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Allocation method for transit lines considering the user equilibrium for operators☆

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ABSTRACT

The purpose of this study is to address the allocation of transit lines problem in operation-sharing. An allocation method for urban transit lines is proposed to guide public authorities to pursue an optimal plan considering the User Equilibrium for operators (UE-O). The method utilizes the concepts from mathematical programming and game theory to present the UE-O and proposes a set partitioning formulation considering the benefits of both passengers and operators. A branch-and-price algorithm employing both column generation and branch-and-bound is used to tackle the problem. The proposed method is validated through a case study using data from the Development District of Dalian. Results show that the proposed line allocation method considering the UE-O can reduce the potential competitions among operators. This method and findings can provide a guidance to the problems in operation-sharing regarding allocation of transit lines.

1. Introduction

In recent years, reform in the transit sector takes place in many countries, including changing the operation of transit from monopoly to sharing. Government monopoly of transit lines is no longer a favorable strategy due to poor service level and low efficiency caused by monopoly. Compared with monopoly, operation-sharing of transit lines could introduce competition to maintain the service level and efficiency. Demsetz (1968) mentioned that competition of the market is a way to enhance efficiency. Opening the market to competition has been necessary to create competitiveness in the transit sector. If transit lines are operated by multiple operators, competitive pressure will be induced among operators. Thus, operators will improve service levels and reduce operating costs to increase their competitiveness. Many countries allocating transit lines by introducing competition have so far been successful (Preston, 2005; Wu et al., 2012; Zhao and Huang, 2016).

However, there are some problems caused by introducing transit competition in operation-sharing. Each operator's individual goal is to maximize his own benefit. Hence, operators may adjust operational strategies of their lines to gain more benefits after the authority allocating lines (Zhou et al., 2005). In general, the main operational strategy is to adjust the departure frequency of each transit line. When an operator is allocated a line to run, competition exists with other operators running on lines "attractive" to the same passengers. On the competitive line, operators tend to increase the departure frequency to compete for passengers, such as

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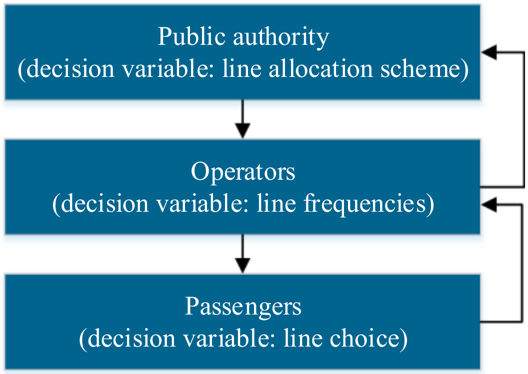


Fig. 1. The hierarchical structure of the players in the transit market.

deploy more vehicles on these competitive lines. And an equilibrium state is obtained when no operator can increase his profit by changing his departure frequency. That is the UE-O presented in the paper. The competition among operators can cause some problems. On one hand, the increase of vehicles can result in heavier traffic in the competitive lines and serious waste of vehicle resources. On the other hand, vehicles of less profitable lines are transferred to the competitive lines due to the fixed number of vehicles available for each operator, which results in poor service level of transit on the less profitable lines. Therefore, though operators may have high profits under competitive market (Filippini et al., 2015; Zha et al., 2016), the service levels of less profitable lines are very low and passenger satisfaction is poor. It is obviously not the global optimal solution desired by public authorities.

Competition among operators results in that the actual departure frequency of each line is not the global optimal solution desired by public authorities. Therefore, when public authorities allocate transit lines considering the UE-O, an allocation mechanism avoiding competition as much as possible should be designed to guarantee the service level. In the proposed model, three types of players are considered: the public authority, operators, and passengers. As shown in Fig. 1, the interactions among the three players are depicted as a hierarchical system. In the hierarchical system, the public authority aims to optimize the allocation of transit lines so as to minimize the total transit time of passengers. Operators are assumed to compete with each other by changing line frequency, while passengers have corresponding responses and adaptations on the changing line frequency considering operators' services. In turn, passenger responses ultimately determine the decision of the public authority and operators. In the proposed model, the optimal frequencies of lines maximizing each operator's profit on each line allocation scheme is obtained considering the UE-O. The competition on the competitive lines is portrayed as a non-cooperative game by changing the departure frequency of each competitive transit line separately so as to maximize the profit of each operator.

In this paper, the problem is formulated as a set partitioning formulation. Then, a branch-and-price algorithm employing both column generation and branch-and-bound is used to find an optimal solution. Finally, the optimal line allocate plan of Development District of Dalian is obtained by using the proposed model and algorithm.

The paper is organized as follows: Section 2 discusses the main findings of the literature on the operation of transit lines, operation plans optimization, and transit competition models applying game theory. Section 3 provides a formal description of the problem. In Section 4, we specify the model. Section 5 presents the solution approach. An instance is studied in Section 6 and we draw some conclusions in Section 7.

2. Literature reviews

In this section, we review research in the operation of transit lines, operation plans optimization, and transit competition models applying game theory.

In the literature of transit lines operation, a lot of works are devoted to studying operation policies, e.g., some scholars assessed the benefits and drawbacks of transit deregulation policy in private or public transit systems (Mackie et al., 1995; White, 1995; Preston and Almutairi, 2013; Li et al., 2015; Tang et al., 2012; Zhang et al., 2016; Liu et al., 2014; Liao et al., 2013; 2014; Liao, 2016; Yu et al., 2018). Wall and Mcdonald (2007) studied the influence of a set of measures to the bus service quality in Winchester, and the measures included introducing new buses, building up new bus infrastructure, increasing the frequency of bus routes in the main city center. Ida and Talit (2015) discussed the structural reforms in the regulation of public bus services in Israel. There are also plenty of literature on optimizing operation plans (Schöbel, 2012; Farahani et al., 2013; Martínez et al., 2014; Ibarra-Rojas et al., 2015; Zeng et al., 2017). Berrebi et al. (2015) proposed a real-time holding mechanism to dispatch buses on a loop-shaped route by using real-time information so as to minimize passengers‘ waiting time. Yu et al. (2015) developed a bi-level programming model to solve the design problem for bus line distribution in multi-modal transport networks. Traffic equilibrium can also be found in lots of literature (Bin et al., 2017; Wardrop and Whitehead, 1952a,b; Yang et al., 2012; Zhang and Yang, 2015; Yildirimoglu et al., 2015; Chen et al., 2017; Zhang et al., 2017).

There are quite a few transit competition models applying game theory in earlier literature. Williams and Abdulaal (1993) proposed a model of competition between transit services operating on a single route. Fisk (1984) modeled the competition among operators for intercity passenger travel by using Nash non-cooperative game. Viton (1982) analyzed the competition between two firms on a simplified corridor by developing equilibrium models. A network equilibrium model was proposed by Fernadez and Marcotte (1992) to model the free market competition among operators. Based on the above model, Zubieta (1998) modeled the situation where the transit market was controlled by several companies. In addition, the interactions between the operators was modeled by Wang and Yang (2005), and the game theoretical approach was used to model the competition over price and service frequency between the operators. By setting the fare structure of transit lines as decision variable, Zhou et al., (2005) presented a bilevel transit fare equilibrium model for an oligopolistic transit market. They proved that there exists a generalized Nash game between transit operators. Schöbel and Schwarze (2006) presented a game theoretic model for the line planning problem, in which each transit line acted as a player and aimed to minimize their own cost by changing their own frequencies. Similar to the work by Schöbel and Schwarze (2006), Kontogiannis and Zaroliagis (2008) also proposed a game theoretic approach for line planning problem. In their paper, different line operators with fixed lines were involved to maximize their personal benefits and a network operator was used to ensure a fair usage of the infrastructure (i.e. socially optimal) by maximizing the sum of benefits of the operators. From a different perspective, Li et al. (2012) proposed three interrelated sub-models to simultaneously optimize the number of transit operators and the allocation of new transit lines in a deregulated transit market.

To summarize, the literature are rich of researches on the operation of transit lines, transit line design, frequency setting, and line allocation. With the introduction of competition mechanism in the transit sector, all operators want to maximize their own benefit. Under the premise of service level, how to allocate transit lines fairly to each operator is a quite difficult task. By designing an allocation mechanism considering the User Equilibrium for the operators of transit lines, this paper aims at making an interesting addition to literature on transit line allocation in a competitive transit network. And the proposed model was examined using the data in the Development District of Dalian.

3. Problem description

There exists a non-cooperative game among operators in sharing-operation of transit lines as shown in Fig. 2. The transit line 1 and line 2 are allocated to operator *A*, while line 3, line 4 and line 5 are allocated to operator *B*. Competition exists between operator *A* and operator *B* because line 1 and line 3 attract passengers from similar locations. On the competitive lines 1 and 3, operator *A* tends to deploy more vehicles on line 1 to compete for more passengers, and operator *B* adopts the same strategies on line 3 as operator *A*. Assume that the number of vehicles available for each operator is fixed, so the vehicles deployed on a less profitable line will be transferred to the competitive lines. For instance, if line 2 and line 4 are less profitable lines, vehicles on line 2 will be transferred to line 1 by operator *A* and vehicles on line 4 will be transferred to line 3 by operator *B*. The operators will not change their strategies if the UE-O is reached. In other words, all operators are best responding to each other.

We use r_g to denote the marginal revenue of operator g to adjust the departure frequency of their transit lines, c_g to denote the marginal cost of operator g, and f_{gl} to denote the departure frequency of allocated transit line l of operator g. We thus use GST ($\boldsymbol{R}$, $\boldsymbol{C}$, $\boldsymbol{F}$) to represent the game among operators in sharing-operation of transit lines, with marginal revenue vector $\boldsymbol{R} = (\ldots, r_g, \ldots)$, marginal cost vector $\boldsymbol{C} = (\ldots, c_g, \ldots)$and departure frequency vector $\boldsymbol{F} = (\ldots, f_{gl}, \ldots)$.

As with the definition of principle of maximizing profits in economics (Lipsey, 1975), in this paper, UE-O is defined as follows:

Definition 1.. For GST (***R, C, F***), the sharing-operation is at **User Equilibrium for operators (UE-O)** if and only if for any operator g and his allocated transit line l, $r_g = c_g$ and f_{gl} cannot be changed to gain more benefits.

The competition among operators can result in heavy traffic and serious waste of vehicle resources on line 1 and line 3 as well as poor level of transit service on line 2 and line 4. Therefore, an allocation method considering the benefits of both passengers and operators is proposed in the paper. As shown in Fig. 3, the mathematical formulation of lines allocation problem consists of two parts.

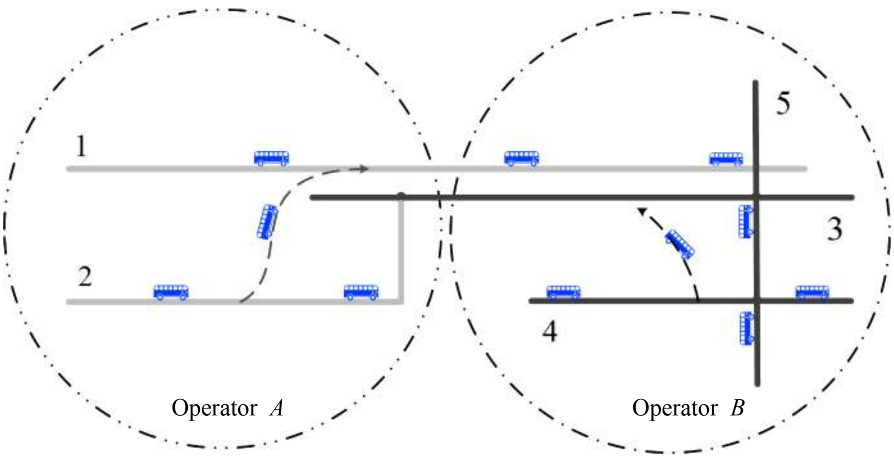


Fig. 2. Non-cooperative game among operators.

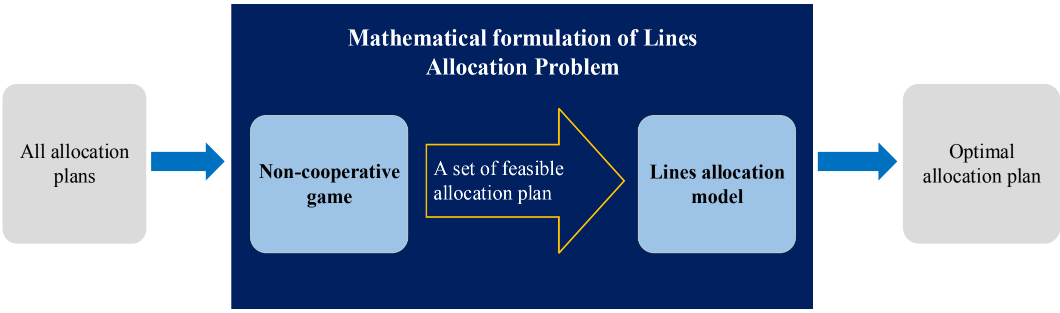


Fig. 3. The line allocation method.

The first part is a non-cooperative game model. The strategy of each operator is the set of frequencies of bus lines. Operators will constantly change their strategies in order to maximize their own benefits until the UE-O is reached. Therefore, the departure frequency of each line under each line allocation plan can be calculated by the non-cooperative game. In other words, a set of feasible allocation plan and frequencies of lines is obtained considering the benefits of operators. In the second part, the optimal allocation plan considering the benefits of passengers is calculated by the line allocation model based on the feasible allocation plan.

4. Model specification

4.1. Notation and assumptions

The model proposed in this paper is intended to be computed at the stage of service planning. It is assumed that each line is only operated by one operator and the number of vehicles available for each operator is fixed. In reality, different vehicle types may be adopted by different public agencies. However, how to efficiently distribute different vehicle types to different transit routes becomes another new problem. To simplify the problem, in this paper, the type of vehicles is assumed to be fixed. The data used in the model to estimate the service demand are considered fixed and represented by an OD matrix. The consideration of fixed data may limit the model representativeness of reality. However, the main target of the proposed service is commuters and compared with other trip purposes, commuting patterns have considerable stability. This limitation may not has significantly impact on the results. Actually, the use of a stochastic demand would not change any of the components of the model proposed in this paper; they would result only in a modification of the data input specification. The limitation can be solved by updating the allocation plan. It is not recommended to change the allocation plan very often because passengers need to get used to the system. Allocation plan are redesigned only when the demand has suffered a significant change or other great changes occur. In addition, in previous literature on transit network design problem, a given OD matrix is always being the simplest, standard, and most convenient way to estimate demand. Thus, in this paper, the demand is also considered fixed. For each OD pair, competition among bus lines is generated by the frequency of services, as each passenger has several strategies for reaching his/her destination. It is assumed that operators' only decision variable is the frequency (the number of vehicles to run per unit of time) of each line in order to maximize profit. The notations used in the paper are shown as follows:

Sets	
$\boldsymbol{G}$	The set of operators indexed by $g; g = 1, 2, ..., G$
$\boldsymbol{L}$	The set of lines indexed by l; $l = 1, 2, ..., L$
$\boldsymbol{K}_g$	The set of all possible feasible allocations of lines to operator g
Decision variables	
$z_{g,k}$	Binary variable to denote whether a feasible allocation $\boldsymbol{H}_{g,k}$ is selected by operator g
$f_{g,l}$	Frequency of line l operated by operator g
Auxiliary variables	
$w_{g,l}$	Total waiting time of passengers of line l operated by operator g
$t_{g,l}^{in}$	Total in-vehicle travel time of passengers of line l operated by operator g
$p_{i,i+1,g,l}^{in}$	Number of passengers in vehicles between stop i and stop $i+1$ of line l operated by operator g
$p_{g,l}^{total}$	Number of passengers of line l operated by operator g
$p_{i,g,l}^{board}$	Number of boarding passengers at stop i of line l operated by operator g
$o_{g,l}$	Total operational cost of line l operated by operator g
$\boldsymbol{H}_{g,k}$	A feasible allocation k selected by operator g
$h_{g,l,k}$	Binary variable to denote whether line l in the feasible allocation k is selected by operator g

Parameters

N	Number of lines
Q	Capacity of a vehicle
d_l	Length of line l
v_l	Average speed of vehicles of line l
n_g	Number of vehicles belonging to operator g
OC	Operational cost per hour per vehicle
$t_{i,i+1,l}^{\text{average}}$	Average travel time from stop i to stop $i+1$ of line l
$\bar{t}_l$	The upper bound of headway of line l
$\underline{t}_l$	The lower bound of headway of line l
T	Time period (for example, rush hour)

4.2. The set partitioning formulation of line allocation problem

The line allocation problem can be stated as follows: Given a set of transit lines with fixed stops and fixed type of vehicles to deploy, known passenger demand between each OD pair, and several operators, determine which lines are operated by operator g and the frequency of each line l operated by operator g, in order to minimize the total transit time of passengers (in-vehicle travel time and waiting time), while considering the benefits of operators. To formulate the line allocation problem as a set partitioning problem, we firstly define:

Let $\boldsymbol{K}_g = \{\boldsymbol{H}_{g,1}, \boldsymbol{H}_{g,2}, ..., \boldsymbol{H}_{g,k}\}$ be the set of all possible feasible allocations of lines to operator g, i.e., $\boldsymbol{H}_{g,k} = \{h_{g,1,k}, h_{g,2,k}, ..., h_{g,N,k}\}$ is a feasible solution to

$$\sum_l h_{g,l,k} \times \frac{2 \times d_l \times f_{g,l}}{v_l} \leqslant n_g, \quad \forall\, g \in \boldsymbol{G} \tag{1}$$

$$h_{g,l,k} = \{0, 1\}, \quad l \in \{1, 2, ..., L\} \tag{2}$$

where constraint condition (1) imposes the number of vehicles constraint for each operator, and constraint (2) defines $h_{g,l,k}$ as a binary variable.

Let $z_{g,k}$ for $g \in \boldsymbol{G}$ and $k \in \boldsymbol{K}_g$ be a binary variable indicating whether a feasible allocation $\boldsymbol{H}_{g,k}$ is selected for operator g ($z_{g,k} = 1$) or not ($z_{g,k} = 0$). Let $\tau_{g,l}$ be the total transit time of passengers. Then, the line allocation problem can now be formulated as follows [P1].

$$[\text{P1}] \quad \min \sum_{g \in \boldsymbol{G}} \sum_{k \in \boldsymbol{K}_g} \left(\sum_{l \in \boldsymbol{L}} \tau_{g,l} \times h_{g,l,k} \right) \times z_{g,k} \tag{3}$$

In which,

$$\tau_{g,l} = w_{g,l} + t_{g,l}^{\text{in}} \tag{4}$$

$$t_{g,l}^{\text{in}} = \sum_i p_{i,i+1,g,l}^{\text{in}} \times t_{i,i+1,l}^{\text{average}}, \quad \forall\, l \in \boldsymbol{L}, \forall\, g \in \boldsymbol{G} \tag{5}$$

$$w_{g,l} = \frac{1}{2 f_{g,l}} \times p_{g,l}^{\text{total}}, \quad \forall\, l \in \boldsymbol{L}, \forall\, g \in \boldsymbol{G} \tag{6}$$

$$p_{g,l}^{\text{total}} = \sum_i p_{i,g,l}^{\text{board}}, \quad \forall\, l \in \boldsymbol{L}, \forall\, g \in \boldsymbol{G} \tag{7}$$

$$\text{s.t.} \sum_{g \in \boldsymbol{G}} \sum_{k \in \boldsymbol{K}_g} h_{g,l,k} \times z_{g,k} = 1, \quad l \in \boldsymbol{L} \tag{8}$$

$$\sum_{k \in \boldsymbol{K}_g} z_{g,k} = 1, \quad g \in \boldsymbol{G} \tag{9}$$

$$z_{g,k} \in \{0, 1\}, \quad g \in \boldsymbol{G}, k \in \boldsymbol{K}_g \tag{10}$$

The objective function (3) minimizes the total transit time of passengers which consists of the in-vehicle travel time and waiting time. Assume that the arrivals of passengers are random and all transit services are on time, then the waiting time of all the passengers on line l operated by operator g can be calculated by formula (6) (Wang and Yang, 2005; Yao et al., 2014). In addition, the number of passengers in vehicles between stop i and stop $i+1$ of line l operated by operator g, $p_{i,i+1,g,l}^{\text{in}}$ and the number of boarding passengers at stop i of line l operated by operator g, $p_{i,g,l}^{\text{board}}$, can be calculated by the optimal strategies assignment model based on the frequencies and the OD matrix (Spiess and Florian, 1989; Yu et al., 2012). The constraint (8) enforces that each line is allocated to one operator and constraint (9) ensures that at most one feasible allocation is selected for each operator. Constraint (10) defines $z_{g,k}$ as a binary variable.

4.3. Non-cooperative game

In this paper, the optimal line frequency of each line on an allocation scheme is obtained through games. On each allocation scheme, if lines operated by different operators attract the same passengers, i.e. the competition among operators cannot be avoided completely, there exists a non-cooperative game among operators on the competitive lines. The competition among operators on the competitive lines is portrayed as a non-cooperative game by changing the frequency of each competitive line separately so as to maximize the profit of each operator. That is to say, operators' strategy is the frequency of each line.

The operators will increase the number of vehicles of the competitive lines to compete for more passengers, and an equilibrium state is obtained when no operators can increase his profit by changing the departure frequency. The operators can be seen as players of the non-cooperative game for which a Nash equilibrium is wanted to be sought, and the state of UE-O is equivalent to the Nash equilibrium.

The game among operators takes on the following form (Hollander and Prashker, 2006; Scutari et al., 2007; Kamalinia et al., 2014).

- Each operator g has a finite set of strategies $\boldsymbol{E}_g$, $\forall \ g \in \{1, 2, ..., G\}$
- Each operator g chooses (simultaneously) a strategy $\boldsymbol{s}_g \in \boldsymbol{E}_g$ and receives a pay-off u_g. $\boldsymbol{s}_g$ is a set of departure frequencies of all lines operated by operator g, $\boldsymbol{s}_g = \{f_{g,l}\}$. And each operator g has a pay-off utility function $u_g \in \boldsymbol{E}_1 \times \boldsymbol{E}_g \times ... \times \boldsymbol{E}_G \rightarrow R$
- Each operator's pay-off u_g is presented as the profit of each operator in this paper

The Nash equilibrium can be defined as a set of strategies $\{\boldsymbol{s}_1^*, \boldsymbol{s}_2^*, ..., \boldsymbol{s}_G^*\}$ such that

$$u_g(\boldsymbol{s}_1^*, \boldsymbol{s}_2^*, ..., \boldsymbol{s}_G^*) \geqslant u_g(\boldsymbol{s}_1, \boldsymbol{s}_g, ..., \boldsymbol{s}_G), \quad \forall \ g \in \{1, 2, ..., G\} \tag{11}$$

The $\boldsymbol{s}_g^*$ is an optimal solution of the following problem:

$$[\text{P2}] \quad \max u_g(\boldsymbol{s}_g) \tag{12}$$

s.t. (7)

$$\sum_l \frac{2 \times d_l \times f_{g,l}}{v_l} \times h_{g,l} \leqslant n_g, \quad \forall g \in \boldsymbol{G} \tag{13}$$

$$\frac{1}{\sum_g f_{g,l}} \leqslant \bar{t}_l, \quad \forall l \in \boldsymbol{L} \tag{14}$$

$$\frac{1}{\sum_g f_{g,l}} \geqslant \underline{t}_l, \quad \forall l \in \boldsymbol{L} \tag{15}$$

$$f_{g,l} \times \text{T} \times Q \geqslant p_{g,l}^{\text{total}}, \quad \forall l \in \boldsymbol{L}, \ \forall g \in \boldsymbol{G} \tag{16}$$

$$f_{g,l} \in R^+, \quad \forall l \in \boldsymbol{L}, \ \forall g \in \boldsymbol{G} \tag{17}$$

The constraint condition (13) imposes the number of vehicles constraint for each operator. Constraint (14) and (15) define the upper bound and the lower bound of headway for each line, respectively. Constraint (16) ensures that all passenger demand is satisfied. Constraint (17) defines $f_{g,l}$ as a positive integer variable.

Every finite non-cooperative game has an equilibrium point (Nash, 1950), so that the equilibrium point $\{\boldsymbol{s}_1^*, \boldsymbol{s}_2^*, ..., \boldsymbol{s}_G^*\}$ exists. Each operator's pay-off u_g is presented as the profit of each operator. The profit of each operator is defined as the sum of profits of all the lines operated by the operator:

$$u_g = \sum_{l \in \boldsymbol{L}} u_{g,l} \times h_{g,l}, \quad \forall g \in \boldsymbol{G} \tag{18}$$

Profit of each line is determined by the number of its carried passengers, fares, and the operating cost. Assume that the fare of each line ϕ_l is constant, then the profit of line l operated by operator g can be calculated as:

$$u_{g,l} = p_{g,l}^{\text{total}} \times \phi_l - o_{g,l}, \quad \forall l \in \boldsymbol{L}, \ \forall g \in \boldsymbol{G} \tag{19}$$

In which,

$$o_{g,l} = OC \times \frac{2 \times d_l \times f_{g,l}}{v_l}, \quad \forall l \in \boldsymbol{L}, \ \forall g \in \boldsymbol{G} \tag{20}$$

In general, the private transit operator aims to maximize its own profit, which is the gross fare revenue minus the total transit operating cost. However, in this paper, the number of vehicles of each operator is given and all operators' vehicles can be seen as running to serve the demand during the peak time period of a working day. The operation cost of each operator g is deemed to be fixed in this paper.

In the above proposed model, we assume a flat fare for each boarding adopted by all operators. However, in reality, there may exist other fare strategies, like the distance-based fare and single fare in BRT systems. Interesting results could be obtained if these combined fare strategies are considered in the transit passenger assignment and line allocation problems. To realistically consider the effect of different fare strategies on the line allocation plan and by introducing the idea of generalized cost, we could provide an enhanced model in Appendix B. In this case, the fare of transit line depending on the distance traveled is discussed. It is worth noting that, in both fare strategies proposed in this paper, passengers will be charged one more time when they have a transfer. Actually, the model used in this paper is a special case of the model in Appendix B.

5. Solution method

In the integer model [P1], the number of feasible allocations is so large that it is impractical to solve the model by using mathematical programming software directly. Column generation algorithm is a method to solve such problems having plenty of variables (Barnhart et al., 1998; Gamache et al., 1999; Desrochers et al., 1992). And when solving large-scale integer programming problem, it was proved that column generation was a successful method as a complement of integer programming approach, such as branch-and-price algorithm (Barnhart et al., 1998; Vanderbeck, 1994, 2011; Rönnberg and Larsson, 2014). Thus, the approach used to solve the model proposed in this paper is based on the column generation algorithm and the branch-and-price algorithm. The branch-and-bound scheme is applied, and the lower bounds are found using column generation.

Column generation was introduced first by Gilmore and Gomory (1961) in the context of cutting stock problems, and used by some researchers as one of most effective approaches to solve set partitioning (or set covering) problems. In the line allocation problem, each variable of the set partitioning formulation represents a feasible allocation. In the column generation procedure, a restricted linear master problem (RLMP) is solved iteratively. And the RLMP consists of columns (feasible allocations) generated by solving the pricing problems.

The branch and price search tree is initialized by adding columns to the RLMP to obtain a feasible solution firstly at the root node. At each node of the search tree, the feasible solution is calculated by the column generation. If the calculated solution of the RLMP is integer, the solution is a feasible solution to the master problem. Then, it is compared with the incumbent solution. If the calculated solution of the RLMP is not integer, branching occurs to cut off the fractional point. Finally, the optimal integer solution of the original problem is the incumbent solution found. In Sections 5.1–5.4, the branch-and-price algorithm is described.

5.1. Master problem

The restricted master problem (RMP) is yielded by relaxing constraint (10) of model [P1]. The RMP is formulated as follows:

$$\text{(RMP)} \quad \min \sum_{g\in \boldsymbol{G}} \sum_{k\in \boldsymbol{K}_g} \left(\sum_{l\in \boldsymbol{L}} \tau_{g,l} \times h_{g,l,k}\right) \times z_{g,k} \tag{21}$$

$$\text{s.t.} \quad \sum_{g\in \boldsymbol{G}} \sum_{k\in \boldsymbol{K}_g} h_{g,l,k} \times z_{g,k} = 1, \quad l \in \boldsymbol{L} \tag{22}$$

$$\sum_{k\in \boldsymbol{K}_g} z_{g,k} = 1, \quad g \in \boldsymbol{G} \tag{23}$$

$$0 \leqslant z_{g,k} \leqslant 1, \quad g \in \boldsymbol{G}, \ k \in \boldsymbol{K}_g \tag{24}$$

5.2. Initialization

To get the first dual variables of the RLMP, the actual line allocation plan is used as the initial column in the paper.

5.3. Pricing problem

The master problem cannot be solved directly because the number of columns (feasible allocations) is exponential. However, a restricted master problem that considers only a subset of columns can be solved directly. Additional columns with negative reduced cost for the restricted master problem can be generated by solving the pricing problem (Savelsbergh, 1997). In this paper, the pricing problem is used to generate new feasible allocations and is decomposed into several subproblems and each subproblem is a two-stage model. The first stage is a 0–1 knapsack problem finding out a feasible line allocation plan. And the second stage is the non-cooperative game model (model [P2]) determining the frequencies of each line on the base of the line allocation plan.

The unrestricted dual variables u_l is associated with each constraint in set (22), v_g with each constraint in set (23). Then, the reduced cost associated with variable $z_{g,k}$ is $\sum_{l\in \boldsymbol{L}} h_{g,l,k}(\tau_{g,l} - u_l) - v_g$. The first stage of the pricing problem is as follows:

$$\min \sum_{l\in \boldsymbol{L}} h_{g,l,k}(\tau_{g,l} - u_l) - v_g \tag{25}$$

s.t.

$$\sum_{l} \frac{2 \times d_l \times f_{g,l}}{v_l} \times h_{g,l,k} \leqslant n_g, \quad \forall\, g \in \boldsymbol{G},\ \forall\, k \in \boldsymbol{K}_g \tag{26}$$

$$\sum_{g} h_{g,l,k} = 1, \quad \forall\, l \in \boldsymbol{L},\ \forall\, k \in \boldsymbol{K}_g \tag{27}$$

$$h_{g,l,k} \in \{0,\ 1\} \quad \forall\, g \in \boldsymbol{G},\ \forall\, l \in \boldsymbol{L},\ \forall\, k \in \boldsymbol{K}_g \tag{28}$$

Constraint (26) imposes the number of vehicle constraint for each operator. Constraint (27) ensures that each line is only allocated to one operator. Constraint (28) defines $h_{g,l,k}$ as a binary variable.

The first stage of the sub-problems presented above is a 0–1 knapsack problem, which is one of the most studied NP-hard problems in the last few decades (Wang et al., 2012; Azad et al., 2014). And for the column generation scheme to work, it is not always necessary to select the column with the lowest reduced cost; any column with a negative reduced cost will do (Savelsbergh, 1997). Therefore, a heuristic algorithm is used in the paper to solve the price problems to find solutions within a reasonable time frame. Define *NC* as the number of generated negative reduced cost columns, the process of the heuristic algorithm is presented as follows.

Algorithm 1 Solution of two-stage subproblem

Repeat
  $NC = 0$;
  Set the set of lines operated by each operator;
  Set strategy set of each operator;
  Solve the passenger assignment model of each strategy set by using the label-setting algorithm (Spiess and Florian, 1989);
  Solve the following mathematical programming for each g to obtain $\boldsymbol{s}_g = \{f_{g,l}\}$ such that
    $\max\ u_g(\boldsymbol{s}_g)$
  Get departure frequencies of each line under the lines allocation plan;
  Solve the problem (25);
  Update *NC*;
  If $NC > 0$ **then**
    Update and Solve the RLMP;
  End if
Until $NC = 0$

5.4. Branching strategy

If the optimal solution calculated by the column generation is integer, the solution is the optimal solution to the original problem. Otherwise, a branching scheme is used to cut off the solution's fractional part. The branching strategy used in the paper is based on the classical branching strategy.

The fractional variable $z_{g,k}$ with fractional part closest to 0.5 is branched on. The results are on two branches in the tree, one with the constraint $z_{g,k} = 1$ and the other with $z_{g,k} = 0$. In the case of ties, the variable with the highest transit time $\tau_{g,l}$ is selected.

6. Case study

An instance based on the Development District of Dalian are conducted in this section to validate the line allocation model proposed in this paper. The instance is solved by CPLEX 12.5 and in each iteration the subproblem is calculated in Visual studio 2010.

6.1. Dataset

There are 8 lines and 214 passenger stops in the Development District of Dalian in 2014. The total length of transit lines is 120.9 km. In reality, the lines are operated by two operators, Shuntong Bus Services Limited Company and Zhongshan Bus Limited Company who have 68 vehicles and 102 vehicles, respectively. The information of these lines is shown in Table 1 and Fig. 4. The passenger flow of each stop in the peak time of working day (6:30–8:30 and 16:30–19:00) is presented in Fig. 5 according to the transit special planning in the Development District of Dalian Jinzhou New Area (Dalian Jinzhou New Area Transportation Bureau, 2014). The study period T is set as the peak time period of a working day (6:30–8:30 and 16:30–19:00). The capacity of a vehicle is 100. The upper bound of headway of line l is set as 10 min and the lower bound of headway of line l is 1 min.

Table 1
Information of lines.

Line No.	Origin-Destination	Operator	Length of lines (km)	Number of stops	Fare (CNY)	Average speed (km/h)
1	Xishan community-Sitongqihua	Shuntong Company	16.2	17	1	14
2	Seashells Square -Songlan Village	Zhongshan Company	19.7	31	1	15
3	Xishan community- DD port	Shuntong Company	14	27	1	17
4	Xinglin community- Lingang community	Shuntong Company	15.2	32	1	14
5	Seashells Square- Dalian University	Zhongshan Company	15.8	34	1	15
6	Seven colours column- Dang Wangi	Shuntong Company	10.6	25	1	13
7	Colorful north gate - Tenth middle school	Shuntong Company	16.1	30	1	16
8	Seashells Square - Jinyang community	Zhongshan Company	13.3	18	1	15

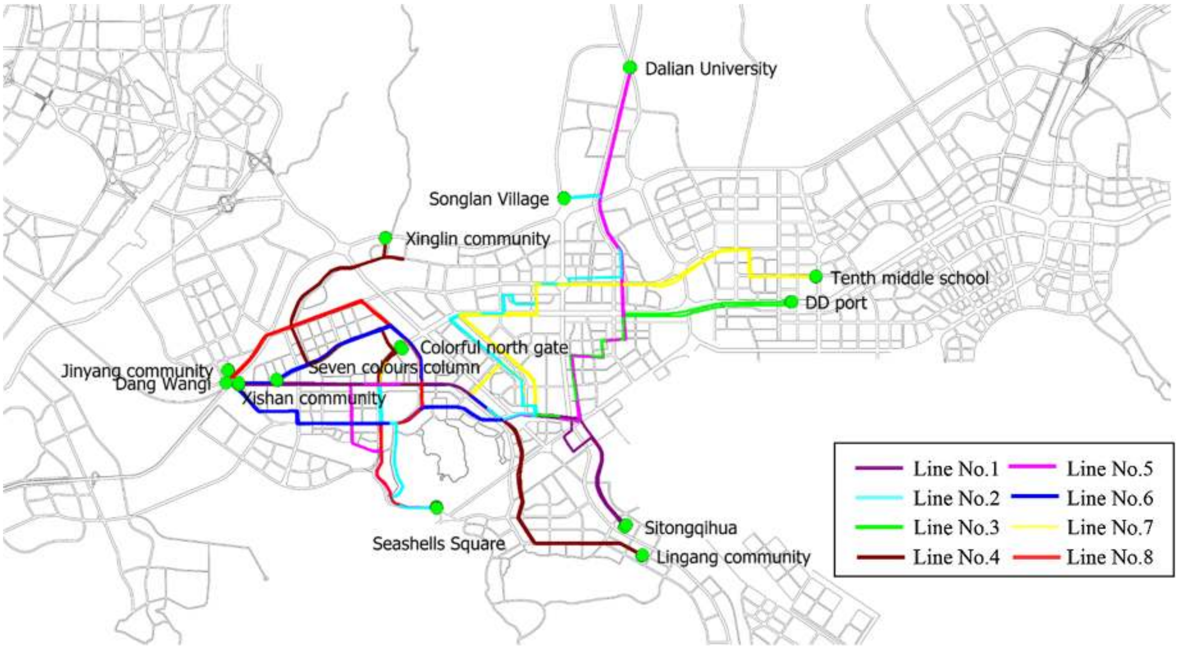


Fig. 4. Network of lines in Development District.

6.2. Results of optimization

On the basis of the data of resident trip survey in 2014 in the Development District of Dalian city, the total transit time of passengers in reality is obtained, and the value is 4282.16 h. Noting that the expected travel time of passengers with current network configuration can also be calculated by using the optimal strategies assignment model introduced in this paper, and the result is 4219.92 h. The error between the actual transit time and the expected travel time is 1.45%. It can conclude that the optimal strategies model is feasible to calculate the total transit time after optimization. Assuming that the number of vehicles of each company is constant, the proposed method is applied. The number of iterations is 155, and the total computing time is 817.52 s. The optimal line allocation obtained by the proposed model is that lines l_1, l_2, l_6 and l_8 are allocated to Shuntong Bus Services Limited Company and lines l_3, l_4, l_5 and l_7 are allocated to Zhongshan Bus Limited Company. The value of the total transit time of passengers after optimization can also be obtained and equals to 4039.27 h (5.67% lower than the actual transit time). Comparison of other operation indexes of each operator, including number of lines, operation mileage, and revenue per hour, before and after optimization is shown in Table 2. In addition, the frequency and passenger flow of each line after optimizing are compared with the actuality in Figs. 6 and 7, respectively.

It can be seen from Table 2 that after optimizing, both operators run 4 lines and the operation mileages are also most the same. It seems to be more equitable for both operators. However, the change of the number of each operator's lines has great influence in the revenue. The total revenue of Shuntong company is down by more than half after optimization, while the revenue of Zhongshan company has a 92 percent increase to about 14,401 yuan.

As shown in Fig. 6, the departure frequencies of line l_4 and l_5 after optimizing are reduced by 26.9% and 17.5%, respectively, compared with the actual plan. As can be seen from Figs. 4 and 5, passenger flows of the competitive stops of line l_4 and l_5 are the largest for all stops, and these two lines are operated by different companies in actuality. Therefore, excess vehicles are deployed on line l_4 and l_5 by the two companies to compete for passengers. After optimizing, the two lines are allocated to the same companies (i.e.

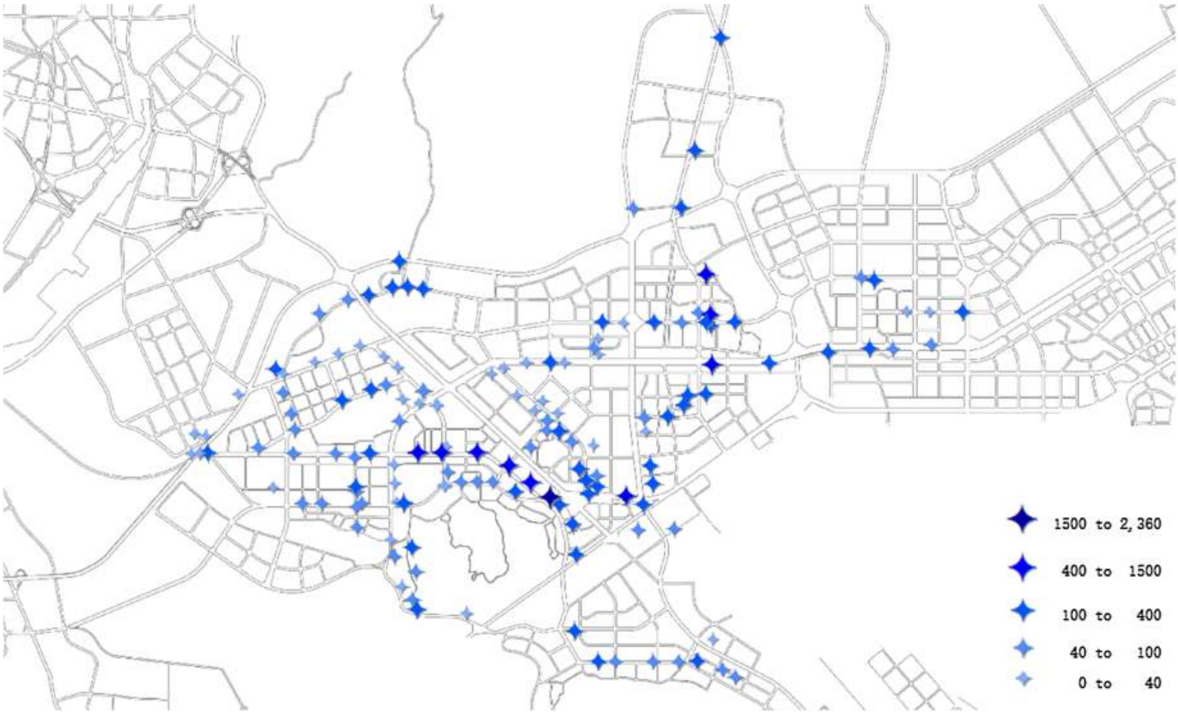


Fig. 5. Passenger flow of each stop in the peak time period of a working day (6:30–8:30 and 16:30–19:00).

Table 2
Comparison of operation indexes of each operator before and after optimization.

Operator	Before Optimization			After Optimization		
	Number of lines	Operation mileage (km)	Revenue (yuan/h)	Number of lines	Operation mileage (km)	Revenue (yuan/h)
Shuntong Company	5	72.1	13,795	4	64.5	6892
Zhongshan Company	3	48.8	7498	4	61.1	14,401

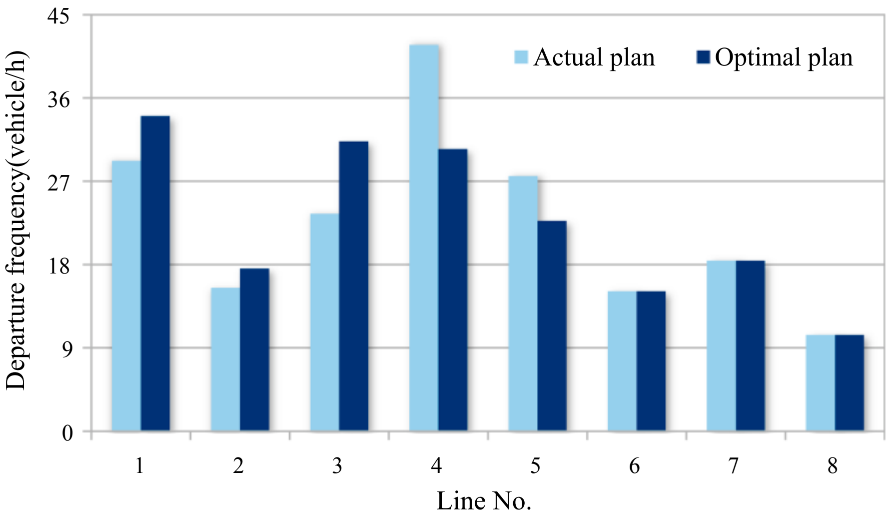


Fig. 6. Comparison of departure frequency of each line.

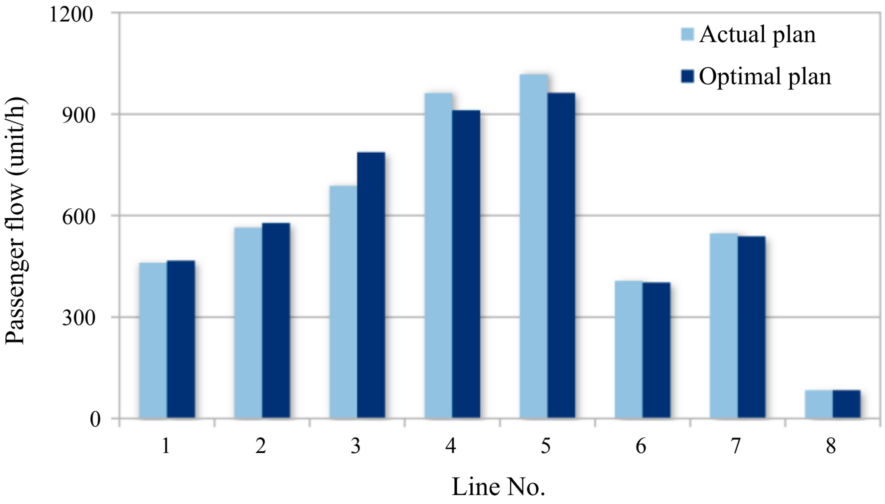


Fig. 7. Comparison of passenger flow of each line.

Zhongshan Company), and there is no competition between line l_4 and l_5. Therefore, the departure frequencies of the two lines are decreased. It also explains why the revenue of Zhongshan Company has increased remarkably. In addition, the comparison of passenger flows of line l_4 and l_5's competitive stops between optimization and actuality is shown in Fig. 8. As can be seen from Figs. 7 and 8 that the reduction of passenger flows of line l_4 and l_5 is small. In other word, obvious reductions of departure frequencies on line l_4 and l_5 do not result in a large reduction of passenger flows, indicating that the number of vehicles allocated to the two lines is larger than the needed level.

From Fig. 6, it can also be seen that the departure frequencies of line l_1 and l_3 after optimizing are increased by 16.7% and 33.3%, respectively, compared with the actual plan. There are some competitive links between line l_1 and l_3, and the two lines are run by the same company in actuality. However, the two lines are allocated to different companies after optimizing. This means the departure frequencies of line l_1 and l_3 are increased. The results show that competition between companies is unavoidable, and the line allocation method considering the UE-O proposed in the paper can be used to reduce the competition as much as possible.

6.3. Sensitivity analysis

This section aims to evaluate the anti-jamming ability of our model under alternative scenarios from a managerial perspective. The first two scenarios are designed to analyze the influences of the number of vehicles and companies on the result of line distribution. The last scenario is used to show how different weighted values of waiting time affect the solution.

6.3.1. Two scenarios modeled by changing the number of vehicles or companies

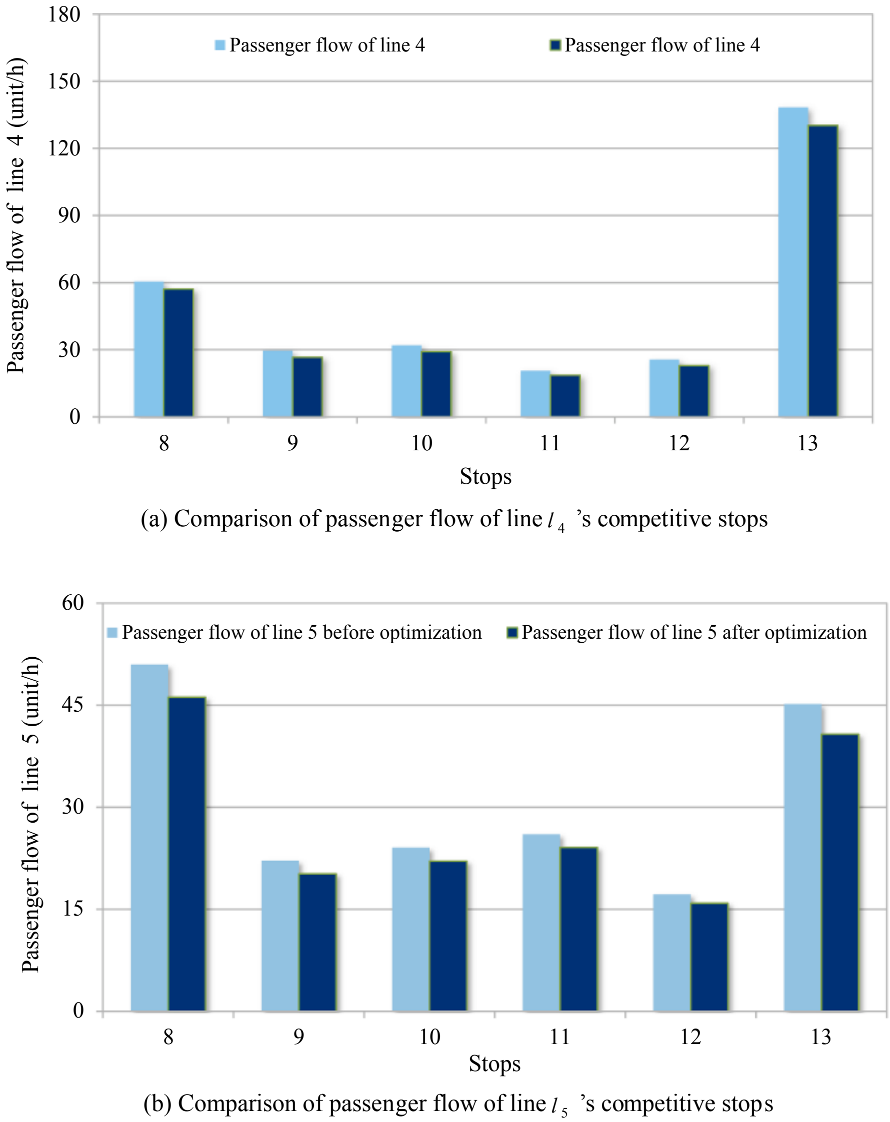
The two scenarios are chosen because the number of vehicles determines the operation strategy of transit operator, while the number of companies reflects the competition level of public transport market (Li et al., 2012). These two factors are together affecting the profit of transit agencies. In scenario 1, all the lines are operated by two companies, and each company has 85 vehicles. In scenario 2, all the lines are operated by three companies, and each company has 57 vehicles.

Table 3 shows the computational results of the two scenarios and the optimal plan as mentioned above. In Table 3, the line allocation plan is given in column 2, the number of iteration is listed in column 3, the computational time in CPU seconds is given in column 4, and the objective value is shown in column 5. It can be seen from Table 3 that if the total number of vehicles is constant, the change of the number of each company's vehicles cannot produce significant impact on the number of iteration and computational time. Besides, as the number of company increases, the number of iteration and computational time increase significantly, while the passengers total transit time has no obvious change. Results also indicate that the computational time of the proposed algorithm to solve the line allocation problem is satisfactory.

In addition, the departure frequency and passenger flow of the two scenarios and the optimal plan are shown in Figs. 9 and 10. As can be seen that, if the companies have the same number of vehicles, passenger flows of the companies are nearly equal. Although the competition among companies is unavoidable in each scenario, the line allocation method considering the UE-O can reduce the competition and maximize the benefits of passengers.

6.3.2. The scenario modeled by changing the weighted value of waiting time

Passengers are sensitive to the waiting time and transfers when they use public transportation. However, it is impossible to satisfy the travel needs of passengers directly (without transfers) because the total operational cost is high with a high number of lines. In transit network design problem, transfers (waiting time) can be included as a penalized term into the objective function. This method is also used in this paper, but the value selection of weight of waiting time with respect to in-vehicle travel time would affect the



(a) Comparison of passenger flow of line l_4 's competitive stops

(b) Comparison of passenger flow of line l_5 's competitive stops

Fig. 8. Comparison of passenger flow of line l_4 and l_5's competitive stops.

Table 3
The computational results of scenarios.

	Line allocation plan			Iteration	CPU(s)	Obj.(h)
	Company 1	Company 2	Company 3			
Optimal plan	1,2,6,8	3,4,5,7	–	155	817.520	4039.27
Scenario 1	1,2,3,6,8	4,5,7	–	155	788.991	4039.24
Scenario 2	2,5	4,7,8	1,3,6	981	3445.800	4033.30

result of line distribution. Thus, this scenario is used to show how different weighted values of waiting time affect the solution. We set the range of the weighted value of waiting time from 1 to 4 and the computational results are shown in Table 4. According to the results in Table 4, the change of weighted value of waiting time does not affect the number of iteration, the computational time and the objective value significantly. However, this value is of importance for the line distribution. It can also be seen that because the waiting time holds a larger proportion in the objective, the objective value slightly increases with the increase in the weighted value of waiting time. Furthermore, when the weighted value of waiting time is set to 1, company 1 and 2 both operate 4 lines and the operation mileages are almost the same.

The departure frequencies for different weighted values of waiting time are also shown in Figs. 11 and 12 shows the

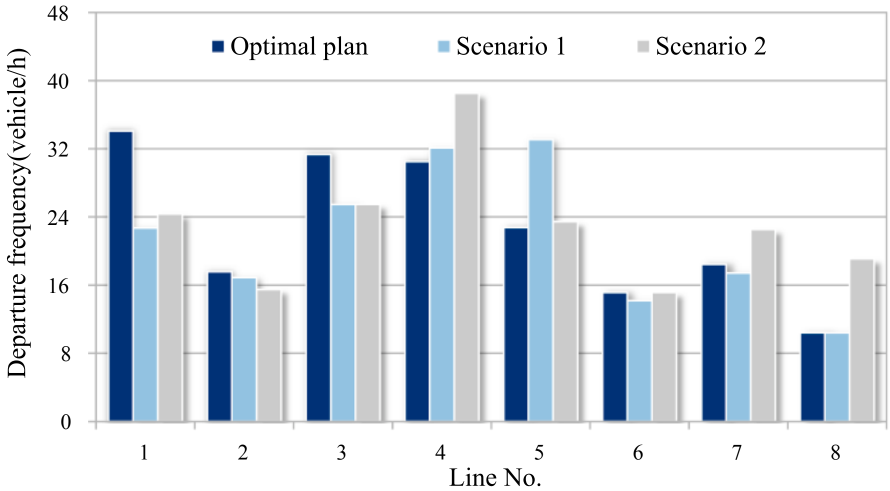


Fig. 9. Comparison of departure frequency.

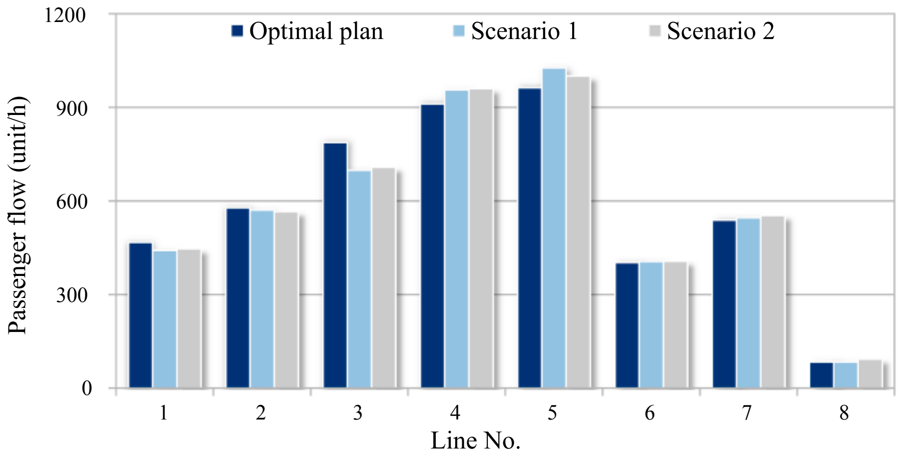


Fig. 10. Comparison of passenger flow.

Table 4
The computational results for different weighted values of waiting time.

Value	Line allocation plan		Iteration	CPU(s)	Obj.(h)
	Company 1	Company 2			
1	1,2,6,8	3,4,5,7	155	817.52	4039.27
2	1,2,7	3,4,5,6,8	160	840.38	4039.34
3	1,3,4	2,5,6,7,8	157	833.44	4039.51
4	5,7	1,2,3,4,6,8	155	820.1	4039.73

configurations of passenger flow. As is shown in Fig. 12, with the increase in the weighted value of waiting time, passengers become more sensitive to the waiting time. Passenger would prefer to reach their destination without any transfers. Those who need a transfer to reach the destination in a relatively short travel time would like to take a longer travel distance but without any transfers.

7. Conclusions

This study focuses on the allocation of transit lines problem in operation-sharing by considering the competition mechanism among operators. While huge literature focuses on the operation of transit lines, few studies incorporated the allocation of transit lines in operation-sharing. Different from the government monopoly, allocating transit lines in operation-sharing reasonably is important for avoiding vicious competition between operators and improving the transit service. This paper presents the concept of UE-

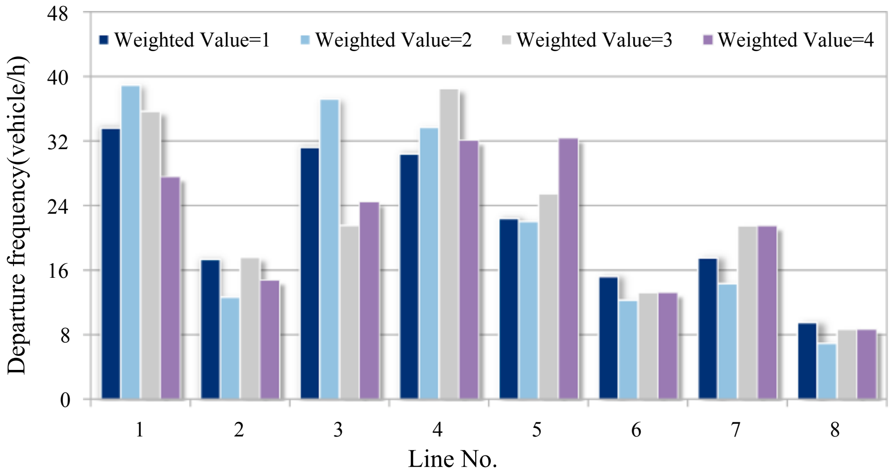


Fig. 11. Departure frequencies for different weighted values of waiting time.

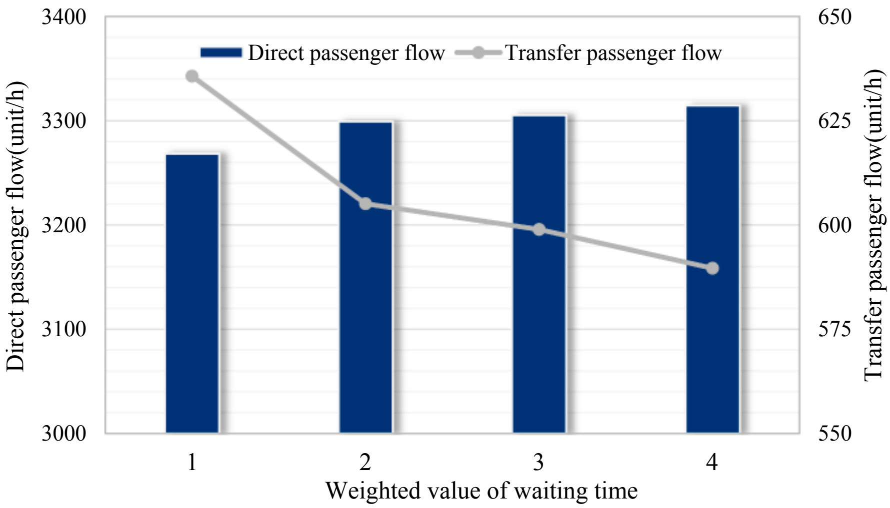


Fig. 12. Configurations of passenger flow for different weighted values of waiting time.

O and proposes a set partitioning formulation considering the benefits of both passengers and operators for the problem of transit line allocation.

In the proposed model, the number of feasible allocation plans is so large that it is impractical to solve the model by using mathematical programming software directly. Therefore, an algorithm based on the column generation and the branch-and-price is proposed in this paper to search the optimal plan of line allocation. Besides, an instance based on the Development District of Dalian is presented in the paper for validating the proposed mixed-integer nonlinear programming model and the branch-and-price algorithm.

Results of the comparison between the optimal line allocation plan and the actual plan indicate that the proposed model and algorithm are able to minimize the transit time of passengers. Although the competition among operators is unavoidable, the line allocation method considering the UE-O proposed in this paper reduces the competition among operators as much as possible.

In reality, different vehicle types may be adopted by different public agencies. In this paper, the type of vehicles is assumed to be fixed. In future study, we will try to consider the effects of different vehicle types on the problem of line allocation. Furthermore, this study mainly focuses on the allocation of transit lines with the non-cooperative game among operators in sharing-operation. There exists cooperative game in sharing-operation. The further study will also consider the allocation of transit lines with cooperative game among operators.

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Appendix A

Algorithm A The scheme of column generation

Repeat
  Solve the RLMP
  Let $u_l$and v_g be the dual variables for corresponding to constraints (22) and (23), respectively;
  Set $NC = 0$;
  Set the set of lines operated by each operator;
  Set strategy set of each operator;
  Solve the passenger assignment model of each strategy set by using the label-setting algorithm (Spiess and Florian, 1989);
  Solve the following mathematical programming for each g to obtain $\boldsymbol{s}_g = \{f_{g,l}\}$ such that
    $\max\ u_g(\boldsymbol{s}_g)$
  Get departure frequencies of each line under the lines allocation plan;
  Solve the problem (25);
  Update NC;
  If $NC > 0$ **then**
    Update NC and Append new column to the RLMP;
  End if
Until $NC = 0$
If needed, **then**
  add branching decisions

Appendix B

Define q_{ij} as the number trips from i to j that should be satisfied by time unit in a given time horizon. Noting that i and j are bus stops. Each trip will be performed by one person that will occupy one place inside a bus operated by one operator. These trips can be calculated by the optimal strategies assignment model based on the frequencies and the OD matrix. We also define m_{ij} as the fare that passengers need to pay for travelling from i to j. m_{ij} can either be a variable associated with travel distance or a constant. Furthermore, a parameter b_p, which used to represent the passenger's unit travel time cost, is also introduced. Thus, we have the following enhanced model to consider the ticket price by using the idea of generalized cost of passengers.

Objective function.

$$\min \sum_{g\in \boldsymbol{G}} \sum_{k\in \boldsymbol{K}_g} \left(b_p \times \sum_{l\in \boldsymbol{L}} \tau_{g,l} + \sum_i \sum_j m_{ij} \times q_{ij} \right) \times h_{g,l,k} \times z_{g,k} \tag{B1}$$

In which,

$$\tau_{g,l} = w_{g,l} + t_{g,l}^{\text{in}} \tag{B2}$$

$$t_{g,l}^{\text{in}} = \sum_i p_{i,i+1,g,l}^{\text{in}} \times t_{i,i+1,l}^{\text{average}},\ \ \forall\ l \in \boldsymbol{L}, \forall\ g \in \boldsymbol{G} \tag{B3}$$

$$w_{g,l} = \frac{1}{2f_{g,l}} \times p_{g,l}^{\text{total}},\ \ \forall\ l \in \boldsymbol{L}, \forall\ g \in \boldsymbol{G} \tag{B4}$$

$$p_{g,l}^{\text{total}} = \sum_i p_{i,g,l}^{\text{board}},\ \ \forall\ l \in \boldsymbol{L}, \forall\ g \in \boldsymbol{G} \tag{B5}$$

Constraints.

$$\sum_{g\in \boldsymbol{G}} \sum_{k\in \boldsymbol{K}_g} h_{g,l,k} \times z_{g,k} = 1,\ \ l \in \boldsymbol{L} \tag{B6}$$

$$\sum_{k\in \boldsymbol{K}_g} z_{g,k} = 1,\ \ g \in \boldsymbol{G} \tag{B7}$$

$$z_{g,k} \in \{0,\ 1\},\ \ g \in \boldsymbol{G}, k \in \boldsymbol{K}_g \tag{B8}$$

In this paper, the number of boarding passengers at stop i of line l operated by operator g, $p_{i,g,l}^{\text{board}}$, is calculated by the optimal strategies assignment model. In this assignment model, it is very difficult to consider the impact of fare on travelers' route choice especially when the transit lines are operated by different operators. However, it can be solved by using other passenger assignment models that can consider both the expected travel time and fare. The use of a different passenger assignment model would not change any of the other components of the transit lines allocation model proposed in this paper.

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