework to investigate how Independent Venture Capitalists (IVCs) and Public Venture Capitalists (PVCs) screen their investment decisions and compared their optimum investment timing. Even when taking into account the incremental revenues which might be lost by not letting IVCs undertaking the investment, we showed that PVCs should be willing to invest earlier than IVCs, not only as they are neutral to taxation (unlike IVCs), but also as they present substantially lower profit triggers when facing investment opportunities featuring high volatility on its profit generation.

Still, given that PVCs hold an efficiency gap when compared to IVCs, Governments might be willing to eliminate this gap by letting PVCs co-invest alongside IVCs or by directly subsidizing Start-up Firms (SuFs) and letting IVCs drive the investment. We showed that both these mechanisms are able to significantly anticipate optimum investment timing, especially when they provide an *equity enhancement* effect to IVCs. Our results highlight that it is not enough for Governments to commit capital on the equity financing market to foster IVC investment. Filling some of the financing gaps demands governments to provide such *equity enhancement* effect to IVCs or private investors in general. In turn, co-investing proved to be more effective in anticipating optimum investment timing than subsidization, except for very high levels of uncertainty, where the opposite is true.

Our framework also offered insights on some of the key drivers of decision-making taken into account by IVCs and how these may influence their optimum investment timing. Surprisingly, we revealed that increasing profit sharing on carried interest may actually postpone optimum investment timing of the IVC fund, and that increasing its underlying hurdle rate anticipates optimum investment timing. As carried interest becomes more valuable when greater profit sharing is in place, General Partners (GPs) postpone their optimum investment timing with the purpose of maximizing the probability of actually earning their carried interest. The opposite effect is observed concerning the hurdle rate. In addition, increasing the equity stake held by GPs on their IVC funds under management was revealed as one of the most significant variables anticipating optimum investment timing.

We tested whether our theoretical predictions regarding PVC investment volumes would obtain empirical support, on a sample of thirteen European countries for the 2007-2014 period. Even though taxation proved not to be correlated with PVC investment volumes, the remaining results offered overall empirical support to our theoretical hypothesis. We also showed that PVC investment volume is not correlated with business cycle and that PVC investment volume is intended to suppress perceived gaps on the equity financing market.

We end by pointing out some relevant insights on future research paths. Although we discussed the *direct* involvement of Governments as a Private Equity (PE) or VC investor (by establishing itself as a GP), we are not able to test – due to data insufficiency – the determinants of their intervention as a Limited Partner (LP), nor the impact of its aggregate direct and indirect intervention on the equity financing market (Buzzacchi et al., 2013, Jääskeläinen et al., 2007).

Potential effects rising from competition on the equity financing market between PVCs and IVCs might be further investigated. We may argue that these could anticipate optimum investment timing. VC *fund economics* also provides GPs running IVC funds with strong incentives to abandon the small business segment, and focus on bigger deals and raising greater amounts of capital per fund. As management fees are proportionately paid to overall fund size, they provide riskless earnings to GPs. Therefore, independent GPs may join the equity financing market by raising small amounts of funds and then, if they succeed, target bigger fund sizes and bigger deals on subsequent funds.

We did not also considered the full range of public stimuli that Governments may offer to foster investment volumes. These may include tax credits, tax allowances, Social Security bonus, or tax based subsidies dependent on incremental employment or allocated to research and development initiatives (Peneder, 2008). Similarly, we have not discussed the nature and effectiveness of different types of Government support (Keuschnigg and Nielsen, 2001, Lee et al., 2013). For example, Lerner (2002) found evidence that a prevalent characteristic among under-achieving firms is the existence of research grants from numerous Government sources, which allow them to avoid accountability.

Our framework assumed that Entrepreneurs did not influence the decision-making process of IVCs and PVCs, except for the amount of capital they would be willing to provide to the SuF. Their option to invest in her or his own SuF may also be modelled as a real option and we would then extend our framework to a two or three party alignment process, in which the Entrepreneur, the IVC and/ or the PVC would jointly determine the outcomes of this EF process.

Finally, PVCs are argued to have a relevant role in mitigating some of the risks rising from information asymmetry (Lerner, 1999, Lerner, 2002), providing certification to investee firms, which may be specially relevant for raising capital on future equity rounds. Considering that staging is one of the features of VC investment processes (Dahiya and Ray, 2012, Elitzur and Gavious, 2003, Gompers, 1995, Hsu, 2010, Leisen, 2012, Li, 2008, Lukas et al., 2016, Tian, 2011, Wang and Zhou, 2004), the role of PVCs as enablers of future investment rounds could also be depicted.

Overall, grounded on a real options approach, we explored the EF decision in a setting where two parties – an Entrepreneur and a VC, or a IVC and a PVC – assess an opportunity to invest in an Entrepreneurial Firm (as in Chapters 2 and 3) or in a SuF (as in Chapter 4). As mentioned in Chapter 1, we face these three elements as the "*Building Blocks of Entrepreneurial Financing Decisions*".

From a broader perspective, our research results point out that financial deal terms covered on EF decisions do not depend as much on the *intrinsic value* of Growth Opportunity they support, but rather on the *relative position* that each of the parties holds against such Growth Opportunity. On Chapters 2 and 3, we learned that optimum up-front share premium and optimum contingent payments depend on the expectations on future profit growth that each of the parties anticipates, on the contribution that each of the parties will provide to the Growth Opportunity and on the current profit generation of the Entrepreneurial Firm assigned to the Entrepreneur. On Chapter 4, we learned that co-investment decisions involving IVCs and PVCs, or subsidization decisions from Governments to SuFs, significantly depend on the specific features of the investment decision process of each of the parties – such as public sector inefficiency or performance incentives – and not only on the specific features of the option to invest in the Growth Opportunity held by the SuF, including volatility on profit generation or capital requirements.

The concept of *alignment* is then determinant to understand the outcomes of EF decisions and demands an understanding beyond the traditional perspective on capital budgeting, whose focus is grounded on assessing the *intrinsic value* of Growth Opportunities and not the *transactional context* in which it takes place. Given the room for further theoretical developments we pointed out on the literature throughout the thesis, and given the specificities underlying the EF decision, we end by advocating that *Entrepreneurial Finance* should form the grounds of an autonomous branch of science, bridging the analytical tools and supply-side view from *Economics* and *Finance* along with the demand-side view from *Entrepreneurship*.

References

Adam, A., Delis, M. and Kammas, P. (2011) 'Public sector efficiency: leveling the playing field between OECD countries', *Public Choice,* 146(1-2), pp. 163-183.

Adelino, M., Schoar, A. and Severino, F. (2015) 'House prices, collateral, and self-employment', *Journal of Financial Economics,* 117(2), pp. 288-306.

Afonso, A., Schuknecht, L. and Tanzi, V. (2005) 'Public sector efficiency: An international comparison', *Public Choice,* 123(3-4), pp. 321-347.

Afonso, A., Schuknecht, L. and Tanzi, V. (2010) 'Public sector efficiency: evidence for new EU member states and emerging markets', *Applied Economics,* 42(17), pp. 2147-2164.

Alperovych, Y. and Hübner, G. (2012) 'Incremental impact of venture capital financing', *Small Business Economics,* 41(3), pp. 651-666.

Andrieu, G. and Groh, A. P. (2012) 'Entrepreneurs' financing choice between independent and bank-affiliated venture capital firms', *Journal of Corporate Finance,* 18(5), pp. 1143-1167.

Ang, J. S. (1992) 'On the theory of finance for privately held firms', *Journal of Small Business Finance,* 1(3), pp. 185-203.

Arthurs, J. D. and Busenitz, L. W. (2003) 'The boundaries and limitations of agency theory and stewardship theory in the VC-entrepreneur relationship', *Entrepreneurship Theory and Practice,* 28, pp. 145–162.

Avnimelech, G., Rosiello, A. and Teubal, M. (2010) 'Evolutionary interpretation of venture capital policy in Israel, Germany, UK and Scotland', *Science and Public Policy,* 37(2), pp. 101-112.

Avnimelech, G. and Teubal, M. (2008) 'From direct support of business sector R&D/innovation to targeting venture capital/private equity: A catching-up innovation and technology policy life cycle perspective', *Economics of Innovation and New Technology,* 17(1), pp. 153-172.

Axelson, U., Strömberg, P. and Weisbach, M. S. (2009) 'Why are buyouts levered? The financial structure of private equity', *The Journal of Finance,* 64(4), pp. 1549-1582.

Ayayi, A. (2004) 'Public policy and venture capital: the Canadian labor-sponsored venture capital funds', *Journal of Small Business Management,* 42(3), pp. 335-345.

Baker, M. and Wurgler, J. (2002) 'Market timing and capital structure', *The Journal of Finance,* 57(1), pp. 1-32.

Baltagi, B. (2008) *Econometric analysis of panel data.* John Wiley & Sons.

Barbopoulos, L. and Sudarsanam, S. (2012) 'Determinants of earnout as acquisition payment currency and bidder’s value gains', *Journal of Banking & Finance,* 36(3), pp. 678-694.

Barbosa, D., Carvalho, V. M. and Pereira, P. J. (2016) 'Public stimulus for private investment: An extended real options model', *Economic Modelling,* 52, pp. 742-748.

Baron, R. A. and Markman, G. D. (1999) 'Beyond social capital: the role of entrepreneurs' social competence in their financial success', *Journal of Business Venturing,* 18(41–60).

Bascha, A. and Walz, U. (2000) 'Convertible securities and optimal exit decisions in venture capital finance', *Journal of Corporate Finance,* 7, pp. 285–306.

Benson, D. and Ziedonis, R. H. (2010) 'Corporate venture capital and the returns to acquiring portfolio companies', *Journal of Financial Economics,* 98(3), pp. 478-499.

Bernstein, S., Giroud, X. and Townsend, R. R. (2015) 'The impact of venture capital monitoring', *The Journal of Finance*, pp. Accepted manuscript on-line.

Bertoni, F., Colombo, M. G. and Grilli, L. (2011) 'Venture capital financing and the growth of high-tech start-ups: disentangling treatment from selection effects', *Research Policy,* 40(7), pp. 1028-1043.

Bertoni, F., D'Adda, D. and Grilli, L. (2016) 'Cherry-picking or frog-kissing? A theoretical analysis of how investors select entrepreneurial ventures in thin venture capital markets', *Small Business Economics,* 46(3), pp. 391-405.

Beuselinck, C. and Manigart, S. (2007) 'Public venture capital across Europe: A 15-year perspective', in Gregoriou, G.N., Kooli, M. & Kraeussl, R. (eds.) *Venture Capital in Europe*. Great Britain: Butterworth-Heinemann, pp. 19-31.

Bitler, M. P., Moskowitz, T. J. and Vissing-Jorgensen, A. (2005) 'Testing agency theory with entrepreneur effort and wealth', *Journal of Finance,* 60(2), pp. 539-576.

Black, F. and Scholes, M. (1973) 'The pricing of options and corporate liabilities', *The Journal of Political Economy,* 81(3), pp. 637-654.

Brander, J. A., Du, Q. and Hellmann, T. (2014) 'The effects of Government-Sponsored Venture Capital: international evidence', *Review of Finance,* 19(2), pp. 571-618.

Brander, J. A., Egan, E. J. and Hellman, T. 2008. Government sponsored versus venture capital: Canadian evidence. *NBER Working Paper.* National Bureau of Economic Research.

Braun, R., Jenkinson, T. and Stoff, I. 2015. How persistent is private equity performance: evidence from deal-level data. *Working Paper.*

Brennan, M. J. and Schwartz, E. S. (1985) 'Evaluating natural resource investments', *The Journal of Business,* 58(2), pp. 135-157.

Bruner, R. F. and Stiegler, S. (2014) 'Technical note on structuring and valuing incentive payments in M&A: earnouts and other contingent payments to the seller', *Darden Business Publishing*, Available: University of Virginia. DOI: doi: 10.1225/UV0465.

Brush, C. G., Edelman, L. F. and Manolova, T. S. (2012) 'Ready for funding? Entrepreneurial ventures and the pursuit of angel financing', *Venture Capital,* 14(2-3), pp. 111-129.

Busenitz, L. W., Moesel, D. D., Fiet, J. O. and Barney, J. B. (1998) 'The framing of perceptions of fairness in the relationship between VCs and new venture teams', *Entrepreneurship Theory and Practice*, pp. 5-21.

Butler, I., Galassi, G. and Ruffo, H. (2015) 'Public funding for startups in Argentina: an impact evaluation', *Small Business Economics,* 46(2), pp. 295-309.

Buzzacchi, L., Scellato, G. and Ughetto, E. (2013) 'The investment strategies of publicly sponsored venture capital funds', *Journal of Banking & Finance,* 37(3), pp. 707-716.

Cable, D. M. and Shane, S. (1997) 'A prisoner's dilemma approach to entrepreneur-venture capitalist relationship', *Academy of Management Review,* 22(1), pp. 142-176.

Cain, M. D., Denis, D. J. and Denis, D. K. (2011) 'Earnouts: a study of financial contracting in acquisition agreements', *Journal of Accounting and Economics,* 51(1-2), pp. 151-170.

Carpentier, C., L'Her, J. F. and Suret, J. M. (2010) 'Stock exchange markets for new ventures', *Journal of Business Venturing,* 25(4), pp. 403-422.

Carter, R. B. and Van Auken, H. E. (1994) 'Venture capital firms' preferences for projects in particular stages of development', *Journal of Small Business Management,* 32(1), pp. 60-73.

Casamatta, C. (2003) 'Financing and advising: optimal financial contracts with venture capitalists', *Journal of Finance,* 58(5), pp. 2059-2085.

Caselli, S., Gatti, S. and Visconti, M. (2006) 'Managing M&A risk with collars, earn-outs and CVRs', *Journal of Applied Corporate Finance,* 18(4), pp. 91-104.

Chaganti, R., DeCarolis, D. and Deeds, D. (1995) 'Predictors of capital structure in small ventures', *Entrepreneurship, Theory and Practice,* 20(2), pp. 7-18.

Chatterji, A. K. and Seamans, R. C. (2012) 'Entrepreneurial finance, credit cards, and race', *Journal of Financial Economics,* 106(1), pp. 182-195.

Chemmanur, T. and Chen, Z. (2014) 'Venture Capitalists versus Angels: the dynamics of private firm financing contracts', *Review of Corporate Finance Studies,* 3(1-2), pp. 39-86.

Chemmanur, T. and Fulghieri, P. (2014) 'Entrepreneurial finance and innovation: an introduction and agenda for future research', *Review of Financial Studies,* 27(1), pp. 1-19.

Chen, J. F., Liao, W. M. and Lu, C. C. (2012) 'The effects of Public Venture Capital investments on corporate governance: evidence from IPO firms in emerging markets', *Abacus-a Journal of Accounting Finance and Business Studies,* 48(1), pp. 86-103.

Choi, A. H. 2015. Facilitating mergers and acquisitions with earnouts and purchase price adjustments. *Working Paper.* SSRN.

Cochrane, J. H. (2005) 'The risk and return of venture capital', *Journal of Financial Economics,* 75(1), pp. 3-52.

Communication from the European Commission to the Council, to the European Parliament, to the Committee of the Regions, and to the European and Social Committee: an action plan to improve access to finance for SMEs. 2011. Brussels: European Commision.

Cornell, B. and Shapiro, A. C. (1987) 'Corporate stakeholders and corporate-finance', *Financial Management,* 16(1), pp. 5-14.

Croce, A., Martí, J. and Murtinu, S. (2013) 'The impact of venture capital on the productivity growth of European entrepreneurial firms: 'Screening' or 'value added' effect?', *Journal of Business Venturing,* 28(4), pp. 489-510.

Cumming, D. (2007) 'Government policy towards entrepreneurial finance: Innovation investment funds', *Journal of Business Venturing,* 22(2), pp. 193-235.

Cumming, D. (2008) 'Contracts and exits in venture capital finance', *Review of Financial Studies,* 21(5), pp. 1947-1982.

Cumming, D. and Johan, S. A. b. (2007) 'Advice and monitoring in venture finance', *Financial Markets and Portfolio Management,* 21(1), pp. 3-43.

Cumming, D., Schmidt, D. and Walz, U. (2010) 'Legality and venture capital governance around the world', *Journal of Business Venturing,* 25(1), pp. 54-72.

Cumming, D. J. and Fischer, E. (2012) 'Publicly funded business advisory services and entrepreneurial outcomes', *Research Policy,* 41(2), pp. 467-481.

Cumming, D. J. and MacIntosh, J. G. (2006) 'Crowding out private equity: Canadian evidence', *Journal of Business Venturing,* 21(5), pp. 569-609.

Cuny, C. J. and Talmor, E. (2007) 'A theory of private equity turnarounds', *Journal of Corporate Finance,* 13(4), pp. 629-646.

Da Rin, M., Nicodano, G. and Sembenelli, A. (2006) 'Public policy and the creation of active venture capital markets', *Journal of Public Economics,* 90(8-9), pp. 1699-1723.

Dahiya, S. and Ray, K. (2012) 'Staged investments in entrepreneurial financing', *Journal of Corporate Finance,* 18(5), pp. 1193-1216.

Datar, S., Frankel, R. and Wolfson, M. (2001) 'Earnouts: the effects of adverse selection and agency costs on acquisition techniques', *The Journal of Law, Economics, & Organization,* 17(1), pp. 201-238.

de Bettignies, J.-E. and Brander, J. A. (2007) 'Financing entrepreneurship: bank finance versus venture capital', *Journal of Business Venturing,* 22(6), pp. 808-832.

De Clercq, D. and Fried, V. H. (2005) 'Executive forum: how entrepreneurial company performance can be improved through venture capitalists' communication and commitment', *Venture Capital,* 7(3), pp. 285-294.

De Clercq, D. and Sapienza, H. J. (2001) 'The creation of relational rents in the venture capitalist-entrepreneur dyads', *Venture Capital,* 3(2), pp. 107-127.

de Meza, D. and Southey, C. (1996) 'The borrowers' curse: optimist, finance and entrepreneurship', *The Economic Journal,* 106, pp. 375-386.

del-Palacio, I., Zhang, X. T. T. and Sole, F. (2012) 'The capital gap for small technology companies: public venture capital to the rescue?', *Small Business Economics,* 38(3), pp. 283-301.

Del Roccili, J. A. and Fuhr Jr, J. P. (2001) 'The pros and cons of earnouts', *Journal of Financial Service Professionals,* 55(6), pp. 88-93.

Dimov, D. P. and Shepherd, D. A. (2005) 'Human capital theory and venture capital firms: exploring “home runs” and “strike outs”', *Journal of Business Venturing,* 20(1), pp. 1-21.

Dixit, A. K. and Pindyck, R. S. (1994) *Investment under uncertainty.* Princeton, USA: Princeton University Press.

Elitzur, R. and Gavious, A. (2003) 'A multi-period game theoretic model of venture capitalists and entrpreneurs', *European Journal of Operational Research,* 144, pp. 440–453.

Elitzur, R. and Gavious, A. (2011) 'Selection of entrepreneurs in the venture capital industry: an asymptotic analysis', *European Journal of Operational Research,* 215(3), pp. 705-712.

Elston, J. A. and Audretsch, D. B. (2009) 'Financing the entrepreneurial decision: an empirical approach using experimental data on risk attitudes', *Small Business Economics,* 36(2), pp. 209-222.

EVCA (2013) *Tax benchmark study 2012*: European Venture Capital Association & KPMG. Available at: http://www.svca.se/wp-content/uploads/2014/03/EVCA-Tax-Benchmark2012.pdf.

EVCA (2014) *Pan-European private equity performance benchmarks study*: European Venture Capital Association & Thomson Reuters. Available at: http://www.investeurope.eu/media/199202/2013-pan-european-private-equity-performance-benchmarks-study-evca-thomson-reuters-final-version.pdf.

Ewens, M. and Rhodes-Kropf, M. (2015) 'Is a VC partnership greater than the sum of its partners?', *Journal of Finance,* 70(3), pp. 1081-1113.

Faccio, M. and Masulis, R. W. (2005) 'The choice of payment method in European mergers and acquisitions', *Journal of Finance,* 60(3), pp. 1345-1388.

Fairchild, R. (2011) 'An entrepreneur's choice of venture capitalist or angel-financing: a behavioral game-theoretic approach', *Journal of Business Venturing,* 26(3), pp. 359-374.

Fisher, P. (1988) 'State Venture Capital Funds as an Economic Development Strategy', *Journal of the American Planning Association,* 54(2), pp. 166-177.

Folta, T. B., Delmar, F. and Wennberg, K. (2010) 'Hybrid Entrepreneurship', *Management Science,* 56(2), pp. 253-269.

Franzoni, F., Nowak, E. and Phalippou, L. (2012) 'Private equity performance and liquidity risk', *Journal of Finance,* 67(6), pp. 2341-2373.

Fraser, S., Bhaumik, S. K. and Wright, M. (2015) 'What do we know about entrepreneurial finance and its relationship with growth?', *International Small Business Journal,* 33(1), pp. 70-88.

Fried, V. H. and Hisrich, R. D. (1994) 'Toward a model of venture capital investment decision making', *Financial Management,* 23(3, Venture Capital Special Issue), pp. 28-37.

Gao, Y. 2015. Democracy and public sector efficiency: an empirical study on the cross-national data. *Working Paper.* Jiangsu, China: Southeast University.

Giacomo, M. d. (2004) 'Public support to entrepreneurial firms: an assessment of the role of Venture Capital in the European experience', *The Journal of Private Equity,* Winter, pp. 18.

Gilson, R. J. (2003) 'Engineering a venture capital market: lessons from the American experience', *Stanford Law Review,* 55(4), pp. 1067-1103.

Gompers, P. 1997. Ownership and control in entrepreneurial firms: an examination of convertible securities in venture capital investments. *Working Paper.* Harvard Business School.

Gompers, P., Kovner, A., Lerner, J. and Scharfstein, D. (2006) 'Skill versus luck in entrepreneurship and venture capital: evidence from serial entrepreneurs', *NBER Working Paper Series,* (12592).

Gompers, P., Kovner, A., Lerner, J. and Scharfstein, D. (2010) 'Performance persistence in entrepreneurship', *Journal of Financial Economics,* 96(1), pp. 18-32.

Gompers, P. and Lerner, J. (1996) 'The use of covenants: an empirical analysis of venture partnership agreements', *Journal of Law & Economics,* 39(2), pp. 463-498.

Gompers, P. and Lerner, J. (2001) 'The venture capital revolution', *Journal of Economic Perspectives,* 15(2), pp. 145-168.

Gompers, P. A. (1995) 'Optimal investment, monitoring and the staging of venture capital', *The Journal of Finance,* 50(5), pp. 1461-1489.

Gompers, P. A. and Lerner, J. (2004) *The venture capital cycle.* MIT press.

Grilli, L. and Murtinu, S. (2014) 'Government, venture capital and the growth of European high-tech entrepreneurial firms', *Research Policy,* 43(9), pp. 1523-1543.

Grilli, L. and Murtinu, S. (2015) 'New technology-based firms in Europe: market penetration, public venture capital, and timing of investment', *Industrial and Corporate Change,* 24(5), pp. 1109-1148.

Guerini, M. and Quas, A. (2016) 'Governmental venture capital in Europe: screening and certification', *Journal of Business Venturing,* 31(2), pp. 175-195.

Hackbarth, D. and Morellec, E. (2008) 'Stock returns in mergers and acquisitions', *The Journal of Finance,* 63(3), pp. 1213-1252.

Hair, J. F., Black, W. C., Babin, B. J. and Anderson, R. E. (2010) *Multivariate data analysis: a global perspective.* United States of America: Pearson.

Harris, R. S., Jenkinson, T. and Kaplan, S. N. (2014) 'Private equity performance: what do we know?', *Journal of Finance,* 69(5), pp. 1851-1882.

Harrison, R. T. and Mason, C. M. (2000) 'Editorial: The role of the public sector in the development of a regional venture capital industry', *Venture Capital,* 2(4), pp. 243-253.

Heaton, J. B. (2002) 'Managerial optimism and corporate finance', *Financial Management,* 31(2), pp. 33-45.

Hechavarría, D. M., Matthews, C. H. and Reynolds, P. D. (2015) 'Does start-up financing influence start-up speed? Evidence from the panel study of entrepreneurial dynamics', *Small Business Economics*, pp. 1-31.

Hellman, T. (1998) 'The allocation of control rights in venture capital contracts', *The RAND Journal of Economics,* 29(1), pp. 57-76.

Hellmann, T. and Puri, M. (2002) 'Venture capital and the professionalization of start-up firms: empirical evidence', *The Journal of Finance,* 57, pp. 169–197.

Hirsch, J. 2006. Public policy and venture capital financed innovation: a contract design approach. *CFS Working Paper.* Center for Financial Studies (CFS), Goethe University Frankfurt.

Hmieleski, K. M. and Baron, R. A. (2009) 'Entrepreneurs' optimism and new venture performance: a social cognitive perspective', *Academy of Management Journal,* 52(3), pp. 473-488.

Holtz-Eakin, D. (2000) 'Public policy toward entrepreneurship', *Small Business Economics,* 15(4), pp. 283-291.

Hood, N. (2000) 'Public venture capital and economic development: the Scottish experience', *Venture Capital,* 2(4), pp. 313-341.

Hsu, D. H. (2004) 'What do entrepreneurs pay for venture capital affiliation', *The Journal of Finance,* 59(4), pp. 1805-1844.

Hsu, D. H. (2007) 'Experienced entrepreneurial founders, organizational capital, and venture capital funding', *Research Policy,* 36(5), pp. 722-741.

Hsu, Y.-W. (2010) 'Staging of venture capital investment: a real options analysis', *Small Business Economics,* 35(3), pp. 265-281.

Hull, J. C. (2012) *Options, Futures, and Other Derivatives.* 8th Edition edn.: Prentice Hall.

Hunter, W. C. and Jagtiani, J. (2003) 'An analysis of advisor choice, fees, and effort in mergers and acquisitions', *Review of Financial Economics,* 12(1), pp. 65-81.

Hvide, H. K. and Panos, G. A. (2014) 'Risk tolerance and entrepreneurship', *Journal of Financial Economics,* 111(1), pp. 200-223.

Inderst, R. and Mueller, H. M. (2009) 'Early-stage financing and firm growth in new industries', *Journal of Financial Economics,* 93(2), pp. 276-291.

Jääskeläinen, M., Maula, M. and Murray, G. (2007) 'Profit distribution and compensation structures in publicly and privately funded hybrid venture capital funds', *Research Policy,* 36(7), pp. 913-929.

Jeng, L. A. and Wells, P. C. (2000) 'The determinants of venture capital funding: evidence across countries', *Journal of Corporate Finance,* 6(3), pp. 241-289.

Jenkins, J. C. and Leicht, K. T. (1996) 'Direct intervention by the subnational state: the development of Public Venture Capital programs in the American states', *Social Problems,* 43(3), pp. 306-326.

Kanniainen, V. and Keuschnigg, C. (2004) 'Start-up investment with scarce venture capital support', *Journal of Banking & Finance,* 28(8), pp. 1935-1959.

Kaplan, S. (1989) 'The effects of management buyouts on operating performance and value', *Journal of Financial Economics,* 24(2), pp. 217-254.

Kaplan, S. N. and Stromberg, P. (2001) 'Venture capitalists as principals: contracting, screening, and monitoring', *American Economic Review,* 91(2), pp. 426-430.

Kaplan, S. N. and Strömberg, P. (2001) 'Venture capitalists as principals: contracting, screening and monitoring', *NBER Working Paper Series,* (8202).

Keuschnigg, C. and Nielsen, S. (2007) 'Public policy, start-up entrepreneurship and the market for venture capital', in Parker, S. (ed.) *The Life Cycle of Entrepreneurial Ventures International Handbook Series on Entrepreneurship*: Springer US, pp. 227-257.

Keuschnigg, C. and Nielsen, S. B. (2001) 'Public policy for venture capital', *International Tax and Public Finance,* 8(4), pp. 557-572.

Kim, J.-H. and Wagman, L. (2016) 'Early-stage entrepreneurial financing: a signaling perspective', *Journal of Banking & Finance,* 67, pp. 12-22.

Kirilenko, A. A. (2001) 'Valuation and control in venture finance', *Journal of Finance,* 56(2), pp. 565-587.

Kohers, N. and Ang, J. (2000) 'Earnouts in mergers: agreeing to disagree and agreeing to stay', *The Journal of Business,* 73(3), pp. 445-476.

Kon, Y. and Storey, D. J. (2003) 'A theory of discouraged borrowers', *Small Business Economics,* 21(1), pp. 37-49.

Korteweg, A. and Nagel, S. (2016) 'Risk-adjusting the returns to venture capital', *The Journal of Finance*, pp. Accepted manuscript on-line.

Kraus, A. and Litzenberger, R. H. (1973) 'A State-Preference Model of Optimal Financial Leverage', *The Journal of Finance,* 28(4), pp. 911-922.

Krug, J. A. and Hegarty, W. H. (2001) 'Predicting who stays and leaves after an acquisition: a study of top managers in multinational firms', *Strategic Management Journal,* 22(2), pp. 185-196.

La Porta, R., Lopez-De-Silanes, F., Shleifer, A. and Vishny, R. M. (1997) 'Legal determinants of external finance', *Journal of Finance,* 52(3), pp. 1131-1150.

Landström, H. (2007) 'Pioneers in venture capital research', in Landström, H. (ed.) *Handbook of research on venture capital*: Edward Elgar Publishing.

Lawton, T. C. (2002) 'Missing the target: assessing the role of government in bridging the European equity gap and enhancing economic growth', *Venture Capital,* 4(1), pp. 7-23.

Lee, S. H., Peng, M. W. and Song, S. (2013) 'Governments, entrepreneurs, and positive externalities: A real options perspective', *European Management Journal,* 31(4), pp. 333-347.

Leicht, K. T. and Jenkins, J. C. (1998) 'Political resources and direct state intervention: the adoption of public venture capital programs in the American states, 1974-1990', *Social Forces,* 76(4), pp. 1323-1345.

Leisen, D. P. J. (2012) 'Staged venture capital contracting with ratchets and liquidation rights', *Review of Financial Economics,* 21(1), pp. 21-30.

Leleux, B. and Surlemont, B. (2003) 'Public versus private venture capital seeding or crowding out: a pan-European analysis', *Journal of Business Venturing,* 18, pp. 81-104.

Lerner, J. (1999) 'The government as Venture Capitalist: the long-run impact of the SBIR program', *The Journal of Business,* 72(3), pp. 285-318.

Lerner, J. (2002) 'When bureaucrats meet entrepreneurs: the design of effective public venture capital programmes', *The Economic Journal,* 112, pp. F73-F84.

Lerner, J. (2010) 'The future of public efforts to boost entrepreneurship and venture capital', *Small Business Economics,* 35(3), pp. 255-264.

Lerner, J. and Schoar, A. (2004) 'The illiquidity puzzle: theory and evidence from private equity', *Journal of Financial Economics,* 72(1), pp. 3-40.

Lerner, J. and Tag, J. (2013) 'Institutions and venture capital', *Industrial and Corporate Change,* 22(1), pp. 153-182.

Lerner, J. and Watson, B. (2008) 'The public venture capital challenge: the Australian case', *Venture Capital,* 10(1), pp. 1-20.

Li, Y. (2008) 'Duration analysis of venture capital staging: A real options perspective', *Journal of Business Venturing,* 23(5), pp. 497-512.

Liu, Y. and Yang, J. Q. (2015) 'Optimal investment of private equity', *Finance Research Letters,* 14, pp. 76-86.

Lukas, E. and Heimann, C. (2014) 'Technological-induced information asymmetry, M&As and earnouts: stock market evidence from Germany', *Applied Financial Economics,* 24(7), pp. 481-493.

Lukas, E., Mölls, S. and Welling, A. (2016) 'Venture capital, staged financing and optimal funding policies under uncertainty', *European Journal of Operational Research,* 250(1), pp. 305-313.

Lukas, E., Reuer, J. and Welling, A. (2012) 'Earnouts in mergers and acquisitions: a game-theoretic option pricing approach', *European Journal of Operational Research,* 223(1), pp. 256-263.

Luukkonen, T., Deschryvere, M. and Bertoni, F. (2013) 'The value added by government venture capital funds compared with independent venture capital funds', *Technovation,* 33(4-5), pp. 154-162.

Macmillan, I. C., Siegel, R. and Narasimha, P. N. S. (1985) 'Criteria used by venture capitalists to evaluate new venture proposals', *Journal of Business Venturing,* 1, pp. 119-128.

Mantecon, T. (2009) 'Mitigating risks in cross-border acquisitions', *Journal of Banking & Finance,* 33(4), pp. 640-651.

Mason, C. and Brown, R. (2013) 'Creating good public policy to support high-growth firms', *Small Business Economics,* 40(2), pp. 211-225.

Mason, C. and Harrison, R. (2003) 'Closing the regional equity gap? A critique of the Department of Trade and Industry's regional venture capital funds initiative', *Regional Studies,* 37(8), pp. 855-868.

Mason, C. and Pierrakis, Y. (2013) 'Venture capital, the regions and public policy: the United Kingdom since the post-2000 technology crash', *Regional Studies,* 47(7), pp. 1156-1171.

Mason, C. M. and Harrison, R. T. (2001) ''Investment readiness': a critique of government proposals to increase the demand for venture capital', *Regional Studies,* 35(7), pp. 663-668.

McDonald, R. and Siegel, D. (1986) 'The value of waiting to invest', *The Quarterly Journal of Economics,* 101(4), pp. 707-728.

McDonald, R. L. and Siegel, D. R. (1985) 'Investment and the valuation of firms when there is an option to shut down', *International Economic Review,* 26(2), pp. 331-349.

McKelvie, A. and Wiklund, J. (2010) 'Advancing firm growth research: a focus on growth mode instead of growth rate', *Entrepreneurship Theory and Practice,* 34(2), pp. 261–288.

Metrick, A. and Yasuda, A. (2011) 'Venture capital and other private equity: a survey', *European Financial Management,* 17(4), pp. 619-654.

Michelacci, C. and Suarez, J. (2004) 'Business creation and the stock market', *Review of Economic Studies,* 71(2), pp. 459-481.

Miller, M. H. (1977) 'Debt and taxes', *The Journal of Finance,* 32(2), pp. 261-275.

Morellec, E. and Zhdanov, A. (2005) 'The dynamics of mergers and acquisitions', *Journal of Financial Economics,* 77(3), pp. 649-672.

Munari, F. and Toschi, L. (2015) 'Assessing the impact of public venture capital programmes in the United Kingdom: do regional characteristics matter?', *Journal of Business Venturing,* 30(2), pp. 205-226.

Murray, G. C. (1998) 'A policy response to regional disparities in the supply of risk capital to new technology based firms in the European Union: The European Seed Capital Fund Scheme', *Regional Studies,* 32(5), pp. 405-419.

Murray, G. C. (2007) 'Venture capital and government policy', *Handbook of Research on Venture Capital*: Edward Elgar Publishing.

Murray, G. C. and Lott, J. (1995) 'Have UK venture capitalists a bias against investment in new technology-based firms?', *Research Policy,* 24(2), pp. 283-299.

Muzyka, D., Birley, S. and Leleux, B. (1999) 'Trade-offs in the investment decisions of European venture capitalists', *Journal of Business Venturing,* 11, pp. 273-287.

Myers, S. C. (1977) 'Determinants of corporate borrowing', *Journal of Financial Economics,* 5, pp. 147-175.

Nahata, R. (2008) 'Venture capital reputation and investment performance', *Journal of Financial Economics,* 90(2), pp. 127-151.

OECD (2015) *United States Policy Brief.* Available at: http://www.oecd.org/policy-briefs/united-states-improve-small-business-financing.pdf (Accessed: 7th March, 2016).

Orser, B. J., Riding, A. L. and Manley, K. (2006) 'Women entrepreneurs and financial capital', *Entrepreneurship Theory and Practice,* 30(5), pp. 643-665.

Parhankangas, A. (2007) 'An overview of research on early stage venture capital: current status and future directions', in Landström, H. (ed.) *Handbook of Research on Venture Capital*: Edward Elgar Publishing, pp. 253-280.

Parry, S. N. (2015) 'The influence of neoliberal economics on small business accounting research: A critical evaluation of agendas and methodologies', *International Small Business Journal*, pp. 1-22.

Peneder, M. (2008) 'The problem of private under-investment in innovation: A policy mind map', *Technovation,* 28(8), pp. 518-530.

Phalippou, L. and Gottschalg, O. (2009) 'The performance of private equity funds', *Review of Financial Studies,* 22(4), pp. 1747-1776.

Popov, A. and Roosenboom, P. (2013) 'Venture capital and new business creation', *Journal of Banking & Finance,* 37(12), pp. 4695-4710.

President Obama announces new partnership with the private sector to strengthen America's Small Business; Renews the Federal Government's QuickPay Initiative (2014), United States of America: The White House. Available at: https://www.whitehouse.gov/the-press-office/2014/07/11/president-obama-announces-new-partnership-private-sector-strengthen-amer (Accessed: 7th March, 2016).

Programme for the Competitiveness of Enterprises and small and medium-sized (2013-2020) and repealing Decision No 1639/ 2006/ EC (347/33), European Parliament & Council of the European Union (2013)

Raffiee, J. and Feng, J. (2014) 'Should I quit my day job? A hybrid path to entrepreneurship', *Academy of Management Journal,* 57(4), pp. 936-963.

Ragozzino, R. and Reuer, J. J. (2009) 'Contingent earnouts in acquisitions of privately held targets', *Journal of Management,* 35(4), pp. 857-879.

Rasmussen, E. and Sørheim, R. (2012) 'Obtaining early-stage financing for technology entrepreneurship: reassessing the demand-side perspective', *Venture Capital,* 14(2-3), pp. 77-89.

Reid, G. C. (1996) 'Fast growing small entrepreneurial firms and their venture capital backers: An applied principal-agent analysis', *Small Business Economics,* 8(3), pp. 235-248.

Reuer, J. J., Shenkar, O. and Ragozzino, R. (2004) 'Mitigating risk in international mergers and acquisitions: the role of contingent payouts', *Journal of International Business Studies,* 35(1), pp. 19-32.

Reum, W. R. and Steele, T. A. (1970) 'Contingent payouts and acquisition risks', *Harvard Business Review,* 48(2), pp. 83-91.

Revest, V. and Sapio, A. (2012) 'Financing technology-based small firms in Europe: what do we know?', *Small Business Economics,* 39(1), pp. 179-205.

Riding, A. L., Madill, J. J. and Haines Jr, G. H. (2007) 'Investment decision making by business angels', *Handbook of Research on Venture Capital*: Edward Elgar Publishing, pp. 332.

Roper, S. and Scott, J. M. (2009) 'Perceived financial barriers and the Start-up decision: an econometric analysis of gender differences using GEM data', *International Small Business Journal,* 27(2), pp. 149-171.

Rubinstein, M. 1992. Exotic options. *Finance Working Paper.* University of California.

Rubinstein, M. and Reiner, E. (1991) 'Breaking down the barriers', *Risk,* 4(8), pp. 28-35.

Sahlman, W. A. (1990) 'The structure and governance of venture-capital organizations', *Journal of Financial Economics,* 27(2), pp. 473-521.

Sapienza, H. J. and Korsgaard, M. A. (1996) 'Procedural justice in entrepreneur-investor relations', *Academy of Management Journal,* 39(3), pp. 544-574.

Sapienza, H. J., Korsgaard, M. A. and Forbes, D. P. (2003) 'The self-determination motive and entrepreneurs' choice of financing', in Katz, J. & Corbett, A.C. (eds.) *Advances in Entrepreneurship, Firm Emergence and Growth*: Emerald Group Publishing, pp. 105-134.

Sapienza, H. J. and Villanueva, J. (2007) 'Conceptual and theoretical reflections on venture capital research', in Landström, H. (ed.) *Handbook of Research on Venture Capital*: Edward Elgar Publishing, pp. 66-85.

Scaling-up SME access to financial services in the developing world (2010): International Finance Corporation & World Bank. Available at: http://www.gpfi.org/sites/default/files/documents/G20_Stocktaking_Report_0.pdf (Accessed: 7th March, 2016).

Scholes, M. (1976) 'Taxes and the Pricing of Options', *The Journal of Finance,* 31(2), pp. 319-332.

Schwienbacher, A. (2007) 'A theoretical analysis of optimal financing strategies for different types of capital-constrained entrepreneurs', *Journal of Business Venturing,* 22(6), pp. 753-781.

Schwienbacher, A. (2013) 'The entrepreneur's investor choice: the impact on later-stage firm development', *Journal of Business Venturing,* 28(4), pp. 528-545.

Shepherd, D. A. (2015) 'Party on! A call for entrepreneurship research that is more interactive, activity based, cognitively hot, compassionate, and prosocial', *Journal of Business Venturing,* 30(4), pp. 489-507.

Shepherd, D. A., Zacharakis, A. and Baron, R. A. (2003) 'VCs' decision processes', *Journal of Business Venturing,* 18(3), pp. 381-401.

Smith, G. (1999) 'How early stage entrepreneurs evaluate venture capitalists', *The Journal of Private Equity,* 4(2), pp. 33-45.

Sorensen, M. and Jagannathan, R. (2015) 'The public market equivalent and private equity performance', *Financial Analysts Journal,* 71(4), pp. 43-50.

Sorensen, M., Wang, N. and Yang, J. Q. (2014) 'Valuing private equity', *Review of Financial Studies,* 27(7), pp. 1977-2021.

Statman, M. (1987) 'How many stocks make a diversified portfolio', *Journal of Financial and Quantitative Analysis,* 22(3), pp. 353-363.

Sunley, P., Klagge, B., Berndt, C. and Martin, R. (2005) 'Venture capital programmes in the UK and Germany: In what sense regional policies?', *Regional Studies,* 39(2), pp. 255-273.

SupplierPay pledge (2014), United States of America: The White House.

Taxation trends in the European Union (2015), Luxembourg: Eurostat, European Commission & DG Taxation and Customs Union.

Tian, X. (2011) 'The causes and consequences of venture capital stage financing', *Journal of Financial Economics,* 101(1), pp. 132-159.

Tourinho, O. 1979. The option value of reserves of natural resources. *Research Program in Finance Working Paper Series.* University of California at Berkeley.

Tyebjee, T. T. and Bruno, A. V. (1984) 'A model of venture capitalist investment activity', *Management Science,* 30(9), pp. 1051-1066.

Ueda, M. (2004) 'Banks versus venture capital: project evaluation, screening and expropriation', *The Journal of Finance,* 59(2), pp. 601-621.

van der Schans, D. (2015) 'The British Business Bank's role in facilitating economic growth by addressing imperfections in SME finance markets', *Venture Capital,* 17(1-2), pp. 7-25.

Venkataraman, S. (1997) 'The distinctive domain of entrepreneurship research', *Advances in Entrepreneurship, Firm Emergence and Growth,* 3, pp. 119-138.

Wang, S. and Zhou, H. (2004) 'Staged financing in venture capital: moral hazard and risks', *Journal of Corporate Finance,* 10(1), pp. 131-155.

Winborg, J. (2000) *Financing small businesses - developing our understanding of financial bootstrapping behavior.* Doctoral Thesis, Lund University [Online] Available at: http://lup.lub.lu.se/record/41124 (Accessed: 15th September, 2015).

Winton, A. and Yerramilli, V. (2008) 'Entrepreneurial finance: banks versus venture capital', *Journal of Financial Economics,* 88(1), pp. 51-79.

Wong, A. (2014) 'The role of government in the venture capital market with asymmetric information', *Quantitative Finance,* 14(6), pp. 1107-1114.

Wright, M., Robbie, K. and Ennew, C. (1997) 'Venture capitalists and serial entrepreneurs', *Journal of Business Venturing,* 12, pp. 227-249.

Wright, M. and Stigliani, I. (2013) 'Entrepreneurship and growth', *International Small Business Journal,* 31(1), pp. 3-22.

Yu, J. and Xu, B. (2011) 'The game analyses to price the target enterprise of merger and acquisition based on the perspective of real options under stochastic surroundings', *Economic Modelling,* 28(4), pp. 1587-1594.

Zacharakis, A. L. and Meyer, G. D. (1998) 'A lack of insight: do venture capitalists know their own decision making process?', *Journal of Business Venturing,* 13, pp. 57–76.

Zahra, S. A. and Allen, S. A. (2007) 'Entrepreneurs' perspective on corporate venture capital (CVC): A relational capital perspective', in Landström, H. (ed.) *Handbook of Research on Venture Capital*: Edward Elgar Publishing, pp. 393-413.

Zou, H., Chen, X., Lam, L. W. R. and Liu, X. (2015) 'Psychological capital and conflict management in the entrepreneur–venture capitalist relationship in China: The entrepreneur perspective', *International Small Business Journal*, pp. 1-22.