Allocation of Optimal Distributed Generation using GA for Minimum System Losses

T.N. Shukla, S.P. Singh and K.B. Naik

Abstract— **Distributed Generation (DG) represents a reliable option for solving major problems of distribution companies, such as load growth, overloaded lines, quality of supply and reliability. Moreover, it has been proven that the additional benefits brought by DG could be substantial if properly used. The DG applications can potentially defer the investments to be made to upgrade the assets of distribution system, extend equipment maintenance intervals, reduce electrical line losses, and improve distribution system reliability. This paper aims to minimize system real power losses. To achieve the objective a Genetic algorithm based optimization methodology has been proposed to calculate the optimal DG size to be allocated at the appropriate location(s) along the feeder decided by loss sensitivity analysis. The method is tested on 69-bus test system, proving that the technique is effective. Numerical results show the effectiveness of the proposed procedure.**

Index Terms— **distributed generation, loss sensitivity, line loss reduction, optimal location and size, radial distribution networks.**

I. INTRODUCTION

Conventionally, the distribution networks have been designed to convey electrical energy from high voltage transmission networks, whereby the majority of electrical generation plants were connected, to the customers. However, the presence of Embedded or Distributed Generation (DG) in the distribution systems radically alters this point of view, since DG changes distribution networks from passive networks with unidirectional power flows from higher to lower voltage levels into active networks with multidirectional power flows [1]. The benefits of DG are numerous [2, 3] and the reasons [1] for implementing DGs are an energy efficiency or rational use of energy, deregulation or competition policy, diversification of energy sources, availability of modular generating plant, ease of finding sites for smaller generators, shorter construction times and lower capital costs of smaller plants and proximity of the generation plant to heavy loads, which reduces transmission costs. Also it is accepted by many countries that the reduction in gaseous emissions (mainly CO2) offered by DGs is major legal driver for DG implementation [4].

The necessity for flexible electric systems, changing regulatory and economic scenarios, energy savings and environmental impact are providing impetus to the development of distributed generation, which is predicted to play an increasing role in the future electric power system; with so much new Distributed Generation (DG) being installed, it is critical that the power system impacts be assessed accurately so that DG can be applied in a manner that avoids causing degradation of power quality, reliability and control of the utility system.

Traditionally, load growth is forecasted by distribution companies until a predetermined amount is reached, whereby a new capacity must be added to the network [5, 6]. This new capacity is usually the addition of new substations or expanding existing substations capacities and their associated new feeders or both. However, the flexibility, technologies, benefits and concepts of DGs is challenging this state of matter and gaining credibility as a solution to the distribution planning problems [7, 8] with the prohibitively high cost of power curtailment (un-served loads), enhancing DGs as an attractive distribution planning option.

The distribution planning problem is to identify a combination of expansion projects that satisfy load growth constraints without violating any system constraints such as equipment overloading [9]. Distribution network planning is to identify the least cost network investment that satisfies load growth requirements without violating any system and operational constraints. Due to their high efficiency, small size, low investment cost, modularity and ability to exploit renewable energy sources, are increasingly becoming an attractive alternative to network reinforcement and expansion. Numerous studies used different approaches to evaluate the benefits from DGs to a network in the form of loss reduction, loading level reduction [10-12]. Presence of DGs in the distribution network can also extend equipment maintenance intervals, reduce electrical line losses, and improve distribution system reliability, all with cost savings to utilities. Naresh Acharya *et al* suggested a heuristic method in [13] to select appropriate location and to calculate DG size for minimum real power losses. Though the method is effective in selecting location, it requires more computational efforts. The optimal value of DG for minimum system losses is calculated at each bus. Placing the calculated DG size for the buses one by one, corresponding system losses are calculated and compared to decide the appropriate location. Also the method used to

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calculate DG size based on approximate loss formula may lead to an inappropriate solution.

In this paper, a GA based technique has been developed and an attempt is made to determine the optimal size(s) of DG to minimize real power losses in distribution systems and to calculate the overall savings derived from it as the benefit to the utility. The DG(s) optimal operating point(s) has been decided from the loss sensitivity factors calculated at various buses. It is to be noted that only real power injections through DG(s) has been considered as they relate to system losses. To test the effectiveness of proposed method, results are compared with the results of a heuristic method reported in [13]. It is observed that the proposed method yield more saving as compared to heuristic method.

II. LINE LOSS REDUCTION ANALYSIS

The loss saving equations is presented in this section and optimization issues are discussed. The instantaneous base case losses for a three-phase distribution system can be expressed as

$$Loss_{(B)} = \frac{rL\left(P_i^2 + Q_i^2\right)}{3V_P^2}. \tag{1}$$

Where *r* is the system resistance per unit length, *L* is the total length of the line, and P_i and Q_i are the real and reactive loads at i^{th} bus respectively and V_P the system phase voltage.

Now The output current of DG supplying complex power $\{S_{Gi} = P_{Gi} + jQ_{Gi}\}$ *is* given by

$$I_G = \left(\frac{P_{Gi} - jQ_{Gi}}{3V_P}\right) \tag{2}$$

The line loss with the integration of DG is a combination (sum) of two parts:

1. Line loss from source to the location of DG.
2. Line loss from DG location to the location of load.

In presence of DG, the feeder current I_S will be the difference of load current I_L and DG output current I_G. The total line loss in presence of DG placed at a '*X*' distance from the source can be expressed as

$$Loss_{(AT)} = \frac{R}{3V_P^2}\left[P_i^2 + Q_i^2 + \left(P_{Gi}^2 + Q_{Gi}^2 - 2P_iP_{Gi} - 2Q_iQ_{Gi}\right)\left(\frac{X}{L}\right)\right] \tag{3}$$

Where, $R = rL$; total resistance of the line.

The instantaneous loss savings (*LS)* at any point on a feeder is the difference between losses without DG and losses with DG and can be represented as

$LS = Loss_{(B)} - Loss_{(AT)}.$

Hence,

$$LS = \frac{RX}{3V_P^2 L}\left(2P_iP_{Gi} + 2Q_iQ_{Gi} - P_{Gi}^2 - Q_{Gi}^2\right) \tag{4}$$

The positive sign of *LS* indicates that system loss reduces with the integration of DG but the negative sign implies that DG causes higher loss in the system.

The loss savings are classified as either capacity or energy loss savings. Capacity loss savings reduce load on T&D and generation system equipment. This lessens the need for capital upgrades. They are calculated by developing feeder and transformer loss saving equations of the form of (4) and evaluating them during peak load conditions. Energy loss savings reduce electricity generation requirements. Their value is the cost savings realized by reducing operation and maintenance expenses of existing plants. In this work, only the real power injection is considered as they relate to system losses.

III. ANALYTICAL METHOD

The analytical method proposed in [13] for sizing of DG(s) at various locations has been expressed in brief in this section.

The total power loss against injected power is a parabolic function and at minimum loss the rate of change of loss with respect to injected power becomes zero.

$$\frac{\partial P_L}{\partial P_i} = 2\sum_{j=1}^{n}\alpha_{ij}P_j - \beta_{ij}Q_j = 0 \tag{5}$$

where, P_L is the real power loss; $Z_{ij} = r_{ij} + jx_{ij}$; ij^{th} element of $[Z_{BUS}]$ matrix and the loss coefficients

$$\alpha_{ij} = r_{ij}\cos(\delta_i - \delta_j)/V_iV_j$$

$$\beta_{ij} = r_{ij}Sin(\delta_i - \delta_j)/V_iV_j$$

From equation (5) it follows that

$$\alpha_{ii}P_i - \beta_{ii}Q_i + 2\sum_{j=1, j\neq i}^{n}(\alpha_{ij}P_j - \beta_{ij}Q_j) = 0 \tag{6}$$

From the above equation, the injected power $P_{inj,i}$, the difference between real power generation and the real power demand at bus *i,* can be expressed that

$$P_{inj,i} = P_{DGi} - P_i \tag{7}$$

$$P_{inj,i} = \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_{inj,i} + \sum_{j=1, j\neq i}^{n}\left(\alpha_{ij}P_{inj,j} - \beta_{ij}Q_{inj,j}\right)\right] \tag{8}$$

Where, P_{DGi} is the real power injection from DG placed at node *i*, and P_i is the load demand at node *i*. By combining equations (2) and (3), the optimum size of DG for each bus *i* for minimum system loss can be expressed as

$$P_{DGi} = P_i + \frac{1}{\alpha_{ii}}\left[\beta_{ii}Q_{inj,i} + \sum_{j=1, j\neq i}^{n}\left(\alpha_{ij}P_{inj,j} - \beta_{ij}Q_{inj,j}\right)\right] \tag{9}$$

The loss, however, is a function of loss coefficient α and β. When DG is installed in the system, the values of loss coefficients will change, as it depends on the state variable voltage and angle. Updating values of α and β again requires another load flow calculation. As accuracy gained in the DG size by updating α and β is small and is negligible. With this assumption, the optimum size of DG for each bus, given by relation (9) can be calculated from the base case load flow (i.e. without DG).

IV. PROBLEM FORMULATION

Since the impacts of distributed generation on system performance depend on system operating conditions and the characteristics of the distributed generation, it is necessary to use some solutions in planning and operation to attain the best performance. In large distribution systems to select best place(s) for installation of optimum size DG units is a complex combinatorial optimization problem. The installation of DG units at non-optimal places can result in an increase in system losses, implying in an increase in costs and, therefore, having an effect opposite to the desired.

Among the many benefits of distributed generation, reduction in system line losses is one of them. System loss reduction by strategically placed DG along the network feeder, can be very useful if the decision maker is committed to reduce losses and to improve network performance (e.g. on the level of losses and/or reliability) maintaining investments to a reasonable low level. This feature may be very useful in case of revenue recovered by DISCO which is not only based on the asset value but also on network performance. The object of this paper is therefore, to minimize system power loss and quantify its benefits to the utilities by injecting real power through DG(s) placing at appropriate operating location(s) in the radial distribution feeders.

A. Selection of location

In order to reduce the search space it is a priori to select best place(s) in the system. Loss sensitivity approach can be used for the purpose which is described in [14]. Accordingly, change in active power loss of the system due to change in active power injection at a node is expressed as

$$\frac{\partial P_{Loss}}{\partial P_j} = 2\sum_{j=1}^{n}\left(\alpha_{ij} P_j - \beta_{ij} Q_j\right) \tag{10}$$

For placing DG at more than one location, successive loss sensitivity analysis is carried out placing DG at earlier selected location(s).

B. Loss Minimization

An established method using loss coefficients α and β popularly referred as exact loss formula [14] used for the calculation of real power loss, is represented

$$P_{Loss}=\sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{ij}\left(P_iP_j+Q_iQ_j\right)+\beta_{ij}\left(Q_iP_j-P_iQ_j\right) \tag{11}$$

The objective function is to minimize total real power losses which can be calculated from loss quation (11)

Minimize

$$P_{Loss} = \sum_{k=1}^{LN} Loss_k \tag{12}$$

Subject to

$$\sum_{i=1}^{n} P_{DGi} \le \sum_{i=1}^{n}\left(P_i + P_L\right) \tag{13}$$

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{14}$$

$$\left|I_{ij}\right| \le \left|I_{ij}\right|_{scheduled} \tag{15}$$

Where,

LN - Total number of line sections,

n - Number of buses

$Loss_k$ -Real power loss in section k,

P_{Loss} -Total real power line losses in the system

P_{DGi} -real power generation by the DG placed at i^{th} bus.

P_i -real power demand at i^{th} bus.

Q_i -reactive power demand at i^{th} bus.

$V_i^{\min} and V_i^{\max}$ defines the voltage limits.

$\left|I_{ij}\right|_{scheduled}$ -Current carrying capacity of line section ij

The resistive components (mainly of the lines) cause the real power loss, while reactive power loss is produced due to the reactive elements. Normally, the real power loss draws more attention for the utilities, as it reduces the efficiency of transmitting energy to customers. Nevertheless, reactive power loss is obviously not less important. This is due to the fact that reactive power flow in the system needs to be maintained at a certain amount for sufficient voltage level. Consequently, reactive power makes it possible to transfer real power through transmission and distribution lines to customers. So in the present work only real power injection through DGs are considered to achieve the objective.

C. Demand Curve

Energy losses are usually accounted per year. Since constant loading condition of a distribution system considered previously, is not realistic and variation of load demand can also not be predicted, load duration curve can be constructed using the demand curve data and can be approximated in discrete levels. Day to day demand curves vary as per the demand of loads. Seasonal variations, social commotion, economic and environmental aspects also dictate the changes in demand curve. Since the final solution depends on proper choice of demand curve, a careful analysis is required. A steady demand curve is considered in this work wherein annual demand curve is approximated by 360 identical daily demand variation curves and is used to compute energy loss. The piecewise linear load duration curve is assumed in this study to include the variation of loads. The load duration curve is divided into three load levels as average, scheduled and peak load conditions of 62.5%, 100% and 125% of scheduled system loads with the duration of 1000 hr., 6760 hrs and 1000 hrs respectively.

V. GENETIC ALGORITHM

For the advantages of parallel searching, robust searching, and searching mechanism based on the principle of natural evolution, genetic algorithm has found applications in many areas and has become one of the most successful optimization algorithms. The GA become particularly suitable for the problem posed here. In this paper a GA based power/energy loss minimization technique is proposed to find best location and optimal size DG in a radial distribution network. The advantages of using GA are that they require no knowledge of gradient information about the response surface; they are resistant to becoming trapped in local optima and can be employed for a wide variety of optimization problems. On the other hand, it is very difficult to achieve analytical relationship between sensitivity of simulated power system and the parameter values to be optimized. Since GA do not need this kind of information, it is suitable in the present optimization task.

A. Coding Strategy

When applying genetic algorithms to optimize DG allocation and sizing problem, an important aspect is coding of potential solutions. In a general way the potential solution is a configuration with the DG units installed in some places. The coded variables are the size of the units at candidate locations for DG installation. GA technique is used to determine the optimal size of distributed generation devices in kW at the appropriate location(s). The coding of active power to be injected at candidate buses through DGs is done in binary form using 12 bits to take care of real power capacity available at the substation for single location and single load level.

GA starts with an initial population whose elements are called chromosomes. Chromosomes consist of a fixed number of variables called genes. In order to evaluate the candidate chromosomes in a population, fitness function based on objective function is to be defined.

B. Fitness Function

Fitness function is a designed function that measures the goodness of a solution. It should be defined in such a way that the better solutions will have a higher fitness value. It plays a major role in selection process. Since the GA proceeds in the direction of evolving better fit strings and the fitness value is only information available to the GA, the performance of the algorithm is highly sensitive to the fitness values. In case of optimization routines, fitness should depend on the objective function to be optimized. Since in the present work, objective function is to minimize loss, the fitness function is defined as inverse of the objective function.

An evolutionary strategy needs to be adopted in order to generate individuals for the next generation. The individuals are arranged by their fitness and only the best of them are taken unchanged into the next generation. In this way good individuals are retained during a run. Other children come from crossover and mutation.

C. Termination of GA

Since the GA is a stochastic search method, it is difficult to formally specify convergence criteria. As the fitness of the population may remain static for a number of generations before a superior individual is found, the application of conventional termination criteria becomes problematic. The common practice is to terminate the GA after a pre-specified number of generations and then test the quality of best members of the population against the problem definition. If no acceptable solutions are found, the GA may be restarted or a fresh search is initiated.

The GA parameters used for optimization are:

- No. of generations: 50
- Population size: 100 individuals
- Cross-over probability: 86%
- Mutation probability: 0.6%
- Selection type: tournament (two individuals)
- Cross-over type: one point cross-over
- Mutation type: constant

VI. IMPLEMENTATION

The GA explained above has been implemented using following steps:

Step 1: Determination of candidate locations:

- Input the distribution system branch impedances and complex bus powers.
- Determine the sensitivity values.
- Arrange the buses in descending order of their sensitivities

Step 2: Input genetic algorithm control data.

Step 3: Initialize population with random strings and copy into mating pool.

Step 4: Do while generation number is less than maximum number of generation taken

- Do while population number is less than population size
- Pick up the string corresponding to population number from mating pool and decode it into test configuration
- Apply load demand
- Call distribution load flow solver
- Check voltage constraints
- Compute fitness function
- Increment population number by one
- Use mating pool to create new population for next generation
- Carry out reproduction, cross over and mutation in mating pool
- Increment generation number by one

Step5: Obtain desired solution i.e. optimal DG size, minimum system loss and savings

Step 6**:** Stop.

VII. RESULTS AND DISCUSSION

A. General Description

In order to demonstrate the capability of the proposed methodology to solve the problem of optimal DG allocation, the well studied 69-bus test system [15] has been considered. The period taken into consideration for the planning study is 10 years long, with all nodes existing at the beginning of the period. It is assumed that active power (sum of total connected load and the base system losses) available at the source node is, at the beginning of the period, about 4.025MW. For each node a constant power demand growth rate of 2.5% per year has been assumed. This assumption has been made for the sake of clarity but there are no restrictions to define a power demand growth rate differentiated for each node. DG energy sources are considered always available throughout.

In the proposed application, size of each generator unit of 500 kW has been adopted. The cost of each unit is assumed equal to $1500.0 which includes DG cost, installation, operation & maintenance charges. The price of the energy purchased from the wholesale electricity market and the price of energy supplied by DG has been assumed equal to $0.05/kWh. These prices may be considered acceptable and has been adopted in order to stress the effectiveness of the methodology on the system. It should be highlighted that, in presence of a liberalized electricity market, different retail sales rate of the energy produced by a DG units may be considered. These retail sales depend on the technology adopted (mini gas turbine, CHP, wind turbine, etc.), the regulatory actions and the willingness to harness renewable.

At the minimum system losses, the energy loss cost which includes purchase of DGs with installation, operation and maintenance costs are evaluated. Saving is calculated as the difference between energy loss cost without DG and the sum of energy loss cost with DG.

B. Discussion

The base case system loss is 225kW calculated by the load flow solver [16]. Loss sensitivity analysis is performed to select best location(s) and results are shown in Table 1. Bus no. 50 (the same bus numbered as 61 in [13]) for base case and 11 (from successive loss sensitivity analysis) are selected in order to place optimal size DG(s) calculated by proposed G.A. The sensitivities of top five buses is tabulated in Table 1. This table contains the successive sensitivity values also for two locations.

To test the effectiveness of the proposed methodology, same loading conditions [13] have been considered for comparison in respect of location, DG capacity and percentage loss reduction. One year savings are also calculated with the intention to observe the savings as the benefits to the utility in terms of money towards in operation and maintenance expenses of existing equipments and the deferral of reinforcement and up-gradation investments. From the results shown in Table 2, it can be seen that the optimal size of DG at node 50 is 1872 kW (1810kW reported in the literature; the method used to calculate DG size based on approximate loss formula may be the reason of difference) which reduces the system loss by 62.50% with the monitory benefits as saving equal to $55570 which the utility can curtail from operation and maintenance expenses of existing plants and avoid reinforcement and up-gradation investments. It is $1469.00 more than the savings could be obtained by heuristic method which shows that the proposed method is better effective and applicable. The loss reduction is also marginally more by proposed method than the Heuristic method.

The methodology to calculate one year energy loss cost and savings is used for two-location DG with same system loads. Results are tabulated in Table 3. It is observed that the losses are further reduced to a minimum of 73.3975 kW. Because of additional loss reduction, the savings in case of two-location DG is $57402.00 in comparison to $55570.00 in case of one location DG. This shows that presence of optimal size DG resources at more than one location is more beneficial.

Using approximated demand curve to consider time varying loads, savings in presence of DGs at single location and two locations are calculated for ten year period. Results are tabulated in Table 4. The ten year energy loss cost without DG is $1275065.00, with DG at one location it is $491868.27 while DG(s) placed at two locations the energy loss cost is $434469.27 and the savings calculated for of two cases are $778039.75 and $833559.77 respectively. It can be seen that in case of DG units placed at two locations the savings benefit is $57399.00 more compared to one location.

Table 1
Sensitivity Analysis Results

	Bus No.	Sensitivity values
Base case Sensitivity results	**50**	0.0219983
	53	0.0119409
	11	0.0094506
	10	0.0041633
	37	0.0037352
Successive sensitivity results	**50**	0.0219983
	11	0.0063301
	53	0.0059686
	54	0.0038651
	10	0.0034569

Table 2
Comparison of results with constant (rated) loads

Methodology	Location (bus no.)	DG size (in kW)	Power Loss (in kW)		Loss reduction (kW)	Savings (in $)
			Without DG	With DG		
Heuristic [13]	50	1810	219.28	81.44	137.84	54101.0
Proposed GA	50	1872	225.00	84.43	140.57	**55570.0**

Table 3
Loss and one year savings at rated loads

	without DG	Single Location	Two Locations
DG value(in kW)	-	1872	1732 & 831
Losses (kW)	-	84. 44	73.3975
Energy loss cost ($)	98550.00	42980.78	41148.00
Saving ($)	-	**55570.00**	**57402.00**

Table 4
Savings for 10 year Period with time varying loads

	without DG	Single Location	Two Locations
DG value(in kW)	-	2047	2047 & 1570
Losses (kW)	-	84. 431	73.3975
Energy loss cost ($)	1275065.00	497025.25	441505.234
Saving ($)	-	**778039.75**	**833559.77**

VIII. CONCLUSION

Size and location of DG are crucial factors in the application of DG for loss minimization. This paper makes use of sensitivity to decide the appropriate location of DG and GA to calculate the optimum size of DG reducing total power losses in primary distribution network. The results show that the integration of DG is highly effective in reducing power losses in the distribution networks. Loss reduction in presence of DG can relieve some of overloaded system equipments and may be very useful in case of revenue recovered by utility based on improved network performance (improved efficiency and reliability). The studies also reveal that maximum benefits from DG can be obtained only if proper DG planning is performed. Though the numerical results presented in this paper are related with a specific system, optimal DG model can vary from system to system, depending on the system configurations, type of loads on the system and a trade-off among the objectives of DG usage.

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