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Alpha-Particle Reduced Widths in Heavy Nuclei

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The α-particle reduced widths γ_α^2 for the ground state in Po^{210} and Po^{212} are calculated on the basis of the nuclear shell model. The calculations are made taking the boundary condition in an approximate way into consideration. The effects of the configuration mixing of the parent and daughter nucleus wave functions on γ_α^2 are examined and it is found that they give quite large contributions to γ_α^2. Some features of the distortion of the two nucleon wave functions arising from the configuration mixing are discussed graphically. It is shown that the wave function of relative motion with the mixing of the level which is lowered from the upper band by spin-orbit force would correspond to the bound electron-pair in the superconducting metals. Although there are some unavoidable uncertainties in the course of the calculations, it is concluded that the wave functions derived by conventional shell model calculations can explain the major part of the experimental values of γ_α^2.

§ 1. Introduction

It is well known that the independent particle model is remarkably successful in explaining the various problems of the nucleus, including the nuclear reactions. The justifications of the independent particle model on the basis of the strong nuclear forces between nucleons have been made by Brueckner et al.[1] However, their theories treat only the case of the infinite nuclear matter and do not describe the behavior of nucleons in the low density region of nuclear surface. Although there are some studies for a dilute Fermi gas,[2] it is not clear whether the conclusions from the studies of the endless dilute gas are applicable to the structure of nuclear surface without any modification. Perhaps the true theory of nuclear surface will have the finite size effect as a matter of first consideration.[3] Unfortunately, there are scarcely any clear-cut experimental facts concerning the characteristic nucleon-correlation at the nuclear surface.[4]

As is well known, the physical quantity which gives a simple measure of nucleon-correlation at the nuclear surface is the reduced widths for the composite particle channels, such as for deuteron or α-particle. In this paper we shall examine to what extent the experimental values of γ_α^2 are explained by the independent particle model within a finite well, such as harmonic oscillator shell model. In the case of light nuclei, we can expect large γ_α^2 also from the independent particle model,[5] since both protons and neutrons are in the same

orbit and the size of the α-particle and light nuclei are not so different. Therefore, we shall examine the cases of heavy α-decaying nuclei in which one might be able to expect large difference between γ_α^2(I.P.M.) and γ_α^2(exp.) if the nucleus has nucleon-correlation at the nuclear surface.

Recently, Mang[6] has calculated the γ_α^2's for nuclei around Pb^{208} on the basis of the nuclear shell model, and obtained good agreement with the experimental data. However, he discussed only the relative magnitude of γ_α^2 and not the absolute values, because he neglected the boundary condition at the nuclear surface. Since for the evaluation of the absolute magnitude of γ_α^2 the boundary condition at the nuclear surface is very important, we have tried to calculate γ_α^2 taking into account the boundary condition as far as possible.

Moreover, we have examined the effect of configuration mixing on the value of γ_α^2 and found that it gives quite large contributions to the γ_α^2. For the case of Po^{210} decay, it is concluded that the nuclear shell model with configuration mixing can explain the experimental value of γ_α^2 satisfactorily, and we need not assume any other mechanism of "clustering" of particles at the nuclear surface.

§ 2. Method of calculation

The most general theory of α-decay is developed by Thomas[7] from the time-independent R-matrix theory of nuclear reactions. He derived the relation between the decay constant λ and the reduced widths γ_α^2 in the following form:

$$\lambda=\frac{2\gamma_{\alpha L}^2}{\hbar}P_L \tag{1}$$

where P_L is the familiar barrier penetrability. The reduced widths amplitude $\gamma_{\alpha L}$ can be expressed as

$$\gamma_{\alpha L}=\sqrt{\frac{\hbar^2 R_0}{2M}}\int\varphi_{JM}^{*}\chi_\alpha(x_\alpha)\sum_m(Ljm M-m|JM)\,Y_L^m\left(\frac{\mathbb{R}}{R}\right)\Psi_{jM-m}(x_\kappa)\,dx_\kappa\,dx_\alpha\,d\Omega_R \tag{2}$$

where φ_{JM} and ψ_{jM-m} are the wave functions of the parent and the daughter nuclei. χ_α and $Y_L^m(\mathbb{R}/R)$ are the internal wave function and the angular part of the centre-of-mass motion of α-particle, respectively.

As is well known, the barrier penetrability P_L is very sensitive to R_0, beyond which α-particle is not influenced by the nuclear force. However, the recent optical model analyses of α-nucleus scatterings have determined the shape of the nuclear potential at the nuclear surface fairly definitely. In Fig. 1 we reproduce the nuclear optical potential of Pb^{208} for 40 Mev α-particle determined by Igo.[8] Due to the strong absorbent of nucleus for α-particle, the inner parts of the potential are not determined, but all potential-sets which have the same form at the nuclear surface can give good agreement with the experimental data. Assuming that the shape of the potential-tail will not be sensitive to the energy, although the main body of the optical potential is, we have determined

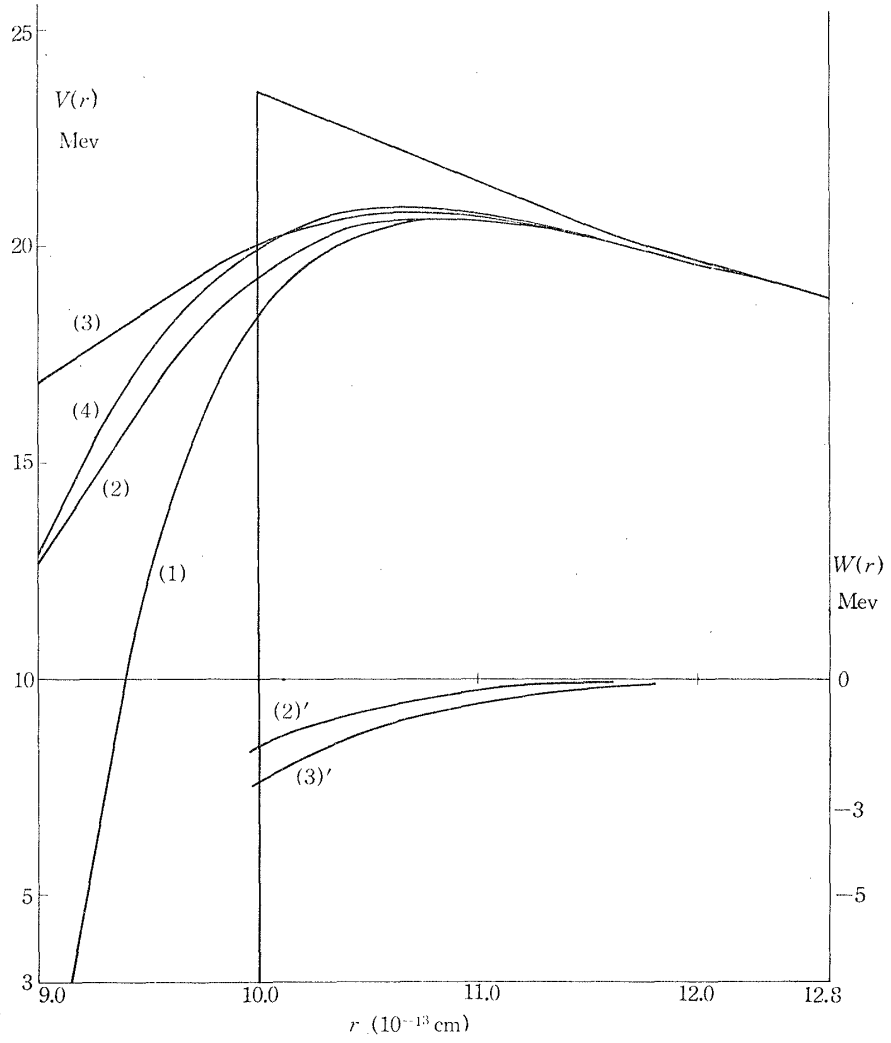

Fig. 1. Various optical potentials of Pb^{208} for 40 Mev α-particle which can give equally good agreement with the experimental data. For real and imaginary parts the left and right scales should be taken, respectively. These have been determined by G. Igo.[8]

the interaction radius R_0 as 10 fermi ($=1.4A^{1/3}+1.6$ fermi) for the cases of α-decay of Po^{210} and Po^{212} in which the energies of α-particle are several Mev. Idealizing the nuclear potential as the schematic one drawn in Fig. 1, we calculate the barrier penetrability P_L, and then from the experimental values of the decay constant, we derive the α-particle reduced widths for ground state of heavy nuclei which are to be compared with the theoretical values.*

* The reduced widths δ^2 given by Rasmussen[9] are derived under a different definition from ours. His definition and its relation to ours are

$$\lambda=(\delta^2/h)\,e^{-2G},$$

$$\delta^2=4\pi k R_0\,\gamma_\alpha{}^2\simeq 120\,\gamma_\alpha{}^2$$

where

$$k=\frac{\sqrt{2M_\alpha E_\alpha}}{\hbar}\,.$$

As mentioned in the Introduction, our aim is to calculate the $\gamma_{\alpha L}$ by using Eq. (2) on the basis of the nuclear shell model. For this purpose, the following assumptions are made for the wave functions.*

(1) The α-particle wave function is of Gaussian type.

$$\chi_\alpha(1234)=\left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2}\exp\left[-\frac{\beta}{2}(\xi_1^2+\xi_2^2+\xi_3^2)\right](4\pi)^{-3/2}\chi_0^0(12)\,\chi_0^0(34), \tag{3}$$

$$\xi_1=\frac{1}{\sqrt{2}}(r_1-r_2),\quad \xi_2=\frac{1}{\sqrt{2}}(r_3-r_4),\quad \xi_3=\frac{1}{2}(r_1+r_2-r_3-r_4)$$

χ_0^0: spin singlet function.

The constant β is chosen so that the *rms* radius of the charge density is equal to the measured value (1.6 fermi).[10] From the equation

$$R_\alpha^2=\frac{1}{4}\sum_i\langle\chi_\alpha(r_i-R)^2\chi_\alpha\rangle, \tag{4}$$

we have

$$\beta=0.44\ \text{fermi}^{-2}.$$

(2) The wave functions of the parent and daughter nucleus are shell model wave functions with seniority zero (in this paper, we treat only even-even nuclei: Po^{210}, Po^{212}). For the single particle wave functions, harmonic oscillator functions are used:

$$\varphi_{\nu lj}^m=\sqrt{\frac{2\nu!\,b^{3/2}}{(\nu+l+1/2)!}}L_\nu^{l+1/2}(br^2)(\sqrt{b}\,r)^l e^{-(1/2)br^2}\sum\left(l\ \frac{1}{2}\ \mu m-\mu|jm\right)Y_l^\mu\left(\frac{r}{r}\right)\chi_{1/2}^{m-\mu}, \tag{5}$$

where

$$L_\nu^{l+1/2}(br^2)=\sum_{h=0}^{\nu}\binom{\nu+l+1/2}{\nu-h}\frac{(-)^h}{h!}(br^2)^h.$$

With the above assumptions, we get the reduced widths[11] for the ground state transitions of even-even nuclei, in which cases all angular momenta of the parent, daughter nuclei and of the outgoing α-particle are equal to zero. It is

$$\gamma_\alpha\Big/\sqrt{\frac{\hbar^2R_0}{2M}}=\left(\frac{A_1!}{2!\,(A_1-2)!}\,\frac{A_3!}{2!\,(A_3-2)!}\right)^{1/2}$$
$$\times\sqrt{\frac{2j_1+3-A_1}{(A_1-1)(2j_1+1)}}\sqrt{\frac{2j_3+3-A_3}{(A_3-1)(2j_3+1)}}\times\mathcal{O}(R_0). \tag{6}$$

$A_1(A_3)$ is the number of protons (neutrons) in the unfilled subshell with quantum number $\nu_1l_1j_1(\nu_3l_3j_3)$ in the parent nucleus. The second factor is the double

* We adopt the same notation as Mang's.

parentage coefficient.[12] The last factor is the overlap integral between four nucleons in the j_1 and j_3 orbits and the α-particle leaving the daughter nucleus with the orbital angular momentum zero:

$$\mathcal{O}(R_0)=\int \psi_p^*(j_1^2,\ J_p=0)\psi_n^*(j_3^2,\ J_n=0)\chi_\alpha(1234)\,Y_0^0\left(\frac{\boldsymbol{R}}{R}\right)dx_\alpha d\Omega_R. \tag{7}$$

Integration implies also the summation over spin variables. Since the $\chi_\alpha(1234)$ includes the spin-singlet function, we must transform the j–j coupling representation of each two-nucleon wave function to the L–S coupling representation. Then

$$\mathcal{O}(R_0)=T(00\,;\,j_1j_1)\,T(00\,;\,j_3j_3)\int \psi_p^*(L=0,\ S=0)\,\psi_n^*(L=0,\ S=0)\chi_\alpha\,Y_0^0\,dx_\alpha d\Omega_R, \tag{8}$$

where

$$T(L=0,\ S=0\,;\,j_1j_1)=\frac{(-)^{(l_1+1/2-j_1)}}{2}(2j_1+1)\,W\left(j_1l_1j_1l_1\,;\ \frac{1}{2}\ 0\right). \tag{9}$$

By using Talmi transformation coefficients $\langle NL,\ nl\,;\ \bar{L}|N_1l_1,\ N_2l_2\,;\ \bar{L}\rangle$,[13] the following expression is derived,

$$\begin{aligned}
\mathcal{O}(R_0)&=T(00\,;\,j_1j_1)\,T(00\,;\,j_3j_3)\int\sum_{Nn_1n_2}\langle NSn_3s\,;\,0|N_1SN_2S\,;\,0\rangle\\
&\times\langle N_1Sn_1s\,;\,0|\nu_1l_1\nu_1l_1\,;\,0\rangle\langle N_2Sn_2s\,;\,0|\nu_3l_3\nu_3l_3\,;\,0\rangle\\
&\times\psi_{n_1s}(\tilde{\xi}_1)\,\psi_{n_2s}(\tilde{\xi}_2)\,\psi_{n_3s}(\tilde{\xi}_3)\,\psi_{NS}(R_0)\\
&\times\left(\frac{2\beta^{3/2}}{(1/2)!}\right)^{3/2}\exp\left[-\frac{\beta}{2}\,(\tilde{\xi}_1^{\,2}+\tilde{\xi}_2^{\,2}+\tilde{\xi}_3^{\,2})\right](4\pi)^{-3/2}\\
&\times\tilde{\xi}_1^{\,2}\tilde{\xi}_2^{\,2}\tilde{\xi}_3^{\,2}d\tilde{\xi}_1\,d\tilde{\xi}_2\,d\tilde{\xi}_3(4\pi)^3\,d\Omega_R\\
&=T(00\,;\,j_1j_1)T(00\,;\,j_3j_3)\sum_{Nn_1n_2}\langle NSn_3s\,;\,0|N_1SN_2S\,;\,0\rangle\\
&\times\langle N_1Sn_1s\,;\,0|\nu_1l_1\nu_1l_1\,;\,0\rangle\langle N_2Sn_2s\,;\,0|\nu_3l_3\nu_3l_3\,;\,0\rangle\\
&\times\left(\frac{1}{2}\,!\right)^{-3/2}\left[\frac{(n_1+1/2)!\,(n_2+1/2)!(n_3+1/2)!}{n_1!\,n_2!\,n_3!}\right]^{1/2}\\
&\times\left(\frac{2\sqrt{\beta b}}{\beta+b}\right)^{9/2}\left(\frac{\beta-b}{\beta+b}\right)^{n_1+n_2+n_3}\sqrt{\frac{2\cdot N!\,(4b)^{3/2}}{(N+1/2)!}}\,L_N^{1/2}(4bR_0^{\,2})\,e^{-2bR_0^2}\\
&\equiv\sum_N C_N\left(\frac{\beta-b}{\beta+b}\right)^{N_m-N}\psi_{NS}(R_0),
\end{aligned} \tag{10}$$

where N_m is the maximum of N and is given by

$$N_m=2\nu_1+l_1+2\nu_3+l_3.$$

Table I.

	$x=\frac{1}{2}R_0^3\psi_{NS}^2(R_0)$	
NL	$Po^{210}(S=-18.5)$*	$Po^{210}(S=-16.7)$*
11 S	0.785	0.855
10 S	0.756	0.803
9 S	0.719	0.757

* S is the well-known shift factor and is expressed as

$$S=-R_0\frac{\sqrt{2M_\alpha(V-E_\alpha)}}{\hbar}$$

in the W. K. B. approximation.

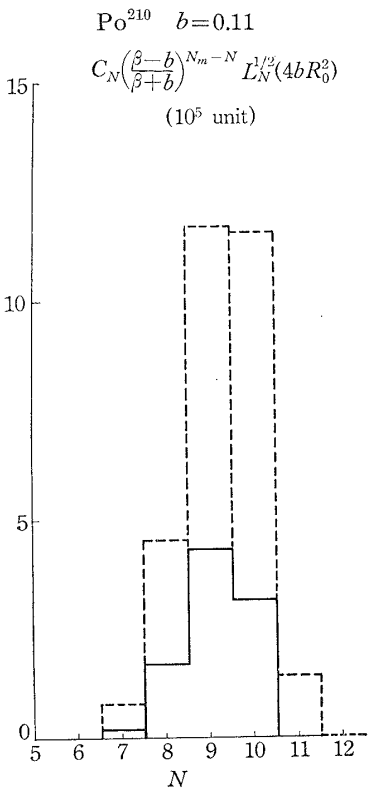

Fig. 2. Rates of the contribution to γ_α from each $\psi_{NS}(R_0)$ in Po^{210}. The full and dotted lines are the ones for pure configuration and mixed configuration, respectively.

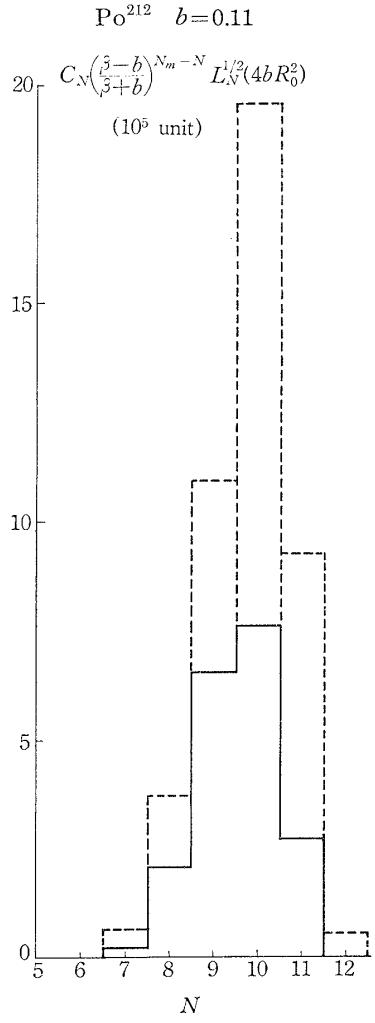

Fig. 3. Rates of the contribution to γ_α from each $\psi_{NS}(R_0)$ in Po^{212}. The full and dotted lines are the ones for pure configuration and mixed configuration, respectively.

The size parameter b of the harmonic oscillator is determined as follows. The eigenstate in the infinite harmonic oscillator well which is regular at infinity cannot have an open channel. Therefore it is obvious that the shell model wave function does not satisfy the boundary condition which we normally impose upon a compound state in resonance reaction theory. Then, instead of fitting the logarithmic derivative of the internal wave function to the external one, we determine the size parameter b so that the value of the harmonic oscillator wave function at R_0 is the same as that of the square well wave function satisfying the boundary condition at R_0 exactly. At this stage we regard the size parameter b as a free parameter which adjusts only the magnitude of wave function at R_0, and forget its relation to the eigenenergy $\hbar\omega$.

The values of the square well wave func-

tions at R_0, which satisfy the boundary conditions, are easily calculated[14] and are given in Table I. On the other hand, we see in Eq. (10) that γ_α is written as a sum over $\psi_{NS}(R_0)$'s with numerical coefficients dependent N. The relative amount of the contribution to γ_α from each term is mainly determined by transformation coefficients, but it also depends on the size parameter b through the factor $\left(\frac{\beta-b}{\beta+b}\right)^{N_m-N}\psi_{NS}(R_0)$. However, it is found that the latter dependence on b is rather weak within the reasonable range of b, and thus could be neglected. In Figs. 2 and 3 we show the magnitude of each term of γ_α as a function of N when $b=0.11$, as an example. It is thus seen that the maximum contributing wave function to γ_α may be taken as $\psi_{10S}(R_0)$ in both cases of Po^{210} and Po^{212}, and b is determined as 0.106, so that the above boundary condition is satisfied for this partial wave, which gives 0.8 for $x=1/2\cdot R_0^3\psi_{10S}^2(R_0)$.

§ 3. Numerical calculation for the cases of Po^{210} and Po^{212}

To evaluate $\mathcal{O}(R_0)$ in Eq. (10) we have first to calculate a number of Talmi transformation coefficients which appear in the coefficients C_N. We carry out these tedious calculations using the formula derived by Arima and Terasawa.[13] The Talmi transformation coefficient which appears as the first factor in the summand of Eq. (10) may be written in a rather compact form:

$$\langle NS,\, ns;\, 0|N_1S,\, N_2S;\, 0\rangle$$

$$=\frac{1}{2^{N+n}}\sqrt{\frac{N!\,(2N+1)!!}{N_1!\,N_2!\,(2N_1+1)!!\,(2N_2+1)!!}}\sqrt{\frac{n!}{(2n+1)!!}}\,I_n,$$

$$I_n=2^n\left\{\sum_{i=0}^{n}\frac{1}{i!\,(n-i)!}\,\frac{(2N_1+1)!!}{\{2(N_1-i)+1\}!!}\,\frac{N_1!}{(N_1-i)!}\right.$$

$$\left.\times\frac{(2N_2+1)!!}{\{2(N_2-n+i)+1\}!!}\,\frac{N_2!}{(N_2-n+i)!}\right\}$$

$$-\sum_{i=0}^{n-1}\frac{1}{(n-i)!}\,\frac{\{2(N-i)+1\}!!}{\{2(N-n)+1\}!!}\,\frac{(N-i)!}{(N-n)!}\,I_i,$$

$$I_0=1.$$

The numbers of the sets (N, n_1, n_2, n_3) that satisfy the condition of the energy conservation are 216 and 273 for the cases of Po^{210} and Po^{212}, even when pure configurations are assumed. The experimental and calculated values of γ_α^2 are given in Table II.

It is worth while to point out that in the case of pure configuration the calculated C_N's change their signs alternately with N. Consequently, all the summands in Eq. (10) have a same sign and thus are added without any can-

Table II.

	Po^{210}		Po^{212}	
E_α	5.33 Mev		8.81 Mev	
P_0	1.22×10^{-24}		5.31×10^{-12}	
S	−18.51		−16.66	
$\gamma^2_{exp.}$	15.7 ev		143 ev	
$\gamma^2_{exp.}/\frac{\hbar^2}{MR_0^2}$	0.88×10^{-4}		0.80×10^{-3}	
$\gamma^2_{theor.}$	0.31 ev	(3.3 ev)	1.3 ev	(7.1 ev)
$\gamma^2_{theor}/\gamma^2_{exp}$	$\frac{1}{50}$	$\left(\frac{1}{4.7}\right)$	$\frac{1}{110}$	$\left(\frac{1}{20}\right)$

cellation, because $\psi_{NS}(R)$ also changes its sign successively with N at the surface.* Nevertheless, it is seen from the last row of Table II that the γ^2_{theor} is rather too small compared with γ^2_{exp}.

In order to see whether the configuration mixing will increase the γ^2_{theor}, we take the case of $Po^{210}\rightarrow Pb^{206}$ decay, in which case the elaborate shell model calculations have been done by several authors.[15]** Employing the wave functions obtained by them:

$$|Po^{210}) = 0.943|(h_{9/2})^2 J=0) + 0.101|(f_{7/2})^2 J=0) - 0.317|(i_{13/2})^2 J=0),$$

$$|Pb^{206}) = 0.865|(p_{1/2})^2 J=0) + 0.308|(f_{5/2})^2 J=0) + 0.377|(p_{3/2})^2 J=0) - 0.122|(i_{13/2})^2 J=0),$$

lengthy calculations are made with the result shown in parentheses in Table II. It is seen that the γ^2_{theor} is increased by almost one order of magnitude compared with the case without configuration mixing.

In the course of the numerical calculations, we found that all the products of C_N's and the mixing coefficients have the same sign as C_N's for the pure configuration, thus the summand in Eq. (10) are again added without any cancellation. To see more clearly, this strikingly large contributions caused by the configuration mixing, it will be interesting to compare the relative wave functions of two protons and two neutrons for the cases with and without the configuration mixing. We do this for the wave functions in which the centre-of-mass motion lies in $5S$ and $4S$ state, since in these cases overlapping with

* On the contrary, in the inner region the summation consists of terms partially cancelling each other.

** Strictly speaking, the nuclear force (Serber, Gauss) and the size parameter used in the calculations are slightly different in each case. Following the procedure of Horie and Sasaki,[16] we recalculate the Po^{210} case taking the same parameter as in Pb^{206}, and the result is

$$|Po^{210}) = 0.946|(h_{9/2})^2 J=0) + 0.143|(f_{7/2})^2 J=0) - 0.290|(i_{13/2})^2 J=0).$$

The minor changes of mixing coefficients which resulted from this recalculation are not important, since there are inherent uncertainties in the experimental reduced widths.

the α-particle wave function becomes maximum. They are written as:

proton $(5S)$

$$\begin{cases}\text{no mixing}: \psi(r)=\underset{0.0630}{\langle 5S.\ 0s|0h,\ 0h\rangle}\psi_{0s}(r)\\ \text{with mixing}: \psi(r)=0.943\underset{0.0630}{\langle 5S,\ 0s|0h,\ 0h\rangle}\psi_{0s}(r)+0.101\underset{0.2764}{\langle 5S,\ 0s|1f,\ 1f\rangle}\psi_{0s}(r)\\ \qquad -0.317\underset{-0.1677}{\langle 5S,\ 1s|0i,\ 0i\rangle}\psi_{1s}(r),\end{cases}$$

proton $(4S)$

$$\begin{cases}\text{no mixing}: \psi(r)=\underset{-0.2697}{\langle 4S,\ 1s|0h,\ 0h\rangle}\psi_{1s}(r)\\ \text{with mixing}: \psi(r)=0.943\underset{-0.2697}{\langle 4S,\ 1s|0h,\ 0h\rangle}\psi_{1s}(r)+0.101\underset{-0.4088}{\langle 4S,\ 1s|1f,\ 1f\rangle}\psi_{1s}(r)\\ \qquad -0.317\underset{0.3934}{\langle 4S,\ 2s|0i,\ 0i\rangle}\psi_{2s}(r),\end{cases}$$

neutron $(5S)$

$$\begin{cases}\text{no mixing}: \psi(r)=\underset{0.4071}{\langle 5S,\ 0s|2p,\ 2p\rangle}\psi_{0s}(r)\\ \text{with mixing}: \psi(r)=1.242\underset{0.4071}{\langle 5S,\ 0s|2p,\ 2p\rangle}\psi_{0s}(r)+0.308\underset{0.2764}{\langle 5S,\ 0s|1f,\ 1f\rangle}\psi_{0s}(r)\\ \qquad -0.122\underset{-0.1677}{\langle 5S,\ 1s|0i,\ 0i\rangle}\psi_{1s}(r),\end{cases}$$

neutron $(4S)$

$$\begin{cases}\text{no mixing}: \psi(r)=\underset{0.03169}{\langle 4S,\ 1s|2p,\ 2p\rangle}\psi_{1s}(r)\\ \text{with mixing}: \psi(r)=1.242\underset{0.03169}{\langle 4S,\ 1s|2p,\ 2p\rangle}\psi_{1s}(r)+0.308\underset{-0.4088}{\langle 4S,\ 1s|1f,\ 1f\rangle}\psi_{1s}(r)\\ \qquad -0.122\underset{0.3934}{\langle 4S,\ 2s|0i,\ 0i\rangle}\psi_{2s}(r).\end{cases}$$

The numbers written below the transformation coefficients are their numerical values. These relative wave functions are shown in Fig. 4 and Fig. 5, together with the wave function of the α-particle for the purpose of comparison.

From Fig. 4, we see that the configuration mixing makes the overlap approximately two times as large as that of the case of no mixing for $5S$ centre-of-mass motion, and that the overlap is rather insensitive to the adopted size parameter, which is seen by looking at the α-particle wave functions given for $\beta=2b$ and $4b$ in Fig. 4. The relative wave function for $4S$ centre-of-mass motion undergoes less distortion due to the configuration mixing than that for $5S$ one, since it has already a large $1s$ component, i.e. $(4S,\ 1s|0h,\ 0h)$ is large, even in the case of no mixing.

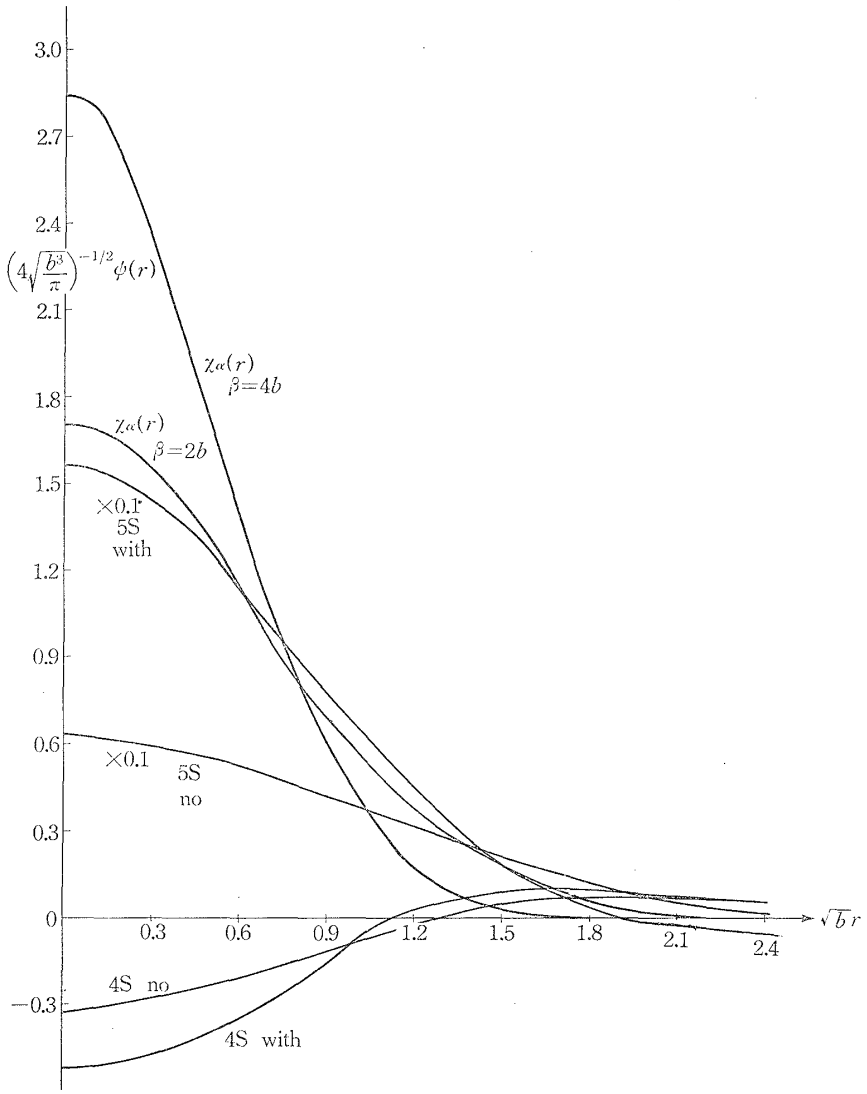

Fig. 4. The relative wave function of two protons.

Similar situations are seen in the case of two-neutrons. In this case, however, the relative wave function for $4S$ centre-of-mass motion changes its sign from plus to minus due to the configuration mixing. Because of the two facts (i) that the centre-of-mass wave function $4S$ and $5S$ have opposite signs at the nuclear surface R_0, and (ii) that the relative wave function corresponding to the $5S$ centre-of-mass has the plus sign, this change of sign is favorable for the construction of di-neutron.

It is noteworthy, in both $4S$ and $5S$ cases, the distortion of the relative wave function, by which the latter resembles more to the α-particle wave function than no mixing case, is mainly caused by the mixing of the $(i_{13/2})^2$ configuration which inherently belongs to the upper band, $6\hbar\omega$. Owing to this, mixed relative wave function contains high frequency wave. This situation corresponds to the bound electron-pairs[17] in the superconducting metals.

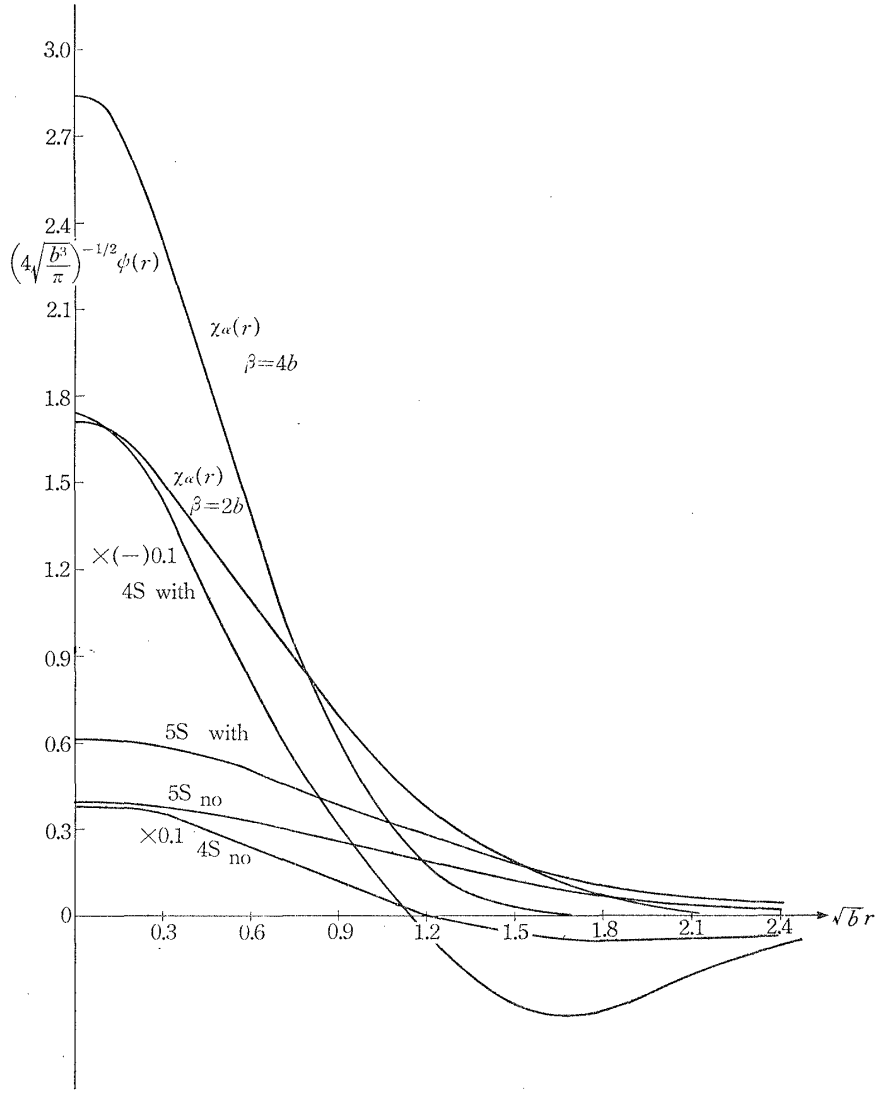

Fig. 5. The relative wave function of two neutrons.

For the case of $Po^{210}\rightarrow Pb^{208}$ decay, a reliable wave function of Po^{212} is not available because of the lack of definite knowledge about the position of the single particle neutron levels which belong to the $6\hbar\omega$ band. For example, the $j_{15/2}$ level predicted by Nilsson[18] has not yet been observed experimentally.[19] Banerjee and Zeh calculated the Po^{210} and Pb^{210} wave function assuming the Serber type δ-force for p–p and n–n force by neglecting the mixture of $(j_{15/2})^2$. Their results are:

$$|Po^{210}) = 0.914(h_{9/2})^2 + 0.256(f_{7/2})^2 - 0.317(i_{13/2})^2,$$

$$|Pb^{210}) = 0.935(g_{9/2})^2 + 0.325(i_{11/2})^2 + 0.141(d_{5/2})^2.$$

Although the wave function $|Pb^{210})$ is less certain in view of the above-mentioned circumstance, in order to obtain a rough estimation for the effect of mixing

for Po^{212} we take $|Po^{210}) \times |Pb^{210})$, each factor being defined above, for $|Po^{212})$ as the first approximation and calculate γ_α^2. This amounts to neglecting the n–p force. The result is also shown in parentheses in Table II.

In this case, the agreement with the experimental value is not so good compared to the Po^{210} case. However, there is a plausible reason for it, that is, the neglect of the mixing of the $j_{15/2}$ level and the n–p force. As is seen in the Po^{210} case, the $j_{15/2}$ level is expected to play an important role for the construction of the neutron-pair. Presumably, the n–p force will act partly for the formation of p–p and n–n pairs, and partly against it. The latter effect will contribute to the formation of n–p pairs. The questions relevant to the n–p force will be studied in a later paper.

§ 4. Discussion and conclusion

It is found from the calculation of the α-particle reduced widths that the wave function derived by the usual shell model calculation with a limited amount of configuration mixing can account for the major part of the experimental values γ_α^2, if we further take into consideration the boundary condition at the nuclear surface appropriately. In other words it is found that the configuration mixing strongly reinforces the formation of α-particle in nucleus. The reason why the configuration mixing has a large effect on the γ_α^2 is that only the s-wave part of the relative motion contributes to γ_α^2, and the amplitude of this s-wave part is much increased under the influence of the short-range nuclear force which caused the configuration mixing. In particular, it is stressed that the mixing of the level which is lowered from the upper band by the spin-orbit force plays an important role for nucleon-clustering. It may thus be concluded that, although some unavoidable uncertainties enter into the course of the calculations, the independent particle model with mixed configurations can explain the experimental values of γ_α^2.

Finally we should discuss R_0 dependence of the experimental and theoretical γ_α^2. For reference, we give the numerical values of penetrability P_0 and shift factor S at the $R_0 = 10$ fermi and 10.5 fermi in the following Table.

Table III.

	R_0	Po^{210}	Po^{212}
P_0	10.0	1.2×10^{-24}	0.5×10^{-11}
	10.5	1.8×10^{-23}	2.7×10^{-11}
S	10.0	−18.5	−16.6
	10.5	−18.8	−16.8

From the above Table it can be seen that γ_α^2(exp.) is very sensitive to R_0, but γ_α^2(theor.) is not so sensitive. Therefore our quantitative conclusion does depend

rather critically on the assumption of $R_0=10$ fermi. However, since one cannot possibly take a smaller value than 10 fermi for R_0 (see Fig. 1), which means that our $\gamma^2_{theor}/\gamma^2_{exp}$ given in Table II would be the lower limit, the conclusion in this paper need not be changed.

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