NASA-CR-172,526

NASA Contractor Report 172526

ALTERNATIVE APPROXIMATION CONCEPTS
FOR SPACE FRAME SYNTHESIS

Robert V. Lust and Lucien A. Schmit

UNIVERSITY OF CALIFORNIA
Los Angeles, California

Grant NSG-1490
March 1985

NASA

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

Alternative Approximation Concepts for Space Frame Synthesis

Robert V. Lust and Lucien A. Schmit

University of California, Los Angeles

Los Angeles, California

NASA Grant NSG - 1490

February 1985

N85-21150#

CONTENTS

LIST OF FIGURES

LIST OF TABLES

NOMENCLATURE

B	Number of generalized design variables.
c	Numerical constant.
c_{bq}	Partial derivative of the q-th retained constraint with respect to the b-th generalized design variable.
d_1	Move limit on the design element cross sectional dimensions.
d_2	Move limit on the design element reciprocal section properties.
$\bar{D},\bar{D}^U,\bar{D}^L$	Vector of design variables, upper and lower bounds.
$[D]$	Transformation matrix relating changes in the reciprocal section properties and cross sectional dimensions to changes in the generalized design variables for a single design element.
E^n	Euclidean n-space.
$f(\bar{Z})$	A general function of $\bar{Z}$.
$\tilde{f}_I(\bar{Z})$	Explicit first order approximation of $f(Z)$ in terms of the reciprocal variables $1/Z_b$.
$\tilde{f}_L(\bar{Z})$	Explicit linear approximation of $f(Z)$ in terms of Z_b.
$\tilde{f}_M(\bar{Z})$	Explicit mixed variable (hybrid) approximation of $f(\bar{Z})$.
$\bar{F}$	Vector of element end forces.
$\{F_i\}_k^e$	Vector of end forces corresponding to the i-th element for the k-th load set in the local coordinate system.
$\{FEF_i\}_k^e$	Vector of fixed end forces due to distributed applied loads corresponding to the i-th element for the k-th load set in the local coordinate system.

$\hat{g}_q$	Constant term in the explicit constraint approximation for the q-th retained constraint.
$g(\bar{X}), g(\bar{X},\bar{Y})$	Implicit behavior constraint function.
$\tilde{g}_q(\bar{Z})$	Explicit approximation of the q-th retained behavior constraint.
$\tilde{g}_L(\bar{X}), \tilde{g}_L(\bar{X},\bar{Y})$	Explicit linear approximation of a retained behavior constraint.
$\tilde{g}_M(\bar{X}), \tilde{g}_M(\overline{XY})$	Explicit mixed variable (hybrid) approximation of a retained behavior constant.
$\bar{G}(\bar{D}), \bar{G}(\bar{X},\bar{Y}), \bar{G}(\bar{Z})$	Vector implicit behavior constraint functions.
$\tilde{h}_q(\bar{Z})$	Explicit approximation of the q-th retained behavior or side constraint in dual formulation.
$\bar{H}(\bar{X},\bar{Y})$	Vector of equality constraint functions.
[H]	Matrix of partial derivatives of the cross sectional dimensions with respect to the reciprocal section properties for a single design element.
I	Number of reciprocal section properties.
[I]	Identity matrix.
J	Number of cross sectional dimensions.
[J]	Matrix of partial derivatives of the reciprocal section properties with respect to the cross sectional dimensions for a single design element.
K	Number of independent loading conditions.
[K]	System level (global) stiffness matrix.
$[K_i]^e$	i-th element stiffness matrix in the local coordinate system.
$[K_i]^g$	i-th element stiffness matrix in the global coordinate system.
$\ell(\bar{\lambda})$	Dual function.

$L(\bar{Z},\bar{\lambda}), L_b(Z_b,\bar{\lambda})$	Lagrangian function, portion of the Lagrangian corresponding to the b-th generalized design variable.
$[L]$	Lower triangular matrix.
m_b	Partial derivative of the objective function (structural mass) with respect to the b-th generalized design variable.
$M(\bar{D}), M(\bar{X}), M(\bar{Z})$	Objective function (structural mass).
$\tilde{M}(\bar{Z})$	Explicit approximation of the objective function.
$\tilde{M}_L(\bar{X}), \tilde{M}_L(\bar{Z})$	Explicit linear approximation of the objective function.
$\tilde{M}_M(\bar{X}), \tilde{M}_M(\bar{Z})$	Explicit mixed variable (hybrid) approximation of the objective function.
$\hat{M}$	Constant term in the explicit objective function approximation.
p	Numerical constant.
$\{P\}_k$	Vector of applied nodal loads corresponding to the k-th loading condition, in the global coordinate system.
$\{P_i\}_k^e$	Vector of applied nodal loads corresponding to the i-th element for the k-th loading condition, in the local coordinate system.
$\{P_i\}_k^g$	Vector of applied nodal loads corresponding to the i-th element for the k-th loading condition, in the global coordinate system.
Q_R	Set of retained behavior constaints.
Q_1	Number of nodal displacement constraints.
Q_2	Number of element strength constraints.
Q	Set of retained behavior and side constraints.
R_c	Behavior constraint cutoff value.

R_q	Response ratio for the q-th constraint.
$[R_\alpha],[R_\beta],[R_\theta]$	Coordinate system rotation matrices.
$T(\overline{Z})$	Design element sizing variable recovery transformation.
$\widetilde{T}(\overline{Z})$	Explicit approximation of the design element sizing variable recovery transformation.
$[T_i]$	Local to global coordinate transformation matrix for the i-th element.
$\overline{u}$	Vector of nodal displacements.
$\overline{u}_0$	Vector of nodal displacements at the beginning of a design stage.
$\{u\}_k$	Vector of nodal displacements for the k-th loading condition, in the global coordinate system.
$\{u_i\}_k$	Vector of nodal displacements corresponding to the i-th element for the k-th loading condition, in the global coordinate system.
$\{u_i\}_k^e$	Vector of nodal displacements corresponding to the i-th element for the k-th loading condition, in the local coordinate system.
$X_i,\overline{X},\overline{X}^U,\overline{X}^L$	i-th reciprocal section property, vector of reciprocal section properties, vectors of upper and lower bounds.
$X_{0_i},\overline{X}_0$	i-th reciprocal section property at the beginning of a design stage, vector of reciprocal section properties at the beginning of a design stage.
$X_{A_i},\overline{X}_A$	Approximate value of the i-th reciprocal section property at the end of a design stage, vector of approximate values of the reciprocal section properties at the end of a design stage.

$X_{E_i}, \overline{X}_E$	Exact value of the i-th reciprocal section property at the end of a design stage, vector of exact values of the reciprocal section properties at the end of a design stage.
$\overline{\tilde{X}}^U, \overline{\tilde{X}}^L$	Vectors of upper and lower bounds on the reciprocal section properties during a design stage.
$\overline{X}(\overline{Z})$	Vector of implicit functions of the reciprocal section properties in terms of the generalized design variables.
$\overline{\tilde{X}}(\overline{Z})$	Vector of explicit approximations of the reciprocal section properties in terms of the generalized design variables.
$\{X\}$	Vector of reciprocal section properties for a single design element.
$\{X_B\}$	Vector of dependent (basic) reciprocal section properties for a single design element.
$\{X_F\}$	Vector of independent (free) reciprocal section properties for a single design element.
$\{X_0\}$	Vector of reciprocal section properties at the beginning of a design stage, for a single design element.
$Y_j, \overline{Y}, \overline{Y}^U, \overline{Y}^L$	j-th cross sectional dimension, vector of cross sectional dimensions, vectors of upper and lower bounds.
$Y_{0_j}, \overline{Y}_0$	j-th cross sectional dimension. at the beginning of a design stage, vector of cross sectional dimensions at the beginning of a design stage.
$\overline{\tilde{Y}}^U, \overline{\tilde{Y}}^L$	Vectors of upper and lower bounds on the cross sectional dimensions during a design stage.
$\overline{Y}(\overline{Z})$	Vector of implicit functions of the cross sectional dimensions in terms of the generalized design variables.
$\{Y\}$	Vector of cross sectional dimensions for a single design element.

$\{Y_B\}$	Vector of dependent (basic) cross sectional dimensions for a single design element.
$\{Y_F\}$	Vector of independent (free) cross sectional dimensions for a single design element.
$\{Y_0\}$	Vector of cross sectional dimensions at the beginning of a design stage, for a single design element.
Z_b, Z_b^U, Z_b^L	b-th generalized design variable, upper and lower bounds.
$\overline{Z}, \overline{Z}^U, \overline{Z}^L$	Vector of generalized design variables vectors of upper and lower bounds.
$\overline{Z}_L$	Vector of generalized design variables after linking.
$Z_{0_b}, \overline{Z}_0$	b-th generalized design variable at the beginning of a design stage, vector of generalized design variables at the beginning of a design stage.
$\overline{Z}^*$	Vector of optimal generalized design variables.
$\{Z\}$	Vector of generalized design variables for a single design element.
$\Delta\overline{X}$	Vector of changes in the element reciprocal section properties.
$\{\Delta X\}$	Vector of changes in the element reciprocal section properties for a single design element.
$\{\Delta X_B\}$	Vector of changes in the dependent (basic) reciprocal section properties for a single design element.
$\{\Delta X_F\}$	Vector of changes in the independent (free) reciprocal section properties for a single design element.
$\Delta\overline{Y}$	Vector of changes in the element cross sectional dimensions.

$\{\Delta Y\}$	Vector of changes in the cross sectional dimensions for a single design element.
$\{\Delta Y_B\}$	Vector of changes in the dependent (basic) cross sectional dimensions for a single design element.
$\{\Delta Y_F\}$	Vector of changes in the independent (free) cross sectional dimensions for a single design element.
$\{\Delta Z\}$	Vector of changes in the generalized design variables for a single design element.

Greek

$\lambda_q, \bar{\lambda}$	Dual variable associated with the q-th retained constraint, vector of dual variables.
$\bar{\lambda}^*$	Vector of optimal dual variables.

Subscripts

A : $\bar{X}_A$	Denotes an approximate value of the associated quantity at the end of a design stage.
b : Z_b	Index for generalized design variable.
B : $\bar{X}_B$	Denotes a dependent (basic) quantity.
c : R_c	Denotes a cutoff value.
E : $\bar{X}_E$	Denotes an exact value of the associated quantity at the end of a design stage.
F : $\bar{X}_F$	Denotes an independent (free) quantity.
i : X_i	Index for reciprocal section properties.
: $[K_i]$	Index for analysis elements.

I : $\tilde{f}_I(\bar{Z})$	Denotes an explicit approximation in terms of inverse variables.
j : Y_j	Index for cross sectional dimensions.
k : $\{u\}_k$	Index for independent loading conditions.
L : $\tilde{f}_L(\bar{Z})$	Denotes an explicit approximation in terms of direct variables.
: $\bar{Z}_L$	Denotes a linked quantity.
M : $\tilde{f}_M(\bar{Z})$	Denotes an explicit approximation in terms of a combination of inverse and direct variables.
q : g_q	Index for retained constraints.
0 : $\bar{X}_0$	Denotes the value of the associated quantity at the beginning of a design stage.

Superscripts

e : $[K_i]^e$	Denotes a quantity described in the element (local) coordinate system.
g : $[K_i]^g$	Denotes a quantity described in the global (system) coordinate system.
L : $\bar{X}^L$	Denotes a lower bound quantity.
T : $[L]^T$	Matrix transpose.
U : $\bar{X}^U$	Denotes an upper bound quantity.
-1 : $[J]^{-1}$	Inverse.
* : $\bar{Z}^*$	Denotes an optimal quantity.

Special Symbols

- : $\bar{X}$	Vector quantity.
~ : $\tilde{g}_q(\bar{Z})$	Explicit approximation of the associated quantity.

$\hat{}$: $\hat{Q}$	Distinguishing mark.
Δ : $\Delta\bar{X}$	Denotes a change in the associated quantity.
∂ : $\frac{\partial g}{\partial Z_b}$	Differential operator.
[ ] : [K]	Matrix quantity.
{ } : {X}	Vector quantity.

ACKNOWLEDGMENT

This report presents some results of a continuing research program entitled 'Fundamental Studies of Methods for Structural Synthesis," sponsored by NASA Research Grant No. NSG-1490. The work reported herein was carried out in the School of Engineering and Applied Science at UCLA during the period from June 1982 to February 1985.

The authors want to take this opportunity to express their appreciation to Deborah Haines for her advice, patience and careful attention to detail during the preparation of this report.

ABSTRACT

A structural synthesis methodology for the minimum mass design of three dimensional frame-truss structures under multiple static loading conditions and subject to limits on displacements, rotations, stresses, local buckling and element cross sectional dimensions is presented. A variety of approximation concept options are employed to yield near optimum designs after no more than 10 structural analyses. Available options include: (A) formulation of the nonlinear mathematical programming problem in either reciprocal section property (RSP) or cross sectional dimension (CSD) space; (B) two alternative approximate problem structures in each design space; and (C) three distinct assumptions about element end-force variations. Fixed element, design element linking and temporary constraint deletion features are also included. The solution of each approximate problem, in either its primal or dual form, is obtained using CONMIN, a feasible directions program (n.b., dual formulation not available for all options).

The frame-truss synthesis methodology is implemented in the COMPASS computer program and is used to solve a variety of problems. These problems were chosen so that, in addition to exercising the various approximation concepts options, the results could be compared with previously published work. The types of problems solved include both planar and three dimensional frame-truss structures and contain frame members having various cross sectional shapes including: (1) a thin walled tube; (2) thin walled box sections; (3) an I section; and (4) a solid square section. Finally, the collection of numerical examples are

used to form guidelines for the solution of future problems.

CHAPTER I

Introduction

1.1 Introduction

During the past decade optimization via general nonlinear mathematical programming techniques has become widely accepted as a viable methodology for engineering design. This has been particularly true in the structural engineering field (Refs. 1-2). Here, mathematical programming methods have been coupled with finite element based structural analysis methods to yield a potentially powerful design tool; leading to the emergence of a number of rather general structural synthesis capabilities (Refs. 3-10). The success of many of the methods is due, in large part, to computational efficiencies gained through the application of various approximation concepts pioneered by Schmit, et al. (Refs. 11-12).

While the basic methodology for structural synthesis is in place for a large class of problems, the majority of the reported computational experience has focused on truss and membrane type structures. The fact that many practical structures are of this type, or can be adequately approximated as such, has certainly contributed to this situation. There is, however, a significant class of problems for which a combined bending-membrane element representation must be used to adequately capture the essential structural behavior (e.g. frame-truss structures). The extension of synthesis methodology to the design of these types of structures has been slow and has met with only limited success. The principal difficulty encountered in this case has been that of choosing an appropriate set of design variables for which accurate behavior constraint approximations can be constructed while simultaneously maintaining adequate design freedom.

The objective of the work reported here is twofold. First, modifications to the current structural synthesis methodology are suggested which will enhance its generality and allow for the more efficient solution of bending-membrane structural design problems. Secondly, numerous example problems, selected to be representative of larger problems of more practical interest, are solved to illustrate the effectiveness of the structural synthesis technique described and to provide a body of computational experience upon which the solution of future problems may be based.

1.2 Background

Much of the early work in the area of optimum design of frame structures was motivated by civil engineering applications. Since many of the structural systems typical of such applications are built using standard section members (e.g. wide flange I-beams) it became popular to use assumed size-inertia relationships of the form

$$I = cZ^{p} \tag{1-1}$$

where I is the cross sectional moment of inertia, c and p are constants and Z is some element sizing variable (e.g. cross sectional area, characteristic cross sectional dimension). These relationships represented assumptions governing the geometry of the element cross section during the re-design process and were frequently based on interpolation and/or extrapolation of tabulated values for standard sections. This approach has the advantage of representing a structural element by a single variable and consequently leads to optimum design problems having relatively few design variables. The popularity of this technique is illustrated by its extensive coverage in the literature (e.g. Refs. 13-19).

While the use of assumed size-inertia relationships has the advantage of reducing the design problem size, it also greatly restricts the amount of design freedom. Although this lack of design freedom is not a serious disadvantage in the design of many civil structures, it can be a severe limitation for more weight critical design applications, such as in the aerospace and automotive industries where the structural elements are usually custom fabrications. As a result, a second approach emerged in which some or all of the element cross sectional dimensions (CSD's) were selected as the structural design variables (Refs. 20-24). The increase in design freedom and the generality of structural elements afforded by this technique lead to its application to increasingly complex problems. However, as in the case of earlier work on truss synthesis, it again became apparent that the implementation of approximation concepts would be required in order that the method be computationally viable for large structural systems.

The integration of approximation concepts into the frame design methodology does not, in itself, present any conceptual difficulties.

However, as in the case of the truss design problem, the implementation requires careful attention to the selection of the intermediate design variables so that accurate approximate expressions for the structural behavior can be generated. It has been demonstrated that high quality first order approximations for nodal displacements and element stresses can be constructed using compliance variables (i.e. reciprocal truss areas and membrane thicknesses) for moderately redundant truss and membrane structures (Ref. 25). Indeed, for the statically determinate case these behavior approximations are exact when formed using the compliance variables. For the frame design problem, the element stresses are, in general, complex nonlinear functions of both the element reciprocal section properties (RSP's) and the element CSD's. As a result there is no particular choice of intermediate design variable which will yield generally high quality approximations for element stresses. However, the nodal displacements are well approximated in terms of the element RSP's. Therefore it might be expected that the compliance variables (RSP's) will yield the best overall behavior constraint approximations for many synthesis problems. Several innovative approaches to the frame design problem have emerged which are based on this concept.

One of the most successful of these approaches is based on the observation that in the case of thin walled beam sections having fixed external dimensions and uniform wall thickness the element RSP's are nearly linear in the reciprocal of the wall thickness (Ref. 5). As a result, high quality approximations of the structural behavior are obtained by selecting the design variables to be the reciprocal wall thicknesses of the design elements. This approach suffers somewhat in that the design freedom is obviously limited. The adverse effects of this limitation are minimized for cases where design element external dimensions are fixed by other considerations such as packaging or attachment requirements. This technique has been applied quite successfully to the preliminary design of automotive frame structures (Refs. 26-27). Extension of the method to cases where the external dimensions are also included as design variables has been explored for several alternative choices of intermediate CSD variables with moderate success (Ref. 28). However, for a general multi-variable design element, behavior approximations based on first order expansions generated directly in terms of the element CSD's or their reciprocals may lead to slow convergence and require an excessive number of structural analyses.

An alternative approach to the frame synthesis problem, which has received somewhat less attention for design elements of general cross sectional geometry, is to perform the structural design directly in terms of the element RSP's. The advantage of such a formulation lies in its ability to capitalize on the high quality behavior approximation for nodal displacements in terms of the element RSP's. However, since the RSP's are treated as independent design variables, a fundamental consideration must be how the actual physical dimensions of the design element cross section are to be recovered from the RSP's. In general, explicit relations for the recovery of the element CSD's are not available. In Ref. 29 a technique which coupled the use of RSP's as design variables and an approximate CSD recovery method was suggested. In this case an approximate linear relationship between the element RSP's and CSD's was constructed and used in the CSD recovery process. As originally presented this technique was applicable only when the number of CSD's equaled the number of RSP's. This restriction was subsequently removed thereby making the method more generally applicable (Ref. 30). Unfortunately, even with the available high quality behavior approximations, the initial numerical experience with the method of Ref. 29 indicated the need for larger than expected numbers of structural analyses and some convergence difficulties. This may have been due to the adverse effect that the linear approximation between element CSD's and RSP's has on the net behavior approximation.

CHAPTER II

The Structural Synthesis Problem

2.1 Introduction

Structural synthesis is, by its very nature, a complex, iterative process. Fundamentally, this process consists of the generation and evaluation of a sequence of trial designs. Each successive design represents an attempt to improve some measure of structural performance. Historically, design modifications were based on the experience and insight of the design engineer. Acceptable designs were frequently obtained only after a considerable number of trial designs had been evaluated. This was particularly true for complex design problems such as those encountered in the design of aerospace structures. This situation, together with increased interest in generating designs which were, in some sense, optimal, subsequently lead to the development of several formal design methodologies based on assumptions as to the number and types of critical failure modes (e.g. structural index, fully stressed design and optimality criterion methods). A more general method based on nonlinear inequality constrained mathematical programming was proposed by Schmit in 1960 (Ref. 31) and forms the basis for this work.

2.2 Problem Formulation

A significant class of structural synthesis problems may be stated as follows: seek a minimum mass design such that all structural behavior quantities and design variables remain within specified limits. Mathematically, this can be written in the form of a nonlinear inequality constrained mathematical programming problem as

$$\min_{\bar{D}} \quad M(\bar{D})$$

$$\text{s.t.} \quad \bar{G}(\bar{D}) \leq \bar{0} \tag{2-1}$$

$$\bar{D}^L \leq \bar{D} \leq \bar{D}^U$$

where the objective function M is the structural mass, $\bar{D}$ is a vector of design variables, $\bar{G}$ is a vector of constraints on the structural behavior (e.g. nodal displacements, element stresses) and $\bar{D}^U$ and $\bar{D}^L$ are the vectors of upper and lower bounds on $\bar{D}$. If it is assumed that the structural topology, configuration, materials and loading conditions are prescribed, then the design variables $\bar{D}$ represent element sizing variables. For frame-truss structures the element sizing variables are typically the element cross sectional dimensions (CSD's) and/or element reciprocal section properties (RSP's). The mathematical program represented by Eq. (2-1), then, can be rewritten for the frame-truss synthesis problem as

$$\min_{\bar{X},\bar{Y}} \quad M(\bar{X},\bar{Y})$$

$$\text{s.t.} \quad \bar{G}(\bar{X},\bar{Y}) \leq \bar{0}$$

$$\bar{H}(\bar{X},\bar{Y}) = \bar{0}$$

$$\bar{X}^L \leq \bar{X} \leq \bar{X}^U \tag{2-2}$$

$$\bar{Y}^L \leq \bar{Y} \leq \bar{Y}^U$$

where $\bar{X}$ is the vector of RSP's, $\bar{Y}$ is the vector of CSD's and $\bar{X}^U$, $\bar{X}^L$, $\bar{Y}^U$ and $\bar{Y}^L$ are their corresponding vectors of upper and lower bounds. The equality constraints $\bar{H}(\bar{X},\bar{Y})$ have been introduced to account for any inter-dependence in the set of sizing variables $\{\bar{X},\bar{Y}\}$. Since these constraints are, in general, nonlinear, the solution of the mathematical program represented by Eq. (2-2) may be computationally burdensome. It is therefore useful to rewrite Eq. (2-2) in terms of a vector of independent generalized design variables $\bar{Z}$ as follows:

$$\min_{\bar{Z}} M(\bar{Z})$$

$$\text{s.t.} \quad \bar{G}(\bar{Z}) \leq \bar{0} \tag{2-3}$$

$$\bar{X}^L \leq \bar{X}(\bar{Z}) \leq \bar{X}^U$$

$$\bar{Y}^L \leq \bar{Y}(\bar{Z}) \leq \bar{Y}^U$$

The generalized design variables are, in general, some subset of the element CSD's and RSP's. This design problem can be solved for $\bar{Z}$, with the element CSD's and RSP's being subsequently determined via a recovery transformation of the form

$$(\bar{X},\bar{Y}) = T(\bar{Z}) \tag{2-4}$$

The structural synthesis problem represented by Eqs. (2-3) and (2-4) is, in general, a complex, implicit nonlinear problem in terms of the generalized design variables. As a result, the direct solution of Eqs. (2-3) and (2-4) is computationally impractical even for relatively small structures. A more tractable approach to the solution is to replace this implicit, nonlinear problem with an explicit approximate problem of reduced dimensionality having the following form:

$$\min_{\bar{Z}_L} \tilde{M}(\bar{Z}_L)$$

$$\text{s.t.} \quad \tilde{g}_q(\bar{Z}_L) \leq 0 \; ; \qquad q \in Q_R \tag{2-5}$$

$$\tilde{\bar{X}}^L \leq \tilde{\bar{X}}(\bar{Z}_L) \leq \tilde{\bar{X}}^U$$

$$\tilde{\bar{Y}}^L \leq \tilde{\bar{Y}}(\bar{Z}_L) \leq \tilde{\bar{Y}}^U$$

$$(\bar{X},\bar{Y}) = \tilde{T}(\bar{Z}) \tag{2-6}$$

where, for the general case, $\tilde{M}$ is an explicit approximation of the objective function; the $\tilde{g}_q$ are explicit approximations of subset Q_R of the original constraints $\bar{G}$; $\bar{Z}_L$ is a vector of linked generalized design variables; $\tilde{\bar{X}}^U$, $\tilde{\bar{X}}^L$, $\tilde{\bar{Y}}^U$ and $\tilde{\bar{Y}}^L$ are upper and lower bounds on the RSP's and CSD's chosen to insure the validity of the approximations; and $\tilde{T}$ represents some approximate recovery transformation. The solution to the original problem (Eqs. (2-3) and (2-4)) is obtained via the iterative construction and solution of approximate problems having the form of Eqs. (2-5) and (2-6). Hence, the solution to the implicit, nonlinear design problem is obtained through the solution of a sequence of explicit approximate problems. The generation and solution of each approximate problem consists of the following four phases: 1) structural analysis, 2) approximate problem generation, 3) optimization and 4) detail design recovery. Each of these four phases is described in detail in Chapters III-VI.

CHAPTER III

Structural Analysis

3.1 Introduction

Structural analysis is an integral part of the structural synthesis problem. The solution of the analysis problem yields the structural response quantities (e.g. nodal displacements and element forces) required for the evaluation of the design constraints. Various techniques are available for the linear analysis of frame-truss structures. One of the most widely used techniques, and the one chosen here, is the well known finite element displacement method. This method is particularly attractive in the structural synthesis context because 1) a variety of different structures and loading can be treated in a unified manner, 2) the method is relatively efficient and easy to implement and 3) the method is well suited for subsequent response quantity sensitivity calculations (as will be shown in Chapter IV).

While the finite element method is quite general, the class of problems considered here is frame-truss structures subject to multiple static loading conditions (including discrete nodal loads and loads uniformly distributed along the element) and homogeneous displacement boundary conditions. The underlying analysis equations are described in detail in the next section.

3.2 Static Analysis

The equations governing the response of a linear structural system subject to multiple static loading conditions are of the form

$$[K]\{u\}_k = \{P\}_k \ ; \qquad k = 1,2,\ldots K \tag{3-1}$$

where $[K]$ is the system stiffness matrix, $\{u\}_k$ and $\{P\}_k$ are the vectors of unknown displacements and known applied nodal loads (corresponding to the k-th loading condition), and K is the total number of loading conditions. Eqs. (3-1) represent a set of linear simultaneous equations which can be generated from the element level stiffness matrices $[K_i]^e$ and load vectors $\{P_i\}_k^e$ using an assembly technique known as the direct stiffness method (Ref. 32). The stiffness matrices and work equivalent load vectors (for uniformly distributed loading) for the space frame and truss elements are given in Appendix A.

Prior to the actual assembly of the system stiffness matrix and load vectors the element level quantities $[K_i]^e$ and $\{P_i\}_k^e$ must be expressed in terms of a common system level or global coordinate system. This is accomplished by using the following transformation equations

$$[K_i]^g = [T_i]^T[K_i]^e[T_i]$$

$$\{P_i\}_k^g = [T_i]^T\{P_i\}_k^e \tag{3-2}$$

where $[K_i]^g$ and $\{P_i\}_k^g$ are the element level stiffness matrix and load vector, in global coordinates, for the i-th structural element. The orthogonal transformation matrix $[T_i]$ has the general form

$$[T_i] = \begin{bmatrix} [t_i] & & & \\ & [t_i] & & \\ & & [t_i] & \\ & & & [t_i] \end{bmatrix} \tag{3-3}$$

where (dropping the subscript for convenience)

$$[t] = [R_\alpha][R_\theta][R_\beta] \tag{3-4}$$

and

$$[R_\alpha] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$[R_\theta] = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3-5}$$

$$[R_\beta] = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

The angles α, θ and β, between the local and global coordinate systems, are shown in Fig. 1. It should be noted that the matrix [t] for the space truss element reduces to the form

$$[t] = [R_\theta][R_\beta] \tag{3-6}$$

by virtue of the fact that α may be arbitrarily set to zero making $[R_\alpha]$ an identity matrix.

Once Eqs. (3-1) have been assembled the homogeneous displacement boundary conditions may be applied. Conceptually this is done by eliminating those equations associated with the boundary degrees of freedom (in actuality these equations are never assembled). With the appropriate boundary conditions imposed Eqs. (3-1) represent a positive definite system of equations which can be solved for the unknown displacement vectors $\{u\}_k$. The solution method used here is based on a modified Cholesky decomposition technique which replaces [K] by a factorization of the form

$$[K] = [L][D][L]^T \tag{3-7}$$

where [L] is a lower triangular matrix and [D] is a nonsingular diagonal matrix. Once [K] has been factorized the solution vectors $\{u\}_k$ are obtained through the usual series of forward and backward substitutions. It is important to recognize that significant computational and computer storage savings can be realized by taking advantage of the banded structure of Eqs. (3-1). Therefore, in this study, the solution method described above is implemented for a compact "skyline" storage arrangement of [K] as described in Ref. 33.

Having calculated the nodal displacement vector $\{u\}_k$; $k = 1,2,\ldots K$, the end forces for the i-th structural element are given by

$$\{F_i\}_k^e = [K_i]^e \; \{u_i\}_k^e + \{FEF_i\}_k^e \tag{3-8}$$

where $\{F_i\}_k^e$, $\{u_i\}_k^e$ and $\{FEF_i\}_k^e$ are the forces, displacements and fixed end forces (corresponding to the uniformly distributed loading) associated with the i-th element for the k-th loading condition, written in the local coordinate system. The local displacements $\{u_i\}_k^e$ are calculated from the global displacement vector $\{u_i\}_k$ via the transformation

$$\{u_i\}_k^e = [T_i]\{u_i\}_k \tag{3-9}$$

where it is understood that $\{u_i\}_k$ is the subset of the global displacement vector $\{u\}_k$ associated with the i-th element. The fixed end forces $\{FEF_i\}_k^e$ for the uniformly distributed loading are given by

$$\{FEF_i\}_k^e = - \{P_i\}_k^e \tag{3-10}$$

where $\{P_i\}_k^e$ is the work equivalent loading vector as defined in Eqs. (A-11) and (A-19) for the frame and truss elements, respectively.

3.3 Multiple Boundary Conditions

The consideration of multiple sets of boundary conditions during the analysis of a structural system is quite common in engineering design. These boundary conditions may represent actual physical restraints corresponding to varying service environments or they may represent "artificial" boundary conditions created as part of the modelling process (e.g. using a half model for a symmetric structure). The application of multiple sets of boundary conditions can significantly increase the analysis solution time if the complete system stiffness matrix is decomposed for each boundary condition set. This computational burden can be greatly reduced by recognizing that, for many structures, a significant portion of the system stiffness matrix is unaffected by the changes in the boundary conditions (Ref. 5).

Conceptually, the system stiffness matrix may be partitioned into portions which are independent of (K_{FF}) and dependent on (K_{BB}, K_{FB}) the boundary condition changes as shown in Fig. 2. The free portion, K_{FF}, need be decomposed only once for all boundary condition sets. The decomposition of the entire matrix is then completed separately for each boundary condition set. This represents a considerable computational savings for structures in which the number of degrees of freedom associated with the changing boundary conditions is small compared to the total number of system degrees of freedom. It should be noted that the degrees of freedom associated with the changing boundary conditions do not actually have to be positioned together in the lower portion of the matrix as depicted in Fig. 2. This is important since it eliminates the need to perform row and column interchanges on the matrix and preserves the original matrix bandwidth.

CHAPTER IV

Approximate Problem Generation

4.1 Introduction

The key to a tractable structural synthesis formulation lies in the replacement of the original implicit nonlinear design problem with a sequence of explicit approximate problems of reduced dimensionality. The generation of these approximate problems is accomplished through the application of a variety of techniques commonly referred to as approximation concepts (Refs. 11-12). Primarily, these techniques serve to 1) reduce the numbers of design variables and constraints in the design problem and 2) reduce the required number of detailed (exact) constraint and objective function evaluations. There are various methods available for this purpose. Those implemented here include design variable linking, temporary constraint deletion and explicit first order constraint approximations. These techniques form the foundation of the approximate problem generation procedure which consists of the following steps: 1) design variable selection, 2) design variable linking, 3) constraint evaluation, 4) constraint deletion and 5) objective function and constraint approximation. This procedure is described in detail in the following sections.

4.2 Design Variable Selection

In Chapter II it was shown that the frame-truss structural synthesis problem could be stated, in a general manner, in terms of an independent set of generalized design variables $\bar{Z}$ (Eq. (2-3)). Conceptually, any independent combination of element cross sectional dimensions (CSD's) and element reciprocal section properties (RSP's) may be selected as the design variables as long as the changes in the dependent variables can be determined from the changes in the independent variables. Two such design variable selection schemes are implemented in this study.

Probably the most natural approach to design variable selection is to simply choose the element CSD's as the design variables (CSD design space). This has been popular in much of the reported literature (e.g. Refs. 20-24 and 26-28) primarily due to the fact that changes in the element RSP's ($\Delta\bar{X}$) are easily related to given changes in the element CSD's ($\Delta\bar{Y}$) for any cross section shape. Although it is possible to compute these changes exactly it is useful (in the construction of the explicit objective function and constraint approximations) to use the following approximate linear relationship to reflect $\Delta\bar{X}$ in terms of $\Delta\bar{Y}$ for each element:

$$\{X\} - \{X_0\} = \left[\frac{\partial X}{\partial Y}\right] \left\{\{Y\} - \{Y_0\}\right\} \tag{4-1}$$

where $\left[\frac{\partial X}{\partial Y}\right]$ can be determined either through differentiation of analytic expressions for the RSP's in terms of the CSD's or from finite difference calculations.

The primary disadvantage in choosing the element CSD's as the design variables is that high quality behavior constraint approximations (in terms of the element CSD's) can not be constructed for a variety of cross sections. This difficulty subsequently lead to a second approach in which the element RSP's are choosen as the design variables (Ref. 29). While this technique offers the potential for the construction of high quality behavior constraint approximations (particularily for displacement constraints), the following inherent difficulties must be addressed: 1) changes in the element CSD's are generally not easily determined from changes in the element RSP's, 2) the element RSP's may not represent an independent set of variables (e.g. the number of RSP's is greater than the number of CSD's for a particular cross section) and 3) the element RSP's may not adequately represent the design freedom associated with the CSD's (e.g. the number of RSP's is less than the number of CSD's for a particular cross section).

The difficulty associated with determining $\Delta\bar{Y}$ in terms of $\Delta\bar{X}$ is due to the nonlinearity of the relationship between the CSD's and RSP's which: 1) limits the range of $\Delta\bar{X}$ changes where linearized approximations are useful and 2) admits the possibility that multiple sets of CSD's can be found which will yield the same values for the RSP's. As a result, an exact representation of $\Delta\bar{Y}$ in terms of $\Delta\bar{X}$, for the general case, would be difficult if not impossible to determine.

To overcome the foregoing problem approximate linear relationships are constructed between $\Delta\bar{X}$ and $\Delta\bar{Y}$ for each design element. These relationships are obtained by first rewriting Eq. (4-1) as

$$\{\Delta X\} = \left[\frac{\partial X}{\partial Y}\right] \{\Delta Y\} = [J]\{\Delta Y\} \tag{4-2}$$

where

$$\{\Delta X\} = \{X\} - \{X_0\}$$

$$\{\Delta Y\} = \{Y\} - \{Y_0\} \tag{4-3}$$

The desired approximate relationship may now be obtained if Eq. (4-2) can be solved for $\{\Delta Y\}$. For the special case where $[J]$ is square (i.e. the numbers of CSD's and RSP's are equal) and non-singular (i.e. the RSP's are linearly independent) Eq. (4-3) can be solved directly to yield

$$\{\Delta Y\} = [J]^{-1}\{\Delta X\} \tag{4-4}$$

and the element RSP's are, indeed, selected to be the generalized design variables $\{Z\}$. However, this is clearly not the general case and, therefore, some alternative method must be employed. Such a method has been suggested in Ref. 30 and is described in detail in Appendix B. This technique automatically selects a set of linearly independent (free) variables

$$\{Z\} = [\{X_F\} \ \{Y_F\}]^T \tag{4-5}$$

and constructs a linear transformation between these variables and the remaining dependent (basic) variables $[\{X_B\} \ \{Y_B\}]^T$ of the form

$$[\{\Delta X_B\} \ \{\Delta Y_B\}]^T = [H]\{\Delta Z\} \tag{4-6}$$

where

$$[H] = \begin{bmatrix} \dfrac{\partial X_B}{\partial Z} \\ \dfrac{\partial Y_B}{\partial Z} \end{bmatrix} \tag{4-7}$$

is calculated from [J] as shown in Appendix B. An important feature of this technique is not only that it allows for the changes in the dependent variables to be written in terms of the changes in the independent variables, but that the number of independent design variables selected is always equal to the number of element CSD's. Therefore all three of the difficulties associated with the RSP design space, as described previously, are overcome simultaneously. It should also be noted that the design variables are selected such that the independent element RSP's are chosen first and any additional variables that may be required (to make the total number of design variables equal to the number of element CSD's) are chosen from the element CSD's. Hence, this design variable selection scheme is referred to as the RSP design space.

Expressions relating the changes in both the element CSD's and RSP's to changes in the generalized design variables can now be constructed using Eqs. (4-2) and (4-6). In the CSD design space the following relationship can be written

$$\begin{Bmatrix} \Delta X \\ \Delta Y \end{Bmatrix} = \left[\frac{\partial X}{\partial Z} \;\middle|\; \frac{\partial Y}{\partial Z} \right]^T \{\Delta Z\} = [J \mid I]^T \{\Delta Z\} = [D]\{\Delta Z\} \tag{4-8}$$

Similarily, in the RSP design space we may write

$$\begin{Bmatrix} \Delta X_B \\ \Delta Y_B \\ \Delta X_F \\ \Delta Y_F \end{Bmatrix} = \left[\frac{\partial X_B}{\partial Z} \quad \frac{\partial Y_B}{\partial Z} \;\Big|\; \frac{\partial X_F}{\partial Z} \quad \frac{\partial Y_F}{\partial Z}\right]^T \{\Delta Z\} = [H \mid I]^T \{\Delta Z\} = [D] \; \{\Delta Z\} \tag{4-9}$$

These relationships will prove to be useful in the subsequent construction of the objective function and constraint approximations. It should be noted that frequent updating of these relationships can be obtained at low computational cost because they do not involve finite element analysis or behavior sensitivity analysis. This will prove to be an important observation as will be shown in Section 4.7.

4.3 Design Variable Linking

Once the generalized design variables Z have been selected, as described in the previous section, design variable linking concepts may be employed, thereby reducing the dimensionality of the synthesis problem. Typically, design variable linking is used to reflect actual design requirements and/or to reduce the problem size enough to make its solution tractable. In this latter case the designer is forced to approximate the actual design problem in terms of a reduced number of design variables in much the same manner that the structural analyst must approximate an analysis problem with a limited number of degrees of freedom in order that the analysis problem can be solved. Various forms of linking are conceivable including the fixing of the relative sizes of a given set of design elements of the same type (total linking) and the linking of a single design variable of one element to that of another element (partial linking). In this work, only total linking between elements of the same type, such that they are identical, has been implemented.

4.4 Behavior Constraint Evaluation

The definitions of acceptable structural behavior are central to the structural synthesis problem statement. These definitions are included in the mathematical problem statement in the form of behavior constraints. Two basic types of behavior constraints are included here: 1) constraints on overall structural stiffness (in the form of nodal displacement/rotation constraints) and 2) constraints on local element strength (e.g. stress and local buckling constraints). These constraints may be written in terms of the structural response quantities (nodal displacements ($\bar{u}$) and element forces ($\bar{F}$)) and the element RSP's ($\bar{X}$) and CSD's ($\bar{Y}$) as follows:

$$g_q = R_q(\bar{u}(\bar{X})) - 1 \leq 0 \quad ; \quad q = 1,2,\ldots Q_1 \tag{4-10}$$

for the displacement constraints, and

$$g_q = R_q(\bar{F}(\bar{u}(\bar{X})),\bar{X},\bar{Y}) - 1 \leq 0 \quad ; \quad q = 1,2,\ldots Q_2 \tag{4-11}$$

for the strength constraints, where Q_1 and Q_2 are the numbers of displacement and strength constraints, respectively, and where the response ratio R_q represents the ratio of the behavior value to the associated allowable and approaches unity as the behavior constraint becomes critical. Evaluation of Eqs. (4-10) is clearly straightforward, given the values of a particular nodal displacement and its allowable. However, the evaluation of the strength constraints (Eqs. (4-11)), in general, requires the evaluation of a different expression for each type of design element. The strength constraint formulations for the various design element types are given in Appendix C.

4.5 Constraint Deletion

Proper design of a structural system usually requires the consideration of a substantial number of possible failure modes since, in general, the critical failure modes are not known at the outset of the design process. As a result, the structural synthesis problem statement may contain a large number of inequality constraints. In order to reduce the number of constraints, and the associated computational burden, it is possible to temporarily ignore certain constraints which are not expected to currently participate in the design. In effect, this process reduces the number of constraints by approximating the potentially critical constraint set.

The criteria by which particular constraints are judged to be participating (active) or non-participating (passive) forms the basis of the constraint deletion technique. Various criteria are conceivable, however a relatively simple but effective strategy consists of deleting all constraints with response ratios (R_q) less than a specified constraint truncation parameter CTP. The value of CTP may, in general, be chosen separately for each constraint type and may change during the design process. In this work, a single value for CTP is used for all behavior constraints. Simply stated, the value of CTP is selected so that: 1) constraints with $R_q \geq .7$ are always retained, 2) constraints with $R_q < .3$ are always deleted and 3) constraints with $.3 \leq R_q < .7$ are retained or deleted depending on the value of the response ratio cutoff value R_c. This criteria can be written as

$$CTP = \min \left\{ \max\{R_c, .3\}, \ .7 \right\} \tag{4-12}$$

where R_c is the maximum response ratio rounded down to the nearest tenth (e.g. if $\max_{q \in Q} R_q = .65$ then $R_c = .6$). The value of CTP is updated for each approximate problem.

4.6 Objective Function and Constraint Approximations

A key element to the efficient solution of the structural synthesis problem lies in the construction of accurate explicit function approximations. This is particularily true in the case of the behavior constraint functions because, in general, exact evaluation of these constraints requires that the structural analysis problem be solved. Various methods are available for the construction of these approximations, with the most commonly used techniques requiring only first order derivatives of the functions to be approximated (Refs. 12 and 34-35). Two types of first order approximations are used here.

The first type of approximation consists of expanding the function in a linear first order Taylor series of the form

$$f(\bar{Z}) \cong \tilde{f}_L(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial Z_b} (Z_b - Z_{0_b}) \tag{4-13}$$

where the expansion variables $\bar{Z}$ are chosen so that the resulting approximation is of the highest possible quality. In many cases, however, no single set of expansion variables may be chosen such that all function approximations are of sufficient quality. In this case it has been suggested (Ref. 36) that a hybrid or mixed variable approximation might be a useful alternative. This approximation can be constructed by the comparison of Eq. (4-13) with a first order Taylor series expansion of the form

$$f(\bar{Z}) \cong \tilde{f}_I(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial(1/Z_b)} \left(\frac{1}{Z_b} - \frac{1}{Z_{0_b}}\right) \tag{4-14}$$

or, equivalently,

$$f(\bar{Z}) \cong \tilde{f}_I(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial Z_b} \left(- Z_{0_b}^2 \left(\frac{1}{Z_b} - \frac{1}{Z_{0_b}}\right)\right) \tag{4-15}$$

Subtracting Eq. (4-13) from Eq. (4-15) gives

$$\tilde{f}_I(\bar{Z}) - \tilde{f}_L(\bar{Z}) = - \sum_{b=1}^{B} \frac{\partial f(\bar{Z}_0)}{\partial Z_b} \left(\frac{(Z_b - Z_{0_b})^2}{Z_b}\right) \tag{4-16}$$

For the case where $f(\bar{Z})$ represents an objective function to be minimized or a constraint function of the form $f(\bar{Z}) \leq 0$ Eq. (4-16) indicates that $\tilde{f}_I$ is more conservative than $\tilde{f}_L$ when

$$\frac{1}{Z_b} \frac{\partial f(\bar{Z}_0)}{\partial Z_b} < 0 \tag{4-17}$$

or, if Z_b represents some physical variable known to be positive in sign, when

$$\frac{\partial f(\bar{Z}_0)}{\partial Z_b} < 0 \tag{4-18}$$

Consequently, comparison of $\tilde{f}_I$ and $\tilde{f}_L$ on a term by term basis leads to the following first order mixed variable approximation:

$$f(\bar{Z}) \cong \tilde{f}_M(\bar{Z}) = f(\bar{Z}_0) + \sum_{b=1}^{B} \frac{\partial f(Z_0)}{\partial Z_b} B_b \tag{4-19}$$

where

$$B_b = \begin{cases} (Z_b - Z_{0_b}) & \text{if} \quad \dfrac{\partial f(\bar{Z}_0)}{\partial Z_b} > 0 \\[2em] - Z_{0_b}^2 \left((1/Z_b) - (1/Z_{0_b}) \right) & \text{if} \quad \dfrac{\partial f(\bar{Z}_0)}{\partial Z_b} < 0 \end{cases}$$

This mixed variable approximation $(\tilde{f}_M(\bar{Z}))$ is more conservative than either the pure linear approximation ($\tilde{f}_L(\bar{Z})$, see Eq. (4-13)) or the pure inverse approximation ($\tilde{f}_I(\bar{Z})$, see Eq. (4-14) or (4-15)). In this work two types of approximations (Eq. (4-13) and Eq. (4-19)) form the basis for the objective function and constraint approximations described in the following subsections.

4.6.1 Objective Function Approximations

The objective function (structural mass) can be written explicitly in terms of the element RSP's as

$$M(\overline{X}) = \sum_{i=1}^{I} \frac{\rho_i L_i}{X_{1_i}} \tag{4-20}$$

where ρ_i, L_i and X_{1_i} are the mass density, length, and reciprocal cross sectional area, respectively, for the i-th design element. Clearly the objective function is easy to evaluate (exactly) when the design is carried out in the RSP design space and, therefore, no approximation is required. However, in the CSD design space Eq. (4-20) cannot be evaluated directly to yield the exact value for M. It would first be necessary to calculate the design element areas from the element CSD's. While this computation is certainly not as burdensome as the detailed evaluation of the behavior constraints, it is, never the less, useful to replace Eq. (4-20) with an explicit approximation in this case. The linear and mixed variable (hybrid) approximations for the objective function can be written in terms of the I RSP's as

$$M(\overline{X}) \cong \tilde{M}_L(\overline{X}) = M(\overline{X}_0) + \sum_{i=1}^{I} \frac{\partial M(\overline{X}_0)}{\partial X_i} (X_i - X_{0_i}) \tag{4-21}$$

and

$$M(\overline{X}) \cong \tilde{M}_M(\overline{X}) = M(\overline{X}_0) + \sum_{i=1}^{I} \frac{\partial M(\overline{X}_0)}{\partial X_i} B_i \tag{4-22}$$

where

$$B_i = \begin{cases} [X_i - X_{0_i}] & \text{if } \dfrac{\partial M(\bar{X}_0)}{\partial X_i} > 0 \\ - X_{0_i}^2 \left((1/X_i) - (1/X_{0_i}) \right) & \text{if } \dfrac{\partial M(\bar{X}_0)}{\partial X_i} < 0 \end{cases}$$

Using Eq. (4-8), the approximations given by Eqs. (4-21) and (4-22) can be rewritten in terms of the B generalized design variables (Z_b) as

$$M(\bar{X}) \cong \tilde{M}_L(\bar{Z}) = M(\bar{X}_0) + \sum_{b=1}^{B} \frac{\partial M(\bar{X}_0)}{\partial Z_b} (Z_b - Z_{0_b}) \tag{4-23}$$

and

$$M(\bar{X}) \cong \tilde{M}_M(\bar{Z}) = M(\bar{X}_0) + \sum_{b=1}^{B} \frac{\partial M(\bar{X}_0)}{\partial Z_b} B_b \tag{4-24}$$

where

$$\frac{\partial M(\bar{X}_0)}{\partial Z_b} = \sum_{i=1}^{I} \frac{\partial M(\bar{X}_0)}{\partial X_i} \frac{\partial X_i(\bar{Z}_0)}{\partial Z_b}$$

$$B_b = \begin{cases} [Z_b - Z_{0_b}] & \text{if } \dfrac{\partial M(\bar{X}_0)}{\partial Z_b} > 0 \\ - Z_{0_b}^2 \left((1/Z_b) - (1/Z_{0_b}) \right) & \text{if } \dfrac{\partial M(\bar{X}_0)}{\partial Z_b} < 0 \end{cases}$$

4.6.2 Behavior Constraint Approximations

The linear and mixed variable approximations for the structural displacement constraints represented by Eq. (4-10) may be written in terms of the I RSP's as follows:

$$g(\bar{X}) \cong \tilde{g}_L(\bar{X}) = g(\bar{u}_0) + \sum_{i=1}^{I} \frac{\partial g(\bar{u}_0)}{\partial \bar{u}} \frac{\partial \bar{u}}{\partial X_i} (X_i - X_{0_i}) \tag{4-25}$$

and

$$g(\bar{X}) \cong \tilde{g}_M(\bar{X}) = g(\bar{u}_0) + \sum_{i=1}^{I} \frac{\partial g(\bar{u}_0)}{\partial \bar{u}} \frac{\partial \bar{u}}{\partial X_i} B_i \tag{4-26}$$

where

$$B_i = \begin{cases} [X_i - X_0] & \text{if} \quad \dfrac{\partial g(\bar{u}_0)}{\partial \bar{u}} \dfrac{\partial \bar{u}}{\partial X_i} > 0 \\ \\ - X_{0_i}^2 \left((1/X_i) - (1/X_{0_i}) \right) & \text{if} \quad \dfrac{\partial g(\bar{u}_0)}{\partial \bar{u}} \dfrac{\partial \bar{u}}{\partial X_i} < 0 \end{cases}$$

Similarily, approximations for the element strength constraints (e.g. stress and local buckling) represented by Eq. (4-11) may be written in terms of the element RSP's and CSD's as

$$g(\overline{X},\overline{Y}) \cong \tilde{g}_L(\overline{X},\overline{Y}) = g(\overline{F}_0,\overline{X}_0,\overline{Y}_0) + \sum_{i=1}^{I} \frac{\partial g}{\partial \overline{F}} \frac{\partial \overline{F}}{\partial X_i} + \frac{\partial g}{\partial X_i} \quad (X_i - X_{0_i})$$

$$+ \sum_{j=1}^{J} \frac{\partial g}{\partial Y_j} (Y_j - Y_{0_j}) \tag{4-27}$$

and

$$g(\overline{X},\overline{Y}) \cong \tilde{g}_M(\overline{X},\overline{Y}) = g(\overline{F}_0,\overline{X}_0,\overline{Y}_0) + \sum_{i=1}^{I} \left[\frac{\partial g}{\partial \overline{F}} \frac{\partial \overline{F}}{\partial X_i} + \frac{\partial g}{\partial X_i}\right] B_i$$

$$+ \sum_{j=1}^{J} \frac{\partial g}{\partial Y_j} C_j \tag{4-28}$$

where

$$B_i = \begin{cases} [X_i - X_{0_i}] & \text{if} \quad \dfrac{\partial g}{\partial \overline{F}} \dfrac{\partial \overline{F}}{\partial X_i} + \dfrac{\partial g}{\partial X_i} > 0 \\ \\ - X_{0_i}^2 \left((1/X_i) - (1/X_{0_i}) \right) & \text{if} \quad \dfrac{\partial g}{\partial \overline{F}} \dfrac{\partial \overline{F}}{\partial X_i} + \dfrac{\partial g}{\partial X_i} < 0 \end{cases}$$

$$C_j = \begin{cases} [Y_j - Y_{0_j}] & \text{if} \quad \dfrac{\partial g}{\partial Y_j} > 0 \\ \\ - Y_{0_j}^2 \left((1/Y_j) - (1/Y_{0_j}) \right) & \text{if} \quad \dfrac{\partial g}{\partial Y_j} < 0 \end{cases}$$

and where it is understood that the summation over J includes only those CSD's corresponding to the design element in which the strength constraint is located. Both the displacement and element strength constraint approximations can be written in terms of the generalized design variables (using either Eq. (4-8) or (4-9) depending on the selection of the design space) as follows:

$$g(\bar{Z}) \cong \tilde{g}_L(\bar{Z}) = g(\bar{X}_0,\bar{Y}_0) + \sum_{b=1}^{B} \frac{\partial g}{\partial Z_b} (Z_b - Z_{0_b}) \tag{4-29}$$

$$g(\bar{Z}) \cong \tilde{g}_M(\bar{Z}) = g(\bar{X}_0,\bar{Y}_0) + \sum_{b=1}^{B} \frac{\partial g}{\partial Z_b} B_b \tag{4-30}$$

where

$$B_b = \begin{cases} [Z_b - Z_{0_b}] & \text{if} \quad \dfrac{\partial g}{\partial Z_b} > 0 \\ \\ -Z_{0_b}^2 \left((1/Z_b) - (1/Z_{o_b})\right) & \text{if} \quad \dfrac{\partial g}{\partial Z_b} < 0 \end{cases}$$

and where

$$\frac{\partial g}{\partial Z_b} = \begin{cases} \displaystyle\sum_{i=1}^{I} \left(\frac{\partial g}{\partial \bar{u}} \frac{\partial \bar{u}}{\partial X_i}\right) \frac{\partial X_i}{\partial Z_b} & ; \quad q \epsilon Q_1 \\ \\ \displaystyle\sum_{i=1}^{I} \left[\left(\frac{\partial g}{\partial \bar{F}} \frac{\partial \bar{F}}{\partial X_i}\right) + \frac{\partial g}{\partial X_i}\right] \frac{\partial X_i}{\partial Z_b} + \sum_{j=1}^{J} \frac{\partial g}{\partial Y_j} \frac{\partial Y_j}{\partial Z_b} & ; \quad q \epsilon Q_2 \end{cases}$$

Construction of the approximations given by Eqs. (4-29) and (4-30) clearly requires the computation of the partial derivatives of the structural response quantities with respect to the element RSP's (i.e $\frac{\partial \bar{u}}{\partial \bar{X}}$, $\frac{\partial \bar{F}}{\partial \bar{X}}$). Various methods are available for the computation of these quantities (Refs. 37-38). The technique used in this work is based on the direct implicit differentiation of the equations governing the structural response and it is commonly referred to as the pseudo-load method. This method is particularily easy to implement and it relatively efficient (particularily when implemented using a partial inverse technique as described in Ref. 12) when only first order sensitivities are required. A detailed formulation of the method is contained in Appendix D.

4.6.3 Side Constraint Approximations

Side constraints on the element RSP's and CSD's of the form

$$\bar{X}^L \leq \bar{X} \leq \bar{X}^U$$

$$\bar{Y}^L \leq \bar{Y} \leq \bar{Y}^U \tag{4-31}$$

where $\bar{X}^U$, $\bar{X}^L$, $\bar{Y}^U$ and $\bar{Y}^L$ are the upper and lower bounds on the element RSP's and CSD's, respectively, play two important roles in the solution of the structural synthesis problem. Primarily, the side constraints represent bounds on the design element sizing variables corresponding to physical design requirements (e.g. packaging limitations, manufacturability). Secondly, the side constraints can be used to limit design changes during the solution of each approximate problem so as to protect the accuracy of the objective function and behavior constraints approximations. In this latter case the global upper and lower bounds ($\bar{X}^U$, $\bar{X}^L$, $\bar{Y}^U$, $\bar{Y}^L$) are replaced by the stepwise bounds ($\tilde{\bar{X}}^U$, $\tilde{\bar{X}}^L$, $\tilde{\bar{Y}}^U$, $\tilde{\bar{Y}}^L$) and Eqs. (4-31) become

$$\tilde{\tilde{\bar{X}}}^L \leq \bar{X} \leq \tilde{\tilde{\bar{X}}}^U$$

$$\tilde{\tilde{\bar{Y}}}^L \leq \bar{Y} \leq \tilde{\tilde{\bar{Y}}}^U \tag{4-32}$$

The stepwise bounds are calculated from move limits (d_1, d_2) supplied by the designer as follows:

$$\tilde{X}_i^L = \max[X_i^L, X_i - d_1 X_i]$$

$$\tilde{X}_i^U = \min[X_i^U, X_i + d_1 X_i]$$

$$\tilde{Y}_j^L = \max[Y_j^L, Y_j - d_2 Y_j] \tag{4-33}$$

$$\tilde{Y}_j^U = \min[Y_j^U, Y_j + d_2 Y_j]$$

where X_i and Y_j are the values of the i-th RSP and j-th CSD at the beginning of each approximate problem stage.

Evaluation of these constraints (Eqs. (4-32)) during the approximate problem solution requires that they be rewritten in terms of the generalized design variables. For the case in which this solution is performed in CSD design space we may write

$$\bar{Y} = \bar{Z}$$

$$\bar{X} = \bar{X}(\bar{Y}) = \bar{X}(\bar{Z}) \tag{4-34}$$

and Eq. (4-32) becomes

$$\overset{\cong}{X}^L \leq \bar{X}(\bar{Z}) \leq \overset{\cong}{X}^U$$

$$\overset{\cong}{Y}^L \leq \bar{Z} \leq \overset{\cong}{Y}^U \tag{4-35}$$

Under the additional assumption that the primary element sizing variables are the element CSD's, the side constraints on the RSP's may be ignored yielding (without approximation) the following form for the side constraints in CSD design space:

$$\overset{\cong}{Y}^L \leq \bar{Z} \leq \overset{\cong}{Y}^U \tag{4-36}$$

In the RSP design space the side constraints given by Eq. (4-32) must be rewritten in terms of the dependent (basic) and independent (free) variables as follows:

$$\overset{\cong}{X}{}^L_B \leq \bar{X}_B(\bar{X}_F, \bar{Y}_F) \leq \overset{\cong}{X}{}^U_B$$

$$\overset{\cong}{Y}{}^L_B \leq \bar{Y}_B(\bar{X}_F, \bar{Y}_F) \leq \overset{\cong}{Y}{}^U_B$$

$$\overset{\cong}{X}{}^L_F \leq \bar{X}_F \leq \overset{\cong}{X}{}^U_F \tag{4-37}$$

$$\overset{\cong}{Y}{}^L_F \leq \bar{Y}_F \leq \overset{\cong}{Y}{}^U_F$$

Using Eq. (4-9) the following first order approximation of the side constraints can be written:

$$
\begin{Bmatrix} \tilde{\bar{X}}_B^L \\ \tilde{\bar{Y}}_B^L \\ \tilde{\bar{X}}_F^L \\ \tilde{\bar{Y}}_F^L \end{Bmatrix} \leq \begin{Bmatrix} \bar{X}_B \\ \bar{Y}_B \\ \bar{X}_F \\ \bar{Y}_F \end{Bmatrix}_0 + [D]\{\Delta\bar{Z}\} \leq \begin{Bmatrix} \tilde{\bar{X}}_B^U \\ \tilde{\bar{Y}}_B^U \\ \tilde{\bar{X}}_F^U \\ \tilde{\bar{Y}}_F^U \end{Bmatrix} \tag{4-38}
$$

where $\{\bar{X}_B\ \bar{Y}_B\ \bar{X}_F\ \bar{Y}_F\}_0^T$ contains the values of the element RSP's and CSD's at the beginning of the design stage.

4.6.4 Selective Constraint Dependence

A significant amount of the computational effort associated with the generation of the approximate design problem is expended during the calculation of the partial derivatives of the structural response quantities with respect to the element RSP's (i.e. $\frac{\partial \bar{u}}{\partial \bar{X}}$, $\frac{\partial \bar{F}}{\partial \bar{X}}$), which are required for the construction of the behavior constraint approximations (Eqs. (4-29) and (4-30)). For some structural synthesis problems significant reductions in the computational effort can be realized by using a selective constraint dependence technique (Ref. 5). This technique is based on the observation that in many practical design problems a single behavior constraint may be strongly dependent on only a relatively few design elements. If these design elements can be identified, then the partial derivatives of the constraint with respect to the other design variables (for elements upon which the constraint is weakly dependent) may be ignored. This technique often leads to dramatic reductions in the required number of derivative calculations. Conceptually, selective constraint dependence may be applied to both the system displacement and the element strength constraints. However, selection of the design elements which strongly influence the displacement constraints may be difficult, in general, and it is certainly problem dependent. On the other hand, the element strength constraints are well suited to the application of three special cases of the selective constraint dependence concept.

Examination of the element strength constraint approximations given by Eqs. (4-27) and (4-28) clearly shows that the strength constraints for a given design element are coupled to the design variables associated with all other design elements only through the element force derivatives $(\frac{\partial \bar{F}}{\partial \bar{X}})$. It is therefore possible to apply the selective constraint dependence technique via assumptions made as to the expected nature of any element force redistribution which may occur during the design process. The following three assumptions are considered here: 1) the element forces are invariant during the current stage in the design process, 2) changes in the forces on a given design element are primarily dependent on the design variables associated with that element and 3) the element forces are strongly dependent on all of the structural design variables. These assumptions lead to the following hierarchy of element force sensitivity calculations: 1) no element force derivatives are calculated, 2) element force derivatives are calculated only with respect to the RSP's associated with that element and 3) element force derivatives are calculated with respect to the RSP's of all elements.

4.7 Updating the Approximate Problem

The extent to which it is possible to minimize the number of structural analyses and response quantity sensitivity calculations ($\frac{\partial \bar{u}}{\partial \bar{X}}$, $\frac{\partial \bar{F}}{\partial \bar{X}}$) required during the design process clearly depends on the quality of the approximate design problems. Approximate problem statements for which the underlying constraint approximations are of high quality are valid over larger changes in the design variables and, as a result, fewer such problems are required to obtain the solution to the actual synthesis problem. As discussed previously, the generation of the response quantity sensitivities in terms of the element RSP's yields values of $\frac{\partial \bar{u}}{\partial \bar{X}}$ and $\frac{\partial \bar{F}}{\partial \bar{X}}$ which are relatively accurate over large changes in $\bar{X}$. This, in turn, tends to improve the quality of the behavior constraint approximations, particularily for displacement constraints in the RSP design space. However, the overall or net quality of the constraint approximations depend not only on the response quantity sensitivities but also on the approximated relationships between the element CSD's/RSP's and the generalized design variables (e.g. see Eqs. (4-29) and (4-30)). Frequently these latter quantities are accurate only for small changes in $\bar{X}$ and $\bar{Y}$ because of the highly nonlinear relationships between the element CSD's and RSP's. Hence, the net approximations may be accurate over smaller than desired changes in the design variables. To a degree, this problem is less severe when the mixed variable constraint approximations are employed. However, additional computational savings may be realized via a procedure which periodically updates the approximate problem without recourse to structural re-analysis or response quantity sensitivity calculations.

The motivation for the approximate problem update procedure described in this section lies in the desire to utilize, to the fullest extent possible, the quality of the response quantity sensitivities; since it is these derivatives which are computationally burdensome to generate. To this end the partial derivatives of the constraints with respect to the element RSP's and CSD's, $(\frac{\partial g}{\partial \bar{X}}, \frac{\partial g}{\partial \bar{Y}})$ are assumed to be of high quality and are saved during the approximate problem generation for use in the subsequent approximate problem update procedure. This procedure consists of the following steps:

1. calculate new objective function derivatives with respect to element RSP's $\frac{\partial M}{\partial \bar{X}}$

2. update the approximate behavior constraint values (to compensate for the approximate relationship between the element CSD's and RSP's) using the equation

$$g = g(\bar{X}_A) + \sum_{i=1}^{I} \frac{\partial g}{\partial X_i} (X_{E_i} - X_{A_i}) \qquad (4\text{-}39)$$

   where $\bar{X}_E$ and $\bar{X}_A$ are the exact and approximate values of the RSP's corresponding to the current values of the CSD's.

3. calculate new values for $\frac{\partial \bar{X}}{\partial \bar{Z}}$ and $\frac{\partial \bar{Y}}{\partial \bar{Z}}$ (see Eq. (4-8) or Eq. (4-9)).

4. form new side constraint approximations (Eq. (4-36) or (4-38))

5. calculate new objective function and behavior constraint derivatives using

$$\frac{\partial M}{\partial \bar{Z}} = \frac{\partial M}{\partial \bar{X}} \frac{\partial \bar{X}}{\partial \bar{Z}}$$

$$\frac{\partial g}{\partial \bar{Z}} = \frac{\partial g}{\partial \bar{X}} \frac{\partial \bar{X}}{\partial \bar{Z}} + \frac{\partial g}{\partial \bar{Y}} \frac{\partial \bar{Y}}{\partial \bar{Z}} \tag{4-40}$$

6. form new objective function and behavior constraint approximations (Eqs. (4-23, 4-24) and (4-29, 4-30)).

Using this procedure it is now possible to update and solve the approximate problem repeatedly wihtout recourse to structural re-analysis or response quantity sensitivity generation. It should be noted that, in practice, the number of times which this update procedure may be performed depends on the quality of the reponse quantity sensitivities. Therefore, it is of paramount importance that the structural response quantity sensitivities be generated directly in terms of those variables which yield response gradients of the highest possible quality, irrespective of the final choice of design variables.

CHAPTER V

Optimization

5.1 Introduction

The approximate problem generation techniques discussed in Chapter IV make it possible to replace the implicit nonlinear frame-truss synthesis problem (Eq. (2-2)) with a sequence of explicit approximate design problems, each having the form

$$
\begin{aligned}
&\min_{\bar{Z}} \tilde{M}(\bar{Z}) \\
&\text{s.t} \quad \tilde{g}_q(\bar{Z}) \leq 0 \quad ; \quad q \in Q_R \\
&\qquad \tilde{\tilde{X}}^L \leq \tilde{\tilde{X}}(\bar{Z}) \leq \tilde{\tilde{X}}^U \\
&\qquad \tilde{\tilde{Y}}^L \leq \tilde{\tilde{Y}}(\bar{Z}) \leq \tilde{\tilde{Y}}^U
\end{aligned}
\tag{5-1}
$$

where $\tilde{M}$, $\tilde{g}_q$, $\tilde{\tilde{X}}$ and $\tilde{\tilde{Y}}$ are, in general, explicit approximations of the structural mass, retained behavior constraints, design element reciprocal section properties (RSP's) and design element cross sectional dimensions (CSD's) in terms of the generalized design variables ($\bar{Z}$). When these approximate problems are constructed using the objective function and constraint approximations described in Chapter IV, Eq. (5-1) represents an explicit, separable, convex inequality constrained mathematical programming problem. As such, Eq. (5-1) can be solved via any number of well known, nonlinear constrained minimization techniques (Ref. 39). Each of these techniques can be classified as either a primal or a dual method, depending on whether the solution is carried out in terms of the primal variables ($\bar{Z}$) or the dual variables ($\bar{\lambda}$). Two methods, one of each type, have been implemented here and they are described in the following sections.

5.2 A Primal Solution Method

Numerous primal methods are available for the solution of the mathematical programming problem represented by Eq. (5-1), including both direct and transformation (e.g. penalty, barrier) methods. The method chosen here is based on the feasible directions method of Zoutendijk (Ref. 40-41) with modifications to improve numerical stability and efficiently solve initially infeasible problems (Ref. 42); as implemented in the CONMIN (Ref. 43) optimization program. This technique was selected for the following reasons: 1) the method is applicable to the rather general class of problems represented by Eq. (5-1) and 2) the implementation of the method, in the form of the CONMIN program, is reliable and relatively efficient for the class of problems considered here.

The feasible directions method serves as the primary solution technique and can be used to solve any of the approximate problem formulations shown in Figs. 3 and 4. During the CONMIN solution process some computational efficiencies are realized by identifying all side constraint approximations as being linear. Similarily, additional computational savings may be gained by identifying the behavior constraint approximation as being linear for problem formulations 1-3 and 7-9. This is, however, optional and the default case is to treat the behavior constraint approximations as being nonlinear, thereby causing the solution to be "pushed off" somewhat from the constraint surfaces.

5.3 A Dual Solution Method

An alternative procedure for solving the mathematical programming problem represented by Eq. (5-1) consists of replacing this primal problem by its dual mathematical programming statement and solving the resulting problem in terms of the dual variables $(\bar{\lambda})$. This may be done by first rewriting Eq. (5-1) in the following slightly more general form:

$$\min_{\bar{Z}} \tilde{M}(\bar{Z})$$

$$\text{s.t} \quad \tilde{h}_q(\bar{Z}) \leq 0 \quad ; \quad q \in \hat{Q}$$

$$\bar{Z}^L \leq \bar{Z} \leq \bar{Z}^U \tag{5-2}$$

where $\tilde{h}_q$; $q \in \hat{Q}$ represent the approximated behavior and side constraints and where $\bar{Z}^U$ and $\bar{Z}^L$ are the upper and lower bounds on the generalized design variables. The dual of Eq. (5-2) may now be written as

$$\max_{\bar{\lambda} \geq 0} \left\{ \min_{\bar{Z}^L \leq \bar{Z} \leq \bar{Z}^U} L(\bar{Z}, \bar{\lambda}) \right\} \tag{5-3}$$

where

$$L(\overline{Z},\overline{\lambda}) = \widetilde{M}(\overline{Z}) + \sum_{q \in Q} \lambda_q \widetilde{h}_q \tag{5-4}$$

Alternatively, Eq. (5-3) may be written as

$$\max_{\overline{\lambda} \geq 0} \ell(\overline{\lambda}) \tag{5-5}$$

where

$$\ell(\overline{\lambda}) = \min_{\overline{Z}^L \leq \overline{Z} \leq \overline{Z}^U} L(\overline{Z},\overline{\lambda}) \tag{5-6}$$

is defined as the dual function. This procedure is viable if it can be demonstrated that the dual maximization problem represented by Eq. (5-5) has a unique solution (saddle point). It is well known that if the primal problem (Eq. (5-2)) is a convex program (i.e. $\widetilde{M}(\overline{Z})$ and $\widetilde{h}_q(\overline{Z})$; $q \in \widehat{Q}$ are convex functions and $\overline{Z}$ is contained in a convex subset of E^n) and has at least one strictly feasible solution (i.e. there exists some $\overline{Z}$ s.t. $\widetilde{h}_q(\overline{Z}) < 0$; $q \in \widehat{Q}$) then the dual problem has a unique saddle point $(\overline{Z}^* \ \overline{\lambda}^*)$. If this saddle point can be found, $\overline{Z}^*$ is the solution to the primal problem (Ref. 44). The existence of a saddle point can be demonstrated when Eq. (5-2) represents one of the approximate design problems described previously, under the assumption that a strictly feasible solution exists. Therefore, in principle, the dual solution method may be applied to any of the approximate problems shown in Figs. 3 and 4.

While the question of saddle point existence is certainly crucial in determining the applicability of the dual solution method, another important consideration concerns the computational efficiency of solving the dual problem (Eq. (5-5)). Clearly, for the general case, maximization of the dual function is considerably complicated by the imbedded Lagrangian minimization represented by Eq. (5-6). However, if $L(\bar{Z}, \bar{\lambda})$ is additively separable then the solvability of Eq. (5-5) is enhanced by the fact that the Lagrangian minimization can be performed as a sequence of smaller minimization problems (Ref. 44). The attractiveness of the dual solution method is further enhanced by the recognition that for the types of approximate problems constructed here the minimization of the Lagrangian simply consists of solving a sequence of explicit single variable minimization problems. This important observation was first made in Ref. 45 in the context of a generalized optimality criteria method and subsequently coupled with approximation concepts in Refs. 46 and 47.

A final consideration in the application of the dual solution technique concerns the method by which the dual function maximization is to be performed. Gradient methods are particularily attractive since it is well known that the first derivatives of the dual function with respect to the dual variables are immediately available from the primal constraint values, i.e.:

$$\frac{\partial \ell(\bar{\lambda})}{\partial \lambda_q} = \tilde{h}_q \quad ; \quad q \epsilon \hat{Q} \tag{5-7}$$

However, if $\ell(\bar{\lambda})$ does not possess continuous first derivatives such gradient techniques may exhibit slow or nonconvergent behavior unless special precautions are taken. The dual function can be shown to be continuously differentiable under the following conditions; 1) $\bar{Z}$ is contained in a closed and bounded subset of E^n (S), 2) $\tilde{M}$ and $\tilde{h}_q$; $q \in \hat{Q}$ are continuous on S and 3) $L(\bar{Z}, \bar{\lambda})$ is minimized over S at a unique point $\bar{Z}(\bar{\lambda})$ for all $\bar{\lambda} \geq 0$. It can be shown that these conditions are satisfied for the case in which the design element CSD's are selected to be the generalized design variables and the objective function is approximated via a mixed variable (hybrid) approximation. Therefore, an explicit mixed variable dual problem can be formulated and solved, via an existing first order technique, for approximate problem options 10-12 shown in Fig. 4. The mixed variable dual formulation given in the following was originally presented in Ref. 48.

The dual problem can be constructed by first writing the approximate primal problems (10-12) as

$$\begin{aligned} &\min_{\bar{Z}} \tilde{M}(\bar{Z}) \\ &\text{s.t} \quad \tilde{g}_q(\bar{Z}) \leq 0 \quad ; \quad q \in Q_R \\ &\qquad \bar{Z}^L \leq \bar{Z} \leq \bar{Z}^U \end{aligned} \tag{5-8}$$

where it is recognized that $\tilde{g}_q(\bar{Z})$; $q \in Q_R$ are the approximations of the retained behavior constraints and $\bar{Z}^U$ and $\bar{Z}^L$ are the stepwise upper and lower bounds on the design variables (element CSD's). Introducing the mixed variable approximations for M and g_q ; $q \in Q_R$ (see Eqs. (4-26) and

(4-30)) Eq. (5-8) can be rewritten as

$$\min_{\bar{Z}} \sum_{m_b>0} m_b Z_b - \sum_{m_b<0} \frac{m_b}{Z_b} Z_{0_b}^2 + \hat{M}$$

$$\text{s.t.} \sum_{c_{bq}>0} c_{bq} Z_b - \sum_{c_{bq}<0} \frac{c_{bq}}{Z_b} Z_{0_b}^2 + \hat{g}_q \leq 0 \quad ; \quad q = 1,2,\ldots Q_R$$

$$Z_b^L \leq Z_b \leq Z_b^U \quad ; \quad b = 1,2,\ldots B \tag{5-9}$$

where

$$m_b = \frac{\partial M}{\partial Z_b}$$

$$c_{bq} = \frac{\partial g_q}{\partial Z_b}$$

$$\hat{M} = M(\bar{Z}_0) - \sum_{m_b>0} m_b Z_{0_b} + \sum_{m_b<0} m_b Z_{0_b}$$

$$\hat{g}_q = g_q(\bar{Z}_0) - \sum_{c_{bq}>0} c_{bq} Z_{0_b} + \sum_{c_{bq}<0} c_{bq} Z_{0_b}$$

The dual problem (Eq. (5-3)) may now be written as

$$\max_{\bar{\lambda} \geq 0} \left\{ \min_{\bar{Z}^L \leq \bar{Z} \leq \bar{Z}^U} L(\bar{Z},\bar{\lambda}) \right\} \tag{5-10}$$

where

$$L(\overline{Z},\overline{\lambda}) = \sum_{m_b>0} m_b Z_b - \sum_{m_b<0} \frac{m_b}{Z_b} Z_{0_b}^2 + \hat{M} +$$

$$\sum_{q\in Q_R} \lambda_q \left\{ \sum_{c_{bq}>0} c_{bq} Z_b - \sum_{c_{bq}<0} \frac{c_{bq}}{Z_b} Z_{0_b}^2 + \hat{g}_q \right\} \tag{5-11}$$

Interchanging the order of the double summation in the fourth and fifth terms, Eq. (5-11) can be rewritten as

$$L(\overline{Z},\overline{\lambda}) = \sum_{m_b>0} m_b Z_b - \sum_{m_b<0} \frac{m_b}{Z_b} Z_{0_b}^2 +$$

$$\sum_{c_{bq}>0} \left\{ \sum_{q\in Q_R} \lambda_q c_{bq} \right\} Z_b -$$

$$\sum_{c_{bq}<0} \left\{ \sum_{q\in Q_R} \lambda_q c_{bq} \right\} \frac{Z_{0_b}^2}{Z_b} + \sum_{q\in Q_R} \lambda_q \hat{g}_q + \hat{M} \tag{5-12}$$

Letting

$$n_b = - m_b Z_{0_b}^2 \quad ; \quad m_b<0$$

$$C_b = \sum_{q\in Q_R} \lambda_q c_{bq} \quad ; \quad c_{bq}>0$$

$$D_b = - \sum_{q\in Q_R} \lambda_q c_{bq} Z_{0_b}^2 \quad ; \quad c_{bq}<0 \tag{5-13}$$

and substituting into Eq. (5-12) yields

$$L(\overline{Z},\overline{\lambda}) = \sum_{m_b>0} m_b Z_b + \sum_{m_b<0} \frac{n_b}{Z_b} + \sum_{c_{bq}>0} C_b Z_b +$$

$$\sum_{c_{bq}<0} \frac{D_b}{Z_b} + \sum_{q \in Q_R} \lambda_q \hat{g}_q + \hat{M} \tag{5-14}$$

Recognizing that the last two terms of Eq. (5-14) are constant and that the remaining terms are additively separable, the minimization of $L(\overline{Z},\overline{\lambda})$ can be performed via B single variable minimizations, i.e.:

$$\min_{\overline{Z}^L \leq \overline{Z} \leq \overline{Z}^U} L(\overline{Z},\overline{\lambda}) = \sum_{b=1}^{B} \left\{ \min_{Z_b^L \leq Z_b \leq Z_b^U} L_b(Z_b,\overline{\lambda}) \right\} \tag{5-15}$$

where

$$L_b(Z_b,\overline{\lambda}) = \begin{cases} m_b Z_b + C_b Z_b + \dfrac{D_b}{Z_b} & ; \quad m_b>0 \\ \\ \dfrac{n_b}{Z_b} + C_b Z_b + \dfrac{D_b}{Z_b} & ; \quad m_b<0 \end{cases} \tag{5-16}$$

The solution to the b-th single variable minimization, (temporarily ignoring the side constraints on Z_b), is given by

$$
Z_b^2 = \begin{cases} \dfrac{D_b}{m_b + C_b} & ; \quad m_b > 0 \\[2ex] \dfrac{D_b + n_b}{C_b} & ; \quad m_b < 0 \end{cases} \tag{5-17}
$$

Taking the side constraints into consideration, the solution to the b-th single variable minimization becomes

$$
Z_b = \begin{cases} \alpha_b & \text{if} \quad (Z_b^L)^2 \leq \alpha_b^2 \leq (Z_b^U)^2 \\ Z_b^L & \text{if} \quad \alpha_b^2 \leq (Z_b^L)^2 \\ Z_b^U & \text{if} \quad \alpha_b^2 \geq (Z_b^U)^2 \end{cases} \qquad ; \quad \text{for} \quad m_b > 0
$$

$$
Z_b = \begin{cases} \beta_b & \text{if} \quad (Z_b^L)^2 \leq \beta_b^2 \leq (Z_b^U)^2 \\ Z_b^L & \text{if} \quad \beta_b^2 \leq (Z_b^L)^2 \\ Z_b^U & \text{if} \quad \beta_b^2 \geq (Z_b^U)^2 \end{cases} \qquad ; \quad \text{for} \quad m_b < 0 \tag{5-18}
$$

where

$$
\alpha_b^2 = \frac{D_b}{m_b + C_b}
$$

$$
\beta_b^2 = \frac{D_b + n_b}{C_b}
$$

Note also the following special cases:

$$Z_b = Z_b^L \quad \text{if} \quad m_b > 0 \quad \text{and} \quad D_b = 0$$

$$Z_b = Z_b^U \quad \text{if} \quad m_b < 0 \quad \text{and} \quad C_b = 0 \tag{5-19}$$

Finally, using Eqs. (5-18) and (5-19), the dual problem may be written as an explicit problem in terms of $\bar{\lambda}$ as

$$\max_{\bar{\lambda} \geq 0} \ \ell(\bar{Z}(\bar{\lambda}), \bar{\lambda}) \tag{5-20}$$

Equation (5-20) represents a relatively unconstrained maximization problem of a differentiable concave function and, as such, may be solved using a gradient based maximization algorithm. The method used here is the feasible directions method described previously, where the only constraints are the non-negativity constraints on $\bar{\lambda}$. Since the solution is carried out in terms of the full set of dual variables ($\bar{\lambda}$) the dimensionality, of the optimization problem is n x n where n is the number of retained behavior constraints. Therefore, the dual solution method is generally more efficient than the primal method if the number of retained constraints is less than the number of primal design variables ($\bar{Z}$). However, it should be noted that specialized solution schemes for the dual problem (e.g. Ref. 46 where the dimensionality of the dual space is gradually increased but does not exceed the number of truly critical behavior constraints) can make the dual solution method more efficient than the primal method even when the number of retained constraints exceeds the number of primal design variables.

CHAPTER VI

Detail Design Recovery

6.1 Introduction

A fundamental consideration in structural synthesis is that of how the actual detail design quantities (sizing variables) are to be determined from the structural design variables. When the relationships between the design variables and sizing variables are simple and explicit (as in the case of a truss design element when selecting A or 1/A as the design variables, or a frame design element when selecting the cross sectional dimensions (CSD's) or their reciprocals as the design variables) then the detailed design recovery process is, of course, trivial and is rarely mentioned as a distinct part of the structural design problem. However, when the relationship between the design variables and sizing variables is not explicit, as in the case where the design variables are the element reciprocal section properties (RSP's) and the sizing variables are the element CSD's, then the detail design recovery process can be quite complex and must be treated as a separate phase of the structural synthesis methodology. Two basic types of detail design recovery techniques are discussed in the following section for use in conjunction with the RSP design space option described in Chapter IV.

6.2 The Recovery Process

The detail design recovery process for the frame–truss synthesis problem (in which the element sizing variables are the design element CSD's) can be viewed as a procedure for calculating the element CSD's ($\bar{Y}$) from the vector of optimal generalized design variables $\bar{Z}^*$ corresponding to the solution of each approximate problem. In general, such a procedure seeks the solution to the set of nonlinear equations

$$\bar{Z}(\bar{Y}) = \bar{Z}^* \tag{6-1}$$

subject to the following restrictions on the element CSD's

$$\bar{Y}^L \leq \bar{Y} \leq \bar{Y}^U \tag{6-2}$$

For the case where the element CSD's are selected as the generalized design variables (CSD design space option) the solution to Eq. (6-1) is immediately given by

$$\bar{Y} = \bar{Z}^* \tag{6-3}$$

where it is recognized that the restrictions on the element CSD's represented by Eq. (6-2) have already been accounted for in the form of bounds on $\bar{Z}$ (see Eq. (4-35)). Clearly, in this case, the recovery process is computationally trivial and may be carried out without recourse to iterative or approximate techniques. This is not, however, the case when the synthesis procedure is carried out using the RSP design space option. In this case the recovery process must attempt to solve Eqs. (6-1) (subject to the constraints represented by Eq. (6-2)) directly.

It should be recognized that Eqs. (6-1) may not possess a solution within the acceptable domain defined by Eq. (6-2) and, therefore, any potential solution procedure must be capable of dealing with this possibility. As a result, the design recovery procedure for the RSP design space option will, in general, be approximate. Two such recovery procedures are discussed below.

Possibly the most natural method for recovering the element sizing variables from the generalized design variables is to formulate the recovery process as an element level optimization problem of the form

$$\min_{\eta \geq 0} \ \eta$$

$$\text{s.t.} \quad -\eta \leq Z_i^* - Z_i(\overline{Y}) \leq \eta \quad ; \quad i = 1,2,\ldots M$$

$$Y_j^L \leq Y_j \leq Y_j^U \quad ; \quad j = 1,2,\ldots N \tag{6-4}$$

where M and N are, respectively, the numbers of generalized design variables and cross sectional dimensions associated with a given design element. This procedure seeks the solution to Eqs. (6-1), one design element at a time, so as to minimize the maximum error in any one equation while forcing the solution ($\overline{Y}$) to lie within the allowable upper and lower bounds (a similar method is suggested in Ref. 23). This procedure will tend to yield the "best" approximate solution to Eqs. (6-1) and, as a result, has the advantage of preserving, to the extent possible, the quality of the behavior constraint approximations. Unfortunately, the recovery scheme summarized by Eqs. (6-4) would require the solution of many (equal to the number of design elements for each approximate problem stage) nonlinear mathematical programming subproblems. This would be computationally burdensome and therefore the recovery scheme represented by Eqs. (6-4) has not been implemented in this work.

An alternative recovery procedure, which has been implemented here, was first used in Ref. 29. This procedure makes direct use of the previously constructed approximate linear relationships between the changes in the element CSD's $\{\Delta Y\}$ and the changes in the generalized design variables $\{\Delta Z\}$ (see Eq. (4-9)). This relationship can be written as

$$\{\Delta Y\} = \left[\frac{\partial Y}{\partial Z}\right]\{\Delta Z\} \tag{6-5}$$

where $\left[\frac{\partial Y}{\partial Z}\right]$ is constructed as described in Chapter IV. Using Eq. (6-5) the actual recovered values for the element CSD's are given by

$$\{Y\} = \{Y_0\} + \{\Delta Y\} \tag{6-6}$$

where $\{Y_0\}$ contains the values of the element CSD's at the beginning of the design stage. Clearly, this procedure requires few additional computations and, therefore can be applied efficiently to large problems. The main disadvantage of the method lies in the fact that the linear approximation (Eq. (6-5)) may be valid for only relatively small changes in the generalized design variables. This can require the use of tight move limits which, in turn, can make it necessary to construct and solve an excessively large number of approximate design problems. Fortunately, this difficulty can be effectively overcome by using the approximate problem update technique (described in Chapter IV) in which the relationships given by Eq. (6-5) are periodically updated during the solution of an approximate design problem while holding the results of the structural analysis and behavior constraint sensitivity analysis invariant (until the beginning of the next stage).

CHAPTER VII

Program Description

7.1 Introduction

The frame-truss synthesis methodology described in Chapters II-VI has been implemented in the COMPASS (Computer Program for Analysis and Synthesis of Space-frames) computer program. This program is intended to serve as a research code for the study and development of practical and efficient synthesis techniques for structural systems whose essential structural behavior requires a bending-membrane element representation. While primarily a research tool, the program is capable of solving problems large enough to be of some practical interest. Although its primary function is structural design, the program can be used for basic structural analysis, with or without design modelling data. A command oriented input data structure makes the program relatively easy to use. Also, the programs modular organization and in-core storage management system serve to facilitate future expansion and development efforts.

7.2 Scope of Program

The COMPASS program is currently capable of determining the minimum mass design of three dimensional frame-truss structures subject to multiple static loading conditions. The structural topology, configuration, material and loading information is supplied in the form of a finite element-analysis model and is assumed to be invariant during the design process. The structural behavior (nodal displacements and member end forces) is determined via a linear displacement method finite element technique, using a combination of space frame and truss elements (see Appendix A).

The structural synthesis problem is solved using the sequence of approximate problems approach pioneered by Schmit et al. (Refs. 11-12). Both a first order Taylor's series approximation and a mixed variable approximation (Ref. 36) are available for construction of the explicit behavior constraint functions. Each approximate problem is constructed and solved in a generalized design variable space, consisting of either cross sectional dimensions (CSD's) or a combination of reciprocal section properties (RSP's) and CSD's, with the ultimate goal being that of determining optimum values for the element sizing variables. These sizing variables are associated with the various design element cross section shapes described in Appendix C. User specified bounds on the element sizing variables prevent the design from assuming unrealistic dimensions. Move limits can also be applied to the design variables to ensure that the behavior of all candidate designs is well represented by the approximate problem. A design element linking capability which links all sizing variables between selected design elements is also available.

The COMPASS program allows the user to design against a variety of failure modes. Limits on nodal displacements and rotations and on design element strength (e.g. stresses and local buckling) can be treated. Differences in the design element cross section shapes require that, in general, the strength failure criteria be tailored to the specific design element. The failure criteria for the various design elements are described in Appendix C.

The approximate design problem may be solved using either a primal or dual mathematical programming algorithm depending on the users choice of design space and constraint approximation technique. The CONMIN (Ref. 43) computer program, based on a feasible directions method (Ref. 43), is used to solve the primal form of the approximate design problem. The solution to the dual form of the design problem is based on the development described in Ref. 48; with the CONMIN program being used to perform the actual dual function maximization. The available combinations of optimization method, design space and constraint approximation techniques are illustrated in Fig. 5.

7.3 Organization

The basic program organization is shown in Fig. 6. Pre and post processing routines are provided to perform one time input and output data processing functions. The design control routine directs the execution of the following four primary design functions; (1) structural analysis, (2) approximate problem generation, (3) optimization and (4) detail design recovery.

The design process begins with the structural analysis phase from which exact values for the structural behavior quantities (nodal displacements and element end forces) are obtained based on the initial design input. The approximate problem generation routine then performs the following operations; (1) design variable selection and linking (2) constraint evaluation, (3) constraint deletion, and (4) objective function and retained constraint approximation. The approximate design problem is then passed on to the optimization routine where the redesign function is performed. Finally, the design element sizing variables are calculated from the design variables in the detail design recovery phase. The new design is then passed back to the design control block where the entire process is repeated until design convergence is achieved.

Under certain circumstances the user may wish to periodically bypass the structural analysis phase, proceeding directly to the approximate problem generator as depicted by the dashed line in Fig. 6. In this case the approximate problem is updated without recourse to structural analysis or response quantity sensitivity calculations as outlined in Chapter IV. The approximate problem updating procedure allows the high quality displacement derivatives to be utilized over larger changes in the design variables than would otherwise be possible. It is important to note that considerable gains in solution efficiency can be realized by bypassing the structural analysis and displacement derivative calculations in this way.

7.4 Storage Management

Program data storage management is one of the most important considerations in program development. Unfortunately, in research programming it is often times ignored. It is generally agreed that some type of centralized data base management system (Ref. 49-50) is essential to the development of modern large scale engineering analysis/synthesis programs. However, it is less clear that the implementation of such data base management schemes in a research code is time and/or cost effective. The COMPASS program utilizes an in-core storage management facility which attempts to provide some of the benefits of general data base management within the constraints of the research environment.

The basic concept-behind the storage management system implemented here is shown in Fig. 7. The storage manager consists of three parts; (1) a data vector, (2) a dictionary and (3) a void area table. All program data is stored within the data vector while the associated descriptions, locations and lengths are stored in the dictionary. The void area table contains the locations and lengths of unused portions of the data vector that may appear as the program data storage is altered during execution. While the storing of program data in a single data vector is quite common in engineering programming, the associated descriptive information is usually maintained outside of the program. Here, the programmer is allowed complete control over the access, creation, deletion and alteration of data storage from within the program through the use of a variety of storage management commands (Fig. 8). Such a capability leads to increased programming flexibility and facilitates program maintenance and development. Finally, although the use of such a storage management system does increase solution time, computational experience indicates that these increases are small when the system is efficiently applied.

7.5 Implementation

The COMPASS program is operational as a stand alone program on the IBM 3033 computer using the MVS/SP operating system. The program consists of approximately 14,000 Fortran, 1000 PLI and 50 Assembler statements. The current version of the program requires approximately 650 K bytes of memory, excluding data storage. Program storage requirements may be considerably reduced, however, by taking advantage of the program's structure through the use of overlay or segmentation techniques. The detailed flow diagrams shown in Figs. 9-17 are provided as a guide for the overlay process. It should be noted, however, that memory savings realized through the overlay process may not be significant for problems in which the data storage is large compared to the program storage.

The standard I/O device unit designations (FT05 and FT06 for Fortran, SYSIN and SYSPRINT for PLI) are used for program input and output. Auxiliary external files are also required for certain program options. File numbers 10 and 11 are required for all problems for which multiple boundary condition sets are specified. File number 12 is required if the approximate problem generation update procedure is enabled. For problems requesting program data checkpoint or restart options the user must define the files associated with the checkpoint or restart file numbers.

The implementation of the COMPASS program on other computer systems would require some program modifications. For computer systems on which the PLI programming language is available the required modifications are relatively minor and confined to the storage management subroutines. On systems where the PLI programming language is not available the input data subroutines would have to be rewritten in Fortran or some other suitable language. The small amount of Assembler code is used only for CPU timing and is easily replaced by any equivalent system CPU timing routine.

7.6 Input Data Commands

The COMPASS program input data format is designed to be easily used and highly flexible. A problem oriented free format command language is coupled with a data scanning feature to provide a data entry method which is essentially free of organizational and formatting restrictions. For convenience, the data input stream is divided into three sections; (1) analysis data, (2) design data and (3) control data. Each section is headed by a data block command. Commands which describe the structure and its loading are supplied in the analysis data block. This data is similar to that provided to most finite element structural analysis programs. The design data block is used to supply the information associated with the structural design problem (e.g. initial design data, side constraints, and behavior allowables). Finally, all of the program control information is supplied via the control data block.

Three input data command forms are shown in Fig. 18. All of the program data commands appear in one of these three basic forms (or slight variations thereof). In form 1 the command is followed on the same line by its associated data. A command followed by several separate lines of data is shown in form 2. Finally, form 3 shows a command followed by several lines of sub-commands and data. In all cases, only the underlined portion of the commands or sub-commands need to be given. However, inclusion of the full command phrase is allowed. Some commands contain optional data (denoted by a quantity enclosed in brackets, e.g. [data]) which may or may not be specified at the user's discretion as outlined in the command description. In this case a default value is assigned for the missing data item. Comment cards are allowed in the data stream and are designated by placing a dollar sign ($) in column 1 of the data card. Data may also be continued from one card to the next by placing a continuation character (-) in column 72 of the card which is to be continued. Data entity (element, node, load set, etc.) numbering is also unrestricted. For example, node point numbering does not have to begin with the number 1 or be in ascending order. The available data commands are described in Appendix E. A set of sample data input is shown in Fig. 19.

7.7 Restrictions and Limitations

The COMPASS program has relatively few operational restrictions. The major restriction is that of problem size. Like most programs which make extensive use of an in-core data structure, the maximum solvable problem size is dependent on the amount of computer memory available. The use of the storage management system described in Section 7.4 helps to alleviate this problem but does not eliminate it.

The dynamic nature of the synthesis problem makes data storage requirements difficult to estimate. In general, however, the problem data requirements are most effected by the number of degrees of freedom and bandwidth in the analysis model and the number of design variables and constraints in the design model. Careful attention to node numbering will help reduce the analysis model bandwidth. To the extent that it is possible, design variable linking can help to reduce the dimensionality of the design space.

CHAPTER VIII

Numerical Examples

8.1 Introduction

In this chapter, the detailed results for numerous structural synthesis problems are presented. Each problem has been solved using the previously described frame-truss synthesis methodology as implemented in the COMPASS computer program on the IBM 3033 at UCLA. The example problems were selected so that, in addition to exercising the various solution options outlined in Table 1, the results could be compared with previously published work. The types of problems solved include both planar and three dimensional frame-truss structures subject to multiple static loading conditions with constraints on nodal displacements/rotations and on element strength (e.g. stress and local buckling). These problems contain frame members having various cross sectional shapes including: 1) a thin walled box beam with 4 cross sections dimensions (CSD's) (B, H, t_b, t_h), 2) an I beam symmetric about the x-y plane with 6 CSD's (B_1, B_2, H, t_1, t_2, t_3), 3) a solid square beam with 1 CSD (B), 4) a thin walled tubular beam with 2 CSD's (R,t) and 5) a thin walled box beam with 3 CSD's (B, H, t) (see Appendix C, Figs. C3, C5-C8).

8.2 Tied Cantilevered Beam (Problem 1)

Figure 20 depicts a tied cantilevered beam subject to two independent loading conditions. The structure is modelled using one frame element (member 1) and one truss element (member 2). The members are made of materials having the same modulus of elasticity and weight density but having different yield stress allowables. The truss member (design element type 1) is described by its cross sectional area while the frame member (design element type 13) has a square cross section with one sizing variable (B). This structure is designed for minimum weight subject to element stress constraints at the ends of each member and side constraints on the element sizing variables. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 2. The design element descriptions are given in Appendix C.

This problem was solved using four different solution options. The iteration history data for these runs is given in Table 3. The corresponding iteration history plots are shown in Figs. 21-24. In the first three runs (options 1(P), 2(P) and 3(P)) the design is carried out in the RSP design space using linear constraint approximations and 40% move limits on the RSP's (d_1 = 0., d_2 = .4). Each approximate problem is solved via a primal solution method (CONMIN). The differences between runs 1, 2 and 3 lie in the assumptions made regarding the element end force variations during the solution of each approximate problem. Comparison of the results for runs 1, 2 and 3 indicates little difference in the number of analyses required for convergence as the amount of element end force sensitivity information included in the stress constraint approximations is increased. However, the final design weight for run 3 is approximately 9% less than that of runs 1 and 2. This result is not unexpected since this problem is known to possess several local minimum solutions (Ref. 51). It is, however, interesting to note that this solution was obtained as a result of an improvement in the quality of the constraint approximations.

The fourth run (option 6(P)) for this problem is the same as run 3 except that the element stress constraints are approximated via a mixed variable (hybrid) approximation. There is no improvement in the convergence rate in this case; indeed the results are identical to those in run 3. This is due to the simple form of the stress constraints which are well approximated in terms of the design element RSP's.

The final designs and critical* constraints for runs 1-4 are shown, along with those of the reference solution (Ref. 51), in Tables 4 and 5. The three reference solutions, designated as Method I, Method II-B and Method IV-B, were obtained using the same assumptions regarding the element end force variations as runs 1, 2 and 3, respectively. The comparisons between the final designs for runs 1-3 and the corresponding reference solutions is quite good in terms of both final weight and material distribution. The largest differences occur for run 2 where the reference solution is approximately 3.3 lb. lighter. However, in this case the stress constraints for the reference solution are slightly

* A critical constraint is defined here as any constraint having an associated response ratio (ratio of the response value to its allowable) greater than or equal to .95.

violated (see Ref. 51). Also, it is interesting to observe that the solution obtained in runs 3 and 4 is distinctly different than that of runs 1 and 2 both in terms of material distribution and critical constraints. Again, this is not unexpected since the structure clearly has two competing primary load paths.

8.3 Two Member Frame (Problem 2)

A two member plane frame subject to a single out of plane load is shown in Fig. 25. This structure is modelled using two thin walled box section frame elements (type 11) having four sizing variables (B, H, t_b, t_h). Both members are made of the same material. The two member frame is designed for minimum mass subject to two independent sets of constraints. The first set (Case A) includes stress constraints on both members and side constraints on the element sizing variables. The second set (Case B) consists of the constraints included in Case A with the addition of constraints against local wall buckling of the members. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 6. The design element is described in Appendix C.

8.3.1 Case A: Stress and Side Constraints

This case was solved using eight different solution options. The iteration history data for these runs is given in Table 7. The iteration history plots are shown in Figs. 26-33. The first three runs (options 1(P), 2(P) and 3(P)) are made using the RSP design space option and linear constraint approximations. Each approximate problem is solved via a primal solution method (CONMIN) with 40% move limits on the RSP's ($d_1 = 0.$, $d_2 = .4$). The differences between runs 1, 2 and 3 lie in the assumptions made regarding the element end force variations during the solution of each approximate problem. Comparison of the results for runs 1-3 shows little change in terms of the number of analyses required for convergence. However, the results do improve in terms of the maximum constraint violation for intermediate designs as the amount of element end force sensitivity information contained in the approximate problem is increased. Indeed, in the case where the element end forces are assumed to be dependent on all of the design variables (run 3) most of the intermediate designs are feasible. Again, this is not surprising since the structure clearly has two competing load paths.

Due to the lack of significant convergence improvement for either the local or global element end force variation options, the remaining runs were made assuming that the element end forces are invariant during the solution of an approximate problem. Run 4 (option 4(P)) is the same as run 1 except a mixed variable (hybrid) approximation is used for the stress constraints and the move limits are increased to 50% ($d_1 = 0.$, $d_2 = .5$) on the RSP's. Comparison of the iteration histories for runs 1 and 4 shows a slight improvement in convergence rate and a significant improvement in terms of maximum constraint violation for intermediate designs, especially considering the more liberal move limits. This improvement can be attributed to the conservativeness of the mixed variable approximation as compared to the pure linear approximation used in run 1. It is also interesting to note that the iteration history for run 4 compares quite favorably with that of run 3 indicating that, at least for some problems, the mixed variable approximation may be used successfully in place of higher levels of element end force sensitivity information (without the associated computational expense).

In runs 5 and 6 (options 7(P) and 10(P)) the design is carried out in CSD space with 25% move limits on the CSD's (d_1 = .25, d_2 = 1.0). Each approximate problem is solved using a primal solution method. In run 5 the stress constraints are approximated using a linear approximation while in run 6 the mixed variable approximation is used. Comparison of the iteration histories for these runs shows superior results in terms of both convergence rate and maximum constraint violation for run 6. However, run 6 does have a slightly higher final mass resulting from a different and slightly less efficient material distribution. Also, run 6 compares well with run 4 except, again, for the slightly higher final mass.

Run 7 (option 10(D)) is the same as run 6 except that each approximate problem is solved using a dual solution method. Comparing the iteration histories for runs 6 and 7 reveals little significant difference. For this problem the dual solution method appears to be more efficient even though the numbers of retained constraints and design variables are the same.

The final run (option 1(PU)) is the same as run 1 except that the approximate problem update procedure is used. In this case the approximate problem is reconstructed, without recourse to structural analysis or response quantity sensitivity calculations, once between each complete approximate problem generation. Comparison of the iteration histories for runs 1 and 8 shows an improvement in the convergence rate by a factor of two while maintaining comparable maximum constraint violations for the intermediate designs. The number of structural analyses required for convergence in this case is 7, resulting in the best convergence rate of all solution options used for this problem.

The final designs and critical* constraints for runs 1-8 are given, along with those of the reference solution (Ref. 29) in Tables 8 and 9. All of the final designs represent the same (intuitively correct) design concept. The sizing variables of the longer member (1) are at their lower bounds, with most of the load being carried through the shorter member (2) and the critical stresses occurring at the fixed end of this member. While these designs are conceptually the same it is interesting to note that there are two competing means of carrying the load through member 2. In the final designs for runs 1-5 and 8 member 2 achieves nearly the maximum allowable outer dimensions (B,H) and minimum thickness (t_b, t_h). However, the final designs for the reference solution and runs 6 and 7 have a significantly smaller base dimension (B) and a larger wall thickness (t_b). While the first design concept is more efficient (and intuitively more satisfying), the final design masses corresponding to the second design concept are only slightly higher (1-3%). This is not too surprising since it has become well recognized that many structural design problems do exhibit practical local minima having relatively close values of the objective function. As in this case, these local minima are often associated with distinct design concepts.

8.3.2 Case B: Stress, Buckling and Side Constraints

This case was solved using seven different solution options. The iteration history data for these runs is given in Table 10. The iteration history plots are shown in Figs. 34-40. As in Case A, the first three runs (options 1(P), 2(P) and 3(P)) are made using the RSP design space option and linear constraint approximations. Each approximate problem is solved using a primal solution method with 40% move limits on the RSP's (d_1 = 0., d_2 = .4). Comparison of the iteration histories for runs 1-3 indicates no change in the convergence rate and only slight improvement in the amount of constraint violation for intermediate designs as the amount of element end force sensitivity information contained in the approximate problem is increased.

Due to the lack of significant convergence improvement for either the local or global element end force variation options, the remaining runs were made assuming that the element end forces are invariant during the solution of an approximate problem. Run 4 (option 4(P)) is the same as run 1 except that a mixed variable (hybrid) approximation is used for the stress and local buckling constraints and the move limits on the RSP's are increased to 50% (d_1 = 0., d_2 = .5). Comparison of runs 1 and 4 shows a slight improvement in convergence rate and a significant improvement in the maximum constraint violation history, especially considering the larger move limits.

In runs 5 and 6 (options 10(P) and 10(D)) the design is carried out using the CSD design space option with mixed variable stress and local buckling constraint approximations and 25% move limits on the CSD's (d_1 = .25, d_2 = 0.). The approximate problems are solved using a primal and a dual solution method, respectively. In this case, comparison of the iteration histories with those of the designs obtained using the RSP space design option (runs 1-4) indicates only slightly better convergence rate but a significantly improved maximum constraint violation history. Indeed, in both runs 5 and 6 most of the intermediate designs are feasible. However, it should be noted that both of these runs have final mass values slightly greater than runs 1-4, resulting from a somewhat less efficient material distribution (see Table 11).

The final run (option 1(PU)) is the same as run 1 except that the approximate problem update procedure is used. As in run 8, Case A, the approximate problem is reconstructed once between each complete approximate problem generation. Again, the use of this procedure results in a significant improvement in the convergence rate, although the maximum constraint violation for the intermediate designs are quite large. The number of structural analysis required for convergence in this case is 7, the best of all solutions options used for this problem.

The final designs and critical* constraints for runs 1-7 are given, along with those of the reference solution (Ref. 29), in Tables 11 and 12. Again, as in Case A, all of the final designs represent the same overall design concept with the load being carried through the shorter member (2). Also, as in Case A, the same two competing means of carrying the load through member 2 are present. They are represented by the designs from runs 1-4, 7 and the reference solution and runs 5-6, respectively. It is interesting to note, however, that in this case these competing design concepts not only have different final masses but they also are associated with different sets of critical constraints (see Table 12). In runs 1-3 and 7, the large base dimension (B) and small base thickness (t_b) lead to both critical stress and local wall buckling constraints. On the other hand, the narrower and thicker base wall dimensions (B, t_b) for the final designs of the reference solution and runs 4-6 preclude criticality of the wall buckling constraint.

8.4 Three Member Frame (Problem 3)

Figure 41 depicts a three member planar frame subject to two simultaneous out of plane loads. This structure is constructed of three thin walled box section frame elements (type 15) each having three sizing variables (B, H, t). All members are made of the same material. The three member frame is designed for minimum material volume subject to constraints on the maximum allowable member stresses. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 13. The design element is described in Appendix C.

This problem was solved using seven different solution options. The iteration history data for these runs is given in Table 14. The iteration history plots are shown in Figs. 42-48. The first three runs (options 1(P), 2(P) and 3(P)) are made using the RSP design space option and linear constraint approximations. Each approximate problem is solved via a primal solution method with 50% move limits on the RSP's ($d_1 = 0.$, $d_2 = .5$). The differences between runs 1, 2 and 3 lie in the assumptions made regarding the element end force variations during the solution of each approximate problem. Comparison of the iteration histories for these runs reveals no difference in the convergence rate for the three solution options. However, there is a substantial difference between runs 1 and 3, and run 2 in terms of the maximum constraint violation for several of the intermediate designs. Two interesting observations can be made here. First, the performance of the solution options 1(P) and 3(P) are nearly identical even though it would appear that, for a statically indeterminate structure such as this, the global element end force variation option (3(P)) would yield superior results. Secondly, the local element end force variation option (2(P)) yields substantially poorer results in terms of maximum constraint violation than the invariant option (1(P)), particularly during the early stages of the design. This behavior can be attributed to the fact that, although the structure is statically indeterminate, the symmetry of the structure and loading leads to a synthesis problem in which the coupling member (2) tends to vanish, thereby reducing the problem to the design of two determinate cantilevered beams (members 1 and 3). It should now be recognized that the local element end force variation option may, in some cases, lead to constraint approximations which are of poorer quality than those resulting from the assumption that the element end forces are invariant during the solution of the approximate problem.

Based on the results of runs 1-3, the remaining runs for the problem were made using the invariance assumption for the element end forces. Run 4 (option 4(P)) is identical to run 1 except that a mixed variable (hybrid) approximation is employed for the stress constraints and the move limits are increased to 60% on the RSP's (d_1 = 0., d_2 = .6). Comparing the iteration history for run 4 with those of runs 1-3 shows a moderate improvement in the convergence rate and, considering the more liberal move limits, a significant improvement in maximum constraint violation history.

In runs 5 and 6 (options 10(P) and 10(D)) the design is carried out in the CSD design space with 30% move limits on the CSD's (d_1 = .3, d_2 = 0.). Mixed variable constraint approximations are employed for the stress constraints. For run 5 a primal method is used to solve each approximate problem, while a dual method is used in run 6. Both runs yield essentially the same iteration histories, comparing favorably with those generated via the RSP design space option (runs 1-4). Note that the maximum constraint violations for the intermediate designs are quite small with nearly all of these designs being feasible. Also, it is interesting to note that the dual solution method proves to be more efficient than the primal even though the number of retained constraints is greater than the number of design variables (16 as compared to 9).

The final run (option 1(PU)) is the same as run 1 except that the approximate problem update procedure is employed. As in the previous problem (Two Member Frame) the approximate problem is reconstructed, without recourse to structural analysis or response quantity sensitivity calculations, once between each complete approximate problem generation. Comparison of the iteration histories for runs 1 and 7 indicates an improvement in the convergence rate by a factor of two while maintaining comparable maximum constraint violations for the intermediate designs (except in stage 1 where the violated constraint was not retained). Again, convergence was achieved after only 7 structural analyses.

The final designs and critical* constraints for runs 1-7 are shown, along with those of the reference solution (Ref. 52), in Tables 15 and 16. All of these designs represent the same (intuitively correct) design concept in which member 2 achieves its minimum allowable dimensions and the loads are carried through to the supports by members 1 and 3. All of the designs have essentially the same final material volume and material distribution. The somewhat smaller material volume (less than 1%) of the reference solution can be attributed to a slightly different stress constraint formulation (compare Ref. 52 and Appendix C).

8.5 Seven Member Frame (Problem 4)

A seven member planar frame structure subject to two independent in-plane loading conditions is shown in Fig. 49. This structure is modelled with seven thin walled box section frame elements (type 15) having three sizing variables (B, H, t). All members are made of the same material. The framework is designed for minimum mass subject to limits on the vertical displacements at node 3 and the allowable stresses at the ends of the members. It should be noted that the reference solution (Ref. 28) also included constraints on the lowest fundamental frequency, however this constraint did not participate in the design process. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 17. The design element is described in Appendix C.

The seven member frame problem was run using seven different solution options. The iteration history data for these runs is given in Table 18. The iteration history plots for runs 2-7 are shown in Figs. 50-55. In the first three runs (options 1(P)), 2(P) and 3(P)) the RSP design space option and linear constraint approximations are employed. Each approximate problem is solved via a primal solution method using move limits of 40% on the CSD's and 100% on the RSP's (d_1 = .4, d_2 = 1.0.). The differences between runs 1, 2 and 3 lie in the assumptions made regarding the element end force variations during the solution of each approximate problem. In run 1 the element end forces were assumed to be invariant during the solution of the approximate problem. In this case, convergence was not attained within 20 design stages. The strong coupling among the members in the structure was not adequately represented by the constraint approximations. For runs 2 and 3, where the local and global element end force variation options are used, convergence is achieved after 16 structural analyses. Comparison of the iteration histories for runs 2 and 3 shows significantly improved maximum constraint violations for run 3, especially during the early stages of the design, as well as a slightly improved final design mass.

Based on the results of the first three runs, the remaining runs were made using the assumption that the element end forces are dependent on all design variables in the structure. Run 4 (option 6(P)) is the same as run 3 except that mixed variable (hybrid) constraint approximations are employed. In this case there is no convergence rate improvement over run 3 and only a small improvement in the maximum constraint violation for intermediate designs.

The CSD design space option and mixed variable constraint approximations are used in runs 5 and 6 (options 12(P) and 12(D)). For run 5, 40% move limits are placed on the element CSD's ($d_1 = .4$, $d_2 = 0.$) and each approximate problem is solved via a primal solution method. In run 6 the move limits are 30% on the CSD's ($d_1 = .3$, $d_2 = 0.$) and a dual solution method is employed. Comparison of the iteration histories for runs 4 and 5 shows little difference in performance, between the RSP and CSD design space options. There is, however, an improvement in the convergence rate in run 6. It should be noted, however, that while the convergence rate is improved the solution time is significantly greater due to the fact that there are over twice as many retained constraints as design variables (and therefore the dimensionality of the dual space is much larger than the primal space).

The final run (option 3(PU)) is identical to run 3 except that the approximate problem update procedure is employed. As in the previous two problems, the approximate problem is updated once between each complete approximate problem generation. Comparing the iteration histories for runs 3 and 7 one can see that run 7 requires 50% fewer structural analyses for convergence while maintaining comparable performance in terms of the maximum constraint violation for intermediate designs.

The final designs and critical* constraints for all runs are shown, along with those of the reference solution (Ref. 28), in Tables 19 and 20. While all of the designs have nearly the same final mass values there is considerable difference in the material distributions and a slight variation in the critical constraint sets. This type of behavior is not uncommon and was also observed in Ref. 28. Aside from this, two other observations can be made here. First, even though the material distributions are quite different, members 5 and 6 are consistently small for all cases. Secondly, the final design which most closely matches the reference solution (run 2) utilizes the same element end force variation assumption used to generate the reference solution.

8.6 Portal Frame (Problem 5)

Figure 56 depicts a three member planar frame subject to two independent loading conditions. All of the members are made of the same material and have the same cross section (symmetric I section with six sizing variables (B_1, B_2, H, t_1, t_2, t_3)). The structure is designed for minimum material volume subject to constraints on the lateral displacement and in-plane rotation at node number 3, stress constraints at the ends of each member and local buckling constraints for the web and flanges of the members. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the sizing variables is given in Table 21. The design element is described in Appendix C.

This problem was solved using five solution options. The iteration history data for these runs is given in Table 22. The corresponding iteration history plots are shown in Figs. 57-61. In the first three runs (options 1(P), 2(P) and 3(P)) the design is carried out in the RSP design space (note that since the cross section has six CSD's the actual design variables include 4 RSP's and 2 CSD's) using linear approximations of the constraints. Each approximate problem is solved via a primal solution method with move limits of 40% on the CSD's and 60% on the RSP's (d_1 = .4, d_2 = .6). Comparing the iteration histories for runs 1-3 one can clearly see that the superior results (in terms of the number of analyses required for convergence) are obtained in run 3 where the element end forces are assumed to be dependent on all of the structural design variables. This is not unexpected since the structure clearly has two competing load paths. While run 3 exhibits the best convergence, it does result in a final design which has a material volume 4% greater than that obtained in run 1. Again, this is not too surprising as this problem is known to possess several local minimum solutions (Ref. 13).

Based on the results of runs 1-3, runs 4 and 5 (options 12(P) and 12(D)) were made using the global element end force variation option. The CSD design space option and mixed variable (hybrid) behavior constraint approximations are employed. In both cases 40% move limits are placed on the element CSD's ($d_1 = .4$, $d_2 = 0.$). Each approximate problem is solved via a primal solution method in run 4 and a dual solution method in run 5. Comparison of the iteration histories for these runs reveals relatively little difference in convergence rate and overall maximum constraint violation for the intermediate designs. The final design material volume for run 5 is slightly less than that of run 4 but the design is also slightly infeasible. Comparing results of run 4-5 with runs 1-3 reveals a significant difference in the final design material volumes between those obtained from the CSD design space option and those obtained from RSP space (10.2% - 13.6%). While, again, this is not unexpected for this problem it is interesting to note that alternative design was obtained as a result of a change in the design space.

The final designs and critical* constraints for runs 1-5 are given, along with those of the reference solution (Ref. 13), in Tables 23 and 24. Comparison of these final designs, both in terms of final material volume and material distribution, clearly reveals the existence of at least two local minimum solutions associated with distinct design concepts. In the case of the reference solution and runs 1-3 both primary load paths (members 1-2, member 3) contribute significantly to the load carrying capacity of the structure. On the other hand, in runs 4 and 5 the load path through member 3 is clearly abandoned in favor of the apparently more efficient path through members 1 and 2.

8.7 One Bay/Two Story Frame (Problem 6)

A one bay/two story frame structure subject to two independent loading conditions is shown in Fig. 62. This structure is modelled with eight thin walled tube elements (type 14), each having two sizing variables (R,t). All of the members are made of the same material. This structure is designed for minimum weight subject to two independent sets of constraints. The first set (Case A) includes stress constraints at both ends of each member, side constraints on the element sizing variables and constraints against both local and column buckling of each member. The local buckling constraints are applied in the form of upper bounds on the member R/t ratios. The second set of constraints (Case B) includes all of the constraints in Case A with the addition of constraints on the lateral displacements at node numbers 2-7. It should be noted that while these two sets of constraints are quite similar to those used in generating the reference solution (Ref. 21), they are not identical. Additional constraints on the stresses and displacements at intermediate points along the members were included in Ref. 21, however, none of these constraints were reported as being critical in the final design. Also, the effective column length parameters used in the column buckling constraint calculation were periodically updated in Ref. 21 while these parameters were held constant (equal to the values corresponding to the final design of Ref. 21) in this work. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 25. The design element is described in Appendix C.

8.7.1 Case A: Stress, Buckling and Side Constraints

The solution for Case A was obtained using five different solution options. The iteration history data for these runs is given in Table 26. The corresponding iteration history plots are shown in Figs. 63-67. The first two runs (options 1(P) and 3(P)) for this problem are made using the RSP design space option and linear constraint approximations. The approximate problems are solved via a primal solution method with 40% move limits on the RSP's ($d_1 = 0.$, $d_2 = .4$). Comparison of the iteration histories for runs 1 and 2 shows a slight improvement in the maximum constraint violation for the intermediate designs when the global element end force variation option is employed (run 2). It is, however, worthwhile to note the considerable increase in the solution time resulting from the required element end force response quantity calculations.

Run 3 (option 6(P)), in this case, is the same as run 2 except that mixed variable (hybrid) approximations are used for the behavior constraints and the move limits are increased to 50% on the RSP's ($d_1 = 0.$, $d_2 = .5$). Comparing these results with run 2 indicates little significant difference in terms of either convergence rate or maximum constraint violation history.

The final two runs for this problem utilize the CSD design space option and mixed variable behavior constraint approximations. The approximate problems are solved via a primal method in run 4 and a dual method in run 5 with move limits of 30% (d_1 = .3, d_2 = 0.) and 15% (d_1 = .15, d_2 = 0.) on the element CSD's, respectively. Comparing the iteration histories for these two runs one can observe significantly poorer performance in the case where the dual solution method was used. This can basically be attributed to large numbers of retained constraints (as many as 5-6 times the number of design variables) resulting in poor convergence of the optimizer. As a result of the optimizer convergence problems, relatively small move limits were required leading to an increase in the number of design stages needed for overall problem convergence. Comparison of runs 3 and 4 reveals that, although the design spaces are difference, there is little difference in the convergence rate. There is, however, significant difference in the maximum constraint violation histories, especially during the early stages of the design process.

The final designs and critical* constraints for this case are given, along with those of the reference solution (Ref. 21), in Tables 27 and 28. All of the designs have essentially the same final weight with the largest difference (between strictly feasible designs) occurring between the reference solution and run 2 (less than 1.4%). The critical constraints are also nearly identical. It is interesting to note that the material distributions for all of the final designs are remarkably similar considering the considerable variations that one often encounters for frame type problems. This behavior is most likely due to the fact that the R/t constraints are critical for all of the members in the structure thereby effectively reducing the design freedom to one variable per member.

8.7.2 Case B: Displacement, Stress, Buckling and Side Constraints

This case was solved using five different solution options. The iteration history data is given in Table 29 and the corresponding iteration history plots are shown in Figs. 68-72. Runs 1 and 2 (options 1(P) and 3(P)) are made using the RSP design space option and linear constraint approximations. Each approximate problem is solved via a primal solution method with move limits of 40% on the RSP's (d_1 = 0., d_2 = .4). The results for these runs are essentially the same, both in terms of convergence rate and maximum constraint violation for the intermediate designs. It is important to note that, contrary to Case A where an improvement in the maximum constraint violation history was observed when the global element end force variation option was employed (compare runs 1 and 2, Table 26), here there is little constraint violation improvement for run 2 as compared to run 1. This is due to the fact that the column buckling constraints play only a small role in the design process (i.e., the design is essentially displacement and R/t constrained, see Table 31).

Based on the results of runs 1 and 2, the remaining runs for this case are made using the invariance assumption for the element end forces. Run 3 (option 4(P)) is the same as run 1 except that the behavior constraints are approximated via a mixed variable approximation and the move limits are increased to 50% on the RSP's ($d_1 = 0.$, $d_2 = .5$). Comparison of the iteration histories for runs 1 and 3 reveals an improvement in the maximum constraint violation history for run 3 (considering the more liberal move limits) but no substantial difference in the convergence rate.

The final two solutions (options 10(P) and 10(D)) are obtained using the CSD design space option and mixed variable behavior constraint approximations. The approximate problems are solved via a primal method in run 4 and a dual method in run 5 with move limits of 30% ($d_1 = .3$, $d_2 = 0.$) and 15% ($d_1 = .15$, $d_2 = 0.$) on the element CSD's. As in Case A, the use of the dual solution method yields poorer results due to the large numbers of retained constraints (3-4 times the number of design variables). Also, as in Case A, run 4 compares well with run 3 in terms of convergence rate but has a less attractive maximum constraint violation history.

The final designs and critical* constraints for runs 1-5 are given, along with those of the reference solution (Ref. 21), in Tables 30 and 31. Again, as in Case A, all of these designs have essentially the same final weight, material distribution and critical constraint sets.

8.8 2 x 5 Grillage (Problem 7)

Figure 73 depicts a 2 x 5 grillage subject to a single loading condition. This structure is modelled with thin walled box section frame elements (type 11) each having four sizing variables (B, H, t_b, t_h). All members are made of the same material. Since both the structure and its loading are symmetric a half model can be used in solving the design problem. The grillage is designed for minimum material volume subject to two independent sets of constraints. In the first case (Case A) constraints are imposed on the vertical displacements at nodes numbers 4, 7 and 10 and side constraints are placed on the element sizing variables. The second set of constraints (Case B) includes all of those in Case A with the addition of member stress and local buckling constraints. A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 32. The design element is described in Appendix C.

8.8.1 Case A: Displacement and Side Constraints

This problem was solved using five different solution options. Since there are no stress or buckling constraints considered for this case all of the solution options used here utilize the element end force invariance assumption. The iteration history data for these runs is given in Table 33. The corresponding iteration history plots are shown in Figs. 74-78. The first two runs (options 1(P) and 4(P)) for this case are made using the RSP design space option. In run 1 linear constraint approximations are employed while in run 2 mixed variable (hybrid) approximations are constructed for the displacement constraints. Each approximate problem is solved via a primal solution method with 40% move limits on the RSP's ($d_1 = 0.$, $d_2 = .4$). Comparison of the iteration history data for these runs reveals a slight improvement in the convergence rate for run 2 but no significant improvement in the maximum constraint violation for the intermediate designs.

In runs 3 and 4 (options 10(P) and 10(D)) the design is carried out in the CSD design space using mixed variable approximations for the displacement constraints. The approximate problems are solved via a primal method in run 3 and a dual method in run 4 with 40% move limits on the CSD's. The results for these runs compare well with those of runs 1 and 2 both in terms of convergence rate and maximum constraint violation. The final design material volume is slightly smaller (.3 - 1.0%) for the designs generated using the CSD space option. The final run (option 1(PU)) for this problem is the same as run 1 except that the approximate problem update procedure is employed. Here, the approximate problem is updated, without recourse to structural analysis or response quantity sensitivity calculations, twice between each complete approximate problem generation. Use of the update procedure leads to an improvement in the convergence rate by a factor of 2-2.7 while at the same time yielding a slightly improved constraint violation history and a final design material volume only 2.9% greater than run 1. It should be noted here that, due to the absence of stress and buckling constraints, it is possible, in this case, to update the approximate problem more than once between complete approximate problem generations thereby making greater use of the high quality nodal displacement sensitivities.

The final designs and critical* constraints for this case are given, along with those of the reference solution (Ref. 29) in Tables 34 and 35. In all runs the critical constraint set at the final design includes the vertical displacements at nodes 7 and 10. In runs 3 and 4 the vertical displacement at node 4 is also critical. The material volume for the various final designs varies only slightly (the maximum variation is less than 3%), however, the corresponding material distributions are significantly different. Again, this result is not unexpected and can be attributed to a "flatness" of the design space in the neighborhood of the optimum design which has been observed to exist for many statically indeterminate problems.

8.8.2 Case B: Displacement, Stress, Buckling and Side Constraints

This problem was solved using four different solution options. Since it is expected that the local buckling constraints will play an important role in the design process and that the element end forces are strongly dependent on all of the problem design variables, all of the solution options employ the global force variation option. The iteration history data for these runs is given in Table 36. The corresponding iteration history plots are shown in Figs. 79-82. In the first two runs (options 3(P) and 6(P)) the design is carried out in the RSP design space using linear and mixed variable (hybrid) behavior constraint approximations, respectively. Each approximate problem is solved via a primal solution method. In run 1 move limits of 20% and 40% ($d_1 = .2$, $d_2 = .4$) are placed on the CSD's and RSP's, respectively, while in run 2 the move limits are 20% and 50% ($d_1 = .2$, $d_2 = .5$). Comparing the results for runs 1 and 2 shows little difference in the convergence rate and only a small improvement in the maximum constraint violation history when the mixed variable approximation is used (run 2).

In run 3 (option 12(P)) the design is carried out using the CSD design space option and mixed variable approximation for the behavior constraints. The approximate problems are solved via a primal solution method with move limits of 30% on the element CSD's ($d_1 = .3$, $d_2 = 0.$). The dual solution option is not used here due to the large number of retained constraints. The iteration history data for this run compares well with that of run 2 both in terms of the convergence rate and overall maximum constraint violation.

The final run (option 3(PU)) for this problem is the same as run 1 except that the approximate problem update procedure is used to update the approximate problem once between each complete approximate problem generation. The convergence rate for this run is improved over that for runs 1-3, however, the maximum constraint violation history is slightly less attractive.

The final designs and critical* constraints for this case are given, along with those of the reference solution (Ref. 29), in Tables 37 and 38. Again, as in Case A, the material volumes corresponding to the final designs are quite close, with the maximum variation being approximately 5.6%. Also, as was the previous case, there are significant variations in the material distribution and slight differences in the critical constraint sets.

8.9 Two Bay/Six Story Frame (Problem 8)

A two bay/six story planar frame structure is shown in Fig. 83. This structure is subjected to two independent loading conditions consisting of both concentrated nodal loading and uniform loading of the structural members. The structure is modelled with 30 thin walled tube elements (type 14), each having two sizing variables (R,t). All of the members are made of the same material. This structure is designed for minimum weight subject to two independent sets of constraints. The first set (Case A) includes stress constraints at both ends of each member, side constraints on the element sizing variables and constraints against both local and column buckling of each member. The local buckling constraints are applied as upper bounds on the member R/t ratios. The second set of constraints (Case B) includes all of the constraints in Case A with the addition of constraints on the lateral displacements at node numbers 1-18. As was mentioned in the previous discussion of Problem 6, these two sets of constraints are not identical to those used in generating the reference solution (Ref. 21). Additional constraints on stresses and displacements at intermediate points along the members were included in Ref. 21, however, none of these constraints were reported as being critical in the final design. Also, the effective column length parameters used in the column buckling constraint calculations were periodically updated in Ref. 21, while, in this work, these parameters were held constant (equal to the values corresponding to the final design of Ref. 21). A summary of the material properties, loading conditions, constraint allowables, initial design and bounds on the element sizing variables is given in Table 39. The design element is described in Appendix C.

8.9.1 Case A: Stress, Buckling and Side Constraints

The solution for Case A was obtained using five different solution options. The iteration history data for these runs is given in Table 40. The corresponding iteration history plots are shown in Figs. 84-88. The first two runs (options 1(P) and 3(P)) for this problem are made using the RSP design space option and linear constraint approximations. Each approximate problem is solved via a primal solution method with 40% move limits on the element RSP's (d_1 = 0., d_2 = .4). Comparison of the iteration histories for runs 1 and 2 shows a moderate improvement in both the convergence rate and maximum constraint violation for the intermediate designs when the global element end force variation option is used (run 2). However, it is important to note the considerable increase in solution time resulting from the required element end force sensitivity calculations.

Run 3 (option 6(P)) is the same as run 2 except that mixed variable (hybrid) approximations are used for the constraints and the move limits are increased to 50% on the RSP's (d_1 = 0., d_2 = .5). Comparing these results with run 2 reveals a decrease in the number of analyses required for convergence and an improvement in the overall maximum constraint violation history. However, the final design weight for run 3 is 1.4% greater than that of run 2.

The fourth run (option 12(P)) utilizes the CSD design space option and mixed variable approximations for the behavior constraints. The approximate problems are solved via a primal solution method with 40% move limits on the CSD's (d_1 = .4, d_2 = 0.). Comparison of the iteration history data for this run with that of run 3 reveals an increase in the number of analyses required for convergence and a slight deterioration in the maximum constraint violation history. The final design weight is, however, approximately 1% lower than that of run 3.

The omparing approximate problem update procedure is used. Here, the approximate problem is updated once between each complete approximate problem generation. In this case the use of the update procedure results in only a slightly improved convergence rate.

The final designs and critical* constraints for this case are given, along with those of the reference solution (Ref. 21), in Tables 41 and 42. All of the final designs have essentially the same weight with the largest difference occurring between runs 3 and 5 (approximately 1.6%). The material distributions and critical constraint sets are also very similar.

8.9.2 Case B: Displacement, Stress, Buckling and Side Constraints

Case B was solved using five different solution options. The iteration history data for these runs is given in Table 43. The corresponding iteration history plots are shown in Figs. 89-93. The first two runs (options 1(P) and 3(P)) for this problem are made using the RSP design option and linear constraint approximations. Each approximate problem is solved via a primal solution method with 40% move limits on the RSP's ($d_1 = 0.$, $d_2 = .4$). Comparison of the iteration histories for runs 1 and 2 reveals a slight increase in the number of analyses required for convergence and a small improvement in the maximum constraint violation for the intermediate designs when the global element end force variation option is used (run 2). It is important to note that, contrary to Case A where the overall design process was improved through the use of the global element end force variation option (compare runs 1 and 2, Table 40), here there is little significant improvement. This is basically due to the fact that in Case B the displacement constraints (whose approximations do not depend on the choice of the element end force variation option) play and important role in the design process (see Table 45).

Based on the results of runs 1 and 2, the remaining runs for this case are made using the element end force invariance assumption. Run 3 (option 4(P)) is the same as run 1 except that mixed variable (hybrid) approximations are utilized for the behavior constraints and the move limits are increased to 50% on the element RSP's ($d_1 = 0.$, $d_2 = .5$). Comparison of the iteration histories for runs 1 and 3 reveals a significant improvement in the maximum constraint violation history for run 3 (especially when considering the more liberal move limits) but little difference in the convergence rate.

The fourth run (option 10(P)) utilizes the CSD design space option and mixed variable approximations for the behavior constraints. The approximate problems are solved via a primal solution method with 40% move limits on the element CSD's ($d_1 = .4$, $d_2 = 0$). Comparison of the iteration history data for run 4 with that for runs 1-3 reveals a significant improvement in the convergence rate for run 4 while maintaining a comparable maximum constraint violation history.

The final run for Case B is the same as run 1 except that the approximate problem update procedure is employed. Here, the approximate problem is updated once between each complete approximate problem generation. Comparing the results of run 5 with runs 1-3 reveals a substantial improvement in the convergence rate for run 5 with only a small increase in the maximum constraint violation for the intermediate designs.

The final designs and critical* constraints for this case are given, along with those of the reference solution (Ref. 21), in Tables 44 and 45. As in Case A, all of the final designs have essentially the same weight with the largest difference being less than 1.6%. The material distributions and critical constraints for the final designs are also quite similar.

8.10 Helicopter Tail Boom (Problem 9)

Figure 94 depicts a space frame idealization of a helicopter tail boom subject to a single loading condition. All members are made of the same material and have the same cross section (thin-walled tube with two sizing variables (R,t)). This structure is designed for minimum weight subject to constraints on the nodal displacements in the y and z directions at node numbers 5-28 and side constraints on the element sizing variables. Stress constraints are also imposed at both ends of each member along with column buckling and local wall buckling constraints (in the form of R/t constraints). A summary of the material properties, constraint allowables, loading conditions, initial design and bounds on the element sizing variables is given in Table 46. The design element (type 14) is described in Appendix C.

This problem was solved using five different solution options. Since this problem is expected to be essentially displacement and R/t constrained all of the solution options chosen make use of the element end force invariance assumption. The iteration history data is given in Table 47. The corresponding iteration history plots are shown in Figs. 95-99. In the first two runs (options 1(P) and 4(P)) the design is carried out in the RSP design space using linear and mixed variable (hybrid) behavior constraint approximations, respectively. Each approximate problem is solved via a primal solution method with move limits on the RSP's of 60% (d_1 = 0., d_2 = .6) for run 1 and 70% (d_1 = 0., d_2 = .7) for run 2. Comparison of the iteration histories for these runs reveals no difference in the convergence rate and only a slight change in the maximum constraint violation for the intermediate designs. It is interesting, however, to observe that, even though the initial design is highly infeasible (211.6%), near feasible designs are achieved after only three design stages.

In runs 3 and 4 (options 10(P) and 10(D)) the design is performed using the CSD design space option and mixed variable behavior constraint approximations. The approximate problems are solved via a primal method in run 3 and a dual solution method in run 4 with 40% move limits on the CSD's (d_1 = .4, d_2 = 0.). There is little difference in convergence rate or maximum constraint violation for these runs, however, the dual solution method is less efficient here due to the large number of retained constraints (twice the number of design variables).

The final run (option 1(PU)) for this problem is the same as run 1 except that the approximate problem update procedure is employed. Here, the approximate problem is updated, without recourse to structural analysis or response quantity sensitivity calculations, once between each complete approximate problem generation. Comparison of the iteration history for this run with those of runs 1-4 reveals a dramatic improvement in both the convergence rate and the maximum constraint violation history. Also, it is important to note that the total solution time is improved here due to the fact that the increase in optimization time is more than offset by the decrease in analysis time (which includes the time required for the approximate problem generation). This improvement is expected to be much more dramatic for large practical problems where the solution of the structural analysis problem represents the primary computational burden.

The final designs and critical* constraints for this problem, along with those of the reference solution (Ref. 21), are given in Tables 48 and 49. In all cases, the critical constraint set at the final design includes the displacement constraints at nodes 25 and 27 and the local wall buckling (R/t) constraints for nearly all of the members. Also, the final design weight and material distribution is nearly the same for all runs. Finally, it is interesting to observe the intuitively satisfying result whereby the lighter designs contain slightly larger members at the base (fixed) end of the structure.

CHAPTER IX

Conclusions and Recommendations

9.1 Conclusions

A synthesis methodology for the design of frame-truss structures subject to multiple static loading conditions has been developed. This methodology has been implemented in the COMPASS computer program and has subsequently been used to solve a variety of frame-truss synthesis problems. The numerical results presented here illustrate the feasibility of obtaining near optimum designs after only 5-10 structural analyses for most frame-truss structures.

While it is believed that the primary goal of this study has been achieved, it is important to realize that attaining this goal has required not only the introduction of a full gamut of approximation concepts, but also the proper application of these techniques to the problem at hand. Unlike the truss-membrane synthesis methodology, which tends to employ a rather standard set of approximation concepts to the solution of most problems with uniform success, the key to the efficient solution of a frame-truss design problem, in many cases, lies in the thoughtful selection of appropriate approximation techniques and mathematical programming methods from a set of available options.

Engineering insight and experience has long been a part of traditional design methods and it is not surprising to find that somewhat analogous insights and experiences must now become a part of an efficient frame-truss synthesis methodology. The considerable body of computational experience reported here is intended to provide the design engineer with the foundation needed to build expertise in the use of frame synthesis methodology. While most of the points that will be discussed in the sequel have been previously mentioned in Chapter VIII, it is useful to summarize these results here in the form of guidelines for solving frame design problems.

By now it should be apparent that the efficient solution of a frame-truss synthesis problem involves making important decisions about the construction and solution of the approximate design problems. The items which must be considered include the types of approximations employed, the choice of design space, the optimization method used to solve the mathematical programming problems, the choice of appropriate move limits and even the frequency of performing the structural analyses and response quantity sensitivity calculations. While all of these items are important, probably the most important decisions to make are those related to the construction of the behavior constraint approximations. This involves deciding which design variables the constraint is believed to be dependent on (i.e. choosing the appropriate element force variation option) and selecting the basic character of the approximation to be used (i.e. linear vs. mixed variable). The following guidelines are offered:

1. the element end force invariance option is recommended if

   a. the structure is statically determinate.

b. the structure is statically indeterminate but the coupling between the structural members is believed to be weak (see Problems 2 and 3, Chapter VIII).

c. the design problem is believed to be essentially displacement constrainted (see Problems 6 and 8 (Case B) and Problem 9, Chapter VIII).

2. the local element end force variation option can be useful in cases where the invariance assumption is inadequate and the expense associated with the global force variance option is prohibitive. This option should be used with care as it can, in some cases, yield approximations which are inferior to the force invariance option (see Problem 3, Chapter VIII).

3. the global element end force variation option is recommended if the structural behavior is strongly coupled and the design problem is believed to be essentially strength critical (see Problems 1, 5 and 6, Chapter VIII).

4. the linear constraint approximations are recommended if the constraints controlling the design process are believed to be linear or nearly linear functions of the design variables.

5. the mixed variable (hybrid) approximation generally yields equal or superior performance as compared to the linear approximations (except as noted in item 4) with little additional computational expense.

6. the mixed variable (hybrid) approximations may, in some cases, be used in lieu of higher levels of element end force sensitivity information without the added computational expense (see Problem 2, Chapter VIII).

The guidelines for selecting the appropriate design space option (i.e., either reciprocal section property (RSP) or cross sectional dimension (CSD) space) are not as specific as those for selecting the constraint approximations. For the problems solved in this study the overall performance of the two design spaces has been nearly the same, although distinctly different designs may be generated depending on the design space chosen (see Problem 5, Chapter VIII). Part of the reason that this has been the case can be attributed to the use of the approximate sizing variable recovery transformation, described in Chapter VI, which compromises the quality of the constraint approximations in the RSP design space. It is possible that a more exact recovery method would lead to superior results for the RSP design space option. For the current implementation of the frame-truss synthesis methodology one may wish to consider the following when making the design space selection:

1. the use of the RSP design space generally requires the addition of sizing variable side constraint approximations to the approximate design problem. While this has not been a computational burden for the problems solved here it may be burdensome for problems involving large numbers of sizing variables.

2. the selection of move limits in the CSD design space may be more physically meaningful than in RSP space.

The selection of the mathematical programming method (i.e., primal or dual) for solving the approximate problems is relatively straight forward. Generally, if the number of retained constraints is expected to be less than 1.5 times the number of design variables the dual method will be at least as, if not more, efficient than the primal method (see Problems 2, 3 and 5, Chapter VIII). For problems where the number of retained constraints is expected to be large compared to the number of design variables the primal solution method should be used (see Problems 4, 6 and 9, Chapter VIII). Two additional comments are appropriate here. First, the development and implementation of a specialized dual method (in which the dimensionality of the dual space does not exceed the number of truly critical constraints) could make the dual method more efficient even when the number of retained constraints is large. Lastly, it should be recognized that for large problems, where the structural analysis and approximate problem generator can be expected to represent the main computational effort in the design process, the selection of the optimization method is not expected to seriously effect the efficiency of the overall problem solution.

The selection of appropriate move limits on the structural design variables can be quite difficult in the absence of prior computational experience. Even with experience the selection process is often problem dependent and may require several attempts. Move limits which are either too tight or too loose can seriously effect the problem convergence rate or, in some cases, preclude convergence altogether. For the problems solved in this study the move limits ranged from 20% to 70% on the RSP's and 15% to 50% on the CSD's, with the most frequently used values being 40–50% for the RSP's and 30–40% for the CSD's. The following guidelines are offered for making an initial choice of move limits:

1. problems which are expected to have a large number of critical buckling constraints will generally require tighter move limits than those which are essentially stress and/or displacement constrained.

2. the use of the mixed variable (hybrid) constraint approximation option generally allows for more liberal move limits, especially when the problem is essentially strength critical.

The final decision involves the use of the approximate problem update procedure. Based on the numerical examples presented here it can be concluded that the use of this option generally results in an improvement in the convergence rate of the design problem (i.e. the number of structural analyses required for convergence is reduced). For small problems, however, even though the convergence rate is improved the total solution time can increase when this option is employed (see Problem 2, Case B, Chapter VIII). For larger problems of practical interest the use of this option does result in savings in solution time as well as an improved convergence rate (see Problem 9, Chapter VIII). The solution time savings are expected to be more dramatic as the analysis problem size increases. Aside from the fact that the approximate problem update option is best used for large problems two other comments are appropriate here:

1. the approximate problem can be updated more frequently without

reanalysis if the design problem is essentially displacement critical (see Problem 7, Case A, Chapter VIII).

2. in the case where the design problem is essentially strength critical, the approximate problem may be updated more frequently without reanalysis when the structural behavior is weakly coupled than when it is strongly coupled.

9.2 Recommendations for Future Work

While the frame-truss synthesis methodology presented in this work can be used to efficiently solve a significant class of structural design problems, several areas of future work and investigation can be identified which will broaden the applicability of the method to include a larger class of problems and/or lead to increased solution efficiency. These areas are summarized below:

1. the expansion of the design element library to include a greater variety of cross sectional shapes.

2. the addition of a modal analysis capability for the inclusion of frequency constraints in the design problem.

3. the addition of an optimum design sensitivity capability.

4. the extension of the approximate problem update procedure to include the updating of the stress and buckling constraint partial derivatives which are explicit functions of the CSD's and RSP's.

5. investigate the use of a cumulative constraint formation (or some

other method) as a means of reducing the numbers of stress and buckling constraints retained for each structural member.

6. develop a specialized, efficient dual solution method for solving all forms of the approximate problems.

7. investigate the possibility of replacing the approximate recovery method with an efficient nonlinear recovery technique.

The suggested extensions offered above can be divided into two groups based on the probability that the work can be successfully completed. It is believed that items 1-4 pose relatively little risk in that the work basically entails the implementation of proven concepts. Items 5-7, however, may require considerable investigation and if completed may not produce the desired results.

REFERENCES

1. Schmit, L.A., "Structural Synthesis - Its Genesis and Development," AIAA Journal, Vol. 19, No. 10, 1981, pp. 1249-1263.

2. Vanderplaats, G.N., "Structural Optimization - Past, Present, and Future," AIAA Journal, Vol. 20, No. 7, 1982, pp. 992-1000.

3. Fleury, C., Ramanathan, R.K., Salama, M. and Schmit, L.A., "ACCESS Computer Program for the Synthesis of Large Structural Systems," Proceedings of the International Symposium on Optimum Structural Design, University of Arizona, Tuscon, Arizona, Oct. 1981, pp. 11.1-11.8.

4. Sobieszczanski-Sobieski, Jaroslaw, and Rogers, James L., "A Programming System for Research and Applications in Structural Optimization," Proceedings of the International Symposium on Optimum Structural Design, University of Arizona, Tuscon, Arizona, Oct. 1981, pp. 11.9-11.21.

5. Bennett, J.A. and Nelson, M.R., "An Optimization Capability for Automotive Structures," SAE Transactions, Vol. 88, 1979, pp. 3236-3243.

6. Fleury, C., "Large Scale Structural Optimization by Finite Element," Proceedings of the International Symposium on Optimum Structural Design, University of Arizona, Tuscon, Arizona, Oct. 1981, pp. 11.23-11.41.

7. Haftka, R.T. and Prasad, B., "Programs for Analysis and Resizing

of Complex Structures (PARS)," Computers and Structures, Vol. 10, No. 1-2, March 1979, pp. 323-330.

8. Morris, A.J., Bartholomew, P., and Dennis, J., "A Computer Based System for Structural Design, Analysis and Optimization," AGARD CP-280, Paper 20, 1980.

9. Isakson, G. and Pardo, H., "ASOP-3: A Program for the Minimum-Weight Design of Structures Subject to Strength and Deflection Constraints," AFFDL-TR-76-157, 1976.

10. Giles, G.L., Blackburn, C.L. and Dixon, S.C., "Automated Procedures for Sizing Aerospace Vehicle Structures (SAVES)," Journal of Aircraft, Vol. 9, Dec. 1972, pp. 812-819.

11. Schmit, L.A. and Farshi, B., "Some Approximation Concepts for Structural Synthesis," AIAA Journal, Vol. 12, No. 5, 1974, pp. 692-699.

12. Schmit, L.A., and Miura, H., "Approximation Concepts for Efficient Structural Synthesis," NASA CR 2552, March 1976.

13. Brown, D.M. and Ang, A.H.-S., "Structural Optimization by Nonlinear Programming," Journal of the Structural Division, ASCE, Vol. 92, No. ST6, 1966, pp. 319-340.

14. Moses, F. and Onoda, S., 'Minimum Weight Design of Structures with Application to Elastic Grillages," International Journal for Numerical Methods in Engineering, Vol. 1, 1969, pp. 311-331.

15. Kavalie, D. and Moe, J., "Application of Non-linear Programming to Optimum Grillage Design with Non-Convex Sets of Variables," International Journal for Numerical Methods in Engineering, Vol. 1, 1969, pp. 351-378.

16. Haug, E.J., Pan, K.C., and Streeter, T.D., "A Computational Method for Optimal Structural Design. I. Piecewise Uniform Structures," International Journal for Numerical Methods in Engineering, Vol. 5, 1972, pp. 171-184.

17. Reinschmidt, K.F., and Norabhoompipat, T., "Structural Optimization by Equilibrium Linear Programming," Journal of the Structural Division, ASCE, Vol. 101, No. ST4, 1975, pp. 921-937.

18. Isreb, M., 'Three Dimensional Beam Elements Synthesis Applications with Stress Constraints, Nonlinear Size-Stiffness Relationships and Various Size - Inertia Powers," Computers and Structures, Vol. 7, 1977, pp. 565-569.

19. Fleury, C. and Sander, G., 'Dual Methods for Optimizing Finite Element Flexural Systems," Computer Methods in Applied Mechanics and Engineering, Vol. 37, 1983, pp. 249-275.

20. Bartel, D.L., Optimum Design of Spatial Structures, Ph.D. Dissertation, The University of Iowa, Iowa City, IA., August, 1969.

21. Govil, A.K., Arora, J.S., and Haug, E.J., "Optimal Design of Frames with Substructuring," Computers and Structures, Vol. 12, 1980, pp. 1-10.

22. Bennett, J.A. and Botkin, M.E., "Automated Design for Automotive Structures," Proceedings of ASME Design Automation Conference, Hartford, Connecticut, September, 1981.

23. Yoshimura, M., Hamado, T., Yura, K., and Hitomi, K., "Design Optimization of Machine Tool Structures with Respect to Dynamic Characteristics," ASME Journal of Mechanical Design, Vol. 22, January 1980, pp. 33-40.

24. Ramana, G.V. and Rao, S.S., "Optimum Design of Plano-Milling Machine Structure Using Finite Element Analysis," Computers and Structures, Vol. 10, No. 2, 1984, pp. 247-253.

25. Fuchs, Moshe B., "Linearized Homogeneous Constraints in Structural Design," International Journal of Mechanical Sciences, Vol. 22, 1980, pp. 33-40.

26. Lust, R.V. and Bennett, J.A., "Structural Optimization in the Design Environment," Proceedings of the Fourth International Conference on Vehicle Structural Mechanics, Detroit, Michigan, November 1981, pp. 169-177.

27. Miura, H., Lust, R.V., and Bennett, J.A., "Integrated Panel and Skeleton Automotive Structural Optimization," Proceedings of the Fourth International Conference on Vehicle Structural Mechanics, Detroit, Michigan, November, 1981, pp. 161-168.

28. Bennett, J.A., "Application of Linear Constraint Approximations to Frame Structures," Proceedings of the International Symposium on

Optimum Structural Design," University of Arizona, Tuscon, Arizona, October 1981, pp. 7.9-7.15.

29. Mills-Curran, W.C., Lust, R.V., and Schmit, L.A., "Approximations Method for Space Frame Synthesis," AIAA Journal, Vol. 21, No. 11, November 1983, pp. 1571-1580.

30. Mills-Curran, W.C., Optimization of Structures Subjected to Periodic Loads, Ph.D. Dissertation, University of California, Los Angeles, California, 1983.

31. Schmit, L.A., "Structural Design by Systematic Synthesis," Proceeding, 2nd Conference on Electronic Computation, ASCE, New York, 1960, pp. 105-132.

32. Desai, C.S. and Abel, J.F., Introduction to the Finite Element Method, Van Norstrand Reinhold, New York, 1972, pp. 181-190.

33. Felippa, Carlos A., "Solution of Linear Equations with Skyline-Stored Symmetric Matrix," Computers and Structures, Vol. 5, No. 1, 1975, pp. 13-29.

34. Prasad, B., 'Explicit Constraint Approximation in Structural Optimization - Part I: Analyses and Projections," Computer Methods in Applied Mechanics and Engineering, Vol. 40, Sept. 1983, pp. 1-26.

35. Prasad, B., 'Novel Concepts for Constraint Treatments and Approximations in Efficient Structural Synthesis," AIAA Journal, Vol. 22, No. 7, July 1984, pp. 957-966.

36. Starnes, J.H., Jr., and Haftka, R.T., "Preliminary Design of Composite Wings for Buckling, Stress and Displacement Constraints," Journal of Aircraft, Vol. 16, August 1979, pp. 564-570.

37. Arora, J.S., and Haug, E.J., "Methods of Design Sensitivity in Structural Optimization," AIAA Journal, Vol. 17, Sept. 1979, pp. 970-974.

38. Vanderplaats, G.N., "Comment on 'Methods of Design Sensitivity in Structural Optimization,'", AIAA Journal, Vol. 18, Nov. 1980, pp. 1406-1407.

39. Luenberger, D.G., Introduction to Linear and Nonlinear Programming, Addison-Wesley, Reading, 1973.

40. Zoutendijk, G., Methods of Feasible Directions, Elsevier, Amsterdam, 1960.

41. Fox, R.L., Optimization Methods for Engineering Design, Addison-Wesley, Reading, 1971, pp. 179-196.

42. Vanderplaats, G.N. and Moses, F., "Structural Optimization by Methods of Feasible Directions," Computers and Structures, Vol. 3, July 1973, pp. 739-755.

43. Vanderplaats, G.N., "CONMIN - A Fortran Program for Constrained Function Minimization," NASA TM X-62,282, August 1973.

44. Lasdon, L.S., Optimization Theory for Large Systems, Macmillan, New York, 1970, pp. 396-459.

45. Fleury, C., "Structural Weight Optimization by Dual Methods of Convex Programming," International Journal for Numerical Methods in Engineering, Vol. 14, No. 12, 1979, pp. 1761-1783.

46. Schmit, L.A. and Fleury, C., "Structural Synthesis by Combining Approximation Concepts and Dual Methods, AIAA Journal, Vol. 18, No. 10, 1980, pp. 1252-1260.

47. Fleury, C. and Schmit, L.A., "Dual Methods and Approximation Concepts in Structural Synthesis," NASA CR 3226, December 1980.

48. Fleury, C., and Braibant, V., "Structural Optimization - A New Dual Method Using Mixed Variables," LTAS Report SA-115, University of Liege, Liege, Belgium, March 1984.

49. Felippa, Carlos, A., "Database Management in Scientific Computing - I. General Description," Computers and Structures, Vol. 10, No. 1-2, 1979, pp. 53-61.

50. Felippa, Carlos, A., "Database Management in Scientific Computing - II. Data Structures and Program Architecture," Computers and Structures, Vol. 12, No. 1, 1980, pp. 131-145.

51. Reinschmidt, K.F., Cornell, C.A. and Brotchie, J.F., "Iterative Design and Structural Optimization," Journal of the Structural Divison, ASCE, Vol. 92, No. ST6, 1966, pp. 281-319.

52. Haug, E.J. and Arora, J.S., Applied Optimal Design, Wiley, New York, 1979.

53. Sobieszczanski-Sobieski, Jaroslaw, James, Benjamin and Dovi, Augustine, "Structural Optimization by Multilevel Decomposition," Proceedings, AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, Nevada, 1983, pp. 124-143.

54. Strang, G., Linear Algebra and Its Applications, Second Edition, Academic Press, 1980, pp. 55-62.

55. Gerard, G., and Becker, H., "Handbook of Structural Stability, Part I - Buckling of Flat Plates," NACA Technical Note 3781, July 1957.

56. Timoshenko, S.P. and Gere, J.M., Theory of Elastic Stability, Second Edition, McGraw-Hill, New York, 1961, pp. 348-439.

57. Timoshenko, S.P. and Goodier, J.N., Theory of Elasticity, Third Edition, McGraw-Hill, New York, 1970, pp. 309-313.

58. Johnston, B.G. (Editor), Guide to Stability Design Criteria for Metal Structures, Structural Stability Research Council, 3rd Edition, Wiley, New York, 1976.

59. Schilling, C.G., "Buckling Strength of Circular Tubes," Journal of the Structural Divison, ASCE, Vol. 91, No. ST5, 1965, pp. 325-349.

APPENDIX A

Stiffness Matrices and Load Vectors for Structural Elements

A.1 Prismatic Frame Element

The space frame element, shown in Fig. A1, is a two node, twelve degree of freedom element oriented with the longitudinal axis in the local x coordinate direction and the cross section principal axes in the local y and z coordinate directions. The element is assumed to have linear axial and torsional displacement states given by

$$u(x) = [N_1(x), N_2(x)] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{A-1}$$

$$\theta_x(x) = [N_1(x), N_2(x)] \begin{Bmatrix} \theta_{x_1} \\ \theta_{x_2} \end{Bmatrix} \tag{A-2}$$

and cubic bending displacement states of the form

$$v(x) = [N_3(x), N_4(x), N_5(x), N_6(x)] \begin{Bmatrix} v_1 \\ \theta_{z_1} \\ v_2 \\ \theta_{z_2} \end{Bmatrix} \tag{A-3}$$

and

$$w(x) = [N_3(x), N_4(x), N_5(x), N_6(x)] \begin{Bmatrix} w_1 \\ \theta_{y_1} \\ w_2 \\ \theta_{y_2} \end{Bmatrix}$$

(A-4)

where u, v and w are the displacements in the local x, y and z coordinate directions, θ_x, θ_y and θ_z are the rotations about the x,y and z axes and where the displacement shape functions are given by

$$N_1(x) = 1 - \frac{x}{L}$$

$$N_2(x) = \frac{x}{L}$$

$$N_3(x) = 1 - 3(\frac{x}{L})^2 + 2(\frac{x}{L})^3 \qquad \text{(A-5)}$$

$$N_4(x) = L[(\frac{x}{L}) - 2(\frac{x}{L})^2 + (\frac{x}{L})^3]$$

$$N_5(x) = 3(\frac{x}{L})^2 - 2(\frac{x}{L})^3$$

$$N_6(x) = L[- (\frac{x}{L})^2 + (\frac{x}{L})^3]$$

The assumed displacement states (Eqs. (A-1) - (A-4)) lead to the following strain-displacement relations

Axial strain: $\frac{\partial u}{\partial x} = [B_1] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

Torsional strain: $$\frac{\partial\theta_x}{\partial x} = [B_1] \begin{Bmatrix} \theta_{x_1} \\ \theta_{x_2} \end{Bmatrix} \tag{A-6}$$

Curvature: $$\frac{\partial^2 v}{\partial x^2} = [B_2] \begin{Bmatrix} v_1 \\ \theta_{z_1} \\ v_2 \\ \theta_{z_2} \end{Bmatrix}$$

$$\frac{\partial^2 w}{\partial x^2} = [B_2] \begin{Bmatrix} w_1 \\ \theta_{y_1} \\ w_2 \\ \theta_{y_2} \end{Bmatrix}$$

where

$$[B_1] = \left[-\frac{1}{L}, \frac{1}{L} \right]$$

$$[B_2] = \left[-\frac{6}{L^2}\left(1-\frac{2x}{L}\right), -\frac{2}{L}\left(2-\frac{3x}{L}\right), -\frac{6}{L^2}\left(\frac{2x}{L}-1\right), -\frac{2}{L}\left(1-\frac{3x}{L}\right) \right] \tag{A-7}$$

The total strain energy for the frame element can now be written as

$$\bar{U} = \frac{EA}{2}\int_0^L \{\frac{\partial u}{\partial x}\}^T\{\frac{\partial u}{\partial x}\}dx + \frac{GJ}{2}\int_0^L \{\frac{\partial \theta_x}{\partial x}\}^T \{\frac{\partial \theta_x}{\partial x}\}dx +$$

$$\frac{EI_z}{2}\int_0^L \{\frac{\partial^2 v}{\partial x^2}\}^T \{\frac{\partial^2 v}{\partial x^2}\}dx + \frac{EI_y}{2}\int_0^L \{\frac{\partial^2 w}{\partial x^2}\}^T\{\frac{\partial^2 w}{\partial x^2}\}dx$$

$$= \frac{1}{2}\{q\}^T[K]^e\{q\} \qquad \text{(A-8)}$$

where

E	–	material modulus of elasticity
G	–	material shear modulus
A	–	cross sectional area
I_y	–	cross sectional principal moment of inertia about the y axis
I_z	–	cross sectional principal moment of inertia about the z axis

and

$$\{q\}^T = \{u_1\ v_1\ w_1\ \theta_{x_1}\ \theta_{y_1}\ \theta_{z_1}\ u_2\ v_2\ w_2\ \theta_{x_2}\ \theta_{y_2}\ \theta_{z_2}\}$$

Evaluation of the integrals of Eq. (A-8) yields the following form of the element stiffness matrix in local coordinates

$$
[K]^e = \frac{E}{L}\begin{bmatrix}
A & 0 & 0 & 0 & 0 & 0 & -A & 0 & 0 & 0 & 0 & 0 \\
 & \frac{12I_z}{L^2} & 0 & 0 & 0 & \frac{6I_z}{L} & 0 & \frac{-12I_z}{L_2} & 0 & 0 & 0 & \frac{6I_z}{L} \\
 & & \frac{12I_y}{L^2} & 0 & \frac{-6I_y}{L} & 0 & 0 & 0 & \frac{-12I_y}{L^2} & 0 & \frac{-6I_y}{L} & 0 \\
 & & & \frac{GJ}{E} & 0 & 0 & 0 & 0 & 0 & \frac{-GJ}{E} & 0 & 0 \\
 & & & & 4I_y & 0 & 0 & 0 & \frac{6I_y}{L} & 0 & 2I_y & 0 \\
 & & & & & 4I_z & 0 & \frac{-6I_z}{L} & 0 & 0 & 0 & 2I_z \\
 & & & & & & A & 0 & 0 & 0 & 0 & 0 \\
 & & \text{Sym.} & & & & & \frac{12I_z}{L^2} & 0 & 0 & 0 & \frac{-6I_z}{L} \\
 & & & & & & & & \frac{12I_y}{L^2} & 0 & \frac{6I_y}{L} & 0 \\
 & & & & & & & & & \frac{GJ}{E} & 0 & 0 \\
 & & & & & & & & & & 4I_y & 0 \\
 & & & & & & & & & & & 4I_z
\end{bmatrix}
\tag{A-9}
$$

Similarily, the external work expression for the space frame element subject to uniformly distributed loads can be written as

$$
W = \int_o^L \{p_x \; p_y \; p_z \; m_x \; m_y \; m_z\} \begin{Bmatrix} u(x) \\ v(x) \\ w(x) \\ \theta_x(x) \\ -\frac{dw}{dx}(x) \\ \frac{dv}{dx}(x) \end{Bmatrix} dx
$$

$$= \{P\}^{e^T} \{q\} \tag{A-10}$$

where p_x, p_y and p_z are forces per unit length in the x, y and z directions and m_x, m_y, m_z are moments per unit length about the x, y and z axes. Evaluation of Eq. (A-10) using Eqs. ((A-1) - (A-5)) and assuming p_x, p_y, p_z as well as m_x, m_y, m_z, are constants (uniformly distributed) leads to the following form for the work equivalent load vector

$$\{P\}^{e^T} = \left\{ \frac{p_x L}{2}, \; \frac{p_y L}{2} - m_z, \; \frac{p_z L}{2} + m_y, \; \frac{m_x L}{2}, \; -\frac{p_z L^2}{12}, \; \frac{p_y L^2}{12}, \right.$$
$$\left. \frac{p_x L}{2}, \; \frac{p_y L}{2} + m_z, \; \frac{p_z L}{2} - m_y, \; \frac{m_y L}{2}, \; \frac{p_z L^2}{12}, \; -\frac{p_y L^2}{12} \right\} \tag{A-11}$$

A.2 Prismatic Truss Element

The space truss element, shown in Fig. A2, is a two node element oriented with the longitudinal axis in the local x direction. The element is assumed to have a linear displacement state of the form

$$u(x) = [1 - \frac{x}{L}, \frac{x}{L}] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{A-12}$$

where u is the displacement in the x direction. This assumed displacement state leads to the following strain-displacement relation

$$\frac{\partial u(x)}{\partial x} = [\frac{-1}{L}, \frac{1}{L}] \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{A-13}$$

The strain energy for the truss element can now be written as

$$\bar{U} = \frac{EA}{2} \int_o^L \{\frac{\partial u}{\partial x}\}^T \{\frac{\partial u}{\partial x}\} dx$$

$$= \frac{1}{2} \{u_1 \;\; u_2\} [K]^e \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{A-14}$$

which leads to the following expression for the element stiffness matrix in local coordinates

$$[K]^e = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{A-15}$$

Similarly, the external work due to a uniformly distributed axial load p_x can be written as

$$W = \int_o^L p_x \, u(x) dx = \{P\}^{e^T} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \tag{A-16}$$

which leads to the following work equivalent load vector

$$\{P\}^e = \frac{p_x L}{2} \{1 \quad 1\}^T \tag{A-17}$$

Rewriting $[K]^e$ and $\{P\}^e$ in terms of the full twelve nodal degrees of freedom gives

$$[K]^e = \frac{E}{L}\begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & -A & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & A & 0 & 0 & 0 & 0 & 0 \\ & & & & \text{sym} & & & 0 & 0 & 0 & 0 & 0 \\ & & & & & & & & 0 & 0 & 0 & 0 \\ & & & & & & & & & 0 & 0 & 0 \\ & & & & & & & & & & 0 & 0 \\ & & & & & & & & & & & 0 \end{bmatrix} \tag{A-18}$$

and

$$\{P\}^{e^T} = \frac{p_x L}{2}\{1\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\} \tag{A-19}$$

APPENDIX B

Generalized Inverse Transformation

Consider the m-vector {X} and the n-vector {Y} related by the m x n linear transformation matrix [J] as follows:

$$[J]\{Y\} = \{X\} \tag{B-1}$$

For the case where m = n and [J] is non-singular the inverse transformation is clearly given by

$$\{Y\} = [J]^{-1}\{X\} \tag{B-2}$$

For the general case where $m \neq n$ and the rows of [J] are not necessarily linearly independent, an inverse transformation of the form given by Eq. (B-2) does not generally exist. It is, however, possible to determine a set of linearly independent (free) variables $[\{X_F\}, \{Y_F\}]^T$ and a set of dependent (basic) variables $[\{X_B\}, \{Y_B\}]^T$ and to construct a linear transformation between them having the form

$$\begin{bmatrix} \{X_B\} \\ \{Y_B\} \end{bmatrix} = [H] \begin{bmatrix} \{X_F\} \\ \{Y_F\} \end{bmatrix} \tag{B-3}$$

The transformation matrix [H] can be constructed from [J] via two sets of pivot operations as shown in the following construction, which relies heavily on material contained in Ref. 54.

Consider Eq. (B-1) and imagine that it is partitioned in terms of the free $\{Y_F\}$ and basic $\{Y_B\}$ variables as follows:

$$[B \quad F] \begin{bmatrix} \{Y_B\} \\ \{Y_F\} \end{bmatrix} = \{X\} \tag{B-4}$$

Pivoting to determine the basis inverse and allowing for the existence of linearly dependent rows gives

$$\begin{bmatrix} I & B^{-1}F \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \{Y_B\} \\ \{Y_F\} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \{X\} \tag{B-5}$$

where

$$[B^{-1}] = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{B-6}$$

Eq. (B-5) yields the following two equations:

$$\{Y_B\} + [B^{-1}F]\{Y_F\} = [B_1]\{X\} \tag{B-7}$$

$$[B_2]\{X\} = \{0\} \tag{B-8}$$

The existence of Eq. (B-8) indicates that the {X} variables are not linearly independent. Therefore, partitioning Eq. (B-8) in terms of the free and basic {X} variables gives

$$\left[B_{2_B} \;\middle|\; B_{2_F} \right] \begin{bmatrix} \{X_B\} \\ \{X_F\} \end{bmatrix} = \{0\} \tag{B-9}$$

Pivoting yields

$$\left[I \;\middle|\; B_{2_B}^{-1} B_{2_F} \right] \begin{bmatrix} \{X_B\} \\ \{X_F\} \end{bmatrix} = \{0\} \tag{B-10}$$

Solving Eq. (B-10) for $\{X_B\}$ gives

$$\{X_B\} = -[B_{2_B}^{-1}\, B_{2_F}]\{X_F\} = [C]\{X_F\} \tag{B-11}$$

We now have $\{X_B\}$ expressed in terms of $\{X_F\}$. To express $\{Y_B\}$ in terms of the free variables let

$$[B_1] = [B_{1_B}\ B_{1_F}] \tag{B-12}$$

and

$$\{X\} = \begin{bmatrix} \{X_B\} \\ \{X_F\} \end{bmatrix} \tag{B-13}$$

Substituting Eqs. (B-12) and (B-13) into Eq. (B-7) and solving for $\{Y_B\}$ gives

$$\{Y_B\} = [B_{1_B}]\{X_B\} + [B_{1_F}]\{X_F\} - [B^{-1}F]\{Y_F\} \tag{B-14}$$

Substituting Eq. (B-11) for $\{X_B\}$ in Eq. (B-14) yields

$$\{Y_B\} = ([B_{1_F}] + [B_{1_B}][C])\{X_F\} - [B^{-1}F]\{Y_F\} \tag{B-15}$$

Finally, writing Eqs. (B-11) and (B-15) in matrix form yields

$$\begin{bmatrix} \{X_B\} \\ \hline \{Y_B\} \end{bmatrix} = \left[\begin{array}{c|c} -[C] & [0] \\ \hline [B_{1_F}] + [B_{1_B}][C] & -[B^{-1}F] \end{array}\right] \begin{bmatrix} \{X_F\} \\ \hline \{Y_F\} \end{bmatrix} \tag{B-16}$$

$$\begin{bmatrix} \{X_B\} \\ \{Y_B\} \end{bmatrix} = [H] \begin{bmatrix} \{X_F\} \\ \{Y_F\} \end{bmatrix} \tag{B-17}$$

It should be noted that if Eq. (B-1) is written in terms of the variable perturbations $\{\Delta X\}$ and $\{\Delta Y\}$ as

$$[J]\{\Delta Y\} = \{\Delta X\} \tag{B-18}$$

where

$$[J] = \left[\frac{\partial X}{\partial Y}\right] \tag{B-19}$$

then Eq. (B-17) becomes

$$\begin{bmatrix} \{\Delta X_B\} \\ \{\Delta Y_B\} \end{bmatrix} = [H]\ \{\Delta Z\} \tag{B-20}$$

where

$$\{\Delta Z\} = [\{\Delta X_F\}\ \ \{\Delta Y_F\}]^T \tag{B-21}$$

and

$$[H] = \begin{bmatrix} \dfrac{\partial X_B}{\partial Z} \\ \dfrac{\partial Y_B}{\partial Z} \end{bmatrix} \tag{B-22}$$

APPENDIX C

Design Element Library

The design element library contains the descriptions of the various design elements available for constructing the structural design model. Each element is completely described by the following: 1) a basic structural element type (e.g. frame or truss), 2) a description of the element cross section and 3) a set of element level constraints (e.g. stress, local buckling). The element end forces used in the element level constraint calculations are shown in Fig. C1 for both the frame and truss type elements.

C.1 Design Element Descriptions

The following design elements are available and are described in this section.

Element	Type
Truss with one sizing variable	1
Box beam with four sizing variables	11
I-beam with six sizing variables	12
Square beam with one sizing variable	13
Thin walled tube with two sizing variables	14
Box beam with three sizing variables	15

Element: TRUSS

Type: 1

Description: This element is a truss element with the cross section described by the sizing variable A as shown in Fig. C2. One stress constraint is computed for this element.

Reciprocal Section Property

$$X_1 = 1/A \tag{C-1}$$

Stress Constraint

$$g = \frac{|\sigma|}{\sigma_a} - 1 = \frac{|F_x\ X_1|}{\sigma_a} - 1 \leq 0 \tag{C-2}$$

where σ_a is the allowable stress in tension and compression.

Element: BOX BEAM

Type: 11

Description: This element is a frame element with the cross section described by the sizing variables B, H, t_b and t_h as shown in Fig. C3. Eight stress and four local buckling constraints are computed at each end of the element for a total of sixteen stress and eight local buckling constraints. The locations on the cross section at which these constraints are evaluated are shown in Fig. C3 for the first node (n_1) end of the element.

Cross Sectional Dimensions

$$Y_1 = B$$

$$Y_2 = H$$

$$Y_3 = t_h \tag{C-3}$$

$$Y_4 = t_b$$

Reciprocal Section Properties

$$X_1 = \frac{1}{A} = \frac{1}{Y_1Y_2 - (Y_2 - 2Y_4)(Y_1 - 2Y_3)}$$

$$X_2 = \frac{1}{J} = \frac{Y_2Y_4 + Y_1Y_3}{2Y_1^2\ Y_2^2\ Y_3\ Y_4} \tag{C-4}$$

$$X_3 = \frac{1}{I_y} = \frac{12}{Y_2Y_1^3 - (Y_2 - 2Y_4)(Y_1 - 2Y_3)^3}$$

$$X_4 = \frac{1}{I_z} = \frac{12}{Y_1Y_2^3 - (Y_1-2Y_3)(Y_2-2Y_4)^3}$$

Stress Constraints

The stress constraints for this element are based on the Von Mises criterion and have the form

$$g_i = \frac{1}{\sigma_a^2}(\sigma_i^2 + 3\tau_i^2) - 1 \leq 0 \quad ; \quad i = 1,2,\ldots 16 \tag{C-5}$$

where σ_a is the allowable stress. The normal stresses (σ_i) are obtained from technical beam theory and are given by

$$\sigma_i = \begin{cases} -\frac{1}{2} M_z Y_2 X_4 - \frac{1}{2} M_y Y_1 X_3 + F_x X_1 & ; \quad i = 1,2,9,10 \\ \frac{1}{2} M_z Y_2 X_4 - \frac{1}{2} M_y Y_1 X_3 + F_x X_1 & ; \quad i = 3,4,11,12 \\ \frac{1}{2} M_z Y_2 X_4 + \frac{1}{2} M_y Y_1 X_3 + F_x X_1 & ; \quad i = 5,6,13,14 \\ -\frac{1}{2} M_z Y_2 X_4 + \frac{1}{2} M_y Y_1 X_3 + F_x X_1 & ; \quad i = 7,8,15,16 \end{cases} \tag{C-6}$$

The shear stresses are calculated assuming that the box beam is thin walled (i.e. horizontal walls do not resist vertical shearing forces and vice-versa; shear stresses due to shearing forces are uniformly distributed over the appropriate wall areas; shear stresses due to the twisting moment is uniformly distributed over the cross section) and are given by

$$
\tau_i = \begin{cases} \dfrac{M_x}{2Y_1Y_2Y_4} + \dfrac{F_z}{2Y_1Y_4} & ; \quad i = 1,8,9,16 \\[2ex] -\dfrac{M_x}{2Y_1Y_2Y_3} + \dfrac{F_y}{2Y_2Y_3} & ; \quad i = 2,3,10,11 \\[2ex] -\dfrac{M_x}{2Y_1Y_2Y_4} + \dfrac{F_z}{2Y_1Y_4} & ; \quad i = 4,5,12,13 \\[2ex] \dfrac{M_x}{2Y_1Y_2Y_3} + \dfrac{F_y}{2Y_2Y_3} & ; \quad i = 6,7,14,15 \end{cases} \tag{C-7}
$$

<u>Local Buckling Constraints</u>

To protect against local buckling of the box beam walls each side of the member is conservatively modelled as a simply supported infinitely long plate, subject to combined axial, bending and shear stresses as shown in Fig. C4. Defining the buckling stress ratios

$$
\begin{aligned} R_s &= \tau/\tau_{cr} \\ R_b &= \sigma_b/\sigma_{bcr} \\ R_x &= \sigma_x/\sigma_{xcr} \end{aligned} \tag{C-8}
$$

where, for an infinitely long plate of width b and thickness t (Ref. 55)

$$\tau_{cr} = 5.35\ S$$

$$\sigma_{bcr} = 23.8\ S$$

$$\sigma_{xcr} = 4.0\ S \qquad \text{(C-9)}$$

$$S = \frac{E\,\pi^2}{12(1-\nu^2)}\,(t/b^{2}) = C_e\,(t/b)^2$$

and combining Eqs. (C-8) in one interaction formula leads to the following local buckling constraint equation

$$g = R_x + R_b^2 + R_s^2 - 1 \leq 0 \qquad \text{(C-10)}$$

Eq. (C-10) is evaluated at four points on each end of the element for a total of eight constraints. The expressions for σ_x, σ_b, τ and S for each of these constraints is given below:

$$\sigma_{x_j} = \begin{cases} -\tfrac{1}{2}\, M_z Y_2 X_4 - F_x X_1 & ; \quad j = 1,5 \\ \tfrac{1}{2}\, M_y Y_1 X_3 - F_x X_1 & ; \quad j = 2,6 \\ \tfrac{1}{2}\, M_z Y_2 X_4 - F_x X_1 & ; \quad j = 3,7 \\ -\tfrac{1}{2}\, M_y Y_1 Y_3 - F_x X_1 & ; \quad j = 4,8 \end{cases} \qquad \text{(C-11)}$$

where $\sigma_{x_j} > 0$ is a compressive stress,

$$\sigma_{b_j} = \begin{cases} \frac{1}{2}\, M_y Y_1 Y_3 & ; \quad j = 1,5 \\ \frac{1}{2}\, M_z Y_2 X_4 & ; \quad j = 2,6 \\ \frac{1}{2}\, M_y Y_1 X_3 & ; \quad j = 3,7 \\ \frac{1}{2}\, M_z Y_2 Y_4 & ; \quad j = 4,8 \end{cases} \tag{C-12}$$

$$\tau_j = \begin{cases} \dfrac{M_x}{2Y_1Y_2Y_4} + \dfrac{F_z}{2Y_1Y_4} & ; \quad j = 1,5 \\[2ex] \dfrac{-M_x}{2Y_1Y_2Y_3} + \dfrac{F_y}{2Y_2Y_3} & ; \quad j = 2,6 \\[2ex] \dfrac{-M_x}{2Y_1Y_2Y_4} + \dfrac{F_z}{2Y_1Y_4} & ; \quad j = 3,7 \\[2ex] \dfrac{M_x}{2Y_1Y_2Y_3} + \dfrac{F_y}{2Y_2Y_3} & ; \quad j = 4,8 \end{cases} \tag{C-13}$$

$$S_j = \begin{cases} C_e(Y_4/Y_1)^2 & ; \quad j = 1,3,5,7 \\ C_e(Y_3/Y_2)^2 & ; \quad j = 2,4,6,8 \end{cases} \tag{C-14}$$

Element: I-BEAM

Type: 12

Description: This element is a frame element with the cross section described by the sizing variables B_1, t_1, B_2, t_2, H, and t_3 as shown in Fig. C5. It is intended primarily for use in planar frame structures. Three stress and three local buckling constraints are computed at each end of the element for a total of six stress and six local buckling constraints. The locations on the cross section at which these constraints are evaluated are shown in Fig. C5 for the first node (n_1) end of the element.

Cross Sectional Dimensions

$$
\begin{aligned}
Y_1 &= B_1 \\
Y_2 &= t_1 \\
Y_3 &= B_2 \\
Y_4 &= t_2 \\
Y_5 &= H \\
Y_6 &= t_3
\end{aligned}
\qquad \text{(C-15)}
$$

Reciprocal Section Properties

$$X_1 = \frac{1}{A} = \frac{1}{Y_1Y_2 + Y_3Y_4 + Y_5Y_6} = \frac{1}{A_1 + A_2 + A_3}$$

$$X_2 = \frac{1}{J} = \frac{3}{A_1Y_2^2 + A_2Y_4^2 + A_3Y_6^2}$$

$$X_3 = \frac{1}{I_y} = \frac{12}{A_1Y_1^2 + A_2Y_3^2 + A_3Y_6^2} \tag{C-16}$$

$$X_4 = \frac{1}{I_z} = \frac{1}{A_1Y_2^2 + A_2Y_4^2 + A_3\ Y_5^2 + 12B}$$

where

$$B = A_1(\bar{y} - \frac{Y_2}{2})^2 + A_2(\bar{y} - Y_2 - Y_5 - \frac{Y_4}{2})^2 + A_3(\bar{y} - Y_2 - \frac{Y_5}{2})^2$$

$$\bar{y} = \frac{A_1(Y_2/2) + A_2(Y_2 + Y_5 + Y_4/2) + A_3(Y_2 + Y_5/2)}{A} \tag{C-17}$$

Stress Constraints

Constraints on normal stress for the flanges and on shear stress for the web are applied at each end of the element. These constraints have the form

$$g_i = \begin{cases} \dfrac{|\sigma_i|}{\sigma_a} - 1 \leq 0 \quad ; \quad i = 1,2,4,5 \\ \\ \dfrac{|\tau_i|}{\tau_a} - 1 \leq 0 \quad ; \quad i = 3,6 \end{cases} \tag{C-18}$$

where σ_a and τ_a are the normal and shear stress allowables. The expressions for σ_i and τ_i are obtained from technical beam theory and are given by

$$\sigma_i = \begin{cases} -M_z\ (Y_2 + Y_4 + Y_5 - \bar{y})X_4 + F_x\ X_1 & ; \quad i = 1,4 \\ \\ M_z\ \bar{y}\ X_4 & ; \quad i = 2,5 \end{cases} \tag{C-19}$$

and

$$\tau_i = F_y X_4 \left[\frac{\bar{y}^2}{2} + \frac{Y_1 Y_2 \bar{y}}{Y_6} - \frac{Y_1 Y_2^2}{2Y_6} - Y_2 \bar{y} + \frac{Y_2^2}{2} \right] \quad ; \quad i = 3,6 \tag{C-20}$$

It should be noted that since I-sections are generally not designed to carry twisting moments, bending moments about the vertical axis or horizontal shear forces, the stress computations do not include these effects. Therefore this element should not be used for structures in which these effects are expected to be significant.

<u>Local Buckling Constraints</u>

To protect against local buckling of the element flanges and web local buckling stress constraints are evaluated for both the flanges and the web at each end of the element. The flanges are assumed to experience only normal compressive stresses and each half of the flange is conservatively modelled as an infinitely long plate, simply supported at the ends and along one side, and free along the remaining side. The web is assumed to be subject only to shear stresses and is conservatively modelled as an infinitely long simply supported plate. The buckling constraints have the form

$$
g_i = \begin{cases} \dfrac{\sigma_i}{\sigma_{cr_i}} - 1 \leq 0 & ; \quad i = 1,2,4,5 \\[2ex] \dfrac{|\tau_i|}{\tau_{cr_i}} - 1 \leq 0 & ; \quad i = 3,6 \end{cases} \tag{C-21}
$$

where σ_{cr_i} and τ_{cr_i} are given by (Ref. 56)

$$
\sigma_{cr_i} = \begin{cases} -\dfrac{0.4\pi^2 E}{12(1- \ ^2)} \left[\dfrac{2Y_4}{Y_3}\right]^2 & ; \quad i = 1,4 \\[3ex] -\dfrac{0.4\pi^2 E}{12(1- \ ^2)} \left[\dfrac{2Y_2}{Y_1}\right]^2 & ; \quad i = 2,5 \end{cases}
$$

$$
\tau_{cr_i} = \frac{5.5\ \pi^2\ E}{12(1- \ ^2)} \left[\frac{Y_6}{Y_5}\right]^2 \quad ; \quad i = 3,6
$$

where E is the material modulus of elasticity and is the Poissons ratio.

Element: SQUARE BEAM

Type: 13

Description: This element is a frame element with the cross section described by the sizing variable B as shown in Fig. C6. Four normal stress and four shear stress constraints are calculated at each end of the element for a total of sixteen stress constraints. The locations on the cross section at which these contraints are evaluated are shown in Fig. C6 for the first node (n_1) end of the element.

Cross Sectional Dimensions

$$Y_1 = B \tag{C-22}$$

Reciprocal Section Properties

$$X_1 = \frac{1}{A} = \frac{1}{Y_1^2}$$

$$X_2 = \frac{1}{J} = \frac{1}{.1406\ Y_1^4}$$

$$X_3 = \frac{1}{I_y} = \frac{12}{Y_1^4} \tag{C-23}$$

$$X_4 = \frac{1}{I_z} = \frac{12}{Y_1^4}$$

Stress Constraints

The normal and shear stress constraints for this element have the form

$$g_i = \begin{cases} \dfrac{|\sigma_i|}{\sigma_a} - 1 \leq 0 & ; \quad i = 1\text{-}4, 9\text{-}13 \\ \\ \dfrac{|\tau_i|}{\tau_a} - 1 \leq 0 & ; \quad i = 5\text{-}8, 14\text{-}16 \end{cases} \tag{C-24}$$

where σ_a and τ_a are the normal and shear stress allowables. The expressions for σ_i are obtained from technical beam theory and are given by

$$\sigma_i = \begin{cases} -\tfrac{1}{2} M_y Y_1 X_3 - \tfrac{1}{2} M_z Y_1 X_4 + F_x X_1 & ; \quad i = 1,9 \\ -\tfrac{1}{2} M_y Y_1 X_3 + \tfrac{1}{2} M_z Y_1 X_4 + F_x X_1 & ; \quad i = 2,10 \\ \\ \tfrac{1}{2} M_y Y_1 X_3 + \tfrac{1}{2} M_z Y_1 X_4 + F_x X_1 & ; \quad i = 3,11 \\ \tfrac{1}{2} M_y Y_1 X_3 - \tfrac{1}{2} M_z Y_1 X_4 + F_x X_1 & ; \quad i = 4,12 \end{cases} \tag{C-25}$$

The shear stress τ_i is obtained by the superposition of the parabolic horizontal shear stress distribution from technical beam theory and a torsional shear stress distribution for a solid square bar (Ref. 57). The expressions for τ_i are given by

$$\tau_i = \begin{cases} - c\, M_x Y_1 X_2 + 1/8\, F_y Y_1^2 X_4 & ; \quad i = 5,13 \\ - c\, M_x Y_1 X_2 + 1/8\, F_z Y_1^2 X_3 & ; \quad i = 6,14 \\ \\ c\, M_x Y_1 X_2 + 1/8\, F_y Y_1^2 X_4 & ; \quad i = 7,15 \\ c\, M_x Y_1 X_2 + 1/8\, F_z Y_1^2 X_3 & ; \quad i = 8,16 \end{cases} \qquad \text{(C-26)}$$

where c = .676

Element: THIN TUBE

Type: 14

Description: This element is a frame element with the cross section described by the sizing variables R and t as shown in Fig. C7. Two stress and two local column buckling constraints are evaluated at each end of the element for a total of four stress and four local column buckling constraints. These constraints are evaluated at the points of maximum normal stress defined by the angle θ, as shown in Fig. C7, where θ is given by

$$\theta = \arctan\,(M_z/M_y) \tag{C-27}$$

It should be noted that θ is assumed to be constant during the solution to each approximate design problem and is updated for each approximate problem. In addition, one local wall buckling constraint is evaluated for this element.

Cross Sectional Dimensions

$$Y_1 = R$$

$$Y_2 = t \tag{C-28}$$

Reciprocal Section Properties

$$X_1 = \frac{1}{A} = \frac{1}{2\pi Y_1 Y_2}$$

$$X_2 = \frac{1}{J} = \frac{2}{\pi(R_o^4 - R_i^4)} \tag{C-29}$$

$$X_3 = \frac{1}{I_y} = \frac{4}{\pi(R_o^4 - R_i^4)}$$

$$X_4 = \frac{1}{I_z} = \frac{4}{\pi(R_o^4 - R_i^4)}$$

where

$$R_o = Y_1 + Y_2/2$$

$$R_i = Y_1 - Y_2/2 \tag{C-30}$$

<u>Stress</u> <u>Constraints</u>

The stress constraints for this element are based on the Von Mises criterion and have the form

$$g_i = \frac{1}{\sigma_a^2}(\sigma_i^2 + 3\tau_i^2) - 1 \leq 0 \quad ; \quad i = 1,2,3,4 \tag{C-31}$$

where σ_a is the allowable stress. The normal stresses σ_i are obtained from technical beam theory and have the form

$$\sigma_i = \begin{cases} M_y Y_1 X_3 \cos\theta + M_z Y_1 X_4 \sin\theta + F_x X_1 & ; \quad i = 1,3 \\ \\ -M_y Y_1 X_3 \cos\theta - M_z Y_1 X_4 \sin\theta + F_x X_1 & ; \quad i = 2,4 \end{cases} \tag{C-32}$$

The shear stresses τ_i are obtained from the superposition of a parabolic horizontal shear stress distribution from technical beam theory and a uniform shear stress distribution due to the twisting moment from thin walled tube theory. The expressions for τ_i are given by

$$\tau_i = \begin{cases} F_y Y_1^2 X_3 \cos\theta - F_z Y_1^2 X_4 \sin\theta + M_x Y_1 X_2 & ; \quad i = 1,3 \\ -F_y Y_1^2 X_3 \cos\theta + F_z Y_1^2 X_4 \sin\theta + M_x Y_1 X_2 i & ; \quad i = 2,4 \end{cases} \tag{C-33}$$

Local Wall Buckling Constraints

To protect against local wall buckling of the thin tube element the following constraint between the wall thickness and mean radius is used (Ref. 58).

$$g = Y_2/Y_2^L - 1 \leq 0 \tag{C-34}$$

with

$$Y_2^L = \left[\frac{\sigma_y}{1.65\text{x}10^6 \text{ psi}}\right] Y_1 \tag{C-35}$$

where σ_y is the material yield stress in tension.

Local Column Buckling Constraints

To protect against column buckling of individual elements subject to axial compressive forces the following buckling stress constraint is used

$$g = \frac{\sigma}{\sigma_{cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 - 1 \leq 0 \tag{C-36}$$

This interaction formula (Eq. (C-36)) is conservative (Ref. 59) when applied to elastically supported columns under combined loading. Equation (C-34) is evaluated at two points on each end of the element as shown in Fig. C7. The values of τ_{cr} and σ_{cr} are given by

$$\sigma_{cr} = \begin{cases} 4.9283 \dfrac{E}{FS} \left(\dfrac{Y_1}{kL}\right)^2 & ; \quad \lambda > \sqrt{2} \\[2em] \left[1 - .0506 \left(\dfrac{kL}{R}\right)^2 \left(\dfrac{\sigma_y}{E}\right)\right] \dfrac{\sigma_y}{FS} & ; \quad \lambda \le \sqrt{2} \end{cases} \tag{C-37}$$

and

$$\tau_{cr}^2 = c \, \frac{E^2 Y_2^3}{Y_1^3 W^2} \tag{C-38}$$

where

$$FS = \begin{cases} 1.92 & ; \quad \lambda > \sqrt{2} \\ 1.67 + .265\,\lambda - .044\,\lambda^3 & ; \quad \lambda \le \sqrt{2} \end{cases}$$

$$\lambda = .450 \left(\frac{Lk}{Y_1}\right) \sqrt{\sigma_y / E}$$

$$W = \min \left[1, \ \frac{.561 (L/2Y_1)^{.5}}{(2Y_1/Y_2)^{.25}} \right]$$

σ_y = material yield strength in tension

E = material modulus of elasticity

L = column length

k = effective length factor

$$c = \begin{cases} .0983 & ; \quad M_x = 0 \\ .0629 & ; \quad M_x \ne 0 \end{cases}$$

The effective length factor k is dependent on the element end restraint stiffnesses. Currently the value of k must be supplied as input data and is assumed to be constant during the design process. It should be noted that λ and W are assumed to be constant during the solution of each approximate problem but they are updated for each approximate problem. Additional details on the column buckling constraint formulation are available in Ref. 21.

Element: BOX BEAM

Type: 15

Description: This element is a frame element with the cross section described by the sizing variables B, H and t as shown in Fig. C8. Eight stress and four local buckling constraints are evaluated at each end of the element for a total of sixteen stress and eight local buckling constraints. The locations on the cross section at which these constraints are evaluated are shown in Fig. C8 for the first node (n_1) end of the element.

Cross Sectional Dimensions

$$Y_1 = B$$

$$Y_2 = H \tag{C-40}$$

$$Y_3 = t$$

Reciprocal Section Properties

$$X_1 = \frac{1}{A} = \frac{1}{Y_1Y_2-(Y_1-2Y_3)(Y_2-2Y_3)}$$

$$X_2 = \frac{1}{J} = \frac{Y_3(Y_1+Y_2)}{2(Y_1Y_2Y_3)^2} \tag{C-41}$$

$$X_3 = \frac{1}{I_y} = \frac{12}{Y_2Y_1^3-(Y_2-2Y_3)(Y_1-2Y_3)^3}$$

$$X_4 = \frac{1}{I_z} = \frac{12}{Y_1Y_2^3-(Y_1-2Y_3)(Y_2-2Y_3)^3}$$

Stress Constraints

The stress constraints for this element are of the same form as those for element type 11 and they are given by Eqs. (C-5) - (C-7) with Y_4 set equal to Y_3.

Local Buckling Constraints

The local buckling constraints for this element are of the same form as those for element type 11 and they are given by Eqs. (C-8) - (C-14) with Y_4 set equal to Y_3.

APPENDIX D

Derivatives of Structural Response Quantities with Respect to Element Reciprocal Section Properties

D.1 Displacement Derivatives

For the linear static structural analysis problem, the displacement derivatives are easily obtained through the implicit differentiation of the governing equilibrium equations with respect to the element reciprocal section properties (RSP's). In general, differentiation of Eq. (3-1) with respect to the j-th RSP of the i-th element (x_{ij}) yields

$$\frac{\partial [K]}{\partial x_{ij}} \{u\}_k + [K] \frac{\partial \{u\}_k}{\partial x_{ij}} = \frac{\partial \{P\}_k}{\partial x_{ij}} \quad ; \quad k = 1,2,\ldots K \tag{D-1}$$

Under the assumption that the external loading is independent of the element RSP's (i.e. $\frac{\partial \{P\}_k}{\partial x_{ij}} = 0$) Eq. (D-1) becomes

$$[K] \frac{\partial \{u\}_k}{\partial x_{ij}} = \bar{V}_{ijk} \quad ; \quad k = 1,2,\ldots K \tag{D-2}$$

where the pseudo load vector $\bar{V}_{ijk}$ is given by

$$\bar{V}_{ijk} = - \frac{\partial [K]}{\partial x_{ij}} \{u\}_k \tag{D-3}$$

Writing the system stiffness matrix [K] as

$$[K] = \sum_{i=1}^{I} [\beta_i]^T \left([T_i]^T [K_i]^e [T_i]\right) [\beta_i] \tag{D-4}$$

where $[K_i]^e$ is the element stiffness matrix in local coordinates, $[T_i]$

is the element coordinate transformation matrix, $[\beta_i]$ is the element local to global degree of freedom transformation matrix and I is the total number of structural elements; substitution into Eq. (D-3) gives

$$\bar{V}_{ijk} = -[\beta_i]^T[T_i]^T \frac{\partial[K_i]^e}{\partial x_{ij}} [T_i][\beta_i] \{u\}_k \tag{D-5}$$

Finally, Eq. (D-5) may be rewritten as

$$\bar{V}_{ijk} = 1/x_{ij}^2([\beta_i]^T[T_i]^T[\tilde{K}_{ij}]^e[T_i][\beta_i] \{u\}_k) \tag{D-6}$$

where it is recognized that

$$[\tilde{K}_{ij}]^e = \frac{\partial[K_i]^e}{\partial(1/x_{ij})} \tag{D-7}$$

is the unit element stiffness matrix formed by assigning the j-th section property a value of unity while the remaining section properties are set to zero.

Using the expression for the pseudo load vector given by Eq. (D-6), Eq. (D-2) can be solved for the unknown displacement derivatives $\frac{\partial\{u\}_k}{\partial x_{ij}}$; k = 1,2,...K via the same procedure used to solve the equilibrium equations (Eq. (3-1)). Solving Eq. (D-2) directly yields the derivative values for all of the displacement degrees of freedom. For the case where the number of displacement degrees of freedom associated with the retained constraint set is fewer than the number of pseudo load vectors associated with Eq. (D-2) it is computationally more efficient to solve Eq. (D-2) using a partial inverse technique represented by the equation

$$\frac{\partial \{u\}_k}{\partial x_{ij}} = [C]\bar{V}_{ijk} \tag{D-8}$$

where $\{u\}_k$ represents the displacement degrees of freedom associated with the retained constraint set. The partial inverse matrix [C] is constructed such that its n-th row contains the vector $\{c\}_n^T$ obtained from the solution of the equation

$$[K]\{c\}_n = \{e\}_n \tag{D-9}$$

where $\{e\}_n$ is a unit vector corresponding to the n-th degree of freedom associated with the retained constraint set. It should be noted that the solution of either Eq. (D-2) or Eq. (D-9) requires only the back substitution of the vectors $\bar{V}_{ijk}$ or $\{e\}_n$ if the decomposed stiffness matrix has been saved from the previous structural analysis.

D.2 Element Force Derivatives

The element force derivatives are obtained through the implicit differentiation of the element force-displacement relations. Rewriting Eq. (3-8) in terms of the global coordinate system gives

$$\{F_r\}_k^g = [K_r]^g \ \{u_r\}_k + \{FEF_r\}_k^g \tag{D-10}$$

where $\{F_r\}_k^g$, $[K_r]^g$, $\{u_r\}_k$ and $\{FEF_r\}_k^g$ are the element force vector, stiffness matrix, vector of nodal displacements and fixed end force vector for the r-th element and k-th load set. Differentiation of Eq. (D-10) with respect to x_{ij} gives

$$
\frac{\partial\{F_r\}_k^g}{\partial x_{ij}} = \begin{cases} \dfrac{\partial[K_r]^g}{\partial x_{ij}}\{u_r\}_k + [K_r]^g \dfrac{\partial\{u_r\}_k}{\partial x_{ij}} + \dfrac{\partial\{FEF_r\}_k^g}{\partial x_{ij}} & ; \quad i = r \\ \\ [K_r]^g \dfrac{\partial\{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \end{cases} \tag{D-11}
$$

where the displacement derivatives $\frac{\partial\{u_r\}_k}{\partial x_{ij}}$ are calculated as shown previously. Under the assumption that the external loading is independent of the element RSP'S

$$
\frac{\partial\{FEF_i\}_k^g}{\partial x_{ij}} = 0 \tag{D-12}
$$

and Eq. (D-11) becomes

$$
\frac{\partial\{F_r\}_k^g}{\partial x_{ij}} = \begin{cases} \dfrac{\partial[K_r]^g}{\partial x_{ij}}\{u_r\}_k + [K_r]^g \dfrac{\partial\{u_r\}_k}{\partial x_{ij}} & ; \quad i = r \\ \\ [K_r] \dfrac{\partial\{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \end{cases} \tag{D-13}
$$

Rewriting $[K_r]^g$ as

$$
[K_r]^g = [T_r]^T[K_r]^e[T_r] \tag{D-14}
$$

and substituting into Eq. (D-13) yields

$$\frac{\partial \{F_r\}_k^g}{\partial x_{ij}} = \begin{cases} [T_r]^T \dfrac{\partial [K_r]^e}{\partial x_{ij}} [T_r] \{u_r\}_k + [T_r]^T [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i = r \\ \\ [T_r]^T [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \end{cases} \tag{D-15}$$

Introducing the unit element stiffness matrix $[\widetilde{K}_{rj}]^e$, Eq. (D-15) becomes

$$\frac{\partial \{F_r\}_k^g}{\partial x_{ij}} = \begin{cases} -\dfrac{1}{x_{ij}^2} \left([T_r]^T [\widetilde{K}_{rj}]^e [T_r]\right) \{u_r\}_k + [T_r]^T [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \\ \\ [T_r]^T [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \end{cases} \tag{D-16}$$

Finally, writing the element force derivatives in the local coordinate system gives

$$\frac{\partial \{F_r\}_k^e}{\partial x_{ij}} = [T_r] \frac{\partial \{F_r\}_k^g}{\partial x_{ij}} \tag{D-17}$$

or

$$\frac{\partial \{F_r\}_k^e}{\partial x_{ij}} = \begin{cases} -\dfrac{1}{x_{ij}^2} [\widetilde{K}_{rj}]^e [T_r] \{u_r\}_k + [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i = r \\ \\ [K_r]^e [T_r] \dfrac{\partial \{u_r\}_k}{\partial x_{ij}} & ; \quad i \neq r \end{cases} \tag{D-18}$$

since, by orthogonality of $[T_r]$,

$$[T_r][T_r]^T = [T_r][T_r]^{-1} = [I] \tag{D-19}$$

APPENDIX E

Data Command Descriptions

E.1 Analysis Data Commands

The synthesis problem structural analysis model may be defined using the analysis data commands listed below.

'ANALYSIS DATA'

'BEAM ELEMENTS'

'BOUNDARY CONDITIONS'

'COORDINATES'

'LOAD CONDITIONS'

'MATERIALS'

'NODAL LOADS'

'TRANSFORMATIONS'

'TRUSS ELEMENTS'

'UNIFORM BEAM LOADING'

These commands are described in this section in alphabetical order.

Command: 'ANALYSIS DATA'

Description: This command denotes the beginning of the analysis input data block and must precede the first occurrence of any other analysis data command.

Command: '<u>BEAM</u> ELEMENTS'

<u>Description</u>: This command allows the user to define the beam element data for the structural analysis model.

<u>Data</u>: $n_1\ m_1\ n1_1\ n2_1\ [A\ J\ IYY\ IZZ\ IYZ]_1 \quad c_1 \begin{Bmatrix} n3_1 \\ \alpha_1 \\ (x,y,z)_1 \end{Bmatrix}$

$\vdots$

$n_k\ m_k\ n1_k\ n2_k\ [A\ J\ IYY\ IZZ\ IYZ]_k \quad c_k \begin{Bmatrix} n3_k \\ \alpha_k \\ (x,y,z)_k \end{Bmatrix}$

<u>Entry</u>	<u>Definition</u>	<u>Note</u>	<u>Variable</u>
n_i	Beam element number		IDBEM(I,1)
m_i	Material specification number	(1)	IBEAM(I,1)
$n1_i$	First end node number	(2)	IBEAM(I,2)
$n2_i$	Second end node number	(2)	IBEAM(I,3)
A_i	Cross sectional area	(3)	PBEAM(I,1)
J_i	Torsional constant	(3)	PBEAM(I,2)
IYY_i	Moment of inertia about local y axis	(3)	PBEAM(I,3)
IZZ_i	Moment of inertia about local z axis	(3)	PBEAM(I,4)
IYZ_i	Product of inertia	(3)	PBEAM(I,5)
c_i	Beam orientation specification code	(4)	PBEAM(I,6)
$n3_i$	Beam orientation node number	(4)	PBEAM(I,7)
α_i	Beam orientation angle	(4)	PBEAM(I,7)

x_i			PBEAM(I,7)
y_i	Components of beam orientation vector	(4)	PBEAM(I,8)
z_i			PBEAM(I,9)
k	Total number of beam elements		NBE

Notes:

1. The corresponding material must be defined by the 'MATERIAL' command.

2. The end nodes must be defined by the 'COORDINATES' command.

3. The beam section properties need not be given if the beam is to be associated with a design element. For the case where the beam is associated with a design element, any section properties supplied by the user will be replaced by section properties calculated from the corresponding design element information.

4. The orientation of the beam element local y axis may be defined by supplying a node or vector which lies in the beam's local x-y plane (Fig. E1) or an angle α (Fig. E2) which is a measure of the angle between the local x-y plane and the $x_{local} - y_{global}$ plane (counter-clockwise looking down the local x-axis). For the special case where the local x-axis and global y-axis are coincident α is the measure of the angle between the local x-y plane and the $x_{local} - z_{global}$ plane (Fig. E3). The means by which the orientation information is supplied is determined from c_i as follows:

$$c_i = 1 \ - \ \text{node point } n3_i$$

$c_i = 2$ – angle α_i

$c_i = 3$ – vector $(x,y,z)_i$

Command: 'BOUNDARY CONDITIONS'

Description: The boundary condition command allows the user to specify the nodal degrees of freedom which are to be considered restrained for the purposes of analysis.

Data: ['SET' k_1]

n_{1_1} SPC_{1_1}

.
.
.

n_{m_1} SPC_{m_1}

.

.

.

['SET' k_L]

n_{1_L} SPC_{1_L}

.
.
.

n_{m_L} SPC_{m_L}

Entry	Definition	Note	Variable
k_j	Boundary condition set number	(1)	IDBCS(J,1)
n_{i_j}	Node number	(2)	ISPCS(I,1,J)
SPC_{i_j}	Degree of freedom code	(3)	ISPCS(I,2,J)
m_j	Number of restrained nodes for boundary condition set k_j	(4)	NCN
L	Number of boundary condition sets		NBS

Notes:

1. The absence of the '<u>SET</u>' descriptor and its associated boundary condition number will cause the following boundary condition specifications to be applied to all boundary condition sets.

2. The use of the '<u>ALL</u>' descriptor <u>in place of</u> the node number will cause the associated boundary condition specification to be applied to all nodes.

3. Any combination of the values 1-6 may be used to specify restraints on the nodal degrees of freedom. The values 1,2,3 and 4,5,6 correspond to the x,y,z translations and rotations, respectively. A negative sign preceeding any <u>individual</u> value will cause a <u>previously set</u> restraint to be removed.

4. The maximum number of restrained nodes for all boundary condition sets is stored in the variable NCN (i.e. NCN = $\max_{j=1,L} m_j$)

Command: 'COORDINATES'

Description: This command allows the user to specify the problem node point data.

Data: $n_1\ x_1\ y_1\ z_1\quad [k_1]$

.

.

.

$n_m\ x_m\ y_m\ z_m\quad [k_m]$

Entry	Definition	Note	Variable
n_i	Node number		IDGRD(I,1)
x_i	X-coordinate		PGRID(I,1)
y_i	Y-coordinate		PGRID(I,2)
z_i	Z-coordinate		PGRID(I,3)
k_i	User coordinate system number	(1)	IGRID(I)
m	Number of nodes		NND

Notes:

1. The user coordinate system number k_i refers to a user supplied coordinate transformation given by the 'TRANSFORMATION' command. All quantities associated with the node except its location are described in the user coordinate system. If the transformation specification includes an origin specification then the node location is also described in the user coordinate system.

Command: 'LOAD CONDITIONS'

Description: The load condition command allows the user to specify the load sets and their corresponding analysis types and boundary condition numbers.

Data: n_1 'type'$_1$ [bc_1]

.

.

.

n_m 'type'$_m$ [bc_m]

Entry	Definition	Note	Variable
n_i	Load set number		IDLOD(I,2)
'type'$_i$	Analysis type	(1)	LOADT(I,1)
bc_i	Boundary condition number	(2)	LOADT(I,2)
m	Number of load sets		NLS

Notes:

1. The descriptor 'type' defines the type of analysis to be performed for the associated load set. Currently only linear static analysis may be performed and therefore 'STATICS' is the only valid descriptor.

2. The absence of the boundary condition number bc_i will cause the load set to be associated with boundary condition number 1.

Command: 'MATERIALS'

Description: This command allows the user to specify material properties to be used for analysis and design purposes.

Data: $m_1\ \rho_1\ E_1\ \nu_1\ G_1\ \bar{\sigma}_1\ \bar{\tau}_1\ [C_1]$

.

.

.

$m_n\ \rho_n\ E_n\ \nu_n\ G_n\ \bar{\sigma}_n\ \bar{\tau}_n\ [C_n]$

Entry	Definition	Note	Variable
m_i	Material number		IDMAT(I,1)
ρ_i	Mass density		PMATE(I,1)
E_i	Modulus of elasticity		PMATE(I,2)
ν_i	Poissons ratio		PMATE(I,3)
G_i	Shear modulus	(1)	PMATE(I,4)
$\bar{\sigma}_i$	Allowable stress in tension and compression		PMATE(I,8)
$\bar{\tau}_i$	Allowable stress in shear		PMATE(I,9)
C_i	Additional material constant	(2)	PMATE(I,10)
n	Total number of materials		NMT

Notes:

1. If a value of 0. is supplied for G the actual value of G will be calculated from

$$G = \frac{E}{2(1+\nu)}$$

2. The additional material constant C_i may be used to supply any additional data required for the element strength constraint calculations (e.g. effective member length for buckling constraint evaluation, factor of safety).

Command: 'NODAL LOADS'

Description: This command allows the user to describe discrete loads to be applied to specific nodal degrees of freedom.

Data: ['SET' k_1]

n_1 DOF_1 P_1

.
.
.

n_{m1} DOF_{m1} P_{m1}

.
.
.

['SET' k_L]

n_{mj+1} DOF_{mj+1} P_{mj+1}

.
.
.

n_{mL} DOF_{mL} P_{mL}

Entry	Definition	Note	Variable
k_i	Load set number	(1)	NLOAD(J,1)
n_j	Node number		NLOAD(J,2)
DOF_j	Degree of freedom specification	(2)	NLOAD(J,3)
P_j	Magnitude of load		FLOAD(J)
L	Number of load sets		NLS
mL	Number of applied nodal loads		NAF

Notes:

1. The absence of the 'SET' descriptor and its associated load set number will cause the following loading data to be associated with load set number 1. The load set number must correspond to a load set defined by the 'LOAD CONDITIONS' command.

2. The degree of freedom specification may be any value between 1 and 6. The values 1, 2, 3 and 4, 5, 6 correspond to the x,y,z translations and rotations respectively.

Command: 'TRANSFORMATIONS'*

Description: This command may be used to to create user defined coordinate systems to facilitate data input and/or to impose boundary conditions and displacement constraints in directions other than the global coordinate directions.

Data: n_1 $'axis1'_1$ $x1_1$ $y1_1$ $z1_1$ $'axis2'_1$ $x2_1$ $y2_1$ $z2_1$ [$'0'$ xo_1 yo_1 zo_1]

.

.

.

n_m $'axis1'_m$ $x1_m$ $y1_m$ $z1_m$ $'axis2'_m$ $x2_m$ $y2_m$ $z2_m$ [$'0'$ xo_m yo_m zo_m]

Entry	Definition	Note	Variable
n_i	Transformation number		IDTRN(I,1)
$'axis1'_i$	First user axis designation	(1)	
$x1_i$			CTRAN(1,I)
$y1_i$	Components of user 'axis1'	(2)	CTRAN(2,I)
$z1_i$			CTRAN(3,I)
$'axis2'_1$	Second user axis designation	(1)	
$x2_i$			CTRAN(4,I)
$y2_i$	Components of user 'axis2'	(2)	CTRAN(5,I)
$z2_i$			CTRAN(6,I)
'0'	Indicates that user coordinate system origin specification follows	(3)	

xo_i	Location of user coordinate system origin	(3)	CTRAN(10,I)
yo_i			CTRAN(11,I)
zo_i			CTRAN(12,I)

Notes:

1. The user axis designations 'axis1' and 'axis2' may be given as 'X', 'Y', or 'Z' and need not be given in cyclic permutation order.

2. The components of the user coordinate axes must be given in terms of the global reference system. The axes 'axis1' and 'axis2' need not be perpendicular. The third user axis ('axis3') is calculated from the cross product of 'axis1' and 'axis2' and then 'axis2' is recomputed from the cross product of 'axis1' and 'axis3'.

3. If the origin specification 'O' is given then the user coordinate system origin (xo,yo,zo) must be given in terms of the global reference system. In this case the node point locations supplied via the 'COORDINATES' command are assumed to be measured with respect to the user coordinate system origin.

* The capabilities available through the use of this command are still under development at this time and have not been fully tested.

Command: 'TRUSS ELEMENTS'

Description: This command allows the user to define the truss element data for the structural analysis model.

Data: n_1 m_1 $n1_1$ $n2_1$ $[A]_1$

.

.

.

n_k m_k $n1_k$ $n2_k$ $[A]_k$

Entry	Definition	Note	Variable
n_i	Truss element number		IDTRS(I,1)
m_i	Material specification number	(1)	ITRUS(I,1)
$n1_i$	First end node number	(2)	ITRUS(I,2)
$n2_i$	Second end node number	(2)	ITRUS(I,3)
A_i	Cross sectional area	(3)	PTRUS(I,1)
k	Total number of truss elements		NTE

Notes:

1. The corresponding material must be defined by the 'MATERIAL' command.

2. The end nodes must be defined by the 'COORDINATES' command.

3. The truss cross sectional area need not be given if the truss is to be associated with a design element. For the case where the

truss is associated with a design element any cross sectional area supplied by the user will be replaced by the cross sectional area calculated from the corresponding design element information.

Command: 'UNIFORM BEAM LOADING'

Description: This command allows the user to specify uniformly distributed loading for any beam element in the structural model. This loading is applied in the form of work equivalent nodal loading.

Data: ['SET' k_1]

n_1 DIR_1 P_1

.
.
.

n_{m1} DIR_{m1} P_{m1}

.

.

.

['SET' k_L]

n_{mj+1} DIR_{mj+1} P_{mj+1}

.
.
.

n_{mL} DIR_{mL} P_{mL}

Entry	Definition	Note	Variable
k_i	Load set number	(1)	IULOD(J,1)
n_j	Beam element number		IULOD(J,2)
DIR_j	Direction specification	(2)	IULOD(J,3)
P_j	Magnitude of loading in load per unit length		ULOAD(J)
L	Number of load sets		NLS
mL	Number of applied uniform beam loads		NULB

Notes:

1. The absence of the '<u>SET</u>' descriptor and its associated load set number will cause the following loading data to be associated with load set number 1. The load set number must correspond to a load set defined by the '<u>LOAD</u> CONDITIONS' command.

2. The direction specification defines the direction of the applied loading in the beam element local coordinate system. The specification may be any value between 1 and 6. The values 1,2,3 and 4,5,6 correspond to the directions of local x,y,z translations and rotations, respectively.

E.2 Design Data Commands

The synthesis problem design model may be defined using the design data commands listed below.

'DESIGN DATA'

'DESIGN ELEMENTS'

'DISPLACEMENT CONSTRAINTS'

'ELEMENT GEOMETRY'

'LOCAL BUCKLING CONSTRAINTS'

'STRESS CONSTRAINTS'

These commands are described in this section in alphabetical order.

Command: 'DESIGN DATA'

Description: This command denotes the beginning of design input data block and must precede the first occurrence of any other design data command.

Command: 'DESIGN ELEMENTS'

Description: This command allows the user to specify the structural elements which are to be considered as design elements during the design process.

Data: $m1_1$ n_1 [$m2_1$]

.

.

.

$m1_k$ n_k [$m2_k$]

Entry	Definition	Note	Variable
$m1_i$	Analysis element number		IDESG(I,1)
n_i	Geometry number	(1)	IDESG(I,2)
$m2_i$	Master element number	(2)	IDESG(I,4)
k	Number of design elements		NDE

Notes:

1. The geometry number n_i must correspond to a design element geometry number specified by the 'ELEMENT GEOMETRY' command.

2. The master element number $m2_i$ allows the user to link several analysis elements together for design purposes. The master element number must appear in the design element list and must not itself refer to another master element. The absence of a user

specified master element causes $m2_i$ to be set equal to $m1_i$.

Command: 'DISPLACEMENT CONSTRAINTS'

Description: This command allows the user to specify allowable upper and lower bounds on nodal translations and rotations.

Data: m_1 n_1 j_1 c_1 LB_1 UB_1

.

.

.

m_k n_k j_k c_k LB_k UB_k

Entry	Definition	Note	Variable
m_i	Load set number		IDISC(I,1)
n_i	Node number		IDISC(I,2)
j_i	Degree of freedom number	(1)	IDISC(I,3)
c_i	Constraint shift value	(2)	RDISC(I,1)
LB_i	Lower bound value	(2)	RDISC(I,2)
UB_i	Upper bound value	(2)	RDISC(I,3)
k	Number of displacement constraints		NDC

Notes:

1. The degree of freedom specification may be any value between 1 and 6. The values 1, 2, 3, and 4, 5, 6 correspond to the x, y, z translations and rotations, respectively.

2. The displacement constraint is written for strictly negative lower

bound and strictly positive upper bound values. For the case where the actual displacement bounds are of the same sign the constraint shift value c_i may be used to shift the constraint value such that the upper and lower bounds will have the correct signs. The form of the displacement constraint is given by

$$(u-c)/u_a \leq 1$$

where u is the displacement value and u_a is the shifted displacement allowable (i.e. LB_i or UB_i).

Command: 'ELEMENT GEOMETRY'

Description: This command allows the user to specify data pertaining to the design element sizing variables.

Data: m_1 n_1

Y_1 YL_1 YU_1

.
.
.

Y_{k1} YL_{k1} YU_{k1}

.
.
.

m_L n_L

Y_{ki+1} YL_{ki+1} YU_{ki+1}

.
.
.

Y_{kL} YL_{kL} YU_{kL}

Entry	Definition	Note	Variable
m_i	Design element geometry number		IDGOM(I,1)
n_i	Design element type	(1)	IGEOM(I,1)
Y_j	Initial value of design element sizing variable		DGEOM(J,1)
YL_j	Lower bound value of design element sizing variable		DGEOM(J,2)
YU_j	Upper bound value of design element sizing variable		DGEOM(J,3)
L	Number of design element geometries		NEG

kL	Total number of design element sizing variables	NCD

<u>Notes</u>:

1. The design element type must correspond to an element type available in the design element library.

Command: 'LOCAL BUCKLING CONSTRAINTS'

Description: This command allows the user to request the calculation of local buckling constraint values for the specified load sets. Local buckling constraint values are calculated for all elements whose corresponding element type has an associated local buckling computation as described in the element library.

Data: k_1

.

.

.

k_n

Entry	Definition	Note	Variable
k_i	Load set number		IBLOD(I)
n	Number of load sets for which buckling constraints are to be calculated		NBL

Command: 'STRESS CONSTRAINTS'

Description: This command allows the user to request the calculation of design element stress constraint values for the specified load sets. Stress constraint values are calculated for all elements whose corresponding element type has an associated stress computation as described in the element library.

Data: k_1

$\cdot$

$\cdot$

$\cdot$

k_n

Entry	Definition	Note	Variable
k_i	Load set number		ISLOD(I)
n	Number of load sets for which stress constraints are to be calculated		NSL

E.3 Control Data Commands

The solution of the structural synthesis problem may be controlled using the commands listed below.

'ANALYSIS'

'CHECKPOINT'

'CONMIN'

'CONTROL DATA'

'CSD '

'DUAL'

'FORCE VARIANCE'

'ITERATIONS'

'MIXED'

'MOVE LIMITS'

'OPTIMIZATION'

'PRINT'

'RESTART'

'SCALE'

'SENSITIVITIES'

'SETUP'

'UPDATE'

These commands are described in this section in alphabetical order.

Command: 'ANALYSIS'

Description: This program function control command causes the program to terminate after the structural analysis has been completed. In this case the program control variable IPCTL is set to 1.

Command: 'CHECKPOINT' [n]

Description: This command causes the program analysis and design data to be written on external file number n upon successful termination of a design run. The external file number is stored in the variable ICKFL. The absence of the file number specification will cause ICKFL to be set to 1.

Command: 'CONMIN' [c]

Description: This command allows the user to specify that the solution to the approximate design problem is to be performed using a primal mathematical programming formulation. The mass minimization problem is solved using the CONMIN optimization program. The parameter c controls the constraint push off factor. If c > 0. then the displacement, stress and local buckling constraints are treated as being nonlinear and c is used as the push off factor. If c = 0. then all constraints are considered to be linear. In the absence of a user specified value c is given a value of 1.0. Specification of the 'CONMIN' command causes the variable IOPTY to be set to 3. The value of c is stored in the variable OPTPRM(1).

Command: 'CONTROL DATA'

Description: This command denotes the beginning of the program control data block and must precede the first occurrence of any other control data command.

Command: 'CSD '

Description: This command allows the user to specify that the element cross sectional dimensions are to be used as the design variables during the solution of each approximate design problem. In the absence of this command either the element reciprocal section properties or a combination of element reciprocal section properties and cross sectional dimensions are chosen as the design variables. Specification of this command causes the variable ICSD to be set to 1.

Command: 'DUAL'

Description: This command allows the user to specify that the solution to the approximate design problem is to be performed using a dual mathematical programming formulation. The dual function maximization problem is solved using the CONMIN optimization program. This feature is currently operational only when used in conjunction with the 'CSD' command and will automatically specify that a mixed variable approximation is to be used for all behavior constraints. Design variable scaling is also activated automatically for this feature. Specification of this command causes the variables IOPTY, IMIX, and ISCALE to be set to 2, 1 and 1, respectively.

Command: 'FORCE VARIANCE' n

Description: This command allows the users to specify that the variation of the design element forces with respect to the problem design variables is to be included in the stress and local buckling constraint approximations during the approximate problem generation. If this command is not specified the element forces are assumed to be invariant during any design step. If n = 1 then the element force sensitivities are calculated only with respect to the design variables associated with that element. If n = 2 then the element force sensitivities are calculated with respect to all design variables. The value of n is stored in the variable IFVAR.

Command: 'ITERATIONS' n

Description: This command allows the user to specify the number of design steps which are to be performed. Each design step consists of the generation and solution of one approximate problem. The number of iterations n is stored in the variable NSTEP.

Command: 'MIXED APPROXIMATIONS'

Description: This command allows the user to specify that a mixed variable approximation will be used for all behavior constraints. Design variable scaling is activated automatically for this feature. Specification of this command causes the variables IMIX and ISCALE to be set to 1.

Command: 'MOVE LIMITS' d_1 d_2

Description: This command allows the user to specify the allowable changes in the problem variables during any single design step. The move limits d_1 and d_2 are applied to the element cross sectional dimensions and reciprocal section properties, respectively. The limiting values (Y^L, Y^U, X^L, X^U) are given by the following equations:

$$Y^L = Y - Yd_1$$

$$Y^U = Y + Yd_1$$

$$X^L = X - Xd_2$$

$$X^U = X + Xd_2$$

If zero is given for either move limit then the limiting values for the corresponding variables are determined from the overall limits on the element sizing variables supplied via the 'GEOMETRY command. The values of d_1 and d_2 are stored in the variables DMOVE(1) and DMOVE(2), respectively.

Command: 'OPTIMIZATION' [c]

Description: This program function control command causes all major program functions (data processing, analysis, approximate problem generation, optimization and design recovery) to be performed. In this case the program control variable IPCTL is set to 3. The value c controls the diminishing returns convergence criterion on structural mass. The design process is terminated if the relative change in structural mass is less than c for three consecutive design steps. The value of c is stored in the variable DELOBJ. In the absence of a user specified value for c DELOBJ is set to .01.

Command: 'PRINT' n_1 n_2 n_3 n_4 n_5 n_6 n_7 n_8 n_9 n_{10}

Description: This command allows the user to control the program printing options. If the value of n_i is set to 0 then the corresponding printing option is disabled. A value of n_i greater than 0 enables the printing options as described below. The value of n_i is stored in the variable IPS(I).

n_1 - Design data is printed at the beginning of each design step.

n_2 - Analysis data is printed at the beginning of each design step.

n_3 - Not used at this time.

n_4 - Structural analysis results are printed for each design step.

n_5 - Design results are printed for each design step.

n_6 - CPU timing summary is printed for each design step.

n_7 - Not used at this time.

n_8 - Not used at this time.

n_9 - All program data storage is printed at completion of specified program function.

n_{10}* - Controls optimizer print option.

* Currently n_{10} is used to control the CONMIN optimizer output. A value of $n_{10} = 1$ is suggested for normal program operation with higher values (up to $n_{10} = 6$) being used for debug operations.

Command: 'RESTART' [n]

Description: This command allows the user to begin the design process from the termination point of a previous design run by causing the previously written analysis and design data to be read from external file number n. The external file number is stored in the variable IRSFL. The absence of the external file number specification will cause IRSFL to be set to 1. It should be noted that at this time only control data modifications are allowable for restarted design runs.

Command: 'SCALE'

Description: This command will cause the design variables to be scaled to unity at the beginning of each approximate problem stage. Specification of this command causes the variable ISCALE to be set to 1. The default value of ISCALE is 0 and scaling is not performed.

Command: 'SENSITIVITIES'

Description: This program function control command causes the program to terminate after the approximate problem generation has been completed. In this case the program control variable IPCTL is set to 2.

Command: 'SETUP'

Description: This program function control command causes the program to terminate after the input data processing has been completed. In this case the program control variable IPCTL is set to the default value 0.

Command: 'UPDATE' [n]

Description: This command will cause the approximate design problem to be partially reconstructed and re-solved n times between each complete structural analysis and approximate problem generation. The value of n is stored in the variable NUPDAT. If n is not specified NUPDAT is set to 1. In the absence of the 'UPDATE' command NUPDAT is set to 0.

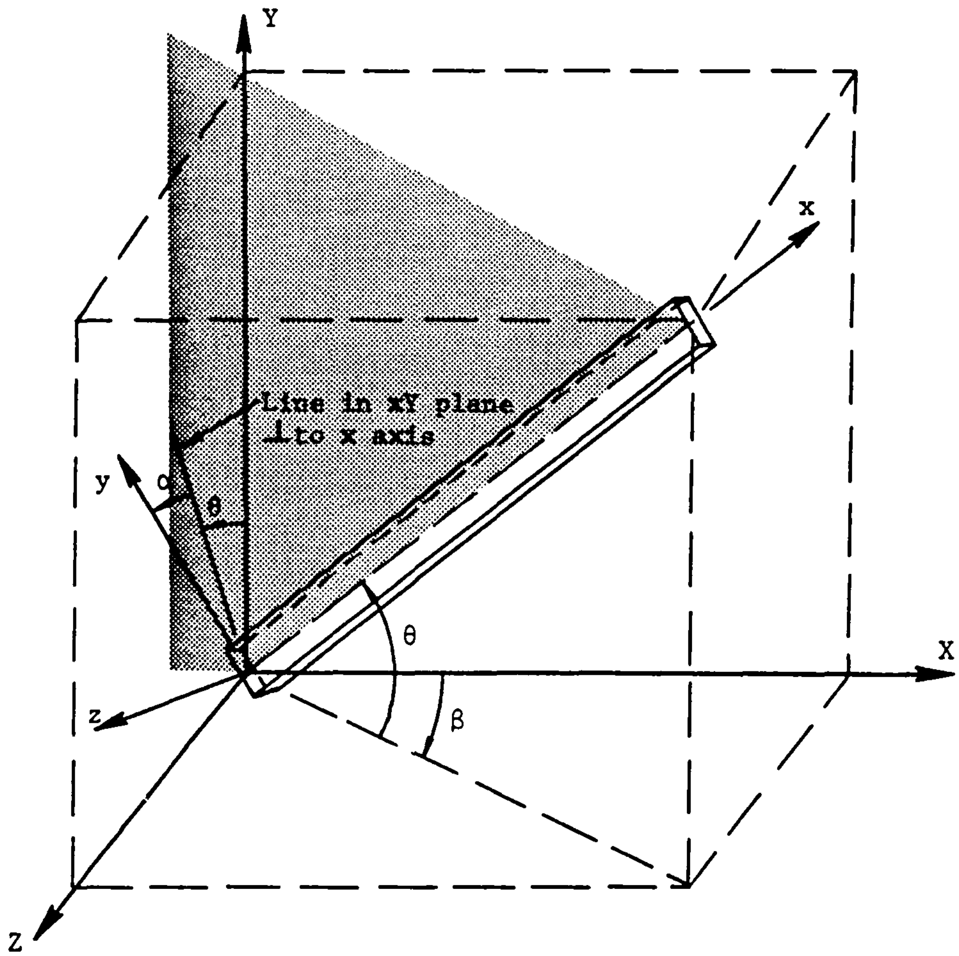


Fig. 1 - Structural Element Orientation

$$\left[\begin{array}{c:c} K_{FF} & K_{FB} \\ \hdashline K_{FB}^{T} & K_{BB} \end{array}\right] \begin{array}{l} \left.\vphantom{K_{FF}}\right\} \text{D.O.F.'s independent of changing boundary conditions} \\ \left.\vphantom{K_{BB}}\right\} \text{D.O.F.'s associated with changing boundary conditions} \end{array}$$

Fig. 2 - Conceptual Matrix Partitioning for Multiple Boundary Conditions

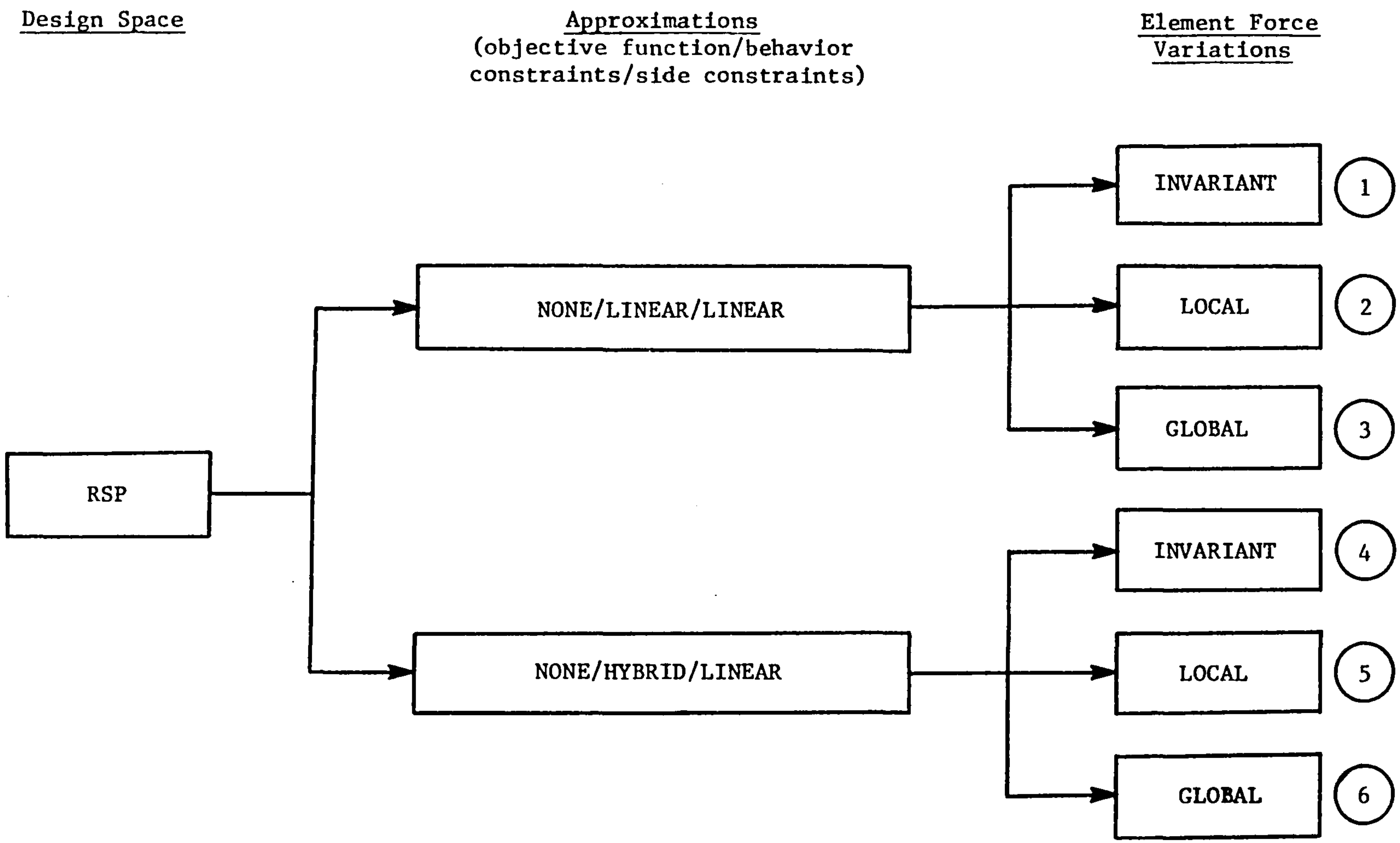


Fig. 3 - Approximate Problem Options in RSP Design Space

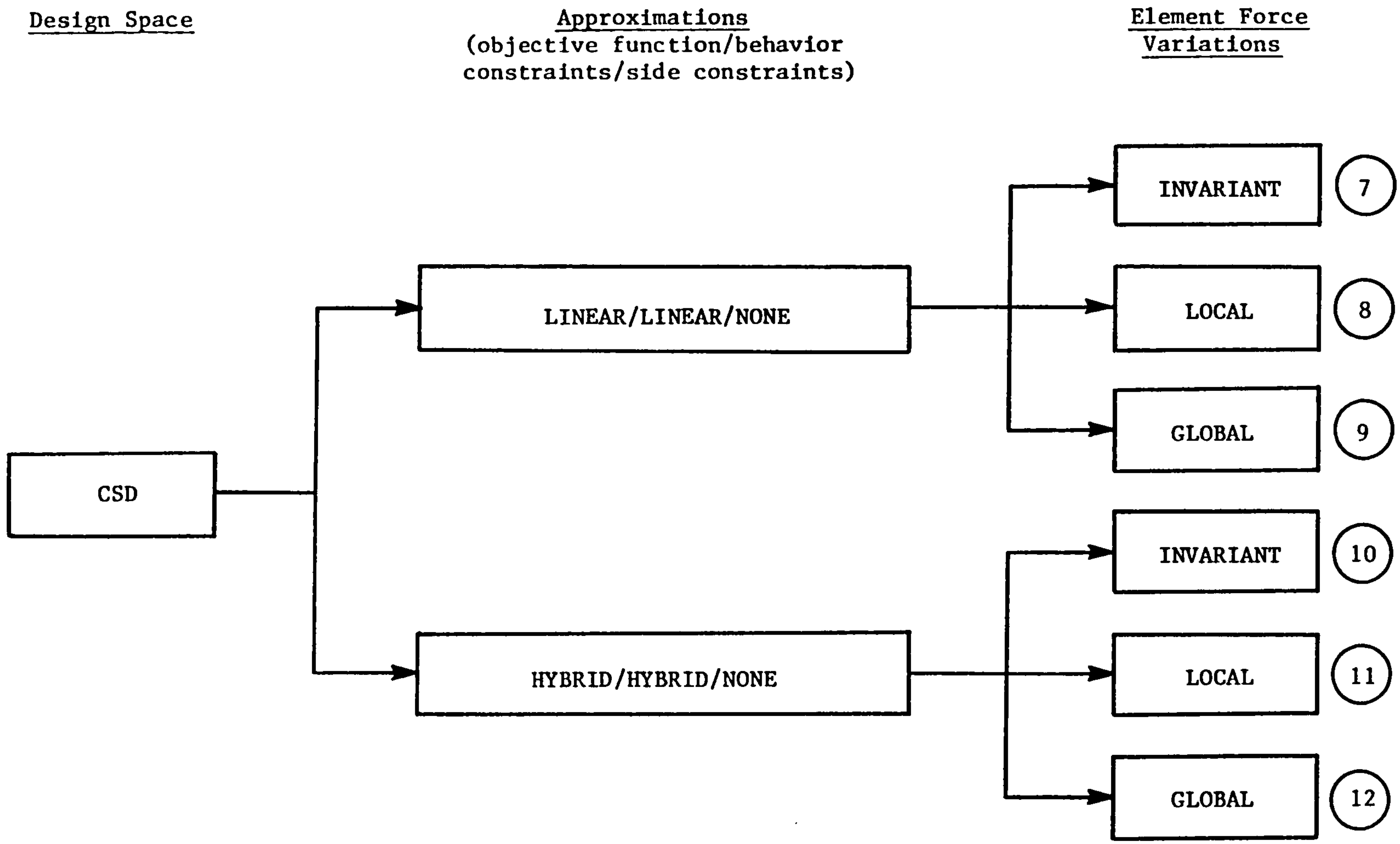


Fig. 4 - Approximate Problem Options in CSD Design Space

Optimization Method \ Design Space	CSD	RSP
PRIMAL	*	*
DUAL	*	--

Optimization Method \ Approximation Type	Linear	Mixed Variable (Hybrid)
PRIMAL	*	*
DUAL	--	*

*available combination

Fig. 5 - Available Optimization Options

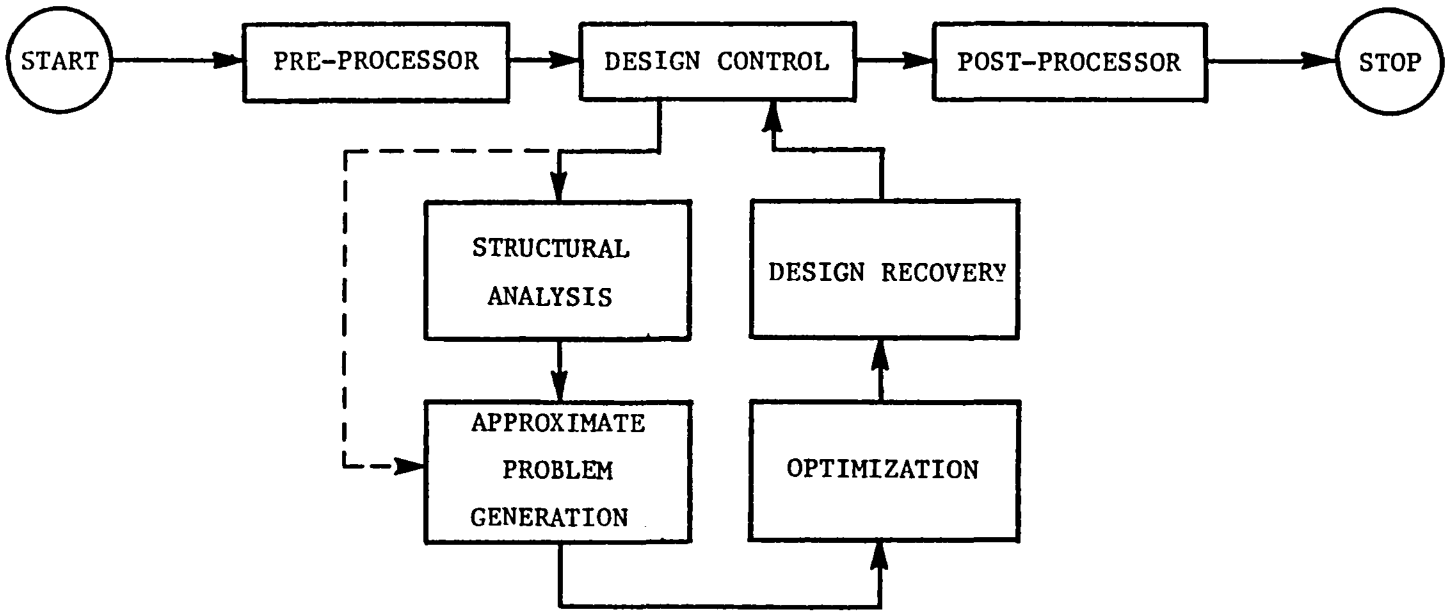


Fig. 6 - Program Organization

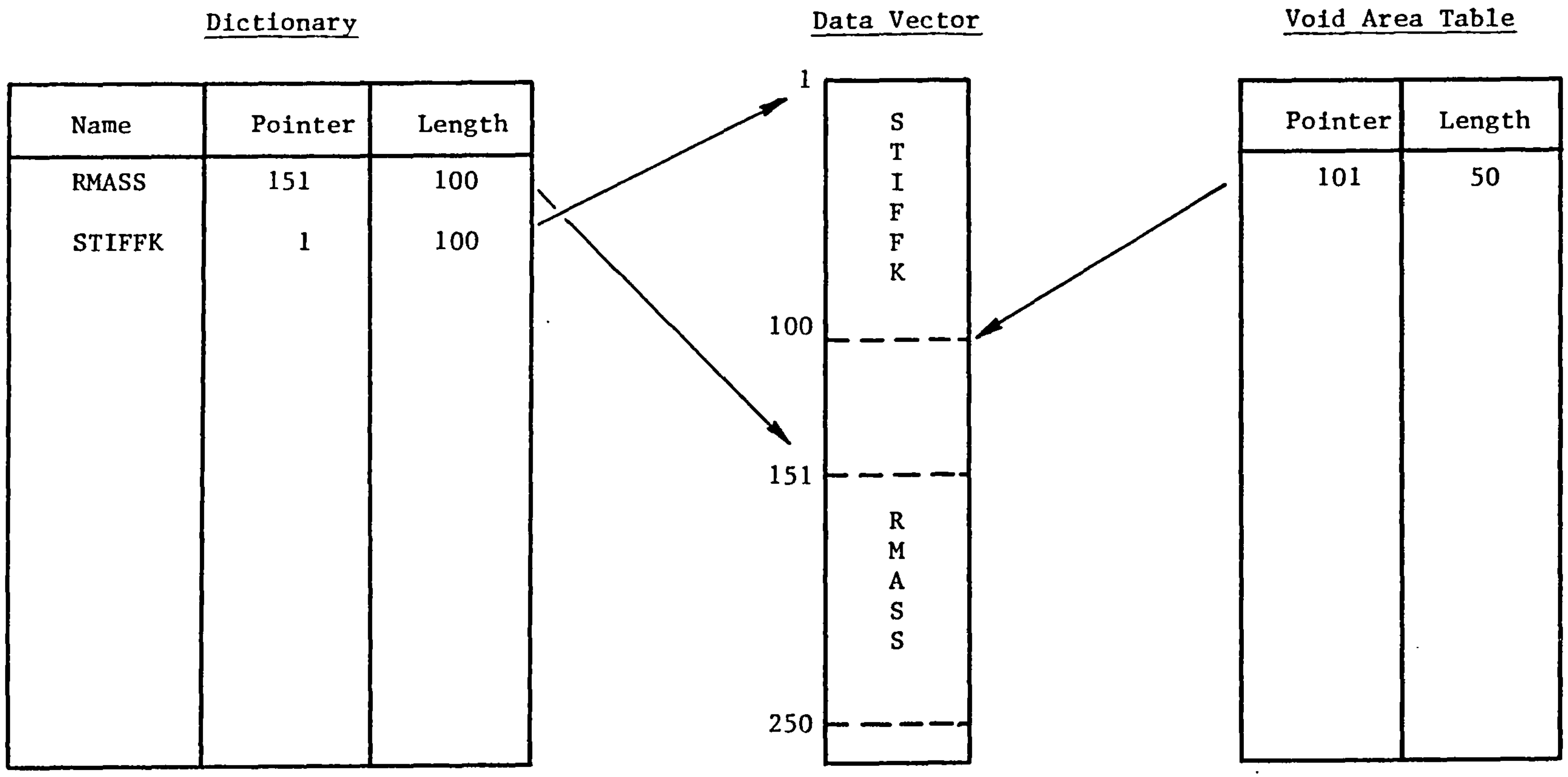


Fig. 7 - Storage Management Scheme

Command	Description
INITIALIZE	Initialize data management dictionary and void area table.
INSERT	Insert new variable in the data vector.
LOCATE	Locate a variable in the data vector.
DELETE	Delete a variable from the data vector.
EXPAND	Increase the storage available for a variable.
CONTRACT	Decrease the storage avaiable for a variable.
COMPRESS	Remove voids from data vector.
CHANGE NAME	Change the name of a variable in storage.
DEBUG	Print data vector, dictionary and void area table.
QUERY	Return maximum storage used.
CHECKPOINT	Write data vector, dictionary and void area table on external file.
RESTART	Read data vector, dictionary and void area table from external file.

Fig. 8. Storage Management Commands

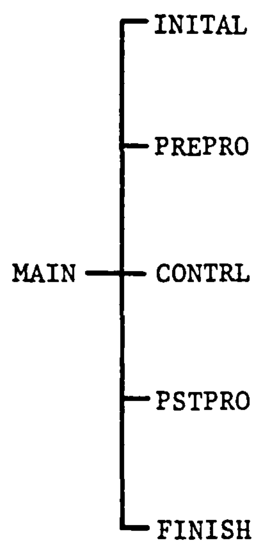


Fig. 9 - Main Routine Flow Diagram

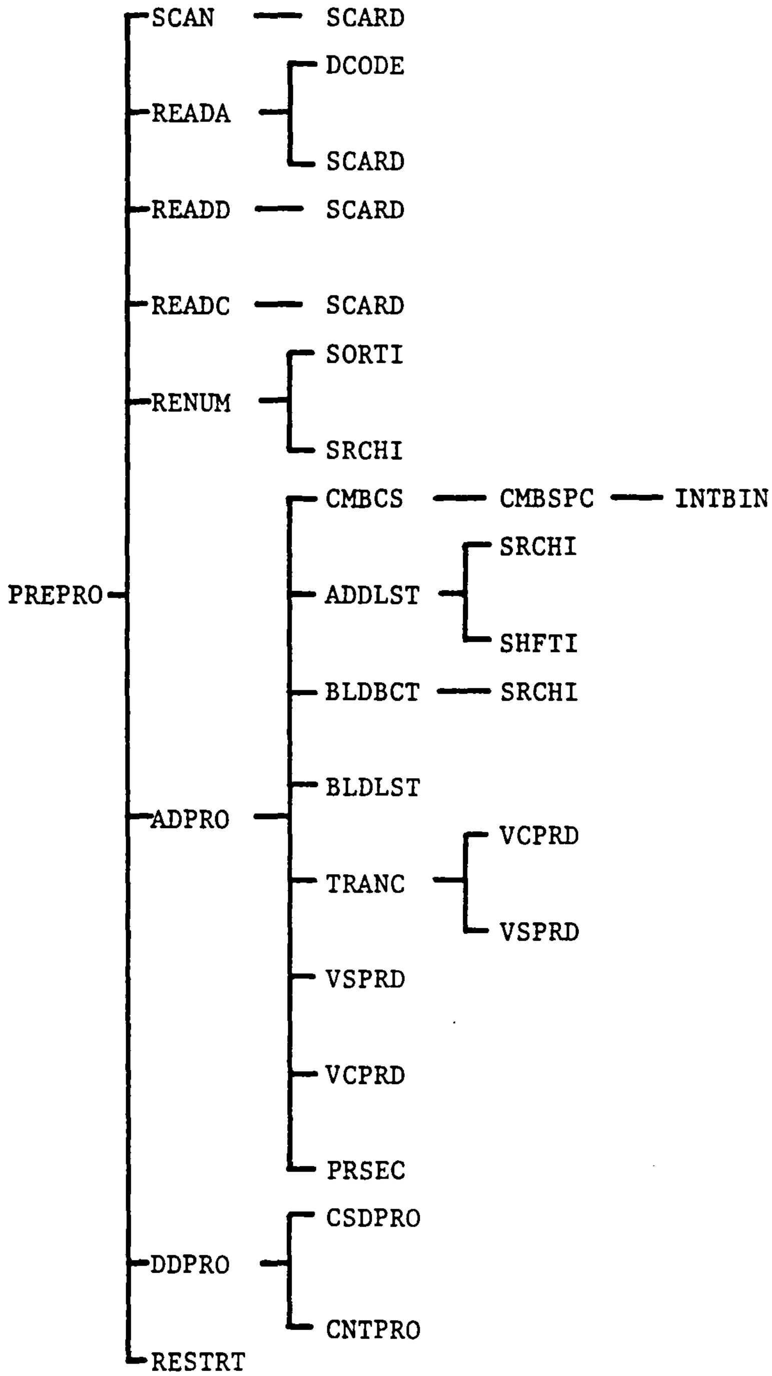


Fig. 10 - Pre-processor Flow Diagram

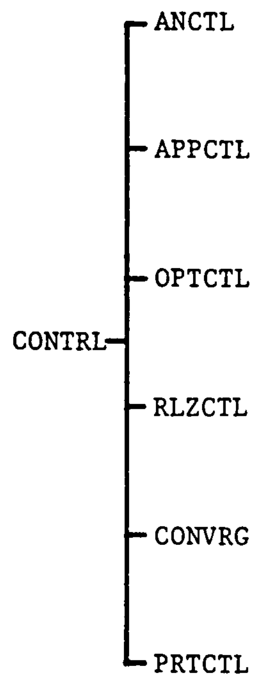


Fig. 11 - Design Control Flow Diagram

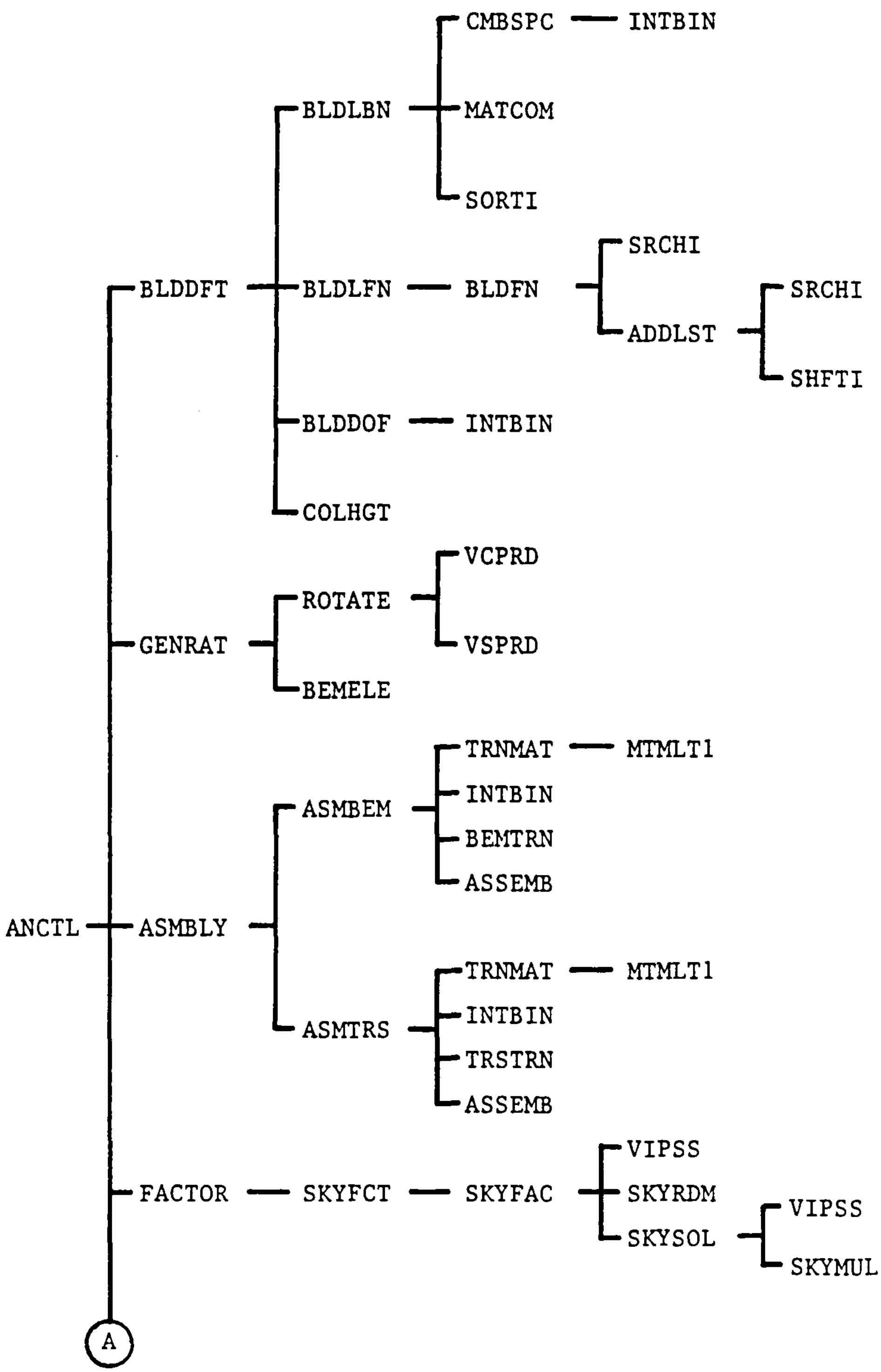


Fig. 12 - Analysis Control Flow Diagram

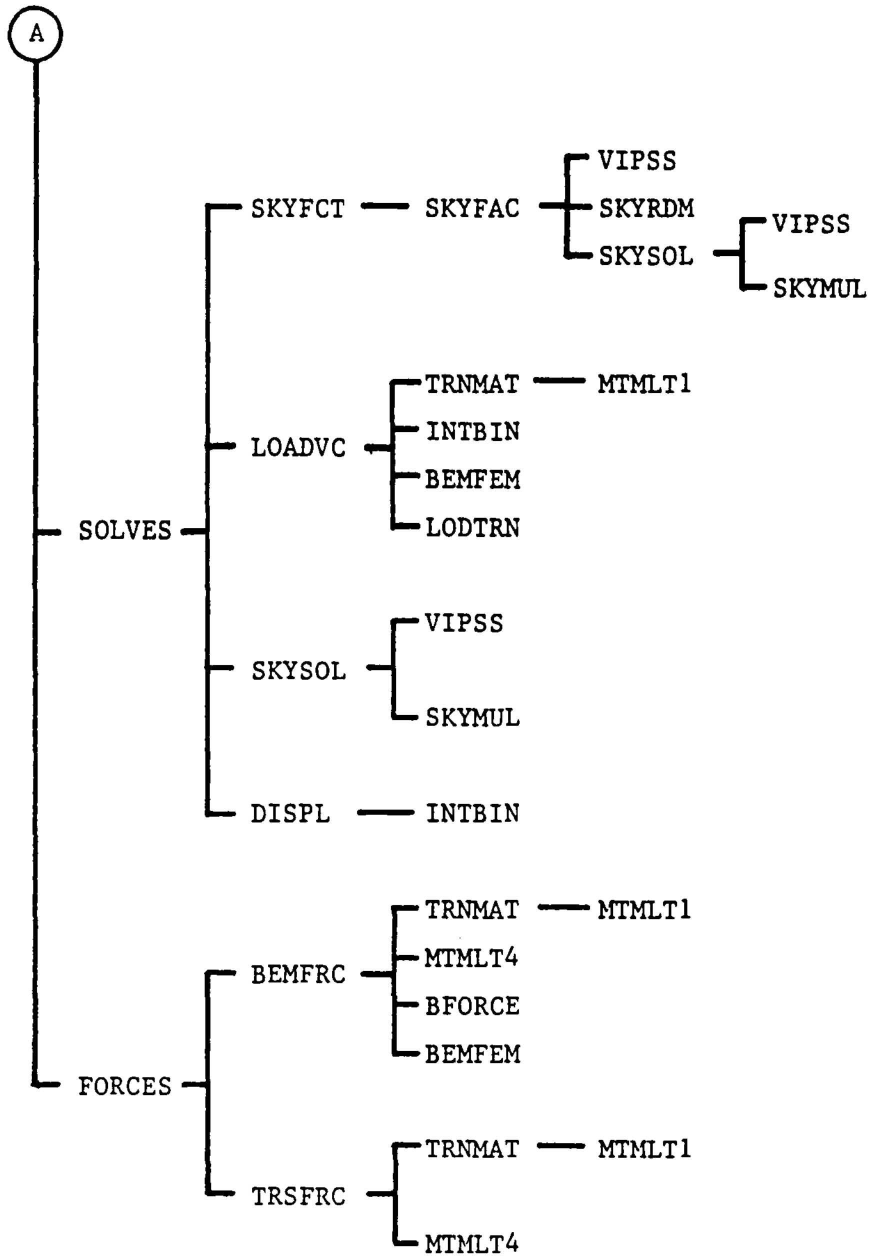


Fig. 12 - Analysis Control Flow Diagram (cont.)

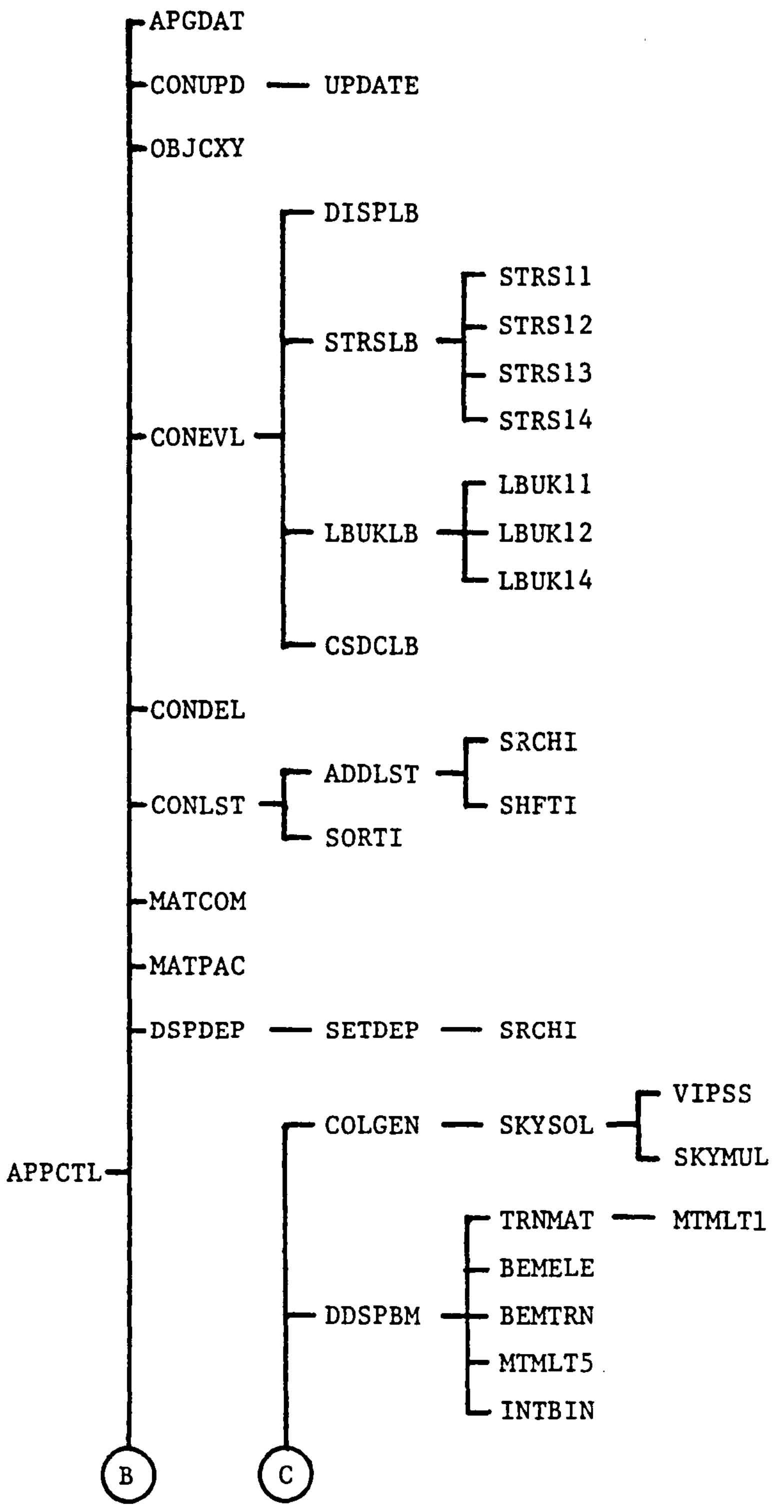


Fig. 13 - Approximate Problem Generation Control Flow Diagram

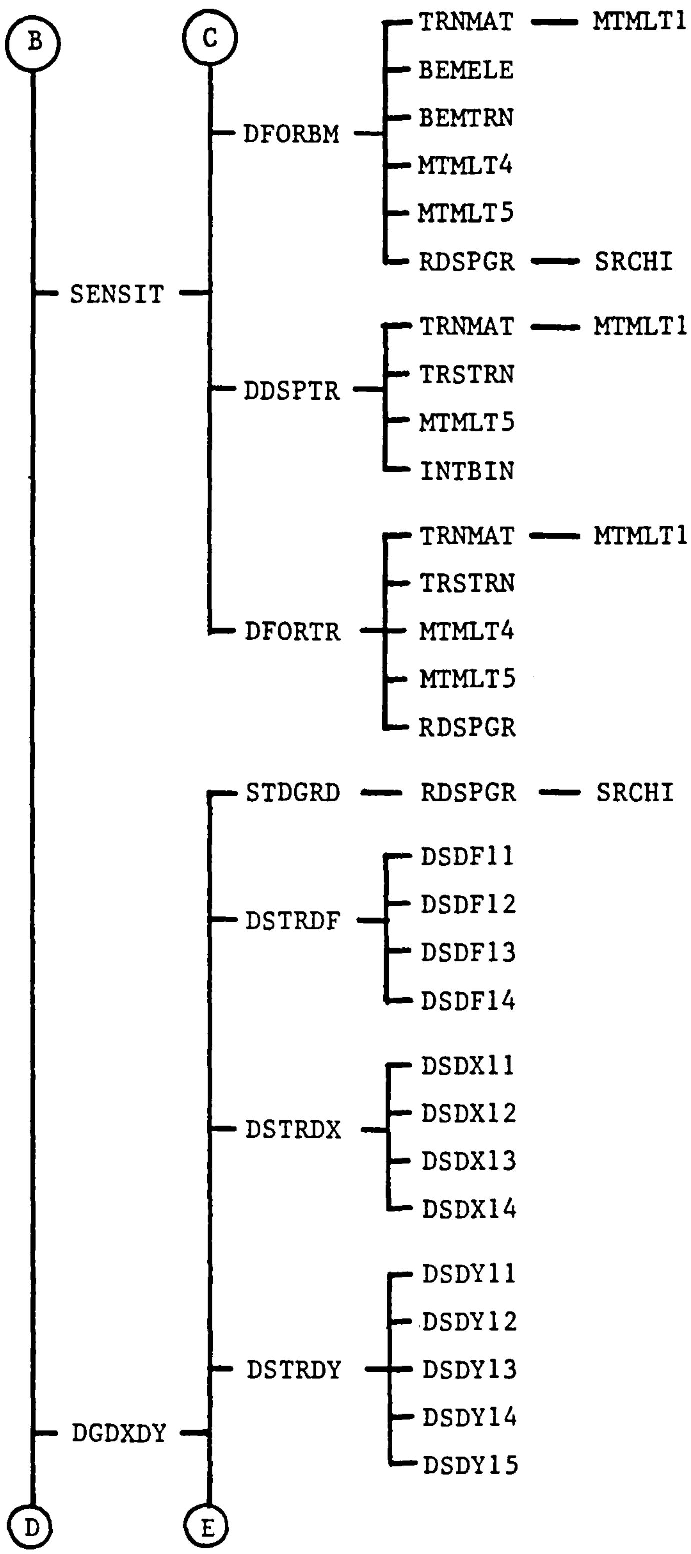


Fig. 13 - Approximate Problem Generation Control Flow Diagram (cont.)

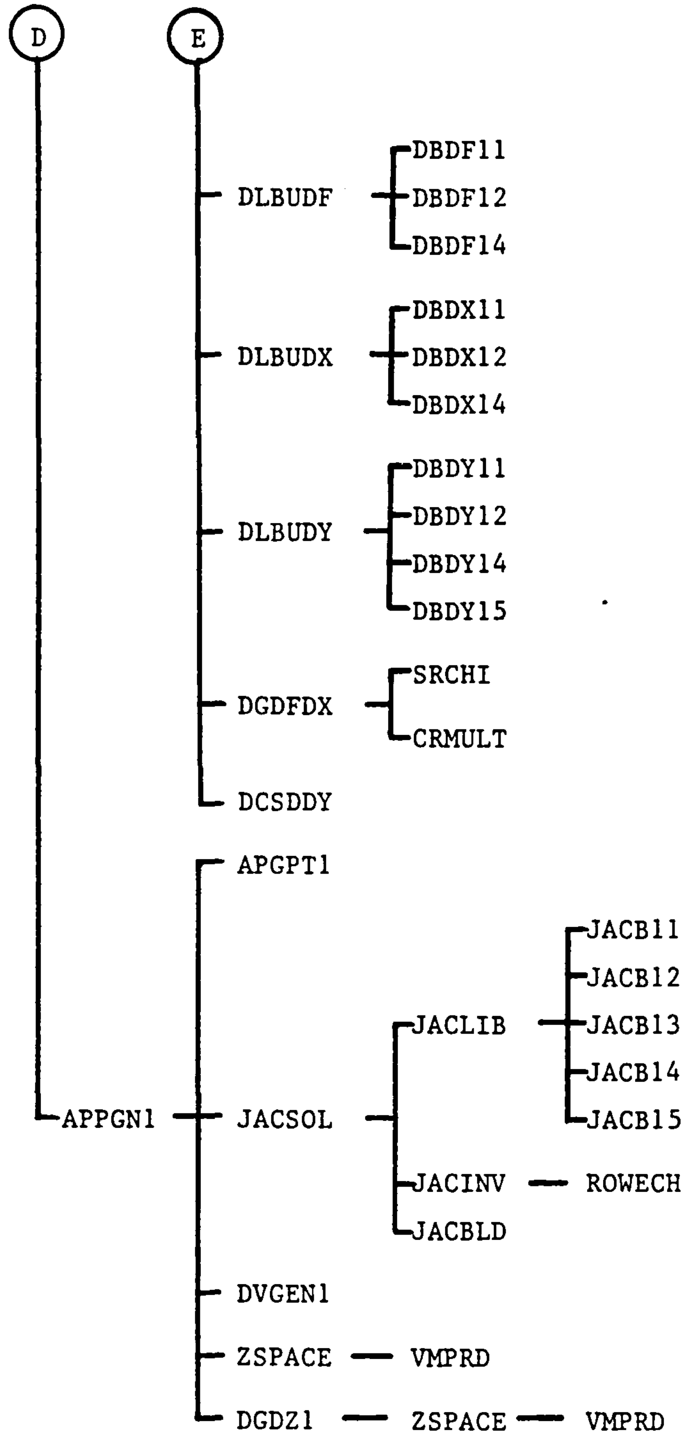


Fig. 13 - Approximate Problem Generation Control Flow Diagram (cont.)

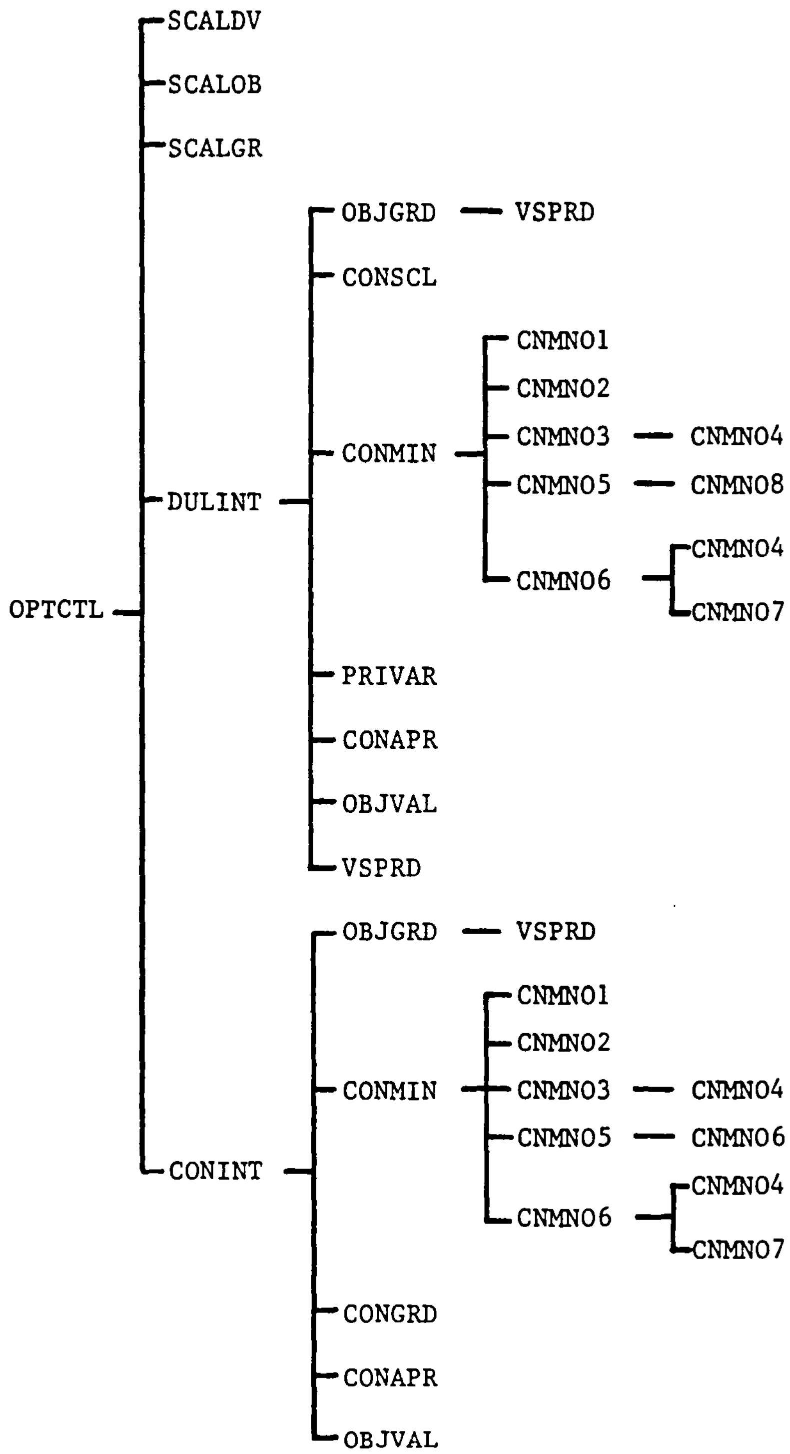


Fig. 14 - Optimization Control Flow Diagram

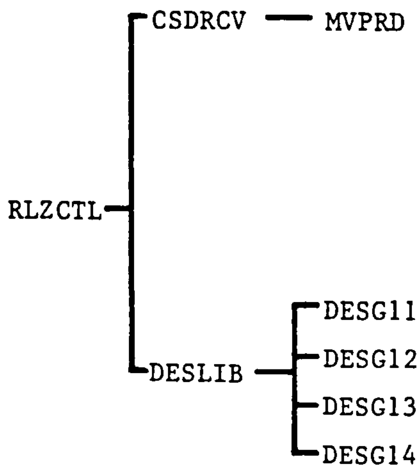


Fig. 15 - Design Recovery Control Flow Diagram

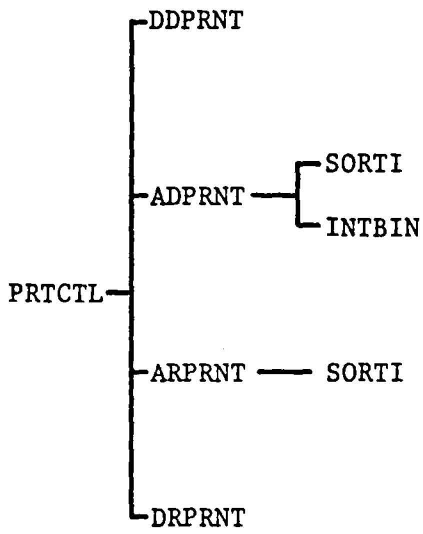


Fig. 16 - Printing Control Flow Diagram

```
         ┌ CLENUP
PSTPRO ──┤
         └ CHECKP
```

Fig. 17 - Post-processor Flow Diagram

Form 1

'COMMAND' data [data]

Form 2

'COMMAND'

data [data]

.
.
.

data [data]

Form 3

'COMMAND'

'SUB-COMMAND'

data [data]

.
.
.

data [data]

Fig. 18 - Data Command Forms

```
$****************************************************************************
$ FRAME OPTIMIZATION PROBLEM - ONE BAY / TWO STORY FRAME
$****************************************************************************
$
$---------------------------------------------------------------------------
$ DESIGN DATA BLOCK
$---------------------------------------------------------------------------
'DESIGN DATA'
$
'DESIGN ELEMENTS'
$
$   ANALYSIS ELEMENT NO.    GEOMETRY NO.    MASTER ELEMENT NO.
$
             1                   1
             2                   1
             3                   1
             5                   1
             8                   1                  1
             7                   1                  2
             4                   1                  3
             6                   1                  5
$
'ELEMENT GEOMETRIES'
$
$   GEOMETRY NO.    GEOMETRY TYPE
$
         1               14
$
$   INITIAL VALUE    LOWER BOUND    UPPER BOUND
$
        16.61           1.00            25.
          .45            .01             5.
$
'STRESS CONSTRAINTS'
$
$   LOAD SET NO.
$
         1
         2
$
'LOCAL BUCKLING CONSTRAINTS'
$
$   LOAD SET NO.
$
         1
         2
$
'DISPLACEMENT CONSTRAINTS'
$
```

Fig. 19 - Sample Program Input Data

```
$   LOAD SET NO.   NODE NO.   DOF   SHIFT   LOWER BOUND   UPPER BOUND
$
          2            2        1    0.0        -.36           .36
          2            3        1    0.0        -.72           .72
          2            4        1    0.0        -.36           .36
          2            5        1    0.0        -.72           .72
          2            6        1    0.0        -.36           .36
          2            7        1    0.0        -.72           .72
$----------------------------------------------------------------------
$   ANALYSIS DATA BLOCK
$----------------------------------------------------------------------
'ANALYSIS DATA'
$
'BOUNDARY CONDITIONS'
$
$   NODE NO.    DEGREE OF FREEDOM SPECIFICATION
$
     'ALL'                  123456
      2                    -126
      3                    -126
      4                    -126
      5                    -126
      6                    -126
      7                    -126
$
'MATERIALS'
$
$   MATERIAL NO.    RHO       E      NU      G      SIGY     FS     K-EFF
$
          1        .2836  30.00E6 0.30 0.785E7 3.6E04  1.51    1.00
          2        .2836  30.00E6 0.30 0.785E7 3.6E04  1.51    1.14
          3        .2836  30.00E6 0.30 0.785E7 3.6E04  1.51    1.38
          4        .2836  30.00E6 0.30 0.785E7 3.6E04  1.51    1.37
$
'BEAMS'
$
$   ELEMENT NO.   MATERIAL NO.   NODE 1   NODE 2    ORIENTATION DATA
$
          1            1            1        2         3   1.  0.  0.
          2            2            2        3         3   1.  0.  0.
          3            3            3        5         3   0.  1.  0.
          4            3            5        7         3   0.  1.  0.
          5            4            2        4         3   0.  1.  0.
          6            4            4        6         3   0.  1.  0.
          7            2            6        7         3   1.  0.  0.
          8            1            6        8         3   1.  0.  0.
$
'COORDINATES'
$
```

Fig. 19 - Sample Program Input Data (cont.)

```
$    NODE NO.      X         Y         Z
$
        1          0.0       0.0       0.0
        2          0.0     180.0       0.0
        3          0.0     360.0       0.0
        4        120.0     180.0       0.0
        5        120.0     360.0       0.0
        6        240.0     180.0       0.0
        7        240.0     360.0       0.0
        8        240.0       0.0       0.0
$
'LOAD CONDITIONS'
$
$    LOAD SET NO.     TYPE       BOUNDARY CONDITION NO.
$
          1          'STATICS'                1
          2          'STATICS'                1
$
'NODAL LOADS'
$
$   LOAD SET NO.
$
     'SET' 2
$
$      NODE NO.     DIRECTION     MAGNITUDE
$
          2             1           45000.
          3             1           45000.
$
'UNIFORM LOADS'
$
$   LOAD SET NO.
$
     'SET' 1
$
$      BEAM NO.    DIRECTION     MAGNITUDE
$
          3            2           -500.
          4            2           -500.
          5            2           -500.
          6            2           -500.
$-------------------------------------------------------------------
$   CONTROL DATA BLOCK
$-------------------------------------------------------------------
'CONTROL DATA'
$
'ITERATIONS' 15
'MOVE LIMIT' .0   .5
'CONMIN'
```

Fig. 19 - Sample Program Input Data (cont.)

```
'MIXED'
'SCALE'
'OPTIMIZATION'  .001
'PRINT' 0 0 0 1 1 1 0 0 0 1
$-----------------------------------------------------------------
$   END OF DATA
$-----------------------------------------------------------------
```

Fig. 19 - Sample Program Input Data (cont.)

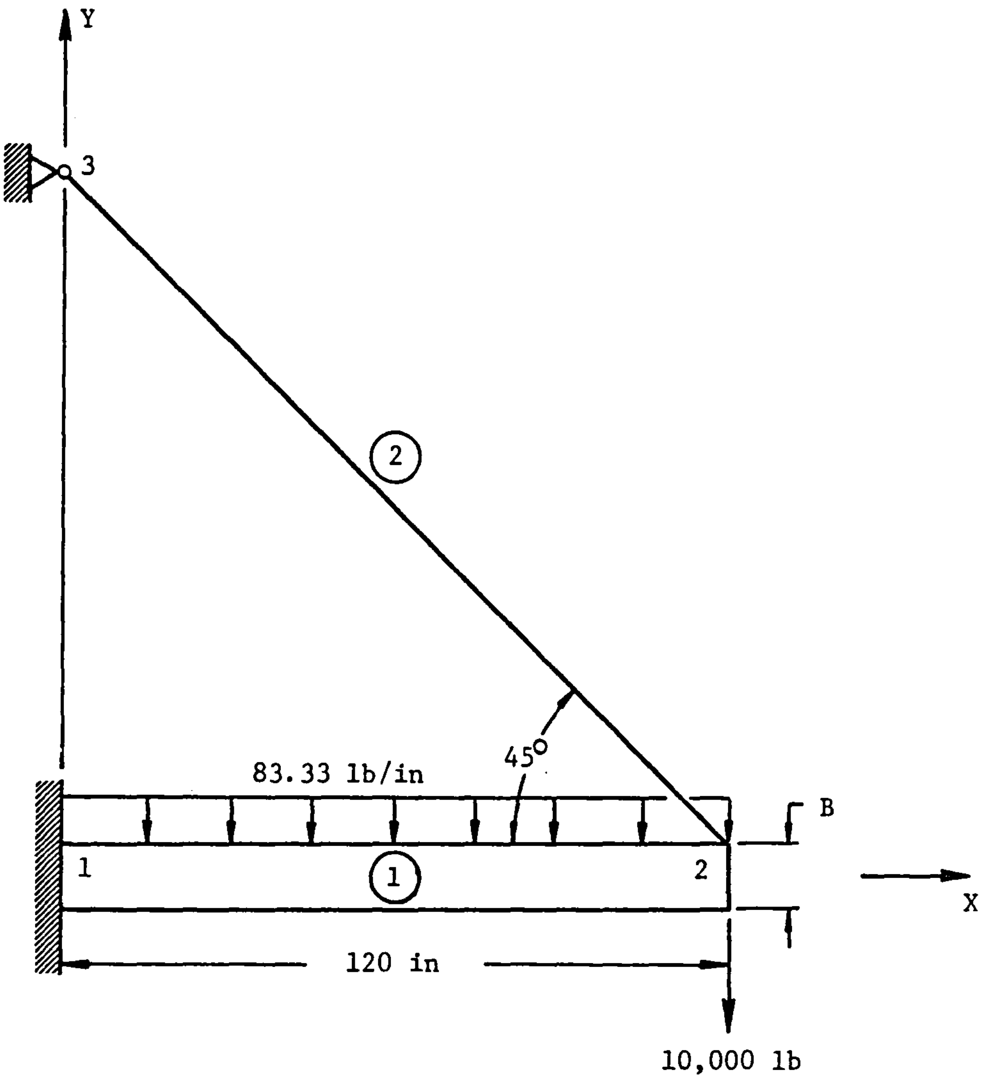


Fig. 20 - Tied Cantilevered Beam (Problem 1)

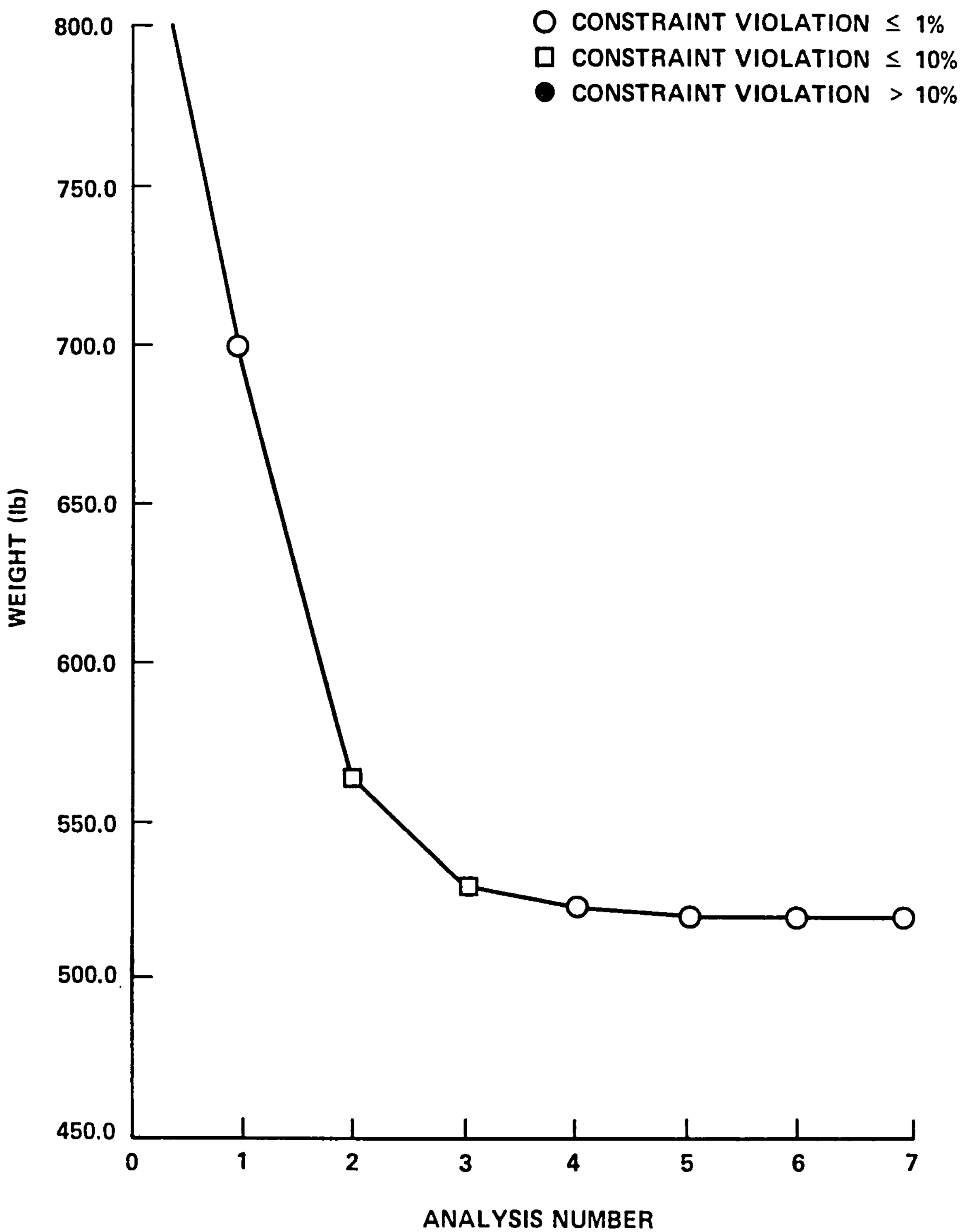


Fig. 21 - Iteration History for Problem 1, Run 1 (Option 1(P))
Tied Cantilevered Beam

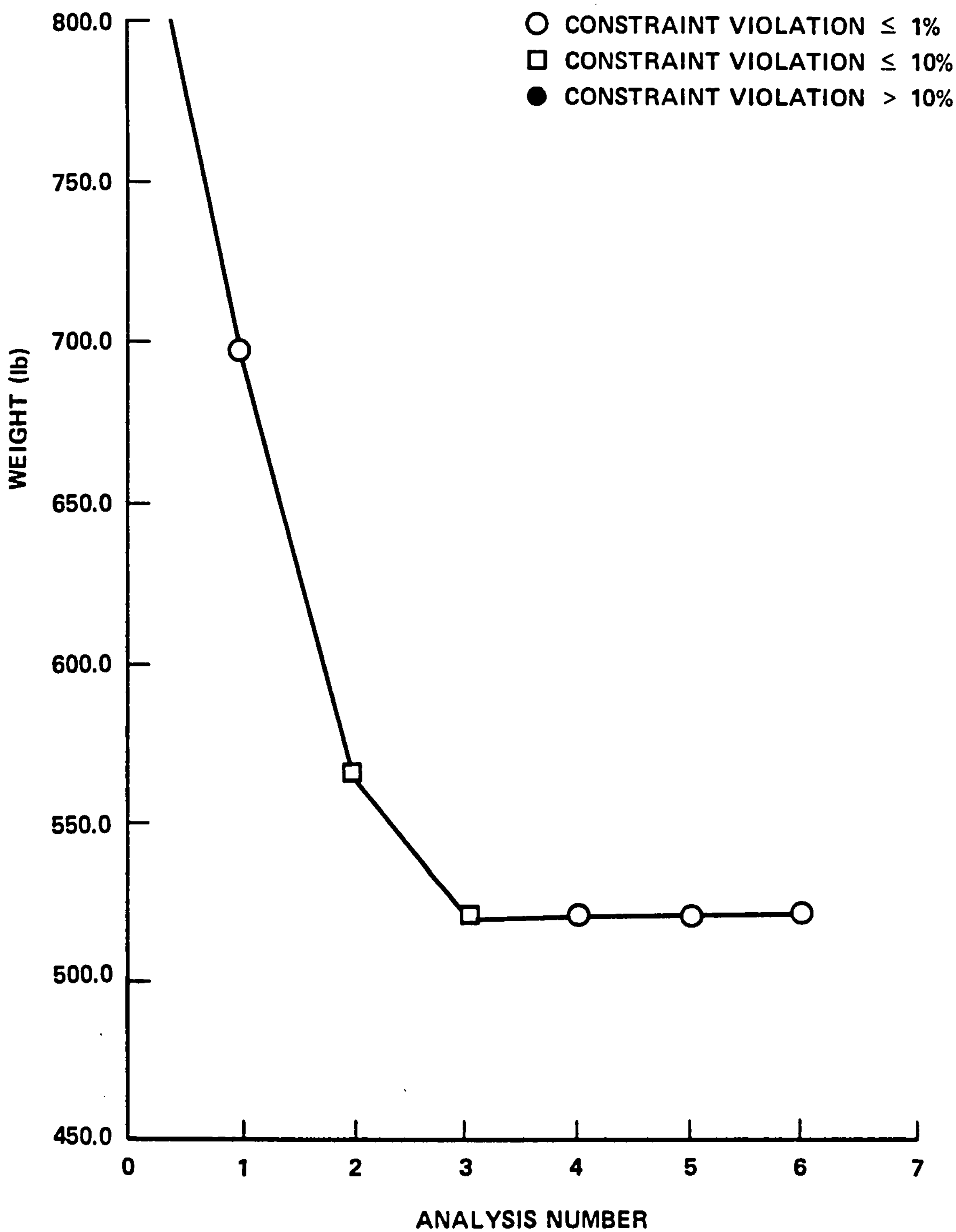


Fig. 22 - Iteration History for Problem 1, Run 2 (Option 2(P))
Tied Cantilevered Beam

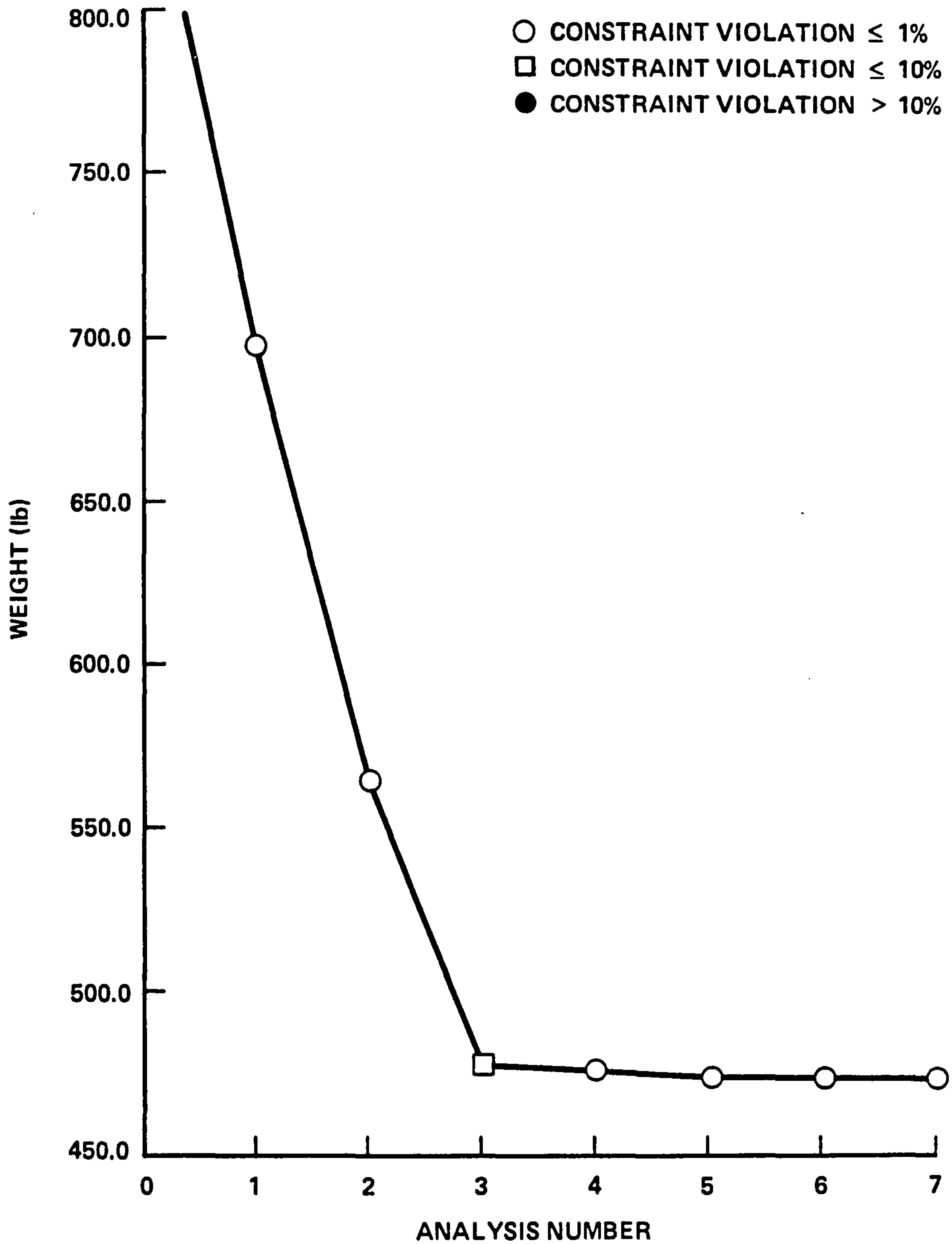


Fig. 23 - Iteration History for Problem 1, Run 3 (Option 3(P))
Tied Cantilevered Beam

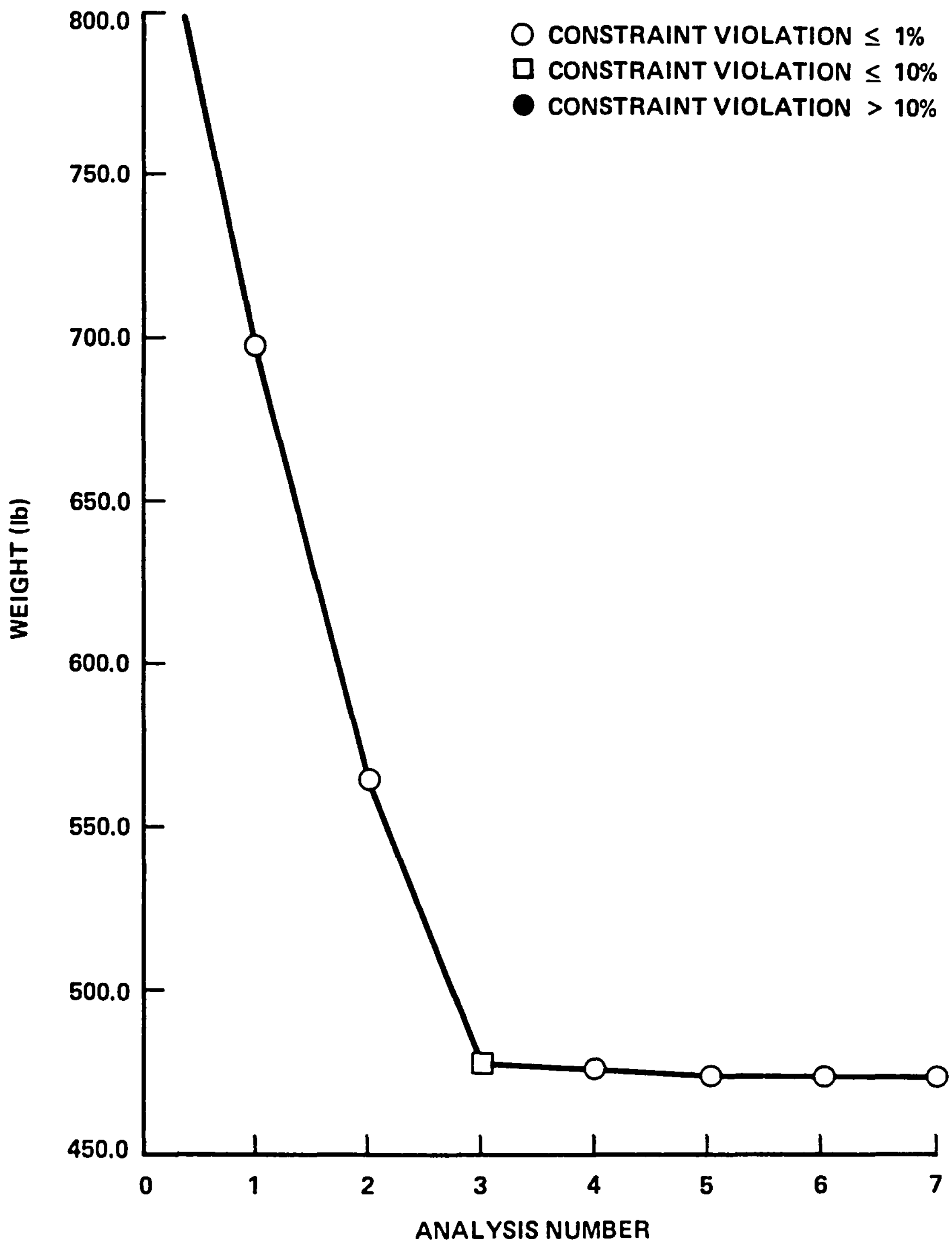


Fig. 24 - Iteration History for Problem 1, Run 4 (Option 6(P))
Tied Cantilevered Beam

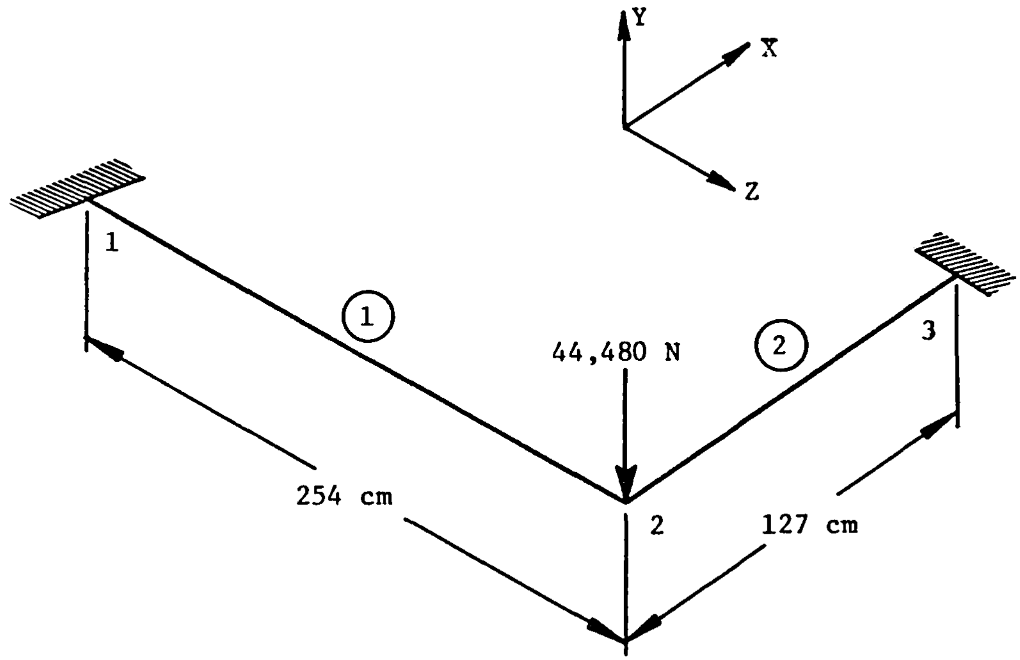


Fig. 25 - Two Member Frame (Problem 2)

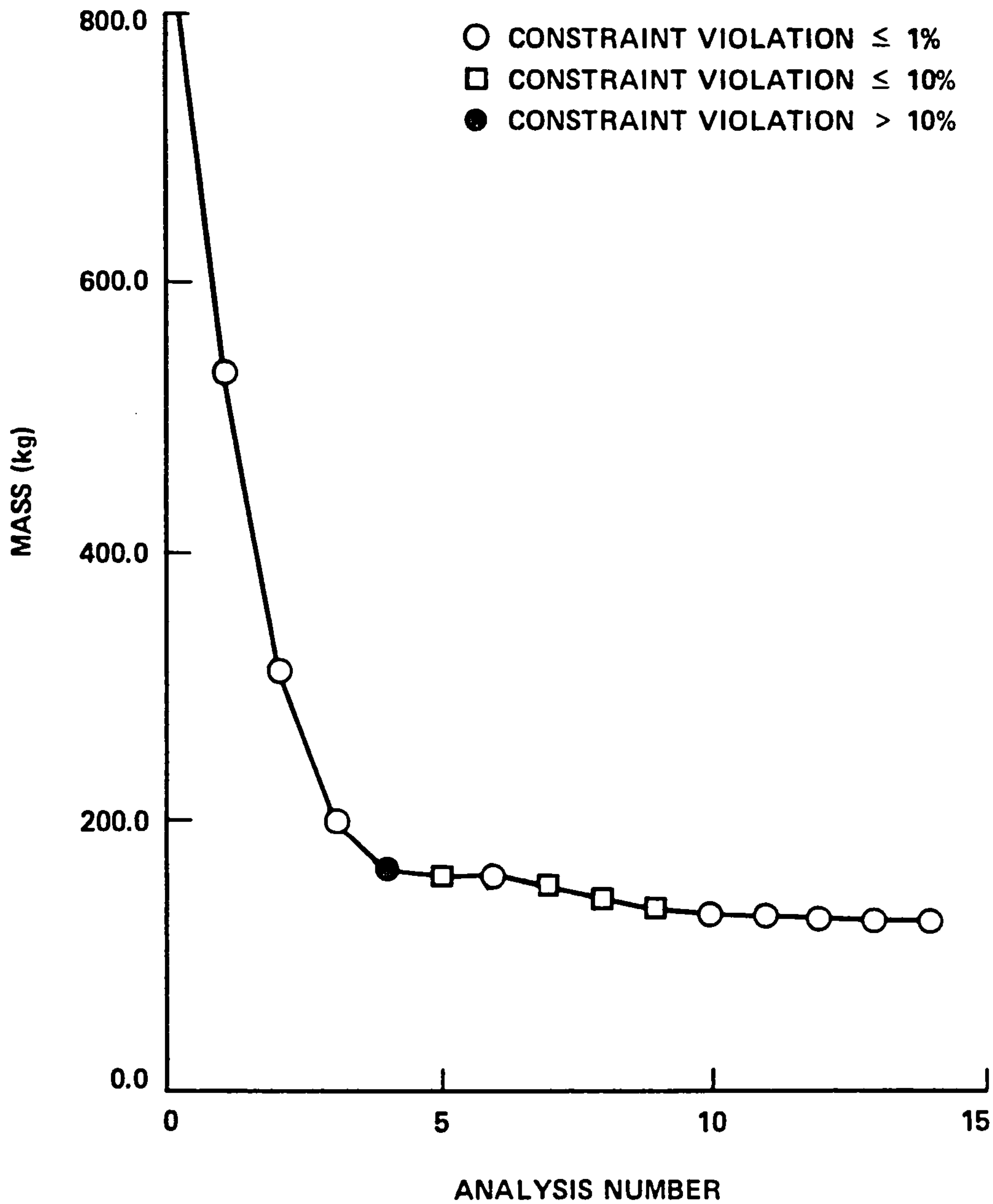


Fig. 26 - Iteration History for Problem 2, Case A, Run 1 (Option 1(P))
Two Member Frame

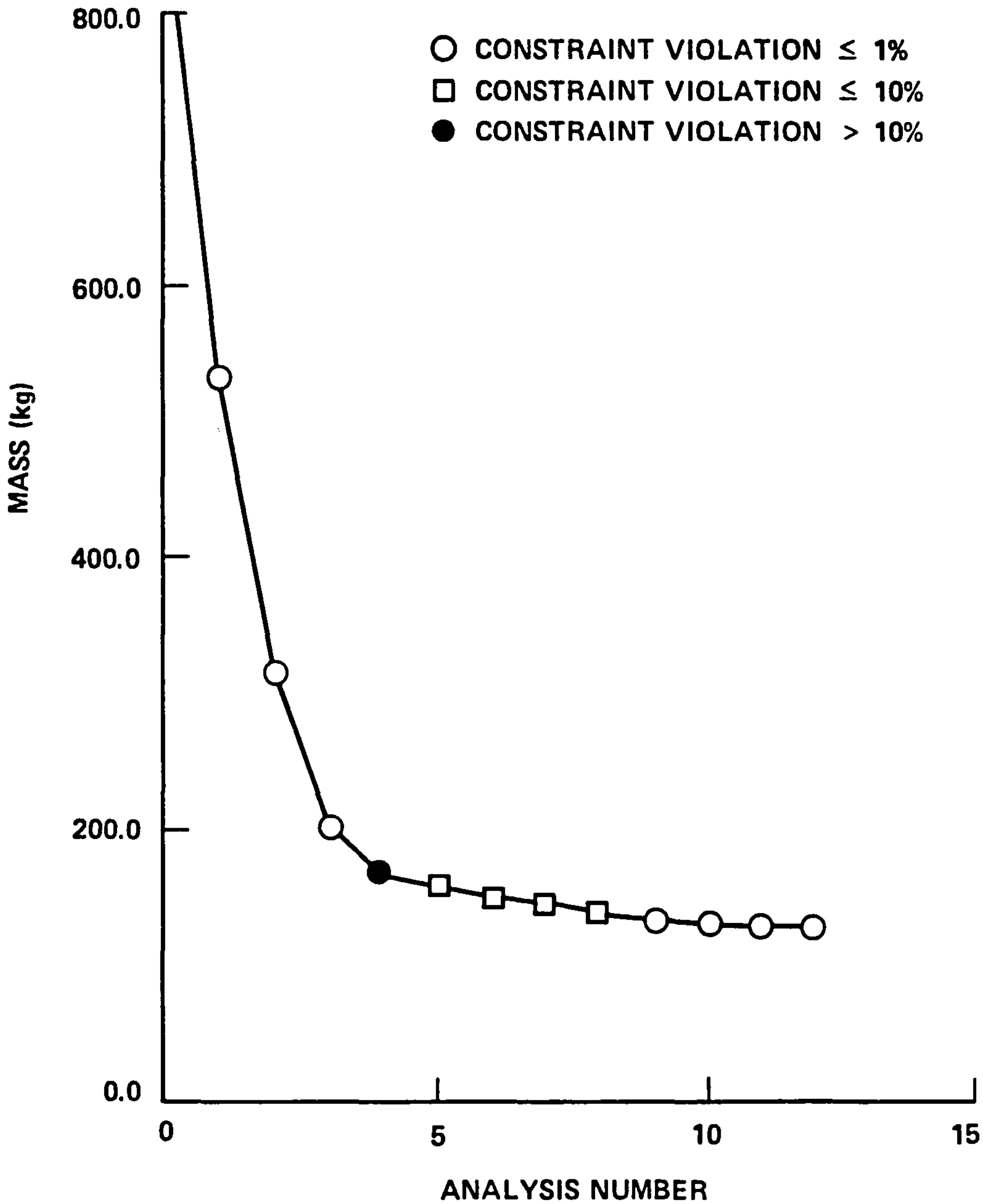


Fig. 27 - Iteration History for Problem 2, Case A, Run 2 (Option 2(P))
Two Member Frame

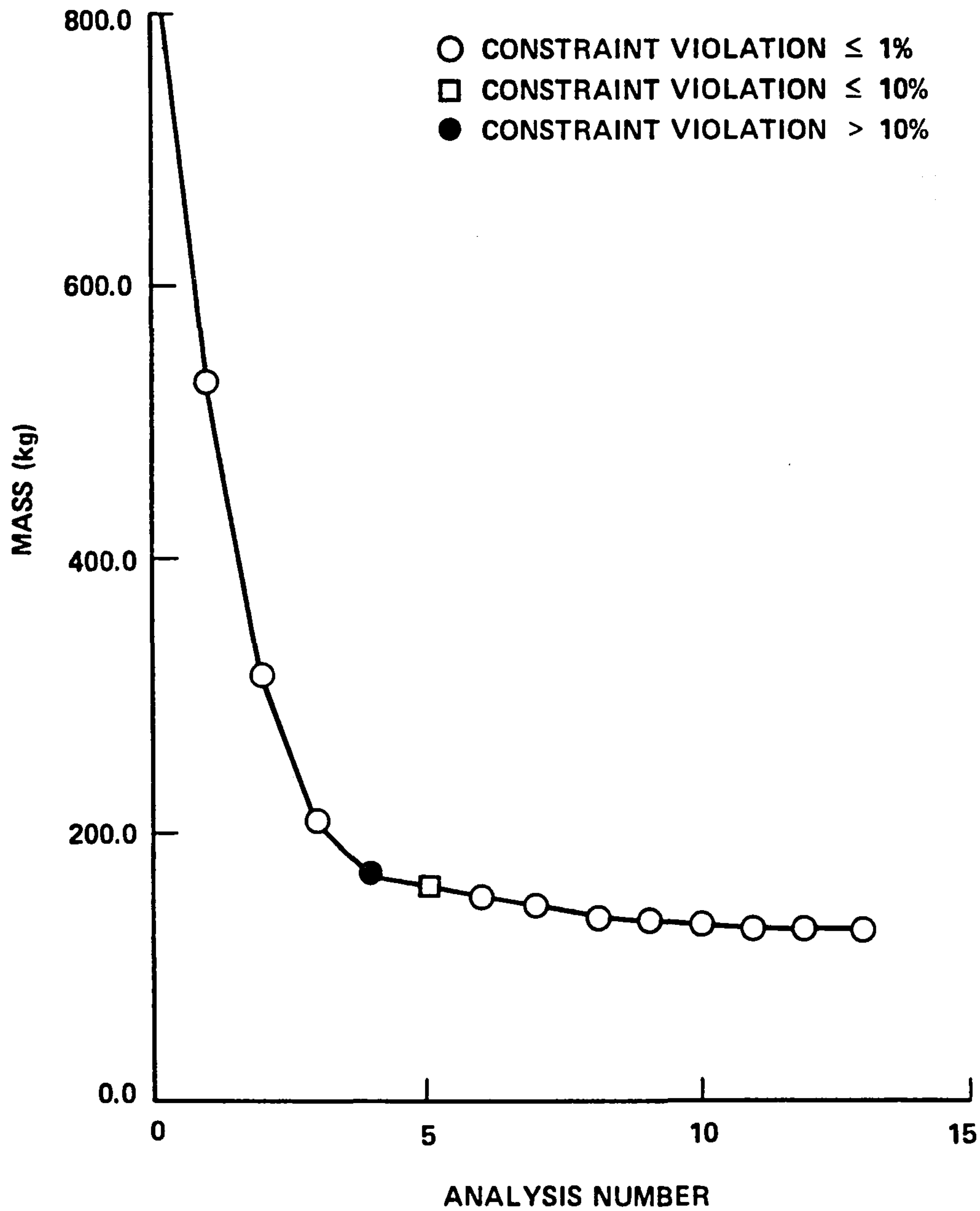


Fig. 28 - Iteration History for Problem 2, Case A, Run 3 (Option 3(P))
Two Member Frame

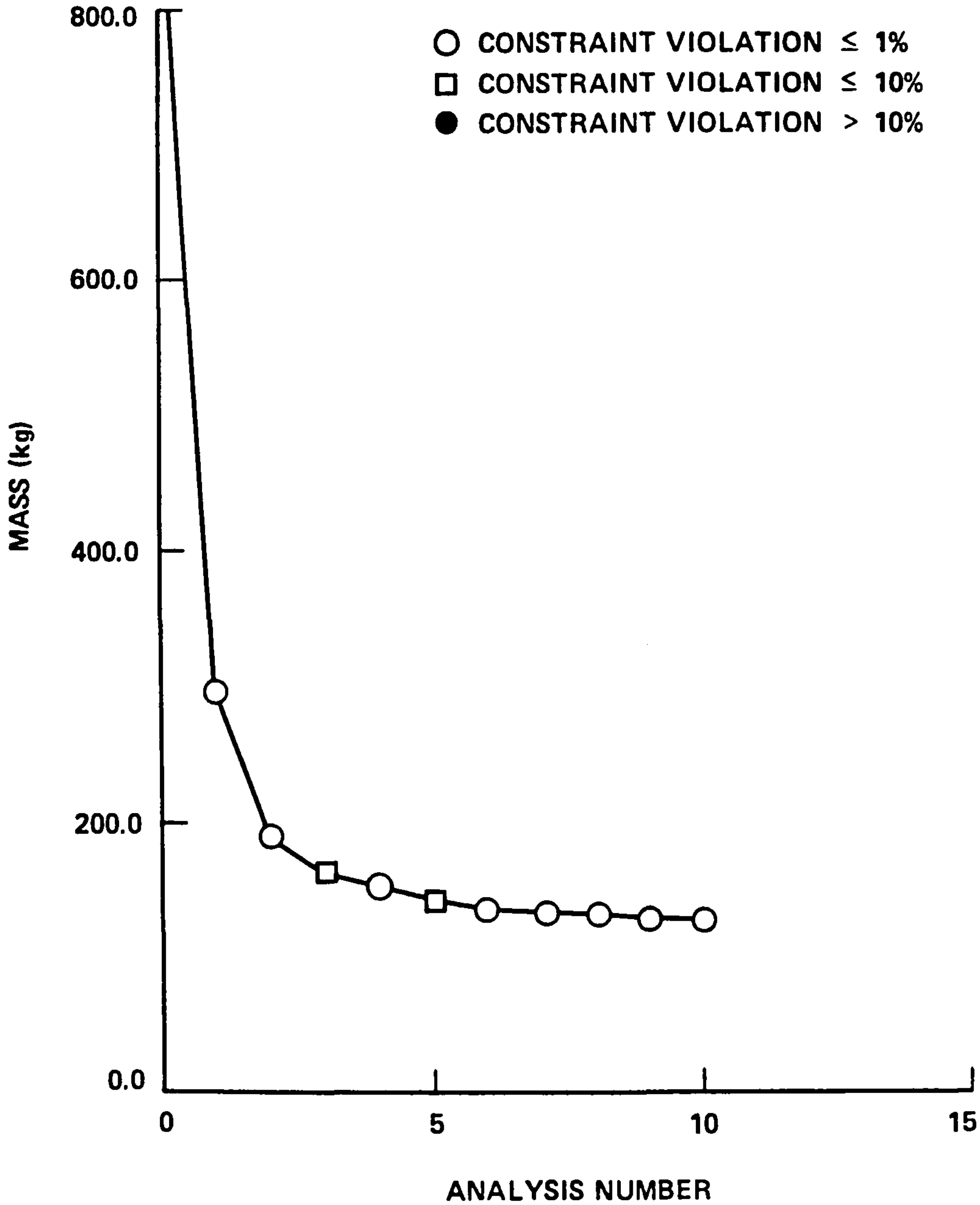


Fig. 29 - Iteration History for Problem 2, Case A, Run 4 (Option 4(P))
Two Member Frame

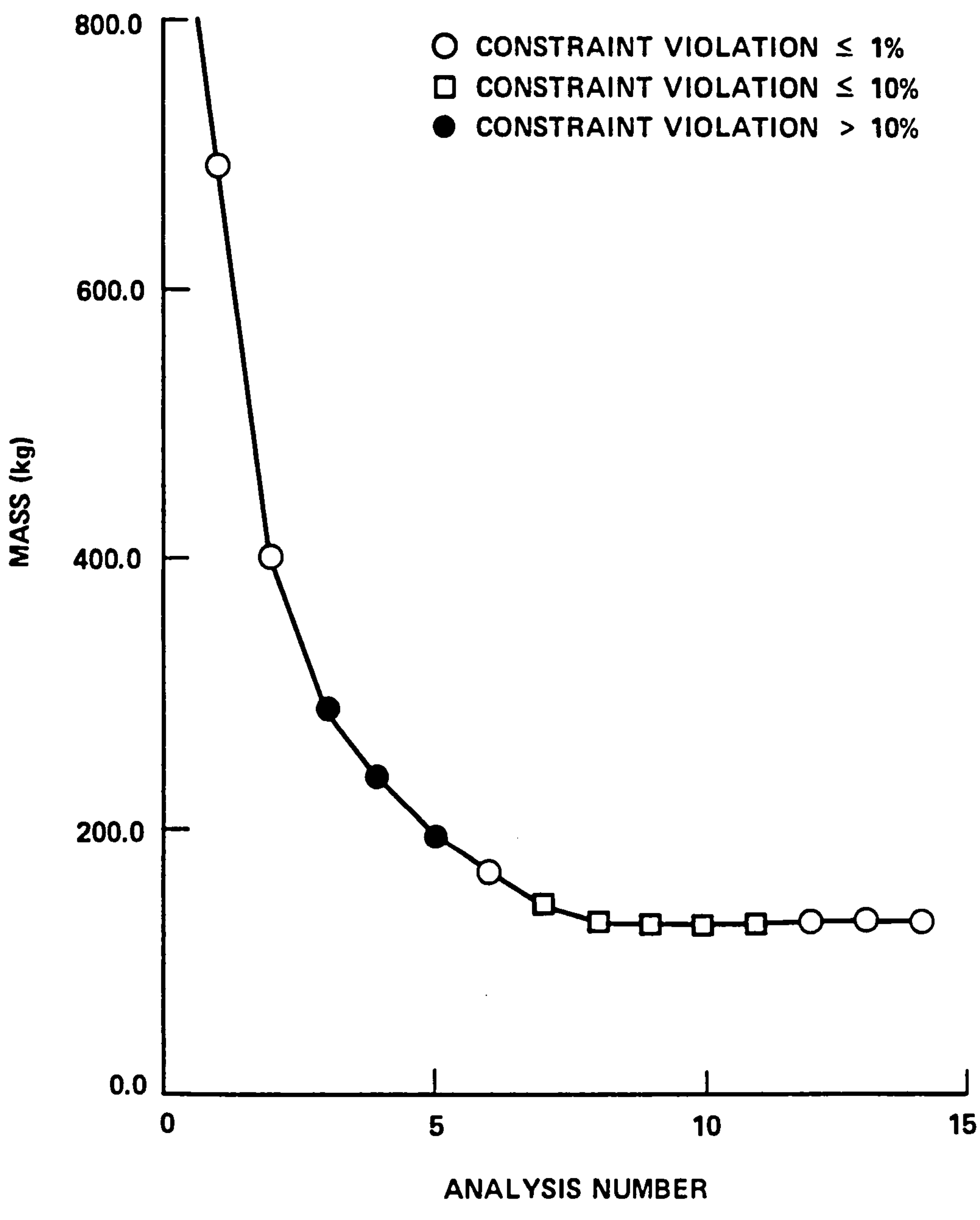


Fig. 30 - Iteration History for Problem 2, Case A, Run 5 (Option 7(P))
Two Member Frame

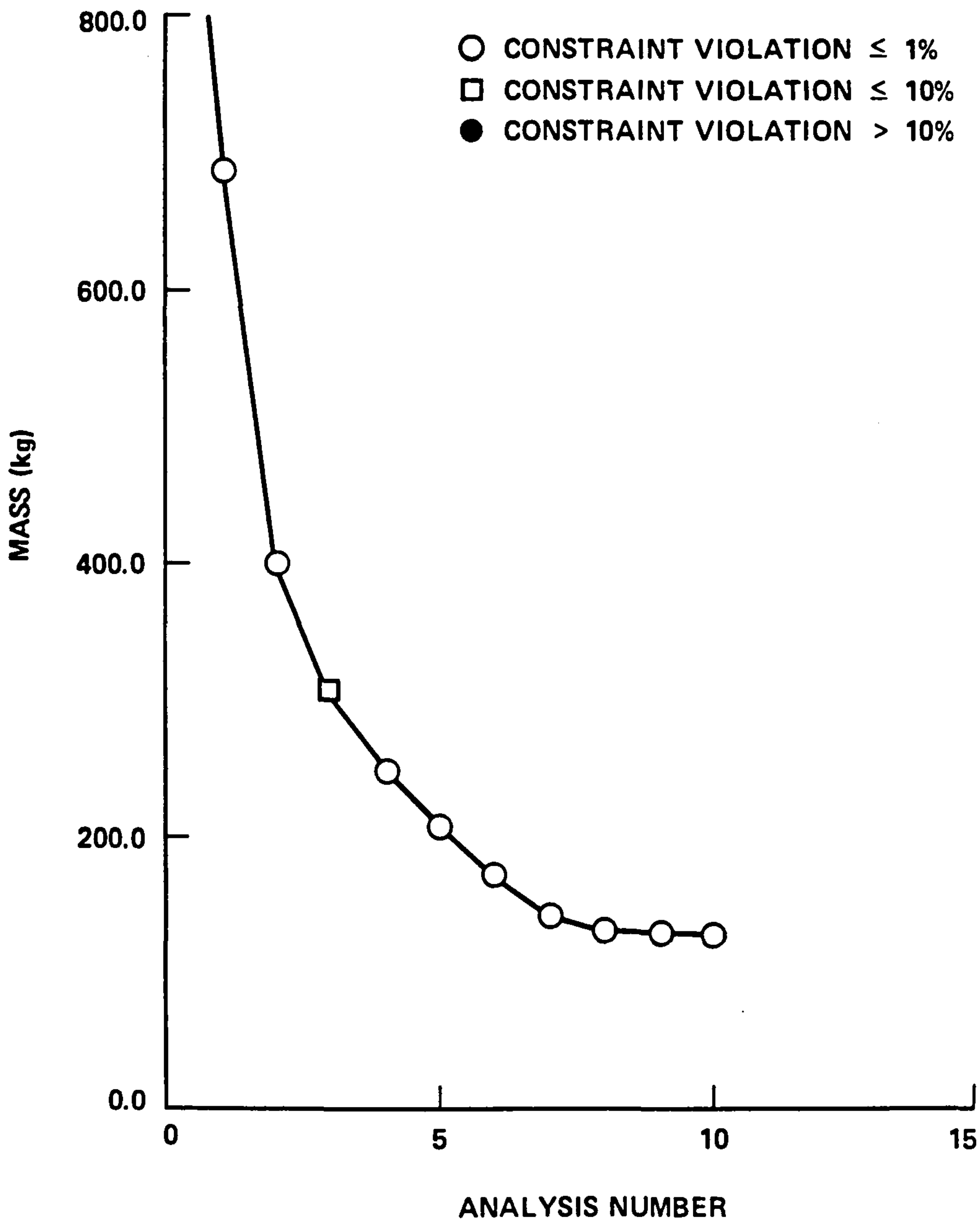


Fig. 31 - Iteration History for Problem 2, Case A, Run 6 (Option 10(P))
Two Member Frame

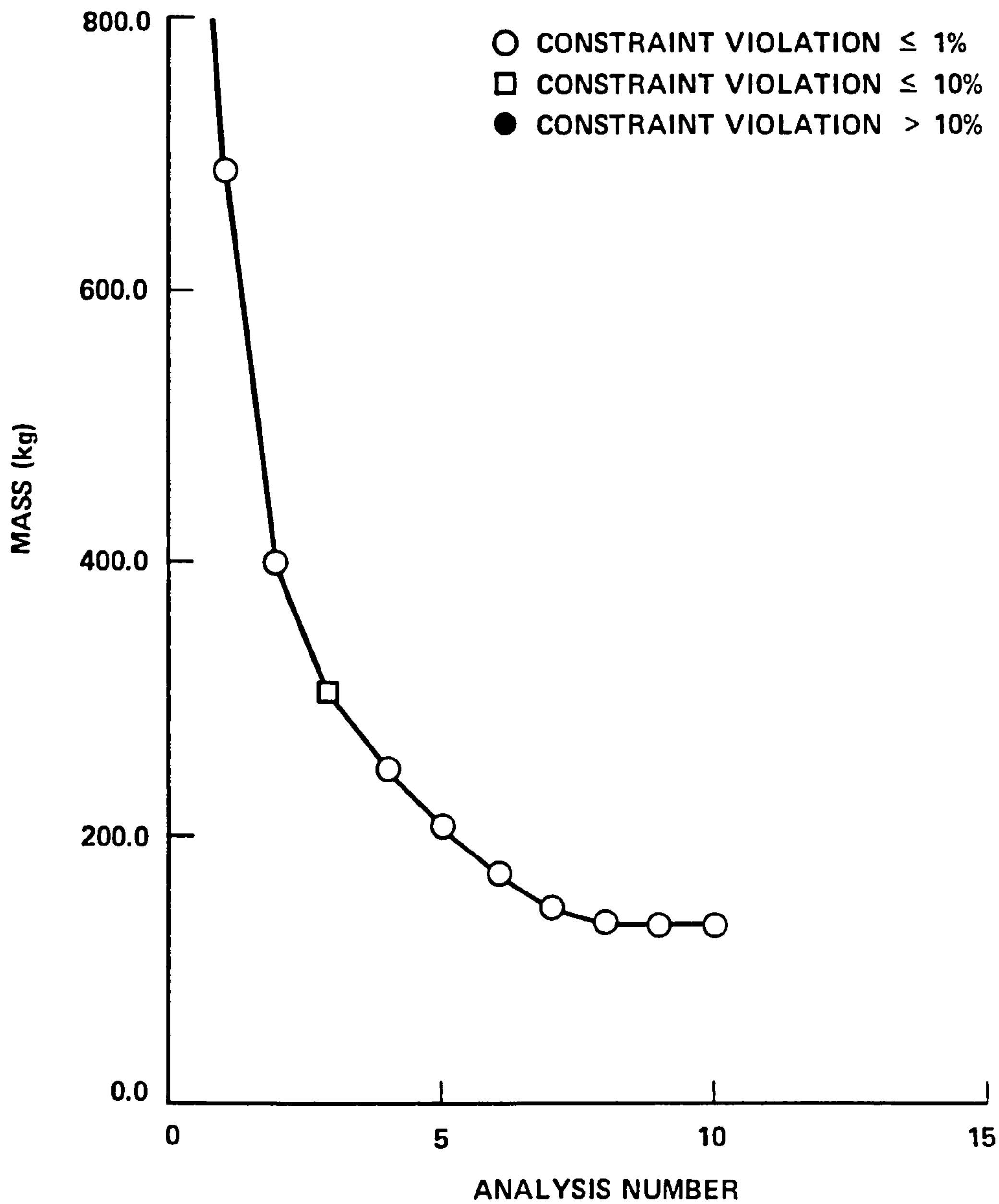


Fig. 32 - Iteration History for Problem 2, Case A, Run 7 (Option 10(D))
Two Member Frame

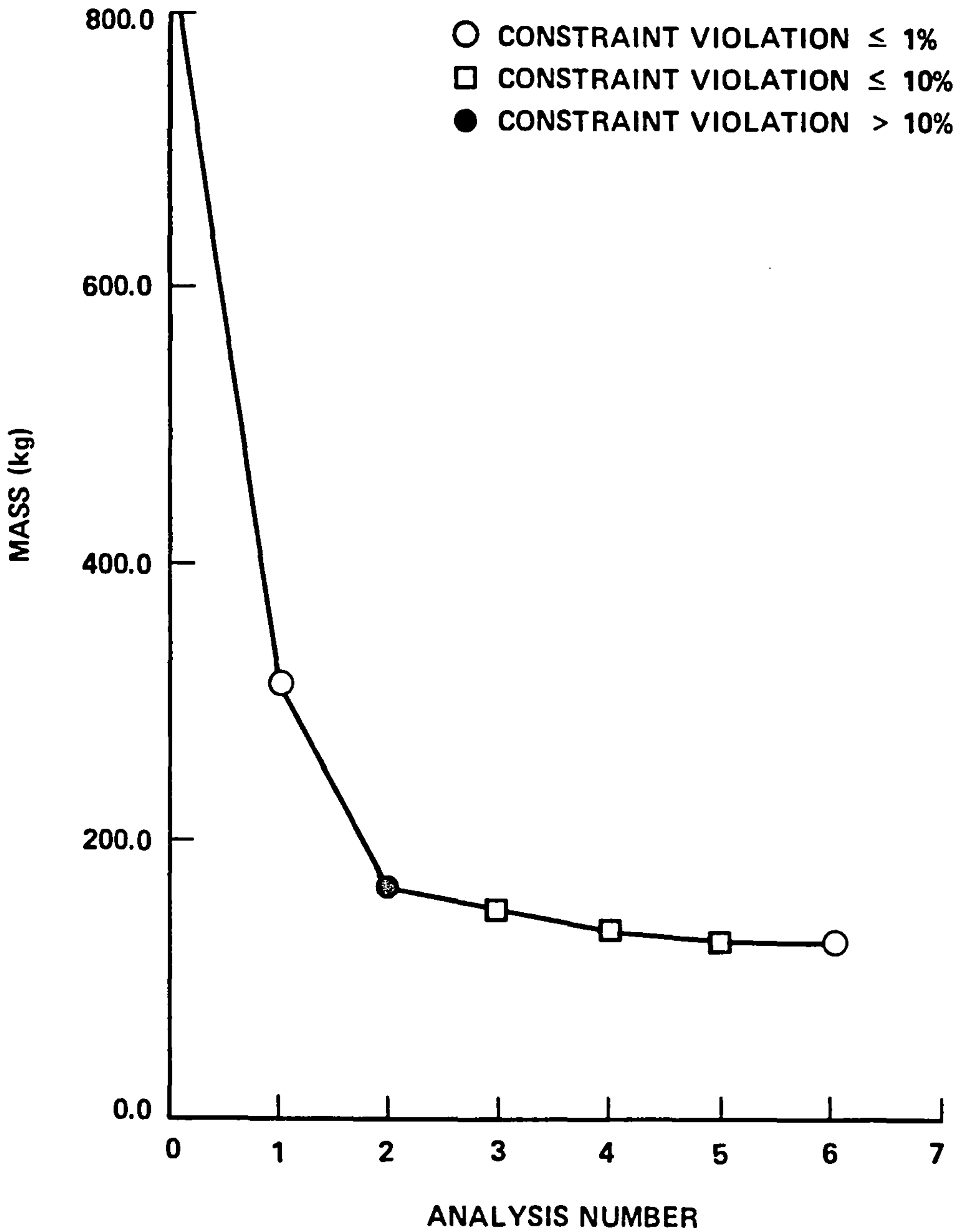


Fig. 33 - Iteration History for Problem 2, Case A, Run 8 (Option 1(PU))
Two Member Frame

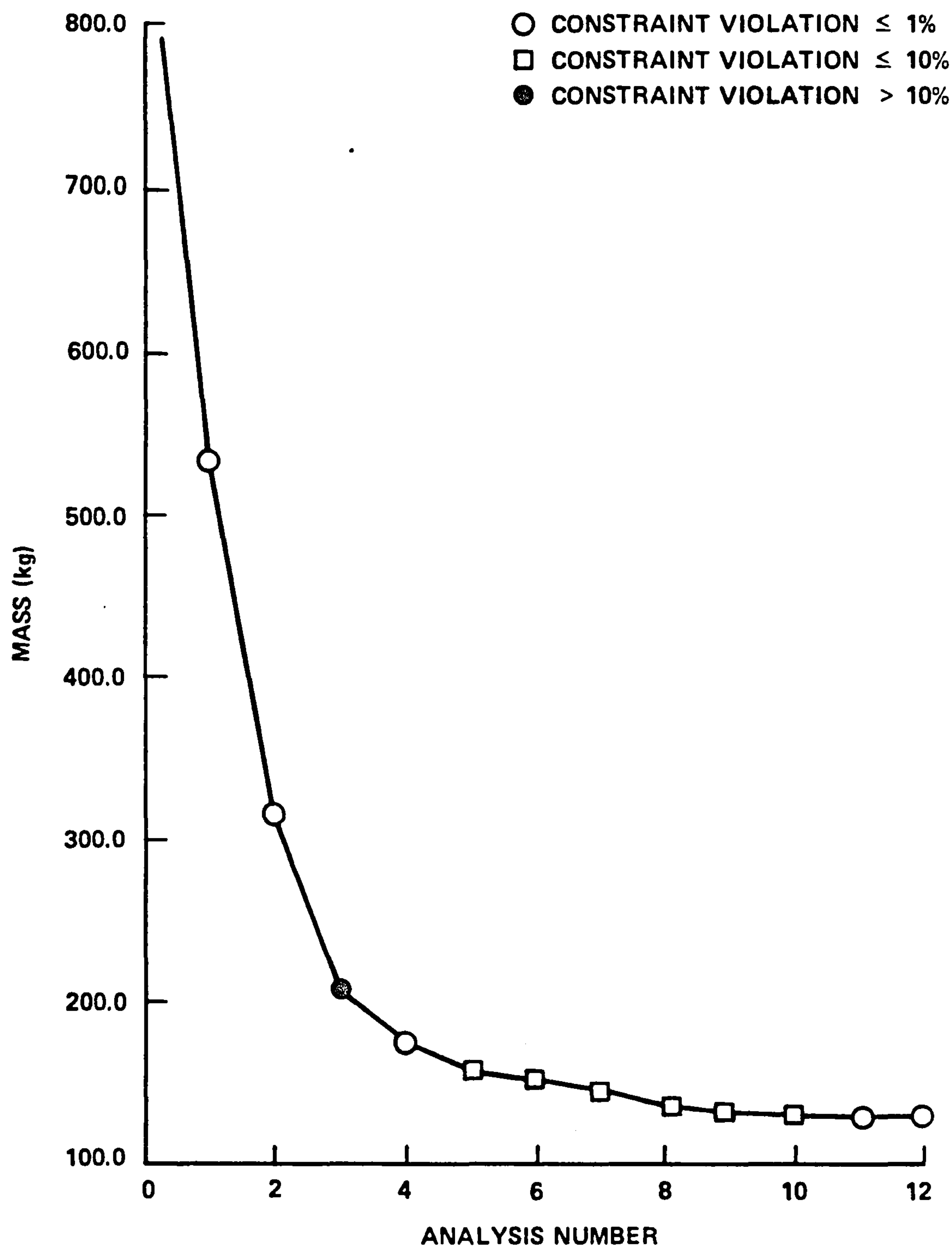


Fig. 34 - Iteration History for Problem 2, Case B, Run 1 (Option 1(P))
Two Member Frame

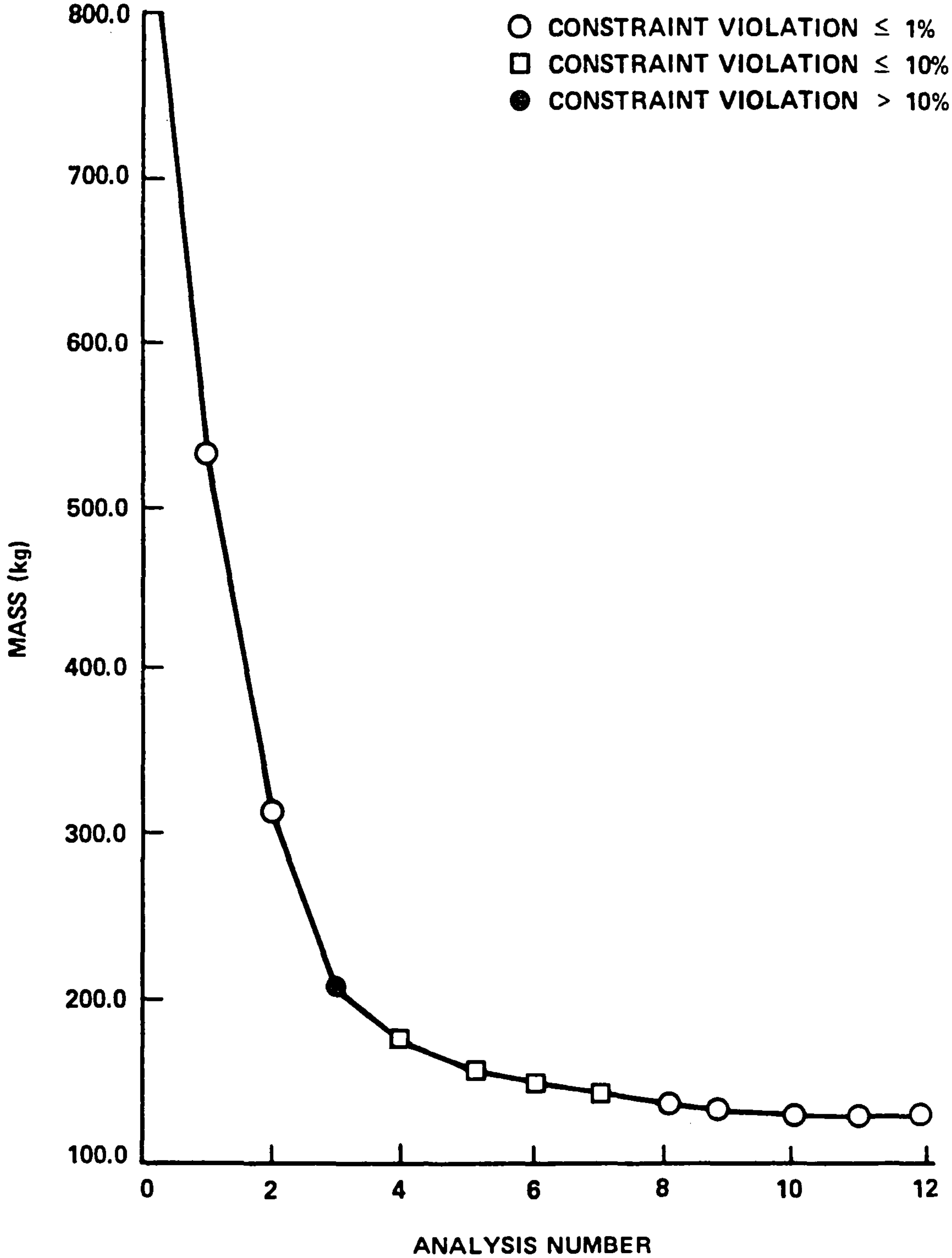


Fig. 35 - Iteration History for Problem 2, Case B, Run 2 (Option 2(F))
Two Member Frame

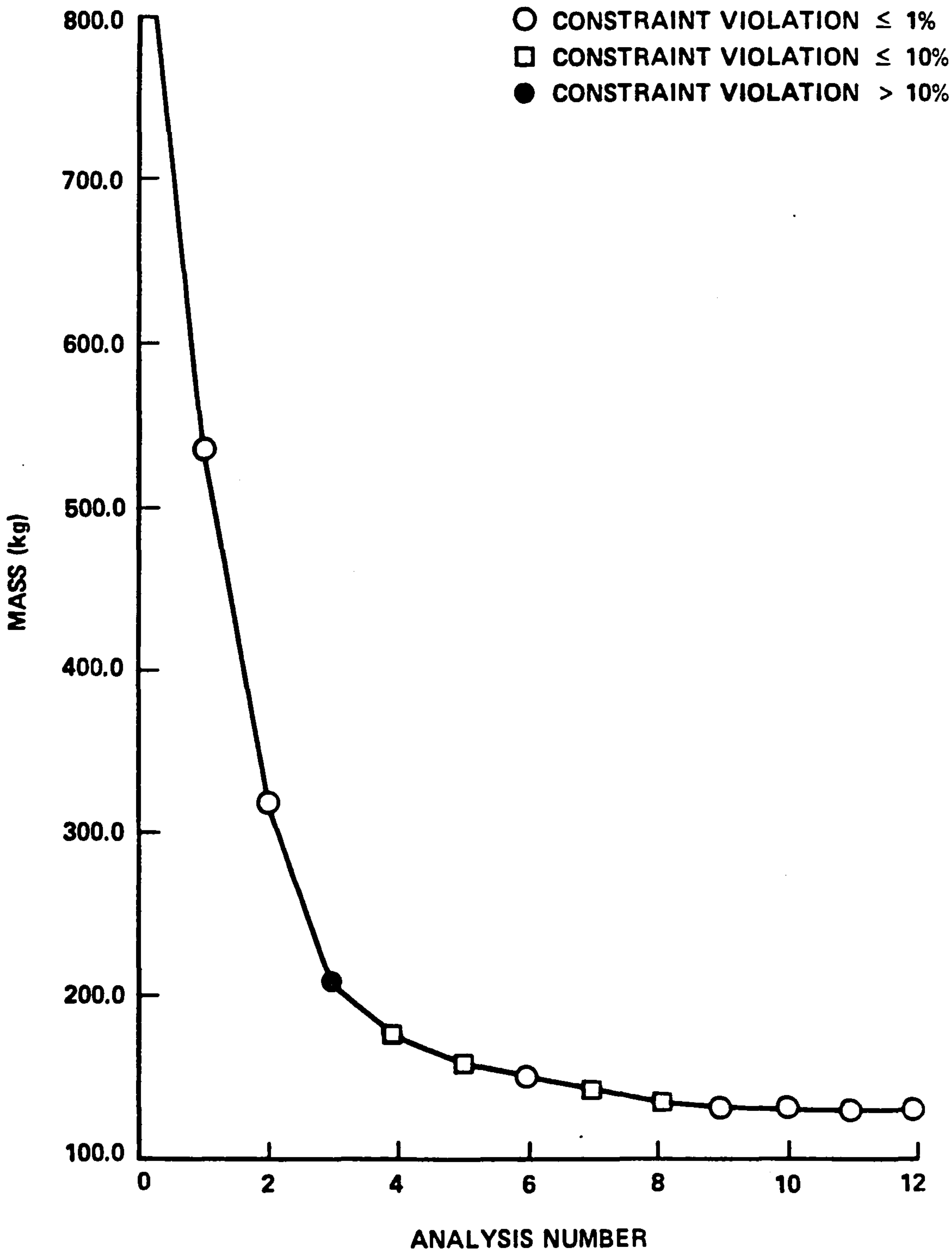


Fig. 36 - Iteration History for Problem 2, Case B, Run 3 (Option 3(P))
Two Member Frame

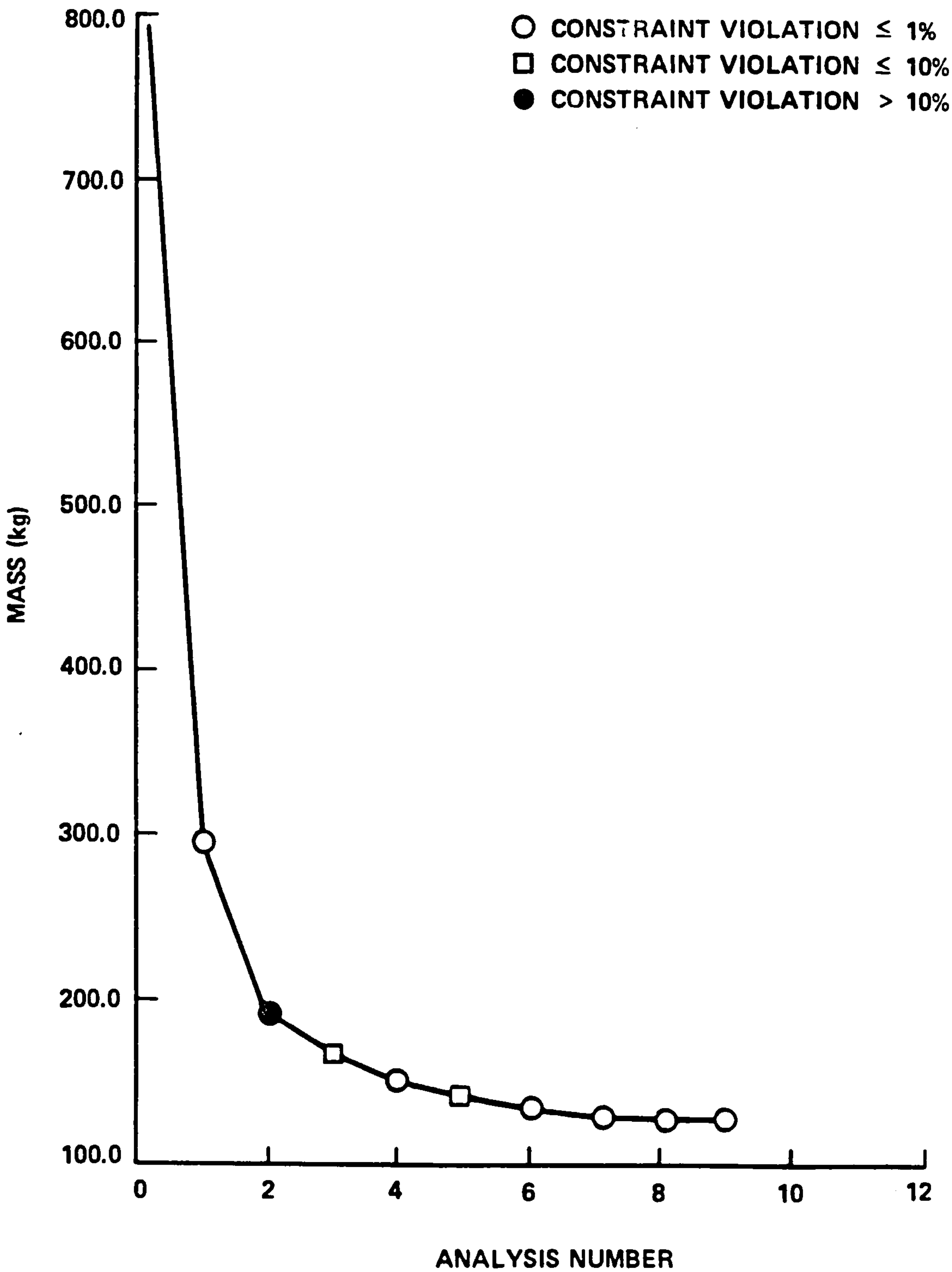


Fig. 37 - Iteration History for Problem 2, Case B, Run 4 (Option 4(P))
Two Member Frame

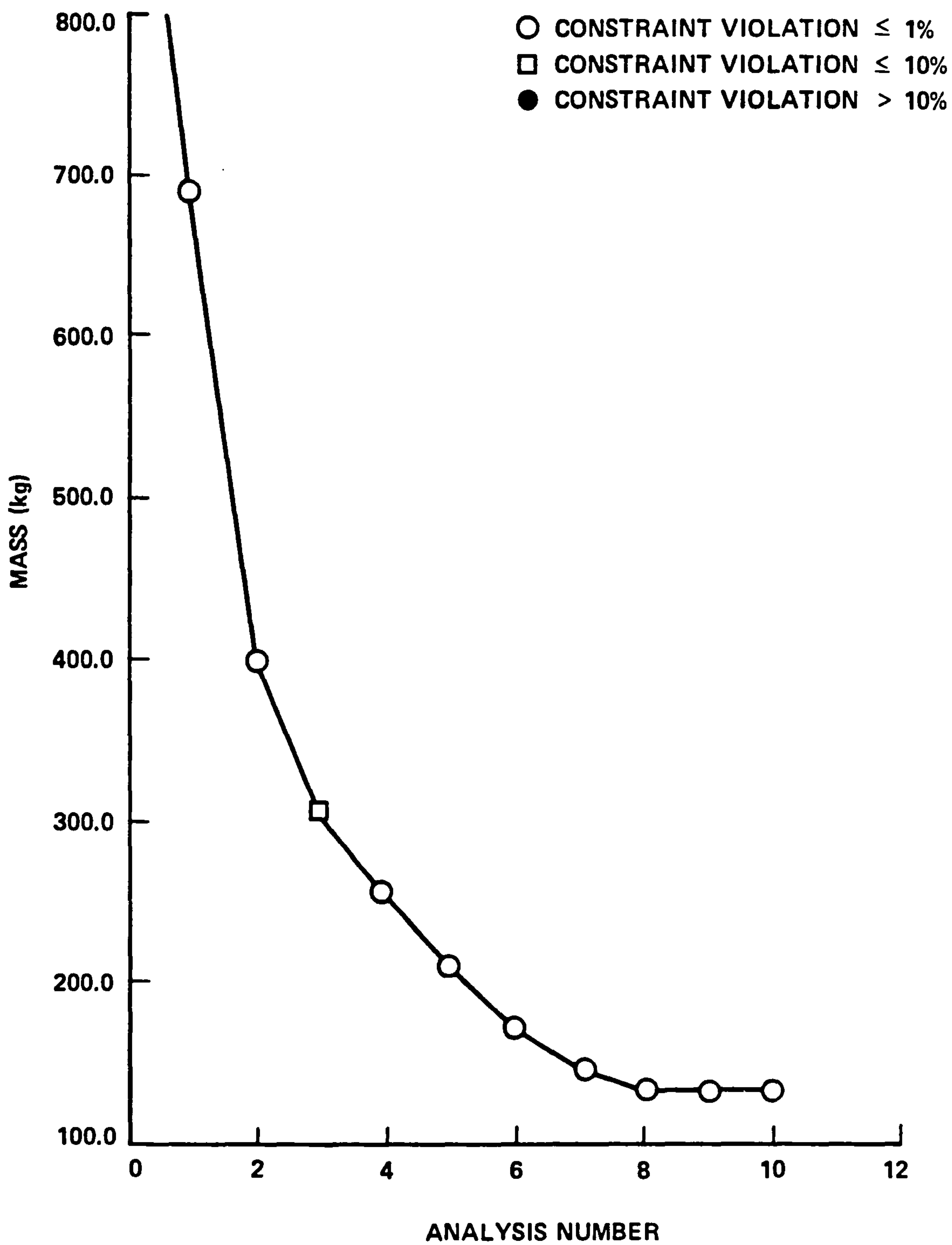


Fig. 38 - Iteration History for Problem 2, Case B, Run 5 (Option 10(P))
Two Member Frame

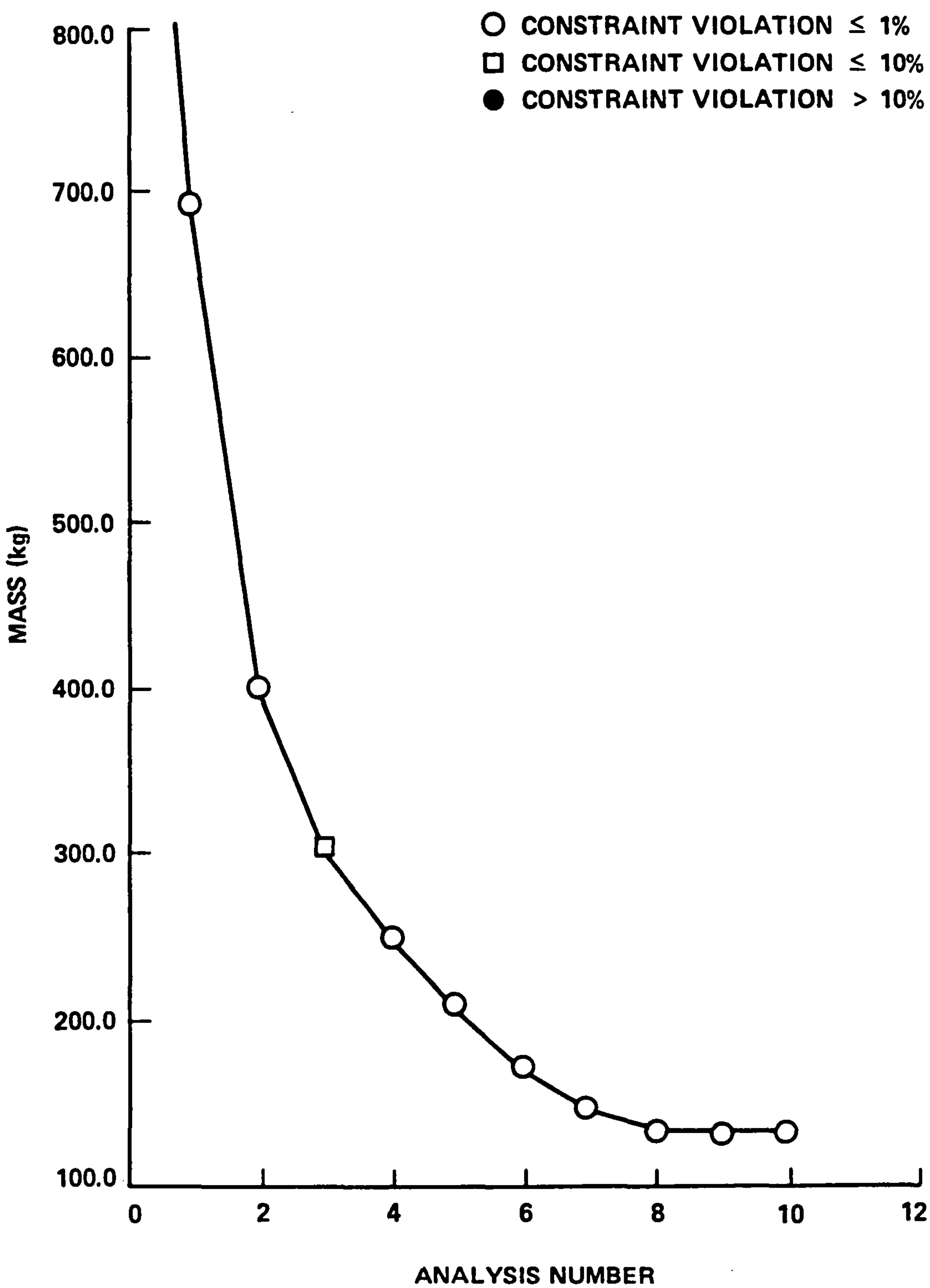


Fig. 39 - Iteration History for Problem 2, Case B, Run 6 (Option 10(D))
Two Member Frame

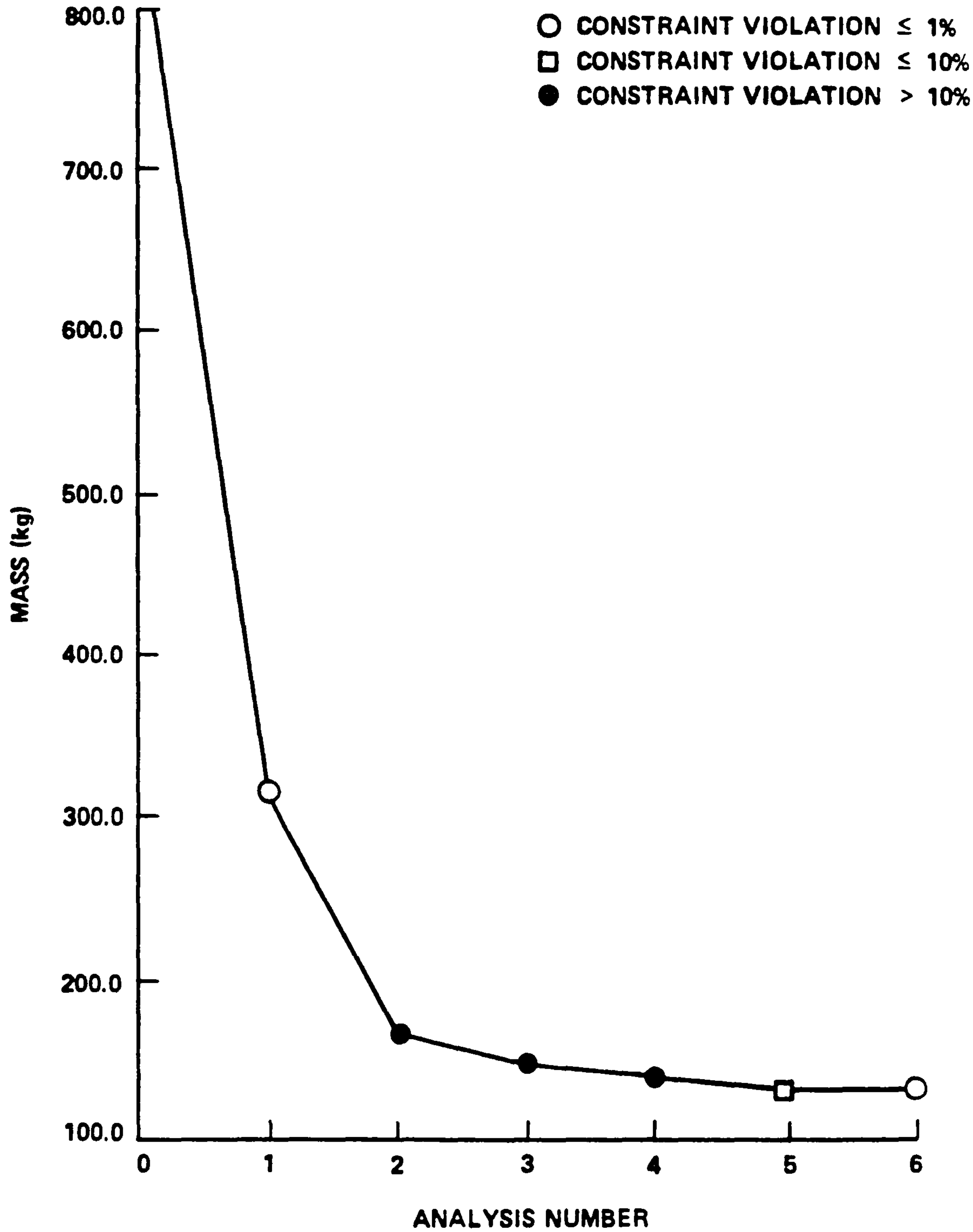


Fig. 40 - Iteration History for Problem 2, Case B, Run 7 (Option 1(PU))
Two Member Frame

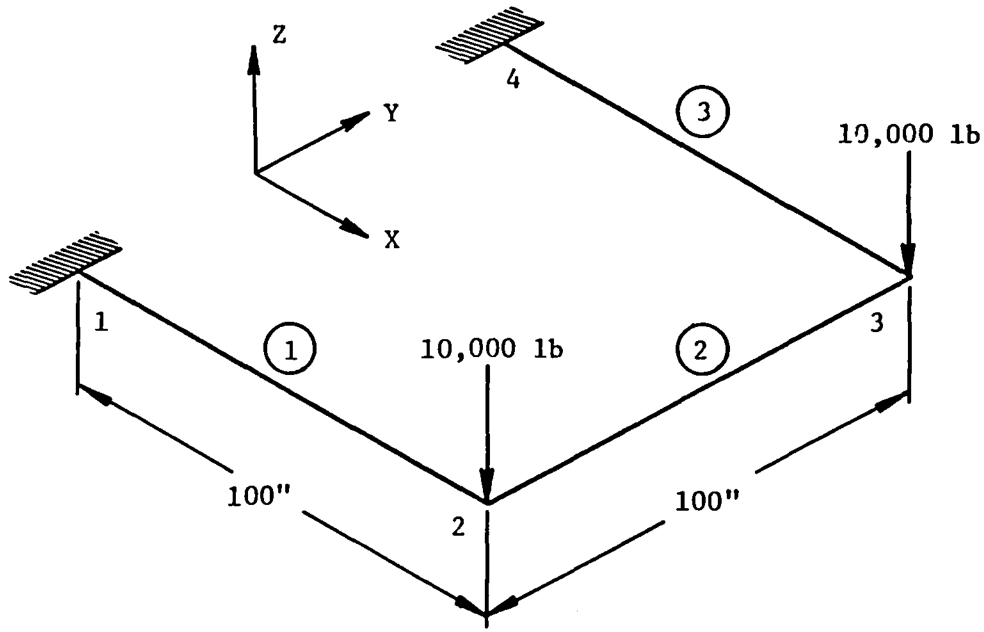


Fig. 41 - Three Member Frame (Problem 3)

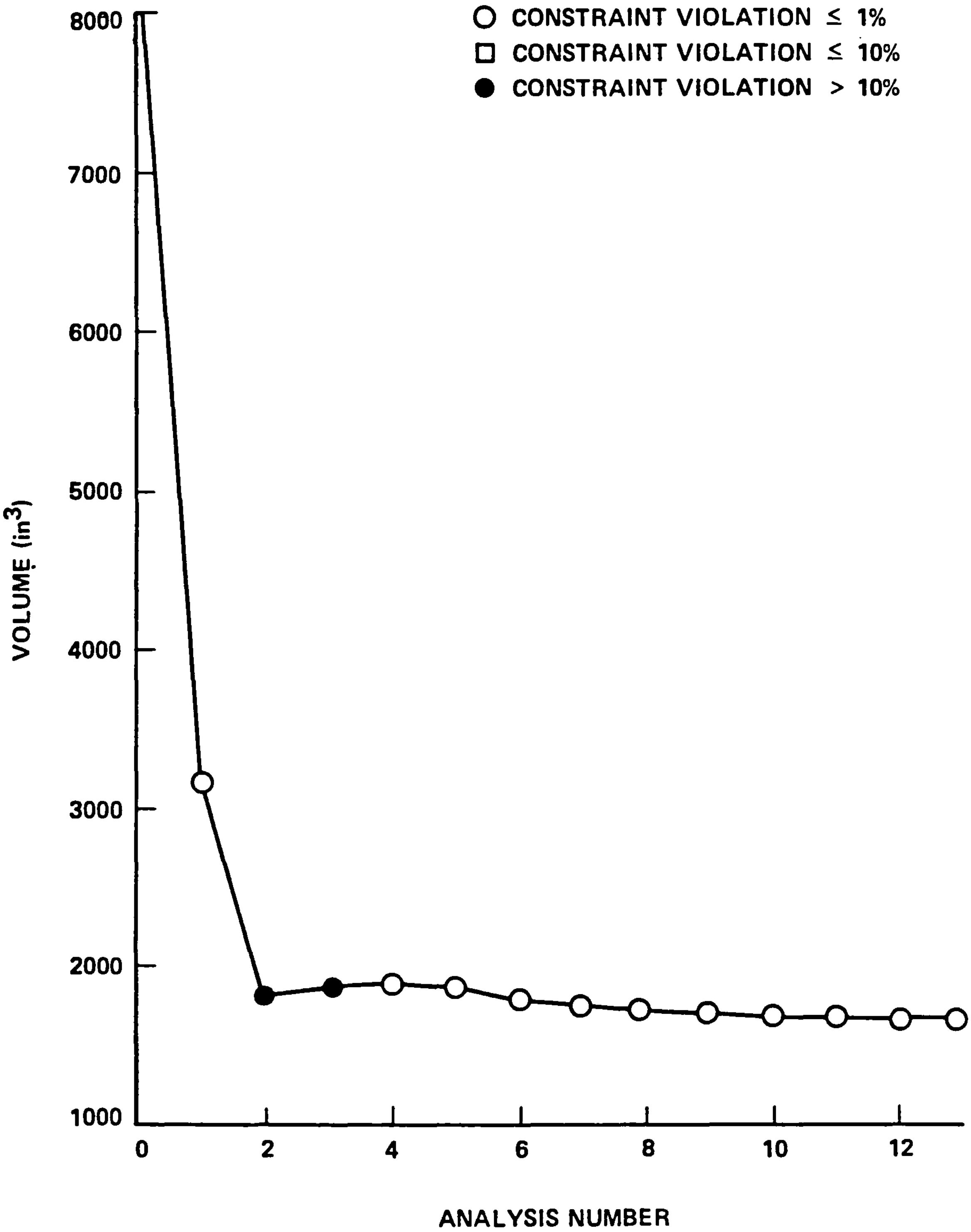


Fig. 42 - Iteration History for Problem 3, Run 1 (Option 1(P))
Three Member Frame

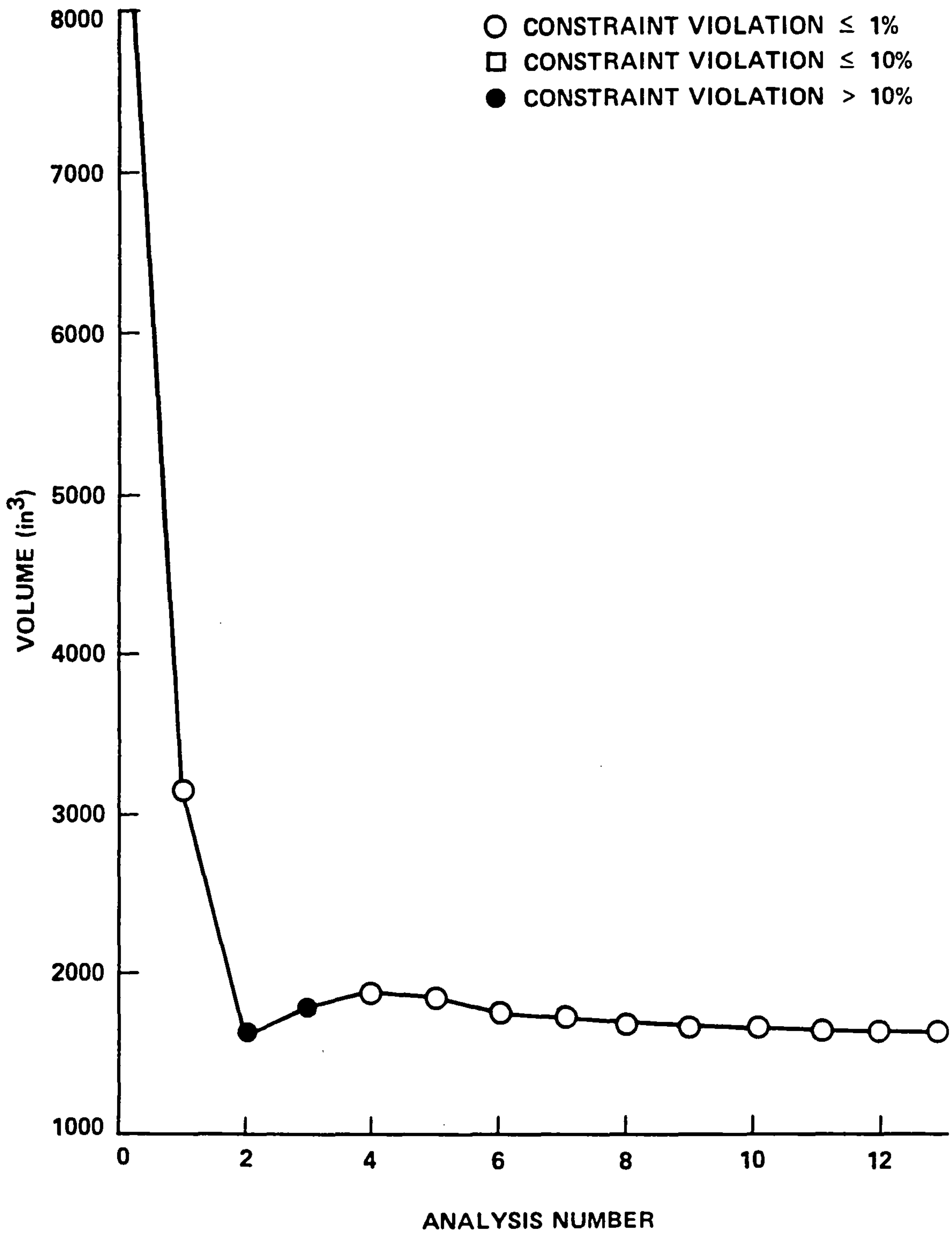


Fig. 43 - Iteration History for Problem 3, Run 2 (Option 2(P))
Three Member Frame

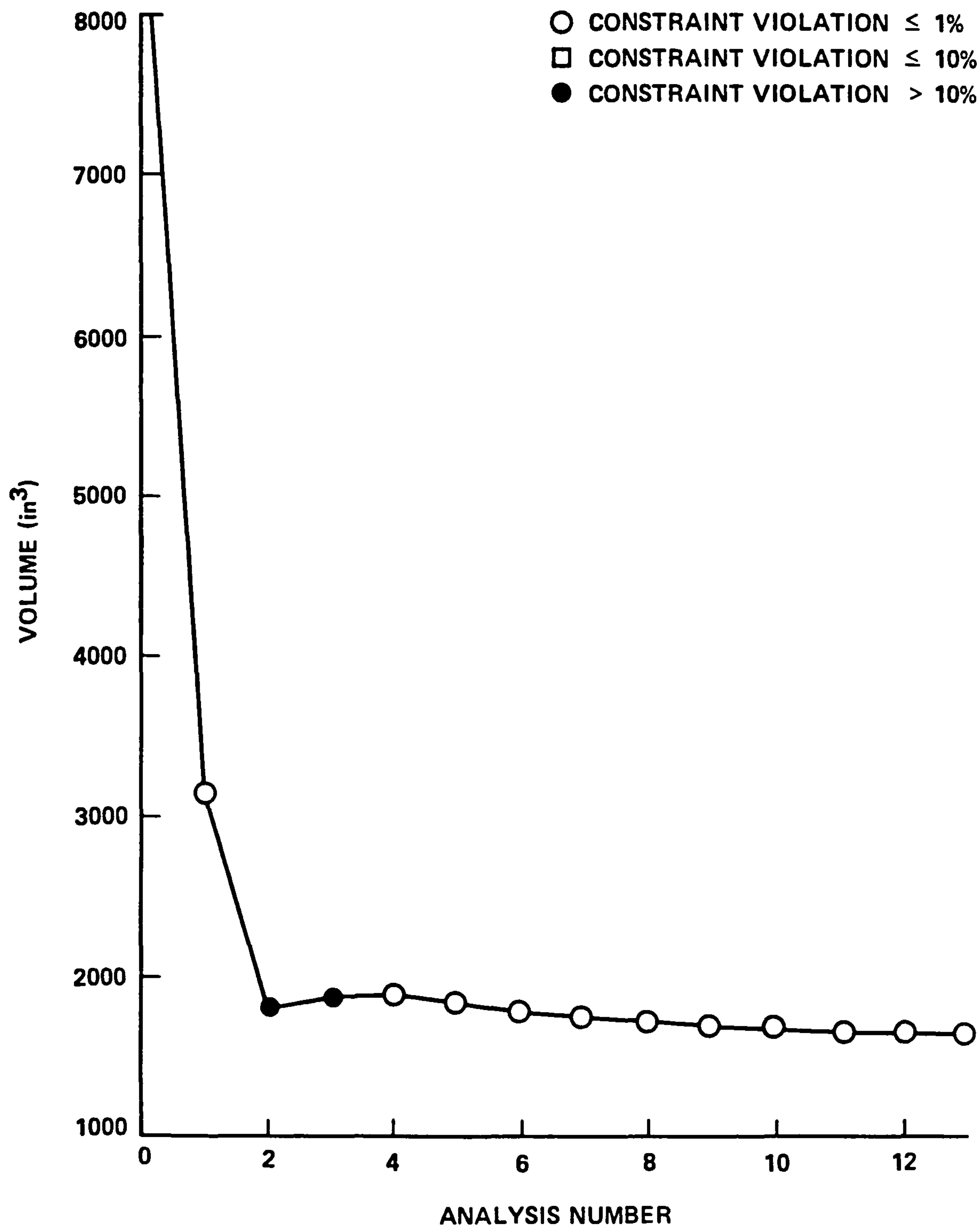


Fig. 44 - Iteration History for Problem 3, Run 3 (Option 3(P))
Three Member Frame

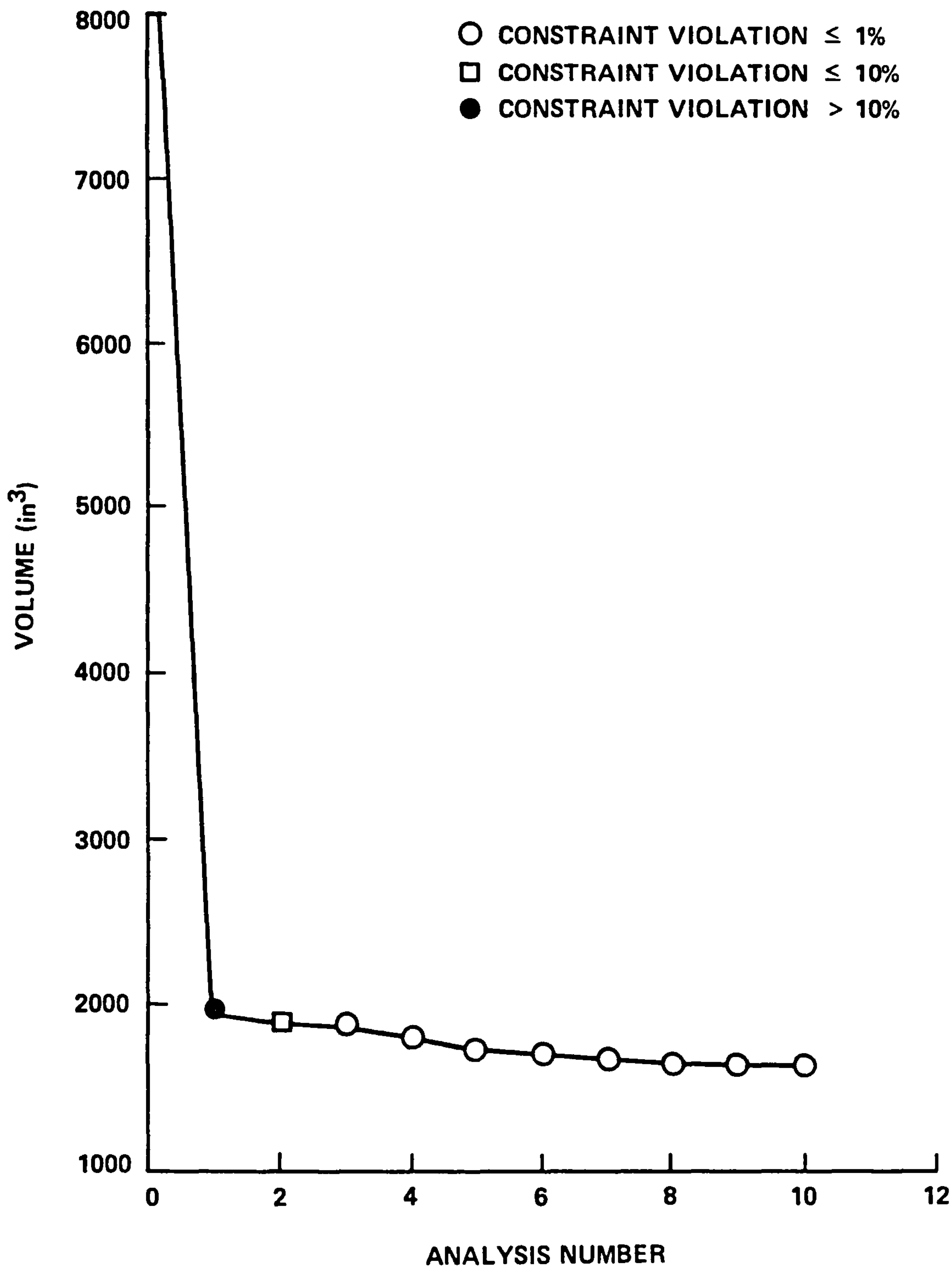


Fig. 45 - Iteration History for Problem 3, Run 4 (Option 4(P))
Three Member Frame

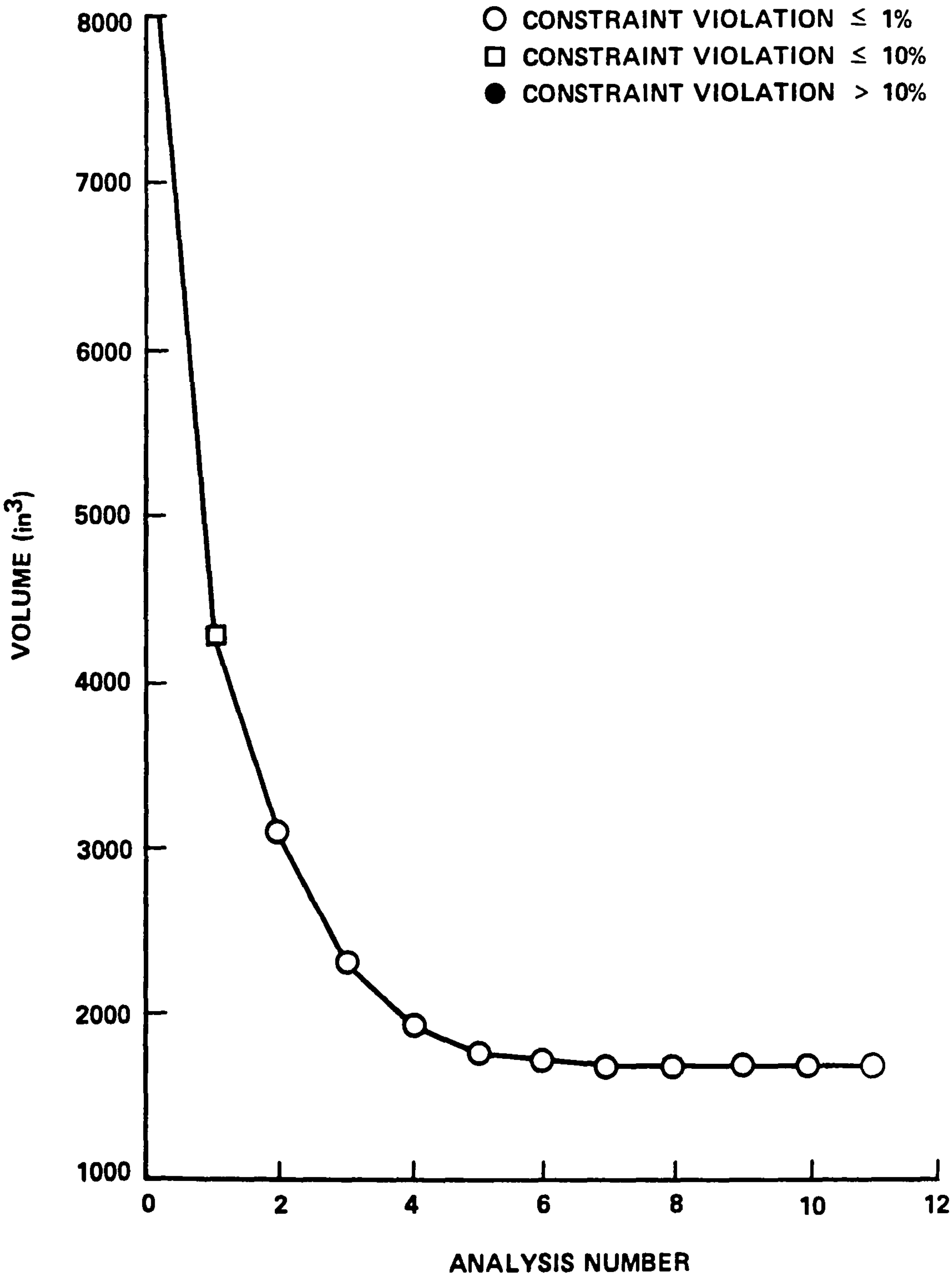


Fig. 46 - Iteration History for Problem 3, Run 5 (Option 10(P))
Three Member Frame

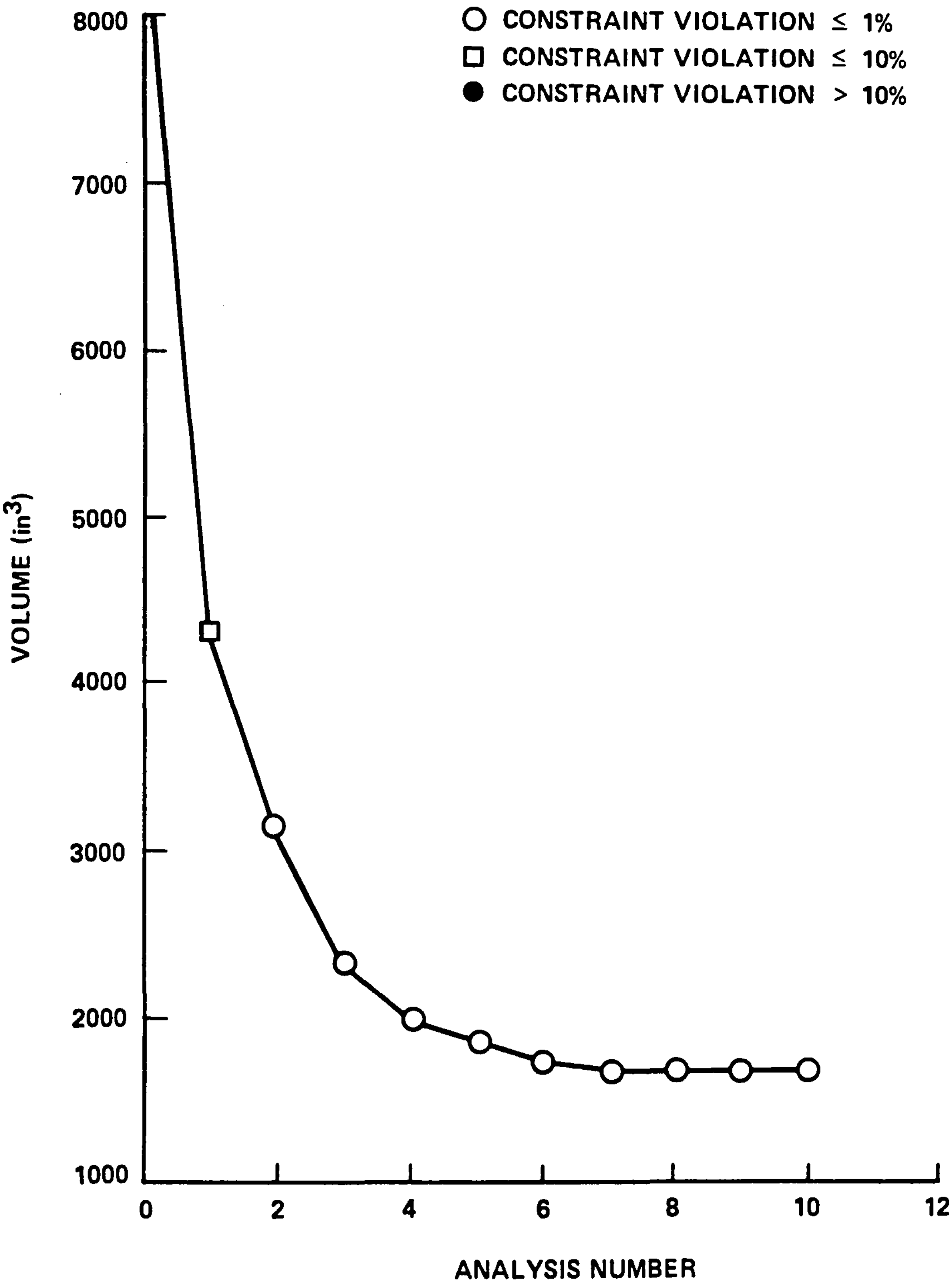


Fig. 47 - Iteration History for Problem 3, Run 6 (Option 10(D))
Three Member Frame

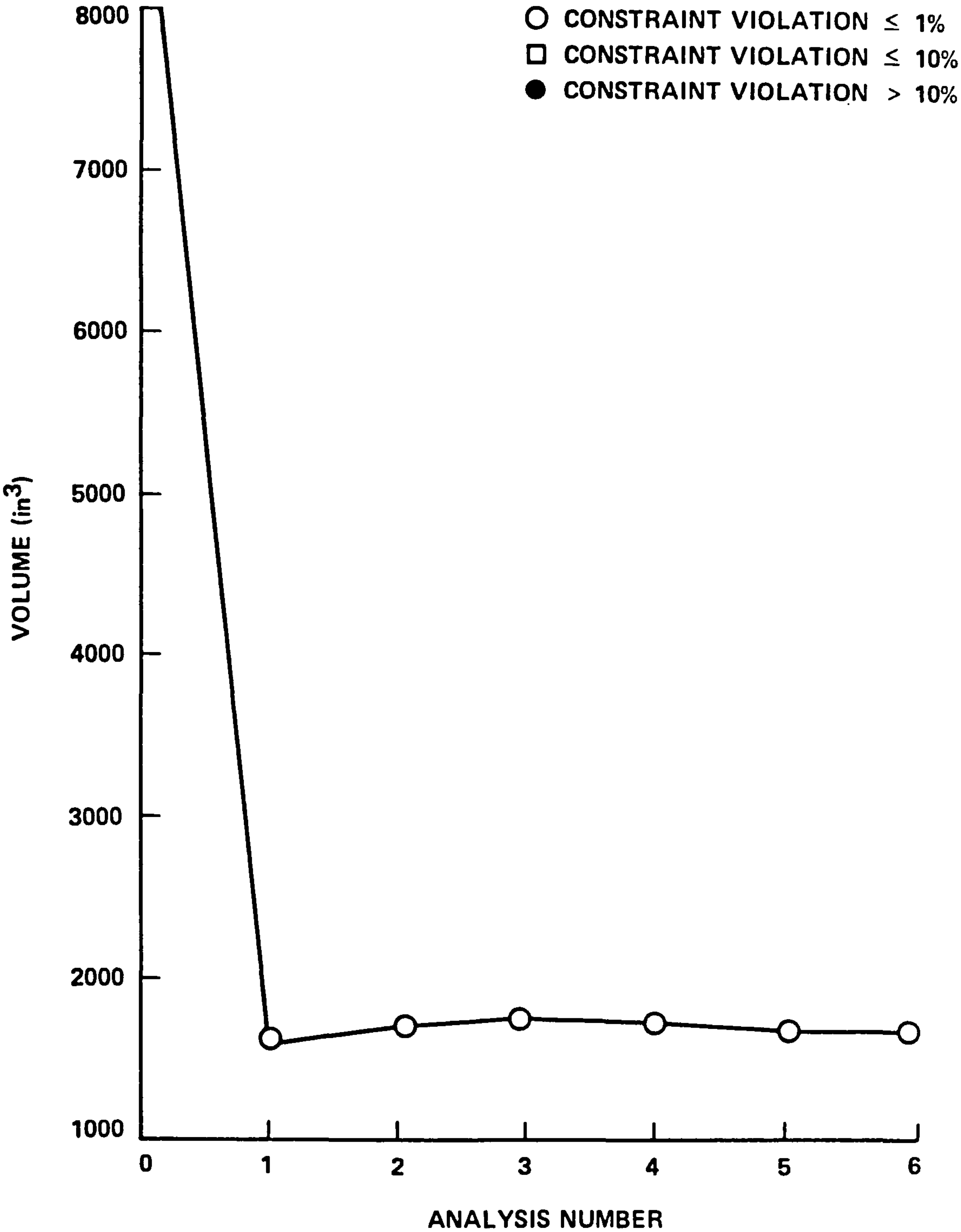
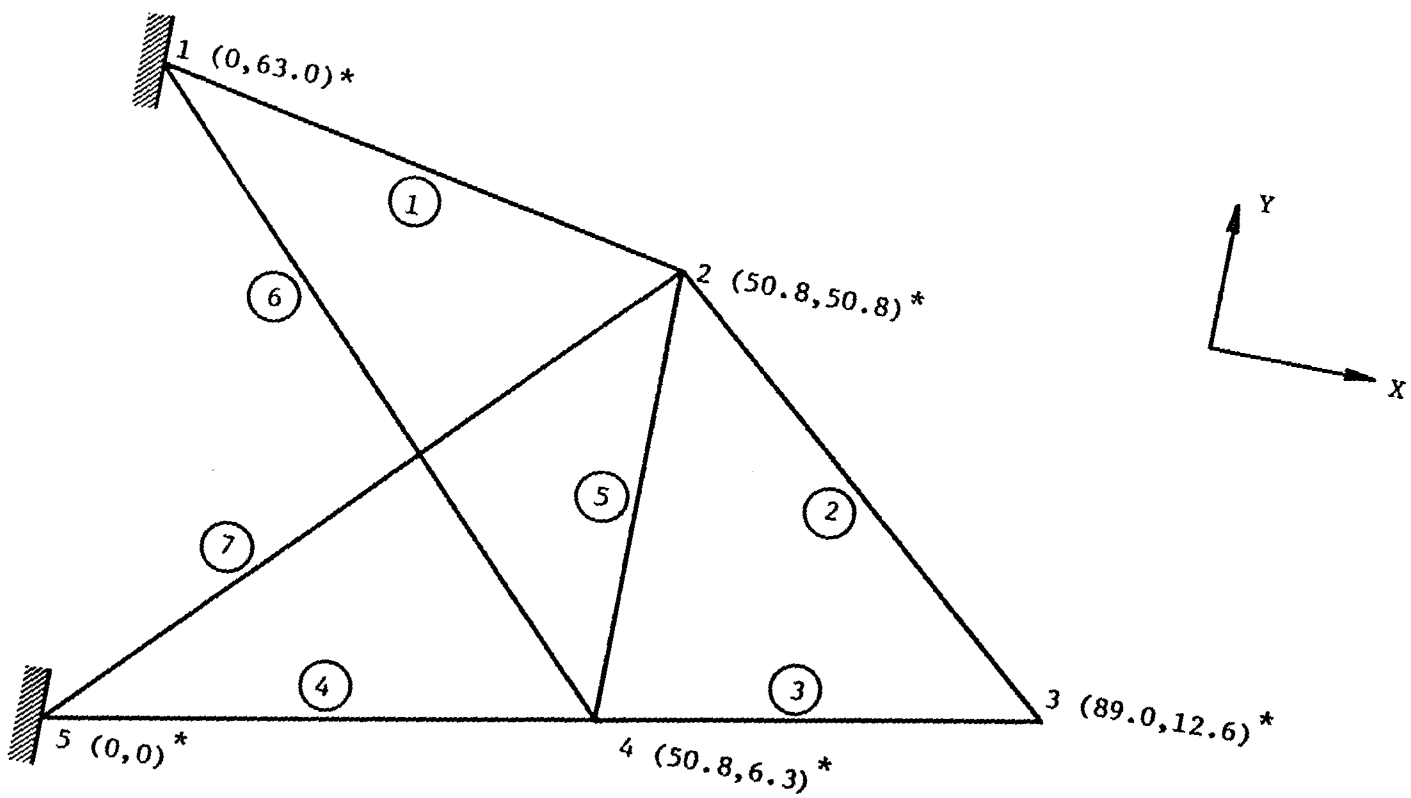


Fig. 48 - Iteration History for Problem 3, Run 7 (Option 1(PU))
Three Member Frame



* Coordinates in cm.

Fig. 49 - Seven Member Frame (Problem 4)

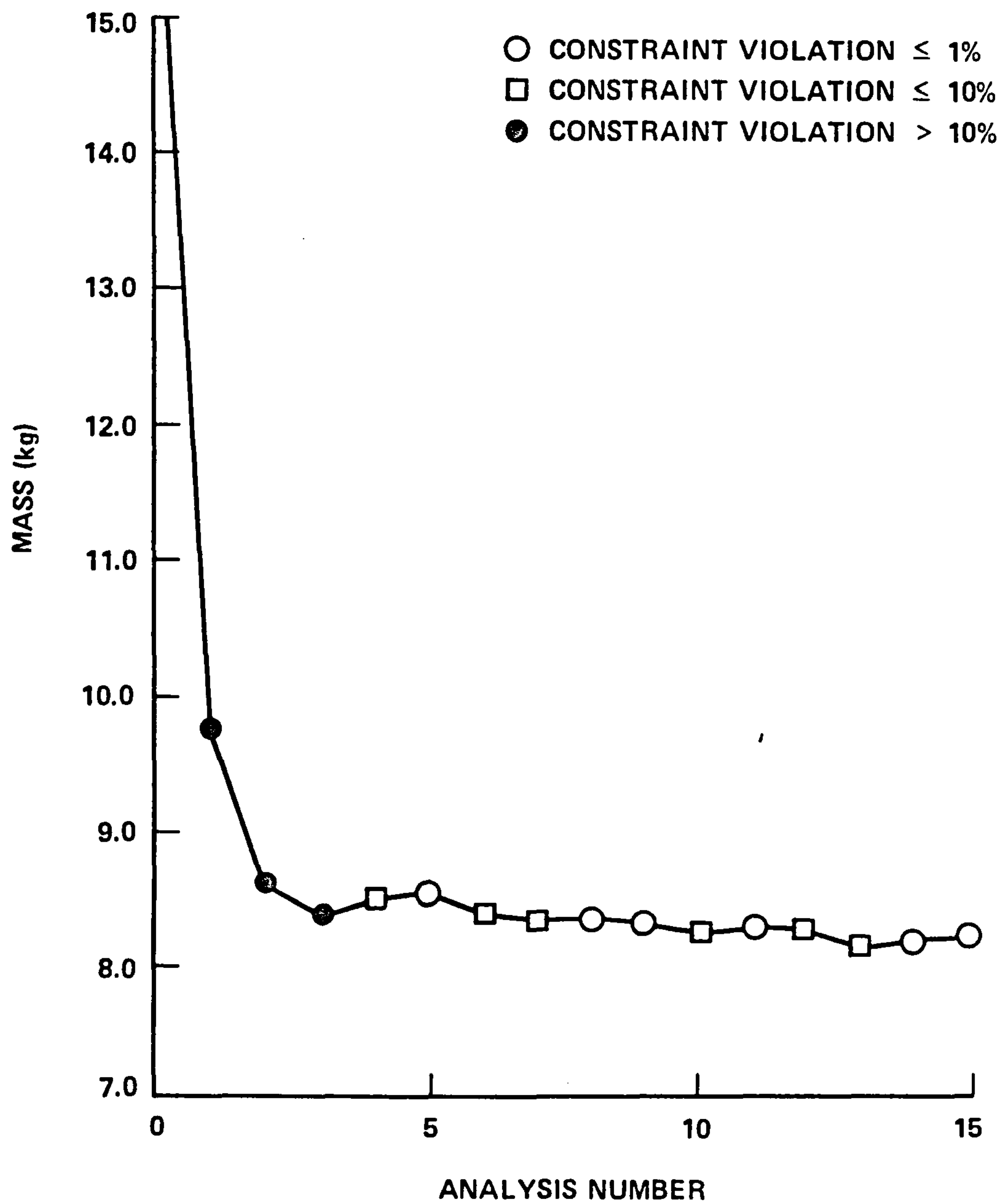


Fig. 50 - Iteration History for Problem 4, Run 2 (Option 2(P))
Seven Member Frame

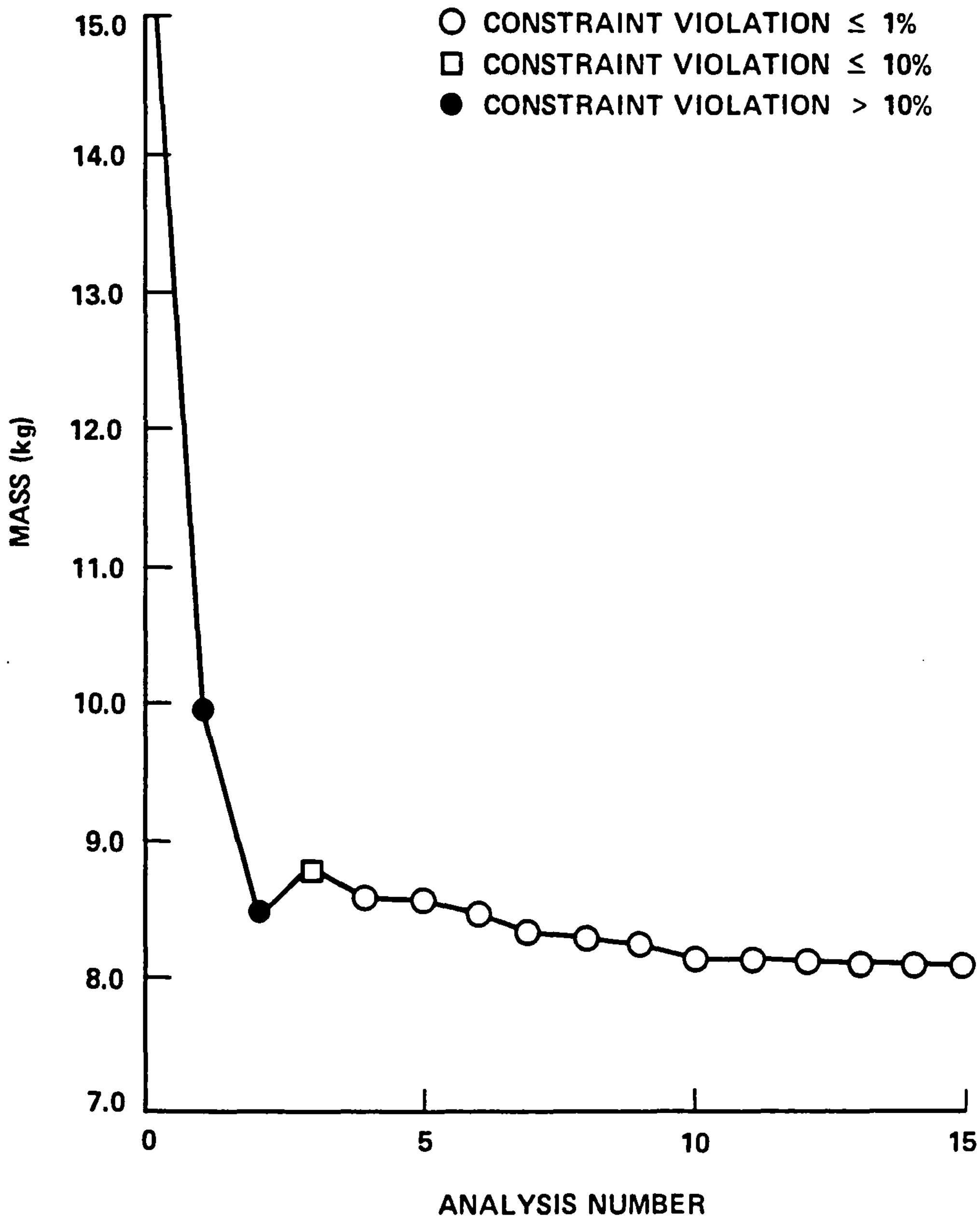


Fig. 51 - Iteration History for Problem 4, Run 3 (Option 3(P))
Seven Member Frame

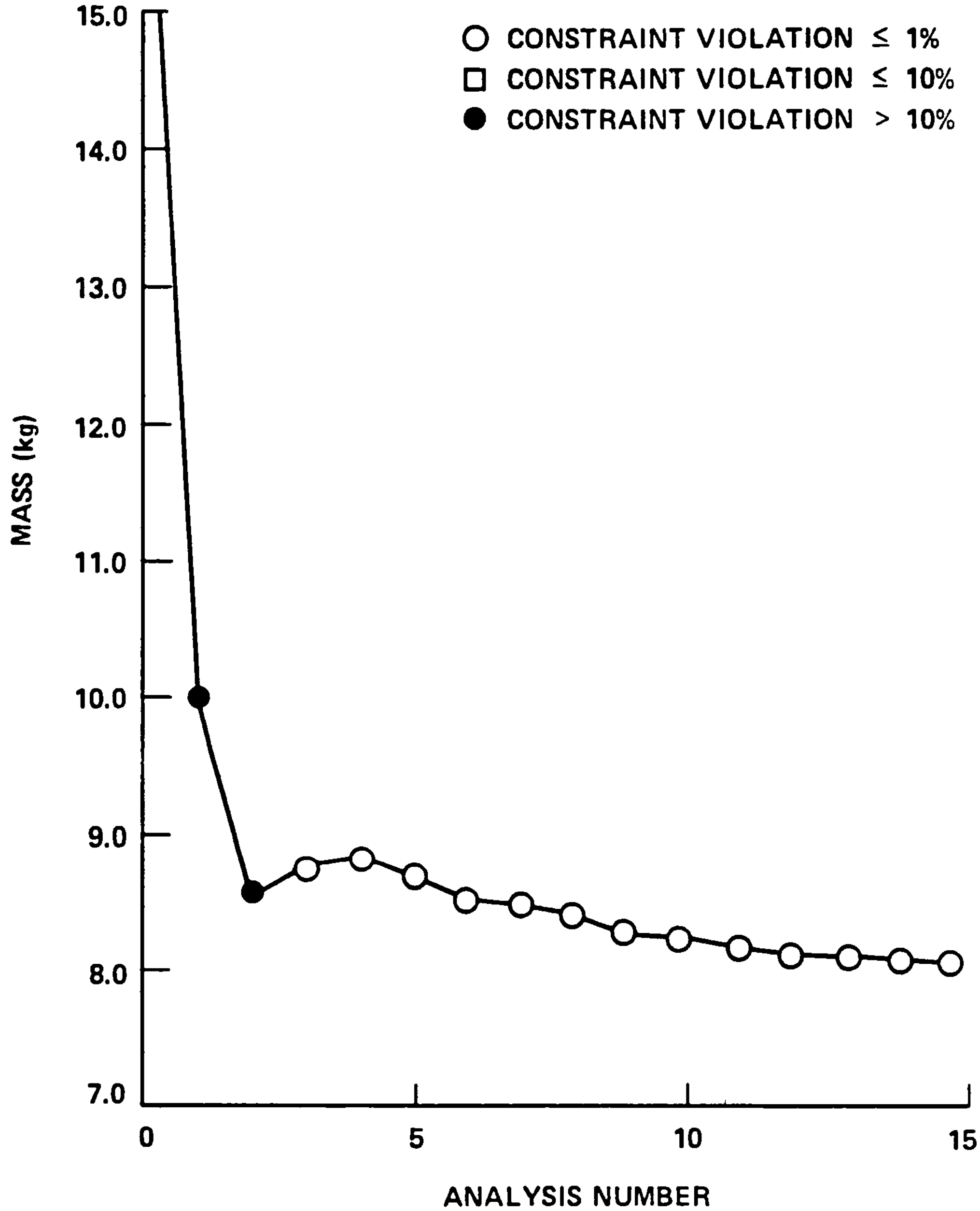


Fig. 52 - Iteration History for Problem 4, Run 4 (Option 6(P))
Seven Member Frame

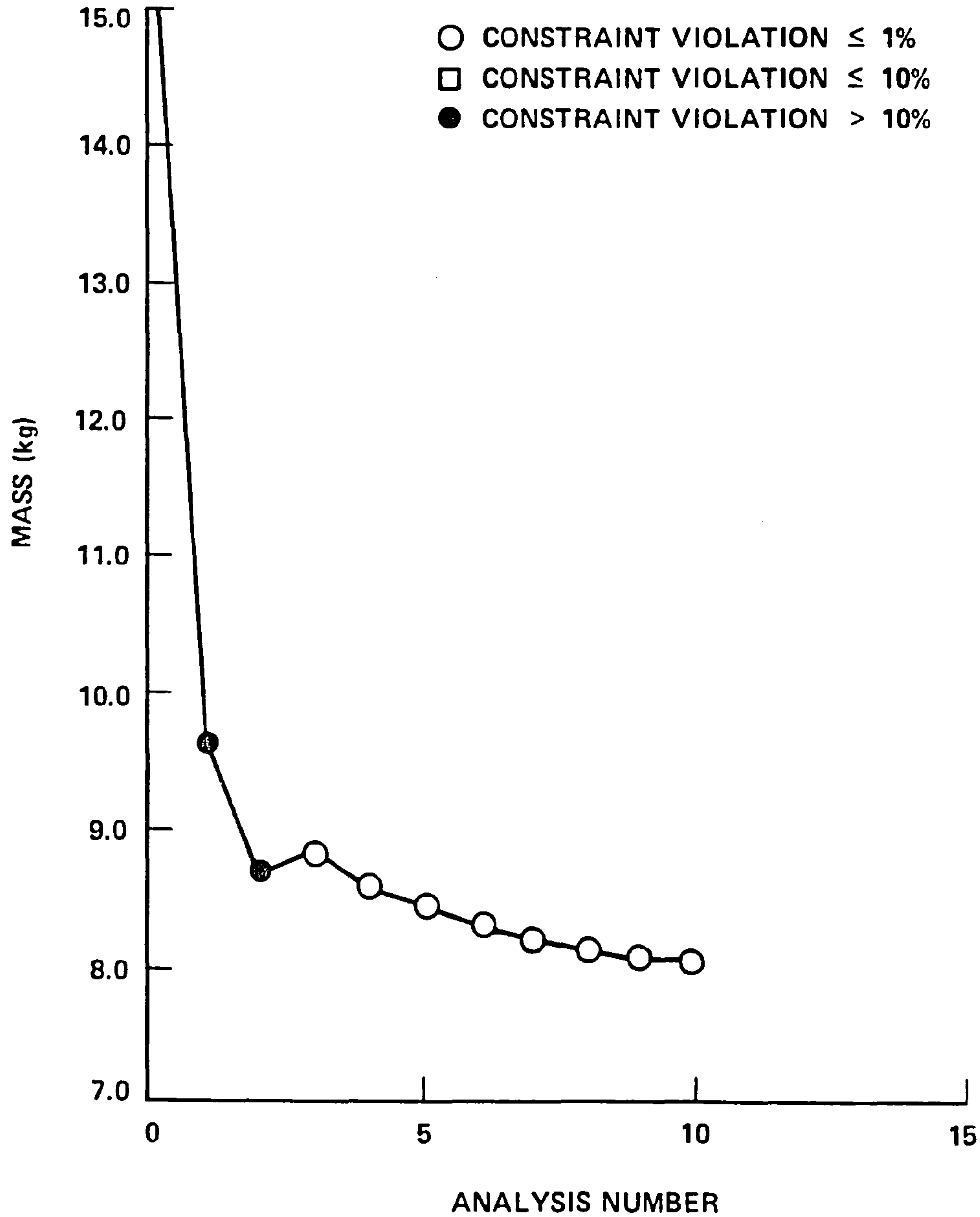


Fig. 53 - Iteration History for Problem 4, Run 5 (Option 12(P))
Seven Member Frame

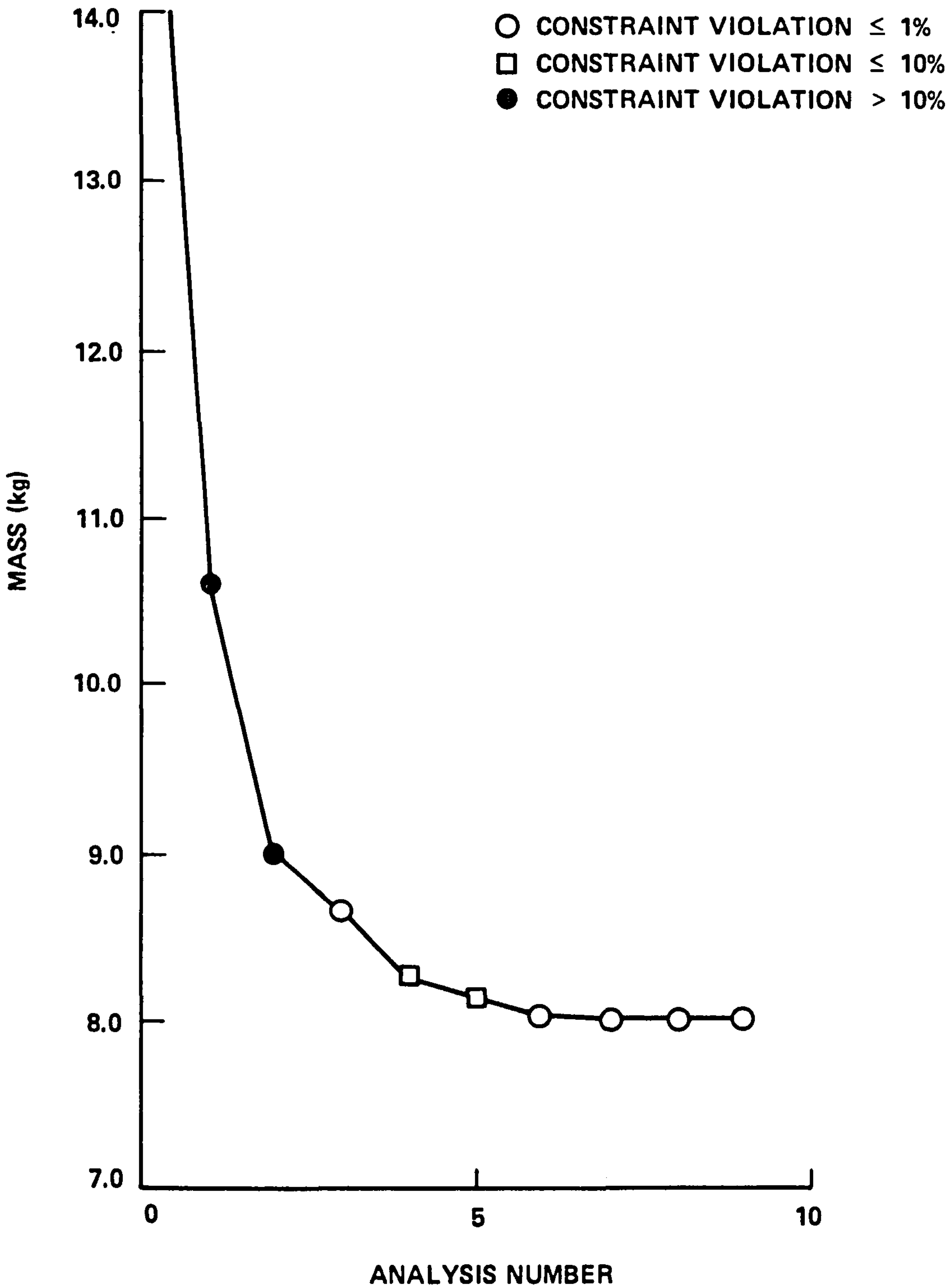


Fig. 54 - Iteration History for Problem 4, Run 6 (Option 12(D))
Seven Member Frame

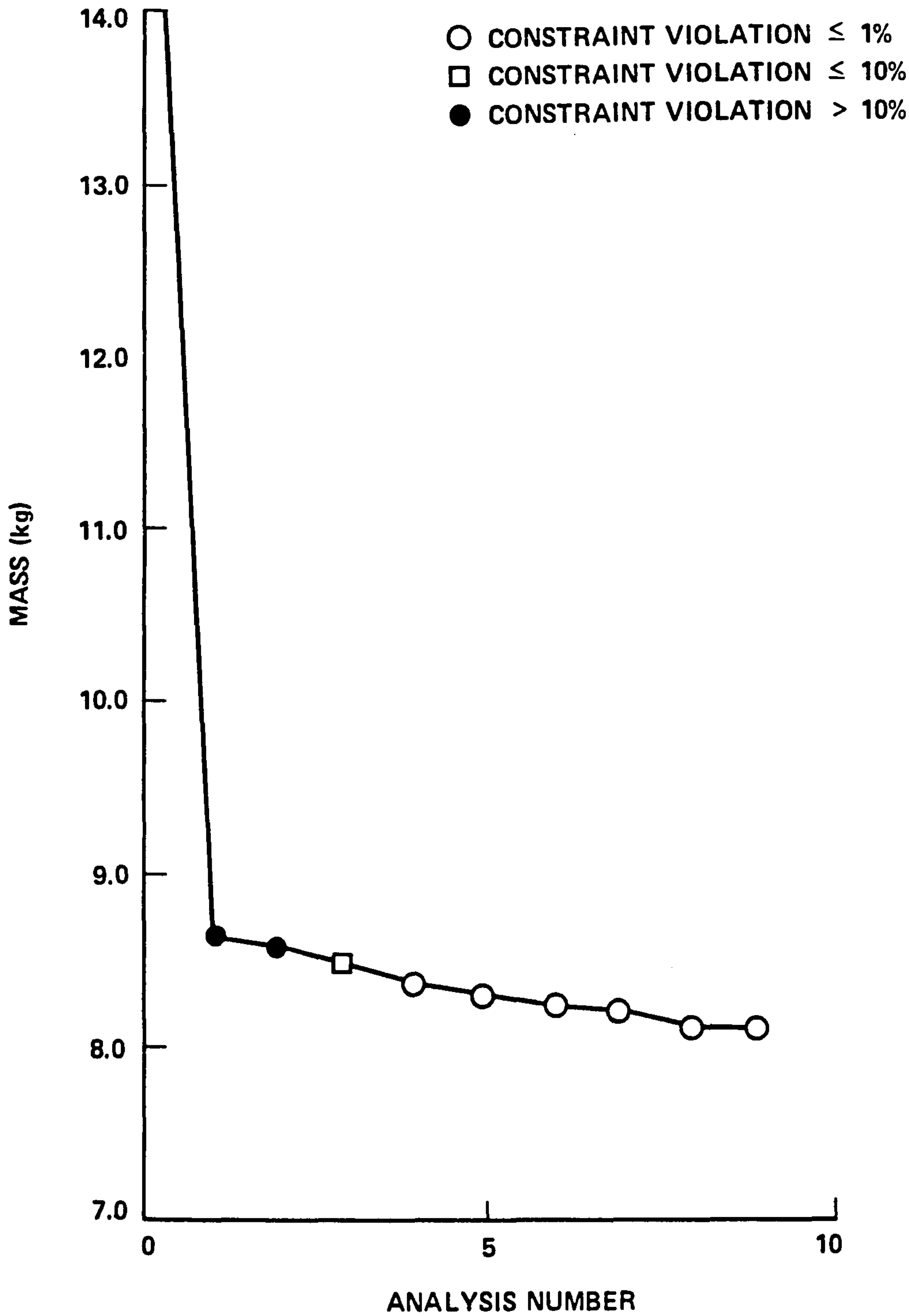


Fig. 55 - Iteration History for Problem 4, Run 7 (Option 3(PU))
Seven Member Frame

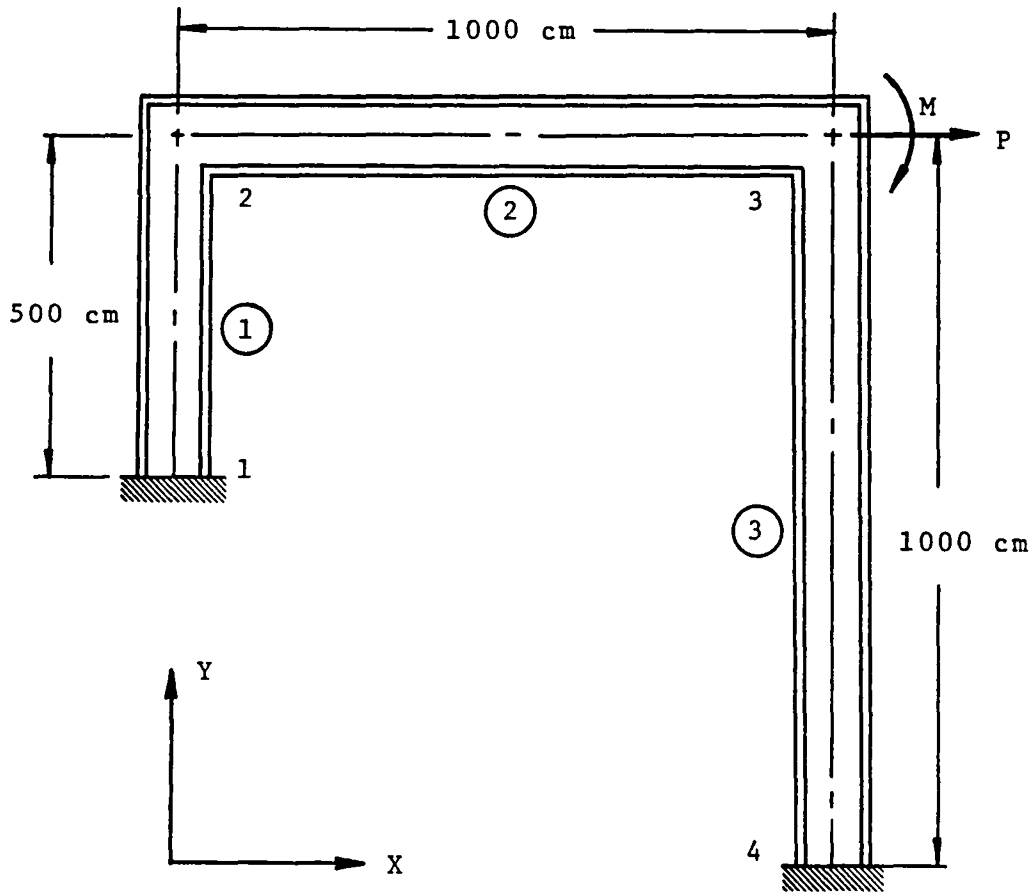


Fig. 56 - Portal Frame (Problem 5)

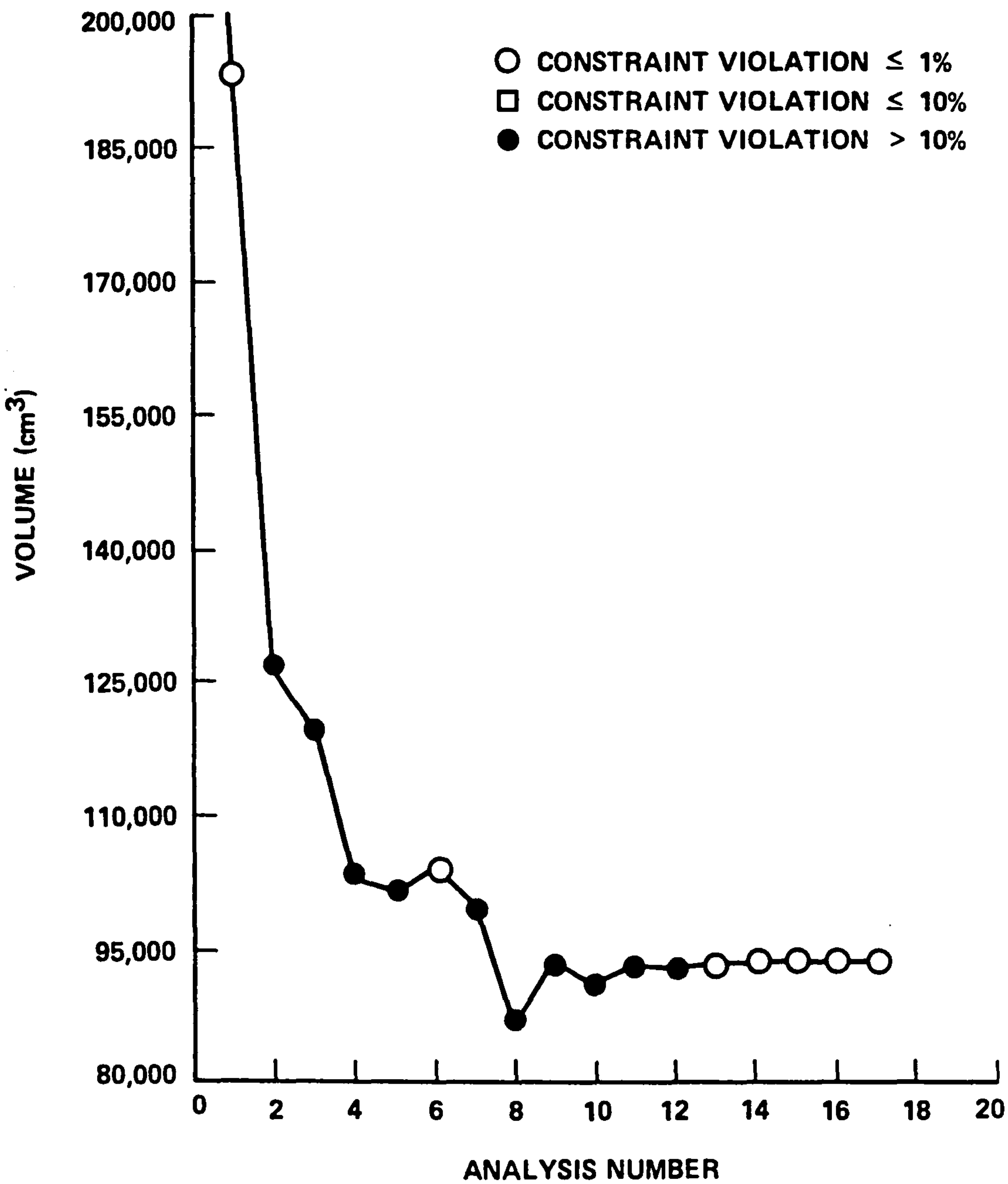


Fig. 57 - Iteration History for Problem 5, Run 1 (Option 1(P))
Portal Frame

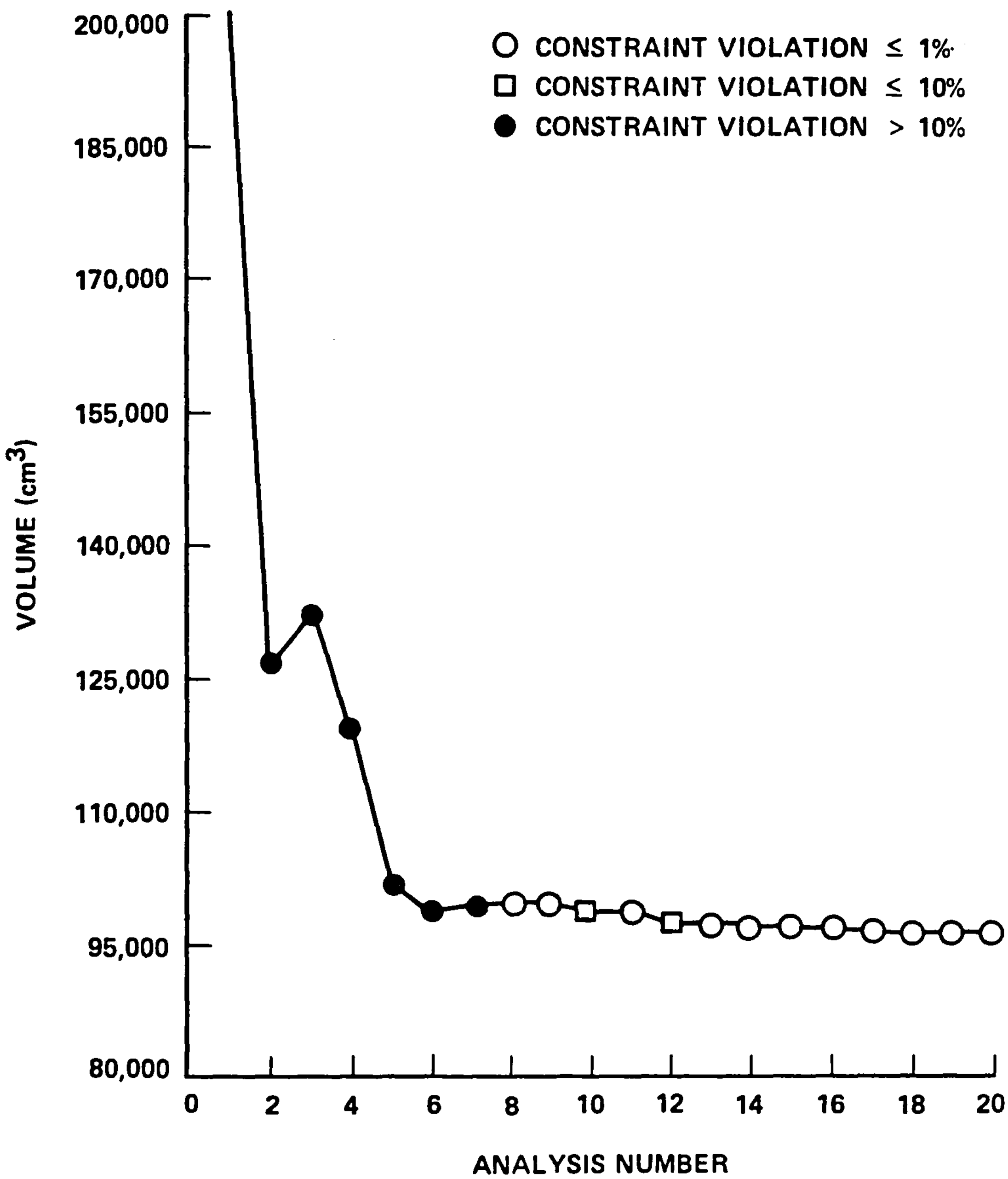


Fig. 58 - Iteration History for Problem 5, Run 2 (Option 2(P))
Portal Frame

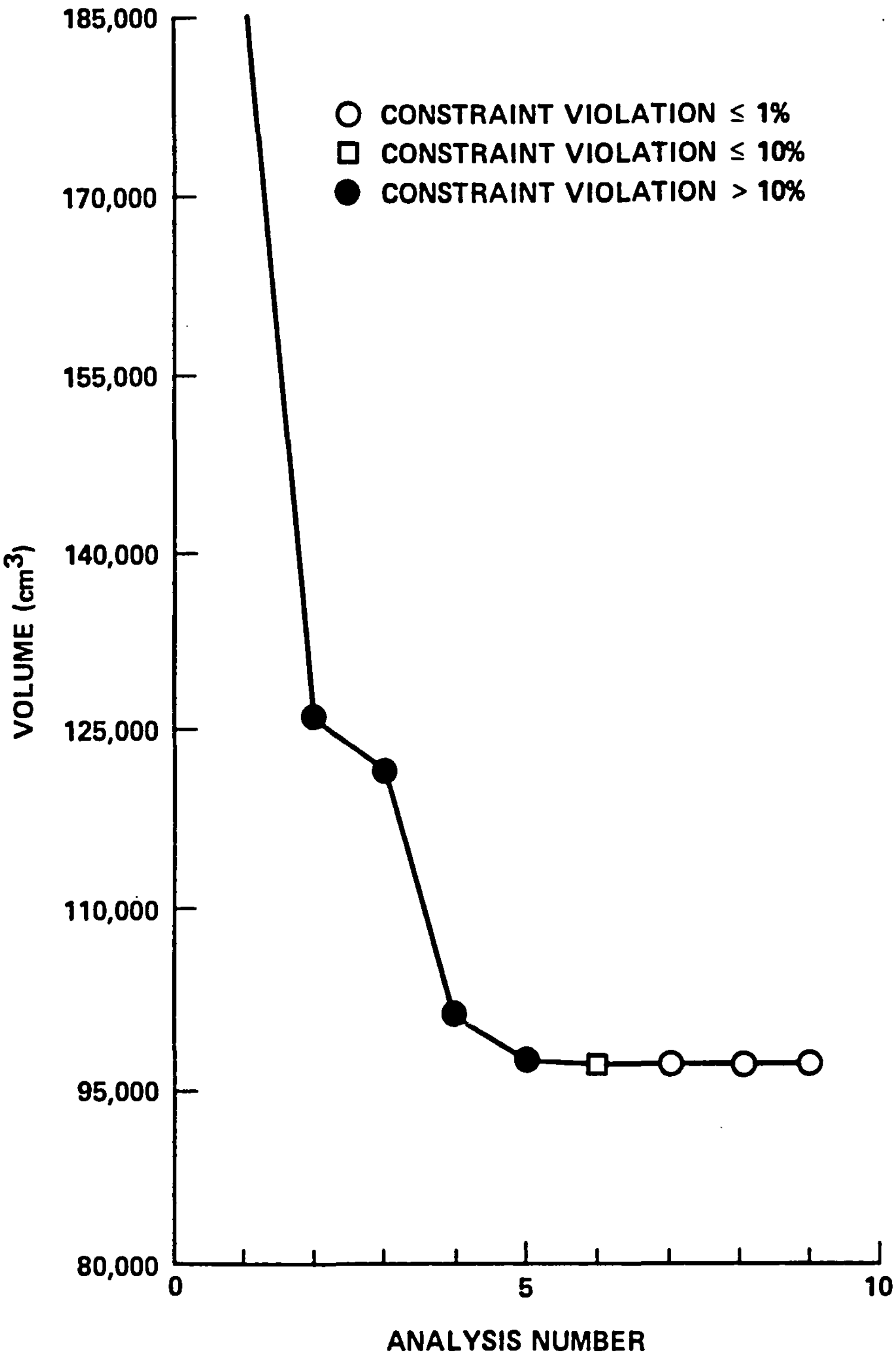


Fig. 59 - Iteration History for Problem 5, Run 3 (Option 3(P)) Portal Frame

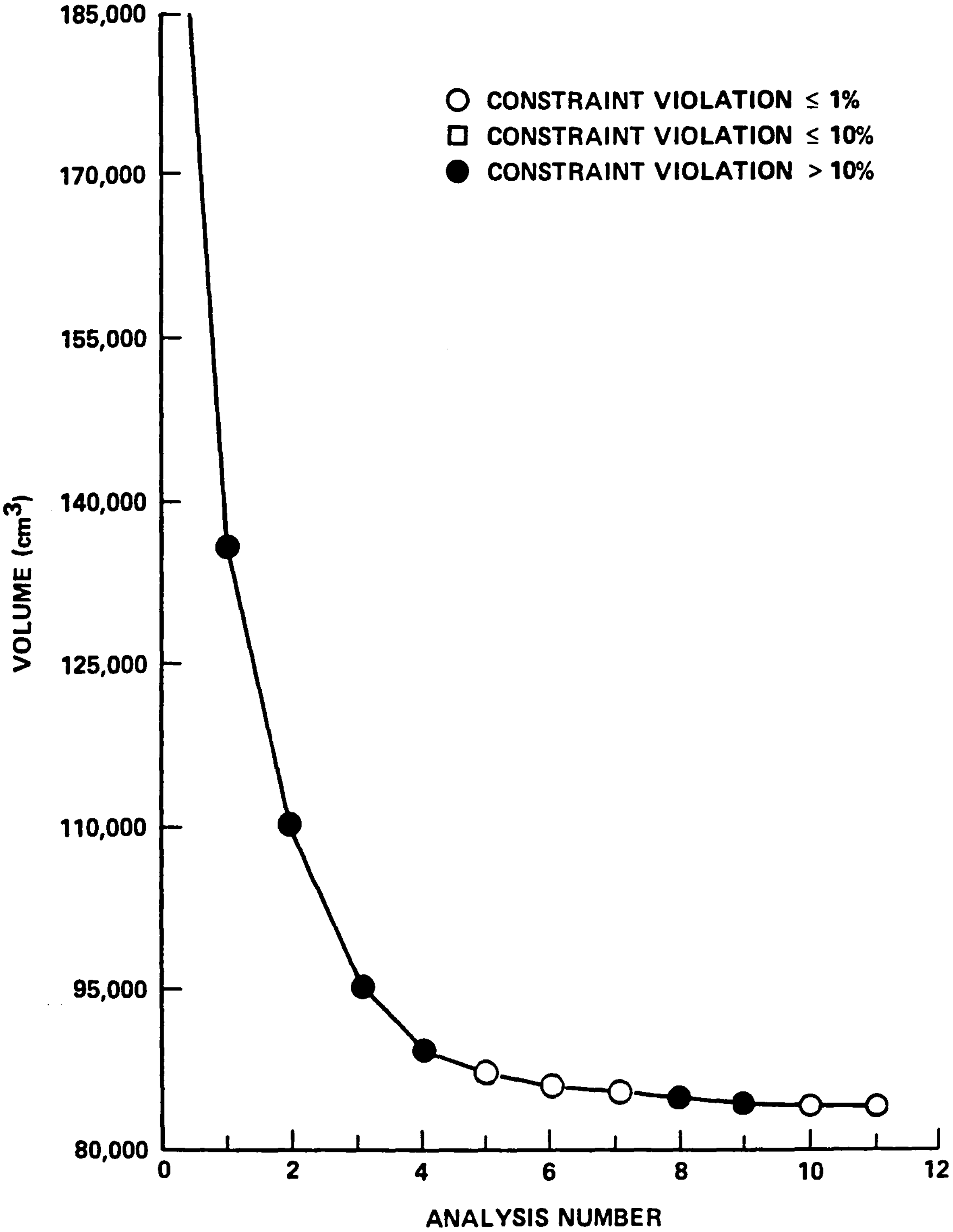


Fig. 60 - Iteration History for Problem 5, Run 4 (Option 12(P))
Portal Frame

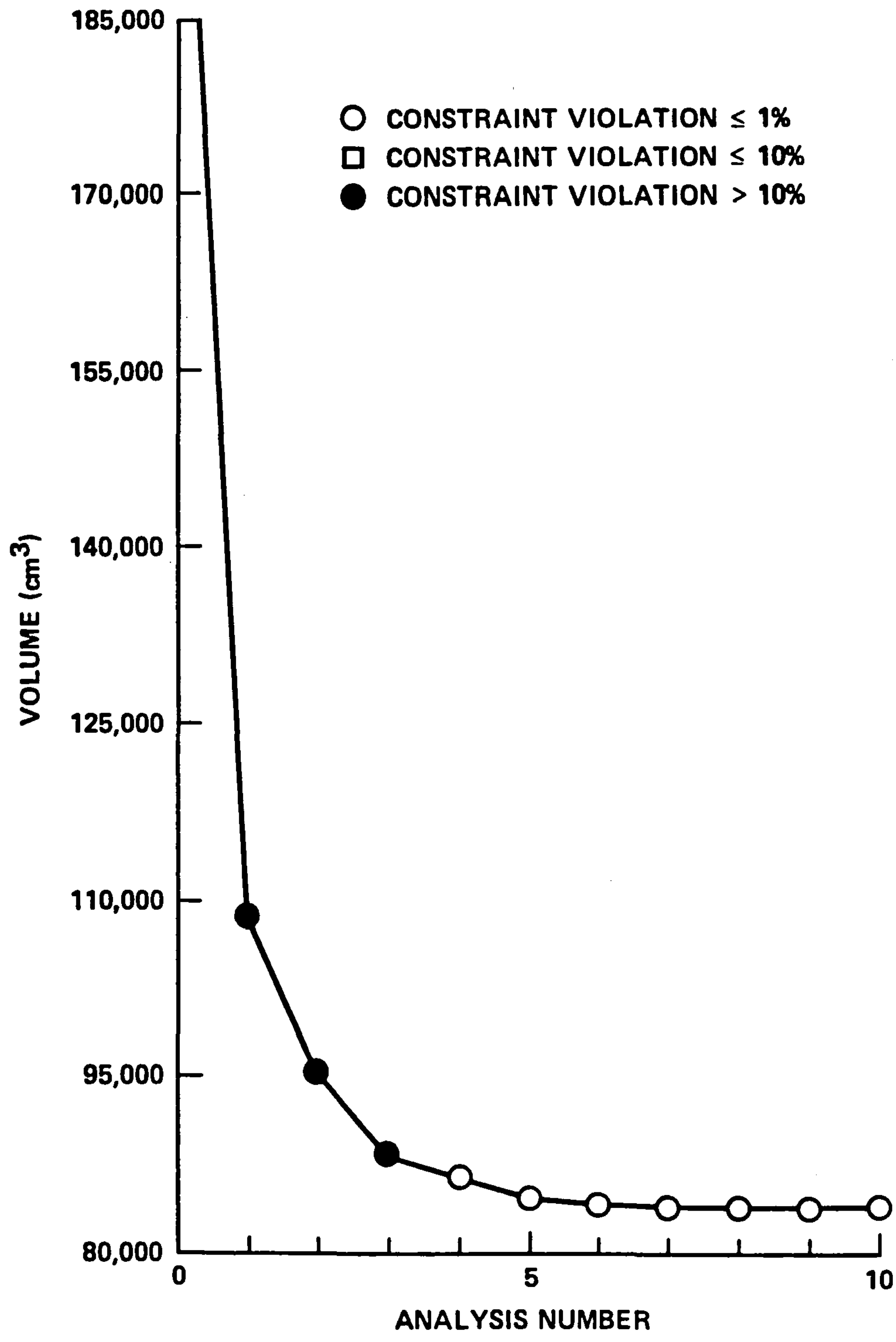


Fig. 61 - Iteration History for Problem 5, Run 5 (Option 12(D)) Portal Frame

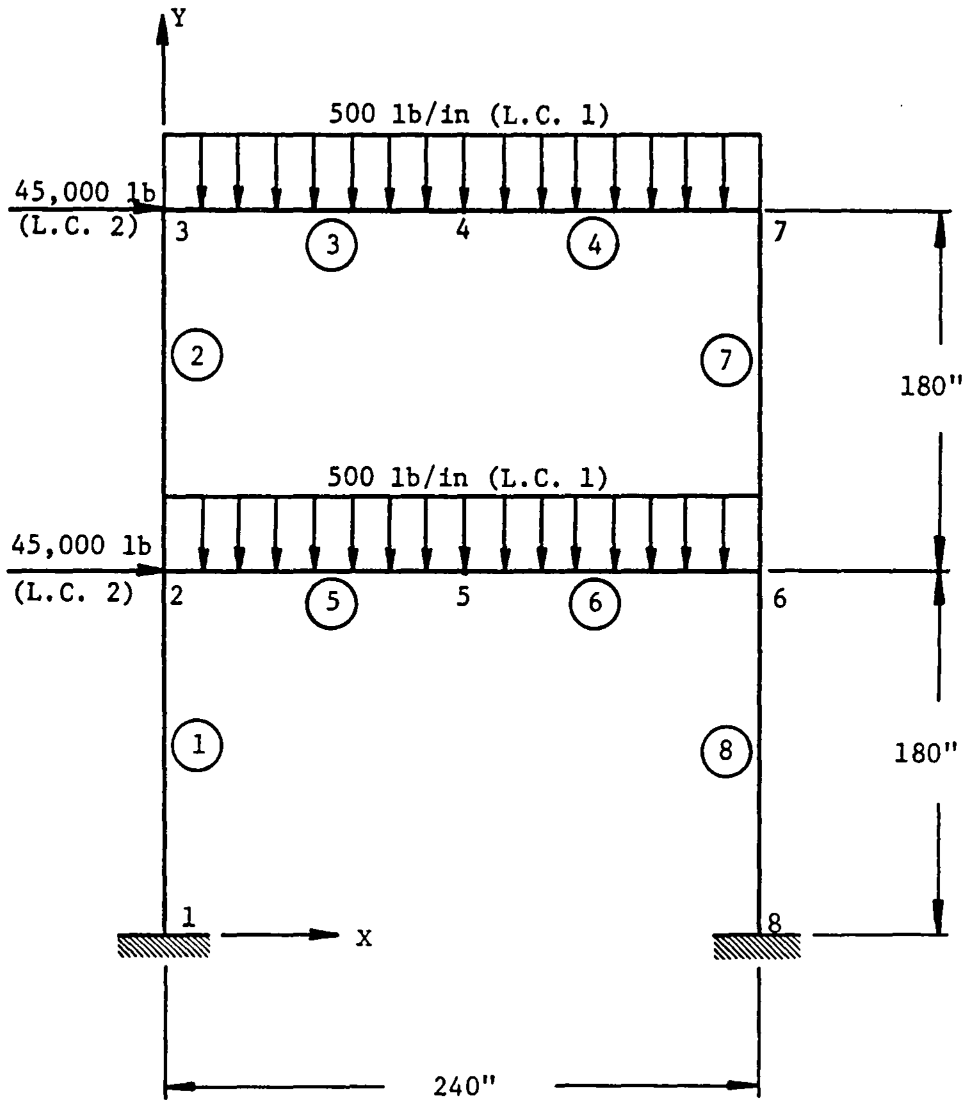


Fig. 62 - One Bay / Two Story Frame (Problem 6)

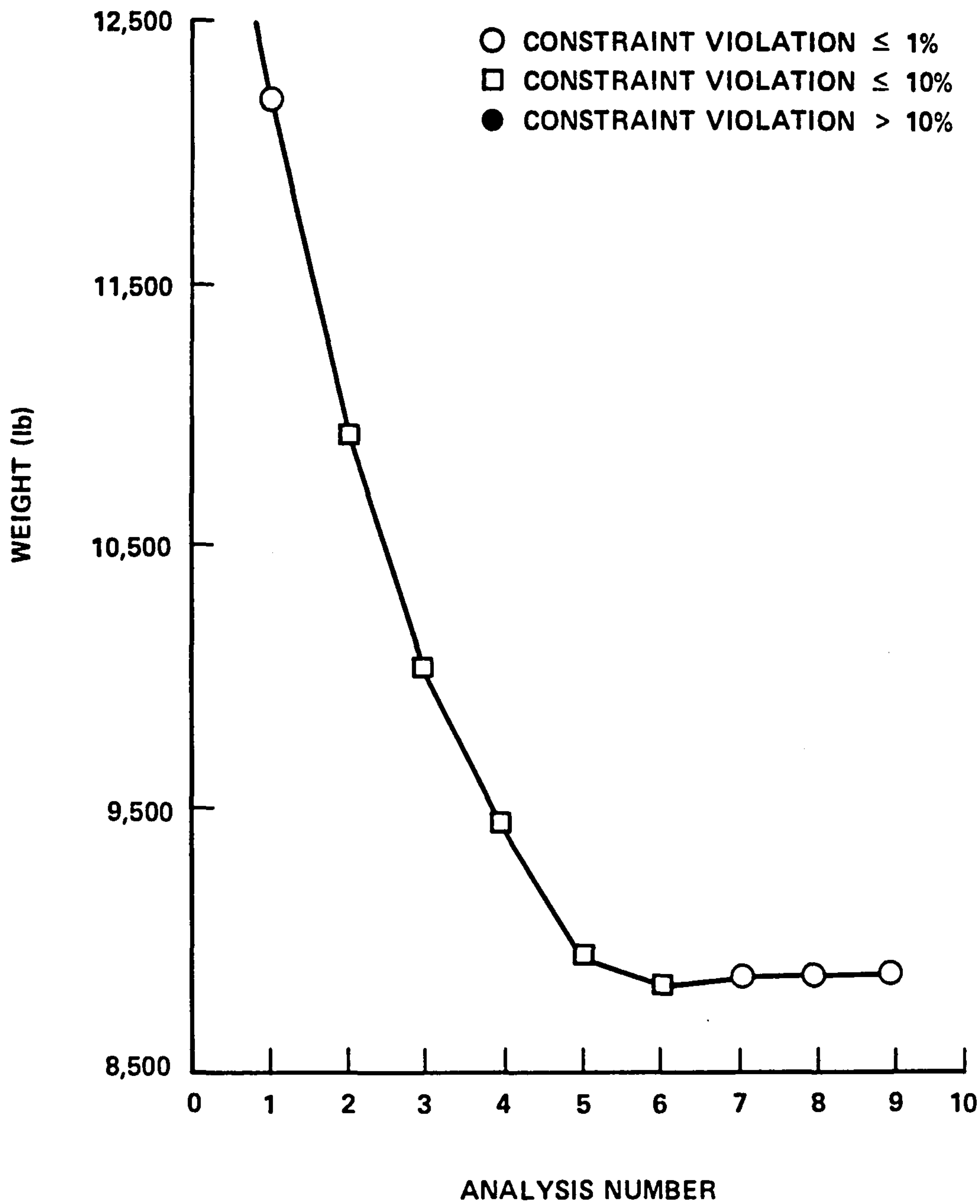


Fig. 63 - Iteration History for Problem 6, Case A, Run 1 (Option 1(P))
One Bay / Two Story Frame

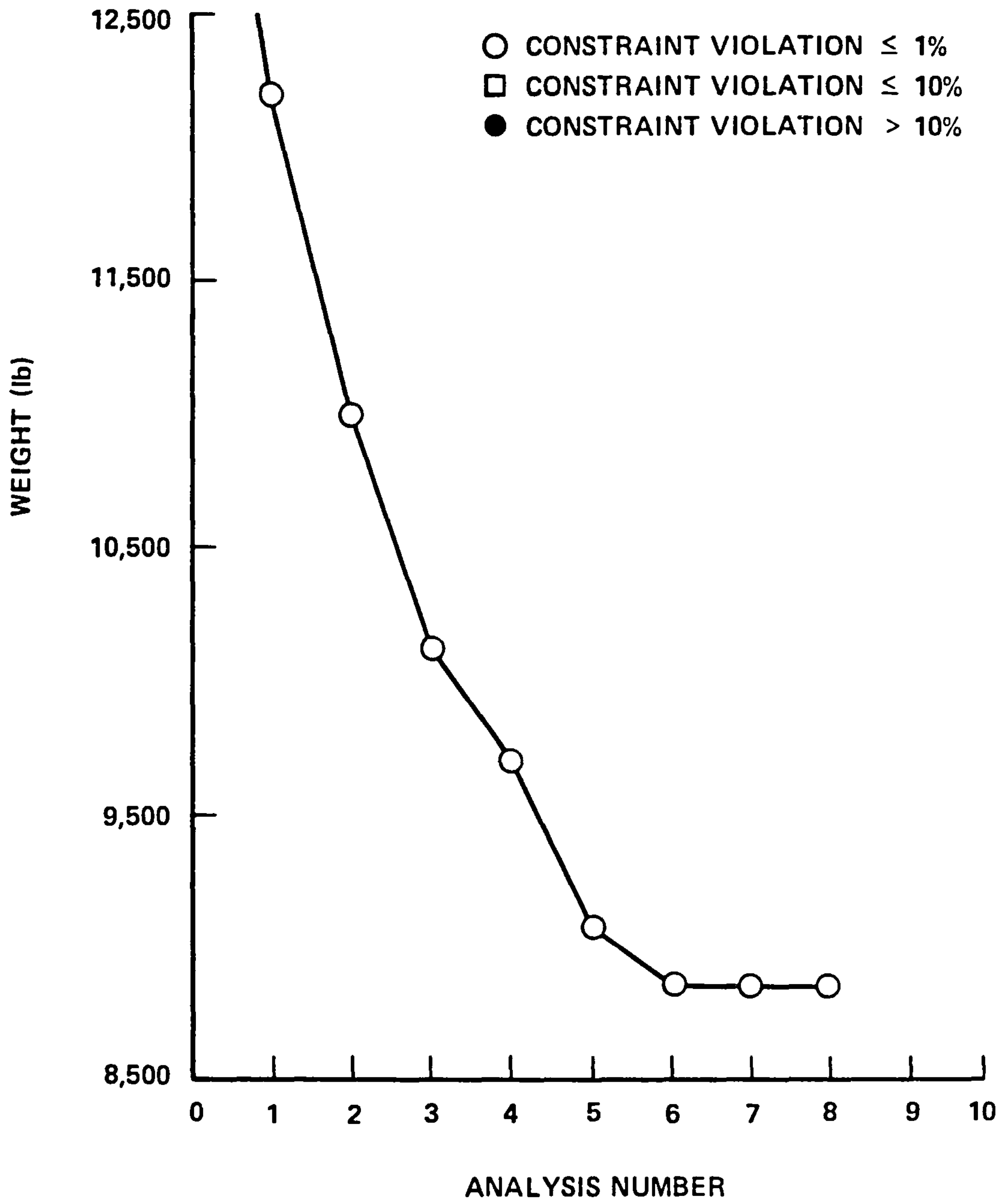


Fig. 64 - Iteration History for Problem 6, Case A, Run 2 (Option 3(P))
One Bay / Two Story Frame

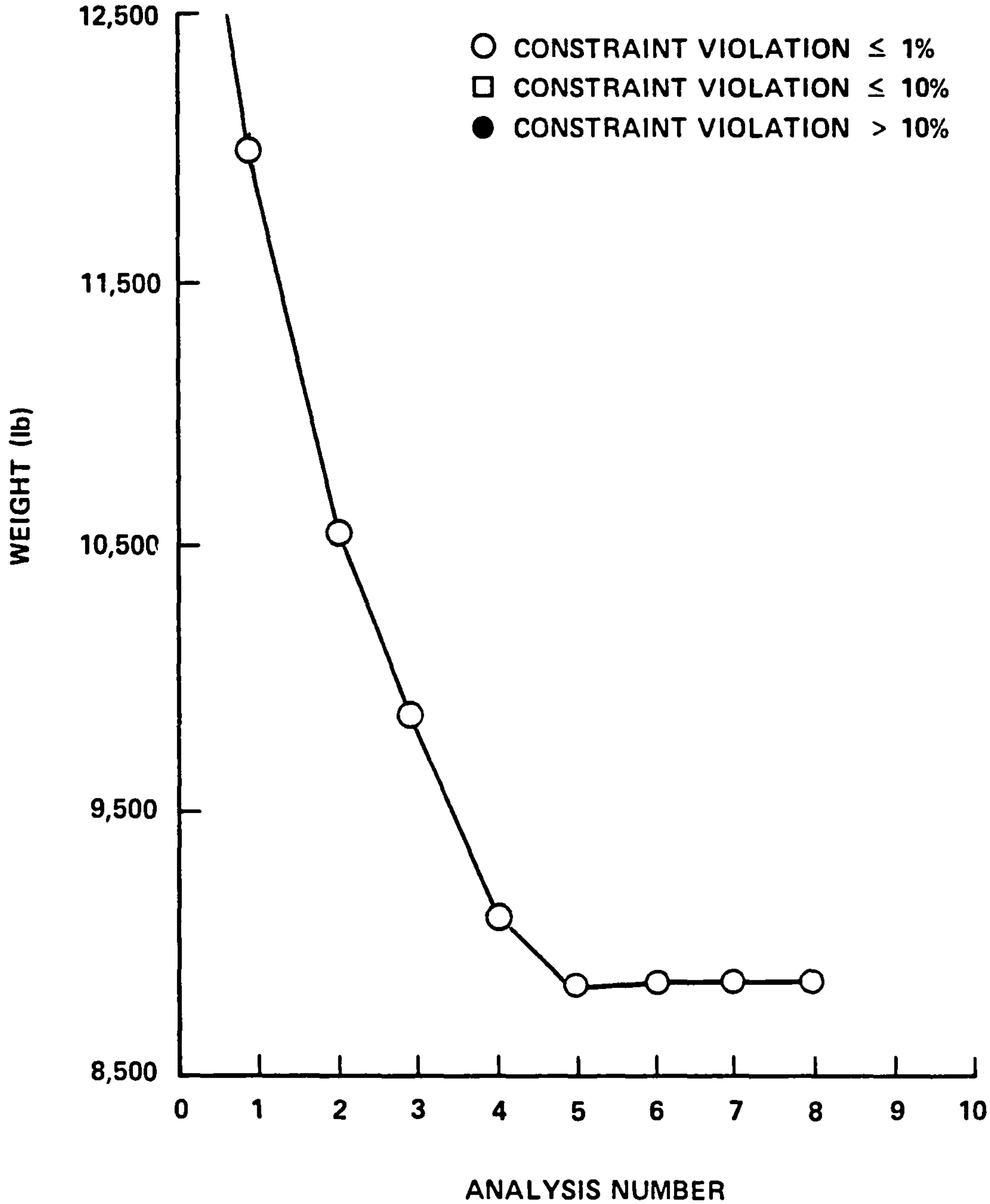


Fig. 65 - Iteration History for Problem 6, Case A, Run 3 (Option 6(P))
One Bay / Two Stroy Frame

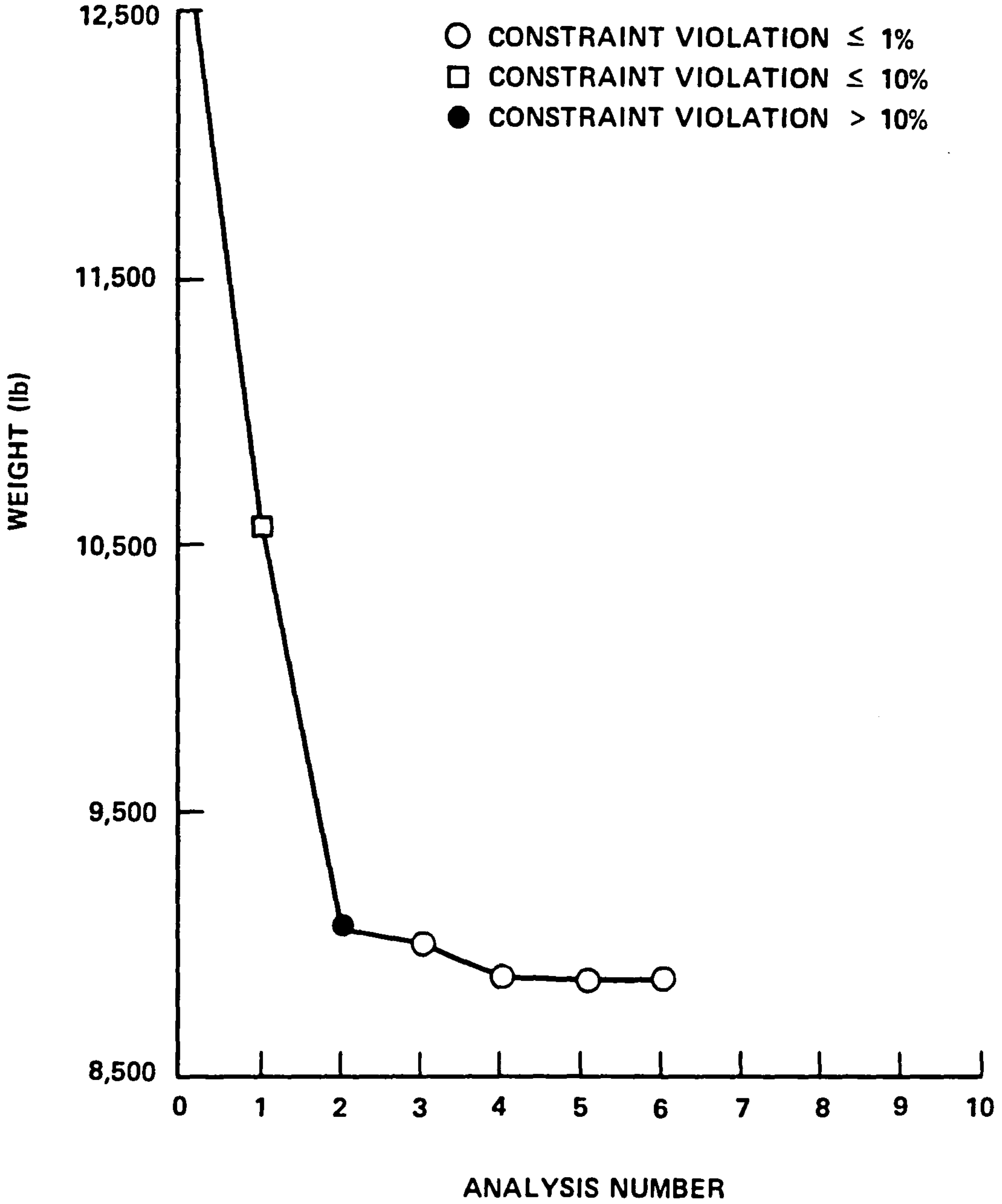


Fig. 66 - Iteration History for Problem 6, Case A, Run 4 (Option 12(P))
One Bay / Two Story Frame

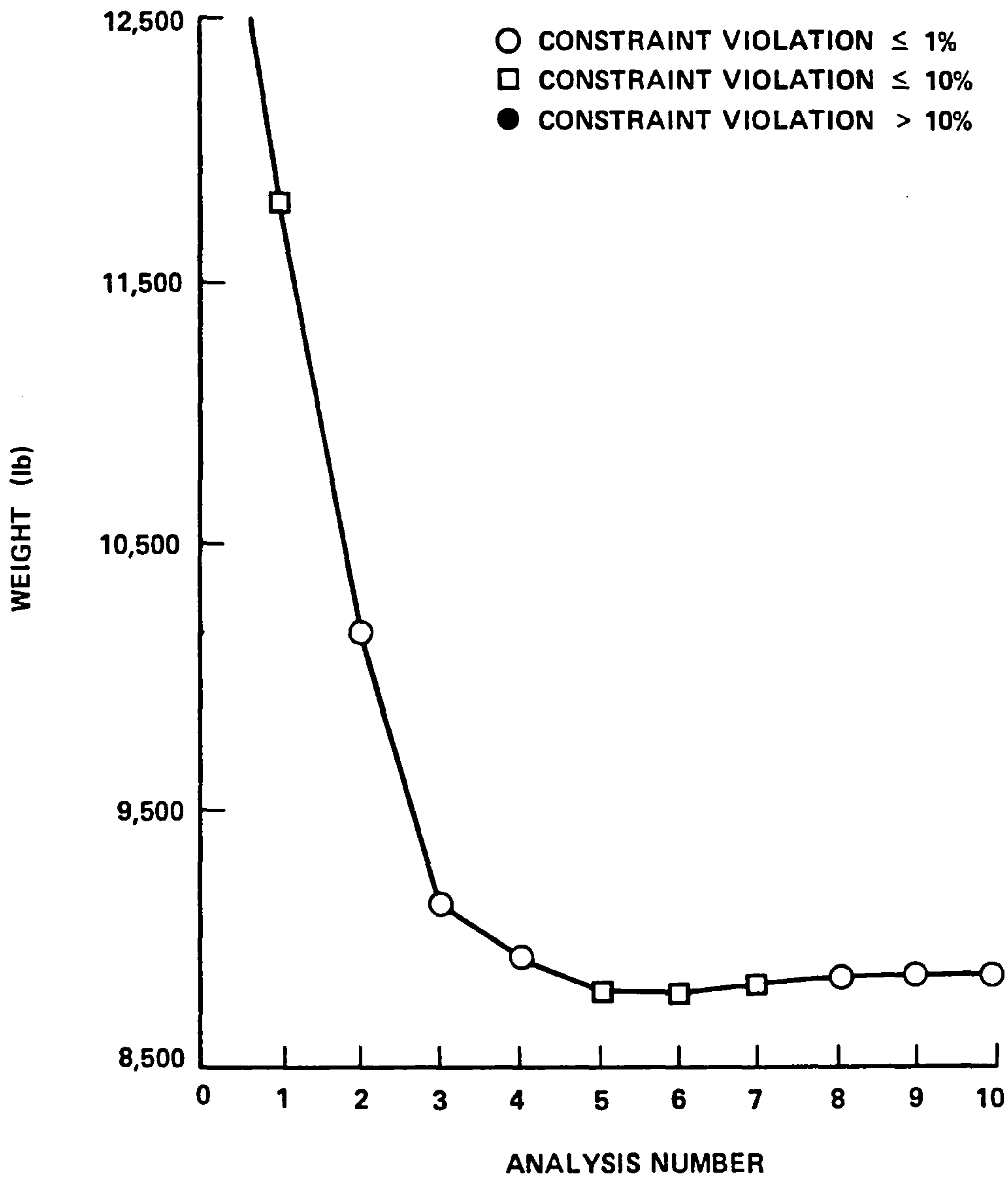


Fig. 67 - Iteration History for Problem 6, Case A, Run 5 (Option 12(D))
One Bay / Two Story Frame

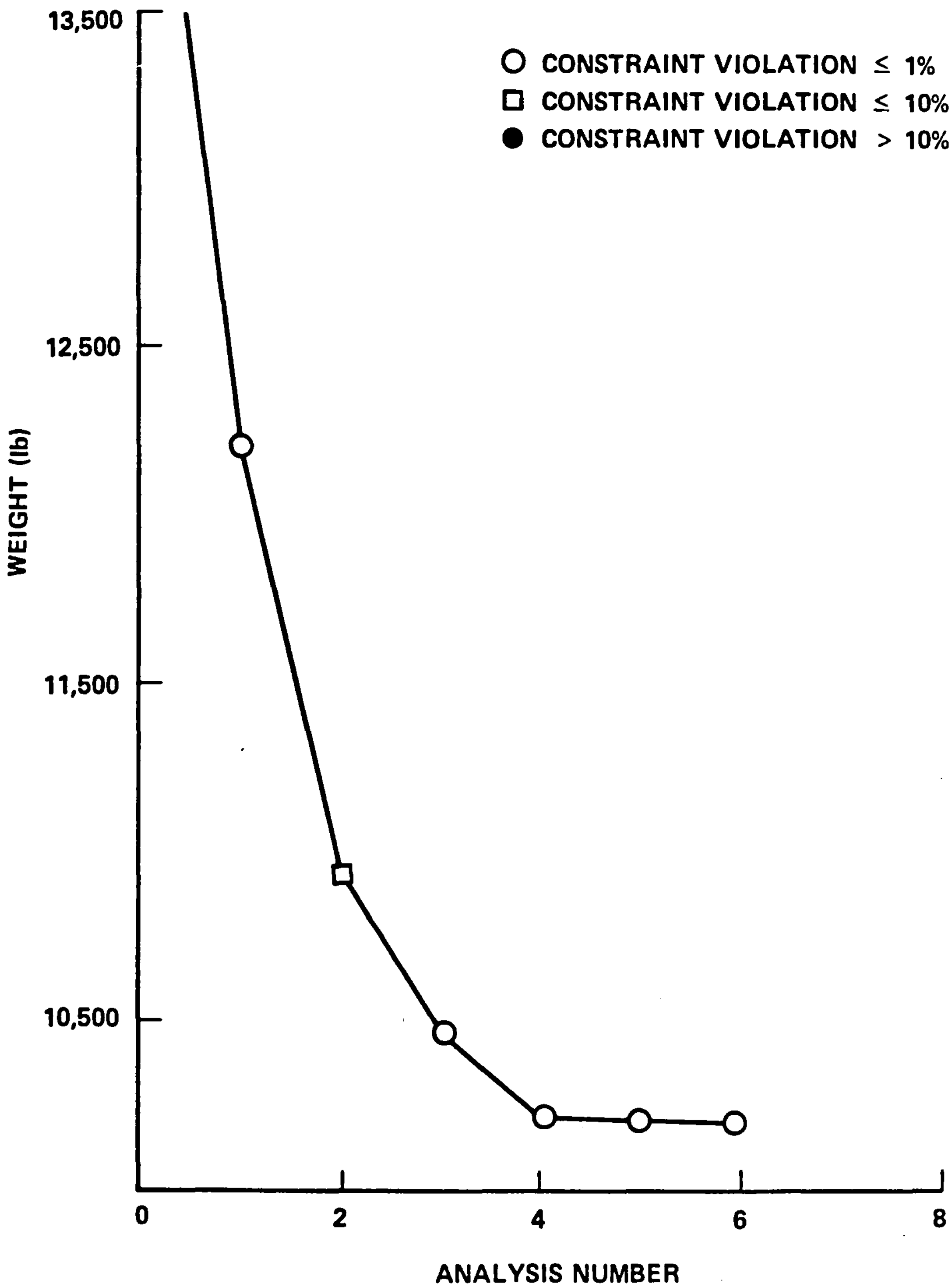


Fig. 68 - Iteration History for Problem 6, Case B, Run 1 (Option 1(P))
One Bay / Two Story Frame

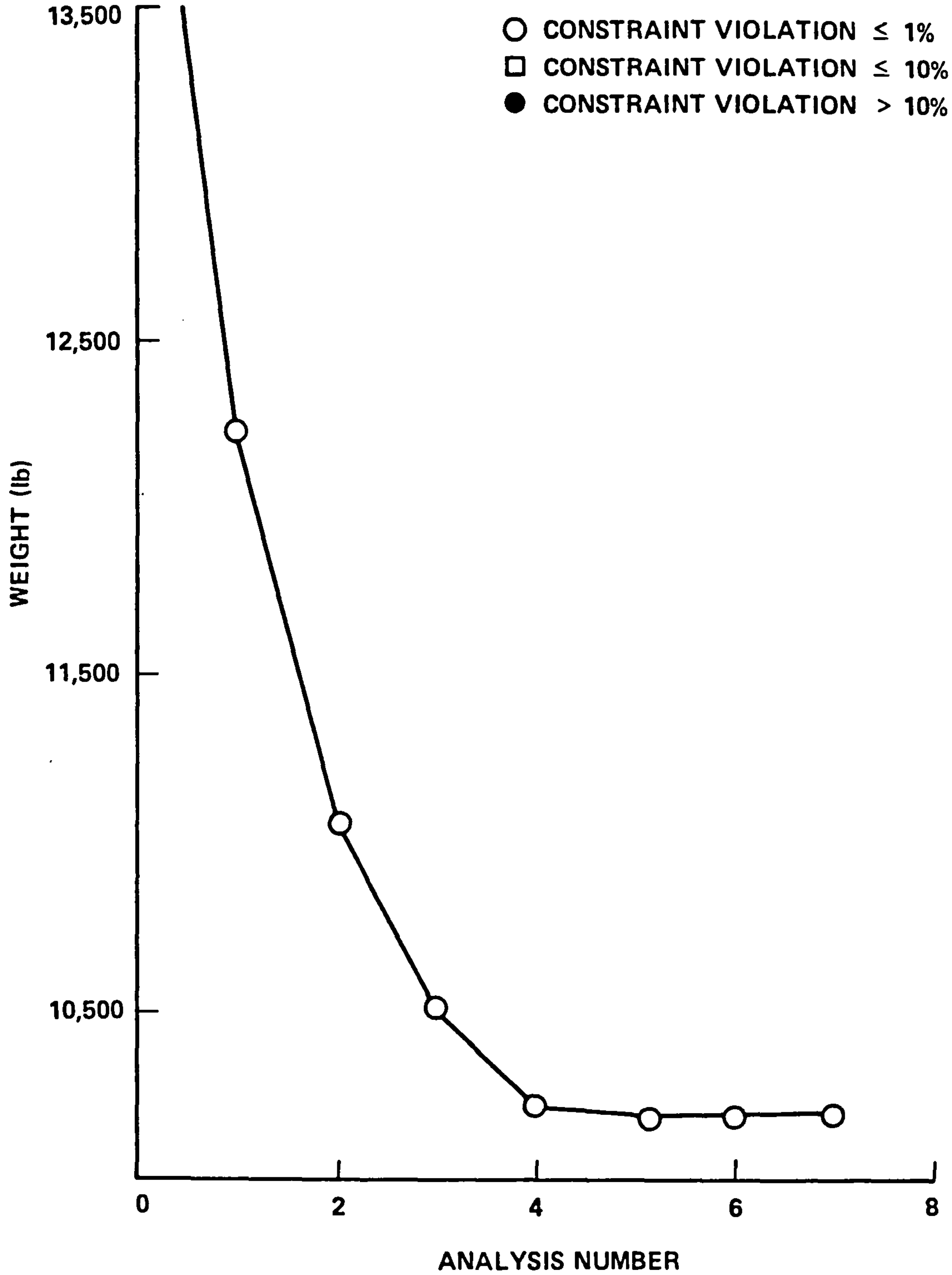


Fig. 69 - Iteration History for Problem 6, Case B, Run 2 (Option 3(P))
One Bay / Two Story Frame

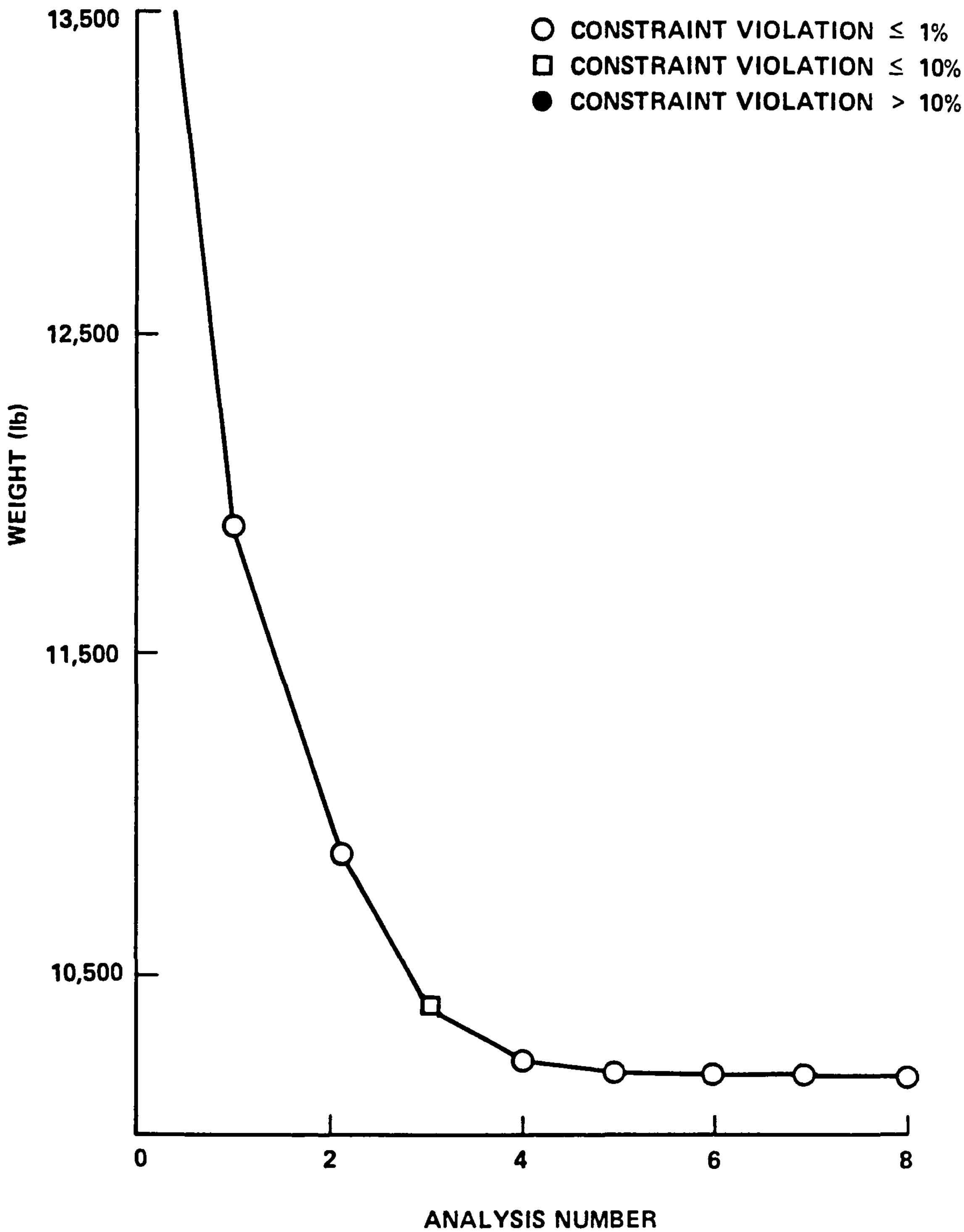


Fig. 70 - Iteration History for Problem 6, Case B, Run 3 (Option 4(P))
One Bay / Two Story Frame

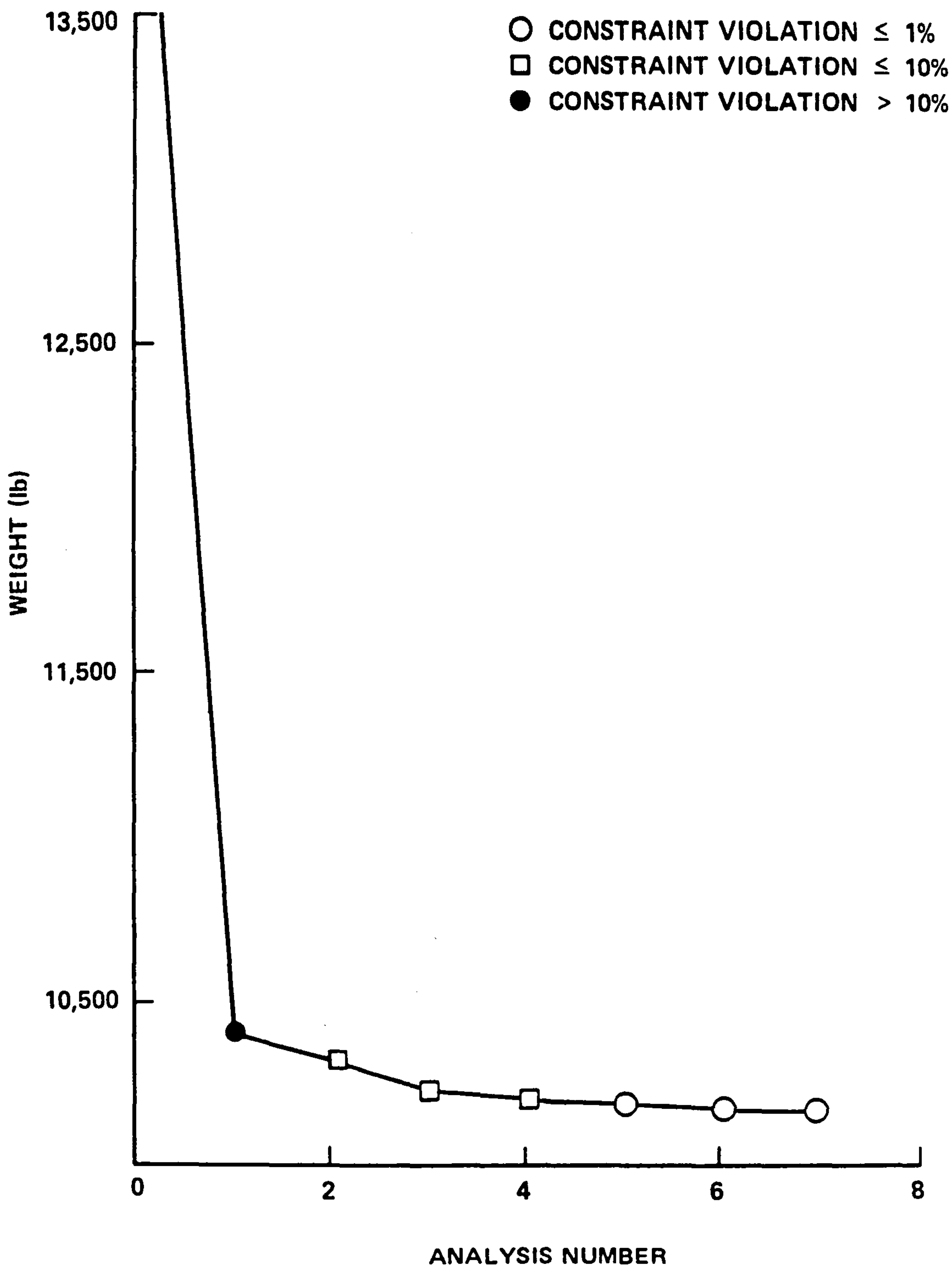


Fig. 71 - Iteration History for Problem 6, Case B, Run 4 (Option 10(P))
One Bay / Two Story Frame

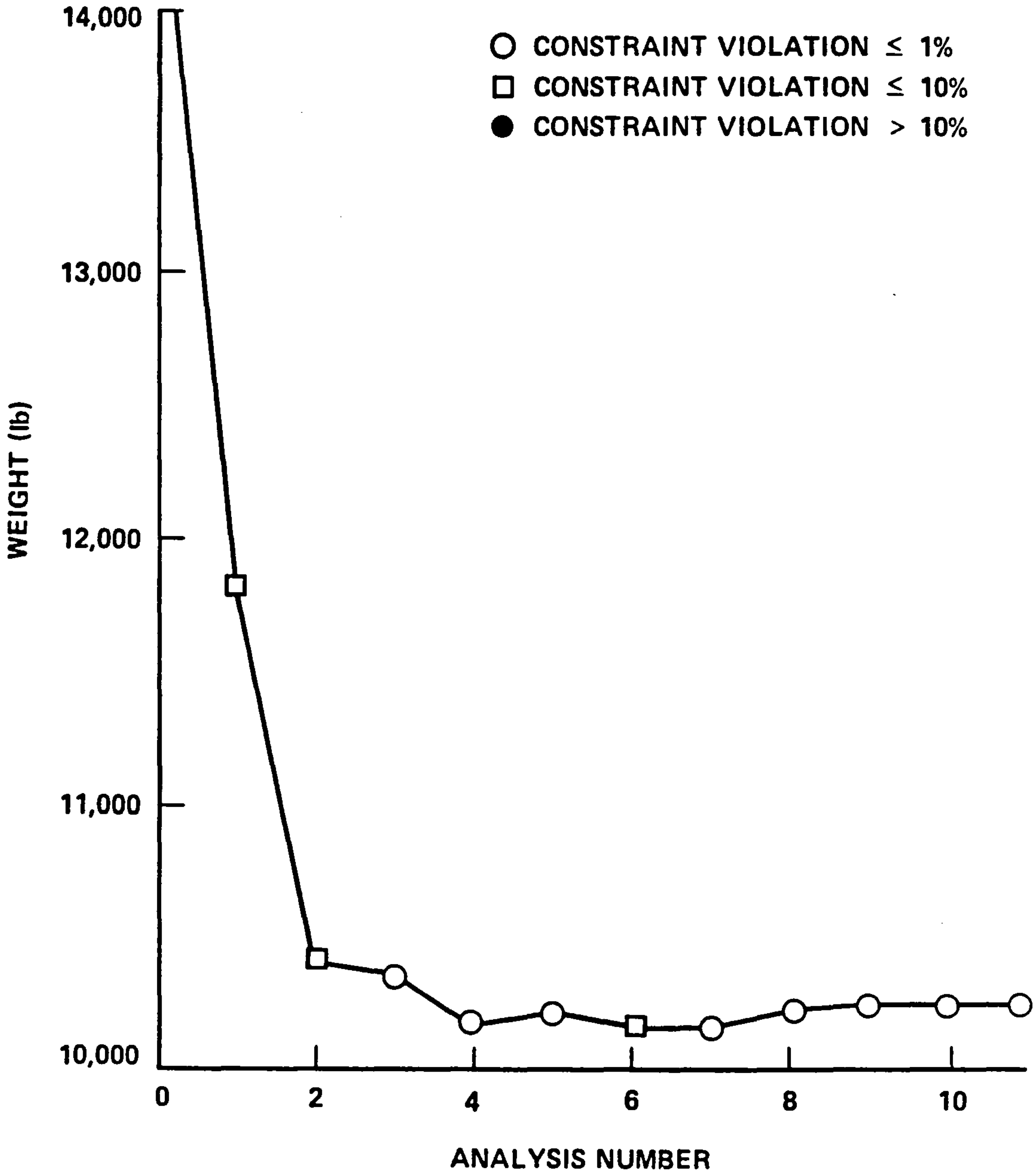


Fig. 72 - Iteration History for Problem 6, Case B, Run 5 (Option 10(D))
One Bay / Two Story Frame

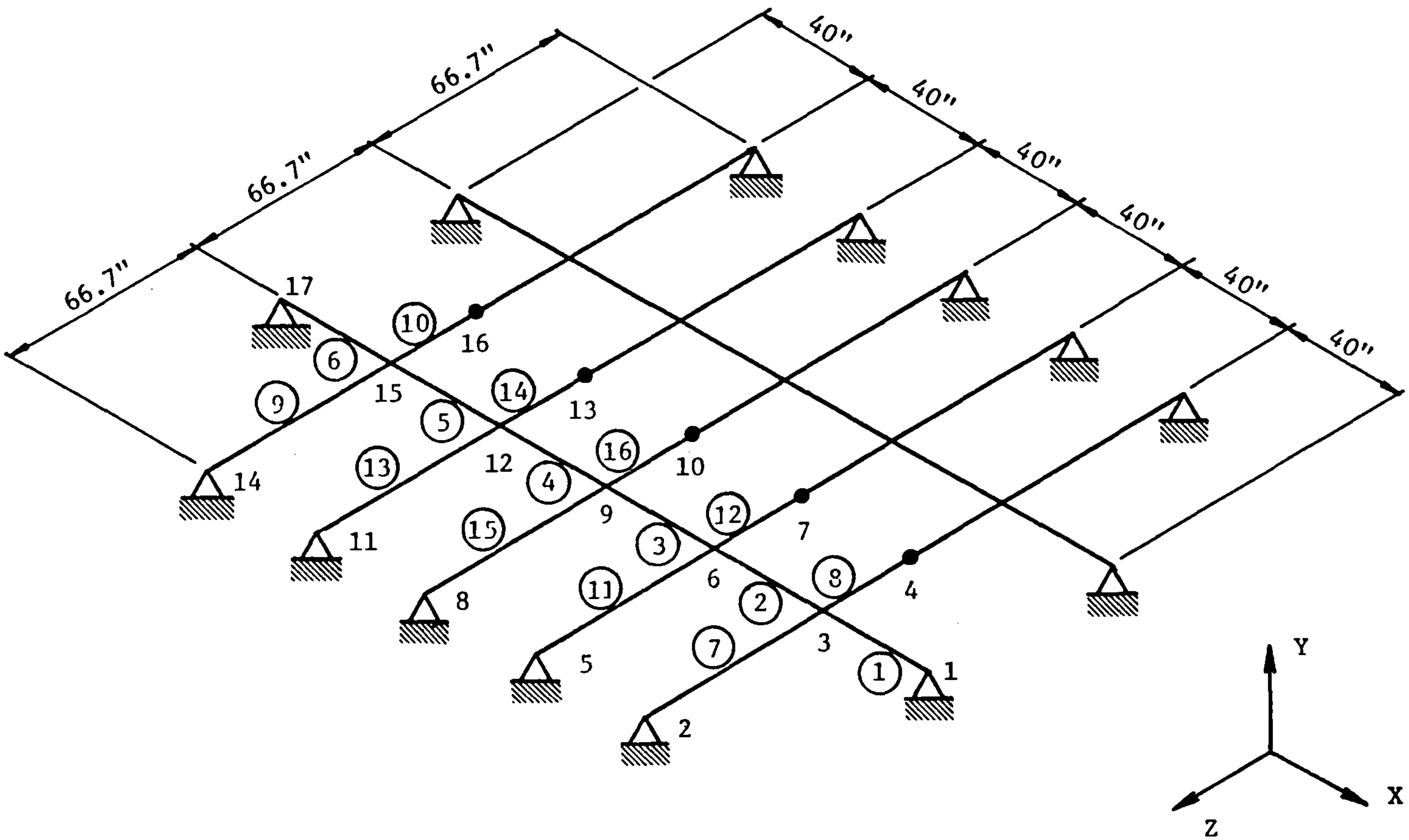


Fig. 73 - 2x5 Grillage (Problem 7)

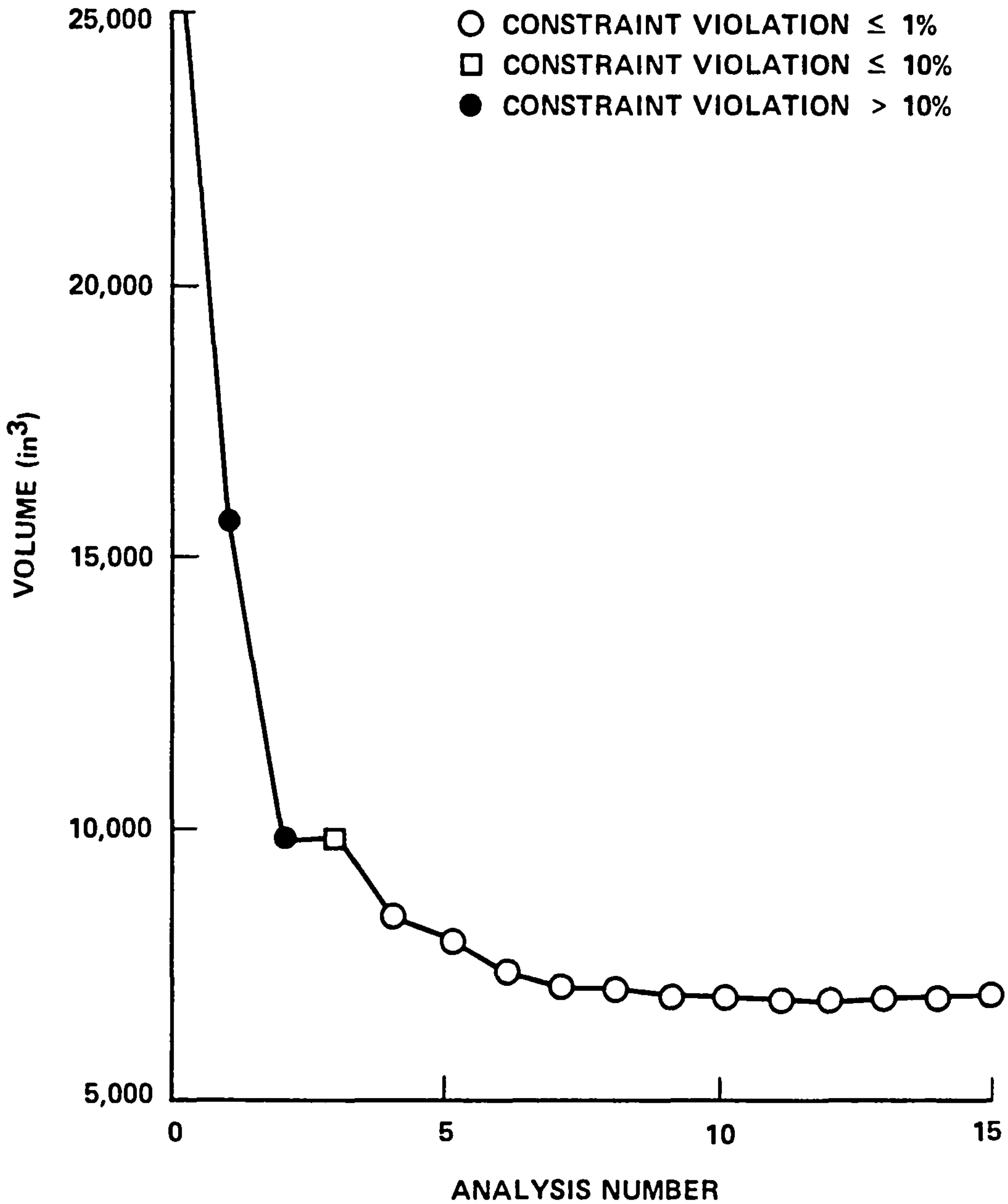


Fig. 74 - Iteration History for Problem 7, Case A, Run 1 (Option 1(P))
2x5 Grillage

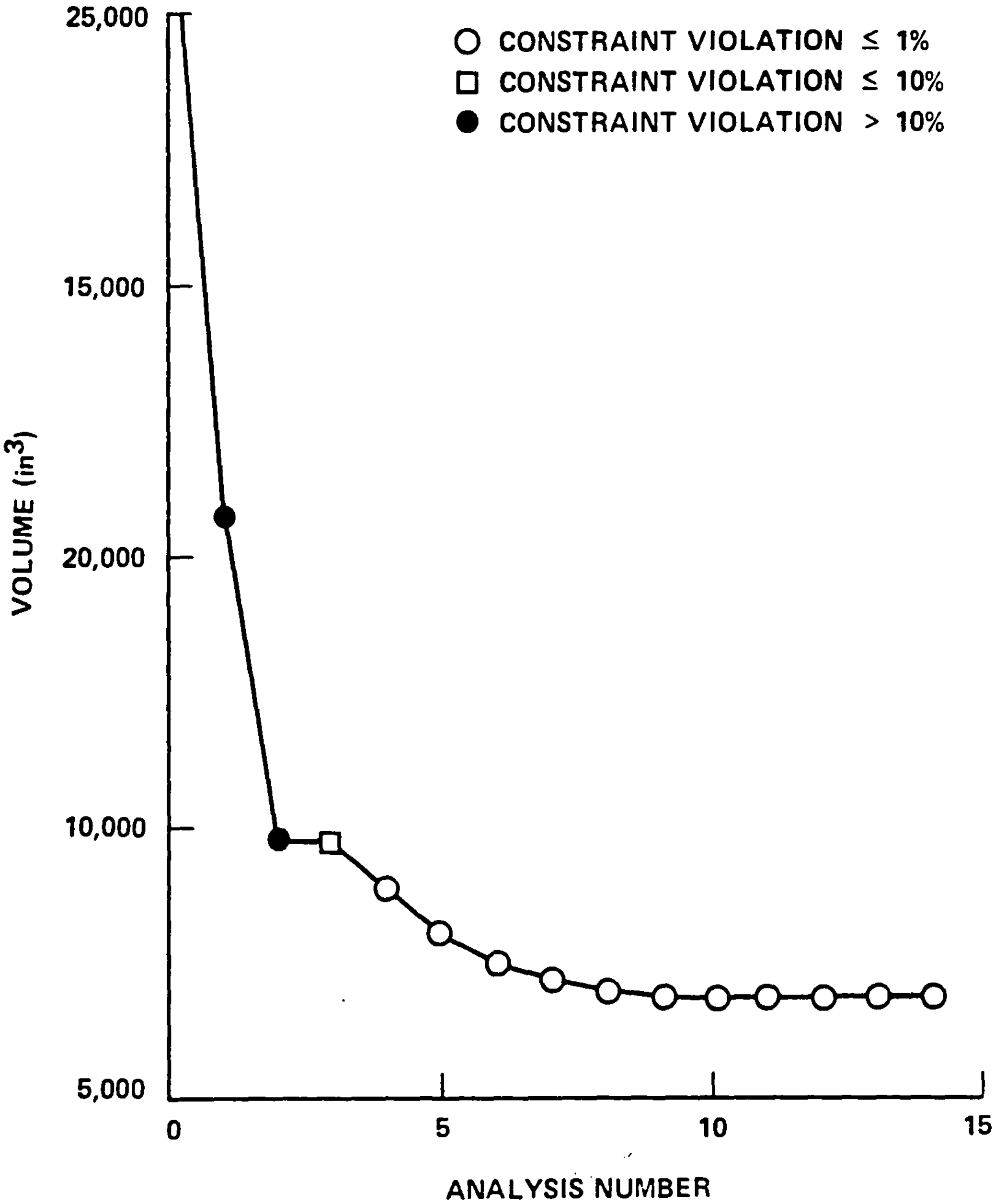


Fig. 75 - Iteration Histroy for Problem 7, Case A, Run 2 (Option 4(P)) 2x5 Grillage

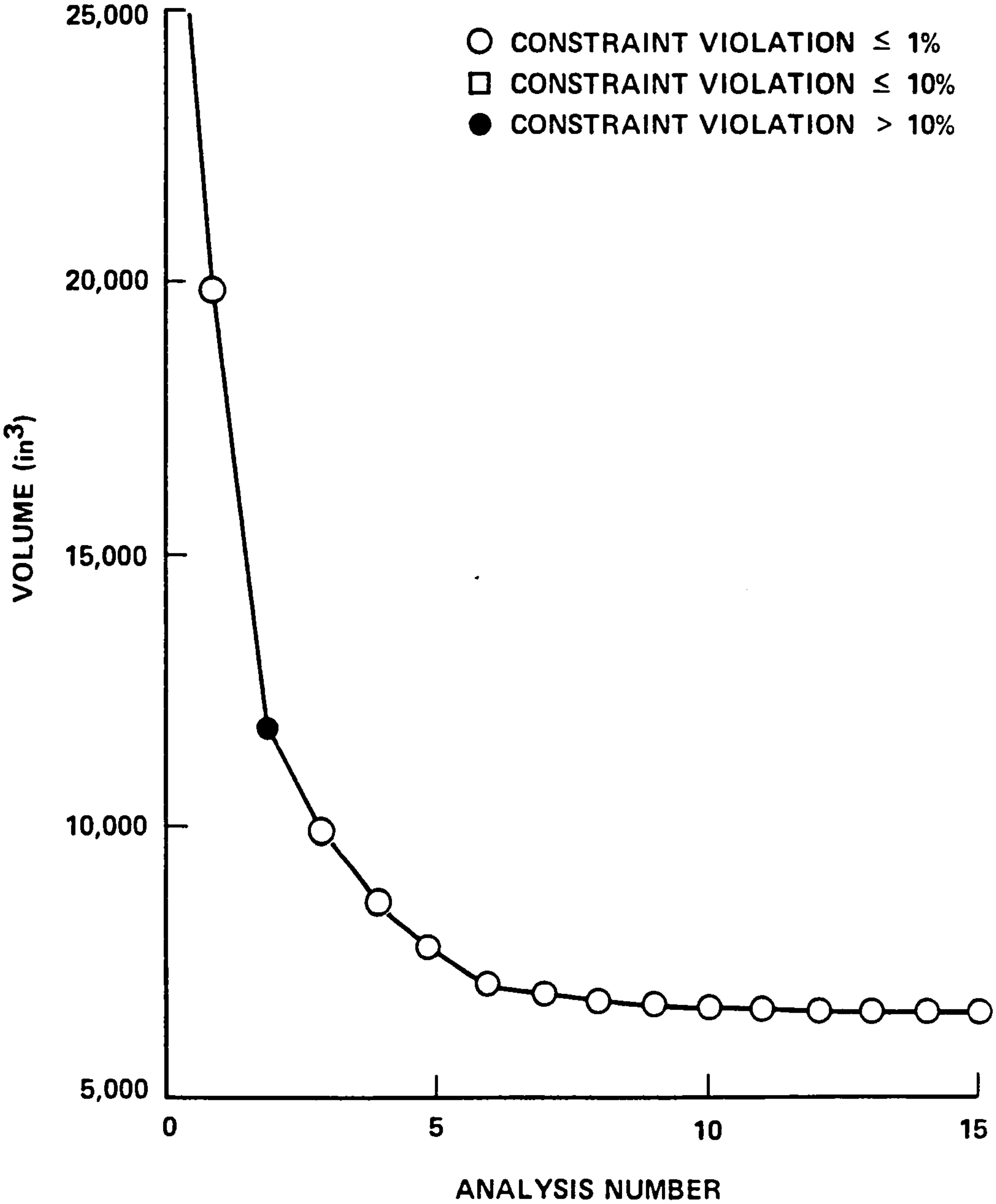


Fig. 76 - Iteration History for Problem 7, Case A, Run 3 (Option 10(P))
2x5 Grillage

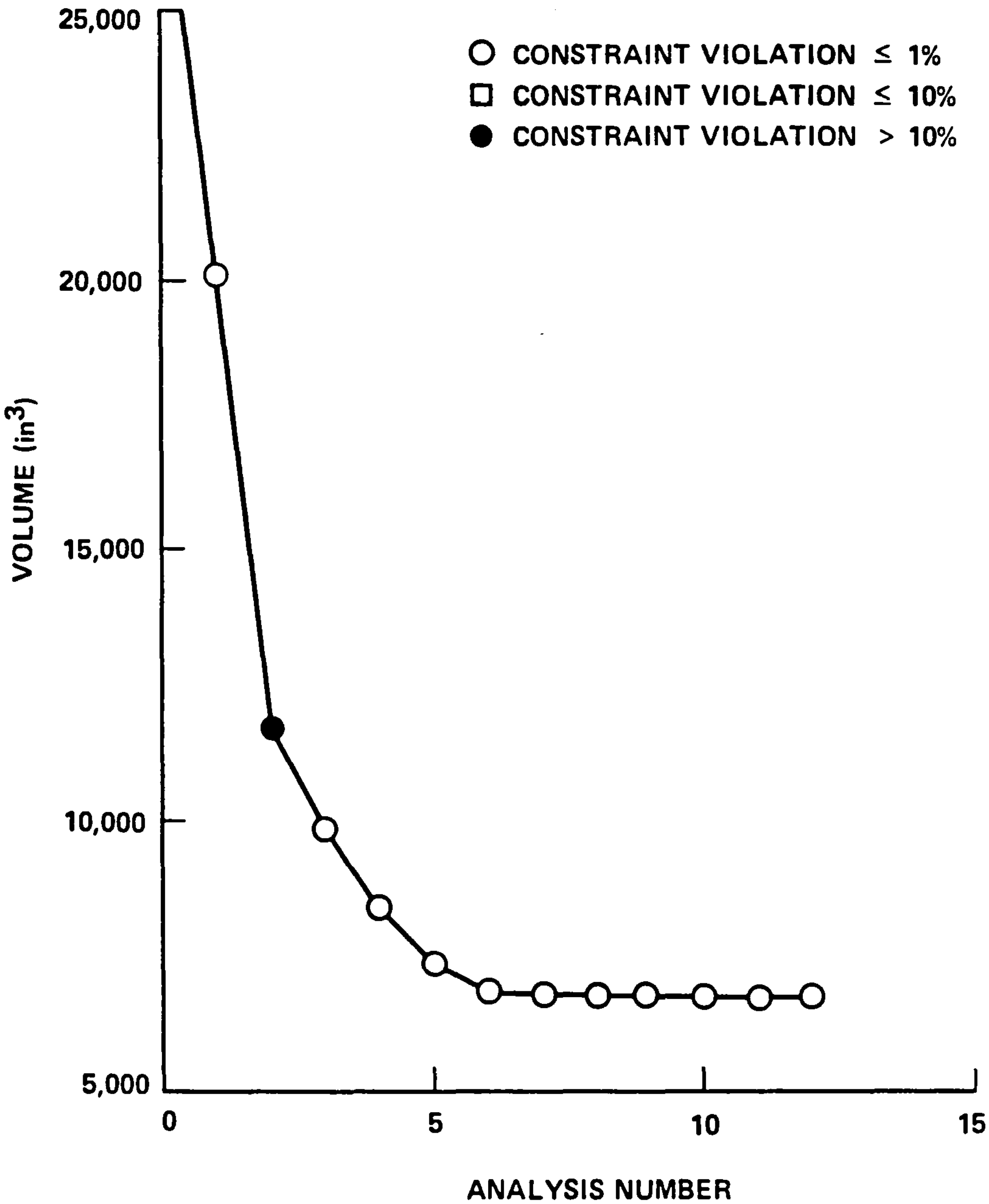


Fig. 77 - Iteration History for Problem 7, Case A, Run 4 (Option 10(D)) 2x5 Grillage

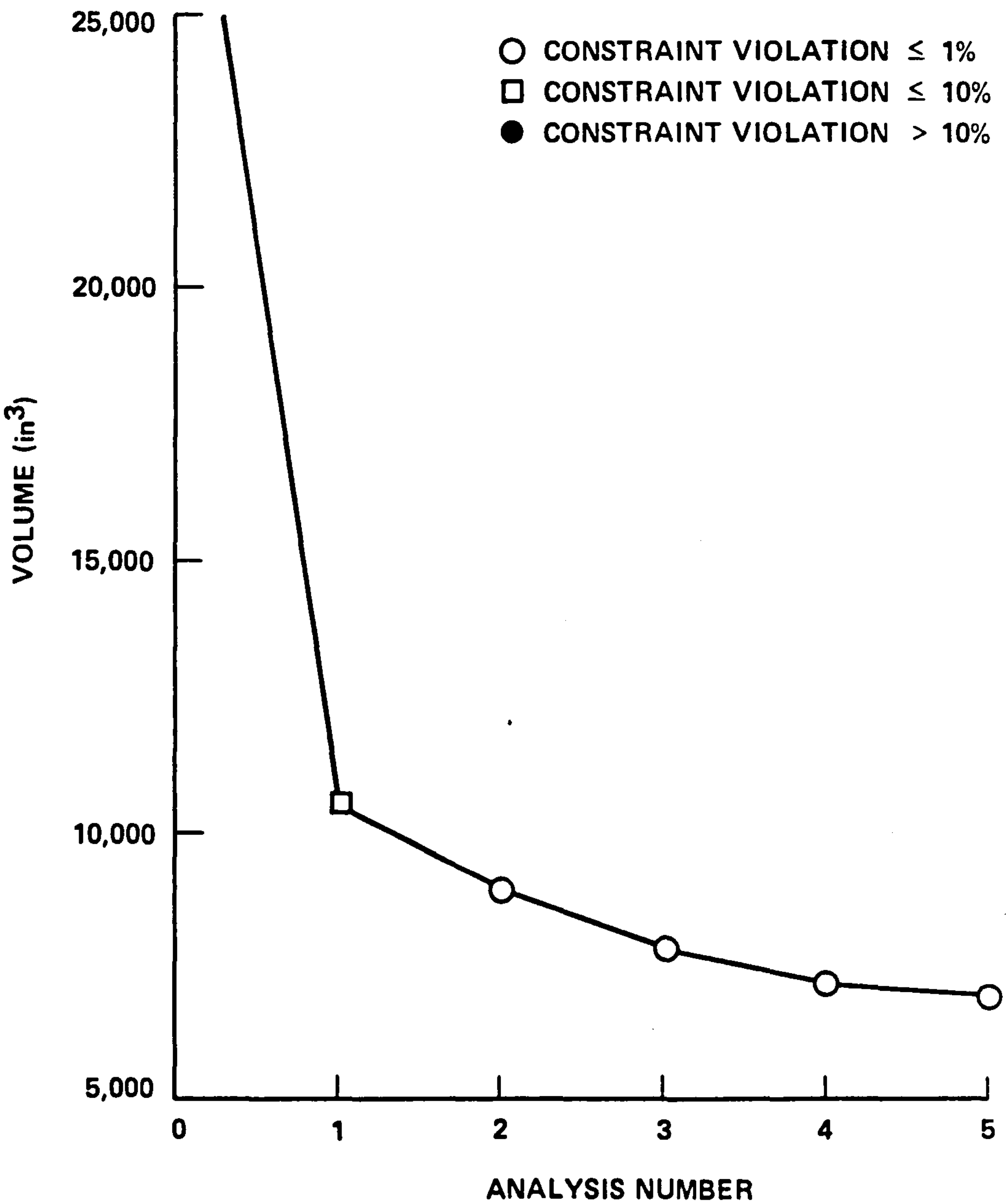


Fig. 78 - Iteration History for Problem 7, Case A, Run 5 (Option 1(PU)) 2x5 Grillage

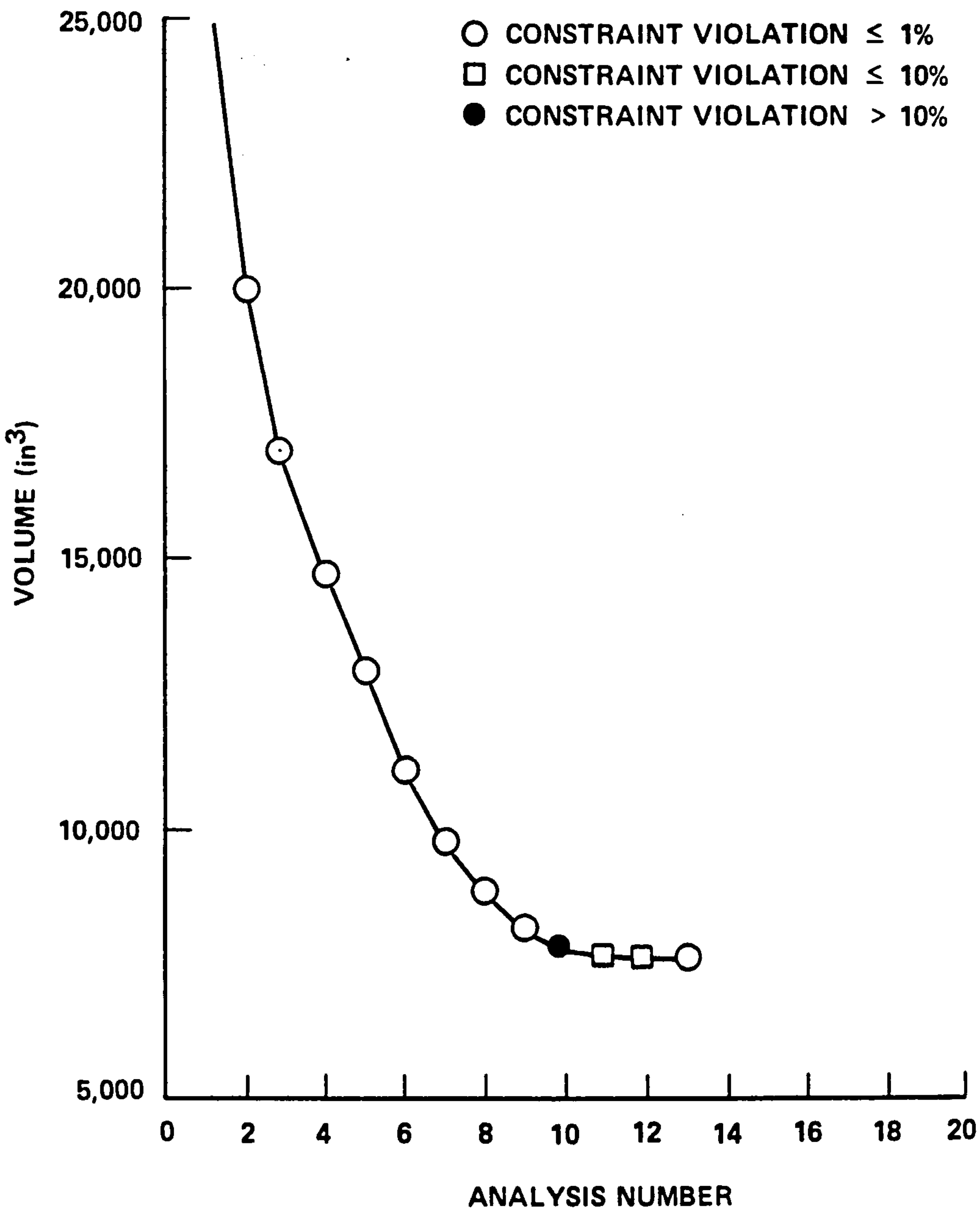


Fig. 79 - Iteration History for Problem 7, Case B, Run 1 (Option 3(P))
2x5 Grillage

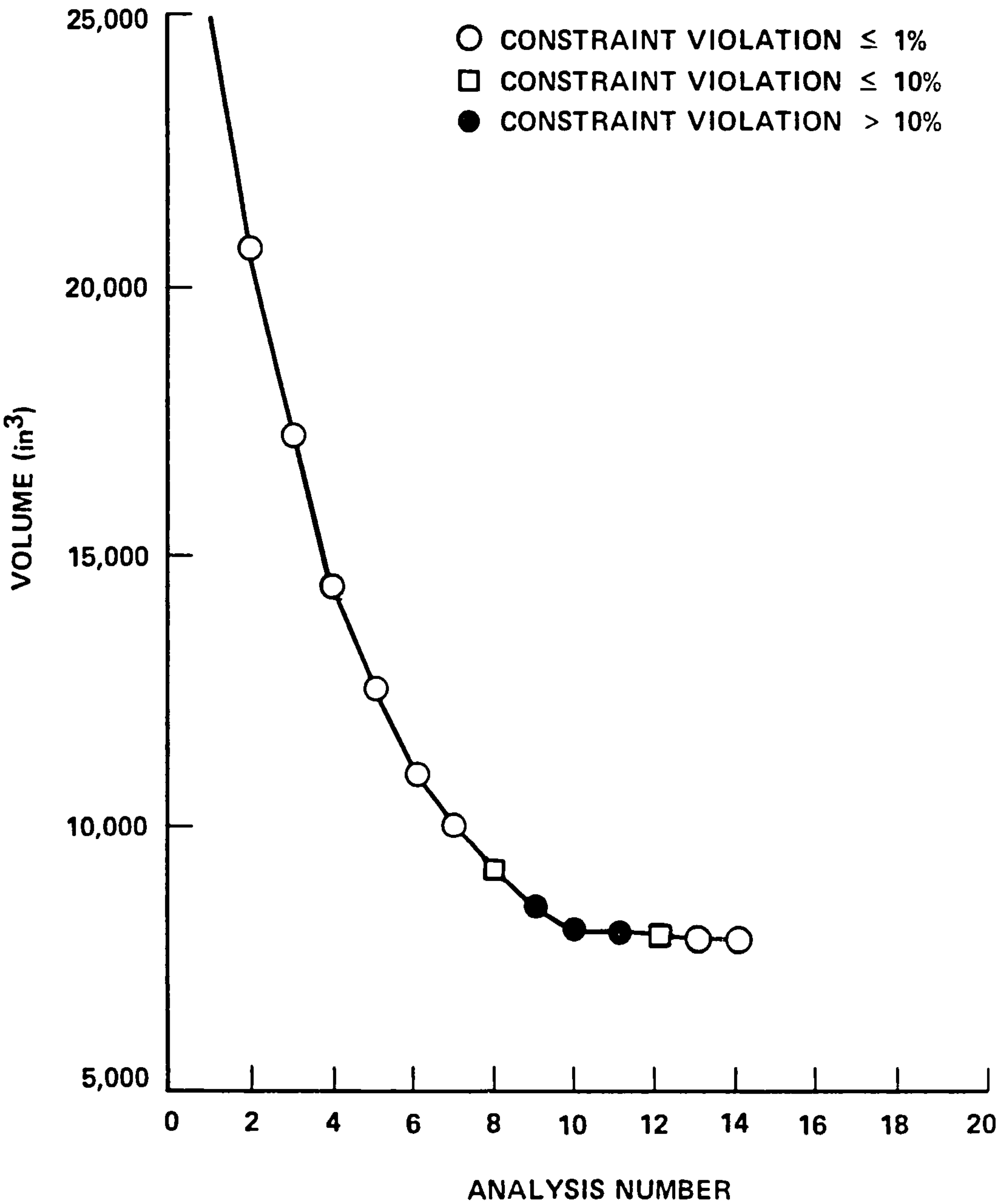


Fig. 80 - Iteration History for Problem 7, Case B, Run 2 (Option 6(P))
2x5 Grillage

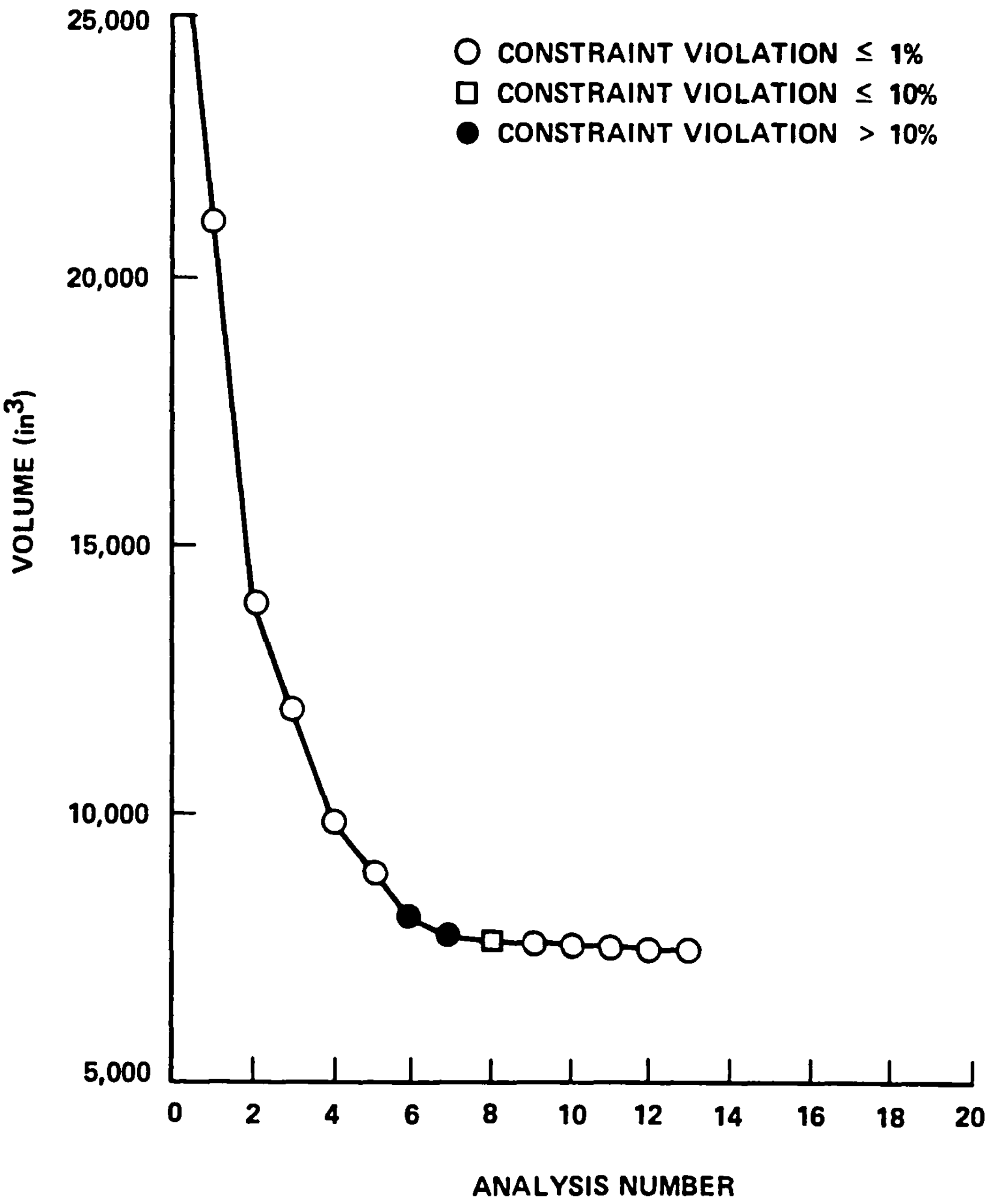


Fig. 81 - Iteration History for Problem 7, Case B, Run 3 (Option 12(P))
2x5 Grillage

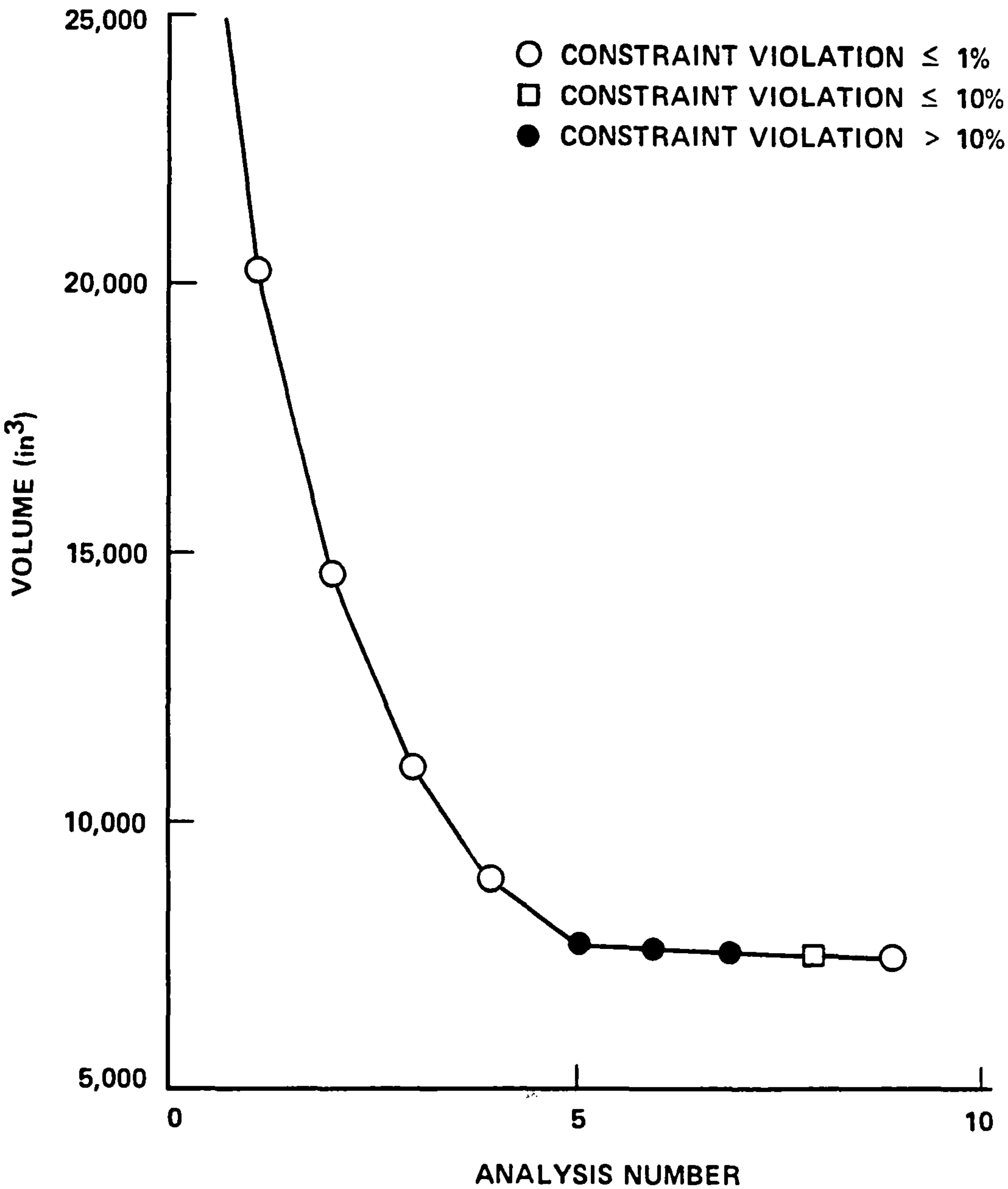


Fig. 82 - Iteration History for Problem 7, Case B, Run 4 (Option 3(PU))
2x5 Grillage

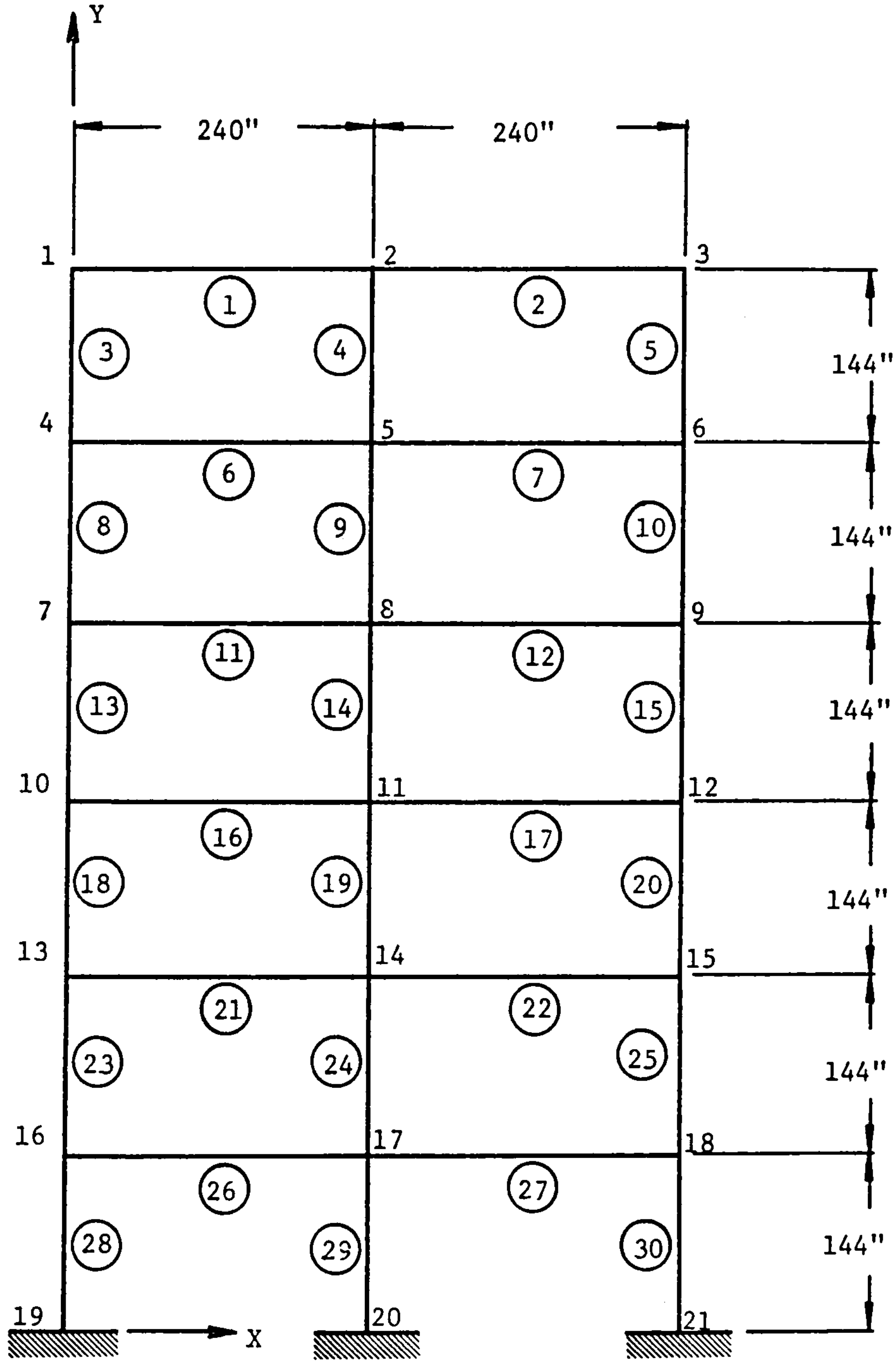


Fig. 83 - Two Bay / Six Story Frame (Problem 8)

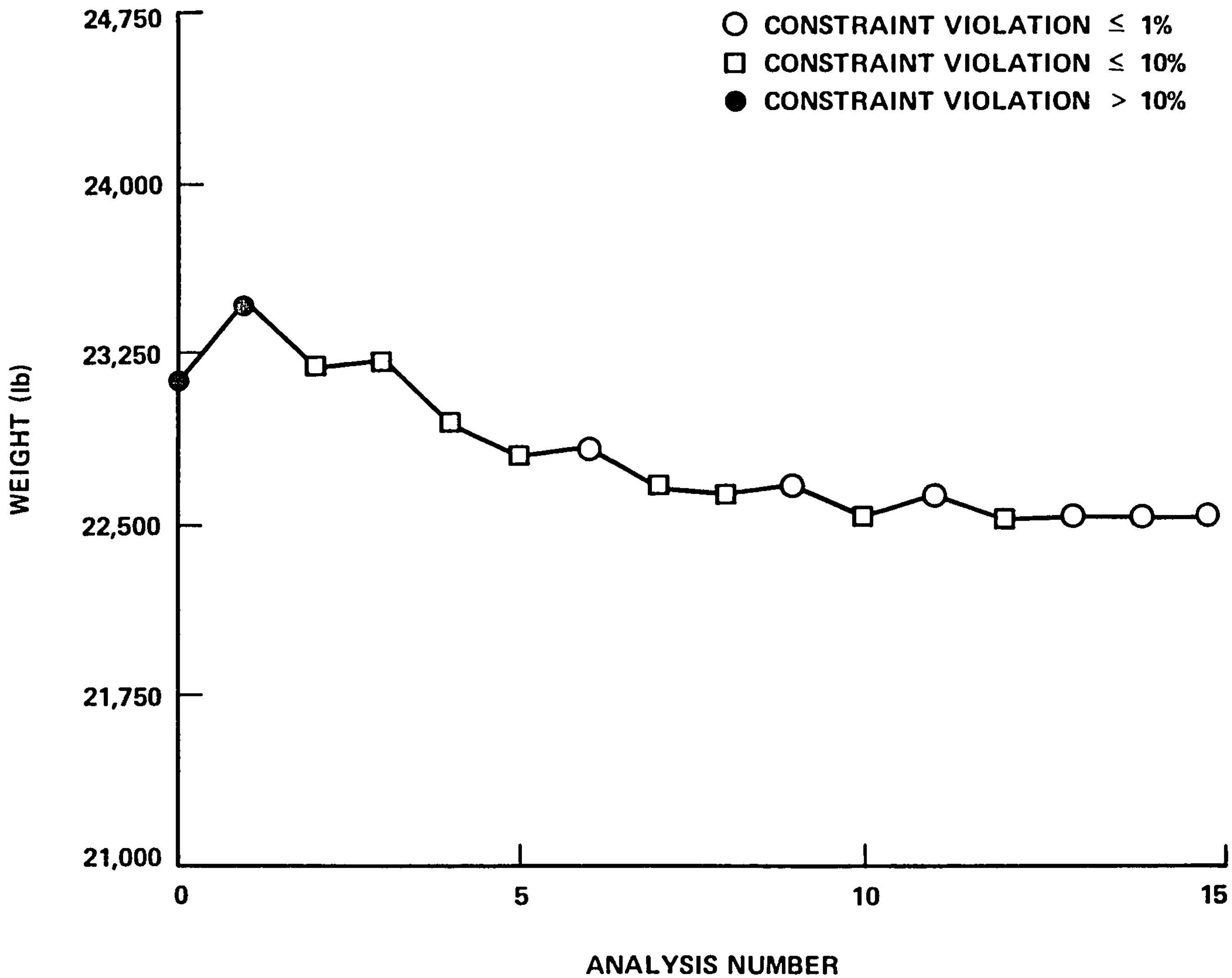


Fig. 84 - Iteration History for Problem 8, Case A, Run 1 (Option 1(P))
Two Bay / Six Story Frame

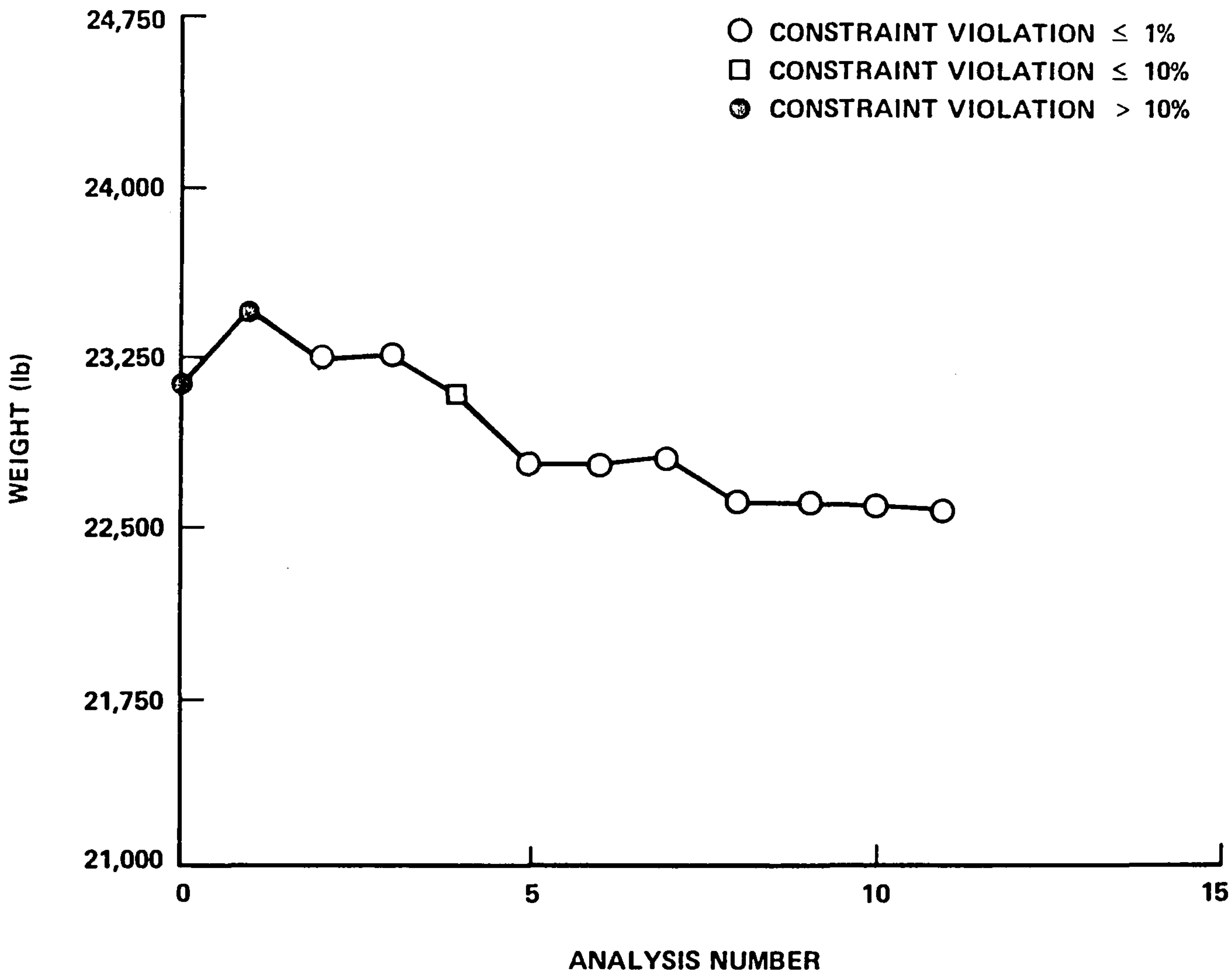


Fig. 85 - Iteration History for Problem 8, Case A, Run 2 (Option 3(P))
Two Bay / Six Story Frame

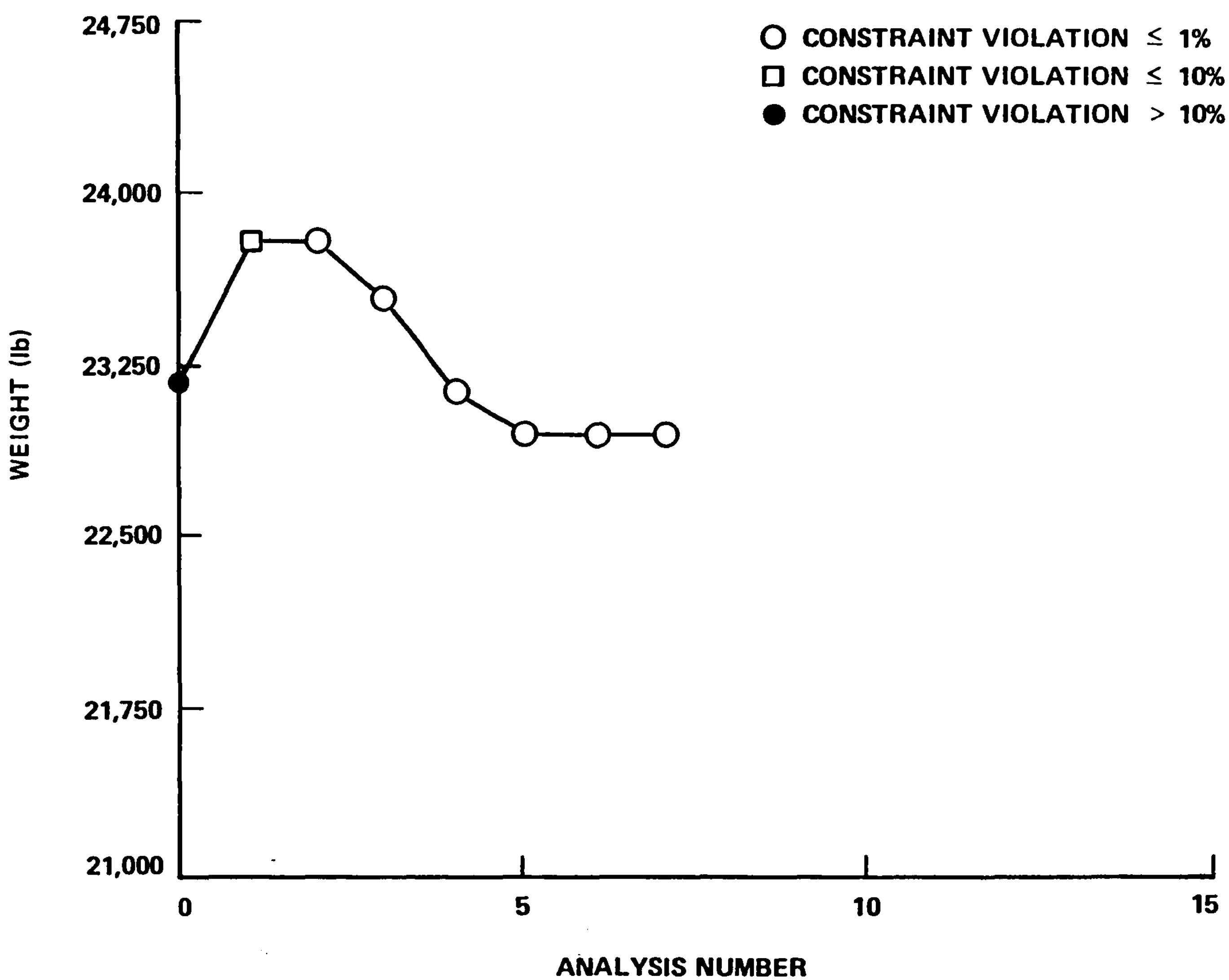


Fig. 86 - Iteration History for Problem 8, Case A, Run 3 (Option 6(P))
Two Bay / Six Story Frame

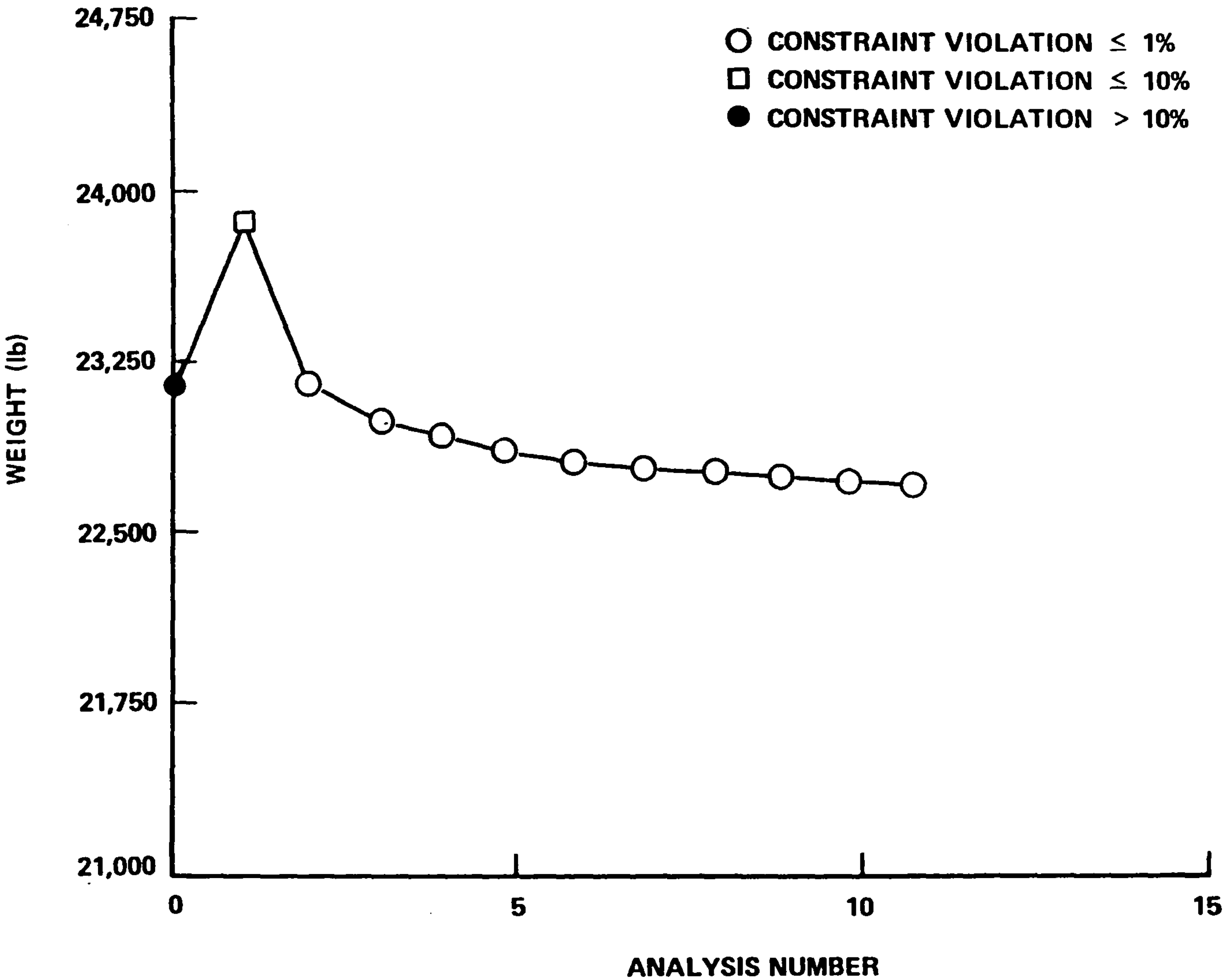


Fig. 87 - Iteration History for Problem 8, Case A, Run 4 (Option 12(P))
Two Bay / Six Story Frame

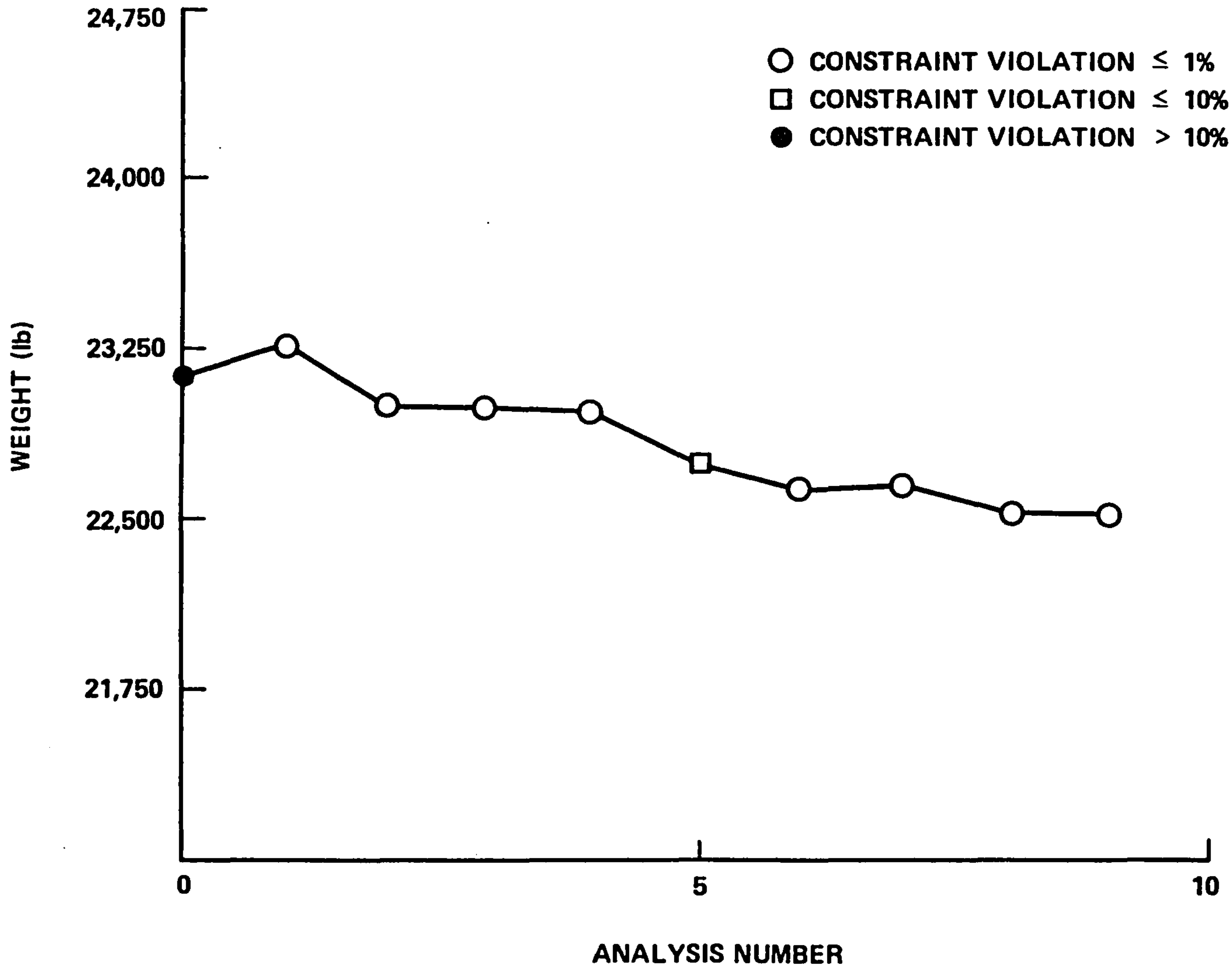


Fig. 88 - Iteration History for Problem 8, Case A, Run 5 (Option 3(PU))
Two Bay / Six Story Frame

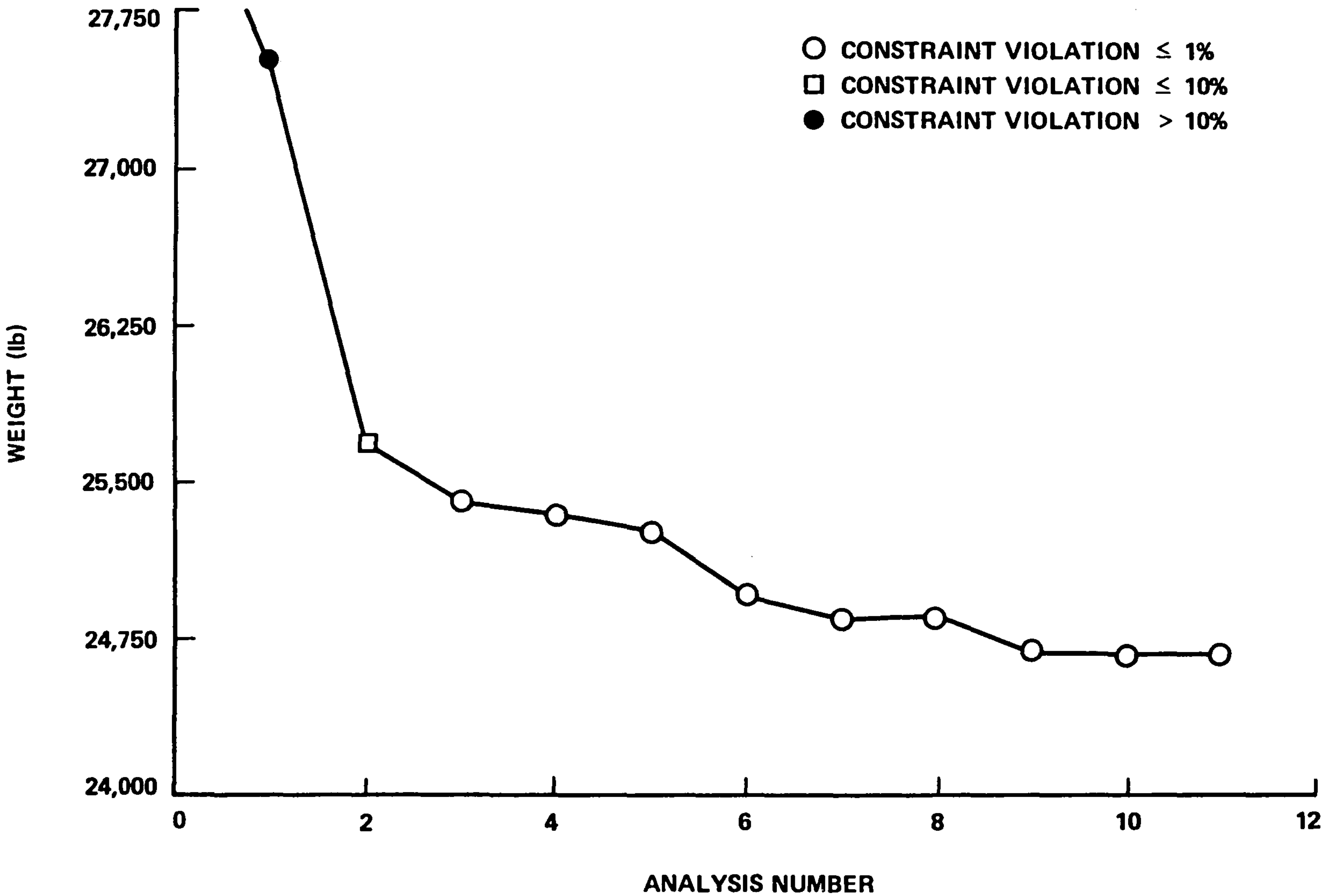


Fig. 89 - Iteration History for Problem 8, Case B, Run 1 (Option 1(P))
Two Bay / Six Story Frame

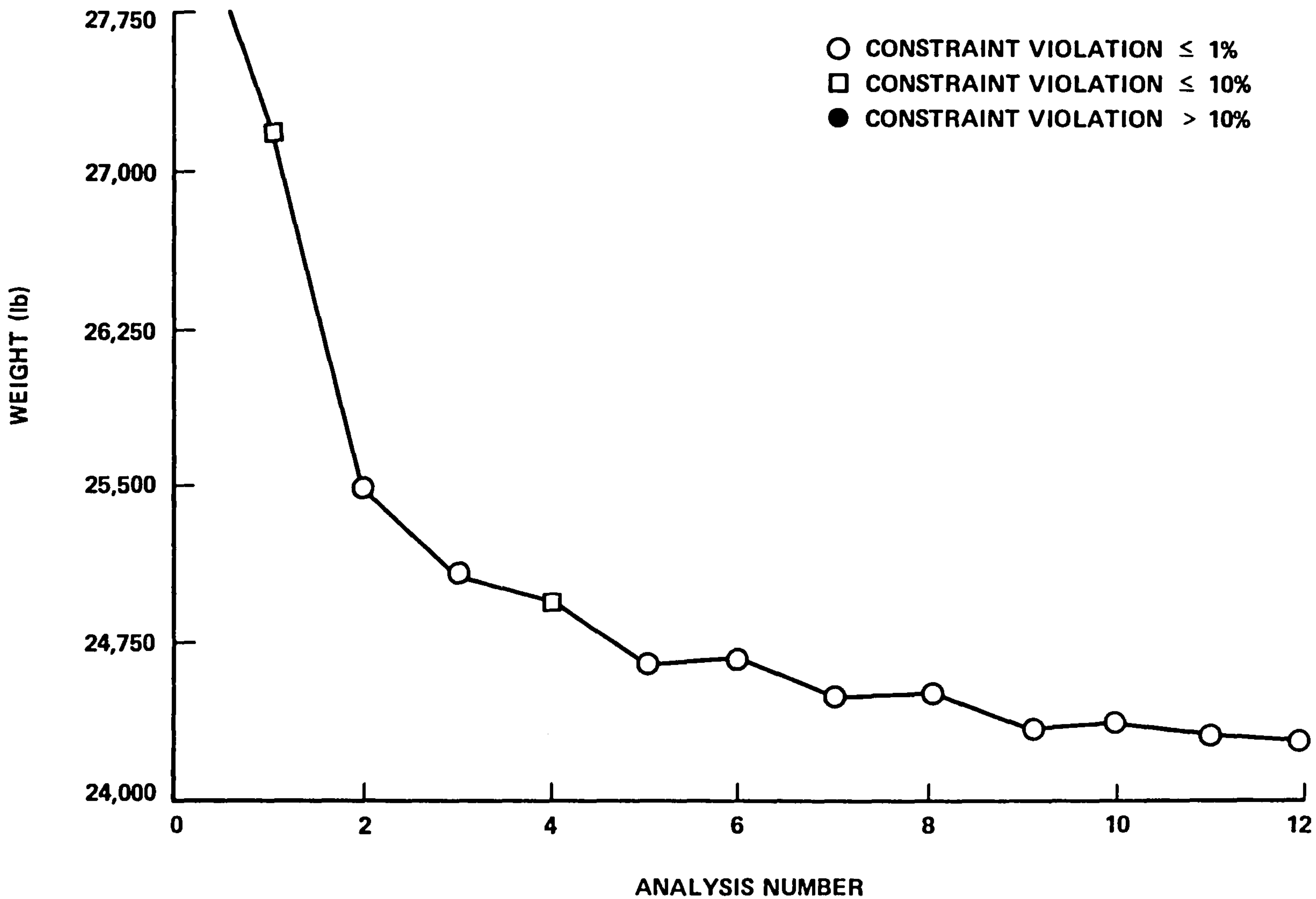


Fig. 90 - Iteration History for Problem 8, Case B, Run 2 (Option 3(P))
Two Bay / Six Story Frame

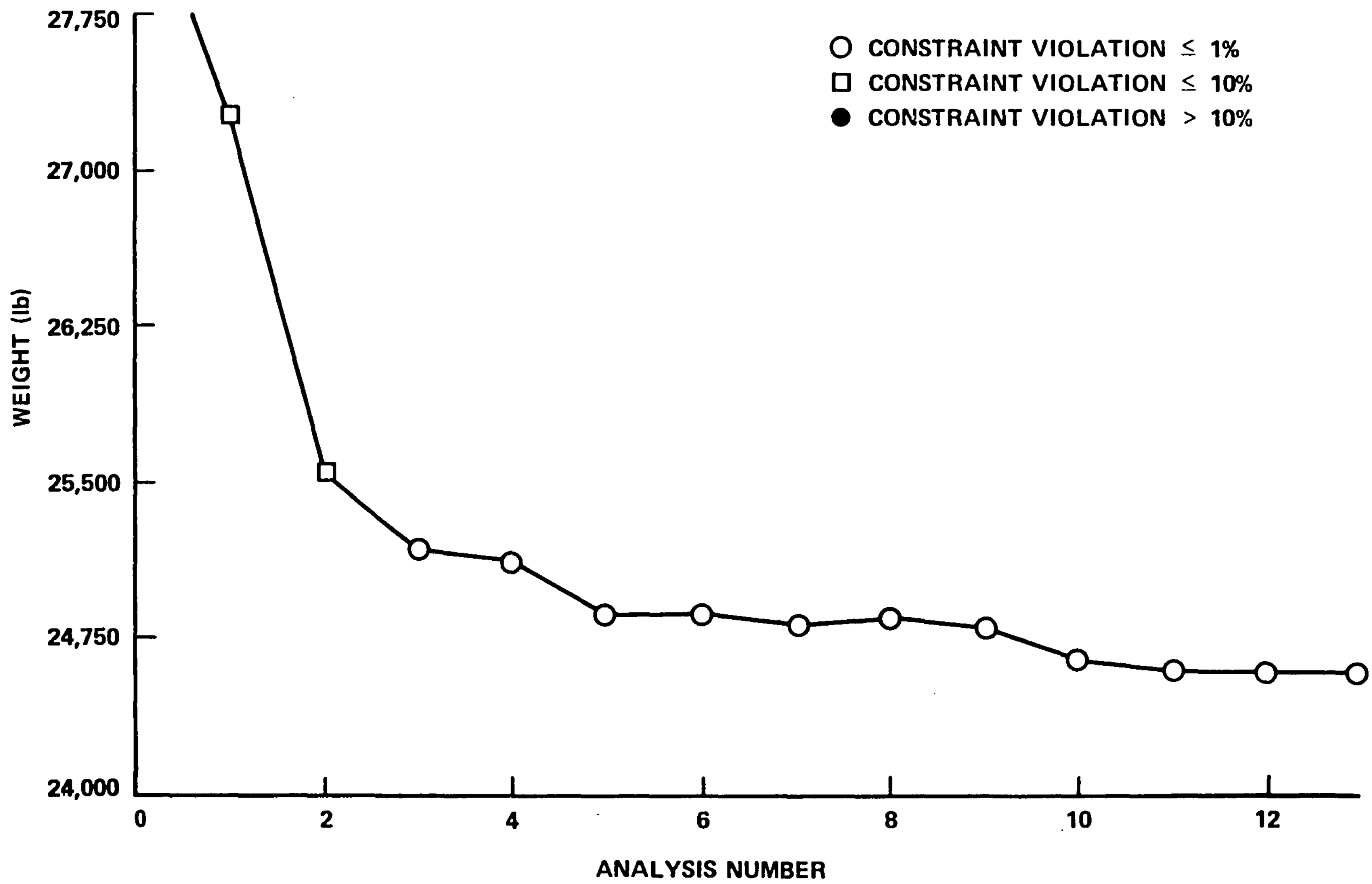


Fig. 91 - Iteration History for Problem 8, Case B, Run 3 (Option 4(P))
Two Bay / Six Story Frame

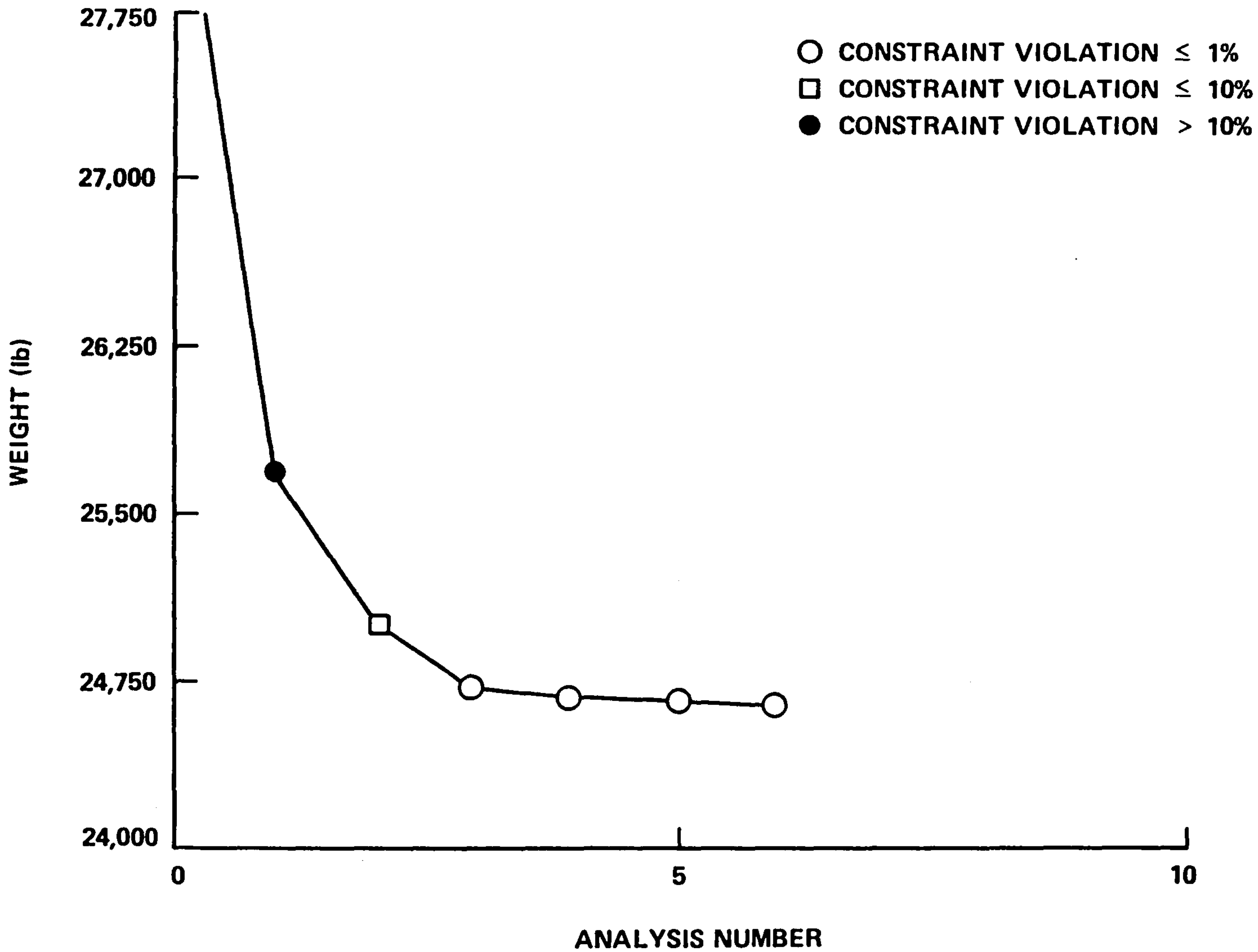


Fig. 92 - Iteration History for Problem 8, Case B, Run 4 (Option 10(P))
Two Bay / Six Story Frame

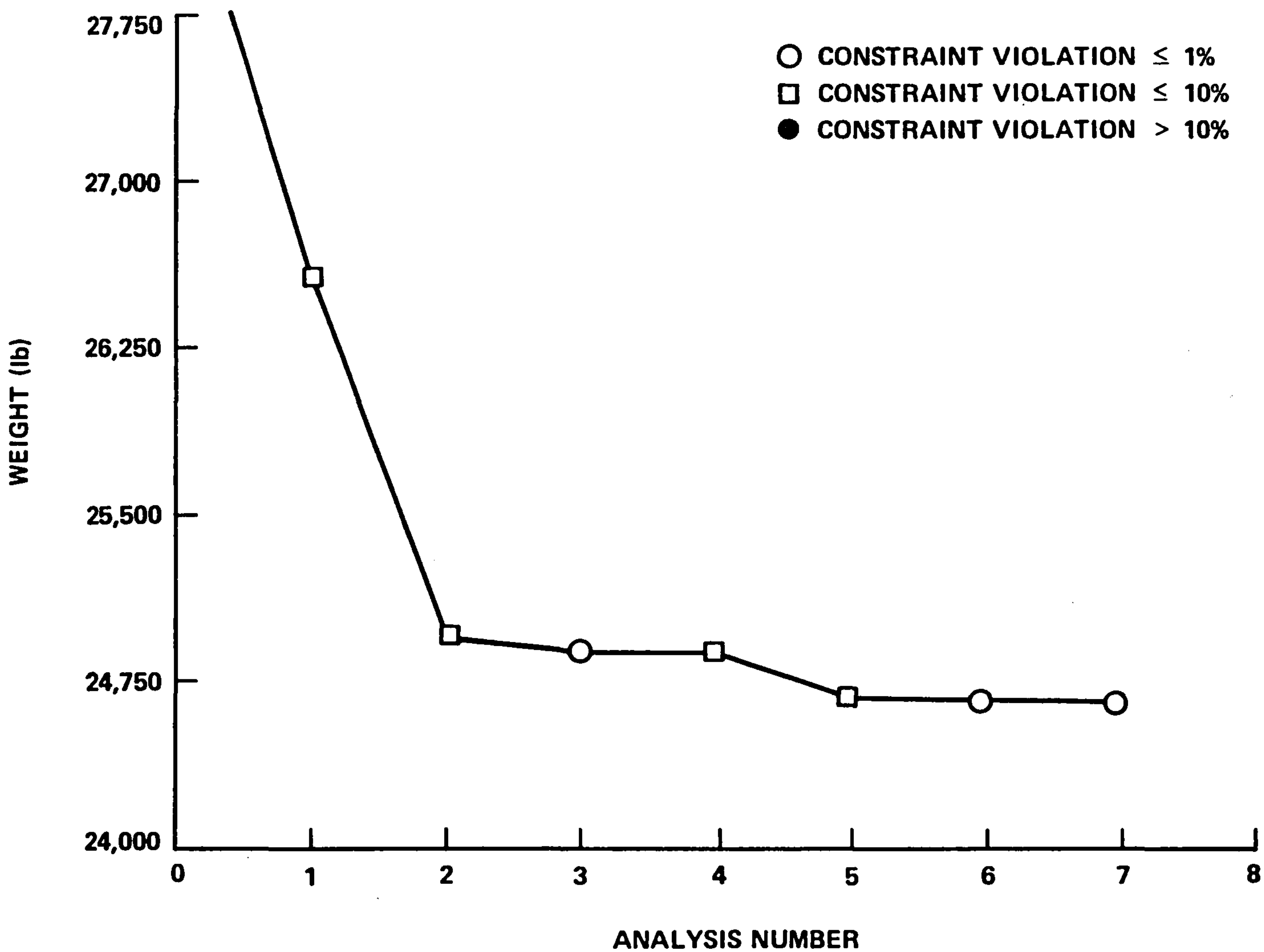


Fig. 93 - Iteration History for Problem 8, Case B, Run 5 (Option 1(PU))
Two Bay / Six Story Frame

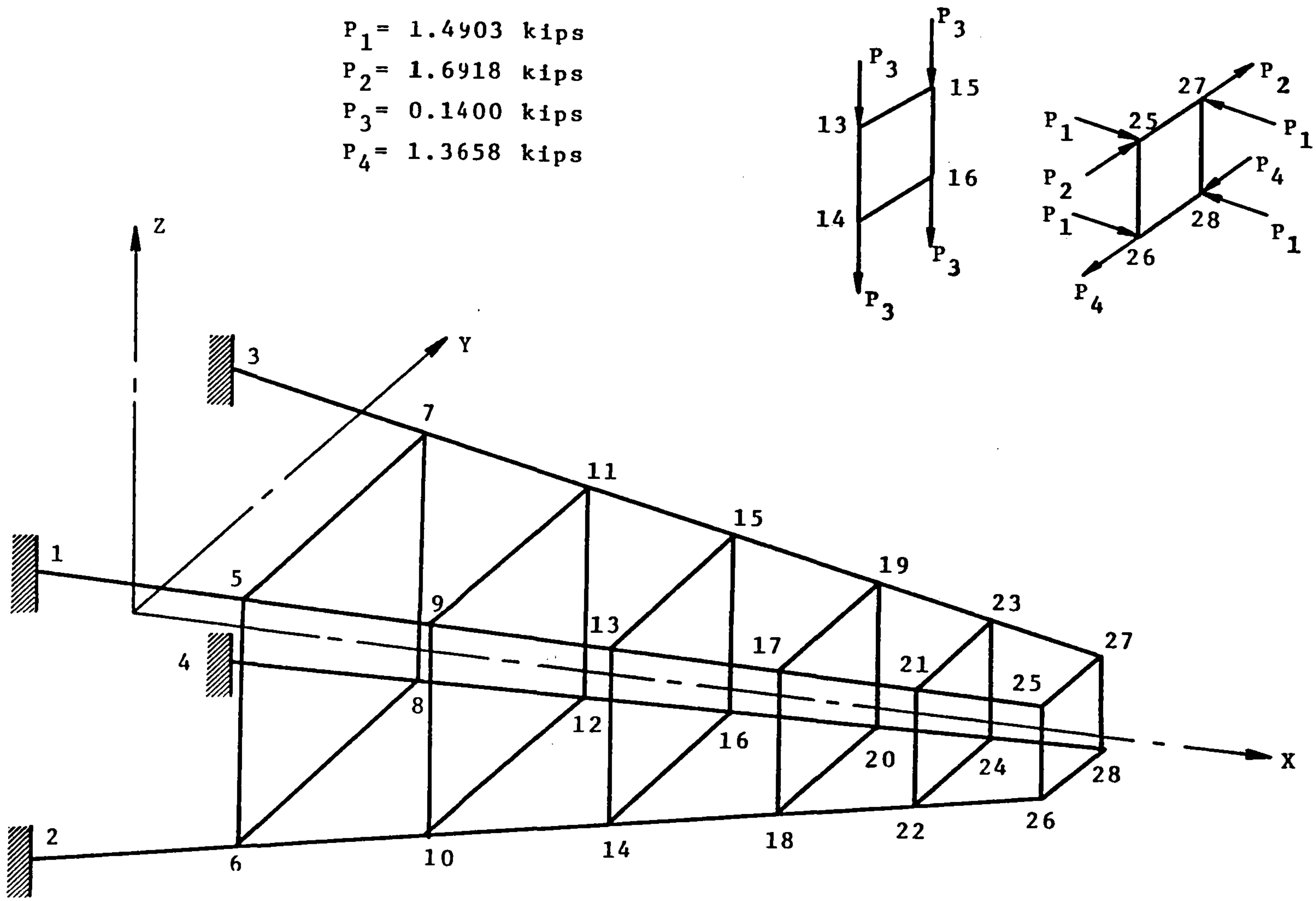


Fig. 94 - Helicopter Tail Boom (Problem 9)

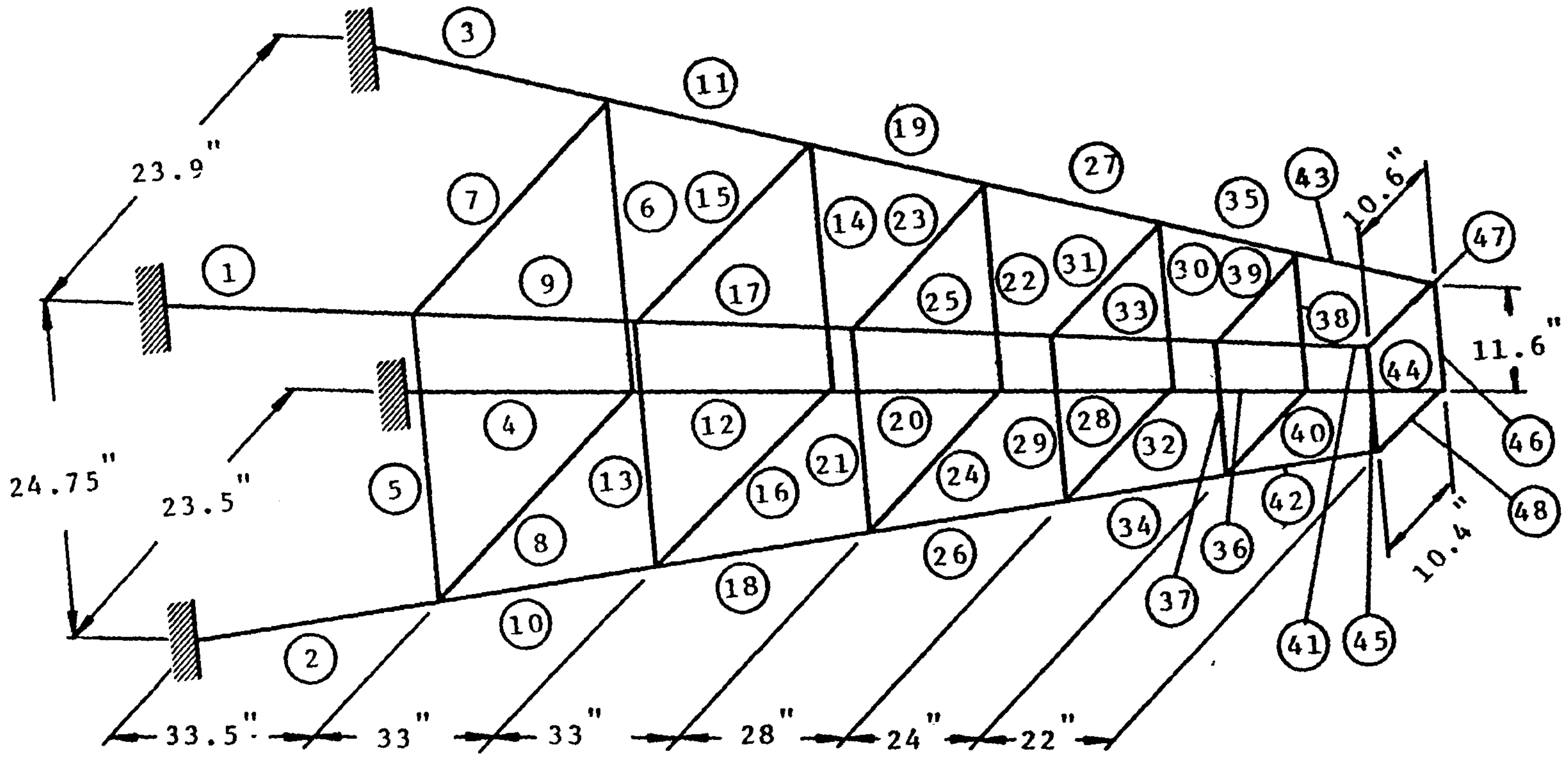


Fig. 94 - Helicopter Tail Boom (Problem 9) (cont.)

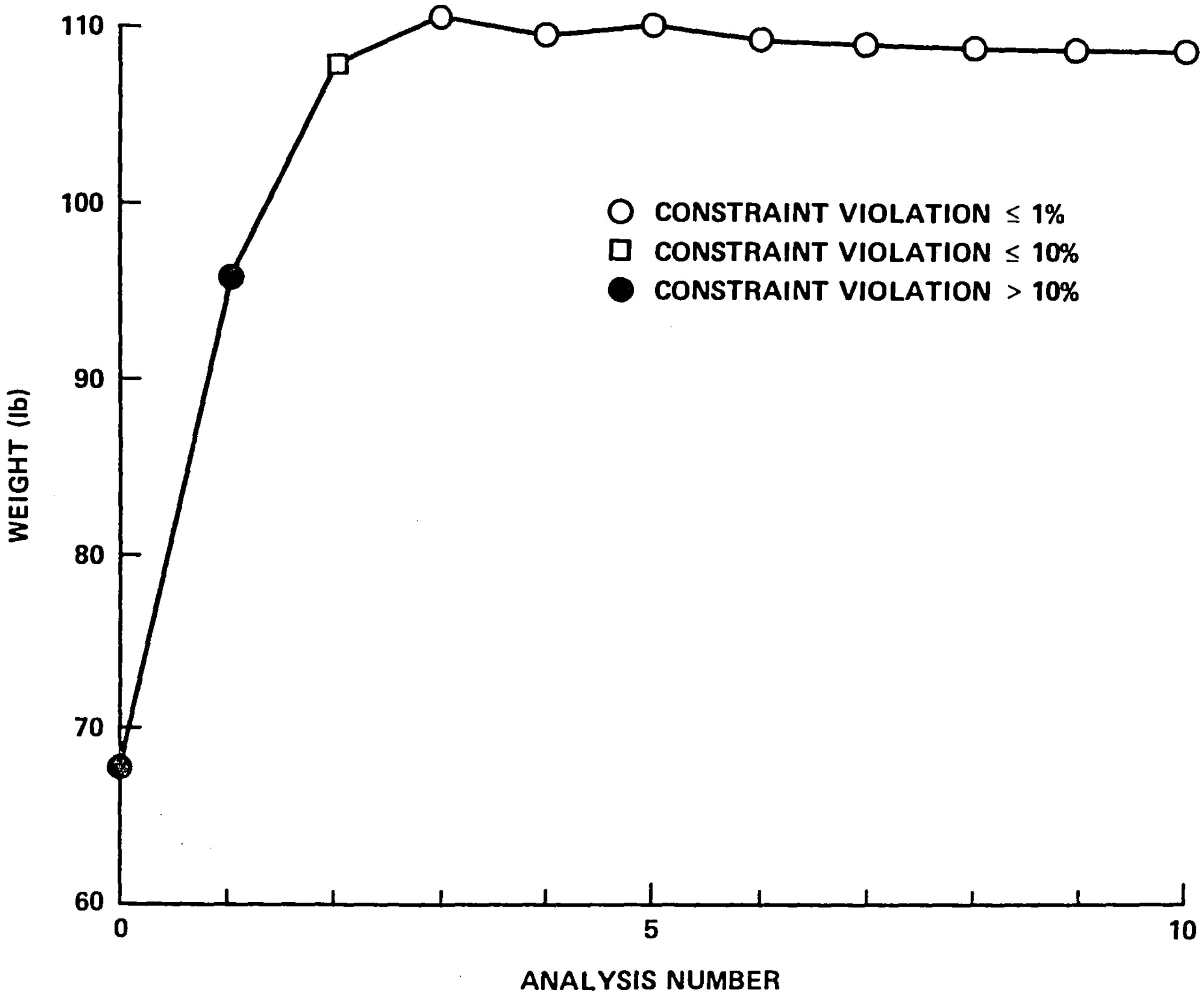


Fig. 95 - Iteration History for Problem 9, Run 1 (Option 1(P))
Helicopter Tail Boom

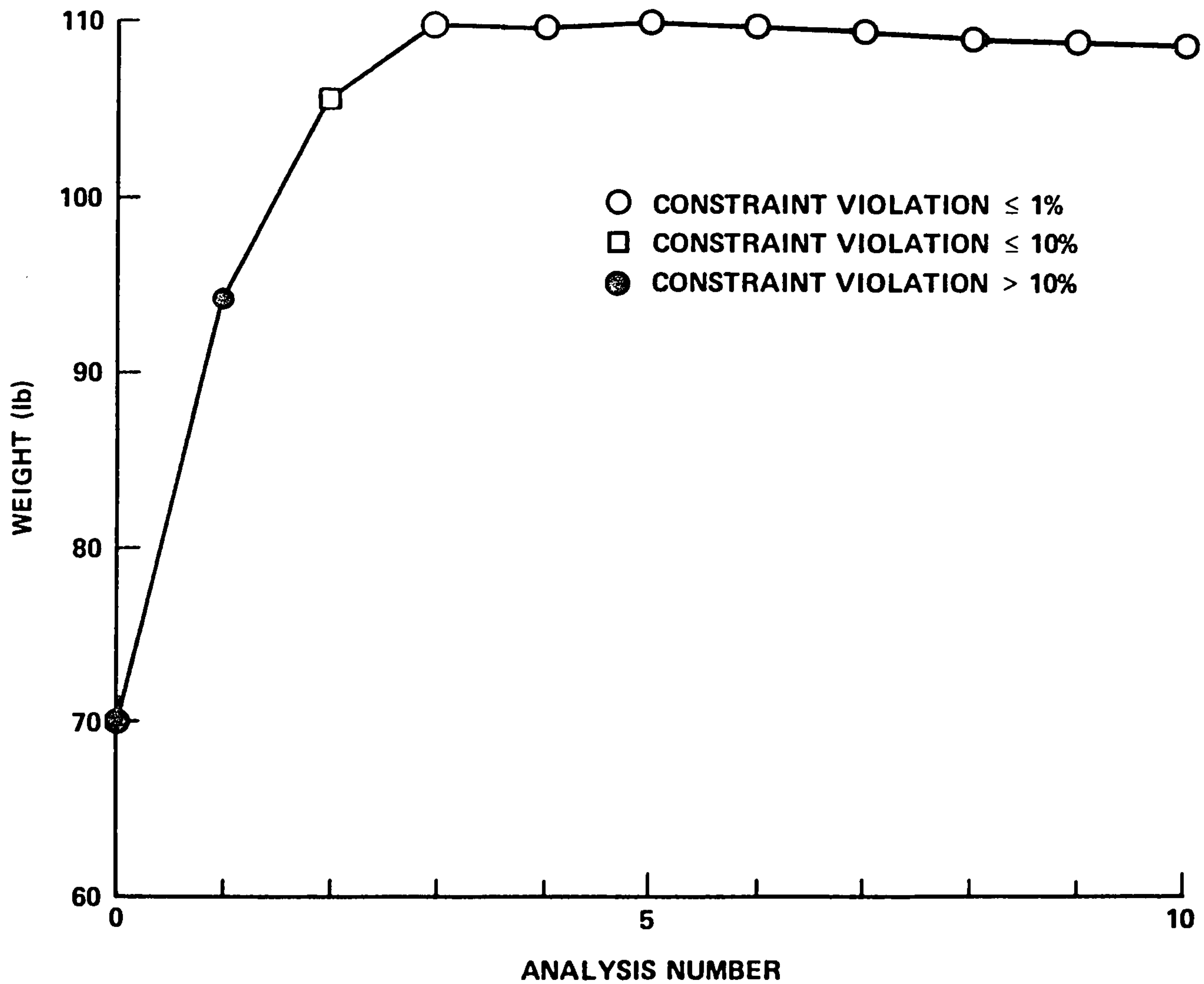


Fig. 96 - Iteration History for Problem 9, Run 2, (Option 4(P))
Helicopter Tail Boom

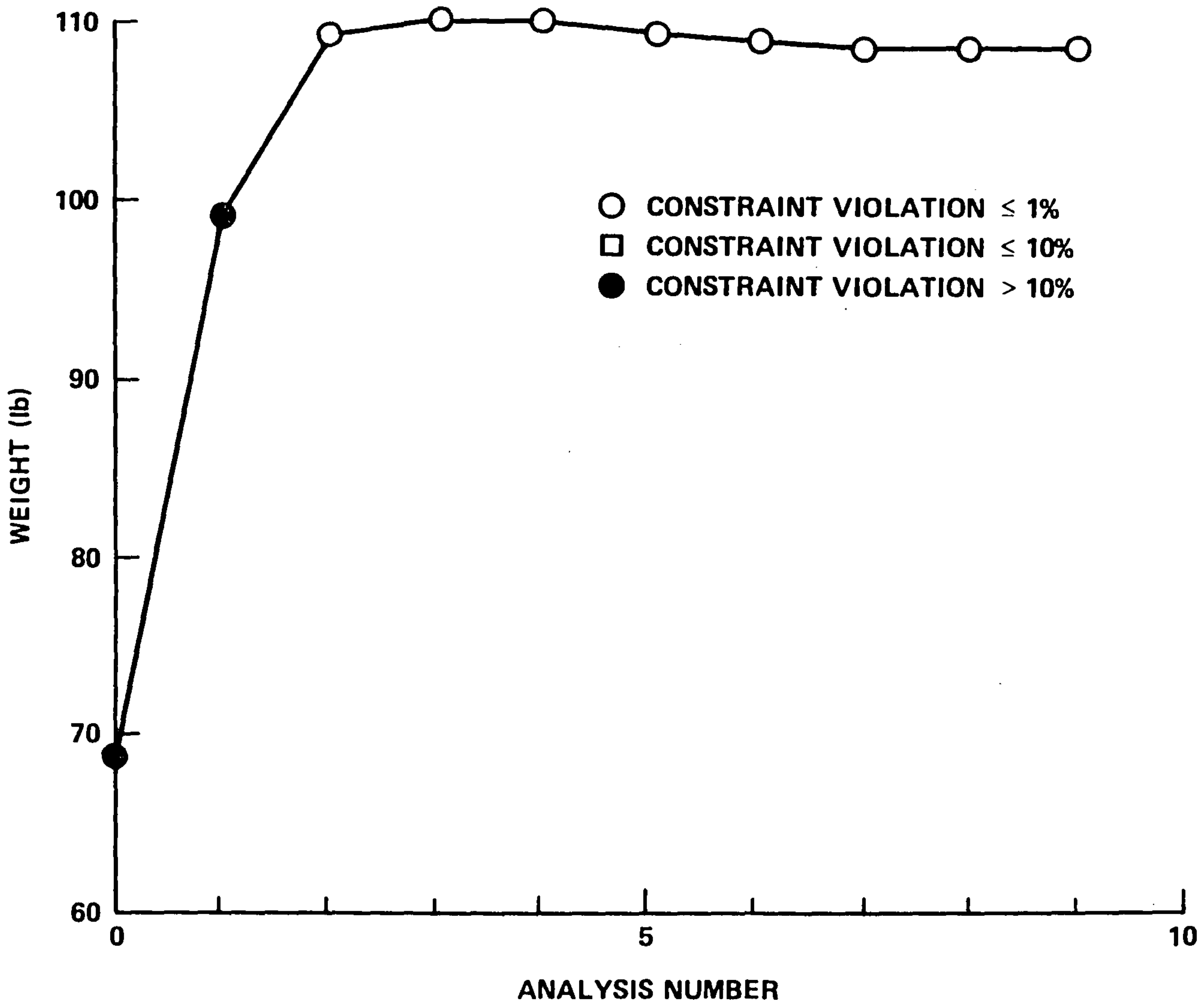


Fig. 97 - Iteration History for Problem 9, Run 3 (Option 10(P))
Helicopter Tail Boom

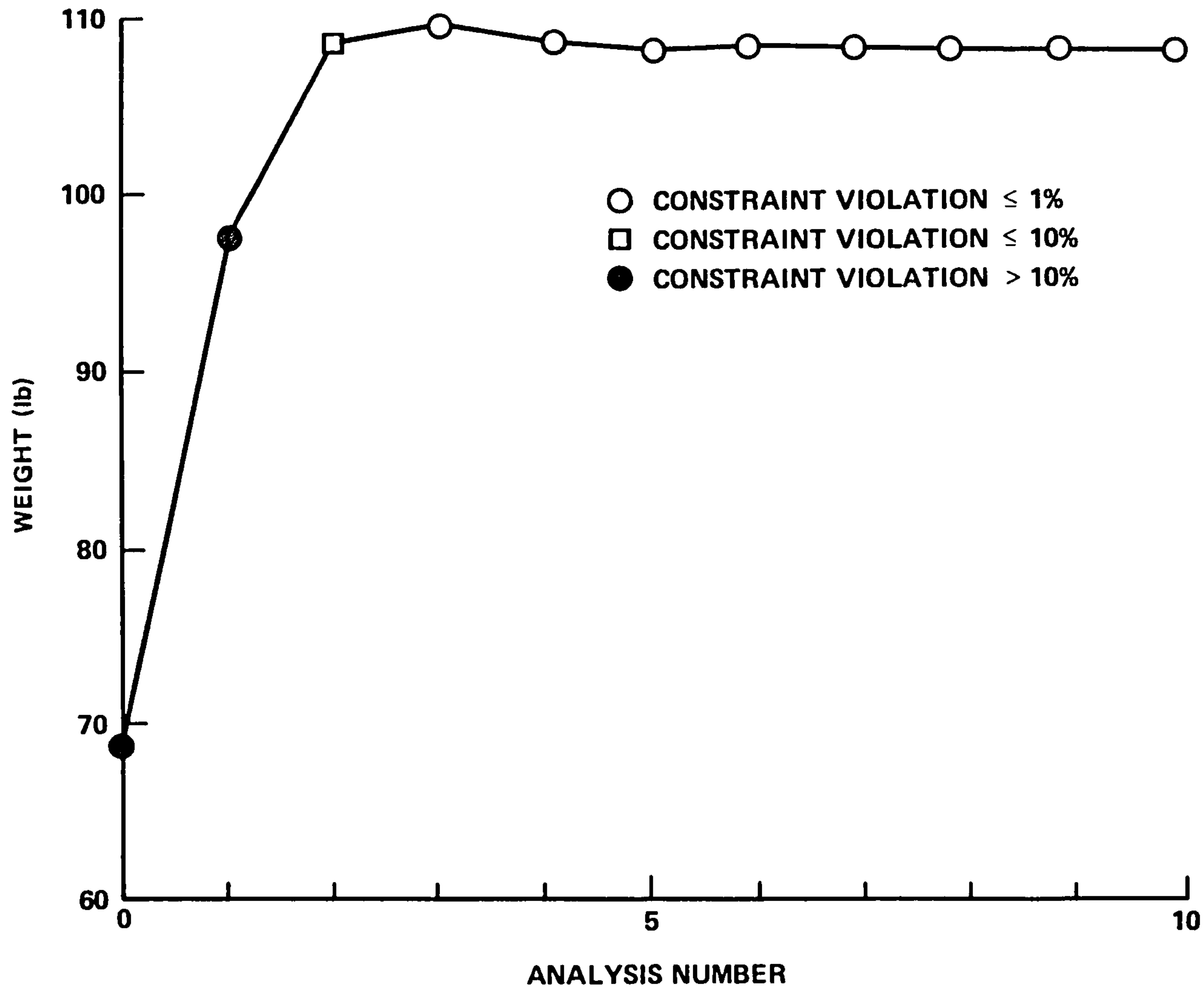


Fig. 98 - Iteration History for Problem 9, Run 4 (Option 10(D))
Helicopter Tail Boom

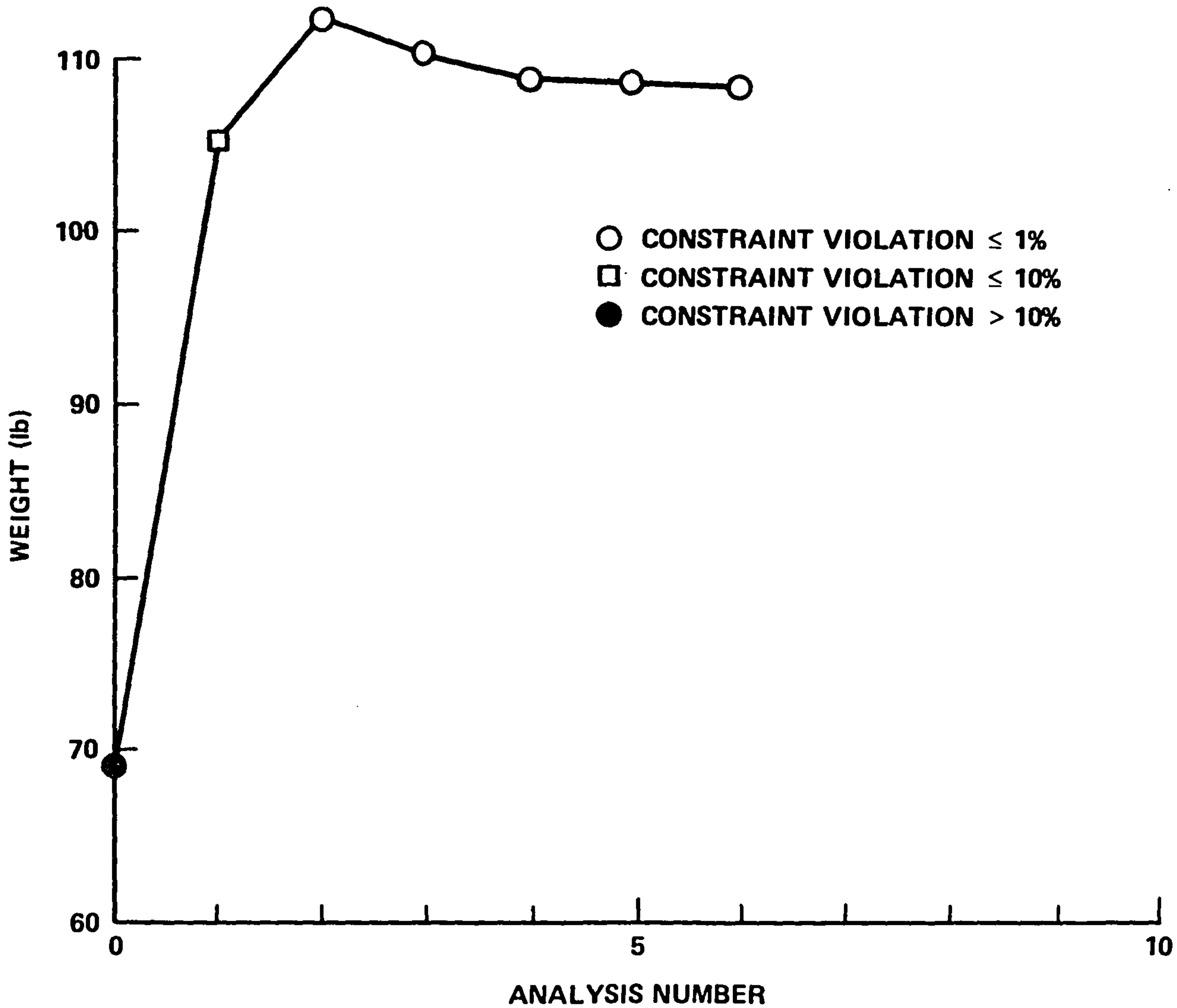


Fig. 99 - Iteration for Problem 9, Run 5 (Option 1(PU))
Helicopter Tail Boom

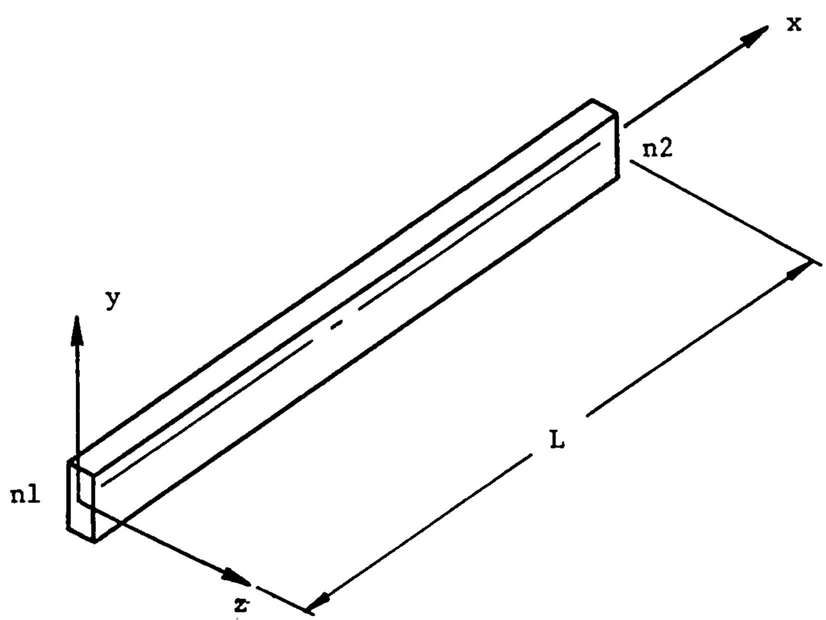


Fig. A1 - Space Frame Element

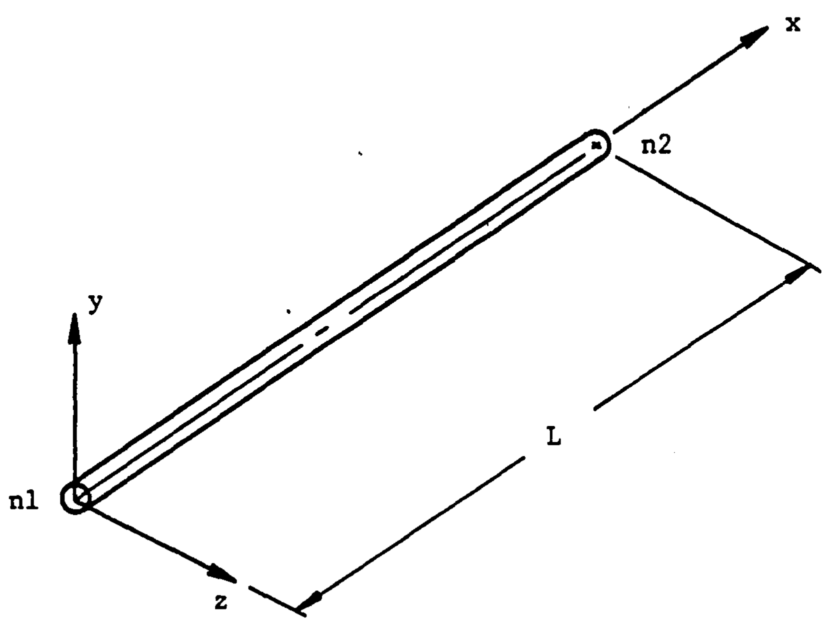
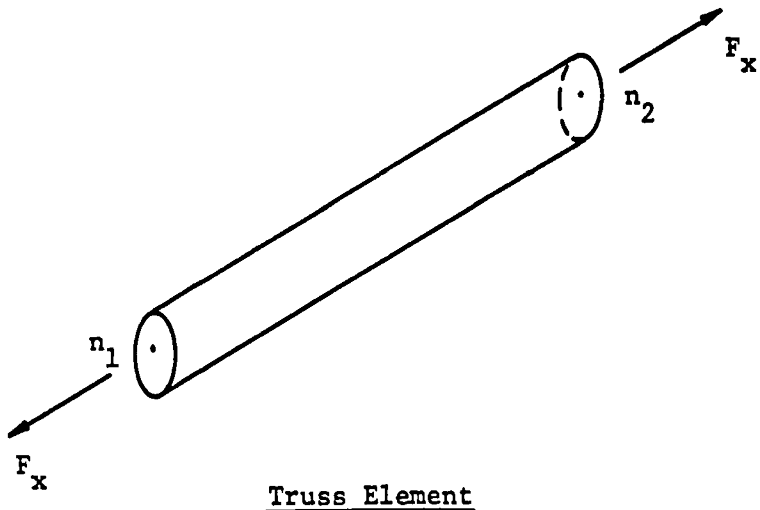
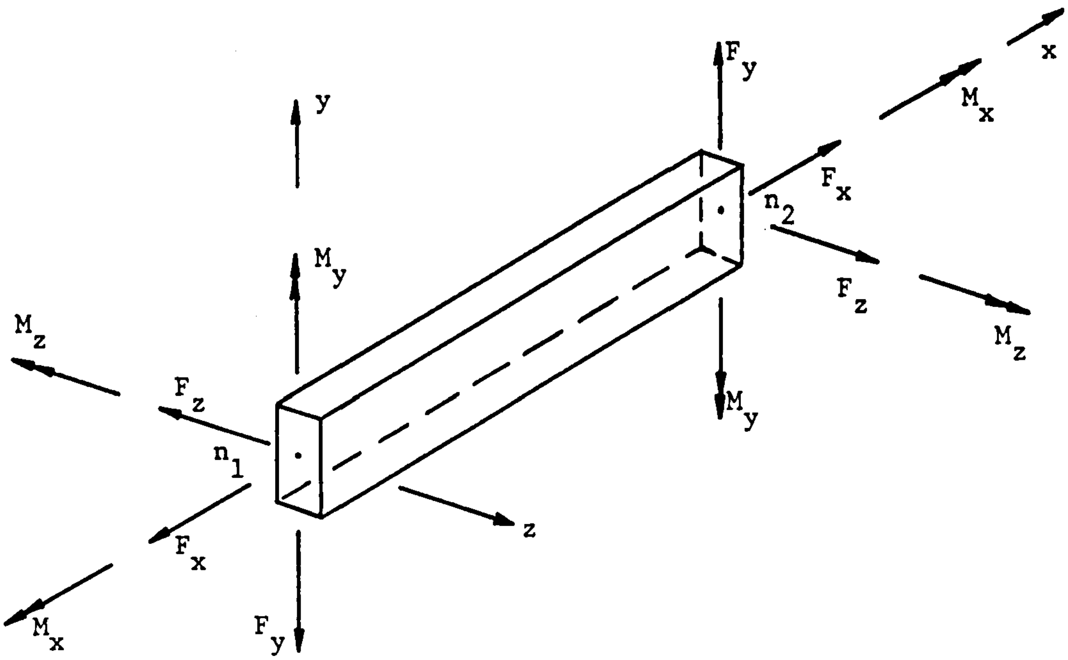


Fig. A2 - Space Truss Element



Truss Element



Frame Element

Fig. C1 - Forces at Element End Nodes

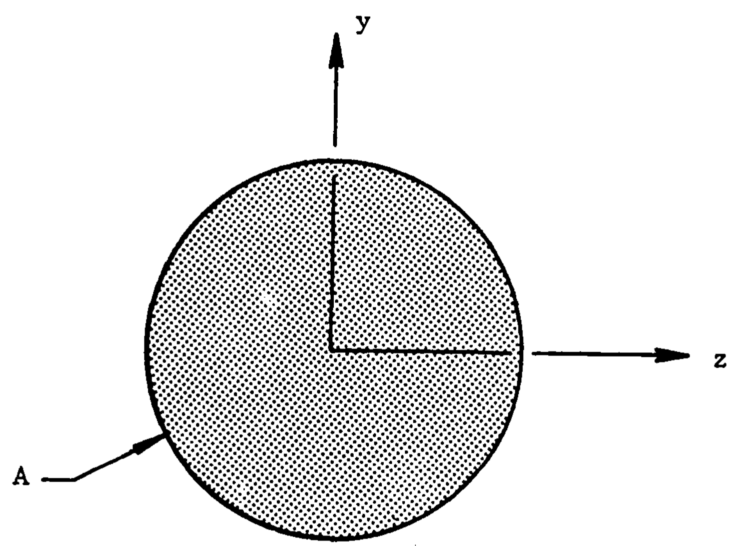
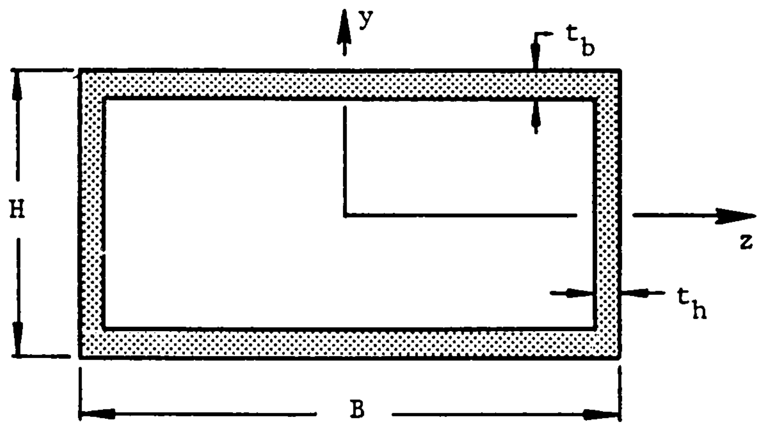
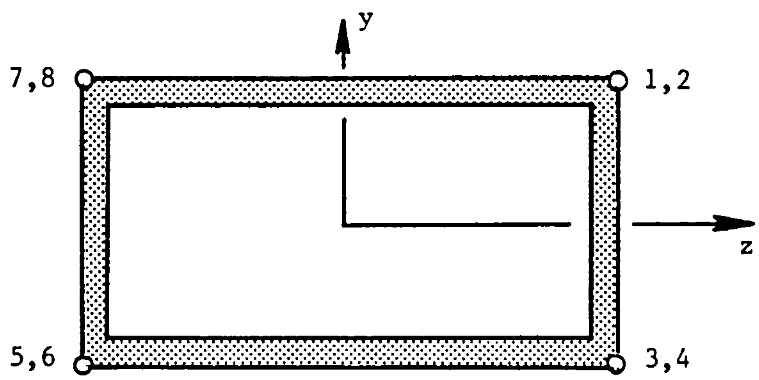


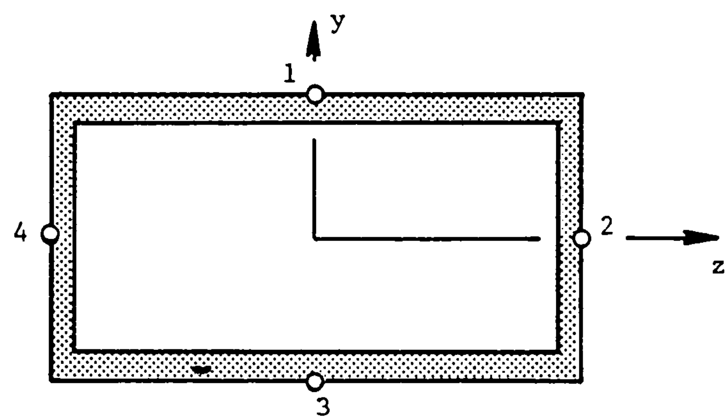
Fig. C2 - Cross Section for Design Element Type 1



Cross Sectional Dimensions



Stress Constraint Evaluation Points



Local Buckling Constraint Evaluation Points

Fig. C3 - Cross Section for Design Element Type 11

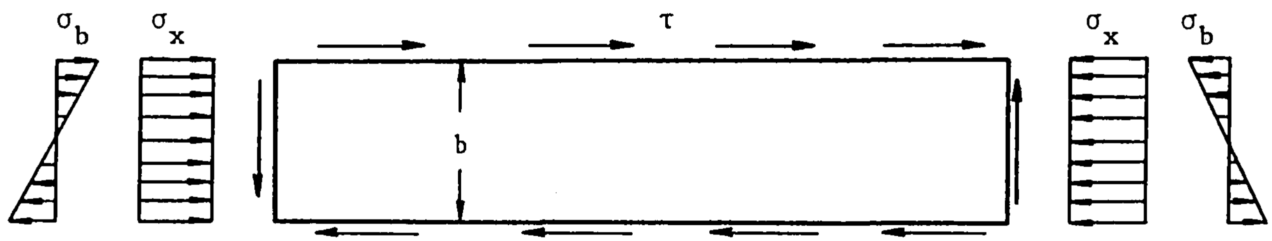
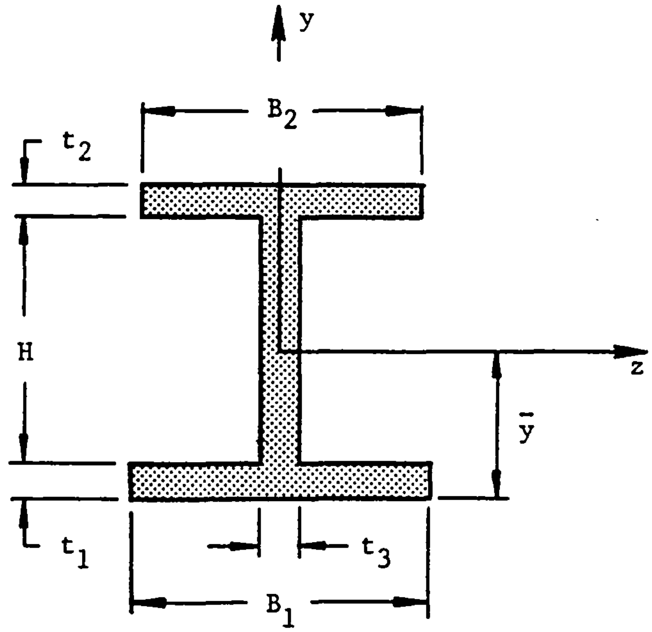
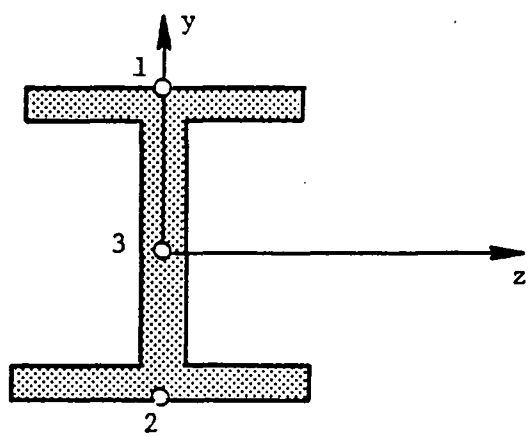


Fig. C4 - Buckling Stresses on a Long Simply Supported Plate

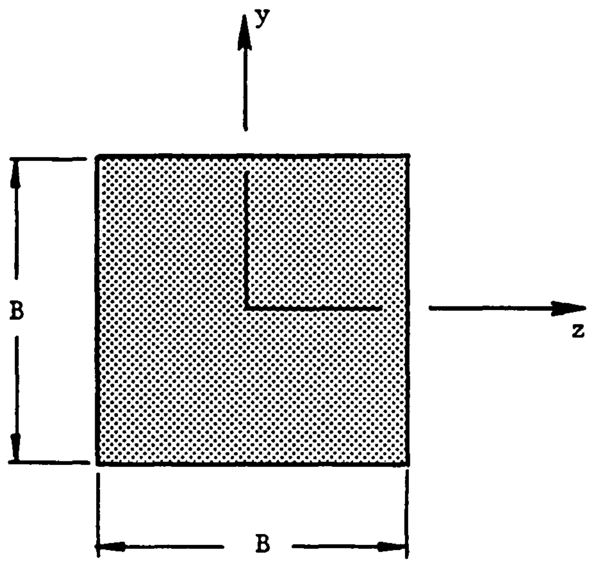


Cross Sectional Dimensions

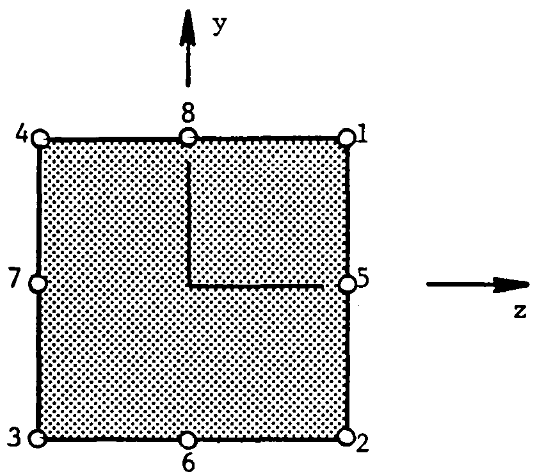


Stress and Local Buckling Constraint
Evaluation Points

Fig. C5 - Cross Section for Design Element Type 12

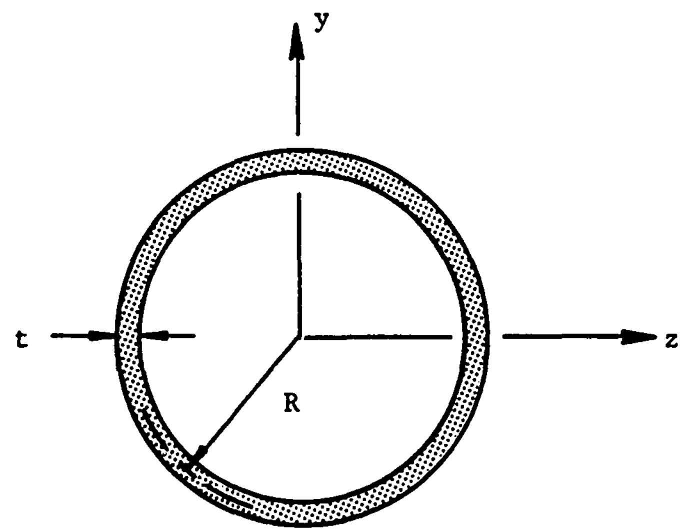


Cross Sectional Dimensions

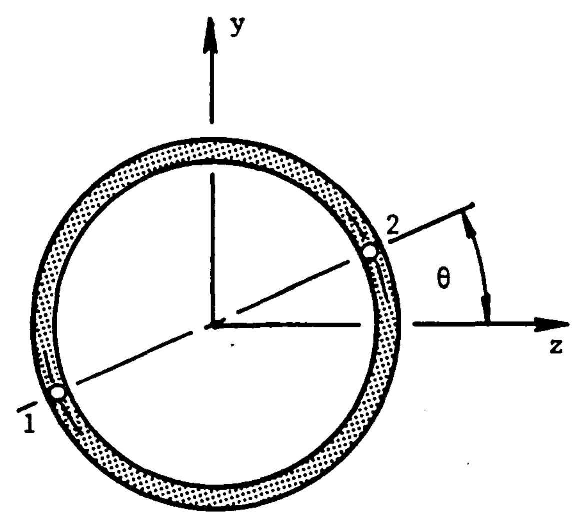


Stress Constraint Evaluation Points

Fig. C6 - Cross Section for Design Element Type 13

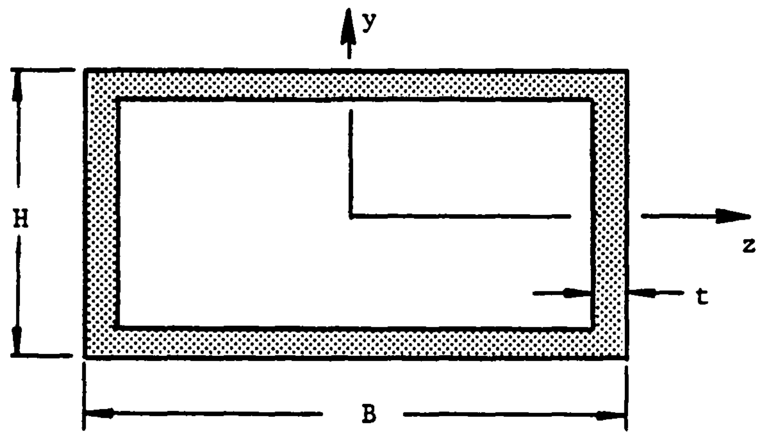


Cross Sectional Dimensions

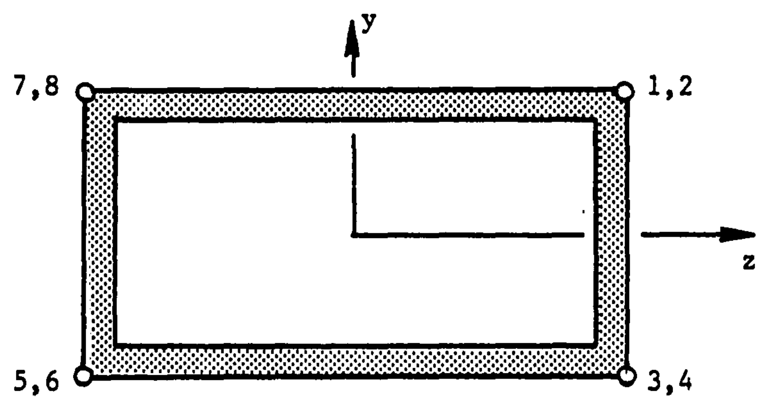


Stress and Buckling Constraint Evaluation Points

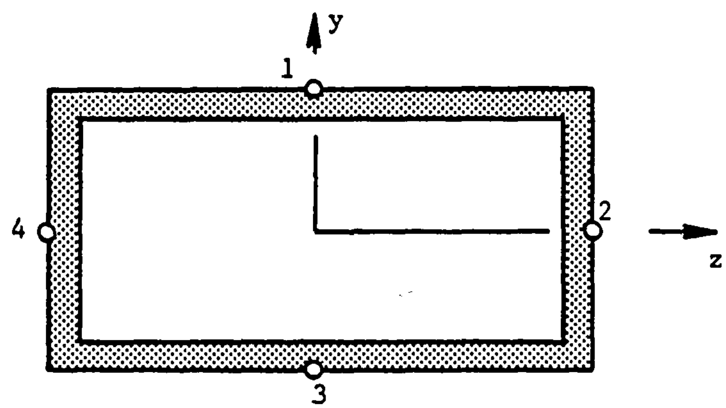
Fig. C7 - Cross Section for Element Type 14



Cross Sectional Dimensions



Stress Constraint Evaluation Points



Local Buckling Constraint Evaluation Points

Fig. C8 - Cross Section for Design Element Type 15

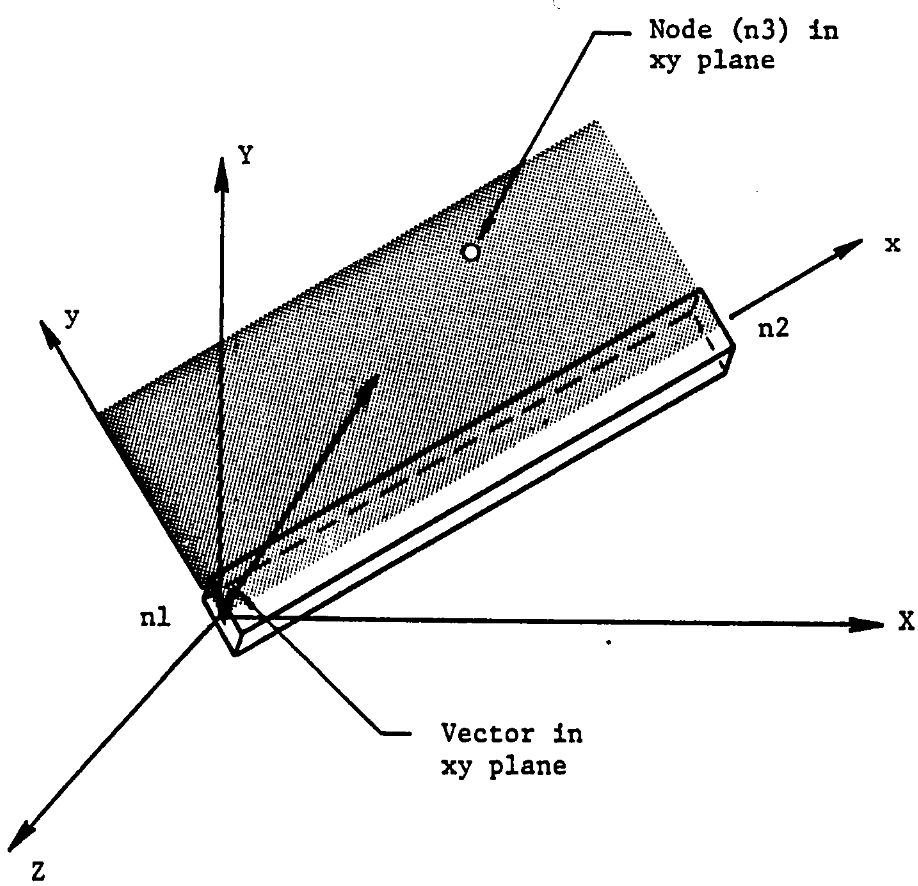


Fig. E1 - Node or Vector Representation of Beam Element Orientation

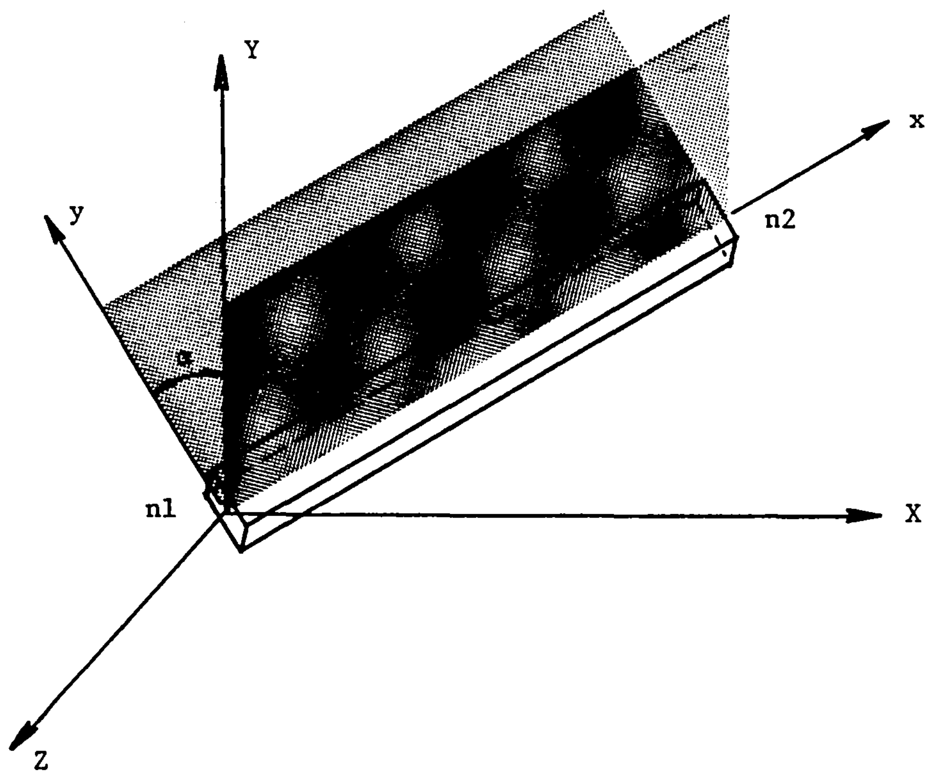


Fig. E2 - Angle Representation of Beam Element Orientation

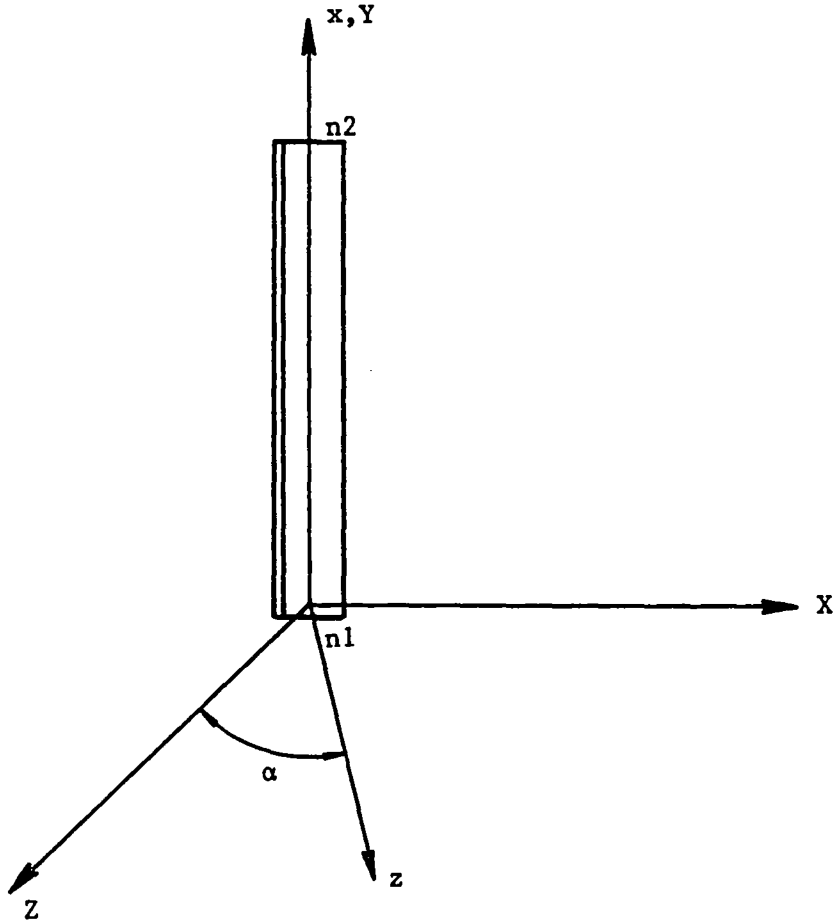


Fig. E3 - Angle Representation of Beam Element Orientation
(Special Case)

Table 1. Descriptions of Design Problem Solution Options

Option*	Design Space	Approximations (objective function/behavior constraints/side constraints)	Element Force Variation
1	RSP	None/Linear/Linear	Invariant
2	RSP	None/Linear/Linear	Local
3	RSP	None/Linear/Linear	Global
4	RSP	None/Hybrid/Linear	Invariant
5	RSP	None/Hybrid/Linear	Local
6	RSP	None/Hybrid/Linear	Global
7	CSD	Linear/Linear/None	Invariant
8	CSD	Linear/Linear/None	Local
9	CSD	Linear/Linear/None	Global
10	CSD	Hybrid/Hybrid/None	Invariant
11	CSD	Hybrid/Hybrid/None	Local
12	CSD	Hybrid/Hybrid/None	Global

* The letter designation following the option number indicates the type of optimization method used to solve the approximate problem ((P)=Primal, (D)=Dual). Also, the designation (U) indicates that the approximate problem update feature was employed.

Table 2. Definition of Problem 1
Tied Cantilevered Beam

Material Properties

Young's Modulus	:	$E = 30.0 \times 10^6$ PSI
Shear Modulus	:	$G = 11.5 \times 10^6$ PSI
Poisson's Ratio	:	$\nu = .3$
Weight Density	:	$\rho = .284$ lb/in^3
Allowable Normal Stress (Truss)	:	$\sigma_a = 120{,}000$ PSI
Allowable Normal Stress (Beam)	:	$\sigma_a = 20{,}000$ PSI
Allowable Shear Stress (Beam)	:	$\tau_a = 10{,}000$ PSI

Nodal Loading

Load Case	Node No.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	2	0.	-10,000	0.	0.	0.	0.

Uniform Loading

Load Case	Member No.	Loading Components (lb/in, in-lb/in)					
		p_x	p_y	p_z	m_x	m_y	m_z
2	1	0.	-83.33	0.	0.	0.	0.

Initial Design and Side Constraints

Member No.	Sizing Variable	Initial Value (in, in^2)	Lower Bound (in, in^2)	Upper Bound (in, in^2)
1	B	5.00	1.00	10.00
2	A	.20	.01	1.00

Table 3. Iteration History Data for Problem 1
Tied Cantilevered Beam

		Weight (lb) [Maximum Constraint Violation (%)]			
Analysis No.		Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 6(P)
0		861.64 [0]	861.64 [0]	861.64 [0]	861.64 [0]
1		697.00 [0]	697.00 [0]	697.00 [0]	697.00 [0]
2		563.97 [1.2]	563.92 [2.2]	563.98 [1.0]	563.98 [1.0]
3		531.34 [1.4]	519.21 [1.8]	476.97 [3.1]	476.97 [3.1]
4		523.20 [.5]	520.14 [0]	475.18 [0]	475.18 [0]
5		520.14 [.2]	520.14 [0]	473.00 [0]	473.00 [0]
6		520.15 [0]	520.14 [0]	473.04 [0]	473.04 [0]
7		520.15 [0]		473.04 [0]	473.04 [0]
CPU Time (sec)	Tot.	.800	.797	1.035	1.033
	Anal.	.131	.196	.249	.249
	Opt.	.166	.142	.192	.192

*See Table 1

Table 4. Final Designs for Problem 1
Tied Cantilevered Beam

			Final Design (in, in^2)			
Linking Group	Member Nos.	Sizing Variable	Ref. 51 Method I	Run 1 Option* 1(P)	Ref. 51 Method II-B	Run 2 Option* 2(P)
1	1	B	3.8850	3.8874	3.8819	3.8874
2	2	A	.1061	.1066	.1062	.1066
Weight (lb)			519.40	520.15	516.80	520.14
Number of Analyses			--	8	--	7

*See Table 1

Table 4. Final Designs for Problem 1
Tied Cantilevered Beam (cont.)

Linking Group	Member Nos.	Sizing Variables	Final Design (in, in^2) Ref. 51 Method IV-B	Run 3 Option* 3(P)	Run 4 Option* 6(P)
1	1	B	3.6394	3.6421	3.6421
2	2	A	.4571	.4354	.4354
Weight (lb)			473.40	473.04	473.04
Number of Analyses			--	8	8

*See Table 1

Table 5. Critical Constraints for Problem 1
Tied Cantilevered Beam

Run No.	Option No.*	Stress Constrained Members	
		Load Case 1	Load Case 2
Ref. 51	--	2	1
1	1(P)	2	1
2	2(P)	2	1
3	3(P)	--	1
4	6(P)	--	1

*See Table 1

Table 6. Definition of Problem 2
Two Member Frame

Material Properties

Young's Modulus : $E = 20.74 \times 10^6$ N/cm^2

Shear Modulus : $G = 7.97 \times 10^6$ N/cm^2

Poisson's Ratio : $\nu = .3$

Mass Density : $\rho = 2.77 \times 10^{-2}$ kg/cm^3

Allowable Stress : $\sigma_a = 2.76 \times 10^4$ N/cm^2

Nodal Loading

Load Case	Node Nos.	Loading Components (N, N-cm)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	2	0.	-44480.	0.	0.	0.	0.

Initial Design and Side Constraints

Member No.	Sizing Variable	Initial Value (cm)	Lower Bound (cm)	Upper Bound (cm)
1	B	15.20	6.350	25.40
	t_b	2.03	.254	2.54
	H	10.20	6.350	25.40
	t_h	2.29	.229	2.54
2	B	22.90	6.350	25.40
	t_b	2.03	.254	2.54
	H	20.30	6.350	25.40
	t_h	2.29	.229	2.54

Table 7. Iteration History Data for Problem 2, Case A
Two Member Frame

Analysis No.		Mass (kg) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 4(P)
0		1220.78 [0]	1220.78 [0]	1220.78 [0]	1220.78 [0]
1		531.31 [0]	531.31 [0]	531.31 [0]	295.74 [0]
2		313.48 [0]	313.48 [0]	313.48 [0]	188.09 [0]
3		204.35 [0]	204.35 [0]	204.35 [0]	163.20 [3.5]
4		165.10 [24.6]	165.23 [23.7]	165.24 [23.6]	151.29 [3.5]
5		161.08 [8.0]	158.88 [6.3]	159.99 [2.5]	141.61 [.6]
6		162.68 [1.0]	150.79 [2.8]	152.69 [0]	133.80 [.3]
7		151.29 [6.8]	142.97 [2.2]	143.70 [0]	133.88 [0]
8		144.37 [2.4]	136.46 [1.4]	136.94 [0]	130.20 [.3]
9		137.49 [1.3]	132.14 [.8]	136.88 [0]	130.32 [0]
10		132.09 [.3]	130.20 [.3]	132.35 [0]	130.32 [0]
11		132.04 [0]	130.20 [.3]	130.47 [0]	
12		130.25 [.1]	130.20 [.3]	130.46 [0]	
13		130.25 [.1]		130.46 [0]	
14		130.25 [.1]			
CPU	Tot.	2.186	2.031	2.192	1.846
Time	Anal.	.293	.375	.467	.217
(sec)	Opt.	.999	.863	.875	.894

*See Table 1

Table 7. Iteration History Data for Problem 2, Case A
Two Member Frame (cont.)

Analysis No.		Mass (kg) [Maximum Constraint Violation (%)] Run 5 Option* 7(P)	Run 6 Option* 10(P)	Run 7 Option* 10(D)	Run 8 Option* 1(PU)
0		1220.78 [0]	1220.78 [0]	1220.78 [0]	1220.78 [0]
1		686.69 [0]	686.69 [0]	686.69 [0]	313.48 [0]
2		397.36 [0]	397.36 [0]	397.36 [0]	165.10 [24.6]**
3		285.02 [33.5]	298.48 [3.6]	298.52 [3.5]	149.25 [8.1]
4		234.82 [25.0]	247.78 [0]	247.69 [0]	137.26 [3.3]
5		193.72 [20.7]	204.49 [0]	204.45 [0]	129.92 [1.2]
6		166.19 [0]	168.88 [0]	168.95 [0]	130.40 [0]
7		139.00 [5.9]	142.71 [0]	142.69 [0]	
8		128.55 [8.2]	132.08 [.1]	132.14 [0]	
9		128.13 [8.4]	132.06 [.1]	132.09 [0]	
10		127.95 [8.2]	132.06 [.1]	132.05 [0]	
11		129.54 [2.3]			
12		130.28 [.1]			
13		130.26 [0]			
14		130.26 [0]			
CPU Time (sec)	Tot.	1.915	1.451	1.146	2.078
	Ana.	.236	.177	.173	.449
	Opt.	.882	.596	.329	.884

* See Table 1
**Constraint was not Retained

Table 8. Final Designs for Problem 2, Case A
Two Member Frame

			Final Design (cm)				
Linking Group	Member Nos.	Sizing Variables	Ref. 29	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 4(P)
1	1	B	6.35^-	6.35^-	6.35^-	6.35^-	6.35^-
		t_b	$.229^-$	$.229^-$	$.229^-$	$.229^-$	$.229^-$
		H	6.35-	6.35^-	6.35^-	6.35^-	6.35^-
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$	$.254^-$
2	2	B	6.35^-	25.36	25.30	25.34	25.36
		t_b	1.14	.248	.248	.249	.249
		H	25.40^+	25.38	25.40^+	25.40^+	25.29
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$	$.254^-$
Mass (kg)			133.70	130.25	130.20	130.46	130.32
Number of Analyses			19	15	13	14	11

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 8. Final Designs for Problem 2, Case A
Two Member Frame (cont.)

Linking Group	Member Nos.	Sizing Variables	Final Design (cm) Run 5 Option* 7(P)	Run 6 Option* 10(P)	Run 7 Option* 10(D)	Run 8 Option* 1(PU)
1	1	B	6.35^-	6.35^-	6.35^-	6.35^-
		t_b	$.229^-$	$.229^-$	$.229^-$	$.229^-$
		H	6.35^-	6.35^-	6.35^-	6.35^-
		t_b	$.254^-$	$.254^-$	$.254^-$	$.254^-$
2	2	B	25.40^+	9.48	9.63	25.31
		t_b	.248	.715	.703	.249
		H	25.40^+	25.40^+	25.40^+	25.40^+
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$
Mass (kg)			130.26	132.06	132.05	130.42
Number of Analyses			15	11	11	7

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 9. Critical Constraints for Problem 2, Case A
Two Member Frame

Run No.	Option No.*	Stress Constrained Members
Ref. 29	--	2
1	1(P)	2
2	2(P)	2
3	3(P)	2
4	4(P)	2
5	7(P)	2
6	10(P)	2
7	10(D)	2
8	1(PU)	2

*See Table 1

Table 10. Iteration History Data for Problem 2, Case B
Two Member Frame

Analysis No.	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 4(P)
	Mass (kg) [Maximum Constraint Violation (%)]			
0	1220.78 [0]	1220.78 [0]	1220.78 [0]	1220.78 [0]
1	531.31 [0]	531.31 [0]	531.31 [0]	295.74 [0]
2	313.48 [0]	313.48 [0]	313.48 [0]	188.09 [>100]**
3	204.35 [42.2]**	204.35 [42.2]**	204.35 [42.2]**	173.83 [6.7]
4	182.90 [0]	182.05 [4.6]	181.71 [7.0]	153.25 [0]
5	159.41 [6.7]	159.55 [6.2]	159.52 [6.3]	142.27 [1.6]
6	153.69 [2.6]	151.42 [2.5]	152.19 [.5]	133.80 [1.0]
7	145.53 [1.7]	143.72 [1.3]	143.92 [6.8]	130.72 [.2]
8	138.31 [1.4]	137.09 [.7]	137.39 [1.8]	130.81 [0]
9	132.27 [1.4]	132.49 [.5]	132.72 [1.0]	130.81 [0]
10	130.22 [3.6]	130.59 [.2]	130.74 [.3]	
11	130.79 [.1]	130.60 [.2]	130.74 [.2]	
12	130.82 [0]	130.60 [.2]	130.75 [.2]	
13				
14				
15				
CPU Time (sec) Tot.	2.051	2.140	2.217	1.662
CPU Time (sec) Anal.	.276	.379	.433	.198
CPU Time (sec) Opt.	.940	.899	.919	.737

*See Table 1 **Constraint was not Retained

Table 10. Iteration History Data for Problem 2, Case B
Two Member Frame (cont.)

Analysis No.		Mass (kg) [Maximum Constraint Violation (%)]		
		Run 5 Option* 10(P)	Run 6 Option* 10(D)	Run 7 Option* 1(PU)
0		1220.78 [0]	1220.78 [0]	1220.78 [0]
1		686.69 [0]	686.69 [0]	313.48 [0]
2		397.37 [0]	397.36 [0]	165.10 [>100]**
3		298.48 [3.6]	299.21 [2.0]	149.55 [77.6]
4		247.78 [0]	247.73 [0]	135.39 [19.6]
5		204.49 [0]	204.42 [0]	130.35 [2.5]
6		168.88 [0]	168.95 [0]	130.75 [0]
7		142.71 [0]	142.73 [0]	
8		132.08 [.1]	132.15 [0]	
9		132.06 [.1]	132.10 [0]	
10		132.06 [.1]	132.15 [0]	
11				
12				
13				
14				
15				
CPU	Tot.	1.526	1.233	2.274
Time	Anal.	.182	.177	.465
(sec)	Opt.	.602	.350	.977

*See Table 1 **Constraint was not retained

Table 11. Final Designs for Problem 2, Case B
Two Member Frame

			Final Design (cm)			
Linking Group	Member Nos.	Sizing Variables	Ref. 29	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)
1	1	B	6.35^-	6.35^-	6.35^-	6.35^-
		t_b	$.229^-$	$.229^-$	$.229^-$	$.229^-$
		H	6.35^-	6.35^-	6.35^-	6.35^-
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$
2	2	B	6.35^-	18.41	18.36	18.41
		t_b	1.14	.348	.348	.348
		H	25.40^+	25.40^+	25.40^+	25.40^+
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$
Mass (kg)			133.70	130.82	130.60	130.75
Number of Analyses			22	13	13	13

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 11. Final Designs for Problem 2, Case B
Two Member Frame (cont.)

Linking Group	Member Nos.	Sizing Variables	Final Design (cm) Run 4 Option* 4(P)	Run 5 Option* 10(P)	Run 6 Option* 10(D)	Run 7 Option* 1(PU)
1	1	B	6.35^-	6.35^-	6.35^-	6.35^-
		t_b	$.229^-$	$.229^-$	$.229^-$	$.229^-$
		H	6.35^-	6.35^-	6.35^-	6.35^-
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$
2	2	B	16.74	9.48	9.64	18.35
		t_b	.385	.715	.704	.349
		H	25.38	25.40^+	25.40^+	25.40^+
		t_h	$.254^-$	$.254^-$	$.254^-$	$.254^-$
Mass (kg)			130.81	132.06	132.15	130.74
Number of Analyses			10	11	11	7

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 12. Critical Constraints for Problem 2, Case B
Two Member Frame

Run No.	Option No.*	Stress Constrained Members	Local Buckling Constrained Members
Ref. 29	--	2	--
1	1(P)	2	2
2	2(P)	2	2
3	3(P)	2	2
4	4(P)	2	--
5	10(P)	2	--
6	10(D)	2	--
7	1(PU)	2	2

*See Table 1

Table 13. Defintion of Problem 3
Three Member Frame

Material Properties

Young's Modulus : $E = 30.0 \times 10^6$ PSI

Shear Modulus : $G = 11.5 \times 10^6$ PSI

Poisson's Ratio : $\nu = .3$

Allowable Stress : $\sigma_a = 40{,}000$ PSI

Nodal Loading

Load Case	Node No.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	2	0.	0.	-10000	0.	0.	0.
	3	0.	0.	-10000	0.	0.	0.

Initial Design and Side Constraints

Member Nos.	Sizing Variable	Initial Value (in)	Lower Bound (in)	Upper Bound (in)
1-3	B	9.0	2.5	10.0
	H	9.0	2.5	10.0
	t	.9	.1	1.0

Table 14. Iteration History Data for Problem 3
Three Member Frame

		Volume (in^3) [Maximum Constraint Violation (%)]			
Analysis No.		Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 4(P)
0		8748.00 [0]	8748.00 [0]	8748.00 [0]	8748.00 [0]
1		3138.34 [0]	3138.34 [0]	3138.34 [0]	1976.35 [40.3]**
2		1808.55 [48.7]	1600.01 [100]	1808.51 [48.7]	1895.92 [8.0]
3		1857.16 [10.1]	1797.94 [19.1]	1857.60 [10.1]	1869.33 [0]
4		1867.32 [.7]	1880.41 [0]	1867.53 [.7]	1806.43 [0]
5		1850.23 [0]	1853.33 [0]	1824.50 [0]	1758.83 [0]
6		1779.38 [0]	1774.28 [.7]	1780.04 [0]	1720.58 [0]
7		1744.35 [0]	1742.52 [.2]	1746.63 [0]	1690.07 [0]
8		1714.93 [0]	1715.20 [0]	1716.11 [0]	1669.84 [0]
9		1690.33 [0]	1690.44 [0]	1691.39 [0]	1668.23 [.1]
10		1673.16 [0]	1682.79 [0]	1682.85 [0]	1667.82 [.1]
11		1670.60 [0]	1666.43 [0]	1667.96 [0]	
12		1670.18 [0]	1666.43 [0]	1667.96 [0]	
13		1670.18 [0]	1666.43 [0]	1667.96 [0]	
CPU Time (sec)	Tot.	3.189	3.423	3.884	2.591
	Anal.	.335	.675	.866	.262
	Opt.	1.935	1.820	2.114	1.551

*See Table 1 **Constraint was not Retained

Table 14. Iteration History Data for Problem 3
Three Member Frame (cont.)

		Volume (in^3) [Maximum Constraint Violation (%)]		
Analysis No.		Run 5 Option* 10(P)	Run 6 Option* 10(D)	Run 7 Option* 1(PU)
0		8748.00 [0]	8748.00 [0]	8748.00 [0]
1		4286.52 [3.5]**	4286.52 [3.5]**	1590.87 [>100]**
2		3066.00 [0]	3084.45 [0]	1751.41 [16.7]
3		2312.58 [0]	2311.07 [0]	1773.40 [1.2]
4		1982.69 [0]	1981.12 [0]	1715.05 [0]
5		1846.23 [0]	1841.54 [0]	1674.29 [0]
6		1739.91 [0]	1727.85 [0]	1671.03 [0]
7		1680.03 [0]	1673.63 [0]	
8		1668.90 [0]	1666.48 [0]	
9		1666.65 [0]	1666.66 [0]	
10		1666.65 [0]	1666.71 [0]	
11		1666.65 [0]		
12				
13				
CPU Time (sec)	Tot.	2.437	1.449	3.321
	Anal.	.243	.221	.507
	Opt.	1.421	.516	1.982

*See Table 1 **Constraint was not Retained

Table 15. Final Designs for Problem 3
Three Member Frame

			Final Design (in)			
Linking Group	Member Nos.	Sizing Variables	Ref. 52	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)
1	1	B	10.00^+	10.00^+	9.99	9.99
		H	10.00^+	10.00^+	10.00^+	10.00^+
		t	.199	.201	.200	.201
2	2	B	2.50^-	2.50^-	2.50^-	2.50^-
		H	2.50^-	2.50^-	2.50^-	2.50^-
		t	$.100^-$	$.100^-$	$.100^-$	$.100^-$
3	3	B	10.00^+	10.00^+	10.00^+	10.00^+
		H	10.00^+	9.99	10.00^+	9.99
		t	.199	.201	.200	.200
Volume (in^3)			1656.96	1670.18	1666.43	1667.96
Number of Analyses			14	14	14	14

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 15. Final Designs for Problem 3
Three Member Frame (cont.)

Linking Group	Member Nos.	Sizing Variables	Final Design (in) Run 4 Option* 4(P)	Run 5 Option* 10(P)	Run 6 Option* 10(D)	Run 7 Option* 1(PU)
1	1	B	10.00^{+}	10.00^{+}	10.00^{+}	10.00^{+}
		H	9.98	10.00^{+}	10.00^{+}	9.98
		t	.201	.200	.200	.201
2	2	B	2.50^{-}	2.50^{-}	2.50^{-}	2.50^{-}
		H	2.50^{-}	2.50^{-}	2.50^{-}	2.50^{-}
		t	.100^{-}	.100^{-}	.100^{-}	.100^{-}
3	3	B	10.00^{+}	10.00^{+}	10.00^{+}	10.00^{+}
		H	9.98	10.00^{+}	10.00^{+}	9.99
		t	.201	.200	.200	.201
Volume (in^3)			1667.82	1666.65	1666.72	1670.77
Number of Analyses			11	12	11	7

*See Table 1

+Sizing Variable at Upper Bound

−Sizing Variable at Lower Bound

Table 16. Critical Constraints for Problem 3
Three Member Frame

Run No.	Option No.*	Stress Constrained Members
Ref. 52	--	1,3
1	1(P)	1,3
2	2(P)	1,3
3	3(P)	1,3
4	4(P)	1,3
5	10(P)	1,3
6	10(D)	1,3
7	1(PU)	1,3

*See Table 1

Table 17. Definition of Problem 4
Seven Member Frame

Material Properties

Young's Modulus : $E = 20.74 \times 10^6\ N/cm^2$

Shear Modulus : $G = 7.85 \times 10^6\ N/cm^2$

Poisson's Ratio : $\nu = .32$

Mass Density : $\rho = 7.81 \times 10^{-3}\ kg/cm^3$

Allowable Stress : $\sigma_a = 20{,}000\ N/cm^2$

Nodal Loading

Load Case	Node No.	Loading Components (N, N-cm)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	3	-40000	-40000	0.	0.	0.	0.
2	3	-50000	0.	0.	0.	0.	0.
	4	0.	-50000	0.	0.	0.	0.

Displacement Constraints

Load Case	Node No.	Direction	Lower Bound	Upper Bound
1	3	y	-.2 cm	.04 cm
2	3	y	-.2 cm	.04 cm

Initial Design and Side Constraints

Member Nos.	Sizing Variable	Initial Value (cm)	Lower Bound (cm)	Upper Bound (cm)
1-7	B	7.62	1.00	10.00
	H	7.62	1.00	10.00
	t	.18	.076	.30

Table 18. Iteration History Data for Problem 4
Seven Member Frame

Analysis No.		Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 6(P)
		Mass (kg) [Maximum Constraint Violation (%)]			
0		16.26 [11.9]	16.26 [11.9]	16.26 [11.9]	16.26 [11.9]
1		9.52 [63.8]**	9.75 [76.7]**	9.93 [74.6]**	10.07 [66.3]**
2		9.25 [20.7]	8.59 [41.9]	8.40 [20.7]	8.59 [16.6]
3		8.75 [62.5]	8.37 [21.2]	8.77 [1.5]	8.85 [.2]
4		8.49 [100]	8.49 [5.1]	8.56 [.8]	8.77 [.3]
5		8.87 [94.4]	8.53 [0]	8.57 [0]	8.65 [.1]
6		9.26 [62.3]	8.38 [2.9]	8.43 [.1]	8.55 [0]
7		9.78 [31.3]	8.34 [1.5]	8.31 [.2]	8.52 [0]
8		10.03 [15.1]	8.36 [0]	8.27 [0]	8.40 [0]
9		10.02 [13.5]	8.29 [.5]	8.23 [0]	8.31 [.1]
10		10.22 [7.8]	8.23 [3.7]	8.12 [.9]	8.28 [.1]
11		10.20 [6.9]	8.29 [0]	8.14 [.2]	8.16 [.4]
12		10.14 [6.0]	8.25 [5.9]	8.11 [0]	8.18 [0]
13		10.06 [5.7]	8.12 [3.4]	8.10 [0]	8.12 [.6]
14		10.00 [4.9]	8.19 [0]	8.08 [0]	8.12 [0]
15		9.94 [4.5]	8.17 [.7]	8.08 [0]	8.10 [.2]
16		9.86 [4.7]			
17		9.82 [3.8]			
18		9.78 [2.9]			
19		9.75 [2.1]			
20		9.72 [1.3]			
CPU Time (sec)	Tot.	No Convergence	10.627	10.615	10.929
	Anal.		1.963	3.388	3.291
	Opt.		6.369	4.921	5.322

*See Table 1 **Constraint was not Retained

Table 18. Iteration History Data for Problem 4
Seven Member Frame (cont.)

Analysis No.		Run 5 Option* 12(P)	Run 6 Option* 12(D)	Run 7 Option* 3(PU)
		Mass (kg) [Maximum Constraint Violation (%)]		
0		16.26 [11.9]	16.26 [11.9]	16.26 [11.9]
1		9.70 [100]**	10.60 [81.8]**	8.62 [95.5]**
2		8.70 [39.6]**	9.00 [10.8]	8.47 [5.6]
3		8.97 [.1]	8.65 [0]	8.37 [0]
4		8.50 [.2]	8.24 [1.3]	8.30 [0]
5		8.30 [0]	8.13 [1.4]	8.23 [.1]
6		8.14 [0]	8.09 [.7]	8.21 [0]
7		8.13 [0]	8.07 [.8]	8.10 [.3]
8		8.12 [0]	8.08 [.3]	8.09 [0]
9		8.10 [0]	8.09 [.1]	
10		8.10 [0]		
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
CPU Time (sec)	Tot.	8.215	17.090	10.333
	Anal.	2.175	1.945	2.589
	Opt.	4.459	13.730	5.307

*See Table 1 **Constraint was not Retained

Table 19. Final Designs for Problem 4
Seven Member Frame

			Final Design (cm)			
Linking Group	Member Nos.	Size Var.	Ref. 28	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 6(P)
1	1	B	10.00^{+}	9.96	8.39	7.22
		H	3.73	2.73	5.18	5.09
		t	.165	.162	.173	.192
2	2	B	10.00^{+}	10.00^{+}	7.91	8.06
		H	10.00^{+}	10.00^{+}	2.10	2.97
		t	.096	.099	.199	.180
3	3	B	7.81	9.97	9.99	9.97
		H	10.00^{+}	10.00^{+}	1.73	2.49
		t	.110	.096	.158	.149
4	4	B	10.00^{+}	10.00^{+}	10.00^{+}	9.99
		H	5.49	8.06	3.08	3.56
		t	.201	.173	.201	.198
5	5	B	1.50	1.07	1.76	2.13
		H	1.32	1.00^{-}	1.06	1.29
		t	$.076^{-}$	$.076^{-}$	$.076^{-}$	$.076^{-}$
6	6	B	1.28	1.00^{-}	1.00^{-}	1.29
		H	1.28	1.00^{-}	1.00^{-}	1.00^{-}
		t	$.076^{-}$	$.076^{-}$	$.076^{-}$	$.076^{-}$
7	7	B	5.31	3.02	2.91	3.06
		H	5.13	7.56	10.00^{+}	10.00^{+}
		t	$.076^{-}$	.087	.081	.079
Mass (kg)			8.25	8.17	8.08	8.10
Number of Analyses			--	16	16	16

*See Table 1

+Sizing Variable at Upper Bound
-Sizing Variable at Lower Bound

Table 19. Final Designs for Problem 4
Seven Member Frame (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (cm) Run 5 Option* 12(P)	Run 6 Option* 12(D)	Run 7 Option* 3(PU)
1	1	B	7.62	8.13	10.00+
		H	4.64	4.18	5.52
		t	.191	.184	.153
2	2	B	6.49	6.88	9.17
		H	6.00	5.56	1.43
		t	.155	.158	.188
3	3	B	7.72	7.49	9.36
		H	3.98	3.81	1.28
		t	.163	.170	.173
4	4	B	7.92	8.88	9.96
		H	6.14	5.40	2.16
		t	.202	.199	.214
5	5	B	1.18	1.07	1.80
		H	1.17	1.11	1.12
		t	.076-	.076-	.076-
6	6	B	1.00-	1.00-	1.15
		H	1.00-	1.00-	1.00-
		t	.076-	.076-	.076-
7	7	B	4.95	4.52	2.99
		H	5.34	4.65	10.00+
		t	.092	.106	.081
Mass (kg)			8.10	8.09	8.09
Number of Analyses			11	10	10

*See Table 1

+Sizing Variable at Upper Bound
-Sizing Variable at Lower Bound

Table 20. Critical Constraints for Problem 4
Seven Member Frame

Run No.	Option No.*	Displacement Constrained Nodes		Stress Constrained Members	
		Load Case 1	Load Case 2	Load Case 1	Load Case 2
Ref. 28	--	--	3	3	1,2,4,7
2	2(P)	--	3	3	1,2,4,7
3	3(P)	--	3	3	1,4
4	6(P)	--	3	3	1,4
5	12(P)	--	3	3	1,4
6	12(D)	--	3	3	1,4
7	3(PU)	--	3	3	1,4

*See Table 1

Table 21. Definition of Problem 5
Portal Frame

Material Properties

Young's Modulus : $E = 7.0 \times 10^6$ N/cm^2

Shear Modulus : $G = 2.7 \times 10^6$ N/cm^2

Poisson's Ratio : $\nu = .3$

Allowable Normal Stress : $\sigma_a = 2.0 \times 10^4$ N/cm^2

Allowable Shear Stress : $\tau_a = 1.16 \times 10^4$ N/cm^2

Nodal Loading

Load Case	Node Nos.	Loading Components (N, N-cm)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	3	5×10^4	0.	0.	0.	0.	0.
2	3	0.	0.	0.	0.	0.	-2×10^7

Displacement Constraints

Load Case	Node Nos.	Direction	Lower Bound	Upper Bound
1	3	x	-4.0 cm	4.0 cm
2	3	θ_z	-.015 rad	.015 rad

Table 21. Definition of Problem 5
Portal Frame (cont.)

Initial Design and Side Constraints

Member No.	Sizing Variable	Initial Value (cm)	Lower Bound (cm)	Upper Bound (cm)
1	B_1	30.0	5.0	100.0
	t_1	1.0	.1	5.0
	B_2	30.0	10.0	100.0
	t_2	1.0	.1	5.0
	H	50.0	50.0	100.0
	t_3	1.0	.1	5.0
2	B_1	30.0	5.0	100.0
	t_1	1.0	.1	5.0
	B_2	30.0	10.0	100.0
	t_2	1.0	.1	5.0
	H	50.0	50.0	100.0
	t_3	1.0	.1	5.0
3	B_1	30.0	5.0	100.0
	t_1	1.0	.1	5.0
	B_2	30.0	10.0	100.0
	t_2	1.0	.1	5.0
	H	50.0	25.0	100.0
	t_3	1.0	.1	5.0

Table 22. Iteration History Data for Problem 5
Portal Frame

		Volume (cm^3) [Maximum Constraint Violation (%)]		
Analysis No.		Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)
0		275,000 [0]	275,000 [0]	275,000 [0]
1		193,179 [0]	193,179 [0]	193,179 [0]
2		126,224 [84.0]**	126,224 [84.0]**	126,224 [84.0]**
3		119,299 [17.6]	133,432 [10.9]	121,745 [21.5]
4		102,977 [>100.0]**	119,212 [12.5]	102,563 [>100.0]**
5		101,275 [55.1]	101,976 [>100]**	101,018 [46.1]
6		104,134 [1.0]	99,087 [70.3]	97,564 [8.6]
7		99,506 [>100.0]**	100,041 [12.9]	97,444 [.3]
8		86,049 [>100.0]**	100,527 [.8]	97,459 [.2]
9		94,010 [>100.0]**	100,547 [.1]	97,460 [0]
10		91,575 [>100.0]	99,083 [1.7]	
11		93,696 [>100.0]	99,110 [.1]	
12		93,575 [37.2]	97,880 [1.3]	
13		93,955 [.2]	97,934 [0]	
14		94,091 [.1]	97,560 [.6]	
15		93,702 [.2]	97,675 [0]	
16		93,773 [.1]	97,574 [0]	
17		93,732 [.3]	97,226 [.6]	
18			97,365 [0]	
19			97,239 [0]	
20			97,240 [0]	
CPU Time (sec)	Tot.	5.339	6.093	3.759
	Anal.	.956	1.613	.953
	Opt.	2.825	2.739	1.804

*See Table 1 **Constraint was not Retained

Table 22. Iteration History Data for Problem 5
Portal Frame (cont.)

	Volume (cm^3) [Maximum Constraint Violation (%)]	
Analysis No.	Run 4 Option* 12(P)	Run 5 Option* 12(D)
0	275,000 [0]	275,000 [0]
1	136,028 [86.2]**	108,464 [>100]**
2	109,962 [>100]**	95,044 [31.1]
3	94,857 [30.2]	88,177 [41.2]**
4	88,267 [43.8]**	86,557 [0]
5	86,899 [0]	84,766 [0]
6	85,469 [0]	84,238 [.8]
7	84,824 [0]	84,109 [.7]
8	84,483 [91.0]**	84,022 [.5]
9	84,265 [10.6]	84,056 [.7]
10	84,215 [.1]	84,058 [.3]
11	84,272 [0]	
12		
CPU Time (sec) Tot.	3.754	3.527
CPU Time (sec) Anal.	1.042	1.069
CPU Time (sec) Opt.	1.672	1.436

*See Table 1 **Constraint was not Retained

Table 23. Final Designs for Problem 5 Portal Frame

Linking Group	Member Nos.	Size Var.	Final Design (cm)					
			Ref. 53	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 12(P)	Run 5 Option* 12(D)
1	1	B_1	13.00	24.26	22.90	22.66	11.55	11.26
		t_1	.450	.624	.567	.550	.415	.410
		H	74.90	66.62	66.83	67.17	77.86	78.21
		t_3	.497	.475	.461	.460	.523	.523
		B_2	12.10	5.00^-	5.00^-	5.00^-	10.40	10.17
		t_2	.487	1.290	1.496	1.610	.463	.456
2	2	B_1	11.40	19.34	18.26	17.57	11.63	11.69
		t_1	.404	.463	.463	.455	.410	.417
		H	89.90	87.81	76.00	72.92	100.00^+	99.47
		t_3	.397	.401	.354	.341	.436	.435
		B_2	10.70	5.00^-	5.00^-	5.00^-	10.71	10.94
		t_2	.435	1.236	1.356	1.430	.446	.447

Table 23. Final Designs for Problem 5
Portal Frame (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (cm) Ref. 53	Run 1 Option* 1(P)	Run 2 Option* 2(P)	Run 3 Option* 3(P)	Run 4 Option* 12(P)	Run 5 Option* 12(D)
3	3	B_1	7.50	14.27	5.00$^-$	5.00$^-$	5.00$^-$	5.00$^-$
		t_1	.268	.356	1.356	1.354	.142	.143
		H	61.90	36.35	55.31	59.21	25.00$^-$	25.00$^-$
		t_3	.250	.152	.262	.282	.100$^-$	.100$^-$
		B_2	10.00$^-$	10.00$^-$	15.37	15.24	10.00$^-$	10.00$^-$
		t_2	.369	.619	.535	.541	.276	.276
Volume (cm^3)			90,592	93,732	97,240	97,460	84,272	84,058
Number of Analyses			--	18	21	10	12	11

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 24. Critical Constraints for Problem 5
Portal Frame

Run No.	Option No.*	Displacement Constrained Nodes		Stress Constrained Members		Local Buckling Constrained Members	
		Load Case 1	Load Case 2	Load Case 1	Load Case 2	Load Case 1	Load Case 2
Ref. 53	--	3	3	--	--	1	2,3
1	1(P)	3	3	--	--	1,2	2,3
2	2(P)	3	3	--	--	1,2	2,3
3	3(P)	3	3	--	--	1,2	2,3
4	12(P)	3	3	--	--	1	2,3
5	12(D)	3	3	--	--	1	2,3

*See Table 1

Table 25. Definition of Problem 6
One Bay / Two Story Frame

Material Properties

Young's Modulus : $E = 30.0 \times 10^6$ PSI

Shear Modulus : $G = 11.5 \times 10^6$ PSI

Poisson's Ratio : $\nu = .3$

Weight Density : $\rho = .2836$ lb/in^3

Yield Stress : $\sigma_a = 36{,}000$ PSI

Factor of Safety : FS = 1.51

Nodal Loading

Load Case	Node No.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
2	2	45000	0.	0.	0.	0.	0.
	3	45000	0.	0.	0.	0.	0.

Uniform Loading

Load Case	Member No.	Loading Components (lb/in, in-lb/in)					
		p_x	p_y	p_z	m_x	m_y	m_z
1	3	0.	-500.	0.	0.	0.	0.
	4	0.	-500.	0.	0.	0.	0.
	5	0.	-500.	0.	0.	0.	0.
	6	0.	-500.	0.	0.	0.	0.

Table 25. Definition of Problem 6
One Bay / Two Story Frame (cont.)

Displacement Constraints

Load Case	Node No.	Direction	Lower Bound	Upper Bound
2	2	x	-.36 in	.36 in
	3	x	-.72 in	.72 in
	4	x	-.36 in	.36 in
	5	x	-.72 in	.72 in
	6	x	-.36 in	.36 in
	7	x	-.72 in	.72 in

Initial Design and Side Constraints

Member Nos.	Sizing Variable	Initial Value (in)	Lower Bound (in)	Upper Bound (in)
1-8	R	16.61	1.00	25.0
	t	.45	.01	5.0

Table 26. Iteration History Data for Problem 6, Case A
One Bay / Two Story Frame

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)
0		15982.6 [0]	15982.6 [0]	15982.6 [0]
1		12210.8 [0]	12210.8 [0]	11906.4 [0]
2		10922.1 [2.7]	10995.8 [0]	10603.6 [0]
3		10011.0 [2.3]	10117.5 [0]	9936.6 [0]
4		9414.1 [2.2]	9701.1 [0]	9126.1 [0]
5		8939.0 [1.9]	9075.9 [0]	8869.6 [.3]
6		8830.8 [1.2]	8857.4 [0]	8885.9 [0]
7		8887.5 [0]	8857.4 [0]	8885.9 [0]
8		8887.1 [0]	8857.4 [0]	8885.9 [0]
9		8888.5 [0]		
10				
CPU Time (sec)	Tot.	2.615	4.410	4.755
	Anal.	.418	2.384	3.000
	Opt.	.906	.848	.902

*See Table 1

Table 26. Iteration History Data for Problem 6, Case A
One Bay / Two Story Frame (cont.)

		Weight (lb) [Maximum Constraint Violation (%)]	
Analysis No.		Run 4 Option* 12(P)	Run 5 Option* 12(D)
0		15982.6 [0]	15982.6 [0]
1		10573.1 [8.3]	11812.6 [5.7]
2		9075.5 [36.4]**	10130.0 [0]
3		9007.6 [0]	9116.0 [0]
4		8886.3 [0]	8915.5 [0]
5		8886.3 [0]	8799.3 [2.5]
6		8886.3 [0]	8791.9 [1.4]
7			8826.9 [1.1]
8			8848.5 [.6]
9			8850.0 [.6]
10			8845.6 [.6]
CPU Time (sec)	Tot.	4.150	16.299
	Anal.	2.203	3.479
	Opt.	.957	11.494

*See Table 1

**Constraint was not Retained

Table 27. Final Designs for Problem 6, Case A
One Bay / Two Story Frame

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)	Run 4 Option* 12(P)	Run 5 Option* 12(D)
1	1,8	R	16.099	16.076	16.008	15.988	15.951	16.005
		t	.3513	.3508	.3496	.3492	.3505	.3493
2	2,7	R	11.825	11.691	11.726	11.779	11.711	11.681
		t	.2580	.2553	.2560	.2570	.2571	.2549
3	3,4	R	11.427	11.432	11.305	11.293	11.238	11.361
		t	.2493	.2494	.2468	.2465	.2499	.2481
4	5,6	R	15.277	15.134	15.166	15.243	15.137	15.158
		t	.3333	.3302	.3314	.3333	.3348	.3308
Weight (lb)			8980.7	8888.5	8857.4	8885.9	8886.3	8845.6
Number of Analyses			16	10	9	9	7	11

*See Table 1

Table 28. Critical Constraints for Problem 6, Case A
One Bay / Two Story Frame

Run No.	Option No.*	Stress Constrained Members		Buckling Constrained Members		R/t Constrained Members
		Load Case 1	Load Case 2	Load Case 1	Load Case 2	
Ref. 21	--	--	--	2,7	3-8	1-8
1	1(P)	--	--	2,7	3-8	1-8
2	3(P)	--	--	2,7	3-8	1-8
3	6(P)	--	--	2,3,4,7	3-8	1-8
4	12(P)	--	--	2,3,4,7	3-8	1-8
5	12(D)	--	--	2,7	3-8	1-8

*See Table 1

Table 29. Iteration History Data for Problem 6, Case B
One Bay / Two Story Frame

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)
0		15982.6 [0]	15982.6 [0]	15982.6 [0]
1		12210.8 [0]	12210.8 [0]	11889.0 [0]
2		10930.2 [2.4]	11055.4 [0]	10633.0 [1.4]
3		10462.8 [0]	10491.1 [0]	10265.0 [.7]
4		10212.0 [0]	10219.2 [0]	10231.1 [0]
5		10201.8 [0]	10195.1 [0]	10224.2 [0]
6		10199.6 [0]	10193.9 [0]	10193.8 [0]
7			10197.3 [0]	10188.3 [0]
8				10188.2 [0]
9				
10				
11				
CPU Time (sec)	Tot.	2.148	3.056	2.700
	Anal.	.616	1.335	.815
	Opt.	.506	.596	.656

*See Table 1

Table 29. Iteration History Data for Problem 6, Case B
One Bay / Two Story Frame (cont.)

		Weight (lb) [Maximum Constraint Violation (%)]	
Analysis No.		Run 4 Option* 10(P)	Run 5 Option* 10(D)
0		15982.6 [0]	15982.6 [0]
1		10400.4 [27.4]**	11817.5 [5.7]
2		10350.2 [5.9]	10398.8 [2.9]
3		10252.0 [4.6]	10364.2 [.3]
4		10238.4 [.9]	10166.8 [0]
5		10181.7 [0]	10214.0 [.7]
6		10181.6 [0]	10156.4 [1.1]
7		10181.5 [0]	10163.6 [.1]
8			10232.4 [0]
9			10253.8 [0]
10			10240.8 [0]
11			10220.7 [0]
CPU Time (sec)	Tot.	2.613	7.309
	Anal.	.696	.996
	Opt.	.794	4.873

*See Table 1 **Constraint not retained

Table 30. Final Designs for Problem 6, Case B
One Bay / Two Story Frame

Linking Group	Member Nos.	Size Var.	Final Design (in)					
			Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)	Run 4 Option* 10(P)	Run 5 Option* 10(D)
1	1,8	R	15.658	15.635	15.599	15.606	15.699	15.730
		t	.3416	.3411	.3404	.3405	.3426	.3435
2	2,7	R	13.206	13.259	13.269	13.277	13.227	13.449
		t	.2881	.2894	.2895	.2902	.2901	.2937
3	3,4	R	13.072	13.109	13.015	13.031	12.934	13.534
		t	.2852	.2871	.2850	.2844	.2822	.2952
4	5,6	R	17.011	17.031	17.133	17.074	17.032	16.401
		t	.3711	.3717	.3740	.3729	.3719	.3582
Weight (lb)			10166.6	10199.6	10197.3	10188.2	10181.5	10220.7
Number of Analyses			22	7	8	9	8	12

*See Table 1

Table 31. Critical Constraints for Problem 6, Case B
One Bay / Two Story Frame

Run No.	Option No.*	Buckling Constrained Members		Displacement Constrained Nodes	R/t Constrained Members
		Load Case 1	Load Case 2	Load Case 2	
Ref. 21	--	--	8	3,5,7	1-8
1	1(P)	--	8	3,5,7	1-8
2	3(P)	--	8	3,5,7	1-8
3	4(P)	--	8	3,5,7	1-8
4	10(P)	--	8	3,5,7	1-8
5	10(D)	--	8	3,5,7	1-8

*See Table 1

Table 32. Definition of Problem 7
2x5 Grillage

Material Properties

Young's Modulus : $E = 30.0 \times 10^6$ PSI

Shear Modulus : $G = 11.5 \times 10^6$ PSI

Poisson's Ratio : $\nu = .2963$

Allowable Stress : $\sigma_a = 20{,}000$ PSI

Nodal Loading

Load Case	Node No.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	1	0.	0.	0.	-13330	0.	0.
	2	0.	0.	0.	0.	37040	0.
	3	0.	0.	-9000	0.	-27780	0.
	4	0.	0.	-3333	0.	0.	0.
	5	0.	0.	0.	0.	37040	0.
	6	0.	0.	-9000	0.	-27780	0.
	7	0.	0.	-3333	0.	0.	0.
	8	0.	0.	0.	0.	37040	0.
	9	0.	0.	-9000	0.	-27780	0.
	10	0.	0.	-3333	0.	0.	0.
	11	0.	0.	0.	0.	37040	0.
	12	0.	0.	-9000	0.	-27780	0.
	13	0.	0.	-3333	0.	0.	0.
	14	0.	0.	0.	0.	37040	0.
	15	0.	0.	-9000	0.	-27780	0.
	16	0.	0.	-3333	0.	0.	0.
	17	0.	0.	0.	13330	0.	0.

Table 32. Definition of Problem 7
2x5 Grillage (cont.)

Displacement Constraints

Load Case	Node No.	Direction	Lower Bound	Upper Bound
	4	z	-0.1 in	1 0 in
1	7	z	-0.1 in	1.0 in
	10	z	-0.1 in	1.0 in

Initial Design and Side Constraints

Member Nos.	Sizing Variable	Initial Value (in)	Lower Bound (in)	Upper Bound (in)
	B	12.00	1.00	19.00
1-16	t_b	.95	.045	1.00
	H	15.00	1.00	20.00
	t_h	.80	.05	.95

Table 33. Iteration History Data for Problem 7, Case A
2x5 Grillage

Analysis No.		Volume (in^3) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 4(P)	Run 3 Option* 10(P)
0		32,382.4 [0]	32,382.4 [0]	32,382.4 [0]
1		15,687.5 [16.2]	15,687.5 [16.2]	19,946.8 [0]
2		9,737.9 [13.6]	9,566.4 [14.9]	11,727.0 [15.5]**
3		9,759.1 [1.9]	9,702.2 [1.9]	9,962.9 [.5]
4		8,853.3 [.2]	8,829.2 [.1]	8,415.9 [0]
5		7,961.3 [0]	7,953.8 [0]	7,440.8 [0]
6		7,418.6 [0]	7,434.6 [0]	6,898.1 [0]
7		7,123.0 [0]	7,100.9 [0]	6,828.7 [0]
8		7,087.6 [0]	6,900.7 [0]	6,812.7 [0]
9		6,872.6 [.2]	6,887.7 [0]	6,800.1 [0]
10		6,861.4 [0]	6,840.0 [0]	6,794.8 [0]
11		6,851.1 [0]	6,771.4 [.7]	6,786.7 [0]
12		6,774.9 [.5]	6,807.4 [0]	6,776.7 [0]
13		6,802.2 [0]	6,801.5 [0]	6,769.7 [0]
14		6,800.3 [0]	6,796.4 [0]	6,762.6 [.1]
15		6,794.9 [0]		6,756.4 [0]
CPU Time (sec)	Tot.	5.094	5.626	4.505
	Anal.	1.695	1.607	1.642
	Opt.	1.955	2.361	1.284

*See Table 1 **Constraint was not Retained

Table 33. Iteration History Data for Problem 7, Case A
2x5 Grillage (cont.)

Analysis No.		Volume (in^3) [Maximum Constraint Violation (%)] Run 4 Option* 10(D)	Run 5 Option* 1(PU)
0		32,382.4 [0]	32,382.4 [0]
1		19,943.8 [0]	10,417.2 [7.1]
2		11,696.8 [16.8]**	8,903.1 [0]
3		9,859.8 [.2]	7,857.5 [0]
4		8,406.6 [0]	7,166.5 [0]
5		7,419.1 [0]	6,989.3 [.1]
6		6,874.5 [0]	
7		6,822.5 [0]	
8		6,800.2 [0]	
9		6,793.7 [0]	
10		6,783.0 [0]	
11		6,778.7 [0]	
12		6,772.8 [0]	
13			
14			
15			
CPU Time (sec)	Tot.	3.304	4.275
	Anal.	1.300	1.105
	Opt.	.738	1.967

*See Table 1 **Constraint was not Retained

Table 34. Final Designs for Problem 7, Case A
2x5 Grillage

			Final Design (in)					
Linking Group	Member Nos.	Size Var.	Ref. 29	Run 1 Option* 1(P)	Run 2 Option* 4(P)	Run 3 Option* 10(P)	Run 4 Option* 10(D)	Run 5 Option* 1(PU)
1	1-6	B	6.31	8.93	9.03	3.35	3.65	19.00^{+}
		t_b	$.045^{-}$	.047	.046	.230	.236	.049
		H	18.90	20.00^{+}	19.97	20.00^{+}	20.00^{+}	20.00^{+}
		t_h	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$
2	7-10	B	6.62	6.49	5.54	1.67	1.00^{-}	17.82
		t_b	$.045^{-}$	$.045^{-}$	$.045^{-}$	.133	.056	$.045^{-}$
		H	15.7	20.00^{+}	19.99	10.27	12.10	19.98
		t_h	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$
3	13-16	B	13.50	18.97	19.00	15.36	13.09	19.00^{+}
		t_b	.822	.482	.500	.558	.639	.381
		H	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}
		t_h	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$

Table 34. Final Designs for Problem 7, Case A
2x5 Grillage (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (in)					
			Ref. 29	Run 1 Option* 1(P)	Run 2 Option* 4(P)	Run 3 Option* 10(P)	Run 4 Option* 10(D)	Run 5 Option* 1(PU)
4	11-12	B	4.89	18.98	19.00^{+}	13.63	13.45	12.95
		t_b	.993	.360	.328	.601	.640	.724
		H	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}	20.00^{+}
		t_h	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$	$.050^{-}$
Volume (in^3)			6971.0	6794.9	6796.5	6756.4	6772.8	6989.3
Number of Analyses			38	16	15	16	13	6

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 35. Critical Constraints for Problem 7, Case A
2x5 Grillage

Run No.	Option No.*	Displacement Constrained Nodes
Ref. 29	--	7,10
1	1(P)	7,10
2	4(P)	7,10
3	10(P)	4,7,10
4	10(D)	4,7,10
5	1(PU)	7,10

*See Table 1

Table 36. Iteration History Data for Problem 7, Case B
2x5 Grillage

Analysis No.		Volume (in^3) [Maximum Constraint Violation (%)] Run 1 Option* 3(P)	Run 2 Option* 6(P)	Run 3 Option* 12(P)	Run 4 Option* 3(PU)
0		32,382.4 [0]	32,382.4 [0]	32,382.4 [0]	32,382.4 [0]
1		25,259.7 [0]	25,268.5 [0]	21,663.8 [0]	20,339.4 [0]
2		19,997.0 [0]	20,724.0 [0]	13,969.4 [0]	14,469.9 [0]
3		17,270.7 [0]	17,578.1 [0]	11,573.5 [0]	11,099.0 [0]
4		14,898.8 [0]	14,605.3 [0]	9,712.1 [0]	8,762.0 [0]
5		12,535.6 [0]	12,916.8 [.6]	8,644.9 [0]	7,544.4 [100]**
6		11,092.8 [.2]	11,463.6 [.7]	7,807.2 [100]**	7,523.9 [100]
7		10,032.6 [.4]	10,630.3 [0]	7,730.0 [20.6]	7,526.1 [42.3]**
8		9,156.0 [.4]	9,115.1 [1.8]	7,717.4 [1.4]	7,525.9 [4.3]
9		8,414.4 [79.1]**	8,753.1 [77.4]**	7,641.5 [0]	7,510.4 [0]
10		7,881.9 [65.5]**	7,959.9 [51.9]**	7,581.5 [0]	7,505.2 [0]
11		7,886.5 [7.7]	7,925.6 [22.2]**	7,557.1 [1.0]	
12		7,884.7 [9.5]	7,914.4 [9.3]	7,550.4 [0]	
13		7,887.6 [0]	7,917.5 [0]	7,545.6 [0]	
14			7,913.5 [0]		
15					
16					
17					
18					
CPU Time (sec)	Tot.	9.032	10.459	10.899	14.065
	Anal.	3.469	3.945	4.979	4.959
	Opt.	2.486	3.263	2.981	5.269

*See Table 1 **Constraint was not retained

Table 37. Final Designs for Problem 7, Case B
2x5 Grillage

Linking Group	Member Nos.	Sizing Variables	Final Design (in) Ref. 29	Run 1 Option* 3(P)	Run 2 Option* 6(P)	Run 3 Option* 12(P)	Run 4 Option* 3(PU)
1	1-6	B	6.10	9.86	10.60	3.26	4.27
		t_b	.159	.096	.105	.224	.234
		H	20.00^+	20.00^+	19.99	20.00^+	20.00^+
		t_h	.093	.091	.090	.096	.094
2	7-10	B	8.28	11.29	11.59	1.91	1.84
		t_b	.074	.093	.095	.148	.131
		H	15.20	19.99	19.98	9.07	9.14
		t_h	.064	.074	.074	.055	.056
3	13-16	B	6.33	15.94	16.85	12.01	11.64
		t_b	1.00^+	.325	.299	.595	.514
		H	20.00^+	20.00^+	20.00^+	20.00^+	20.00^+
		t_h	.098	.095	.094	.103	.098

Table 37. Final Designs for Problem 7, Case B
2x5 Grillage (cont.)

			Final Design (in)				
Linking Group	Member Nos.	Sizing Variables	Ref, 29	Run 1 Option* 3(P)	Run 2 Option* 6(P)	Run 3 Option* 12(P)	Run 4 Option* 3(PU)
4	11-12	B	11.50	17.62	18.81	14.20	17.65
		t_b	1.00^+	.657	.614	.683	.643
		H	20.00^+	19.99	19.99	20.00^+	20.00^+
		t_h	.117	.119	.119	.113	.118
Volume (in^3)			7927.0	7887.6	7913.5	7545.6	7505.3
Number of Analyses			41	14	15	14	11

*See Table 1

+Sizing Variable at Upper Bound

-Sizing Variable at Lower Bound

Table 38. Critical Constraints for Problem 7, Case B
2x5 Grillage

Run No.	Option No.*	Displacement Constrained Nodes	Stress Constrained Members	Local Buckling Constrained Members
Ref. 29	--	7,10	--	1,6,8,9,11,13,16
1	1(P)	7,10	--	1,2,5,6,8,10,11,13,15
2	6(P)	7,10	--	1,6,8,10,11,13,15
3	12(P)	4,7,10	--	1,6,8,10,11,13,15
4	3(PU)	4,7,10	8,10	1,6,8,10,11,13,15

*See Table 1

Table 39. Definition of Problem 8
Two Bay / Six Story Frame

Material Properties

Young's Modulus : $E = 30.0 \times 10^6$ PSI

Shear Modulus : $G = 11.5 \times 10^6$ PSI

Poisson's Ratio : $\nu = .3$

Weight Density : $\rho = .2836$ lb/in^3

Yield Stress : $\sigma_a = 36{,}000$ PSI

Factor of Safety : FS = 1.51

Nodal Loading

Load Case	Node No.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
2	1	9000.	0.	0.	0.	0.	0.
	4	9000.	0.	0.	0.	0.	0.
	7	9000.	0.	0.	0.	0.	0.
	10	9000.	0.	0.	0.	0.	0.
	13	9000.	0.	0.	0.	0.	0.
	16	9000.	0.	0.	0.	0.	0.

Table 39. Definition of Problem 8
Two Bay / Six Story Frame (cont.)

Uniform Loading

Load Case	Member No.	Loading Components (lb/in, in-lb/in)					
		P_x	P_y	P_z	M_x	M_y	M_z
1	1	0.	-333.3	0.	0.	0.	0.
	2	0.	- 83.3	0.	0.	0.	0.
	6	0.	- 83.3	0.	0.	0.	0.
	7	0.	-333.3	0.	0.	0.	0.
	11	0.	-333.3	0.	0.	0.	0.
	12	0.	- 83.3	0.	0.	0.	0.
	16	0.	- 83.3	0.	0.	0.	0.
	17	0.	-333.3	0.	0.	0.	0.
	21	0.	-333.3	0.	0.	0.	0.
	22	0.	- 83.3	0.	0.	0.	0.
	26	0.	- 83.3	0.	0.	0.	0.
	27	0.	-333.3	0.	0.	0.	0.
2	1	0.	- 83.3	0.	0.	0.	0.
	2	0.	- 83.3	0.	0.	0.	0.
	6	0.	- 83.3	0.	0.	0.	0.
	7	0.	- 83.3	0.	0.	0.	0.
	11	0.	- 83.3	0.	0.	0.	0.
	12	0.	- 83.3	0.	0.	0.	0.
	16	0.	- 83.3	0.	0.	0.	0.
	17	0.	- 83.3	0.	0.	0.	0.
	21	0.	- 83.3	0.	0.	0.	0.
	22	0.	- 83.3	0.	0.	0.	0.
	26	0.	- 83.3	0.	0.	0.	0.
	27	0.	- 83.3	0.	0.	0.	0.

Table 39. Definition of Problem 8
Two Bay / Six Story Frame (cont.)

Displacement Constraints

Load Case	Node Nos.	Direction	Lower Bound	Upper Bound
2	1-3	x	-1.728 in	1.728 in
	4-6	x	-1.440 in	1.440 in
	7-9	x	-1.152 in	1.152 in
	10-12	x	- .864 in	.864 in
	13-15	x	- .576 in	.576 in
	16-18	x	- .288 in	.288 in

Initial Design and Side Constraints

Member Nos.	Sizing Variables	Initial Value (in)	Lower Bound (in)	Upper Bound (in)
1-10	R	9.00	1.00	25.0
	t	.20	.01	5.0
11-20	R	11.00	1.00	25.0
	t	.24	.01	5.0
21-30	R	14.00	1.00	25.00
	t	.31	.01	5.0

Table 40. Iteration History Data for Problem 8, Case A
Two Bay / Six Story Frame

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)
0		23,141.4 [63.6]	23,141.4 [63.6]	23,141.4 [63.6]
1		23,503.0 [19.8]	23,475.2 [14.7]	23,792.0 [8.8]
2		23,188.3 [3.5]	23,243.1 [1.0]	23,793.4 [0]
3		23,240.5 [.5]	23,257.5 [.1]	23,536.4 [0]
4		22,945.8 [4.6]	23,083.8 [1.1]	23,117.4 [0]
5		22,804.6 [3.5]	22,793.7 [.2]	22,915.5 [0]
6		22,854.3 [.3]	22,777.9 [0]	22,906.4 [0]
7		22,668.5 [5.3]	22,804.0 [.8]	22,887.8 [0]
8		22,621.7 [2.9]	22,608.7 [0]	
9		22,691.1 [.7]	22,603.1 [.2]	
10		22,559.1 [4.7]	22,579.8 [0]	
11		22,645.2 [.7]	22,566.8 [.1]	
12		22,531.1 [2.0]		
13		22,557.0 [.9]		
14		22,545.7 [.5]		
15		22,568.6 [.6]		
CPU Time (sec)	Tot.	19.281	58.239	40.108
	Anal.	2.050	43.969	29.103
	Opt.	12.201	9.155	8.043

*See Table 1

Table 40. Iteration History Data for Problem 8, Case A
Two Bay / Six Story Frame (cont.)

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 4 Option* 12(P)	Run 5 Option* 3(PU)
0		23,141.4 [63.6]	23,141.4 [63.6]
1		23,919.1 [8.7]	23,270.9 [.8]
2		23,157.4 [.4]	23,014.6 [.7]
3		22,999.2 [.1]	23,027.4 [.3]
4		22,936.1 [0]	22,982.3 [.6]
5		22,866.8 [0]	22,754.3 [2.4]
6		22,841.1 [0]	22,641.6 [0]
7		22,794.7 [.1]	22,655.2 [.2]
8		22,747.5 [.1]	22,534.2 [.1]
9		22,713.3 [.1]	22,523.0 [.1]
10		22,693.4 [.1]	
11		22,674.2 [.1]	
12			
13			
14			
CPU Time (sec)	Tot.	56.484	54.005
	Anal.	42.334	35.074
	Opt.	10.120	14.229

*See Table 1

Table 41. Final Designs for Problem 8, Case A
Two Bay / Six Story Frame

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)	Run 4 Option* 12(P)	Run 5 Option* 3(PU)
1	1,2	R	10.235	10.250	10.192	9.985	10.175	10.215
		t	.2233	.2238	.2225	.2181	.2224	.2229
2	3,5	R	10.571	10.462	10.396	8.732	10.351	10.422
		t	.2306	.2285	.2276	.3030	.2286	.2272
3	4	R	5.547	7.063	7.669	8.565	8.193	7.397
		t	.1210	.1544	.1681	.1869	.1798	.1608
4	6,7	R	9.808	9.813	9.674	9.712	9.742	9.688
		t	.2140	.2143	.2116	.2122	.2132	.2115
5	8,10	R	7.208	7.389	6.802	8.015	7.383	6.786
		t	.1573	.1616	.1496	.1751	.1627	.1493
6	9	R	9.140	8.837	9.047	8.648	8.716	9.096
		t	.1994	.1937	.1978	.1890	.1905	.1992
7	11,12	R	10.082	10.139	10.092	10.019	10.096	10.104
		t	.2200	.2212	.2203	.2192	.2207	.2208

Table 41, Final Designs for Problem 8, Case A
Two Bay / Six Story Frame (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (in)					
			Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)	Run 4 Option* 12(p)	Run 5 Option* 3(PU)
8	13,15	R	10.535	10.356	10.491	8.864	10.115	10.524
		t	.2298	.2262	.2293	.2628	.2332	.2291
9	14	R	9.233	9.212	9.541	9.859	9.688	9.199
		t	.2014	.2013	.2089	.2151	.2118	.2004
10	16,17	R	10.395	10.420	10.432	10.390	10.422	10.478
		t	.2268	.2273	.2278	.2269	.2279	.2293
11	18,20	R	8.973	8.963	8.814	9.238	8.943	8.661
		t	.1958	.1955	.1933	.2022	.1963	.1897
12	19	R	11.336	11.150	10.835	9.918	10.899	11.355
		t	.2473	.2444	.2596	.2713	.2490	.2486
13	21,22	R	11.076	11.176	11.182	11.182	11.174	11.182
		t	.2417	.2439	.2441	.2440	.2442	.2440
14	23,25	R	10.619	10.299	10.349	10.203	10.229	10.444
		t	.2317	.2260	.2258	.2232	.2525	.2273

Table 41. Final Designs for Problem 8, Case A
Two Bay / Six Story Frame (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 6(P)	Run 4 Option* 12(P)	Run 5 Option* 3(PU)
15	24	R	11.236	10.319	11.274	11.588	11.345	11.187
		t	.2452	.2476	.2468	.2539	.2490	.2448
16	26,27	R	10.986	11.014	11.094	10.933	11.016	11.116
		t	.2397	.2408	.2422	.2389	.2408	.2430
17	28,30	R	10.687	11.095	10.989	11.810	11.258	10.790
		t	.2332	.2421	.2412	.2581	.2467	.2365
18	29	R	15.021	14.141	13.168	12.193	13.572	13.352
		t	.3277	.3126	.3455	.3249	.3221	.3508
Weight (lb)			22530.5	22568.6	22566.8	22887.8	22674.2	22523.0
Number of Analyses			23	16	12	8	12	10

*See Table 1

Table 42. Critical Constraints for Problem 8, Case A
Two Bay / Six Story Frame

Run No.	Option No.*	Stress Constrained Members		Buckling Constrained Members		R/t Constrained Members
		Load Case 1	Load Case 2	Load Case 1	Load Case 2	
Ref. 21	--	7	--	1,3,4,11, 13,14,20,24	9,16,19,21, 22,24-26,29	1-30
1	1(P)	7	--	1,3,11,13, 14,20,23	9,16,19-22, 24-26,29,30	1-30
2	3(P)	7	--	1,3,10,11, 13,20,23,24	9,10,16,19-22 24-26,29,30	1-18,20-28,30
3	6(P)	7	--	1,3,11,13, 20,23	16,17,19-22, 24-27,29,30	1,2,4,6-12,14, 16-18,20-28,30
4	12(P)	7	--	1,3,11,13, 20,23,24	9,16,19-22, 24-26,29-30	1-12,14,16-28,30
5	3(PU)	7	--	1,3,10,11,13, 14,20,23,24	9,16,19-22, 24-26,29-30	1-28,30

*See Table 1

Table 43. Iteration History Data for Problem 8, Case B
Two Bay / Six Story Frame

		Weight (lb) [Maximum Constraint Violation (%)]		
Analysis No.		Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)
0		28,536.8 [55.7]	28,536.8 [55.7]	28,536.8 [55.7]
1		27,484.3 [16.9]	27,184.2 [9.9]	27,270.2 [10.0]
2		25,692.2 [4.3]	25,495.6 [.7]	25,546.8 [1.8]
3		25,404.1 [.8]	25,168.4 [0]	25,179.7 [0]
4		25,356.5 [.2]	25,056.6 [1.5]	25,128.6 [.5]
5		25,268.1 [0]	24,651.6 [0]	24,871.5 [.2]
6		24,939.8 [.2]	24,689.4 [0]	24,889.6 [0]
7		24,853.9 [.4]	24,488.8 [0]	24,818.5 [0]
8		24,865.1 [1.0]	24,530.4 [0]	24,855.5 [0]
9		24,672.7 [0]	24,339.1 [0]	24,808.1 [0]
10		24,661.9 [.7]	24,363.1 [0]	24,644.2 [0]
11		24,666.6 [.4]	24,314.3 [0]	24,598.2 [0]
12			24,278.8 [0]	24,618.5 [0]
13				24,608.8 [0]
14				
15				
CPU Time (sec)	Tot.	17.205	54.538	23.801
	Anal.	7.316	39.959	8.639
	Opt.	5.702	10.048	10.435

*See Table 1

Table 43. Iteration History Data for Problem 8, Case b
Two Bay / Six Story Frame (cont.)

		Weight (lb) [Maximum Constraint Violation (%)]	
Analysis No.		Run 4 Option* 10(P)	Run 5 Option* 1(PU)
0		28,536.8 [55.7]	28,536.8 [55.7]
1		25,656.7 [19.1]**	26,576.6 [10.0]
2		24,952.5 [2.5]	24,952.4 [1.7]
3		24,654.2 [.6]	24,889.6 [1.0]
4		24,609.4 [.2]	24,890.1 [1.8]
5		24,624.1 [0]	24,676.1 [1.4]
6		24,615.2 [0]	24,675.9 [.9]
7			24,665.6 [.3]
8			
9			
10			
11			
12			
13			
14			
15			
CPU Time (sec)	Tot.	13.617	19.436
	Anal.	3.749	6.483
	Opt.	7.324	8.018

*See Table 1

Table 44. Final Designs for Problem 8, Case B
Two Bay / Six Story Frame

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)	Run 4 Option* 10(P)	Run 5 Option* 1(PU)
1	1,2	R	10.009	10.071	9.913	10.110	10.008	10.079
		t	.2183	.2203	.2165	.2210	.2226	.2201
2	3,5	R	10.361	10.298	10.137	10.342	10.240	10.296
		t	.2261	.2252	.2214	.2266	.2273	.2250
3	4	R	6.322	7.871	6.494	6.628	6.353	7.062
		t	.1379	.1732	.1432	.1446	.1504	.1709
4	6,7	R	9.682	9.771	9.521	9.690	9.647	9.780
		t	.2112	.2141	.2078	.2120	.2148	.2137
5	8,10	R	7.825	8.338	7.746	7.681	7.655	7.923
		t	.1707	.1826	.1705	.1676	.1706	.1823
6	9	R	10.953	9.748	11.275	11.179	10.796	10.345
		t	.2390	.2128	.2462	.2439	.2407	.2259
7	11,12	R	10.639	10.637	10.813	10.827	10.719	10.652
		t	.2321	.2322	.2363	.2363	.2374	.2324

Table 44. Final Designs for Problem 8, Case B
Two Bay / Six Story Frame (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (in)					
			Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)	Run 4 Option* 10(P)	Run 5 Option* 1(PU)
8	13,15	R	9.891	9.668	9.245	9.781	9.615	9.636
		t	.2158	.2127	.2028	.2135	.2132	.2106
9	14	R	10.763	11.211	11.214	10.532	10.777	11.330
		t	.2348	.2448	.2469	.2298	.2399	.2474
10	16,17	R	11.406	11.687	11.585	11.665	11.642	11.675
		t	.2488	.2557	.2535	.2556	.2578	.2554
11	18,20	R	8.687	9.675	8.782	9.148	9.188	9.440
		t	.1895	.2116	.1947	.2004	.2015	.2106
12	19	R	13.104	11.744	12.826	12.618	12.377	12.208
		t	.2859	.2565	.2801	.2754	.2737	.2666
13	21,22	R	12.114	12.235	12.108	12.217	12.183	12.223
		t	.2643	.2678	.2644	.2674	.2694	.2668
14	23,25	R	10.690	10.417	10.456	10.392	10.309	10.117
		t	.2332	.2287	.2282	.2267	.2268	.2271

Table 44. Final Designs for Problem 8, Case B
Two Bay / Six Story Frame (cont.)

			Final Design (in)					
Linking Group	Member Nos.	Size Var.	Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 3(P)	Run 3 Option* 4(P)	Run 4 Option* 10(P)	Run 5 Option* 1(PU)
15	24	R	11.919	12.441	12.201	12.018	12.092	12.414
		t	.2601	.2715	.2696	.2662	.2694	.2706
16	26,27	R	11.707	11.577	11.225	11.584	11.576	11.729
		t	.2554	.2533	.2519	.2531	.2552	.2559
17	28,30	R	11.413	11.636	9.480	11.493	11.385	11.468
		t	.2490	.2545	.2068	.2513	.2520	.2533
18	29	R	13.915	13.067	16.746	13.801	13.557	13.155
		t	.3036	.2851	.3654	.3013	.2987	.2866
Weight (lb)			24405.4	24666.6	24278.8	24608.8	24615.2	24665.6
Number of Analyses			27	12	13	14	7	8

*See Table 1

Table 45. Critical Constraints for Problem 8, Case B
Two Bay / Six Story Frame

Run No.	Option No.*	Displacement Constrained Nodes	Stress Constrained Members		Buckling Constrained Members		R/t Constrained Members
		Load Case 1	Load Case 1	Load Case 2	Load Case 1	Load Case 2	
Ref. 21	--	1-12	7	--	1,3,4,12,20	25,29	1-30
1	1(P)	4-12	7	--	1,3,13	29,30	1-30
2	3(P)	1-12	7	--	1,3,4,13,30	25	1-30
3	4(P)	4-12	7	--	1,3,13	25,29	1-30
4	10(P)	1-12	7	--	1,3,13	25,29	1-3,5-30
5	1(PU)	4-12	7	--	1,3,13	25,29,30	1-3,5-8, 10-30

*See Table 1

Table 46. Definition of Problem 9
Helicopter Tail Boom

Material Properties

Young's Modulus : $E = 10.5 \times 10^6$ PSI

Shear Modulus : $G = 40.4 \times 10^5$ PSI

Poisson's Ratio : $\nu = .3$

Weight Density : $\rho = .1$ lb/in^3

Allowable Stress : $\sigma_a = 4.2 \times 10^4$ PSI

Factor of Safety : FS = 1.25

Nodal Loading

Load Case	Node Nos.	Loading Components (lb, in-lb)					
		F_x	F_y	F_z	M_x	M_y	M_z
1	13-16	0.	0.	-140.0	0.	0.	0.
	25	1490.3	1691.8	0.	0.	0.	0.
	26	1490.3	-1365.8	0.	0.	0.	0.
	27	-1490.3	1691.8	0.	0.	0.	0.
	28	-1490.3	-1365.8	0.	0.	0.	0.

Displacement Constraints

Load Case	Node Nos.	Direction	Lower Bound	Upper Bound
1	5-28	y	-.5 in	.5 in
	5-28	z	-.5 in	.5 in

Initial Design and Side Constraints

Member Nos.	Sizing Variable	Initial Value (in)	Lower Bound (in)	Upper Bound (in)
1-44	R	2.0	.25	25.0
	t	.051	.001	5.0

Table 47. Iteration History Data for Problem 9
Helicopter Tail Boom

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 1 Option* 1(P)	Run 2 Option* 4(P)	Run 3 Option* 10(P)
0		69.11 [211.6]	69.11 [211.6]	69.11 [211.6]
1		95.86 [34.7]	93.97 [34.0]	99.51 [18.6]
2		107.94 [7.2]	105.77 [7.2]	109.45 [1.0]
3		110.69 [.4]	109.53 [.2]	109.59 [.1]
4		109.51 [.1]	109.26 [0]	109.52 [0]
5		110.13 [0]	109.59 [0]	109.45 [0]
6		109.33 [0]	109.33 [0]	109.40 [0]
7		109.25 [0]	109.12 [0]	109.36 [0]
8		108.83 [0]	109.18 [0]	109.27 [0]
9		108.71 [0]	108.60 [0]	109.22 [0]
10		108.80 [0]	108.66 [0]	
CPU Time (sec)	Tot.	15.819	16.337	13.664
	Anal.	9.368	9.904	8.426
	Opt.	2.661	2.652	1.991

*See Table 1

Table 47. Iteration History Data for Problem 9
Helicopter Tail Boom (cont.)

Analysis No.		Weight (lb) [Maximum Constraint Violation (%)] Run 4 Option* 10(D)	Run 5 Option* 1(PU)
0		69.11 [211.6]	69.11 [211.6]
1		97.87 [20.3]	105.44 [7.3]
2		108.65 [1.7]	112.34 [0]
3		109.62 [0]	110.20 [0]
4		108.74 [.4]	109.15 [0]
5		108.34 [.5]	108.90 [0]
6		108.74 [0]	108.70 [0]
7		108.66 [0]	
8		108.48 [0]	
9		108.52 [0]	
10		108.35 [0]	
CPU Time (sec)	Tot.	41.147	13.498
	Anal.	9.367	6.379
	Opt.	28.039	3.235

*See Table 1

Table 48. Final Designs for Problem 9
Helicopter Tail Boom

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option* 4(P)	Run 3 Option* 10(P)	Run 4 Option* 10(D)	Run 5 Option* 1(PU)
1	1-4	R	2.6695	3.1432	3.0791	3.1473	3.0816	3.1198
		t	.0880	.0801	.0792	.0811	.0782	.0794
2	5-8	R	1.9152	1.2850	1.3675	1.4169	1.3848	1.3364
		t	.0487	.0355	.0377	.0380	0.372	.0386
3	9 -12	R	2.6530	2.8813	2.9242	2.8725	2.8850	2.9110
		t	.0829	.0744	.0744	.0738	.0736	.0737
4	13-16	R	2.1035	2.0071	1.9918	1.9786	1.9927	1.9975
		t	.0535	.0511	.0509	.0512	.0507	.0509
5	17-20	R	2.6784	2.8101	2.8255	2.8328	2.8086	2.8239
		t	.0753	.0717	.0719	.0736	.0716	.0717
6	21-24	R	2.1488	2.0703	2.0709	2.0573	2.0656	2.0756
		t	.0547	.0527	.0527	.0533	.0526	.0529
7	25-28	R	2.6238	2.6724	2.6681	2.6071	2.6848	2.6672
		t	.0673	.0680	.0679	.0674	.0685	.0678
8	29-32	R	2.1569	2.0965	2.0959	2.0862	2.0884	2.0983
		t	.0549	.0535	.0533	.0540	.0532	.0534

Table 48. Final Designs for Problem 9
Helicopter Tail Boom (cont.)

Linking Group	Member Nos.	Size Var.	Final Design (in) Ref. 21	Run 1 Option* 1(P)	Run 2 Option *4(P)	Run 3 Option* 10(P)	Run 4 Option* 10(D)	Run 5 Option* 1(PU)
9	33-36	R t	2.5038 .0637	2.5179 .0642	2.5101 .0639	2.4670 .0638	2.4965 .0637	2.5103 .0640
10	37-40	R t	2.1730 .0553	2.1475 .0548	2.1459 .0546	2.1347 .0533	2.1583 .0550	2.1480 .0547
11	41-44	R t	2.3748 .0604	2.3703 .0605	2.3664 .0602	2.3654 .0603	2.4109 .0615	2.3618 .0602
12	45-48	R t	1.9707 .0502	1.9487 .0497	1.9476 .0496	1.9300 .0501	1.9938 .0508	1.9485 .0496
Weight (lb)			111.20	108.80	108.66	109.22	108.35	108.70
Number of Analyses			13	11	11	10	11	7

*See Table 1

Table 49. Critical Constraints for Problem 9
Helicopter Tail Boom

Run No.	Option No.*	Displacement Constrained Nodes	R/t Constrained Members
Ref. 21	--	25,27	5-8,13-16,21-24,29-48
1	1(P)	25,27	1-4,9-48
2	4(P)	25,27	1-4,9-48
3	10(P)	25,27	1-4,9-48
4	10(D)	25,27	1-4,9-48
5	1(PU)	25,27	1-4,9-48

*See Table 1

☆U.S. GOVERNMENT PRINTING OFFICE: 1985 537-057/10014

1. Report No. NASA CR-172526	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Alternative Approximation Concepts for Space Frame Synthesis		5. Report Date March 1985
		6. Performing Organization Code
7. Author(s) Robert V. Lust and Lucien A. Schmit		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address Department of Engineering and Applied Science University of California Los Angeles, CA 90024		11. Contract or Grant No. NSG-1490
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		14. Sponsoring Agency Code 506-53-53-06

15. Supplementary Notes

Langley Technical Monitor - Dr. Jaroslaw Sobieski

16. Abstract

A structural synthesis methodology for the minimum mass design of 3-dimensional frame-truss structures under multiple static loading conditions and subject to limits on displacements, rotations, stresses, local buckling, and element cross-sectional dimensions is presented. A variety of approximation concept options are employed to yield near optimum designs after no more than 10 structural analyses. Available options include: (A) formulation of the nonlinear mathematical programing problem in either reciprocal section property (RSP) or cross-sectional dimension (CSD) space; (B) two alternative approximate problem structures in each design space; and (C) three distinct assumptions about element end-force variations. Fixed element, design element linking, and temporary constraint deletion features are also included. The solution of each approximate problem, in either its primal or dual form, is obtained using CONMIN, a feasible directions program)n.b., dual formulation not available for all options). The frame-truss synthesis methodology is implemented in the COMPASS computer program and is used to solve a variety of problems. These problems were chosen so that, in addition to exercising the various approximation concepts options, the results could be compared with previously published work. The types of problems solved include both planar and 3-dimensional frame-truss structures and contain frame members having various cross-sectional shapes including: (1) a thin-walled tube; (2) thin-walled box sections; (3) an I section; and (4) a solid square section. Finally, the collection of numerical examples are used to form guidelines for the solution of future problems.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
3-dimensional frame-truss structures, CONMIN, COMPASS computer program, frame-truss synthesis methodology	Unclassified - Unlimited Subject Category 05

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	430	A19

N-305

For sale by the National Technical Information Service, Springfield, Virginia 22161