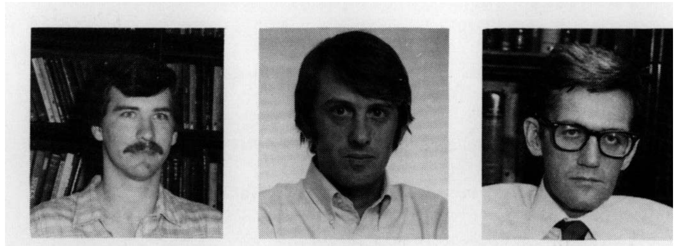
ALL-DIGITAL SIMULATION OF SIMPLE AUTOMOBILE MANEUVERS

Keywords: digital simulation, mechanical engineering, transportation, vehicle dynamics



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ABSTRACT

A new, all-digital simulation describes straight-line acceleration and braking of an automobile. The model has five degrees of freedom: longitudinal velocity, vertical velocity, pitch angular velocity, and the angular velocities of the front and rear wheels. The model is capable of handling severe maneuvers. Special provisions allow the user to include antilock systems, average suspension forces, perform braking studies, and assess the importance of nonlinear terms. A validation study shows a close comparison with results from earlier models. The model is modular, well-documented, and readily available to researchers in a convenient form.

INTRODUCTION

The purpose of analyzing the dynamics of a vehicle is to determine how it responds to control inputs, such as steering or braking, and to external disturbances such as wind gusts. This response depends on a complex interaction between vehicle parameters (such as suspension geometry) and the forces and moments generated between the tires and the road. Because of this complexity, computer simulation techniques are valuable aids to the designer in assessing the effects of system changes on both subjective and objective measures of vehicle ride and handling. Computer simulations can lead to the design of vehicles that are more attractive to the public and can help designers meet new government standards (e.g., requiring a certain level of performance at large longitudinal and lateral accelerations). The manufacturer with an in-house simulation would have a valuable tool to aid in meeting new standards and could reduce the number of expensive field tests. Another group interested in vehicle dynamics consists of investigators concerned with the interactions of the human driver and the automobile. Billing[3] and Hoffmann[8] have provided thorough surveys of such closed-loop systems.

Bernard[2] has described comprehensively the early work on the simulation of vehicle dynamics. Bohn and Keenan[4] have discussed later developments in their report on the hybrid simulation at the Applied Physics Laboratory of the Johns Hopkins University. To avoid the need for a hybrid computer, we have recently developed all-digital simulations for open- and closed-loop automobile maneuvers.[7] As part of that work, we employed simplified vehicle models for simulating open-loop maneuvers involving straight-line acceleration and braking. These models are the subject of the present report.

This work differs from previous efforts in the following major respects:

(1) The simulation is entirely digital and can be run on any large digital computer; a hybrid facility is not necessary.

(2) We have modularized the computer program to facilitate future development. Our aim was to provide a structure in which most expected changes or improvements, such as might--for example--be necessary to model suspension designs, will involve a single module.

(3) The model can handle severe maneuvers involving large longitudinal accelerations or decelerations (in excess of 0.5 g). To handle such maneuvers we had to retain nonlinear terms in the forces and moments generated by the tires and in the vehicle kinematics.

(4) The simulation provides for an antilock system (which can be switched off) in which front and rear brakeline pressures are apportioned to avoid wheel lockup (skidding). This feature was included in the event that antilock systems are used on passenger cars instead of only on large trucks.

(5) The model has extensive documentation to facilitate usage and alteration[11-13]; to our knowledge, no model with these features is available elsewhere in an accessible form such as cards or magnetic tapes.

LIST OF SYMBOLS

a, b	x and y distances between the center of gravity of the sprung mass and the center lines of the front and rear wheels, respectively
F_{APi},	i=1,2 Antipitch forces in the suspensions
F_{Ci},	i=1,2 Circumferential tire forces at wheel i
F_{Ri},	i=1,2 Normal (to ground) tire force at wheel i
F_{1i},	i=1,2 Coulomb damping forces in the suspensions
F_{2i},	i=1,2 Suspension forces produced by springs and bump stops
F_{3i},	i=1,2 Viscous damping forces in the suspensions
g	Acceleration of gravity
I_y	Moment of inertia of the sprung mass about the y axis
M	Total vehicle mass
q	Pitch angular velocity, i.e., angular velocity about the y axis
R_i,	i=1,2 Rolling radius of wheel i
SN	Skid number ratio
S_i,	i=1,2 Suspension force at wheel i
t_s	Vehicle stopping time
u, w	Velocity components of the vehicle in the x and z directions, respectively
u_o	Initial velocity in the x direction
x, y, z	Vehicle fixed axes
z_F, z_R	Static z distance between the center of gravity of the sprung mass and the front and rear wheel centers, respectively
δ_i,	i=1,2 Suspension deflections relative to the vehicle, measured at the center of wheel i, from the position of static equilibrium
θ	Pitch angle

Our models have the following general features:

(1) Five degrees of freedom, namely, longitudinal and vertical velocities, pitch angular velocity, and the angular velocities of the front and rear wheels.

(2) Two versions, one single precision and one double precision. We refer to these as SLABIM (straight-line-acceleration braking integration model) and SLABAM (straight-line-acceleration braking algebraic model). SLABIM integrates the wheel-spin differential equations numerically, whereas SLABAM handles them algebraically. The algebraic method is more economical but less accurate. SLABIM can handle maneuvers involving extremely large accelerations (on the order of 1 g.) requiring a high degree of accuracy.

(3) Use of the tire model developed by APL-CALSPAN (see Bohn and Keenan[4]). This model is a good representation of the measured data and accounts for the nonlinear mechanical behavior of pneumatic tires.

(4) Allowance for averaging suspension forces as a smoothing process. One of the difficulties we had in developing the models was the occurrence of rapid oscillations in the suspension forces. We felt that these oscillations would not affect the overall vehicle dynamics, since it involves much longer periods, and so we averaged them out (an optional feature).

(5) Provisions for braking studies. The models allow the user to specify trapezoidal brakeline pressures and accelerations with variable rise time, peak value, duration, and decay time. The drive torques may be apportioned to both the front and rear wheels, so that rear-wheel, front-wheel, and four-wheel drives can be handled. Brakeline pressures may also be apportioned to the front and rear wheels, as would be required in an antilock system. The skid number ratios may also be altered for front and rear wheels.

(6) Provisions for multiple runs in which the initial velocity, maximum brakeline pressure, or drive torque is incremented automatically for each run. This feature would be useful in performing sensitivity studies.

Outputs include the position, velocity, and accelerations of the sprung mass; the suspension deflections and rates of deflection; the static and dynamic suspension forces; the tire radii; the applied torques at each wheel; the forces and coefficients of longitudinal friction and the slip ratios at the tire road interface; the skid number; and the number of interval subdivisions of the integration.

A program control parameter lets the user linearize some nonlinear terms in the modeling and thereby assess the importance of the nonlinearities.

MATHEMATICAL MODEL OF THE VEHICLE

To give the reader the flavor of the mechanics, we will now outline our mathematical model. The equations of motion of the vehicle are (see Figure 1):

$$\dot{u} = -wq - g\sin\theta + \left[(F_{C1}+F_{C2})\cos\theta + (F_{R1}+F_{R2})\sin\theta\right]/M$$

$$\dot{w} = uq + g\cos\theta + (S_{PR1} + S_{PR2})/M$$

$$q = \Big[S_1 a - S_2 b + (F_{C1}\cos\theta + F_{R1}\sin\theta)(z_F+\delta_1+R_1)$$

$$+ (F_{C2}\cos\theta + F_{R2}\sin\theta)(z_R+\delta_2+R_2)\Big] / I_y$$

$$S_{PRi} = -F_{1i} - F_{2i} - F_{3i} + F_{APi} \quad , \quad i = 1,2$$

Some remarks regarding these equations are in order. Since the equations include trigonometric functions, the vehicle kinematics are not restricted to small angles. The suspensions are taken to move parallel to the z-axis and the forces developed in them include the effects of gravity, Coulomb friction, spring and bump stops, shock-absorber viscous damping, and antipitch devices. Much of the modeling is taken into the nonlinear range, so the simulation should be capable of handling large longitudinal accelerations or decelerations (in excess of 0.5 g.).

Following Bohn and Keenan,[4] we calculate the nonlinear forces F_{2i}, F_{3i}, and F_{APi} by linear interpolation from tabular experimental data. The Coulomb damping forces F_{1i} present special problems since they undergo a finite jump and a sign change when the signs of the suspension velocities change. This discontinuous behavior can lead to numerical instability; two methods to overcome it are available — namely, a piecewise linear fit to the data and a hyperbolic tangent fit. Both overcome the instability, but we have never needed to use the smoother hyperbolic tangent fit. As mentioned earlier, in the initial stages of development we had to use small integration time steps because the suspension forces contained oscillations at frequencies much higher than the fundamental frequencies of the vehicle. We used a smoothing option to average these forces with those of the previous time interval (*single averaging*) or those of the previous two time intervals (*double averaging*). We justify these procedures on the grounds of the simulation validation, which we will discuss later. Double averaging allowed us to double the integration time steps and thereby reduce run costs.

We calculate the tire forces using the method given in Bohn and Keenan.[4] This method is based on curves fitted to the measured data and gives forces as a function of the normal force on the tire and the longitudinal slip angle. We implemented two procedures for calculating the longitudinal slips. The first involves a numerical integration of the differential equations describing wheel spins.

Unfortunately, these differential equations are stiff and integrating them can be very expensive. To reduce costs, we used a scheme in which we integrated the stiff equations separately using a much smaller time step than we used on the other equations. We implemented this approach by setting up two simultaneous integration schemes HPCG1 and HPCG2 (both used the routine HPCG in the IBM Scientific Subroutine Program). In the normal operation of HPCG, the user supplies initial values of the integration variables, step size, the time of the run, the equations to be integrated and an output subroutine OUT, which is called at the end of each time step. HPCG1 is used to integrate the main equations and uses a step size of 2 ms. By making the error criterion large, we ensured that HPCG never changed its step size. We chose 2 ms because experience with small error criteria showed this to be adequate. HPCG1 exits to OUT1, which then calls HPCG2 to integrate the stiff differential equations with a time step of 0.1 ms. Finally, HPCG2 returns control to HPCG1 and the process is repeated.

Though this dual time-step scheme was reasonably efficient, we developed a second (preferred) method for handling the wheel-spin equations following ideas set forth by Bernard.[2] In this method the time interval is taken to be small enough that the rolling radii of the tires, which normally are variables, can be assumed to have constant values. In addition, the nonlinear tire forces are expanded in Taylor series about the local value of the slip angle. These procedures lead to constant-coefficient ordinary differential equations, which of course have closed-form solutions. Thus at each time step only algebraic equations must be solved, and the simulation costs are thereby reduced significantly (comparative costs will be given later).

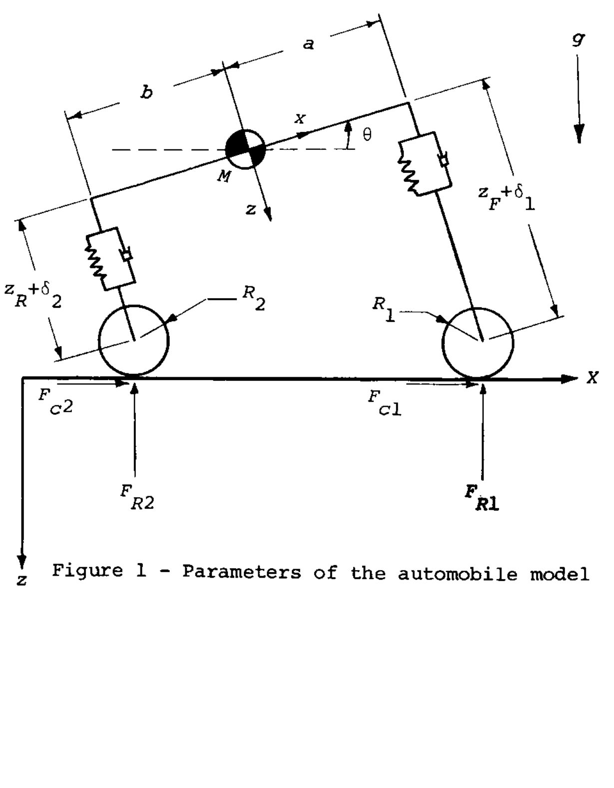

Figure 1 - Parameters of the automobile model

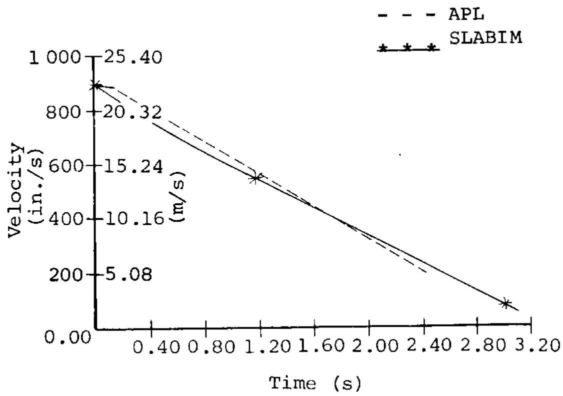

Figure 2 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 475 psi (3 275.13 kPa)]; nonlinear mode

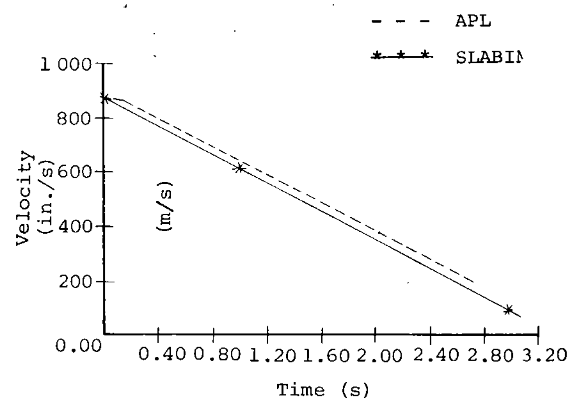

Figure 4 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 475 psi (3275.13 kPa)]; linear mode

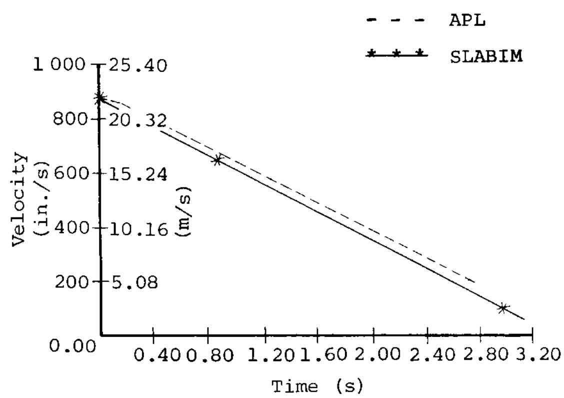

Figure 3 - Forward velocity versus time in ramp braking maneuver [peak pressure = 550 psi (3 792.25 kPa)]; nonlinear mode

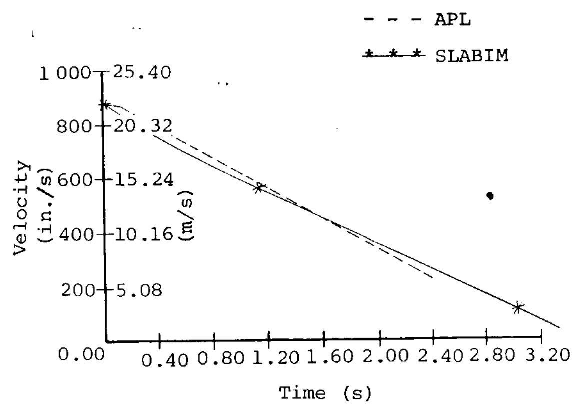

Figure 5 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 550 psi (3 792.25 kPa)]; linear mode

Program control switches allow the user to neglect some nonlinearities in the mathematical modeling. Thus the user can assess the effects of the nonlinearities in various maneuvers, an important area that has received too little attention. The specific assumptions that can be included or excluded are:

(1) The pitch angle is small, so that $\sin\theta \approx \theta$, $\cos\theta \approx 1$.

(2) The relative velocities between the suspensions and the chassis are small. Then in the calculation of the velocities of the tire patch contact points, $\dot{\delta}_1$ and $\dot{\delta}_2$ are smaller than u, w, qz_F, and qz_R and can be neglected.

(3) The suspension deflections δ_1 and δ_2 can be neglected in comparison with z_F, z_R, R_1, and qR_2.

We included assumption (3) because others have considered it. Our experience suggests that it is inaccurate and we recommend against making it. In fact, we found that several of the iterative processes involved in the simulation do not converge if this assumption is made.

(4) Finally, in anticipation of possible future development applications, the model includes an option for antilock systems, as described by Ervin *et al.*[6]

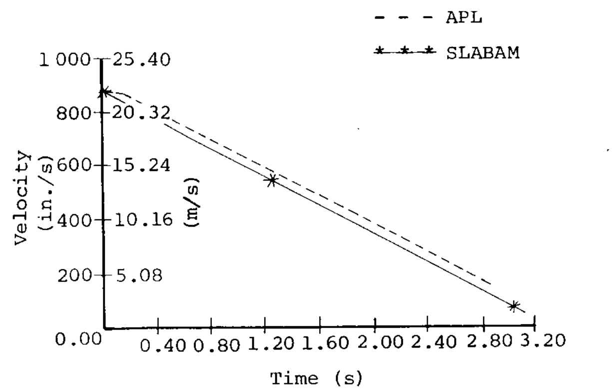

Figure 6 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 475 psi (3275.13 kPa)]; nonlinear mode

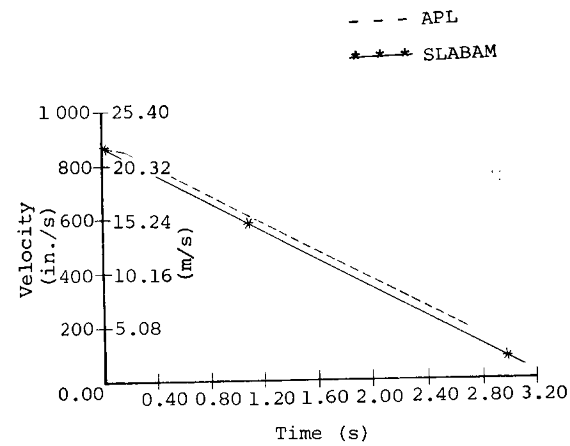

Figure 8 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 475 psi (3275.13 kPa)]; linear mode

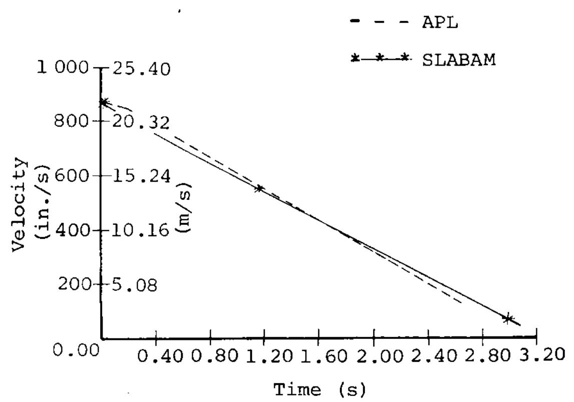

Figure 7 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 550 psi (3 792.25 kPa)]; linear mode

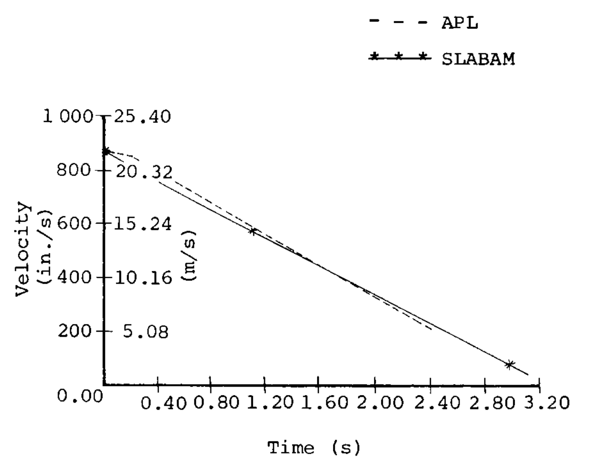

Figure 9 - Forward velocity versus time in a ramp braking maneuver [peak pressure = 550 psi (3 792.25 kPa)]; linear mode

VALIDATION

We validated SLABIM and SLABAM by extensive comparisons with output from the hybrid simulation developed by Bohn and Keenan.[4] This latter simulation has been validated by comparisons with field tests.[10] Also, Ford Motor Company used the APL model to develop an all-digital simulation of open-loop maneuvers.[5] Their field tests showed good agreement with the modeling.

The present test set consisted of a set of straight-line braking maneuvers with increasing degrees of severity. Such maneuvers are of concern from a safety viewpoint. Also, regulations setting performance criteria in this area could be legislated in the future. The vehicle, a 1971 Ford Mustang, is traveling initially at 50 mph (22.35 m/s); ramp brakeline pressure pulses are then applied with rise times of 0.1 s.

Figures 2 through 9 show results for peak pressures of 475 psi (3 275.13 kPa) — moderate braking — and 550 psi (3 792.25 kPa) — severe braking. Note that none of the simulations models the unimportant low-speed regime and all simulations are shut off once a certain velocity has been reached. SLABIM and

Table 1

Cost data for an application of SLABIM to a braking maneuver

Description of the maneuver:

(1) The initial vehicle speed is 55 mph (24.59 m/s).

(2) Ramp braking is applied with a maximum brake line pressure of 480 psi (3 309.60 kPa) and a rise time of 0.1 s.

Quantity	Double precision, no assumptions	Double precision, small-angle assumption	Single precision, no assumptions	Single precision, small-angle assumption
Cost per vehicle second	$0.39	$0.38	$0.35	$0.34
CPU time (s)	5.920	5.842	3.768	3.681
Maximum acceleration (g)	-0.73	-0.73	-0.73	-0.73
Maximum longitudinal slip of front wheel	0.12	0.12	0.12	0.12
Maximum longitudinal slip of rear wheel	0.21	0.21	0.21	0.21
Vehicle stopping distance	45.26 m 148.49 ft	45.26 m 148.49 ft	45.29 m 148.59 ft	45.29 m 148.59 ft
Vehicle stopping time (s)	3.50	3.50	3.50	3.50

Table 2

Cost data for an application of SLABAM to a braking maneuver

Description of the maneuver:

(1) The initial vehicle speed is 55 mph (24.59 m/s).

(2) Ramp braking is applied with a maximum brake line pressure of 480 psi (3 309.60 kPa) and a rise time of 0.1 s.

Quantity	Double precision, no assumptions	Double precision, small-angle (pitch) assumption	Single precision, no assumptions	Single precision, small-angle (pitch) assumption
Cost per vehicle second	$0.29	$0.28	$0.22	$0.21
CPU time (s)	4.555	4.352	3.325	3.303
Maximum acceleration (g)	-0.73	-0.73	-0.73	-0.73
Maximum longitudinal slip of front wheel	0.12	0.12	0.12	0.12
Maximum longitudinal slip of rear wheel	0.20	0.20	0.20	0.20
Vehicle stopping distance	45.11 m 148.00 ft	45.11 m 148.00 ft	44.99 m 147.60 ft	44.99 m 147.60 ft
Vehicle stopping time (s)	3.47	3.47	3.47	3.47

SLABAM offer the slight advantage of a smaller shut-off value than the APL simulation. The main conclusion drawn from the figures is the very close agreement between all the simulations. This validates both the linear and nonlinear versions of SLABIM and SLABAM. The results also show that SLABAM is practically as accurate as SLABIM. In fact, the corresponding output would be almost indistinguishable if plotted on the same graph. Another important point is that the linearized models are very accurate.

SIMULATION SIZE AND RUNNING COSTS

All the programs are written in FORTRAN; the single precision version of SLABAM consists of 1 122 statements, whereas SLABIM consists of 1 472 statements. Tables 1 and 2 present data on simulation run costs for a braking maneuver. These were obtained using the Amdahl 470 system at the University of Michigan. These data must be treated as relative since actual costs depend, of course, on the computer facility and the operating mode. The linearized, single-

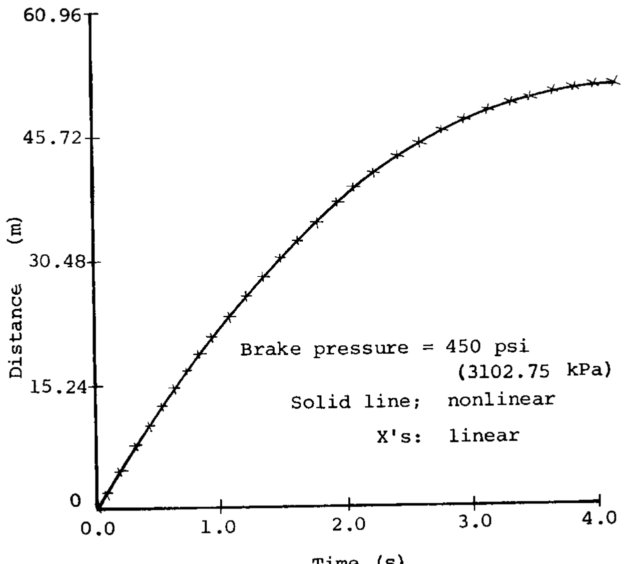

Figure 10 - Distance as a function of time; initial speed 55 mph (24.59 m/s), antilock on

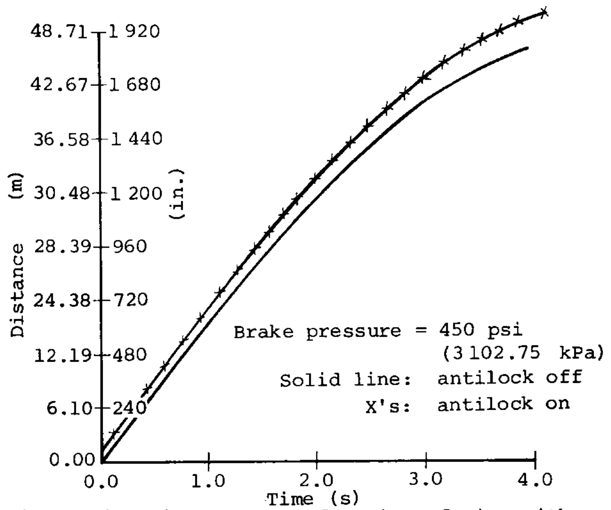

Figure 12 - Distance as a function of time with and without antilock; initial speed 55 mph (24.59 m/s), nonlinear

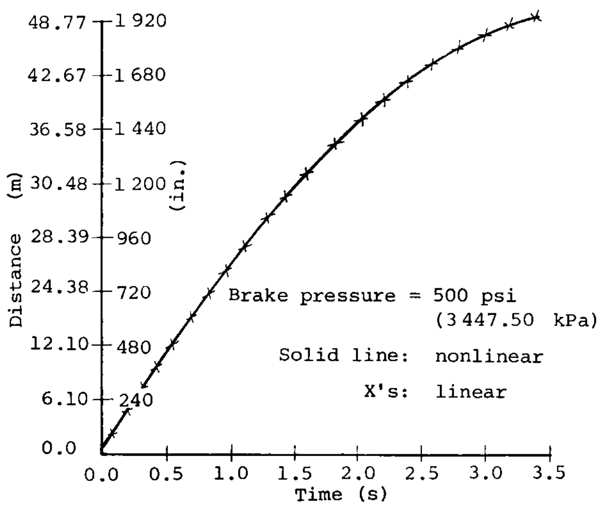

Figure 11 - Distance as a function of time; initial speed 55 mph (24.59 m/s), antilock on

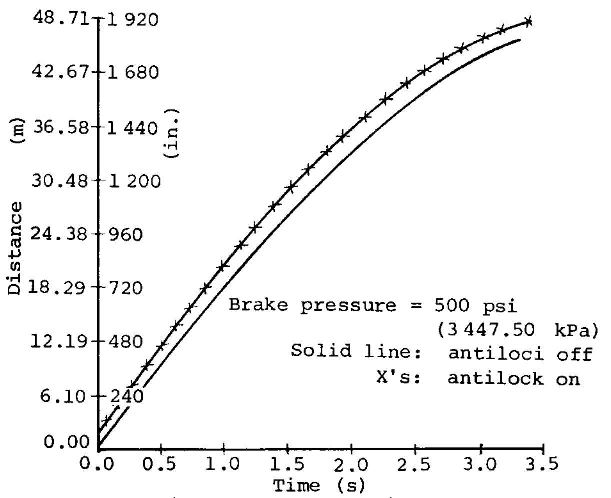

Figure 13 - Distance as a function of time with and without antilock; initial speed 55 mph (24.50 m/s), nonlinear

precision version of SLABAM gives results for the stopping distances and times (the most important variables) that agree to within 2% of those obtained from the double-precision version of SLABIM. Since this is an acceptable level of accuracy, we recommend the former model to the user. However, we should note that the user can retain the nonlinearities at only about a 7% increase in cost.

Figures 10 through 13 provide further information on linear versus nonlinear modeling and on the cost of including the antilock system.

Figures 10 and 11 show that the nonlinearities have little effect on stopping distance. Relative "local" costs were monitored. For 450 psi (3 102.75 kPa), the cost of the linear and nonlinear runs were $1.94 and $1.99, respectively, whereas for the 500 psi (3 447.50 kPa) case, the figures are $1.75 and $1.87, respectively. Thus, for the five-degree-of-freedom models, retaining the nonlinear terms is not expensive.

We also assessed the cost of including an antilock capacity. Figures 12 and 13 give stopping distances for brakeline pressures of 450 psi (3 102.75 kPa) and 500 psi (3 447.50 kPa). For the first case with the antilock off and on, the costs were $1.67 and $1.99, respectively, whereas for the second case the numbers are $1.58 and $1.87, respectively. Clearly, the antilock capacity is more expensive than is retention of the nonlinearities.

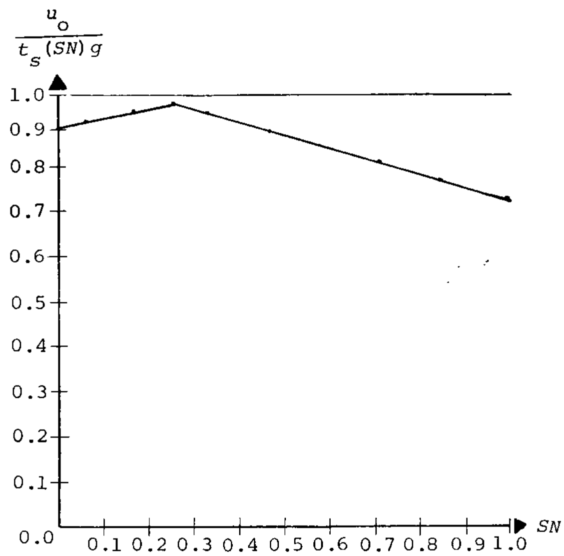

Figure 14 - Plot of normalized initial velocity versus skid number for wheel lockup

Figure 14 shows simulation output from a braking study. The vehicle is traveling initially at a speed u_o (45 mph or 20.12 m/s), and a ramp brakeline pressure pulse is applied with a rise time of 0.1s.

Focusing on a given skid number SN, the peak pressure is increased until wheel lockup occurs. The process is then repeated for several values of SN. Figure 14 plots the normalized velocity $u_o/gt_s(SN)$, where $t_s(SN)$ is the stopping time, versus SN. Front-wheel lockup occurs in the ascending portion of the curve, whereas rear-wheel lockup occurs in the descending portion. This figure, which gives a measure of adhesion utilization, would aid in the design of a braking system trying to achieve simultaneous four-wheel lockup, which leads to maximum deceleration. The overall shape of this curve agrees well with results given in Newcomb and Spurr.[14]

Currently, we are developing vehicle models which include lateral asymmetries, such as imbalanced brake systems, offset payload, and the transfer of normal load, caused by engine torque, from the right rear tire to the left rear tire. We plan to perform sensitivity studies on these additions.

The programs are available through the National Highway Safety Traffic Administration. Also, card decks can be obtained, at cost, from the last author.

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